Project report

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On

"Study of solutions of fractional order differential equations."

Submitted in partial fulfillment of the requirement for the degree of B.Sc. (Hons.) Mathematics

Submitted by

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CERTIFICATE

This is to Certify that ***PARUL CHAUDHARY, MAYANK KALKAL*** and

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"Study of solutions of fractional order differential equations." under my supervision. This work is fit for submission for the award of Bachelor Degree in Mathematics Honors.

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CANDIDATE DECLARATION

WE, hereby declare that the dissertation entitled "***Study of solutions of fractional order differential equation.***" submitted by us in partial fulfillment for the degree of B.Sc. in Mathematics to the Division of Mathematics, school of basic and applied science, Galgotias University, Greater Noida, Uttar Pradesh, India, is our original work. It has not been submitted to a limited extent or full to this University or some other University for the honor of Certificate or Degree.

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Title of the Proposed Thesis:

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ABSTRACT

Here in project, we centralize on the presence and singularity of gentle arrangements, and their approximations of a class of development conditions of vital and fractional orders involving deviating arguments and impulses. Evolution equations are usually dealt with the governing partial differential equations of many physical phenomena in Hilbert spaces or more generally, in Banach spaces and may be viewed as ordinary differential equations in an infinite dimensional state space. Although, the term has no accurate definition, and its significance depends on the actual situation, yet in addition on the detailing of the issue for which it is utilized. Many physical phenomena like reaction diffusion equations, laser optics, coupled oscillators, enzyme kinetics, food webs, control theory, climate models, viscosity materials, population ecology, the heat conduction and the wave propagation in materials are naturally modeled by abstract functional differential equations in Hilbert spaces (or in Banach spaces), where the space variables are merged with the domain of the operator.

Chapter-1

INTRODUCTION

Hasty(Impulsive) impacts are normal in the development cycle where the momentary brothers are to be viewed as whose length is immaterial in examination with the all out term of the first cycle. The administering mathematical problems alike cycles might be demonstrated as hasty (impulsive) differential conditions (mathematical problems).

As of late, obsolete a developing enjoyment in the research of imprudent differential conditions. In which case, the customary starting circumstances u(0) = u_o

are supplanted by the hasty(**impulsive**) circumstances

u(0) =u_o , Δu(t_i) = I_i(u(t_i)), i = 1, 2, ….,

where 0 < t_1< t_2 < · · · , Δu(t_i) = u - u(t_i^-), i = 1, 2, ... , and I_i There are a few administrators. That is, the hasty circumstances are the blends of the conventional starting circumstances and the transient irritations whose span can be unimportant in correlation with the length of the cycle. One of the principal benefits of these situations over the conventional starting with issues(initial value problems) is that these conditions model numerous genuine frameworks and cycles that can't be demonstrated by the customary introductory worth issues, for example, the elements of populaces exposed to unexpected changes (shocks, collecting, illnesses, catastrophic events, and so on.). The examination of presence and uniqueness of gentle answers for incautious differential conditions have been talked about by many creators (see[Rhapsody in fractional (degruyter.com),[17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38] what's more, references referred to in that). In any case, because of hypothetical and viable troubles, the investigation of rash(impulsive) differential conditions with veering(deviating) off contentions has developed rather leisurely. The rash(impulsive) differential conditions with going astray contentions have been concentrated on as of late by a few creators. Guobing et al. in his book [40] laid out the presence of answers for occasional limit esteem issues for a class of hasty nonpartisan differential conditions with multideviation contentions. In [39], Jankowski talked about the presence of answers for second request imprudent differential conditions with going astray contentions (see likewise [16] and the references referred to in that).

Then again, Rash(impulsive) impacts are normal in numerous developmental cycles, where the transient brothers are thought of. The term of these annoyances is irrelevant in correlation with the all out length of the first cycles. The overseeing conditions of such cycles might be demonstrated as imprudent differential conditions. The most recent twenty years have been set apart by a developing interest in rash differential conditions as these situations give a characteristic structure to numerical demonstrating of some certifiable peculiarities, to be specific in the control hypothesis, physical science, science, populace elements, biotechnology, financial matters and clinical fields.

Chapter-2

Sobolev Spaces

Sobolev spaces were introduced and investigated by the Russian mathematician Sergei Lvovich Sobolev. The significance of the Sobolev spaces comes from the way that the arrangements of fractional differential conditions are normally found in Sobolev spaces, as opposed to in spaces of persistent capabilities and with the subordinates grasped in the old style sense. For a decent prologue to this point, we allude the peruser to the books by Steve Taylor [Sobolev spaces], Adams and Fournier [Sobolev Spaces, Scholastic Press, 1975] .

MARK 1.1 Assign Ω ⊂ R^n being an open set and 1 ≤ p ≤ ∞. The Lebesgue Space L^p(Ω) is defined as

L^p(Ω) = {f : Ω → C(or R)| f is a quantifiable capability and $\int_\Omega \quad |f|^p$**dx< ∞, p < ∞}.**

The space L^∞(Ω) is the arrangements of all limited quantifiable capabilities on Ω.

Definition 1.2 Assign Ω ⊂ R^n being an open set and 1 ≤ p ≤ ∞. We describe the Sobolev Space $W^{k,p}$(Ω) as

$W^{k,p}$(Ω)= { u ∈ L^p(Ω) : D^αu ∈L^p(Ω) for all |α| ≤ k},

Where $D^\alpha u = \frac{\partial^{\alpha \vee u}}{\partial {x_1}^{\alpha_1} \partial_{x2}{}^{\alpha_2} \ldots \partial_{x_n}{}^{\alpha_n}}$, |α|= α1 + α2 + · · · + αn, is the distributional derivative of u, for multi-index α with α = (α1, α2,..., αn), α_i≥ 0 integers. The positive integer k is known as the order of the Sobolev space $W^{k,p}$(Ω). Clearly,$W^{k,p}$(Ω) is a vector space.

There are few decisions for a norm for $W^{k,p}$(Ω). The accompanying two are normal and are identical in the feeling of equivalence of norms:

$\|u\|W^{k,p}(\Omega) = \begin{cases} (\sum_{|\alpha|\le k}\|D^{\alpha}u\|^p_{L^p(\Omega)} \\ max_{|\alpha|\le k}\|D^{\alpha}u\|_{L^{\infty}(\Omega)} \end{cases}$, $1 \le p < \infty$, **........(1.1.1)**

And

$$\| u \|'_{W^{k,p}(\Omega)} = \begin{cases} \sum_{|\alpha|\le k} \|D^{\alpha u}\|_{L^p(\Omega)} & 1\le p \le \infty \\ max_{|\alpha|\le k} \| D^{\alpha}u \|_{L^{\infty}(\Omega)}, p = +\infty \end{cases}$$

.........(1.1.2)

Concerning both of these norms, $W^{k,p}(\Omega)$ is a Banach space (see also [21, Theorem 3.3]). For p < +∞, $W^{k,p}(\Omega)$ is likewise a distinguishable space. It is traditional to mean $W^{k,2}(\Omega)$ by $H^k(\Omega)$ that its a Hilbert space along the norm $\|.\|_{W^{k,2}(\Omega)}$.

Currently, discuss an important subspace of the space $W^{k,2}(\Omega)$. Let D(Ω) denote the space of all limitlessly differentiable capabilities.

(denoted by C^{∞}) with conservative help, the support being contained in Ω, defined on Ω. Then, for any k and p, D(Ω) ⊂ $W^{k,p}(\Omega)$. If 1 ≤ p < ∞, we characterize the space $W_0^{k,p}(\Omega)$ as the conclusion of D(Ω) in the $W^{k,p}(\Omega)$norm. Thus $W_0^{k,p}(\Omega)$ is a closed subspace of $W^{k,p}(\Omega)$ and its elements can be approximated in the $W^{k,p}(\Omega)$ norm by C^{∞} functions with compact support.

For p = 2, the spaces play an important role as we shall see later. We will be denoted $W^{k,p}(\Omega)$ by $H^k(\Omega)$ and $W_0^{k,p}(\Omega)$ by $H_0^k(\Omega)$.

Thus $H^k(\Omega)$ = $W^{k,p}(\Omega)$ and $H_0^k(\Omega)$ = $W_0^{k,2}(\Omega)$. We recall the following imbeddings theorem of Sobolev spaces.

Theorem 1.3 Let k ≥ 1 be an integer and 1 ≤ p < ∞. We have the nonstop imbeddings:

(i) if p < $\frac{n}{k}$, then $W^{k,p}(R^n) \to L^q(\ R^n)$, where $\frac{1}{q} = \frac{1}{p} - \frac{k}{n}$;

(ii) if p =$\frac{n}{k}$, then $W^{k,p}(R^n) \to L^q(\ R^n)$, where p ≤ q < ∞;

(iii) if p >$\frac{n}{k}$, then $W^{k,p}(R^n) \to L^\infty(\ R^n)$.

We note that if p >$\frac{n}{k}$, then the following continuous inclusion hold,

$W^{k,p}(R^n) \to C^k(\ R^n)$.

The results are also valid for any open set Ω ⊂R^n for the spaces $W_0^{k,2}$(Ω) , and for $W^{k,p}$(Ω) if Ω is of class C^1 with bounded boundary ∂Ω or Ω =${R^n}_+$. In these cases, the following continuous inclusion hold,

$W^{k,p}(\Omega) \to C^k(\underline{\Omega})$ for p >$\frac{n}{k}$.

Example 1.4 Think about the accompanying fractional differential eqs with a digressing case, for 0 < T < ∞

$$\begin{cases}\frac{\partial^\beta u}{\partial t^\beta} = \frac{\partial^2 u}{\partial t^2} + H(\ x\ ,u\ (x\ , \quad t)\Bigg) + G\big(t,x,u\ (x\ ,t)\big),\end{cases}$$

$$(x,t) \in (0,1) * (0,T\ ),$$

$$u(0,t) = u(1,t) = 0$$

$$u(x,0) = u_0(\ x\ ), x\ \in (0\ ,1), \text{..........}\textbf{(1.4.1)}$$

here β ∈ (0, 1) ,

H˜ (x , u(x , t)) = $\int_0^x K\ (x,y)u(y,g(t)|u(y,t)|dy$, What's more, the capability G : R + × [0 ,1] * R → R a quantifiable within x, centrally H¨older Continuous within t, locally lipschitz continuous within u, consistently in x. suppose that g : R+ → R + a locally H¨older continuous on t along g(0) = 0 , k ∈ C^1([0, 1] × [0, 1]; R).

Take X = L^2((0 , 1) ; R), $A_u = \frac{d^2u}{dx^2}$, D(A) = H^2(0, 1) ∩ H_0^1 (0, 1) and $X_{\frac{1}{2}}$= D($(-A^{\frac{1}{2}})$ = H_0^1 (0, 1) and $X_{\frac{-1}{2}}$= H_0^1 ((0, 1))∗ = H^{-1}(0, 1) ≡ H^{-1}(0 ,1). As x ∈ (0 , 1),

Now characterize f : R_+ × H_0^1(0, 1) × H^1(0, 1) → = L^2(0, 1) by

f(t, φ, ψ) = H ˜ (x, ψ) + G(t , x, φ),

here H ˜(x, ψ(x , t)) = $\int_0^x K(x,y)\,\psi(y,t)dy$.

Thus , we can rewrite (1.4.1) in an abstract form as follows

$\frac{d^\beta \phi}{dx^\phi}$= Aφ + f(t, φ(t), φ[h(t , φ(t))]),

u(0) = u_0, …….(1.4 .2)

here h(t, φ(t)) = g (t)|φ (x, t)|.

Accordingly, we have seen that the development conditions in some sense are the underlying worth issues for the normal differential conditions in the endless layered spaces and are related with the fractional practical and unique differential conditions overseeing specific actual peculiarities.`

Semigroups

In this segment we notice a few significant definitions and properties of the semigroups of limited direct administrators. The hypothesis of semigroups furnishes us with important devices to concentrate on the practical(functional) differential conditions and utilitarian(functional) integro-differential conditions, in quantum mechanics or in limitless layered control hypotheses. All through this segment, we expect that X will being a Banach space.

Definition 2.i A one boundary Family T(t), $0 \leq t < \infty$, of bounded(limited) linear operator from X to X is supposed to be a semigroup of bounded linear operator on X if

(i) T (0) = I, here I is a identity operator (administrator) on X.

(ii) T (t + s) = T(t) T(s) for each t , s $\geq$ 0 (the semigroup thm).

Mark 2.2 Assume T (t), $0 \leq t < \infty$ be a semigroup of bounded linear operator (limited direct administrator) on X. A linear(straight) operator A on X, which is characterized as

D (A) = { x ∈ X : $lim_{t \to 0} \frac{T(t)x - x}{t}$ exist}

Furthermore

Ax = $lim_{t \to 0} \frac{T(t)x - x}{t} = \frac{d^{+T(t)x}}{dt} |_{\ t=0}$ for x ∈ D(A),

supposed to be the infinitesimal generator of semigroup T (t), D (A) is the domain(space) of A

Example 2.3 Assume H = C(−∞,∞) be the space of bounded consistently continuous on (−∞,∞). Characterize T (t) : H → H the linear operator by

(T(t)u)(s) =$\{u(s), t = 0; e^{-\lambda t} \sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!} u(s - n\mu), t > 0.$

here λ, μ > 0. Then $(T(t))_t$≥0 being a semigroup for bounded linear operators on H and the infinitesimal generator is the difference operator A: (Au) (s) = λ(u(s − μ) – u (s)).

A logical(analytical) semigroup is a specific sort of unequivocally persistent semigroup. Logical semigroups are utilized to acquire the arrangements of halfway(partial) differential conditions; contrasted with the unequivocally constant semigroups, insightful semigroups give better consistency of the answers for beginning worth issues, and improved results concerning annoyances of the infinitesimal generator.

Definition 2.4 Let { T(z) : z ∈ Σ} being a group of bounded linear operator on X, where

Σ = {z ∈ C : ϕ1 < arg z < ϕ2, ϕ1 < 0 < ϕ2}.

Then the family {T(z) : z ∈ Σ} is said to be an analytic semigroup in Σ if

(i) z → T(z) is analytic in Σ;

(ii) T (0) = I and $_{z\to 0}$ T(z)x = x for each x∈ X;

(iii) T(z_1 +z_2) = T(z_1) T(z_2) for z_1, z_2 ∈ Σ.

A C_0 -semi-group $(T(t))_t$≥0 of bounded linear operators on X is said to be an analytic Semigroup in the event that it is scientific in some area Σ containing the non-negative genuine pivot(real axis).

Chapter-3

Fractional Calculus

Fractional calculus (Partial Analytics) alludes the integration or differentiation of any (i.e., not-integer) order. The field has a set of experiences as old as math itself , that didn't draw in sufficient consideration for quite a while. Nonetheless, as of late, the idea of the fragmentary math (fractional calculus) and partial differential conditions (fractional differential equations) has turned into a fascinating exploration subject as a result for the wide materialism within mechanism, science , chemistry (demonstrating the irregular dissemination conduct of Brownian particles), electricals designing (electromagnetic waves) etc.

There are numerous ways of characterizing the fractional(partial) integral and fractional derivative subsidiary and these methodologies are called by their creators. For instance, the Grunwald Letnikov meaning of fractional integral and fractional derivative begins from traditional meanings of derivatives and integrals based on infinitesimal division and limit. The drawbacks of this approach are its specialized trouble of the calculations and the evidence and huge limitations on capabilities. Luckily there are other, more rich methodologies like the Riemann-Liouville definition which incorporates the consequences of the past one as an exceptional case.

In this postulation, we will just zero in on the definitions because of Riemann-Liouville and Caputo since these are the most involved ones in applications.

3.1 Fractional integral according to Riemann-Liouville

As per the Riemann-liouville way to deal with fractional calculus, the thought of fractional integral for order α(α>0) for the capability f(t), is an immediate speculation of Cauchy's equation for n-fold integral, which lessens the computation for n - fold integral of the function f(t) of a solitary basic of convolution type.

$$J_t^n f(t) = \frac{1}{(n-1)!}\int_0^t \frac{f(s)ds}{(t-s)^{1-n}}, \text{ n} \in \text{N, t} > 0, \qquad (3.1.1)$$

here N being arrangement(sets) of positive integers. By (3.1.1) it shoud be noted that J_t^nf(t) disappears at t = 0 , mutually within subordinates of order 1, 2, …... , n − 1.

Since (n − 1)! = Γn, Riemann understood that the right hand side of (3 .1.1) could possess significance in any event, when n takes non-integer values,

one characterizes the Riemann-Liouville fractional integration for the order

α > 0

$$J_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t \frac{f(s)}{(t-s)^{1-\alpha}}s, \text{ t} > 0, \ \alpha \in R_+, \qquad (3 .1.2)$$

here R_+, being set of +ve real numbers. For addition we discuss J_t^0= I (identity operator), i.e., we means J_t^0f(t) = f(t).

Comment: We should be noted that the Riemann – Liouville fractional integral fulfill the semigroup properties i.e.

J_t^α oJ_t^β = $J_t^{\alpha+\beta}$, α , β ≥ 0, which hinted the commutative property $J_t^\alpha 0 J_t^\beta$ = $J_t^\beta 0 J_t^\alpha$.

3.2 Fractional derivative according to Riemann-Liouville

Fractional (fractional order) derivative is a speculation of integer-order derivative and integral. it started in the letter for the significance of $\frac{1}{2}$ order derivative from L ‘hospital to Leibnitz in 1695, and is a favorable device for depicting memories peculiarities. The Riemann-Liouville

Fractional derivatives of erratic order α is characterized as follows:

D^{α}f(t) = $D^{m}J^{m-\alpha}$ f(t) along m-1 < α ≤ m,

in particular

D^{α}f(t) = $\{\frac{d^m}{dt^m}\left[\frac{1}{\Gamma(m-\alpha)}\int_0^t \quad \frac{f(\tau)}{(-\tau)^{\alpha-m+1}}\tau,\right]$ m − 1 < α < m,

$\{\frac{d^m}{dt^m}f(t)\ ,\ \alpha = m$ (3.1.3)

where m ∈ N and $D^{m} = \frac{d^m}{dt^m}$.

Remarks:

(i) Define $D^0 = J^0$ = I (Identity operator). Then
D_t^{α}oJ_t^{m} = D_t^{m} o$J_t^{m-\alpha}$oJ_t^{α} = D_t^{m}oJ_t^{α} = I, α ≥ 0.
Furthermore, we obtain
$D^{\alpha}T^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma-\alpha+1)}$, α > 0, γ > −1, t > 0.

(ii) The fractional differentiation D^{α} f should not be zero for the steady function. If f(t) ≡ 1, then

$D^{\alpha}1 = \frac{t^{-\alpha}}{\Gamma(1-\alpha)}$, α ≥ 0, t > 0,

which indistinguishably evaporates for α ∈ N, because of the posts for the Gamma function within the points 0, −1, −2, · · · .

Chapter-4

Caputo Fractional Derivative

In 1967, M. Caputo introduced a new definition of a fractional derivative in his paper [A Model of Dissipation Whose Q Is Almost Frequency Independent], the purported Caputo fractional derivative .in this part now express the meaning and a few theorems of the Caputo fractional derivative.

Definition 3.1 Let f ∈ C^{m-1} ((0, T), X),

($\Theta_{m-\alpha} *$ f) ∈ $W^{m,1}$ ((0, T), X), 0 ≤ m − 1 < α < m , m ∈ N.

Then articulation be

cD_t^{α} f(t) = $D_t^m J_t^{m-\alpha}$ (f(t) − $\sum_0^{m-1}$ $f^i(0)(0)_{i+1}(t)$ (3.1.4)

where $D_t^m = \frac{d^m}{dt^m}$,

$\Theta_\alpha(t)$ = $\{\frac{1}{\Gamma\alpha} t^{\alpha-1}, t > 0; 0, t \leq 0,$

and Θ_0(t) = 0, is known as the Caputo Fractional Derivative of order α of f.

Lemma 3.2 Assign α, β > 0. The following properties hold.

(i) $J_t^{\alpha} J_t^{\beta} f(t)$= $J_t^{\alpha+\beta}$f(t) for each f ∈ L^1 (J; X);

(ii)J_t^{α} (f∗ g) = J_t^{α}f ∗ g for each f, g ∈L^p (J; X)(1 ≤ p < +∞);

(iii) The Caputo fractional derivative cD_t^{α} is a left inverse of J_t^{α} :

$cD_t^{\alpha} J_t^{\alpha}$ = f, for all f ∈ L 1 (J; X)

but in general not a right inverse, in fact, for all f(t) ∈ C^{m-1} (J; X) with (Θm−α ∗ f) ∈ $W^{m,1}$(J, X) (m∈ N, 0 ≤ m − 1 < α < m), one has

J_t^{α} (cD_t^{α})f(t) = f(t) −$\sum_{i=0}^{m-1}$ $f^{(i)}(0)\Theta_{+1}$ (3.1.5)

Model 3.iii Let α ≥ 0, n = ⌈α⌉ and f(x) = $(x - a)^c$. then

cD^{α}f(x) ={$\frac{\Gamma(c+1)}{\Gamma(c+1-\alpha)}(x-a)^{c-a}$, C∈ N, c ≥ n or c ∈ N, c > n − 1,

{ 0 , C ∈ {0, 1, 2, · · · , n-1}, …..(3.1.6)

where ⌈α⌉ means the littlest integers prominent than or equal to α.

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END :

This paper examined the idea of fragmentary subordinate and a prologue to partial differential conditions. Riemann-Liouville and Caputo partial subordinates were returned to and dissected under the radiance of the proposed standards. Regardless of late advancements of partial math and its applications during the most recent twenty years, there are as yet numerous strange issues and open inquiries including (I) a mathematical or actual translation of a subsidiary of Non whole number request, (ii) presence of a Non number request subordinate of a capability which doesn't have number request subsidiaries.

We likewise examined the issue with another class of dynamic differential conditions for which motivations are not immediate with the indiscreet differential condition and presence of gentle arrangements.

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