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| **Name.** ________________________________<br>**Student Admn. No.:** ________________________ | **Printed Pages:01** |

**School of Basic & Applied Sciences**
**Backlog Examination, June 2023**

**[B.Sc. Mathematics ] [Semester: IV ]**

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| **[Batch: ] Course Title: Ring Theory**<br>**Course Code: BSCM423** | **Max Marks: 100**<br>**Time: 3 Hrs.** |

***Instructions:***
1. *All questions are compulsory.*
2. *Assume missing data suitably, if any.*

| | | K Level | COs | Marks |
|---|---|---|---|---|
| | **SECTION-A (15 Marks)** **5 Marks each** | | | |
| **1.** | Find the basis of Q(√3,√5) over Q. | | | 5 |
| **2.** | Give an example of a subring S of a ring R such that S and R have different unity (multiplicative identity). | | | 5 |
| **3.** | Show that the characteristic of an integral domain is either 0 or prime. | | | 5 |
| | **SECTION-B (40 Marks)** **10 Marks each** | | | |
| **4.** | Show that the sum of two subrings of a ring R may not be a subring of R. | | | 10 |
| **5.** | Find all the ring homomorphisms from **Z** to **Z**. | | | 10 |
| **6.** | Prove that every Euclidean domain is a Principal ideal domain. Is the converse true? | | | 10 |
| **7.** | Find the g.c.d. of $x^6+x^3+x+1$ and $x^2+1$ in **Q**[x].<br>OR<br>Describe the splitting field of $x^3$-2 over **Q**, the field of rational numbers. | | | 10 |
| | **SECTION-C (45 Marks)** **15 Marks each** | | | |
| **8.** | Show that the polynomial $x^2 + x + 2$ is irreducible over F = {0, 1, 2} mod 3. Use it to construct a field of 9 elements. | | | 15 |
| **9.** | Define a Euclidean Domain. Further, show that **Z**, the set of integers is a Euclidean domain. | | | 15 |
| 10 | Prove that a finite extension is an algebraic extension. Is the converse true?<br>OR<br>Show that √-5 is a prime element in **Z**[√-5], while 3 is not. Also check whether 3 is an irreducible element or not. | | | 15 |