

School of Basic and Applied Sciences

Mathematics
ETE - Jun 2023

Time : 3 Hours

Marks : 50

Sem II - MSCM203 - Partial Differential Equations

Your answer should be specific to the question asked

Draw neat labeled diagrams wherever necessary

1. Write Charpit's auxiliary equations to solve the following equation: $(p + q^2)y = z$. K2 CO1 (2)
2. Write Charpit's auxiliary equations to solve the following equation: $(p^2 + p + q)y = qz$. K2 CO1 (2)
3. Classify whether the following PDE is linear, quasi-linear or nonlinear:
(a) $zz_x + 2xyz_y = 0$; (b) $z_x^2 - zz_y = 0$. K1 CO1 (2)
4. Classify the following equations into hyperbolic, elliptic or parabolic type.
(A) $8u_{xx} + u_{yy} - u_x + [\log(2 + x^2)]u = 0$.
(B) $xu_{xx} + u_y = 0$. K2 CO1 (2)
5. Write auxiliary equations associated with Jacobi method to solve first-order PDE of the form $f(x, y, z, u_x, u_y, u_z) = 0$. K1 CO1 (2)
6. Reduce the equation $u_{xx} = x^2u_{yy}$ to its canonical form. K3 CO2 (5)
7. Reduce the equation $u_{xx} - x^2u_{yy} = 0$ to its canonical form. K3 CO2 (5)
8. Solve $(mz - ny)p + (nx - lz)q = ly - mx$. K4 CO3 (6)
9. Solve the following IBVP:
 $u_t = 4u_{xx}, \quad 0 \leq x \leq \pi, \quad t > 0,$
 $u(0, t) = u(\pi, t) = 0, \quad t > 0,$
 $u(x, 0) = 3 \sin 2x - 6 \sin 5x, \quad 0 \leq x \leq \pi.$ K3 CO3 (8)
10. Derive the D'Alembert solution to the IVP:
 $u_{tt} = c^2u_{xx}, \quad -\infty < x < \infty, \quad t \geq 0,$
 $u(x, 0) = f(x),$
 $u_t(x, 0) = g(x).$ K4 CO4 (8)
11. Solve the following BVP:
 $u_{xx} + u_{yy} = 0, \quad 0 < x < 1, \quad 0 < y < 1,$
 $u(x, 0) = x(x - 1), \quad u(x, 1) = 0, \quad 0 \leq x \leq 1,$
 $u(0, y) = 0, \quad u(1, y) = 0, \quad 0 \leq y \leq 1.$ K4 CO4 (8)