GALGOTIAS UNIVERSITY

ADMISSION NUMBER

School of Basic Sciences

Bachelor of Science Honours in Chemistry
Semester End Examination - Nov 2023

Duration : 180 Minutes
Max Marks : 100

Sem III - MSCM301 - Functional Analysis

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) Define convergence of a sequence in a Normed linear space. K1 (2)

2) **If H is a Hilbert space,{ei} is an orthonormal set in H & x is an arbitrary vector in H, then Show that** $||x||^2 = \sum |(x, e_i)|^2 \Rightarrow \{e_i\}$ **is complete.** K2 (4)

3) Show that $H(e^t - \pi) = \begin{cases} 0 & t < \log \pi \\ 1 & t > \log \pi \end{cases}$ K2 (6)

4) Apply the theory of norms show that l_∞ the space of all bounded sequences $< x_n >$ is a normed space under the norm $\|x\| = \sup|x_n|$. K3 (9)

5) Apply the theory of norms to show that $l_n^p, 1 \leq p < \infty$, the space of all n-tupples is a normed space under the norm $||x|| = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p}, where\ x = (x_1, x_2, ..., x_n).$ K3 (9)

6) Evaluate that the inner product in a Hilbert space is jointly continuous. That is if $x_n \to x$ and $y_n \to y$ then $(x_n, y_n) \to (x, y)$ as $n \to \infty$. K5 (10)

7) State and prove the spectral mapping theorem for polynomials. K4 (12)

8) Prove that l_∞ the space of all bounded sequences $< x_n >$ is a Banach space under the norm $\|x\| = \sup|x_n|$. K5 (15)

9) Prove that the linear spaces $\mathbb{R}^n$ is a Banach space under the norm $\|x\| = (\sum_{i=1}^{n} |x_i|^2)^{\frac{1}{2}}$, where $x = (x_1, x_2, ... x_n)$. K5 (15)

10) Let a function f(x) be n times continuously differentiable; then prove that K6 (18)

$$f(x)\delta^n(x) = (-1)^n f^n(0)\delta(x) + (-1)^{n-1} n f^{n-1}(0)\delta'(x) + \frac{(-1)^{n-2} n(n-1) f^{n-2}(0)\delta''(x)}{2!} + \dots + f(0)\delta^n(x).$$