

**School of Basic Sciences**  
**Bachelor of Science Honours in Mathematics**  
**Mid Term Examination - Nov 2023**

**Duration : 90 Minutes**  
**Max Marks : 50**

**Sem III - C1UC301T - Multivariable Calculus and Partial Differential equations**

General Instructions

*Answer to the specific question asked*

*Draw neat, labelled diagrams wherever necessary*

*Approved data hand books are allowed subject to verification by the Invigilator*

- 1) Find the gradient of the scalar field  $f(x,y) = y^2 - 4xy$  at the point (1,2). K2 (2)
- 2) Find the value of  $\frac{\partial z}{\partial x}$  at the point (1, 1, 1) if the equation  $xy + z^3x - 2yz = 0$  defines z as a function of the two independent variables x and y. K1 (3)
- 3) Find the second-order partial derivative  $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}$  of the function  $f(x,y) = \sin(xy)$ . K2 (4)
- 4) Find  $\iint_R x \cos(xy) dA$  where  $R: 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq 1$ . K2 (6)
- 5) Find the points of discontinuity of the function  $f(x,y) = \begin{cases} \frac{3x^2y}{x^4+y^4}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ . K3 (6)
- 6) Show that the vector field  $F = 2x(y^2 + z^3)i + 2x^2yj + 3z^2x^2k$  is conservative. Find its potential function. K3 (9)
- 7) Apply Lagrange Multiplier method to find the maximum and minimum values of  $f(x,y) = xy$  on the curve  $3x^2 + y^2 = 6$ . K4 (8)
- 8) Analyse the critical points of the function  $f(x,y) = 8x^3 - 24xy + y^3$  and use second derivative test to classify each point as one where a saddle, local minimum or local maximum occurs. K4 (12)

**OR**

- Analyse the critical points of the function  $f(x,y) = 10xye^{-(x^2+y^2)}$  and use second derivative test to classify each point as one where a saddle, local minimum or local maximum occurs. K4 (12)