

School of Basic Sciences
Bachelor of Science Honours in Mathematics
Mid Term Examination - Nov 2023

Duration : 90 Minutes
Max Marks : 50

Sem III - C1UC302T - Complex analysis

General Instructions

Answer to the specific question asked

Draw neat, labelled diagrams wherever necessary

Approved data hand books are allowed subject to verification by the Invigilator

- 1) Explain whether the function $f(z) = |z|^2$ is analytic at $z=0$ or not. K2 (2)
- 2) Find the Arg $(1 + i)$ and $\arg(1 + i)$. K1 (3)
- 3) Find the value of $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x+y}$. K2 (4)
- 4) Show that the following function is continuous at origin: K2 (6)

$$f(z) = \begin{cases} \frac{x^3 y^5 (x + iy)}{x^4 + y^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$
- 5) Using the concept of analyticity, show that $\sin \bar{z}$ is nowhere analytic. K3 (6)
- 6) Verify the function $u(x,y) = e^x \cos y$ is harmonic. Find its conjugate harmonic function and the corresponding analytic function $f(z)$. K3 (9)
- 7) Categorize which of the following functions are harmonic: (i) e^z (ii) $x^2 + iy^2$ K4 (8)
- 8) Examine, whether the function $f(z) = z + \bar{z}$ is differentiable or not. K4 (12)

OR

Examine, whether the function $f(z) = \frac{1}{(z-1)^3}$ has isolated singularity or not K4 (12)
and find the kind of singularity of the function $f(z) = \frac{e^z}{(z-1)}$.