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**School of Basic Sciences**

Bachelor of Science in General  
Semester End Examination - Nov 2023

Duration : 180 Minutes  
Max Marks : 100

**Sem V - C1UC501T - Group and Ring Theory and Linear Algebra**General Instructions

Answer to the specific question asked

Draw neat, labelled diagrams wherever necessary

Approved data hand books are allowed subject to verification by the Invigilator

- 1) Describe the characteristic polynomial for a matrix with example. K1 (2)
- 2) Estimate the number of conjugacy classes of a non-abelian group of order 343. K2 (4)
- 3) Show that the units of  $R$  and  $R[x]$  are same. K2 (6)
- 4) Verify the rank-nullity theorem for  $T : R^3 \rightarrow R^3$  defined as  $T(x, y, z) = (x - y + 2z, 2x + y, -x - 2y + 2z)$ . K3 (9)
- 5) Verify the rank-nullity theorem for  $T : R^3 \rightarrow R^3$  defined as  $T(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z)$ . K3 (9)
- 6) Show that the polynomial  $x^2 + x + 2$  is irreducible over  $F = \{0, 1, 2\} \text{ mod } 3$ . Use it to construct a field with 9 elements. K5 (10)
- 7) Examine, whether a group of order 33 is simple or not. K4 (12)
- 8) State and prove the rank nullity theorem. K5 (15)
- 9) Prove that the basis is the largest linearly independent and smallest spanning set. K5 (15)
- 10) Conclude that a group of order  $pqr$ , ( $p < q < r$ ) is not simple. K6 (18)