

School of Basic Sciences
Bachelor of Science Honours in Mathematics
Semester End Examination - May 2024

Duration : 180 Minutes
Max Marks : 100

Sem VI - C1UC601T - Metric Spaces and Complex Analysis

General Instructions

Answer to the specific question asked

Draw neat, labelled diagrams wherever necessary

Approved data hand books are allowed subject to verification by the Invigilator

- 1) Find the following balls in discrete metric space (X, d) with $x \in X$: (i) $B(x, 1/2)$, (ii) $B(x, 3/2)$ K1 (3)
- 2) Estimate the value of $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x+y}$. K2 (4)
- 3) Show that the following function is continuous at origin: $f(z) = \sqrt{|xy|}$ K2 (6)
- 4) Show that $\cos \bar{z}$ is nowhere analytic. K3 (6)
- 5) Show that $\sin \bar{z}$ is nowhere analytic K3 (6)
- 6) Verify that the integral of an analytic function along the closed curve is zero. K3 (9)
- 7) Verify the function $u(x,y) = e^x \cos y$ is harmonic. Find its conjugate harmonic function and the corresponding analytic function $f(z)$. K3 (9)
- 8) Show that the singularity of the set of zeros of a function $f(z)$ is an isolated singularities. K4 (8)
- 9) Examine, whether the function $f(z) = \bar{z}$ is differentiable or not. K4 (12)
- 10) Evaluate the integral $\oint \frac{1}{(z^2-z)(z-2)} dz$ over $C: |z| = 2.9$. K5 (10)
- 11) Prove that a set in \mathbb{R} with usual metric is compact iff it is closed and bounded. K5 (15)

OR

Prove the sequential criterion of continuity in a metric space. K5 (15)

- 12) Justify the statement: A function between the metric spaces is continuous if the preimage of every open set is open. K6 (12)

OR

Justify the difference between continuity and uniform continuity by providing examples. K6 (12)