

School of Computing Science and Engineering
Bachelor of Technology in Computer Science and Engineering
Mid Term Examination - May 2024

Duration : 90 Minutes
Max Marks : 50

Sem II - C1UC224T - Discrete Mathematics

General Instructions

Answer to the specific question asked

Draw neat, labelled diagrams wherever necessary

Approved data hand books are allowed subject to verification by the Invigilator

- 1) Let $P(x)$ be the statement " $x = 2x$." If the domain consists of the integers, what are the truth values of the following?
a) $P(2)$ b) $\exists xP(x)$ K2 (2)
- 2) Three cards are chosen one after the other from a 52-card deck. Find the number of ways this can be done: K1 (3)
(a) with replacement;
(b) without replacement.
- 3) What is the converse, contrapositive, and inverse of the conditional statement "If a positive integer has no divisors other than 1 and itself, then it is prime." K2 (4)
- 4) Let R be a relation on the set \mathbb{R} of real numbers defined by $R = \{(x, y) : |x - y| < 1\}$. Show that R is reflexive and symmetric but not transitive. K2 (6)
- 5) How many cards must be selected from a deck of 52 cards to guarantee that at least 5 cards of the same suit are chosen? K3 (6)
- 6) a) Let f and g be the functions K3 (9)
 $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$
Find the composition functions $f \circ g$ and $g \circ f$.
b) Prove that the function $f: \mathbb{R} \rightarrow \mathbb{R}^+$ given by $f(x) = e^x$ and $g: \mathbb{R}^+ \rightarrow \mathbb{R}$ given by $g(x) = \ln(x)$ are inverse of each other.
- 7) Construct a truth table for the compound proposition: $(p \wedge r) \rightarrow (\neg q \vee r)$ K4 (8)
- 8) Let R be the relation defined on the set \mathbb{Z} of integers as follows: for integers a and b , aRb if and only if $a \equiv b \pmod{4}$, that is, aRb if 4 divides $(a-b)$. K4 (12)

1. Show that R is an equivalence relation on \mathbb{Z} .
2. What are the equivalence class containing $[0]$?

OR

1. Prove that if n is an odd integer, then n^2 is odd. K4 (12)
2. Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.