GALGOTIAS UNIVERSITY

ADMISSION NUMBER

School of Basic Sciences

Master of Science in Mathematics
Semester End Examination - Jun 2024

Duration : 180 Minutes
Max Marks : 100

Sem II - C1PM201T - Functional Analysis

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) Define test function. K1(3)

2) Define Similar operators on a Hilbert space H. K2(4)

3) The linear space l_n^{∞} is a Banach space. K2(6)

4) Let (X, $\|.\|$) and (Y, $\|.\|$) be two normed linear spaces over the field K (= $\mathbb{R}$ or $\mathbb{C}$). Let T:(X, $\|.\|$) $\rightarrow$ (Y $\|.\|$) be a continuous linear transformation then show that the range space of T need not be closed. K3(6)

5) Define Orthonormal set in Hilbert space. K3(6)

6) If X is a complex inner product space and $\alpha, \beta, \gamma \in \mathbb{C}$, then $(x, \beta y + \gamma z) = \bar{\beta}(x, y) + \bar{\gamma}(x, z)$. K3(9)

7) If X is a complex inner product space and $\alpha, \beta \in \mathbb{C}$, then $(\alpha x - \beta y, z) = \alpha(x, z) - \beta(y, z)$. K3(9)

8) The linear space C of all convergent sequence $x = \{x_n\}$ with the norm K4(8)

$$\|x\| = \sup_{1 \le n < \infty} |x_n|$$

is a Banach Space.

9) Show that each reflexive space is a Banach space. K4(12)

10) Let (X, $\|.\|$) and (Y, $\|.\|$) be two normed linear spaces over the field K (=$\mathbb{R}$ or $\mathbb{C}$). Let T:(X, $\|.\|$) $\rightarrow$ (Y $\|.\|$) be a linear transformation then prove that the followings are equivalent: K5(10)
(i). T is a bounded linear transformation.
(ii). T is uniformly continuous.
(iii). T is continuous at some $x_0 \in X$.

11) State and prove the Hahn-Banach theorem using Zorn's Lemma. K5(15)

OR

Prove that the conjugate space of l_p is l_q, where $\frac{1}{p} + \frac{1}{q} = 1$ and K5(15)

$1 \leq p < \infty$.

12) Prove that the conjugate space of c_0 is l_1. K6(12)

OR

Prove that the conjugate space of c is l_1 K6(12)