

School of Basic Sciences

**Master of Science in Mathematics
Semester End Examination - Jun 2024**

**Duration : 180 Minutes
Max Marks : 100**

Sem II - C1PM206B - Mathematical Statistics

General Instructions

Answer to the specific question asked

Draw neat, labelled diagrams wherever necessary

Approved data hand books are allowed subject to verification by the Invigilator

- 1) Find the correlation coefficient by Karl Pearson method (6, 9), (2, 11), (10, 7), (4, 8), (8, 7). K1(3)
- 2) The guaranteed average life of a certain type of electric light bulbs is 1000 hours with a standard deviation of 125 hours. It is decided to sample the output so as to ensure that 90 per cent of the bulbs do not fall short of the guaranteed average by more than 2.5 per cent. Estimate the minimum size of the sample. K2(4)
- 3) Show that $\chi^2 = \left(\sum_1^n \frac{o_i^2}{E_i} \right) - N$, where o_i^2 and E_i are observed and expected theoretical frequency. K2(6)
- 4) A coin was tossed 400 times and the head turned up 216 times. Write the hypothesis that the coin is unbiased. K3(6)
- 5) A bag contains defective article, the exact number of which is not known. A sample of 100 from the bag gives 10 defective articles. Develop the limits of the proportion of defective articles in the bag. K3(6)
- 6) Given that K3(9)

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} & 0 < x < \theta \text{ and } \theta > 0 \\ 0 & \text{elsewhere} \end{cases}$$
- 7) Solve for the reciprocal of $nE \left[\left\{ \frac{\partial \log f(x, \theta)}{\partial \theta} \right\}^2 \right]$ K3(9)
If X has a chi-square distribution with n degree of freedom, find for moment generating function.
- 8) A random sample of size 20 from a normal population has mean 42 and standard deviation of 5. Examine the hypothesis that the population mean is 45. Use 5% level of significance. K4(8)

9) In a sample of 600 men from a certain city, 450 are found to be smokers. In a sample of 900 from another city, 450 are found to be smokers. Analyze whether two cities are significantly different with respect to prevalence of smoking habits among men. K4(12)

10) A random sample of size 16 has 53 as mean. The sum of squares of the deviation from mean is 135. Evaluate 95% and 99% confidence limits of the mean of the population. K5(10)

11) A sequence of random variables is defined by K5(15)

$$P[X_n = -\sqrt{2n-1}] = P[X_n = +\sqrt{2n-1}] = \frac{1}{2}$$
 Show that for the sequence $\{X_n\}$ the law of large number does not hold.

OR

Prove that $r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_z^2}{2\sigma_x\sigma_y}$, where r is the coefficient of correlation between x and y σ_x , σ_y and σ_z are concerned standard deviations. K5(15)

12) A random sample of 500 adults who are questioned regarding their political affiliation and opinion on a tax reform bill as per the given table. Test if the political affiliation and their opinion on a tax reform bill are dependent at a 5% level of significance. Design the test statistic. K6(12)

	Favor	Indifferent	Opposed	Total
Democrat	138	83	64	285
Republican	64	67	84	215
Total	202	150	148	500

OR

If X is a Poisson Variate with mean 'm' by assuming $z = \frac{x-m}{\sqrt{m}}$ design the m.g.f. and also find its limit. K6(12)