GALGOTIAS UNIVERSITY

ADMISSION NUMBER

School of Basic Sciences

Master of Science in Physics
Semester End Examination - Jun 2024

Duration : 180 Minutes
Max Marks : 100

Sem II - C1PO207B - Applied Numerical Methods

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) Explain the fundamental principle behind the Birge-Vieta method for finding roots of polynomials. K1(3)

2) Explain the difference between the predictor and corrector steps in Milne's Predictor-Corrector method. K2(4)

3) Describe the difference between interpolation and least squares approximation. Provide examples to illustrate each. K2(6)

4) Use Simpson's 1/3 rule to approximate the integral of $f(x) = \ln(2x + 1)$ from $x = 0$ to $x = 1$. K3(6)

5) Interpolate the points (0; 0), (1; 1), and (3; 9) at x = 2 using Newton's backward interpolation. K3(6)

6) Use the Birge-Vieta method to find the roots of the polynomial $f(x) = 3x^3 - 6x^2 + 4x - 1$. Begin with an initial guess of $x_0 = 1$. Compute the roots with an error tolerance of 10^{-4}. K3(9)

7) Using the method of least squares, find the best quadratic approximation for the data points (1, 1), (2, 4), (3, 9), and (4, 16). K3(9)

8) Given the function $f(x) = \cos(x) - x$, use the Newton-Raphson method to find an approximate root starting from $x_0 = \frac{\pi}{4}$. Calculate the value of x_1 after one iteration. K4(8)

9) Apply the fourth-order Runge-Kutta method to solve the initial value problem $y' = 3x^2 - 2y$, $y(0) = 1$ over the interval $0 \leq x \leq 2$ with a step size of $h = 0.2$. K4(12)

10) Apply the bisection method to find a root of $f(x) = x^3 + 3x^2 - 2x - 4$ in the interval $[-2, 0]$. Use a tolerance of 10^{-5}. K5(10)

11) Given the data points (0; 1), (1; 3), (2; 7), and (3; 15), determine the polynomial using the Newton forward formula. K5(15)

OR

Use Newton's forward difference formula to approximate the first derivative of $f(x) = x^3$ at $x = 2$ using the following data: K5(15)

$$x : 1, 2, 3, 4$$

$$f(x) : 1, 8, 27, 64$$

12) Apply the Runge-Kutta 2nd order method to approximate the solution of $y' = 3y - x$ with $y(0) = 2$ at $x = 0.5$ using a step size of 0.1. K6(12)

OR

Approximate the third derivative of $f(x) = \ln(x)$ at $x = 2$ using Newton's forward difference method with the data: K6(12)

$$x : 1, 1.5, 2, 2.5$$

$$f(x) : 0, 0.4055, 0.6931, 0.9163$$