ADMISSION NUMBER

School of Basic Sciences

Bachelor of Science Honours in Mathematics
Semester End Examination - Jun 2024

Duration : 180 Minutes
Max Marks : 100

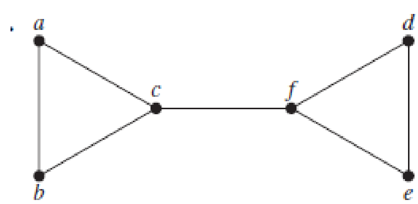
Sem IV - C1UC405T - Discrete Mathematics

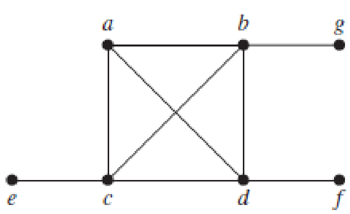
General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) Find the translation of the following statements into English, where *C(x)* is "*x* is a comedian" and *F(x)* is "*x* is funny" and the domain consists of all people. K1(3)
(a) $\forall x\,(C(x) \rightarrow F(x))$
(b) $\forall x\,(C(x) \land F(x))$
(c) $\exists x\,(C(x) \rightarrow F(x))$

2) Show that the value of the prefix expression + - * 3 2 / 8 4 1 is 5. K2(4)

3) Explain whether the given graph has a Hamilton circuit. If it does, find such a circuit. K2(6)

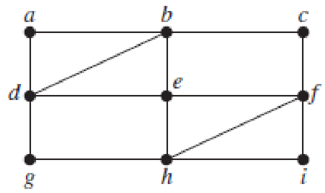
1. 

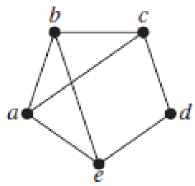
2. 

4) Solve for the greatest lower bound and the least upper bound of the sets {3, 9, 12} and {1, 2, 4, 5, 10},if they exist, in the poset(**Z**+, |). K3(6)

5) Apply the concept of Euler circuit to determine whether the given K3(6)

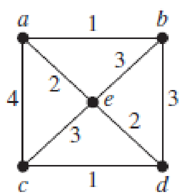
graph has an
Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists:

1. 

2. 

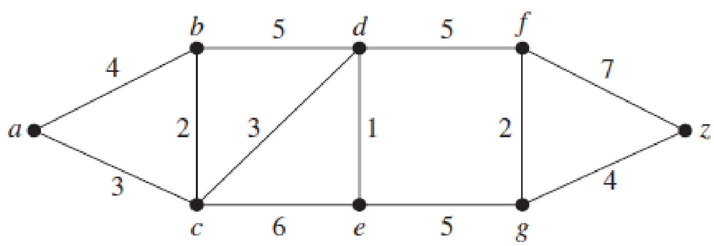
6) Develop a direct proof of the theorem “If n is an odd integer, then n^2 is odd.” K3(9)

7) Apply Prim’s algorithm to find a minimum spanning tree for the given weighted graph. K3(9)



8) Use generating function to examine solution of the the recurrence relation$a_n + 4a_{n-1} + 4a_{n-2} = n^2 - 3n + 5$ K4(8)

9) Use Dijkstra’s Algorithm to examine the length of a shortest path between *a*
and *z* in the given weighted graph. K4(12)



10) Determine the graph represented by the given adjacency matrix: K5(10)

1. $$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

2. $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix}$

3. $\begin{bmatrix} 0 & 2 & 3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$

11) Prove the proposition p that sum of the square of first n positive integers is n(n+1)(2n+1)/6. K5(15)

OR

Prove or disprove that there are always two vertices of the same degree in a finite multi graph having at least two vertices K5(15)

12) Formulate a binary search tree for the words *banana, peach, apple, pear, coconut, mango, and papaya* using alphabetical order. K6(12)

OR

Discus these questions for the poset*(*{3*,* 5*,* 9*,* 15,24*,* 45}*,* |*).* K6(12)

a) Find the maximal elements.
b) Find the minimal elements.
c) Is there a greatest element?
d) Is there a least element?
e) Find all upper bounds of {3*,* 5}*.*
f) Find the least upper bound of {3*,* 5}, if it exists.
g) Find all lower bounds of {15*,* 45}*.*
h**)** Find the greatest lower bound of {15*,* 45}, if it exists.