ADMISSION NUMBER

School of Basic Sciences

Bachelor of Science Honours in Mathematics
Semester End Examination - Jun 2024

Duration : 180 Minutes
Max Marks : 100

Sem II - C1UC204T - Linear Algebra

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1) Check whether $T:\mathbb{R}^2 \to \mathbb{R}^2\ s.t.\ T(x_1,x_2) = (x_2,x_1)$ is a Linear Transformation or not. K1 (3)

2) Show that the minimal polynomial of a Linear operator T divides its characteristic polynomial. K2 (4)

3) Let v and w be eigen vectors of T corresponding to two distinct eigen values of T. Show that v+w cannot be an eigen vector of T. K2 (6)

4) Prove that the determinants of two similar matrices are the same. K3 (6)

5) Prove that the matrix given below is similar to itself: K3 (6)
$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

6) If A = $\begin{bmatrix} -2 & 6 & -6 \\ 0 & 3 & -5 \\ 0 & -3 & 1 \end{bmatrix}$ then find the eigen value of $A^4 + A^2 + 5A$ K3 (9)

7) Find the Geometric and Algebraic multiplicity of the following matrix: K3 (9)
$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

8) Verify the Cayley-Hamilton Theorem for the matrix $A = \begin{pmatrix} 3 & 4 \\ 1 & -2 \end{pmatrix}$. Hence find the A^{-1}. K4 (8)

9) Find $a, b \in \mathbb{R}$ such that $\Sigma_{i=1}^{4}|a + bx_i - y_i|^2$ is minimum K4 (12)

10) State and prove Cauchy-Schwarz inequality K5 (10)

11) State and prove Sylvester's Law. K5 (15)

OR

Let $T:V \to W$ be a L.T. where V and W are two F.D.V.S. with same dimension. Then the following are equivalent K5 (15)

1. T is invertible.
2. T is non singular (i.e. T is 1-1)
3. T is onto (i.e Range T =W)
4. If $\{v_1, v_2, \ldots, v_n\}$ is a basis of V then $\{T(v_1), T(v_2), \ldots, T(v_n)\}$ is a basis of W.

12) A L.T. $T:V \to V$ is one-one if and only if T is onto. K6 (12)

OR

Let V and W be two vector spaces over F. Let $\{v_1, v_2, \ldots, v_n\}$ be a basis of V and $\{w_1, w_2, \ldots, w_n\}$ be any vectors in W then there exists a unique L.T. $T:V \to W \ s.t. T(v_i) = w_i \quad i = 1,2, \ldots, n.$ K6 (12)