GALGOTIAS UNIVERSITY

ADMISSION NUMBER

School of Basic Sciences

Bachelor of Science Honours in Mathematics
Semester End Examination - Jun 2024

Duration : 180 Minutes
Max Marks : 100

Sem IV - C1UC403T - Real analysis-II

General Instructions
Answer to the specific question asked
Draw neat, labelled diagrams wherever necessary
Approved data hand books are allowed subject to verification by the Invigilator

1)	Find the limit $\lim_{n\to\infty}\left(\frac{n^n}{n!}\right)^{1/n}$ as an integral.	K1(3)
2)	Estimate the value of radius of convergence of a power series $\sum_{n=0}^{\infty}(1+\frac{1}{n})^{n^2(-1)^n}x^n$	K2(4)
3)	Show that a function $f : [a,b] \to \mathbb{R}$ be bounded on $[a,b]$, and let f be monotone on $[a,b]$, then f is integrable on [a, b].	K2(6)
4)	Let f be defined on $[0,1]$ by $f(x)=\begin{cases}\frac{1}{2^n}; & \frac{1}{2^{n+1}} < x \le \frac{1}{2^n};\ n=0,1,2,\dots \\ 0; & x=0\end{cases}$ Show that f is Riemann integrable and also find the value of integral on [0, 1].	K3(6)
5)	Using the concept of Riemann integral find the upper and lower integral of the function f(x)=$\begin{cases} sinx, & x\epsilon\mathbb{Q} \\ x, & x\epsilon\mathbb{Q}^c\end{cases}$ on [0, $\frac{\pi}{2}$].	K3(6)
6)	Verify the first mean value theorem of Riemann integrable function.	K3(9)
7)	Verify the following series are uniformly convergent by applying Weierstrass M-test 1. $\sum_{n=1}^{\infty}\frac{\cos nx}{n^4}, x\epsilon(-\infty,\infty)$ 2. $\sum_{n=1}^{\infty}\frac{\sin nx}{n^2},\ x\epsilon(-\infty,\infty)$ 3. $\sum_{n=1}^{\infty}\frac{1}{x^2+n^2},$ $x\epsilon$-∞, ∞.$x\epsilon(-\infty,\infty)$.	K3(9)
8)	Categorize the following power series with finite radius of convergence and those with infinite radius of convergence,	K4(8)

providing reasons for the categorization: (i) $\sum_{n=0}^{\infty}(2+(-1)^n)^n x^n$,
(ii) $\sum_{n=0}^{\infty}\frac{2^n}{n!}x^n$, (iii) $\sum_{n=0}^{\infty} n!\,x^n$, and (iv) $\sum_{n=0}^{\infty}\frac{(x-1)^{2n}}{2^n n^3}$.

9) Examine whether the following sequence of function $\{f_n\}$ are uniformly convergent or not: K4(12)

1. $f_n(x) = \frac{x}{1+nx^2}$, $x\epsilon[0,1], n\epsilon\mathbb{N}$.
2. $f_n(x) = \frac{x}{x+n}$, $x\epsilon[0,\infty), n\epsilon\mathbb{N}$.
3. $f_n(x) = \frac{n^2 x}{1+n^4x^2}$, $x\epsilon[0,1], n\epsilon\mathbb{N}$.

10) Show that $\int_0^{\infty}\left(\frac{1}{1+x}-e^{-x}\right)\frac{dx}{x}$ is convergent. K5(10)

11) Let $<f_n>$ is a sequence of continuous functions on an interval $[a,b]$ and if $f_n \to f$ uniformly on $[a,b]$, then prove that f is continuous on $[a,b]$. K5(15)

OR

Let f is bounded and integrable on $[a,b]$ and M, m are the bounds of f on $[a,b]$. Then prove that $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$, if $b \ge a$ and $m(b-a) \ge \int_a^b f(x)dx \ge M(b-a)$, if $b \le a$. K5(15)

12) Justify that if f is Riemann integrable then f^2 is also Riemann integrable and provide an example which contradict the converse of this statement. K6(12)

OR

Justify the statement, every closed subset of a complete metric space is complete. Further provide an example. K6(12)