# HYDRAULICS IN CIVIL AND ENVIRONMENTAL ENGINEERING
Fourth edition

# SOLUTIONS MANUAL

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This solutions manual is made available free of charge. Details of the accompanying textbook *Hydraulics in Civil and Environmental Engineering* are on the website of the publisher www.sponpress.com and can be ordered from Book.orders@tandf.co.uk
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## Preface

This manual has been prepared for use in conjunction with the textbook *Hydraulics in Civil and Environmental Engineering* (4th edition). The Problems for solution in the book cover the material found in Chapters 1-11.

The *Solutions Manual is* particularly intended for use by course tutors. It provides detailed methods of solution for all of the problems in the 4th edition, so that they can be integrated into the tutorial scheme for a hydraulics lecture programme.

# Chapter 1

***Problem 1***

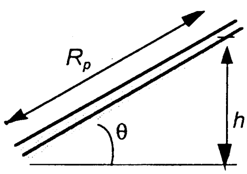
Apply hydrostatic equation for head of oil: $h = \frac{p}{\rho g} = \frac{200000}{850 \times 9.81} = 24 \text{ m}$

Apply hydrostatic equation for head of water: $h = \frac{p}{\rho g} = \frac{200000}{1000 \times 9.81} = 20.4 \text{ m}$

***Problem 2***

From the hydrostatic equation $p = \rho g h$

$$400 = 1800 \times 9.81 \times h$$




therefore:

$h = 0.0226\text{m}$

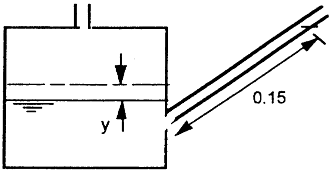
$$\frac{h}{R_p} = \frac{0.0226}{0.15} = 0.151 = sin\theta$$

$$\theta = 8.69^\circ$$

***Problem 3***

Volume of fluid in manometer tube $= A_{TUBE} \times 0.15$

Volume of fluid in reservoir tank $= A_{TANK} \times y$



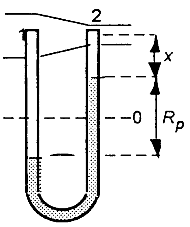

$A_{TUBE} \times 0.15 = A_{TANK} \times y$

$$y = \frac{A_{TUBE} \times 0.15}{A_{TANK}} = \frac{0.15}{40} = 0.00375 = 3.75 \text{ mm}$$

Error in $R_p$ $= \frac{y}{\sin\theta} = \frac{3.75}{0.151} = 24.8 \text{ mm}$

Percentage error $= \frac{24.8}{151} \times 100 = 16.5\%$

***Problem 4***




On left side of manometer

$38000 + 9.81 \times 1000\,(x + R_p)$

On right side of manometer

$-\,50000 + 9.81 \times 1000\,(x) + 9.81 \times 13600\,(R_p)$

Therefore:

$38000 - (-\,5000) = 9.81\,R_p\,(13600 - 1000)$

Therefore: $R_p = 0.712$ m      712 mm

***Problem 5***

$101500 = 9.81 \times 13600 \times H_m$      $H_m = 760.7$ mm

***Problem 6***

Gauge pressure = 103000 - 101500 = 1500 N/m²

$$h = \frac{p}{\rho g} = \frac{1500}{1000 \times 9.81} = 0.153 \text{ m} \qquad = 153 \text{ mm Water}$$

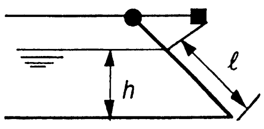
At highest point: Atmospheric pressure $= 101500 - 1.21 \times 9.81 \times 150$
$= 99719$ N/m²

Gas pressure $= 103000 - 0.56 \times 9.81 \times 150$
$= 102176$ N/m²

Gauge pressure $= 102176 - 99719$
$= 2457$ N/m²

$$h = \frac{p}{\rho g} = \frac{2457}{1000 \times 9.81} = 0.250 \text{m} \qquad = 250 \text{ mm Water}$$

***Problem 7***




Moment holding gate shut
= 100 x 9.81 x 0.5 +250 x 9.81 x 0.5
= 1717 N-m

$$\ell = \frac{h}{\sin 45^\circ}$$

Force on immersed surface = $\rho g A\bar{y} = 1000 \times 9.81 \times \left(1 \times \frac{h}{\sin 45^\circ}\right) \times \frac{h}{2}$

$= 6937\ h^2$

Centre of pressure or "point of action" is at $\frac{I}{A\bar{\ell}}$ below water surface

i.e

$$\bar{\ell} = \frac{1 \times \left(h/\sin 45\right)^3 \times 2}{12 \times \left[\left(h/\sin 45\right) \times 1\right] \times h/\sin 45} = 0.2357$$

Distance from hinge to point of action = $\left(\frac{1-h}{\sin 45}\right) + 0.2357h = 1.414 - 1.179h$

Moment of force = $(1.414 - 1.179h)\ 6937h^2$ = 1717 N-m

Therefore $h$ = 0.58 m when gate is on point of opening

***Problem 8***

There are two ways of tackling the problem. One (as below) is to calculate vertical and horizontal components separately. The other is to apply 1st and 2nd moments directly to the two parts (vertical and sloping) of the upstream dam face.

Forces on dam:

**Horizontal**

$F_H$ = force on vertical projected area

$= \rho g A\bar{y} = 1000 \times 9.81 \times 25 \times 200 \times 12.5$

$= 613.13 \times 10^6 N$

$F_H$ acts at centre of pressure, which is ($^2/_3$ x 25) m i.e 16.67 m below water level.

**Vertical**

Volume of water above dam foot, $v = \left[\left(\frac{20 \times 5}{2}\right) + (5 \times 5)\right] \times 20$

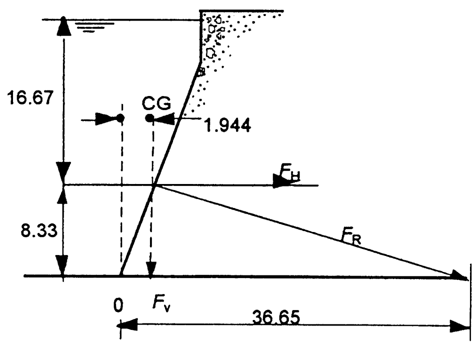
$= 15000\ m^3$

$F_v$ = weight = $\rho g v$ = 1000 × 9.81 × 15000 = 147.15 x $10^6$ N

Force acts through centre of gravity. To find centre of gravity take moments:

$$\left[\left(\frac{20 \times 5}{2}\right) \times \frac{5}{3} + (5 \times 5)\right] \times \frac{5}{2} = 148.53\ m^3$$

$$\bar{x} = \frac{145.83}{\left[\left(\frac{20\times 5}{2}\right)+\left(5\times 5\right)\right]} = 1.944 \text{ to right of vertical through } O$$




Resultant force on dam $F_R$

$$F_R = \left(613.13^2 + 147.15^2\right)^{\frac{1}{2}}$$

$$= 630.5 \text{ MN}$$

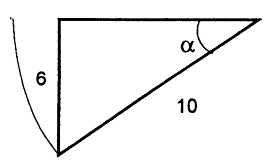
Angle of $F_R$ to horizontal

$$= \tan^{-1}\frac{147.15}{613.13} = 13.5^\circ$$

$$\text{Distance} = 8.33 \times \frac{613.13}{147.15} = 34.7$$

34.7 + 1.944 = 36.65 m

***Problem 9***




$$\alpha = \sin^{-1}\frac{6}{10} = 36.9^\circ$$

$$\text{Area of sector} = \frac{36.9}{360} \times \pi \times 10^2 = 32.2 \text{ m}^2$$

$$\text{Area of triangle} = \frac{1}{2} \times 6 \times 10 = 30 \text{ m}^2$$

Therefore:

$$\text{Area of segment} = 32.2 - 30 = 2.2 \text{ m}^2$$

Volume of water displaced = 2.2 × 10 = 22 m³

So vertical (weight) force = 22 × 9.81 × 1000 = 215820 N

Horizontal force = Force on vertical projected area

$$= \rho g A\bar{y} = 1000 \times 9.81 \times (6 \times 10) \times 3$$

$$= 1765800 \; N$$

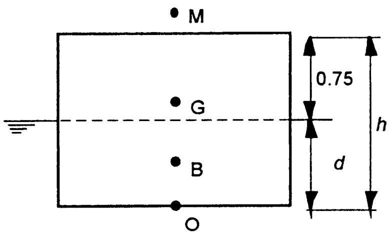
Resultant force $= \sqrt{215820^2 + 1765800^2}$

$$= 1778940 \text{ N}$$

Therefore load on each bearing $= \frac{1778940}{2} = 889470 \text{ N} = 0.889 \text{ MN}$

Force is perpendicular to radial surface and so must pass through bearings.

***Problem 10***




Displacement = 300000/1000 = 300 m³

Draft, $d = \dfrac{\text{Vol}}{bL} = \dfrac{300}{6L}$

therefore centre of buoyancy

$\text{CB} = \dfrac{300}{12L}$ above O

$\overline{\text{MB}} = \dfrac{I}{V} = \dfrac{L \times 6^3}{12 \times 300} = 0.06L$

$h = d + 0.75 = \dfrac{300}{6L + 0.75}$ therefore $\overline{\text{OG}} = \dfrac{h}{2} + 0.3$

$\overline{\text{MG}}$(metacentric height) = 1.5 = $\overline{\text{MB}} + \overline{\text{BO}} - \overline{\text{OG}}$

Therefore

$$1.5 = 0.06L + \frac{300}{12L} - \left(\frac{300}{12L} + 0.375 + 0.3\right)$$

$$= 0.06L - 0.675$$

$$L = 36.25 \text{ m}$$

$$h = \frac{300}{6 \times 36.25} + 0.75 = 2.13 \text{ m}$$

# Chapter 2

### *Problem 1*

Apply energy equation entry to (1) to (2)

$$\frac{p_1}{\rho g}+\frac{V_1}{2g}=\frac{p_0}{\rho g}+\frac{V_2^2}{2g}=\frac{p_{entry}}{\rho g}+\frac{V_{entry}^2}{2g}+4$$

$$\frac{p_{entry}}{\rho g}=2m \quad V_{entry}=0 \quad \frac{p_0}{\rho g}=0$$

$$\therefore \qquad \frac{V_2^2}{2g}=2+4$$

$$V_2=\sqrt{12g}=\underline{10.85\ m/s}$$

To find $p_1$, first find $V_1$

$$V_1A_1=V_2A_2$$

$$V_1=V_2\frac{A_2}{A}=10.85\times\frac{300^2}{200^2}$$

$$V_1=24.41\ m/s$$

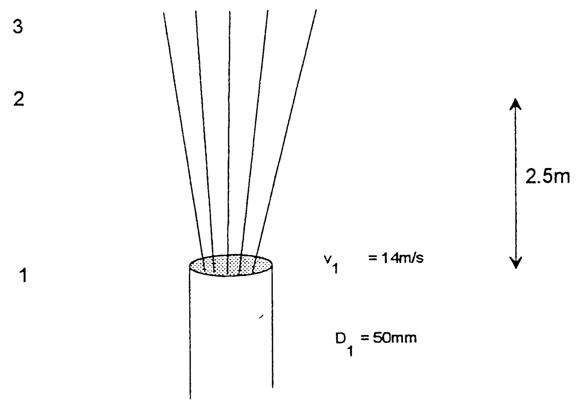
hence $$\frac{p_1}{\rho g}+\frac{24.41^2}{2g}=6$$

$$\frac{p_1}{\rho g}=6-30.37$$

$$p_1=-239\ kN/m^2 \quad !$$

The solution is physically impossible as $p_1$ cannot be less than negative atmospheric pressure. Either frictional losses would reduce the velocities and/or cavitation would occur.

## *Problem 2*




Assume atmospheric pressure at all heights and apply the energy equation.

$$\frac{V_1^2}{2g} + 0 = \frac{V_2^2}{2g} + 2.5 = Z_3$$

$$\therefore \quad \frac{V_2^2}{2g} = \frac{14^2}{2g} - 2.5$$

$$V_2 = 12.12 \ m/s$$

Apply continuity

$$V_1 A_1 = V_2 A_2$$

$$A_2 = \frac{V_1}{V_2} A_1$$

$$\text{or } D_2^2 = \frac{V_1}{V_2} D_1^2 = \frac{14}{12.12} \times 0.05^2 = 0.00289$$

$$D_2 = 53.7m$$

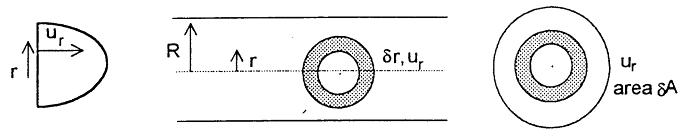
$$Z_3 = \frac{V_1^2}{2g} = 9.99 \ m$$

### *Problem 3*

Given $u_r = K\left(R^2 - r^2\right)$

and $\alpha = \dfrac{1}{\bar{V}^3 A}\int u_r^3 \, dA$

then




$$A = \pi R^2 \qquad \delta A = 2\pi r \, \delta r$$

$$\therefore \quad \int u^3 dA = 2\pi K^3 \int_0^R r\left(R^2 - r^2\right)^3 dr$$

$$= 2\pi K^3 \int r\left(R^4 + r^4 - 2R^2 r^2\right)\left(R^2 - r^2\right) dr$$

$$= 2\pi K^3 \int r\left(R^6 + R^2 r^4 - 2R^4 r^2 - R^4 r^2 - r^6 + 2R^2 r^4\right) dr$$

$$= 2\pi K^3 \int_0^R \left(r R^6 + 3R^2 r^5 - 3R^4 r^3 - r^7\right) dr$$

$$= 2\pi K^3 \left[\frac{r^2 R^6}{2} + \frac{3R^2 r^6}{6} - \frac{3R^4 R^4}{4} - \frac{r^8}{8}\right]_0^R$$

$$= 2\pi K^3 \left[\frac{R^8}{2} + \frac{R^8}{2} - \frac{3R^8}{4} - \frac{R^8}{8}\right]$$

$$= 2\pi K^3 \frac{R^8}{8}$$

$$= \frac{\pi K^3 R^8}{4}$$

$$\bar{V} = \frac{Q}{A} = \frac{\int_0^R u_r 2\pi r \, dr}{\pi R^2}$$

$$= \frac{2}{R^2}\int_0^R K\left(R^2 - r^2\right) r \, dr$$

$$= \frac{2K}{R^2}\int_0^R R^2 r - r^3 \, dr$$

$$= \frac{2K}{R^2}\left[\frac{R^2r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= \frac{2K}{R^2}\left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$\bar{V} = \frac{KR^2}{2}$$

$$\therefore \quad \bar{V}^3 A = \frac{K^3R^6}{8}\pi R^2 = \frac{\pi R^8 K^3}{8}$$

$$\therefore \quad \alpha = \frac{\pi K^3 R^8}{4} . \frac{8}{\pi R^8} K^3$$

$$\alpha = 2$$

---

$$\beta = \frac{1}{\bar{V}^2}A \int u^2 dA$$

$$\int u^2 dA = 2\pi K \int_0^R r\left(R^2 - r^2\right)^2 dr$$

$$= 2\pi K^2 \int_0^R \left(rR^4 + r^5 - 2R^2r^3\right) dr$$

$$= 2\pi K^2\left[\frac{r^2R^4}{2} + \frac{r^6}{6} - \frac{2R^2}{4}r^4\right]_0^R$$

$$= 2\pi K^2\left[\frac{R^6}{2} + \frac{R^6}{6} - \frac{R^6}{2}\right]$$

$$= \frac{\pi K^2}{3}R^6$$

$$\bar{V}^2 A = \frac{K^2}{4}R^4 . \pi R^2 = \frac{K^2\pi}{4}R^6$$

$$\therefore \beta = \frac{\pi K^2}{3}R^6 . \frac{4}{K^2 \pi R^6}$$

$$= \frac{4}{3}$$

## Problem 4

**Momentum forces**

$$F_{mx} = \rho Q\left(V_2 \cos 45 - V_1\right) = +4.36 kN$$

$$F_{my} = \rho Q\left(-V_2 \sin 45 - 0\right) = -5.64 kN$$

**Pressure Forces**

$$F_{px} = p_1 A_1 - p_2 A_2 \cos 45 = 176.9 kN$$

$$F_{py} = 0 + p_2 A_2 \sin 45 = +19.5 kN$$

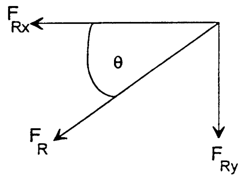
**Reaction Forces**

$\left(F_m = \Sigma F_p + F_R\right)$ $F_{Rx} = -F_{px} + F_{mx} = -176.9 + 4.36 = -172.54$

$$F_{Ry} = -F_{py} + F_{my} = -19.5 - 5.64 = -25.14$$

$$F_R = \sqrt{172.5^2 + 25.14^2} = 174.4 kN$$

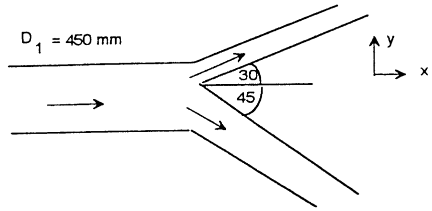
$$\theta = \tan^{-1} \frac{25.14}{172.54} = 8.3°$$



## *Problem 5*

$P = -9 kN/m^2$
$D_2 = 150 mm$
$Q_2 = 0.23 m^3/s$
$V_2 = 13.0 m/s$
$A_2 = 0.018 m^2$

$P_1 = 69 kN/m^2$
$Q_1 = 0.57 m^3/s$
$A_1 = 0.16 m^2$
$V_1 = 3.58 m/s$




$D_3 = 300 mm$
$Q_3 = 0.34 m^3/s$
$V_3 = 4.81 m/s$
$A_3 = 0.071 m^2$
$P_3 63.8 kN/m^2$

Find $V_2$, $V_3$ from continuity
Find $P_2$, $P_3$ from Bernoulli

**Momentum**

x direction $\quad Fmx = \rho Q_2 V_2 \cos 30 + \rho Q_3 V_3 \cos 45 - \rho Q_1 V_1$

$= \rho(2.589 + 1.156 - 2.04)$

$= 1.705$ kN

y direction $\quad Fmy = \rho Q_2 V_2 \sin 30 - \rho Q_3 V_3 \sin 45 - 0$

$= \rho(1.495 - 1.156)$

$= 0.34$ kN

**Pressure**

x direction $\quad Fpx = P_1 A_1 - P_2 A_2 \cos 30 - P_3 A_3 \cos 45$

= 11.04 -0.14-3.203

= + 7.98 kN

y direction $\quad Fpy = -P_2 A_2 \sin 30 + P_3 A_3 \sin 45$

= -(-0.081)+ 3.203

= +3.28 kN

*Problem 5 (continued)*

Total force equations $\left(F_p + F_R = F_m\right)$

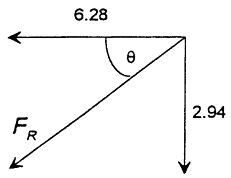
x direction $F_{Rx} = 1.705 - 7.98$

$= -6.28\ kN$

y direction $F_{Ry} = 0.34 - 3.28$

$= -2.94\ kN$

Resultant Force



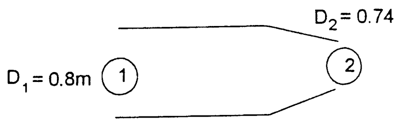

$$F_R = \sqrt{6.28^2 + 2.94^2}$$

$$F_R = 6.93\ kN$$

$$\theta = \tan^{-1}\left(\frac{2.94}{6.28}\right)$$

$$\theta = 25.1$$

## *Problem 6*




$\frac{\Delta p}{\rho g}$ =30 mm water

$\therefore \ \Delta p = 249.3 N/m^2$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\therefore \quad \frac{V_2^2 - V_1^2}{2g} = \left(\frac{p_1 - p_2}{\rho g}\right)$$

$$\frac{V_2^2 - V_1^2}{2} = \frac{\Delta p}{\rho} = \frac{294.3}{1.3} = 226.38$$

also $V_1 A_1 = V_2 A_2$

$$\therefore \quad V_1 = \frac{V_2 A_2}{A_1} = \frac{V_2 \times 0.74^2}{0.8^2} = 0.8556 V_2$$

$$\therefore \quad \frac{V_2^2 - 0.8556^2 V_2^2}{2} = 226.38$$

$$0.134 V_2^2 = 226.38$$

$$V_2 = 41.1 \ m/s$$

Mass flow rate = $\rho Q$

$$Q = V_2 A_2 = \frac{41.1 \times 0.74^2 \times \pi}{4} = 17.68 m^3/s$$

$$\rho Q = 1.3 \times 17.68 = 23.0 \ kg/s$$

***Problem 7***

$$Q = C_d \frac{\pi D_1^2}{4}\left(\frac{1}{\sqrt{m^4 - 1}}\right)\sqrt{2gh^*}$$

$$h^* = R_p\left(\frac{\rho g}{\rho} - 1\right) = 0.015\left(\frac{13.6}{1} - 1\right) = 0.189m$$

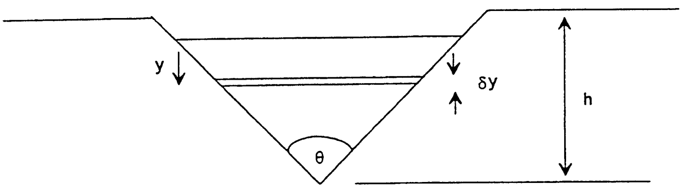
$$m = \frac{D_1}{D_2} = \frac{300}{200} = 1.5$$

$$\therefore \qquad Q = 0.98\frac{\pi \times 0.3^2}{4}\left(\frac{1}{\sqrt{1.5^4 - 1}}\right)\sqrt{2g \times 0.189}$$

$$= \qquad 0.0662\ m^3 s$$

$$= \qquad 66.2\ l/s$$

*Problem 8*




For an elemental strip at depth y

$$\text{velocity } u = \sqrt{2gy}$$

$$\text{area} = 2(h-y)\tan\left(\frac{\theta}{2}\right)\delta y$$

$$\delta Q = \sqrt{2gy}\,2(h-y)\tan\frac{\theta}{2}$$

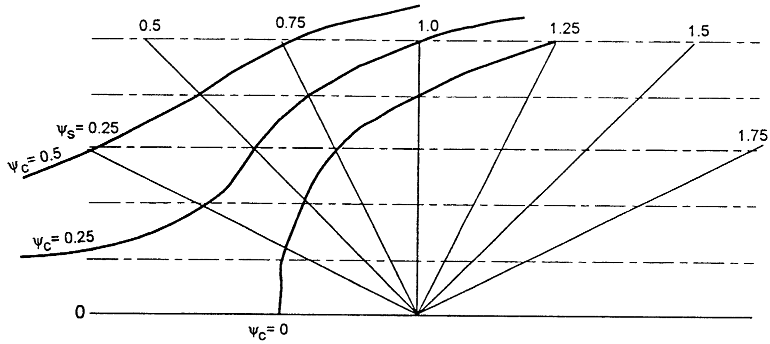
$$\therefore Q = 2\sqrt{2g}\,\tan\frac{\theta}{2}\int_{y=0}^{y=h}(h-y)y^{\frac{1}{2}}dy$$

$$= 2\sqrt{2g}\,\tan\frac{\theta}{2}\int_{y=0}^{y=h}\left(hy^{\frac{1}{2}}-y^{\frac{3}{2}}\right)dy$$

$$= 2\sqrt{2g}\,\tan\frac{\theta}{2}\left[\frac{2}{3}hy^{\frac{3}{2}}-\frac{2}{5}y^{\frac{5}{2}}\right]_{y=0}^{y=h}$$

$$Q_{ideal} = \frac{8}{15}\sqrt{2g}\,\tan\frac{\theta}{2}h^{\frac{5}{2}}$$

***Problem 9***




Velocity → 0 at stagnation point where radial velocity $V_r$ is equal and opposite to velocity of linear flow i.e. $V_r = 5$ m/s.

i.e. $V_r r\theta = \dfrac{Q\theta}{2\pi} = \psi$

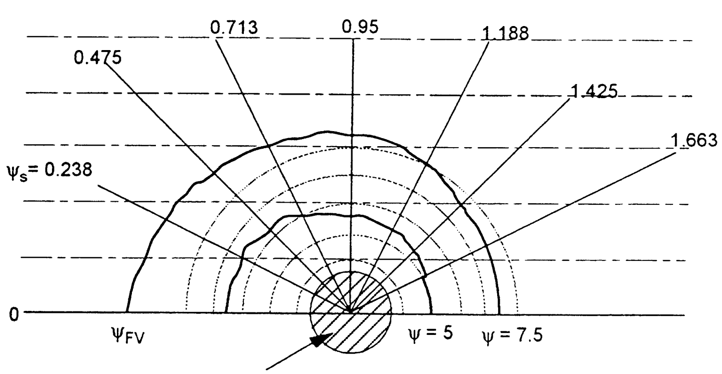
Therefore

$$r = \frac{Q}{2\pi V_r} \quad \text{for } \theta = 0$$

Therefore

$$r = \frac{4}{2\pi \times 5} \qquad = 0.127 \text{ m} \qquad = 127 \text{ mm}$$

***Problem 10***



For forced vortex $V = \omega r \quad = 100 \times 0.15 \quad = 15 \text{ m/s}$

For source $V_r = \frac{3.8}{\pi} \times 0.30 \quad = 4.03 \text{ m/s}$

Resultant velocity $= \sqrt{15^2 + 4.03^2} \quad = 15.53 \text{ m/s}$

$$\frac{dH}{dr} = \frac{2\omega^2 r}{g} \quad \text{therefore } H = \frac{\omega^2 r^2}{g} \quad = \frac{100^2 \times 0.15^2}{9.81} = 22.94 \text{ m}$$

At 0.15 m radius the free vortex begins, so

$$V = \frac{K}{r} \qquad 15 = \frac{K}{0.15} \qquad \text{therefore} \qquad K = 2.25$$

At 250 mm radius $V = \frac{2.25}{0.25} \quad = 9 \text{ m/s}$

$$V_{source} = V_r = \frac{3.8}{2\pi \times 0.25} \qquad = 2.434 \text{ m/s}$$

Resultant velocity $= \sqrt{9^2 + 2.424^2} \quad = 9.32 \text{ m/s}$

$$H = 22.94 \quad = \frac{p}{\rho g} + \frac{V^2}{2g} \quad = \frac{p}{\rho g} + \frac{9.32^2}{2 \times 9.81}$$

therefore $p = 181.6 \text{ kN/m}^2$

# Chapter 3

***Problem 1***

For water $\quad \mathrm{Re} = \dfrac{1000 \times 3 \times 0.5}{1.14 \times 10^{-3}} \quad = 1315789 \quad$ Turbulent flow

For air $\quad \mathrm{Re} = \dfrac{1.24 \times 0.1 \times 0.025}{1.7 \times 10^{-5}} \quad = 182 \quad$ Laminar flow

***Problem 2***

$$\theta = \int_0^\delta \left[\frac{2y}{\delta} - \left(\frac{y}{\delta}\right)^2\right]\left[1 - \left(\frac{2y}{\delta} - \left(\frac{y}{\delta}\right)^2\right)\right] dy \quad = \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right) dy \qquad = \frac{2\delta}{2} - \frac{5\delta}{3} + \frac{4\delta}{4} - \frac{1\delta}{5}$$

$$= 0.133\delta$$

$$\tau_0 = \rho u_\infty^2 \frac{d\theta}{dx} \qquad = \mu \frac{du}{dy} \qquad = \mu \frac{d}{dy} \times u_\infty \left(\frac{2y}{\delta} - \left(\frac{y}{\delta}\right)^2\right)$$

$$= 1000 \times 3^2 \times 0.133 \frac{\mathrm{d}\delta}{\mathrm{d}x} \qquad = 0.001 \frac{\mathrm{d}}{\mathrm{d}y}\left(3 \times \left[\frac{2y}{\delta} - \left(\frac{y}{\delta}\right)^2\right]\right)$$

$$1197 \frac{\mathrm{d}\delta}{\mathrm{d}x} \qquad = \frac{0.006}{\delta}$$

$$\mathrm{d}x = \frac{1197}{0.006}\delta \mathrm{d}\delta \qquad x = 99750\delta^2 + C \qquad x = 0,\ \delta = 0 \text{ therefore } C = 0$$

Therefore

$$\delta = \sqrt{\frac{x}{9970}} \qquad = 0.0071 \ = 7.1 \text{ mm}$$

***Problem 3***

Definitions as in text

$$\frac{u}{u_\infty}=\left(\frac{y}{\delta}\right)^{\frac{1}{7}} \qquad \theta=\int_0^\delta \frac{u}{u_\infty}\left(1-\frac{u}{u_\infty}\right)\mathrm{d}y \;=\int_0^\delta\left(\frac{y}{\delta}\right)^{\frac{1}{7}}\left(1-\left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right)\mathrm{d}y=\frac{7}{72}\delta$$

$$\tau_0=\rho u_\infty^2 0.0225\left(\frac{u}{\rho u_\infty \delta}\right)^{\frac{1}{4}} \qquad =\rho u_\infty^2\frac{\mathrm{d}\theta}{\mathrm{d}x}= \qquad \rho u_\infty^2\frac{7}{72}\frac{\mathrm{d}\delta}{\mathrm{d}x}$$

Therefore

$$0.0225\left(\frac{0.01565}{\delta^{\frac{1}{4}}}\right) \quad =\frac{7}{72}\frac{\mathrm{d}\delta}{\mathrm{d}x}$$

$$3.622\times10^{-3}\mathrm{d}x \qquad =\delta^{1/4}d\delta$$

$$3.622\times10^{-3}x \qquad =\frac{4}{5}\delta^{5/4}$$

$$4.528\times10^{-3}x^{4/5} \qquad =\delta$$

$\delta F=\tau_0 b\ \mathrm{d}x$ = Force on small element of surface area $bdx$

Substituting for $\delta$

$$\tau_0 \quad =\rho u_\infty^2 0.0225\left(\frac{\mu}{\rho u_\infty \delta}\right)^{\frac{1}{4}} \qquad =1000\times20^2\times0.0225\left(\frac{1.2\times10^{-3}}{1000\times20\delta}\right)^{\frac{1}{4}}$$

$$=140.85\times\left(\frac{1}{4.528\times10^{-3}x^{4/5}}\right)^{\frac{1}{4}}=\frac{543}{x^{1/5}}$$

Therefore

$$\delta F=\frac{543}{x^{1/5}}\times0.5\times\delta x$$

therefore

$$F \quad =271.5\int_0^1\frac{1}{x^{1/5}}\mathrm{d}x=\frac{271.5}{8}x^{0.8} \quad =340\ \mathrm{N}$$

***Problem 4***

Chimney projected area $= 30 \times 1 = 30 \text{ m}^2$

$$V = 80 \text{ km/h} = \frac{80}{60 \times 60} = 22.2 \text{ m/s}$$

$$\text{Force} = C_D \times \tfrac{1}{2}\rho A V^2 = 1 \times \tfrac{1}{2} \times 1.24 \times 30 \times 22.2^2$$

$$= 9185 \text{ N}$$

$$f = 0.5 \text{ Hz} \qquad St = \frac{fd}{u_\infty} = \frac{0.5 \times 1}{u_\infty} = 0.198\left(1 - \frac{19.7}{\text{Re}}\right)$$

(assuming Re in range 250 - 2 × 10$^{5}$)

$$\frac{0.5 \times 1}{u_\infty} = 0.198\left(1 - \frac{19.7 \times 1.7 \times 10^{-5}}{1.24 \times u_\infty \times 1}\right)$$

$$u_\infty = 2.525 \text{ m/s} = 9.09 \text{ km/h}$$

# Chapter 4

## *Problem 1*

$$\text{Re} = \frac{\rho D V}{\mu} \leq 2000 \text{ for laminar flow}$$

for $D$ = 12mm  Re = 2000

$$\therefore \quad V_{max} = \frac{2000 \times 1.14 \times 10^{-3}}{1000 \times 0.012}$$

$$= 0.19 \text{ m/s}$$

$$\therefore \quad Q = VA = 0.19 \times \frac{\pi \times 0.012^2}{4}$$

$$Q = 0.0215 \; l/\text{s}$$

---

$$h_f = \frac{32 \mu L V}{\rho g D^2}$$

$$= \frac{32 \times 1.14 \times 10^{-3} \times 1 \times 0.19}{9.81 \times 0.012^2}$$

$$h_f = 4.9 \text{ mm/m}$$

---

$$V\text{max} = 2\bar{V} = 0.38 m/s$$

---

$$\lambda = \frac{64}{\text{Re}} = 0.032$$

---

## *Problem 2*

Essay based on contents of chapter 4.

## *Problem 3*

Computer programme should solve the Colebrook-White equation for $\lambda$ for a range of Re and $k_s$.

***Problem 4*** (Note: example 4.5 is very similar)

Let H = $h_f + h_L$

where $h_L = \Sigma k_\iota V^2/2g$

Iterative Solution

1 assume $h_L = 0$

2 find V from Colebrook-White assuming $h_f = H - h_L$

3 calculate $h_L = \Sigma k_\iota V^2/2g$

4 if $h_f + h_L \approx H$ solution is correct

5 if $h_f + h_L \not\approx H$ then repeat from 2

a) $H_L = 0$

$\therefore h_f = H = 150$

$\therefore S_f = \frac{150}{10,000}$

$$V = -2\sqrt{2gDS_f}\log\left(\frac{k_s}{3.7D} + \frac{2.51\nu}{D\sqrt{2gDS_f}}\right)$$

$$= -2\sqrt{2 \times 9.8 \times 0.5 \times 0.015}\log\left(\frac{0.03 \times 10^{-3}}{3.7 \times 0.5} \times \frac{2.51 \times 1.14 \times 10^{-6}}{0.5\sqrt{2 \times 9.8 \times 0.5 \times 0.015}}\right)$$

$V = 3.46\ m/s$

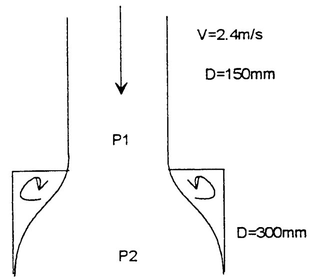
$$Q = \frac{V \times \pi \times 0.5^2}{4} = 0.68\ m^3/s$$

b) similar to part (a) but iterative solution

| $h_f$ | V | $h_L$ | $h_f + h_L$ | Q |
|---|---|---|---|---|
| 10 | 7.07 | 25.5 | 35.5 | 0.5 |
| 5 | 5 | 12.7 | 17.7 | 0.35 |
| 3 | 3.9 | 7.6 | 10.6 | 0.27 |
| 2.4 | 3.5 | 6.1 | 8.5 | 0.24 |
| 2.7 | 3.7 | 6.9 | 9.6 | 0.26 |
| 2.8 | 3.74 | 7.12 | 9.92 | 0.264 |

It may be noted that in this case local losses exceed friction losses and that the iterative procedure is intuitive (as given here).

## *Problem 5*




For a sudden enlargement

$$h_L = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2g}$$

$$= \left(1 - \frac{0.15^2}{0.3^2}\right)^2 \frac{2.4^2}{2g}$$

$$h_L = 0.165m$$

For a sudden contraction with $Cc = \dfrac{Ac}{A_2} = 0.62$

$$h_L = \left(1 - \frac{CcA_1}{A_1}\right)^2 \frac{V_1{}^2}{2g} = 0.11m$$

### Problem 6

a) $D = 1.2m$ $\qquad S_f = \frac{1}{100}$ $\qquad k_S = 0.6$

Using tables, charts or Colebrook-White equation.

$V = 3.75m/s$

$$Q = \frac{V \times \pi \times 1.2^2}{4} = 4.5m^3/s$$

b) for $Q \quad = 0.5m^3/s$

$$Q_p = \frac{0.5}{4.5} = 0.11$$

$$\theta = \left( \frac{0.0006}{1.2} + \frac{1}{3600 \times 1.2 \times 0.01^{\frac{1}{3}}} \right)^{-1}$$

$$\theta = 635$$

using HRS proportional depth/discharge tables

$$d/D = 0.233$$

$$\therefore d = 0.28m$$

and $\frac{V_d}{V_D} = 0.67$

$$\therefore V_d = 2.5m/s$$

There are no grounds, given in the question, upon which the pipe size might be considered unsuitable!

## *Problem 7*

$D = 0.75m \qquad S_f = \frac{1}{200} \qquad n = 0.012 \qquad Q = 0.85 m^3/s$

a) from Manning's equation

$$Q = \frac{1}{n}\frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} S_0^{\frac{1}{2}}$$

$$\therefore \quad \frac{Q_d}{Q_D} = \frac{A_d^{\frac{5}{3}}}{P_d^{\frac{2}{3}}} \cdot \frac{P_D^{\frac{2}{3}}}{A_D^{\frac{5}{3}}}$$

as $\quad \frac{A_d}{A_D} = A_P = \left(\frac{\phi - \sin\phi}{2\pi}\right)$

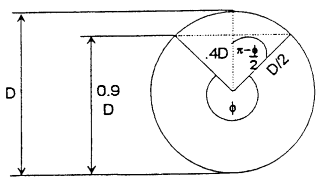
and $\quad P_P = \frac{P_d}{P_D} = \frac{\phi\frac{D}{2}}{\pi D} = \frac{\phi}{2\pi}$

then $\quad Q_P = \left(\frac{\phi - \sin\phi}{2\pi}\right)^{\frac{5}{3}}\left(\frac{2\pi}{\phi}\right)^{\frac{2}{3}}$

for $\quad \frac{d}{D} = 0.9$ and $\cos\left(\pi - \frac{\phi}{2}\right) = \frac{0.4D}{0.5D}$

$\therefore \quad \pi - \frac{\phi}{2} = 0.6435$

$\therefore \quad \phi = 4.996(\text{rad})$




$$\therefore \quad Q_P = \left(\frac{4.996 - \sin 4.996}{2\pi}\right)^{\frac{5}{3}}\left(\frac{2\pi}{4.996}\right)^{\frac{2}{3}}$$

$Q_P = 1.066$

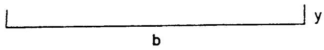
$\therefore \quad Q_d = 1.066 \times 0.85$

$Q_d = 0.906 m^3/s$

(b) Q in (a) is > $Q_{full}$ because wetted perimeter rapidly reduces and A does not for $d/D \rightarrow 1$.

# Chapter 5

## *Problem 1*

for 

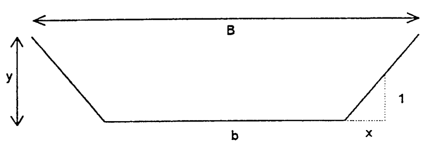

$$P \approx b \qquad\qquad A = by$$

$$\therefore \quad Q \approx \frac{1}{n}\,\frac{(by)^{\frac{5}{3}}}{b^{\frac{2}{3}}}\,S_0^{\frac{1}{2}}$$

$$\therefore \quad \frac{Q}{Q_1} \approx \left(\frac{y}{y_1}\right)^{\frac{5}{3}}$$

$$\text{or} \quad \frac{y}{y_1} \approx \left(\frac{Q}{Q_1}\right)^{\frac{3}{5}} \qquad \text{QED}$$

For a trapezoidal channel




$$A = (b + xy)y \tag{1}$$

$$P = b + 2y\sqrt{1 + x^2} \tag{2}$$

and using Manning's equation $Q = \frac{1}{n}\frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}}S_0^{\frac{1}{2}}$ (3)

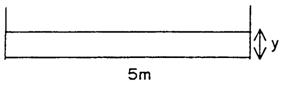
computer program uses (1) (2) and (3) and the following iterative procedure:

1. give an initial guess for $y$ $(y_1)$
2. calculate $Q_1$
3. compare with $Q$
4. if $\frac{|Q_1 - Q|}{Q} > 0.01$ then let $y_2 = y_1\left(\frac{Q}{Q_1}\right)^{\frac{3}{5}}$ and repeat from (2)

***Problem 2***

$$E_S = y + \frac{V^2}{2g} \quad \text{and} \quad V = Q/A$$

for




and $Q = 10\text{m}^3/s$

$$A = 5y \qquad \therefore \qquad V = 10/5y = 2/y$$

$$\therefore \qquad E_S = y + \frac{4}{2gy^2}$$

to draw the graph tabulate $y$ and calulate $E_S$

| $y$ | $E_s$ |
|---|---|
| (m) | (m) |
| 2.0 | 2.05 |
| 1.5 | 1.59 |
| 1.0 | 1.2 |
| $yc \to 0.74$ | 1.11 |
| 0.5 | 1.32 |
| 0.35 | 2.01 |

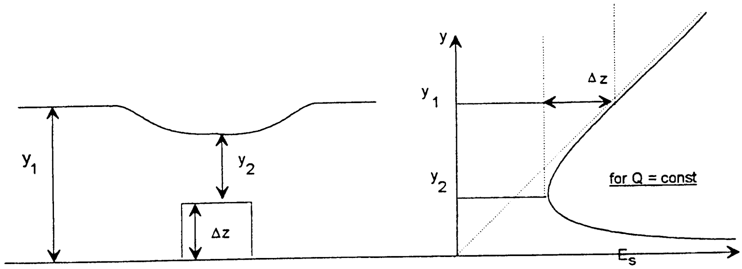
$$yc = \sqrt[3]{\frac{q^2}{g}} \qquad q = \frac{10}{2} = 2$$

$$yc = \sqrt[3]{\frac{2^2}{g}} = 0.74\text{m}$$

Figure 5.10 has been drawn from these tabulated results

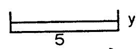
Using the graph $y_2 \approx 1.0$m (note from example 5.4 $y_2 = 0.95$m)

## *Problem 3*




for $y_2 = yc$

First find $Q$

we have 


$$\left.\begin{aligned} A &= 5y &&= 5 \\ P &= 5+2y &&= 7 \end{aligned}\right\} for \quad y = 1$$

$yn = 1\text{m}$

$$Q = \frac{1}{n}\frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}}S^{\frac{1}{2}}$$

$$\therefore Q = \frac{1}{0.02}\frac{5^{\frac{5}{3}}}{7^{\frac{2}{3}}}0.001^{\frac{1}{2}}$$

$$Q = 6.32\text{m}^3/s$$

as $\quad y_2 = yc \quad \& \quad yc = \sqrt[3]{q^2/g}$

then $\quad yc = \sqrt[3]{\left(\frac{6.32}{5}\right)^2/g}$

$$yc = 0.546\text{m} \qquad (3)$$

hence $E_2 = yc + \dfrac{V_2^2}{2g} = 0.546 + \dfrac{\left(\frac{6.32}{5\times.546}\right)}{2g}$

or $\quad yc = \frac{2}{3}Esc$

$$E_2 = 0.819\text{m} \qquad (2)$$

Neglecting energy loss from (1) to (2)

$$H = const = E_1 = E_2 + \Delta Z$$

$$\therefore \quad E_1 = 0.819 + 0.5$$

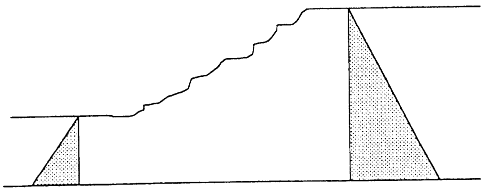
$$E_1 = 1.319m$$

Trial and error solution for $y_1$ from $E_1 = y_1 + \frac{V_1^2}{2g}$

| $y_1$ | $\frac{V_1^2}{2g}$ | $E_1$ |
|---|---|---|
| 1.2 | 0.056 | 1.256 |
| 1.3 | 0.048 | 1.348 |
| <u>1.27</u> | 0.05 | <u>1.32</u> |

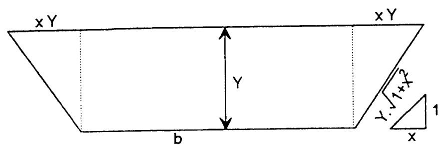
$$y_1 = 1.27m$$

## *Problem 4*

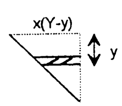


$F + M = const$ across an hydraulic jump

channel is trapezoidal



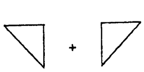

for  $F = \dfrac{\rho g Y^2 b}{2}$


for  $\delta F = \rho g y . x(Y - y)\delta y$


$$\therefore \quad F = \int_{y=0}^{Y} \delta F$$

$$\therefore \quad F = \rho g x \left[ \frac{y^2 Y}{2} - \frac{y^3}{3} \right]$$

$$= \frac{\rho g x Y^3}{6}$$

for  $F = \dfrac{\rho g x Y^3}{3}$

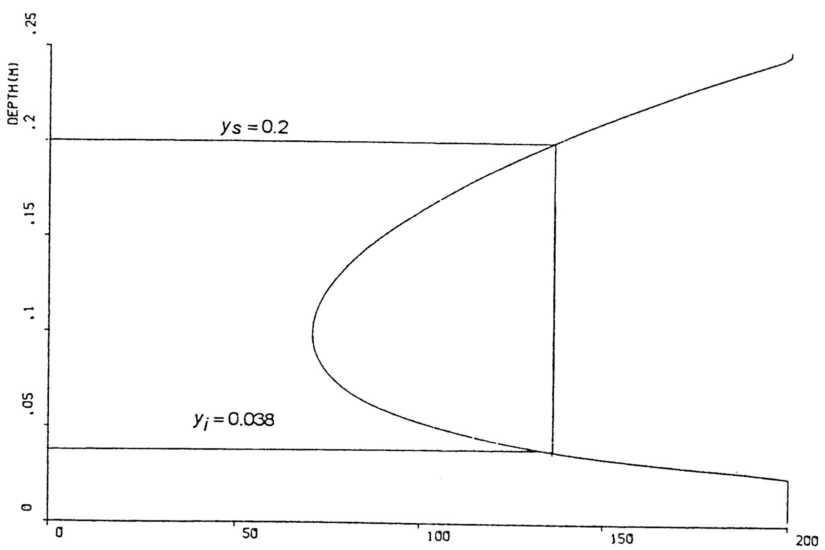

combining $F = \dfrac{\rho g Y_b^2}{2} + \dfrac{\rho g x Y^3}{3}$

$$M = \rho Q V = \rho Q.\frac{Q}{A} = \frac{\rho Q^2}{(b+xY)Y}$$

$$\therefore \quad F + M = const = \rho g\left(\frac{bY^2}{2} + \frac{xY^3}{3}\right) + \frac{\rho Q^2}{(b+xY)Y}$$

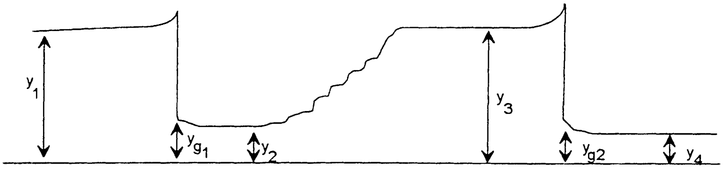
QED

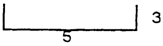
To draw the force momentum curve, first tabulate values of $F + M$ for various values of $y_1$ then plot as show below.




FORCE+MOMENTUM DIAGRAM
Q= 50 L/S B= .46 M SIDE SLOPE 1:1

## *Problem 5*




Given $Q = 15\text{m}^3/s$ & 

$yg_1 = 1\text{m}$

hence $y_2 = 0.61 \times yg_1 = 0.6$

Hydraulic jump equation $y_3 = y_2 \frac{1}{2}\left(\sqrt{1+8Fr_2^2} - 1\right)$

$$Fr_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{15/(5\times 0.61)}{\sqrt{9.81}\times 0.61} = 2.01$$

hence $y_3 = 0.61 \times 0.5\left(\sqrt{1+8\times 2.01^2} - 1\right)$

$y_3 = 1.456\text{m}$

To find $y_4$ apply Bernoulli's equation 3 to 4 (assuming no energy loss)

$$y_3 + \frac{V_3^2}{2g} = y_4 + \frac{V_4^2}{2g}$$

$y_3 = 1.456$

$V_3 = 15/(5 \times 1.456) = 2.06$

$\therefore \quad y_3 + \frac{V_3^2}{2g} = 1.672$

hence $1.672 = y_4 + \frac{\left(15/15\times y_4\right)^2}{2g}$

Solution by trial and error subsitution for $y_4$

| $y_4$ | RHS |
|---|---|
| 0.6 | 1.87 |
| 0.6 | 2.33 |
| 0.7 | 1.636 |
| 0.69 | 1.65 |
| 0.68 | 1.672 |

hence correct solution is $y_4 = 0.68$

hence downstream gate opening $y_{g2} = \frac{0.68}{0.61} = 1.115\text{m}$

## Problem 6

### Part (a)

Referring to figure 5.16

For critical flow at (2)

$$E_1 = E_2 + \Delta Z_1$$

$$y_2 = y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{15^2}{g}} = 2.841\text{m}$$

$$\therefore \quad E_3 = 1.5 y_2 = 4.262\text{m}$$

$E_1$ is that for normal depth at (1)

Using Manning's equation $Q = \frac{1}{0.012} \frac{(10y)^{\frac{5}{3}}}{(10+2y)^{\frac{2}{3}}} 0.001^{\frac{1}{2}}$

| y | Q | | |
|---|---|---|---|
| 3 | 120 | | |
| 3.5 | 149.3 | | |
| 3.51 | 149.9 | OK | $y_n = 3.52$ |

$$E_1 = y_n + \frac{q^2}{2g y_n^2} = 3.51 + \frac{15^2}{2g \times 3.51^2} = 4.441\text{m}$$

$$\therefore \quad \Delta Z = 4.441 - 4.262 = 0.179\text{m}$$

### Part (b)

Using the broad crested weir equation

$$Q = 1.705 \times .88 \times 10 \times H^{\frac{3}{2}}$$

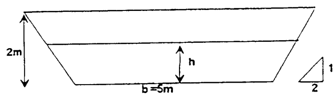
for $Q = 150$

$$\therefore \quad H = \left(\frac{150}{1.705 \times .88 \times 10}\right)^{\frac{2}{3}}$$

$$H = 4.64\text{m}$$

$$\therefore \quad y_1 = 4.64 + 0.5 = 5.14\text{m}$$

## Problem 7


Take $Q = 15 m^3/s$ $\quad S_1 = 0.01$ $\quad n = 0.035$

$S_2 = 0.05$

for critical flow $\quad \frac{Q^2 \beta c}{g A c^3} = 1$

Manning $\quad Q = \frac{1}{n} \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} S^{\frac{1}{2}}$

$A = \left(\frac{2b+4h}{2}\right) h = (b+2h)h$

$P = b + 2\sqrt{5}\, h = b + 4.47h$

(a) <u>Find Normal Depth</u>

using Manning

| | | | |
|---|---|---|---|
| for u/s | guess | h = 1.5 | Q = 34.85 |
| | | h = 1.0 | Q = 16.35 |
| | | h = 0.95 | Q = 14.89 |
| | | h = 0.96 | Q = 15.18 |
| | | h = 0.955 | Q = 15.04 |
| for d/s | | h = 0.5 | Q = 10.66 |
| | | h = 0.6 | Q = 14.66 |
| | | h = 0.61 | Q = 15.09 |

(b) <u>Find Critical Depths</u>

let $Cc = \frac{Q^2 Bc}{g A_c^3} = 1$ (critical flow condition)

| | | | |
|---|---|---|---|
| | $B = b + 4h$ | $A = (b+2h)h$ | |
| try | $h = 0.6$ | $Cc = 3.3$ | |
| | $h = 0.4$ | $Cc = 12.12$ | |
| | $h = 0.7$ | $Cc = 2.99$ | |
| | $h = 0.8$ | $Cc = 1.28$ | |
| | $h = 0.82$ | $Cc = 1.17$ | |
| | $h = 0.84$ | $Cc = 1.09$ | |
| | $h = 0.85$ | $Cc = 1.04$ | |
| | $h = 0.86$ | $Cc = 1.00$ | critical depth |

(c) Find Froude Nos

$Fr = \frac{V}{\sqrt{g dm}}$ where $dm = \frac{A}{B}$

u/s for $h = 0.955$ $A = 6.6$ $B = 8.82$

$V = Q/A = 15/6.6 = 2.273$

$dm = \frac{6.6}{8.82} = 0.748$

u/s $Fr = \frac{2.273}{\sqrt{9.81.\,0.509}} = 0.84$

d/s $h = 0.61$ $A = 3.79$ $B = 7.44$

$V = 15/3.79 = 3.958$

$dm = \frac{3.79}{7.44} = 0.509$

d/s $Fr = \frac{3.958}{\sqrt{9.81.\,0.509}} = 1.77$

(d) The depth at the intersection is the critical depth

Refer to figure 5.18a for the sketch.

## *Problem 8*

For steep slope - discharge controlled by lake i.e. critical depth

mild slope - discharge controlled by channel

For critical flow

$$\frac{Q^2 B}{g A^3} = 1$$

$$\therefore \quad Q = \frac{g A^3}{B} = \frac{g b^3 y^3}{b} = g b^2 y^3$$

For uniform flow

$$Q = \frac{S^{\frac{1}{2}}}{n} \frac{A^{\frac{5}{3}}}{p^{\frac{2}{3}}} = \frac{S^{\frac{1}{2}}}{n} \frac{(by)^{\frac{5}{3}}}{(b+2y)^{\frac{2}{3}}}$$

Specific Energy

$$E = y + V^2/2g$$

$$= y + \frac{Q^2/b^2 y^2}{2g}$$

or $\quad Q = \sqrt{2g}\, by\left(E_0 - y\right)^{\frac{1}{2}}$ for constant $E = E_0$

and $\quad y_c = \frac{2}{3} E_0$

Plot $\quad Q_E$ for const $E$ against $y$

$Q_{s1}$ for $s = 0.1$ against $y$

$Q_{s2}$ for $s = 0.001$ against $y$

| | y | $Q_E$ | supercritical $Q_{s1}$ | $Q_{s2}$ |
|---|---|---|---|---|
| | 0 | 0 | 0 | 0 |
| | 0.5 | 21.0 | 36.0 | 3.6 |
| | 1.0 | 37.6 | | |
| | 1.5 | 48.8 | | |
| $\rightarrow yc =$ | 2.0 | 53.2 | 285.6 | 28.6 |
| | 2.5 | 47.0 | | 38.9 |
| | 3.0 | 0 | | 49.7 |
| | 2.62 | 42.9 | | 41.4 |

note: results shown plotted on next page

where

$$Q_E = \sqrt{2g}\,.\,6\,y\,(3-y)^{\frac{1}{2}}$$

$$= 26.58 \quad y \quad (3-y)^{\frac{1}{2}}$$

$$Q_{s1} = \frac{s_1^{\frac{1}{2}}}{n} \frac{(6y)^{\frac{5}{3}}}{(6+2y)^{\frac{2}{3}}}$$

$$= 21.08 \frac{(6y)^{\frac{5}{3}}}{(6+2y)^{\frac{2}{3}}}$$

$$Q_{s2} = 2.108 \frac{(6y)^{\frac{5}{3}}}{(6+2y)^{\frac{2}{3}}}$$

hence for $S_1 = 0.1 \quad \rightarrow \quad$ critical flow

$$Q = \sqrt{2g}\,A\left(E_0 - yc\right)$$

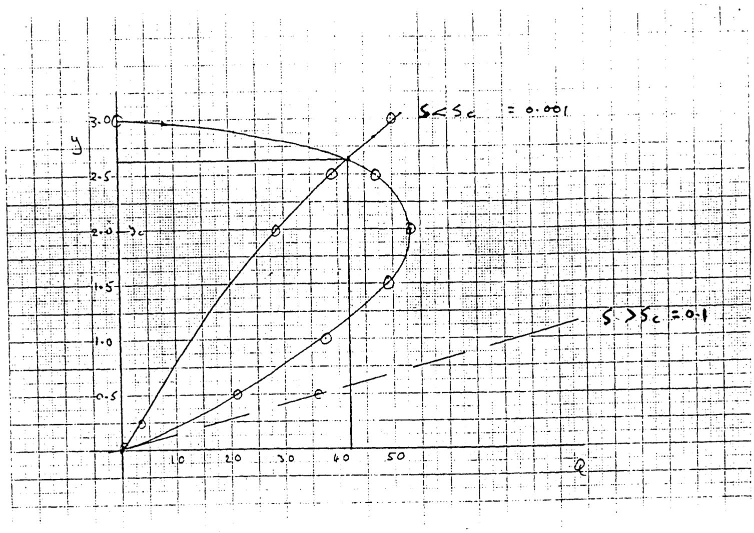
$$yc = 2 \;\therefore\; Q = \sqrt{2g}\,6.2(3-2)$$

$$Q = 53.15 m^3/s \quad y = 2m$$

for $S_2 = 0.001 \rightarrow$ subcritical

$Q \approx 42 \quad y \approx 2.06$

Graph Plot for Problem 8



### *Problem 9*

Use gradually varied flow equation using the direct step method e.g.:

$$\Delta x = \Delta y\left(\frac{1-Fr^2}{S_0-S_f}\right)mean \qquad (1)$$

choose any channel properties such that $y_n > y_c$

then select 3 initial values of y

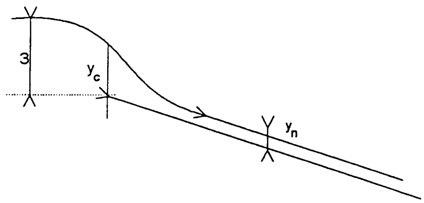
$y_1 > yn$ (Region 1)

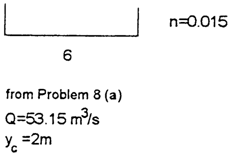
$yn > y_1 > yc$ (Region 2)

$y_1 < yc$ (Region 3)

and apply (1) to confirm.

### *Problem 10*




For $yn$ use an iterative solution to Manning's equation to obtain

$yn = 0.642$

To find the distance use the direct step method. Here a three step solution is given to illustrate the method. 20 steps are necessary to achieve the solution given in the text.

| y | A | P | Fr | $(1-Fr^2)_m$ | $S_f$ | $(S_0 - S_f)_m$ | $\Delta x$ | x |
|---|---|---|---|---|---|---|---|---|
| 2.0 | 12 | 10 | 1 | | 0.0035 | | | 0 |
| | | | | -0.476 | | 0.095 | 2 | |
| 1.6 | 9.6 | 9.2 | 1.4 | | 0.0065 | | | 2 |
| | | | | -2.29 | | 0.0892 | 10.3 | |
| 1.2 | 7.2 | 8.4 | 2.15 | | 0.0151 | | | 12.3 |
| | | | | -9.13 | | 0.067 | 54.4 | |
| 0.8 | 4.8 | 7.6 | 3.95 | | 0.0509 | | | 66.7 |

# Chapter 6

***Problem 1***

$$c = \sqrt{\frac{K}{\rho}} = \left(\frac{2.11\times10^9}{10^3}\right)^{\frac{1}{2}} = 1452 \text{ m/s}$$

$$\delta p = 2.9\times10^6 \text{ N/m}^2 = \rho uc$$

Therefore

$$u = \frac{2.9\times10^6}{1000\times1452} = 2 \text{ m/s}$$

$$Q = 392\ \ell/\text{s}$$

***Problem 2***

$44\ \ell/\text{s} = 0.044\ \text{m}^3/\text{s}$ $\qquad u = 2.49$ m/s

$$T_p = \frac{2L}{c} = \frac{2\times1000}{1400} = 1.43 \text{ s}$$

If time of closure $T_c < T_p$ then $\delta p = \rho uc$

$= 1000 \times 2.49 \times 1400$

$= 3.49 \times 10^6$ N/m²

Allowable pressure rise $= (5.600 - 2.45) \times 10^6$

$= 3.15 \times 10^6$ N/m²

Valve closure must be slower, i.e. $\dfrac{3.15}{3.49} = \dfrac{1.43}{T_c}$ $\qquad T_c = 1.58$ s

***Problem 3***

$$c = \sqrt{\frac{K}{\rho}} = \left(\frac{2.13\times10^9}{10^3}\right)^{\frac{1}{2}} = 1459 \text{ m/s}$$

$$\delta p = \rho uc = 1000\times2.5\times1459 = 3.65 \text{ MN/m}^2$$

$$= 372\ m$$

***Problem 4***

$$\rho c^2 = \frac{1}{1/K + D/Ee} = \frac{1}{\left(1/0.67\times10^9\right)+\left(0.1/205\times10^9\times0.0075\right)}$$

$$= 0.642\times10^9 \text{ kgm}^2/s^2$$

$c \quad = 801 \text{ m/s} \qquad$ (instantaneous closure)

$\delta p \quad = \rho uc \quad = 950\times801\times1 \quad = 761 \text{ kN/m}^2$

# Chapter 7

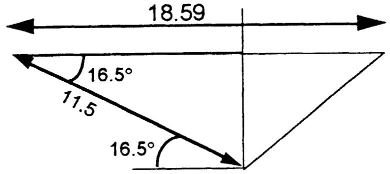
***Problem 1***

Impeller diameter = 0.5   $N$ = 710 rev/min

Tangential speed of impeller = $\pi dn = \pi \times 0.5 \times \frac{710}{60}$

$= 18.59$ m/s

Absolute velocity of water = 11.5 m/s




$V_{W2}$ = 11.5 cos 16.5° = 11.03 m/s

$V_{F2}$ = 11.5 sin 16.5° = 3.27 m/s

Vane angle = $\cot^{-1}\left(\frac{18.59-11.03}{3.27}\right)$

= 23.40°

Hydraulic power = $\rho Q V_{W2} V_{I2}$ = 1000 x 1 x 11.03 x 18.59 = <u>205 kW</u>

***Problem 2***

Head for system = 2 + $h_f$ = $H$

| **Q (ℓ/s)** | ***hf*** | ***H*** |
|---|---|---|
| 20 | 0.434 | 2.434 |
| 40 | 1.735 | 3.735 |
| 60 | 3.90 | 5.90 |
| 80 | 6.94 | 8.94 |
| 100 | 10.84 | 12.84 |

see graphical solution

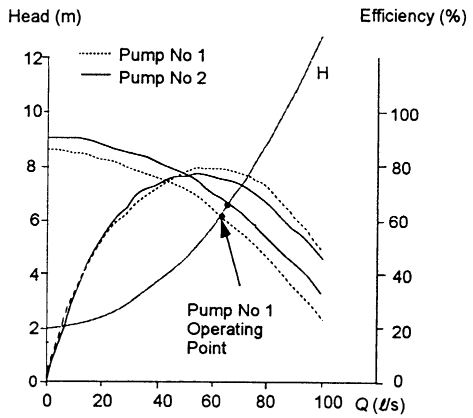
Hydraulic power = $\frac{\rho g Q H}{\text{Efficiency}} = \frac{1000 \times 9.81 \times 0.063 \times 6.15}{0.79}$

= 4811 W

= <u>4.8 kW</u>

$N_s = \frac{1385 \times 0.063^{\frac{1}{2}}}{6.15^{\frac{3}{4}}}$ = <u>89</u>

As 70< 89 <165 this would probably be a *mixed flow* pump




***Problem 3***

$H_p = 31.7 + 7 \ = 38.7 \text{ m}$

$$H_c = \frac{p_A - p_{vap}}{\rho g} \qquad = \frac{101.4 \times 10^3 - 1.82 \times 10^3}{1000 \times 9.81} \qquad = 10.15 \text{ m}$$

$$\sigma_t = \frac{10.15 - 7}{38.7} \qquad = \underline{0.081}$$

For the higher altitude

$$H_c = \frac{93.4 \times 10^3 - 1.4 \times 10^3}{1000 \times 9.81} \quad = 9.4 \text{ m}$$

$$\sigma_t = 0.081 = \frac{9.4 - H_{ps}}{38.7} \qquad \text{Therefore} \qquad H_{ps} = 6.25 \text{ m}$$

$$H_s = 3 - (7 - 6.25) \quad = \underline{2.25 \text{ m}}$$

# Chapter 8

## *Problem 1*

Given $T = 8.5s$

then $C_0 = gT/2\pi = 13.27(m/s)$

$L_0 = C_0 T = 112.8(m)$

For transitional depths need to solve the wave dispersion equation e.g.:

$\omega^2 = gk \tanh(kd)$ (1)

For $d/L_0 = 0.1$ $d = 0.1 \times 112.8 = 11.28$

$\omega^2 = (2\pi/T)^2 = 0.5464$

using Fig 8.4 $C/C_0 \approx 0.7$ for $d/L_0 = 0.1$

and $L/L_0 = C/C_0$

$\therefore$ $L \approx 79$

use trial values of L to satisfy (1).

| L | K | tanh (kd) | gk tanh (kd) |
|---|---|---|---|
| 80 | 0.07854 | 0.7094 | 0.5466 |
| 80.1 | 0.07844 | 0.7088 | 0.05454 |

$\therefore L = 80\text{m}$ $\therefore C/C_0 = \frac{80}{112.8} = 0.709$ $\therefore C = 9.41\text{m/s}$

$C_G = \frac{C}{2}\left(1 + \frac{2kd}{\sinh 2kd}\right) = 7.62\text{m/s}$

## *Problem 2*

Exactly analogous to example 8.1 sample trials given here:

| | $d/L_0$ | $d$ | $C/C_0$ | $C$ | $K_S$ | $\alpha$ | $K_R$ | $H/H_0$ | $H$ | $d_B$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1.0 | 112.8 | 1 | 13.27 | 1 | 30 | 1 | 1 | 5 | 6.4 |
| | 0.107 | 12.1 | 0.73 | 9.6 | 0.93 | 21.3 | 0.96 | 0.9 | 4.5 | 5.7 |
| wave break → | 0.055 | 6.2 | 0.55 | 7.3 | 1 | 16 | 0.95 | 0.96 | 4.8 | 6.1 |

## *Problem 3*

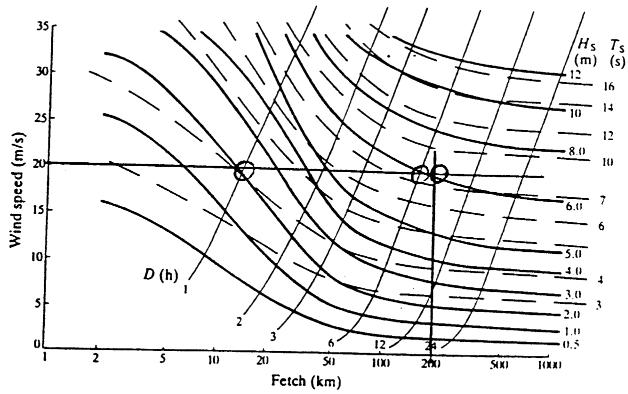



Fig. 8.14 Prediction of significant wave height and period for oceanic waters around the UK

| | | | | |
|---|---|---|---|---|
| for | D=1hr | $Hs = 2.1m$ | $Ts = 4.2s$ | duration limited |
| | D=6hr | $Hs = 6.0m$ | $Ts = 7.4s$ | duration limited |
| | D=24hr | $Hs = 6.3m$ | $Ts = 7.6s$ | fetch limited |

# Chapter 9

***Problem 1***

Assume $F_s = \dfrac{\tau_0}{(\rho_s - \rho)gD} = 0.056$ from Shields' diagram

then $0.056 = \dfrac{\tau_0}{(2650 - 1000) \times 9.81 \times 0.01}$

so $\tau_0 = \underline{9.06 \text{ N/m}^2}$

Check Re$_*$: $u_* = (\tau_0 / \rho)^{\frac{1}{2}} = (9.06/1000)^{\frac{1}{2}} = 0.095 \text{ m/s}$

$$\text{Re}_* = \rho u_* D / \mu = \frac{1000 \times 0.095 \times 0.01}{1.14 \times 10^{-3}} = 833$$

So using $F_S$ = 0.056 is a reasonable assumption

To find Bed-load

If $K_3 = \dfrac{0.54}{g(\rho_s - \rho)}$ (equation 9.8) and $\tau_0 \cong \rho g y s_0$

Then $K_3 = \dfrac{3.874 \times 10^{-8}}{D^{\frac{3}{4}}} = \dfrac{3.874 \times 10^{-8}}{0.01^{\frac{3}{4}}} = 1.225 \times 10^{-6}$

$$\tau_0 = \frac{1000 \times 9.81 \times 8}{2000} = 39.24 \text{ N/m}^2$$

$$Q_s = BK_3\tau_0(\tau_0 - \tau_{cr}) = 100 \times 1.225 \times 10^{-6} \times 39.24(39.24 - 9.06)$$

$= \underline{0.145 \text{ m}^3/\text{s}}$

For limiting sediment diameter

$$\frac{R_{50}}{D} = 0.0924 \qquad \therefore D = \frac{6.89}{2000 \times 0.0924}$$

$$R = \frac{100 \times 8}{\left(100 + (2 \times 8)\right)} \qquad = 0.0373 \text{ m}$$

$$= \underline{6.89 \text{ m}} \qquad = \underline{37.3 \text{ mm}}$$

***Problem 2***

Use Ackers and White formula

$$D_{gr} = D_{50}\left\{\frac{g\left[\left(\frac{\rho_s}{\rho}\right)-1\right]}{v^2}\right\}^{\frac{1}{3}} = 0.0005\left[\frac{9.81\left(\frac{2650}{1000}-1\right)}{\left(1.14 \times 10^{-6}\right)^2}\right]^{\frac{1}{3}}$$

$$= 11.47$$

Therefore

$$n = 1 - 0.56 \log D_{gr} = 0.407 \qquad m = 1.67 + \frac{6.83}{D_{gr}} = 2.265$$

$$A_{gr} = 0.14 + \frac{0.23}{D_{gr}^{\frac{1}{2}}} = 0.208 \qquad \begin{cases} \log C = 2.79 \log D_{gr} - 0.98\left(\log D_{gr}\right)^2 - 3.46 \\ C = 0.0214 \end{cases}$$

$R = 6.89$ (from problem 1)

Therefore

$$u_* = \sqrt{gRs_0} = \left(\frac{9.81 \times 6.89}{2000}\right)^{\frac{1}{2}} = 0.184 \text{ m/s}$$

From Chézy Manning formula (take n = 0.02 for earth channel):

$$Q = \frac{A^{\frac{5}{3}} s_0^{\frac{1}{2}}}{nP^{\frac{2}{3}}} = \frac{\left(100 \times 8\right)^{\frac{5}{3}}}{0.02 \times \left(2000\right)^{\frac{1}{2}} \times \left(100 + 2 \times 8\right)^{\frac{2}{3}}}$$

$$= 3247 \text{ m}^3/s$$

Therefore

$$V = 4.06 \text{ m/s}$$

$$F_{gr} = \frac{0.184^{0.407}}{\left[9.81 \times 0.0005\left(\frac{2650}{1000}-1\right)\right]^{\frac{1}{2}}} \left[\frac{4.06}{\sqrt{32}\, log\left(\frac{10 \times 8}{0.0005}\right)}\right]^{0.593}$$

$$= 1.724$$

$$G_{gr} = \frac{q_s D_m}{qD}\left(\frac{u_*}{V}\right)^n \qquad = C\left(\frac{F_{gr}}{A_{gr}} - 1\right)^m$$

$$= \frac{q_s \times 8}{32.47 \times 0.0005}\left(\frac{0.184}{4.06}\right)^{0.407} \qquad = 0.0214\left(\frac{1.724}{0.208} - 1\right)^{2.265}$$

$q_s \quad = 0.0138\ m^3 / sm$ or $Q_s = 1.38\ m^3 /s$

# Chapter 10

### *Problem 1*

Essay based on concepts introduced at the beginning of Chapter 10.

### *Problem 2*

Rank the data and assign non-exceedence probabilities, return period and reduced variate. Calculate the probability weighted moments.

| $Q_j$ ($m^3/s$) | $j$ | $\frac{(j-1)}{(n-1)}Q_j$ | $\frac{(j-1)(j-2)}{(n-1)(n-2)}Q_j$ | $\frac{(j-1)(j-2)(j-3)}{(n-1)(n-2)(n-3)}Q_j$ | $\frac{Q_j}{QMED}$ | $F(Q)$ | $y$ |
|---|---|---|---|---|---|---|---|
| 38.2 | 1 | | | | 0.491 | 0.040 | -3.187 |
| 40.4 | 2 | 3.108 | | | 0.519 | 0.110 | -2.086 |
| 57.6 | 3 | 8.862 | 0.738 | | 0.740 | 0.181 | -1.508 |
| 58.9 | 4 | 13.592 | 2.265 | 0.206 | 0.757 | 0.252 | -1.087 |
| 66.2 | 5 | 20.369 | 5.092 | 0.926 | 0.850 | 0.323 | -0.740 |
| 66.5 | 6 | 25.577 | 8.526 | 2.325 | 0.854 | 0.394 | -0.432 |
| 73.6 | 7 | 33.969 | 14.154 | 5.147 | 0.945 | 0.465 | -0.142 |
| 82.1 | 8 | 44.208 | 22.104 | 10.047 | 1.055 | 0.535 | 0.142 |
| 84.8 | 9 | 52.185 | 30.441 | 16.604 | 1.089 | 0.606 | 0.432 |
| 97.1 | 10 | 67.223 | 44.815 | 28.519 | 1.247 | 0.677 | 0.740 |
| 120.6 | 11 | 92.769 | 69.577 | 50.601 | 1.549 | 0.748 | 1.087 |
| 142.1 | 12 | 120.238 | 100.199 | 81.981 | 1.825 | 0.819 | 1.508 |
| 159.9 | 13 | 147.600 | 135.300 | 123.000 | 2.054 | 0.890 | 2.086 |
| 206.8 | 14 | 206.800 | 206.800 | 206.800 | 2.656 | 0.960 | 3.187 |
| 1294.800 | | 836.500 | 640.012 | 526.156 | ← **Totals** | | |
| $b_0$ = 92.486 | | $b_1$ = 59.750 | $b_2$ = 45.715 | $b_3$ = 37.583 | | | |

Calculate L-moment ratios and sample estimators for GL distribution.

| | | |
|---|---|---|
| Median | *QMED* | 77.850 |
| L-moments: | $l_1$ | 92.486 |
| | $l_2$ | 27.014 |
| | $l_3$ | 8.276 |
| | $l_4$ | 4.713 |
| L-CV | $t_2$ | 0.292 |
| L-skewness | $t_3$ | 0.306 |
| L-kurtosis | $t_4$ | 0.174 |

GL distribution sample estimators:

| | |
|---|---|
| $k$ | -0.306 |
| $\beta$ | 0.290 |

Fit GL distribution using sample estimators.

| *F* | *x* (Equation 10.12) | *y* (Equation 10.16) | *Q* ($m^3/s$) (Equation 10.10) | *T* (years) (Equation 10.4) |
|---|---|---|---|---|
| 0.05 | 0.438 | -2.944 | 34.094 | 1.053 |
| 0.10 | 0.537 | -2.197 | 41.779 | 1.111 |
| 0.15 | 0.610 | -1.735 | 47.498 | 1.176 |
| 0.20 | 0.673 | -1.386 | 52.371 | 1.250 |
| 0.25 | 0.730 | -1.099 | 56.808 | 1.333 |
| 0.30 | 0.784 | -0.847 | 61.016 | 1.429 |
| 0.35 | 0.837 | -0.619 | 65.130 | 1.538 |
| 0.40 | 0.890 | -0.405 | 69.249 | 1.667 |
| 0.45 | 0.944 | -0.201 | 73.460 | 1.818 |
| 0.50 | 1.000 | 0.000 | 77.850 | 2.000 |
| 0.55 | 1.060 | 0.201 | 82.519 | 2.222 |
| 0.60 | 1.125 | 0.405 | 87.589 | 2.500 |
| 0.65 | 1.198 | 0.619 | 93.226 | 2.857 |
| 0.70 | 1.280 | 0.847 | 99.673 | 3.333 |
| 0.75 | 1.378 | 1.099 | 107.313 | 4.000 |
| 0.80 | 1.500 | 1.386 | 116.811 | 5.000 |
| 0.85 | 1.663 | 1.735 | 129.490 | 6.667 |
| 0.900 | 1.908 | 2.197 | 148.565 | 10.000 |

| $F$ | $x$ (Equation 10.12) | $y$ (Equation 10.16) | Q ($m^3$/s) (Equation 10.10) | $T$ (years) (Equation 10.4) |
|---|---|---|---|---|
| 0.910 | 1.976 | 2.314 | 153.806 | 11.111 |
| 0.920 | 2.053 | 2.442 | 159.822 | 12.500 |
| 0.930 | 2.143 | 2.587 | 166.858 | 14.286 |
| 0.940 | 2.252 | 2.752 | 175.282 | 16.667 |
| 0.950 | 2.385 | 2.944 | 185.697 | 20.000 |
| 0.960 | 2.558 | 3.178 | 199.162 | 25.000 |
| 0.970 | 2.798 | 3.476 | 217.800 | 33.333 |
| 0.980 | 3.170 | 3.892 | 246.811 | 50.000 |

GL predicted $\underline{Q_{50} = 246.8\ m^3/s}$

For histogram select class intervals

| Class | $f_i$ |
|---|---|
| 0-50 | 2 |
| 50-100 | 8 |
| 100-150 | 2 |
| 150-200 | 1 |
| 200-250 | 1 |

calculate $f(Q) = \dfrac{\alpha^{-1} e^{-(1-k)w}}{\left(1 + e^{-w}\right)^2}$ at the centre of each class interval

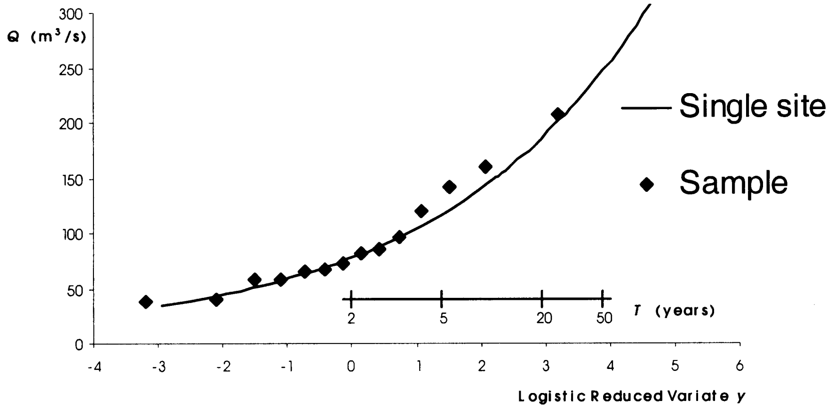
e.g.:

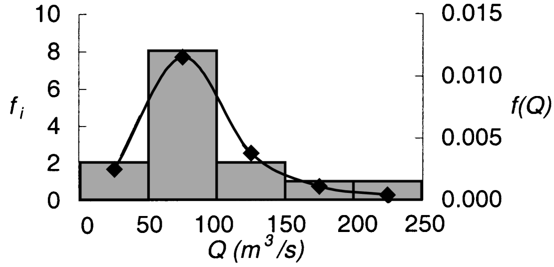
| $Q$ | $f(Q)$ |
|---|---|
| 25 | $2.449 \times 10^{-3}$ |
| 75 | $11.481 \times 10^{-3}$ |
| 125 | $3.738 \times 10^{-3}$ |
| 175 | $1.083 \times 10^{-3}$ |
| 225 | $0.388 \times 10^{-3}$ |

calculate equivalent scale for $N$ = 14 and $\Delta Q$ =50

| $f_i$ | $f(Q) = f_i/N\Delta Q$ |
|---|---|
| 2 | $2.86 \times 10^{-3}$ |
| 4 | $5.71 \times 10^{-3}$ |
| 6 | $8.57 \times 10^{-3}$ |

All data can now be plotted as shown below.




Both GL and histogram/pdf appear to be a reasonably good fit. However, 14 years of data is too short for estimation of 50 year event. FEH would recommend use of a pooling group.

## ***Problem 3***

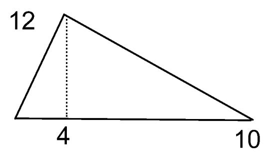
Use a synthetic unit hydrograph

$$Q_p = \frac{2.2 \times 21.82}{4} = 12\text{m}^3/\text{s}$$

$$T_B = 2.52 \times 4 \approx 10\text{hrs}$$

Find peak run off by convolution - only need to calculate one line with max rainfall x max UH ordinate.

UH ordinates




| time (hrs) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| UH ord ($m^3/s$) | 3 | 6 | 9 | 12 | 10 | 8 | 6 | 4 | 2 | 0 |

| time | rainfall | UH ordinates: 3 6 9 (12) 10 8 6 4 2 | q |
|---|---|---|---|
| 1 | 2 | 2x3 | $q_1$ |
| 2 | 5 | 5x3+2x6 | $q_2$ |
| 3 | 8 | | |
| 4 | 11 | ….etc. | |
| 5 | 14 | | |
| 6 | 31 | | |
| 7 | (93) | | |
| 8 | 31 | | |
| 9 | 14 | | |
| 10 | 11 | $\frac{\left(11\times 3+14\times 6+31\times 9+93\times 12+31\times 10+14\times 8+11\times 6+8\times 4+5\times 2\right)\times .68}{10}$ | 138.9 |

Note: net rain = rain x 0.68
Peak runoff = 138.9 + 1 (base flow) = 139.9

$\approx$ 140$m^3/s$

***Problem 4***

Solution based on FEH methods given in Fig 10.14.

(a) Frequency analysis of pooling group data from donor or analogue catchments (for peak Q)
Synthetic UH (for flood hydrograph shape & peak Q)

(b) Frequency analysis of pooling group which includes the subject site (for peak Q)
Derive UH from records (for flood hydrograph)

### *Problem 5*

First find the time to fill the reservoir, then apply reservoir routing technique as given in example 10.5.

For each two hour interval inflow vol = $\left(\dfrac{I_1 + I_2}{2}\right)\Delta t$

| $t$ | $I$ | $\Delta V$ | $V$ |
|---|---|---|---|
| 0 | 5 | | |
| | | $46.8 \times 10^3$ | $46.8 \times 10^3$ |
| 2 | 8 | | |
| | | $82.8 \times 10^3$ | $129.6 \times 10^3$ |
| 4 | 15 | | |
| | | $162 \times 10^3$ | $291.6 \times 10^3$ |
| 6 | 30 | | |
| | | $414 \times 10^3$ | $705.6 \times 10^3$ |
| 8 | 85 | | |

hence after 8 hours the reservoir is full.
Next carry out reservoir routing as in example 10.5.

| $t$ | $I$ | $H$ | $V$ | A | $O$ |
|---|---|---|---|---|---|
| (h) | ($m^3$/s) | (m) | ($m^3$) | ($m^2$) | ($m^3$/s) |
| 8 | 85 | 0.000 | 0.0 | 800000.0 | 0.00 |
| 10 | 160 | 0.848 | 702159.6 | 856516.9 | 49.96 |
| 12 | 140 | 1.427 | 1209538.9 | 895137.9 | 109.11 |
| 14 | 95 | 1.473 | 1250819.2 | 898207.0 | 114.43 |
| 16 | 45 | 1.225 | 1030353.2 | 881691.8 | 86.81 |
| 18 | 0 | 0.846 | 700574.6 | 856393.6 | 49.79 |

Peak outflow is 115 $m^3$/s

### *Problem 6*

| Pipe No | $L$ | Grad | $D$ | $V$ | $t_f$ | $t_c$ | $i$ | $A$ | $C_V$ | $Q_p$ | $Q_{full}$ | Comments |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 70 | 0.0175 | 150<br>175 | 1.33<br>1.47 | 0.88<br>0.79 | 4.88<br>4.79 | 54.5<br>55.3 | 0.1415 | 0.9 | 25.0<br>25.4 | 23.5<br>35.4 | Diameter too small<br>Diameter OK |
| 1.1 | 75 | 0.017 | 225 | 1.71 | 0.73 | 5.52 | 52 | 0.3275 | 0.9 | 55.3 | 67.9 | Diameter OK |
| 1.2 | 92 | 0.0085 | 250 | 1.29 | 1.19 | 6.71 | 47.4 | 0.4085 | 0.9 | 62.9 | 63.2 | Diameter OK |

Take $C_R$ = 1.3

$$Q_p = \frac{C_V C_R i}{0.36} A$$

Solve in tabular form.
Please note $C_R$ = 1.3 has been assumed. Area values are cumulative and standard tables relating grad, $Q$, $V$, $k_s$ have been used.
Rainfall intensity curve has been plotted for interpolation.

# Chapter 11

***Problem 1***

$$Q \quad = f\left(\rho g h \mu b B\right)$$

Dimensionally this may be expressed

$$\frac{L^3}{T} \quad = f\left[\left(\frac{M}{L^3}\right)^a \left(\frac{L}{T^2}\right)^b L^c \left(\frac{M}{LT}\right)^d L^e L^f\right]$$

Equating indices: $M$: $0 = a + d$ $\qquad a = -d$

$T$: $-1 = -2b - d$ $\qquad b = \frac{1}{2} - \frac{d}{2}$

$L$: $3 = -3a + b + c - d + e + f$ $\qquad c = \frac{5}{2} - \frac{3}{2}d - e - f$

$$Q \quad = f\left[\rho^{-d} g^{\left(\frac{1}{2}-\frac{d}{2}\right)} h^{\left(\frac{5}{2}-\frac{3d}{2}-e-f\right)} \mu^d b^e B^f\right]$$

group by index

$$\frac{Q}{g^{\frac{1}{2}} h^{\frac{5}{2}}} \quad = f\left[\frac{\mu}{\rho g^{\frac{1}{2}} y^{\frac{3}{2}}}, \frac{b}{h}, \frac{B}{h}\right]$$

as scale $= \dfrac{1}{50}$ $\qquad \left(\dfrac{b}{h}\right)' = \left(\dfrac{b}{h}\right)'' \quad \left(\dfrac{B}{h}\right)' = \left(\dfrac{B}{h}\right)''$

also assume flow turbulent and therefore independent of Re

Therefore

$$\left(\frac{Q}{g^{\frac{1}{2}} h^{\frac{5}{2}}}\right)' = \left(\frac{Q}{g^{\frac{1}{2}} h^{\frac{5}{2}}}\right)'' \qquad 3.5 \times \left(\frac{h'}{h''}\right)^{\frac{5}{2}} \quad = Q'$$

Therefore

$Q' \quad = 61.871\ \ell / s$

say $62\ m^3 / s$

***Problem 2***

$$F_R = f(\rho u r \mu)$$

Dimensionally

$$\frac{ML}{T^2} = f\left[\left(\frac{M}{L^3}\right)^a\left(\frac{L}{T}\right)^b L^c\left(\frac{M}{LT}\right)^d\right]$$

| | | | |
|---|---|---|---|
| Equating indices: | $M$: | $1 = a + d$ | $a = 1 - d$ |
| | $T$: | $-2 = -b - d$ | $b = 2 - d$ |
| | $L$: | $1 = -3a + b + c - d$ | $c = 2 - d$ |

$$F_R = f\left(\rho^{1-d}u^{2-d}r^{2-d}\mu^d\right)$$

$$\frac{F_R}{\rho u^2 r^2} = f\left(\frac{\mu}{\rho u r}\right)^d \qquad \text{if } F_R = \rho u^2 r^2 f\left(\frac{\mu}{\rho u r}\right)^d \propto u\text{, then } d = 1$$

Hence

$$F_r = K\mu r u$$

Force on particle $= \frac{\pi}{6}D^3 g(\rho_s - \rho) \quad = F_R$ for steady motion

$$F_R = \frac{\pi}{6}\times\left(0.01\times10^{-3}\right)^3\times9.81(2500-1000) \quad = 7.7\times10^{-12}\text{ N}$$

But

$$F_r = 2\pi\mu r u$$

$$= 2\pi\times1.1\times10^{-3}\times0.005\times10^{-3}u \qquad = 7.7\times10^{-12}$$

Therefore

$$u = 2.7\times10^{-4}\text{ m/s}$$

Therefore to travel 3 m

$$= \frac{3}{2.7\times10^{-4}} \quad = 11120\text{ s}$$

$$= 3.1\text{ h}$$

***Problem 3***

$N_1$ = 2950 rev/min  $Q$ = 50 $\ell$/s  $H_p$ = 75 m  Efficiency = 75%

Impeller diameter = 350 mm

$Q_2$ = 450 $\ell$/s  $H_p$ = 117 m

$$N_1 = \frac{2950}{60} = 49.17 \text{ rev/s}$$

From dimensionless group $\Pi_3 = \dfrac{gH}{N^2D^2}$

For pump 1 (as tested) $\Pi_3 = \dfrac{9.81\times 75}{49.17^2 \times 0.35^2} = 2.484$

For pump 2 $\Pi_3 = \dfrac{9.81\times 117}{N^2 \times D^2} = 2.484$

Therefore $\dfrac{462}{N^2} = D^2$  $D = \dfrac{21.5}{N}$

From dimensionless group $\Pi_2 = \dfrac{Q}{ND^3}$

For pump 1 $\dfrac{0.05}{49.17\times 0.35^3} = 0.0237$

For pump 2 $\dfrac{0.45}{ND^3} = 0.0237$

But $D = \dfrac{21.5}{N}$  Therefore $D^3 = \dfrac{9938}{N^3}$

Therefore

$$\frac{0.45\times N^2}{0.0237\times 9938} = 1 \qquad N = 22.87 \text{ rev/s}$$
$$= 1373 \text{ rev/min}$$

$$D = \frac{21.5}{N} = \frac{21.5}{22.87} = 0.94 \text{ m} = 940 \text{ mm}$$

***Problem 4***

Try scale $\lambda_L = \dfrac{1}{250}$ Plan area of model $= A$

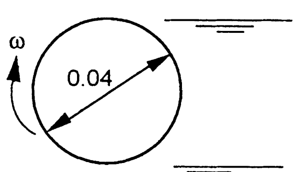
$$A = \frac{1}{2} \times \frac{1500}{250} \times \frac{7000}{250} = 84 \text{ m}^2$$

Tidal period : scale for $T = \dfrac{L}{c}$

$$\text{i.e. } \lambda_T = \frac{\lambda_L}{\lambda_c} = \frac{\lambda_L}{\sqrt{\lambda_L}} = \lambda_L^{\frac{1}{2}} = \left(\frac{1}{250}\right)^{\frac{1}{2}} = \frac{1}{15.8}$$

Therefore model tidal period $= \dfrac{12h \times 60}{15.8} = \underline{45.5 \text{ min}}$

Tidal range $= \dfrac{10}{250} = 0.04 \text{ m}$




'Circular frequency' for tide

$$= \omega = \frac{2\pi}{45 \times 60} = 2.3 \times 10^{-3} \text{ rad/s}$$

Therefore maximum elevation rate for water surface is $2.3 \times 10^{-3} \times \dfrac{0.04}{2}$

Thereforemaximum inflow

$$= 2.3 \times 10^{-3} \times \frac{0.04}{2} \times 84$$
$$= 3.9 \times 10^{-3} \text{ m}^3/s$$
$$= \underline{3.9\ \ell/s}$$

***Problem 5***

$B' = 60$ m, $y' = 7.5$ m, $Q' = 1700 \text{ m}^3/s$

$\lambda_L = \dfrac{20}{5000} = \dfrac{1}{250}$ say $\dfrac{1}{300}$ (allows for working space)

Undistorted model $B'' = \frac{60}{300} = 0.2$ m $y'' = \frac{7.5}{300} = 0.025$

$Q'' = \lambda_Q Q'$: $\lambda_Q = \lambda_L^{2\frac{1}{2}} = \frac{1}{300^{2\frac{1}{2}}}$

Therefore $Q'' = 1.09\ \ell/s$

Larger scale possible

*Distorted* model $Q'' \approx 0.02\ \text{m}^3/s = \lambda_x \lambda_y^{1\frac{1}{2}} Q'$

$\lambda_x = \frac{1}{300}$ $\therefore \lambda_y \approx \frac{1}{43}$ *say* $\frac{1}{50}$

$\lambda_Q = \left(\frac{1}{300}\right)\left(\frac{1}{50}\right)^{1\frac{1}{2}}$ so $Q'' = \left(\frac{1}{300}\right)\left(\frac{1}{50}\right)^{1\frac{1}{2}} 1700$

$= 0.016\ \text{m}^3/s$

$= \underline{16\ \ell/s}$