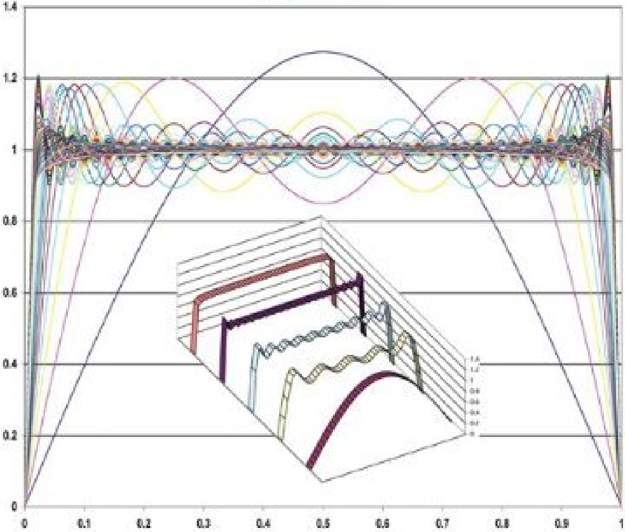
INSTRUCTOR'S SOLUTIONS MANUAL

Experimental Methods for Engineers

Eighth Edition



J.P. Holman

Instructor's Solutions Manual

to accompany

Experimental Methods for Engineers

Eighth Edition

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Chapter 2

2-3

$$\text{Amplitude ratio} = \frac{x_0}{\frac{F_0}{k}} = \frac{1}{\left\{\left[1-\left(\frac{w_1}{w_n}\right)^2\right] + \left[2\left(\frac{a}{c_c}\right)\left(\frac{w_1}{w_n}\right)\right]\right\}}$$

$$= \frac{1}{\{[1-(0.4)^2]^2 + [2(0.7)(0.4)]^2\}^{1/2}}$$

amplitude ratio $= 0.99$ (Use Figure. 2-5)

$F(t) = F_0 \sin w_1 t;\ x(t) = x_0 \sin(w_1 t - \phi)$

time lag $= t_{x_{\max}} - t_{F_{\max}}$

$F(t) = F_0 = \max$ (when $\sin w_1 t = 1$) $\therefore w_1 t = \sin^{-1} 1 = \frac{\pi}{2};\ t_{F_{\max}} = \frac{1}{w_1}\frac{\pi}{2}$

$$t_{F_{\max}} = \left(\frac{1}{40}\right)\left(\frac{\pi}{2}\right)\left(\frac{1}{2\pi}\right) = 0.00625 \sec$$

$x(t) = x_0 = \max$ (when $\sin(w_1 t - \phi) = 1$ $\therefore (w_1 t - \phi) = \sin^{-1} 1 = \frac{\pi}{2}$

$$t_{x_{\max}} = \frac{1}{w_1}\left(\frac{\pi}{2} + \phi\right)$$

$$\phi = \tan^{-1}\frac{2\left(\frac{c}{c_c}\right)\left(\frac{w_1}{w_n}\right)}{1-\left(\frac{w_1}{w_n}\right)^2} = \tan^{-1}\frac{2(0.7)(0.4)}{1-(0.4)^2}$$

$\phi = 33.7°$ (Use Figure 2-6)

$$t_{x_{\max}} = \frac{1}{40}\left[\frac{\pi}{2} + 33.7\left(\frac{\pi}{180}\right)\right] = 0.054 \sec$$

$\therefore$ time lag $= 0.54 - 0.000625$

time lag $= 0.0478$ sec

2-4

$$\frac{1}{\left\{\left[1-\left(\frac{w_1}{w_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{w_1}{w_n}\right)^2\right]\right\}^{1/2}} = \frac{x_0}{\frac{F_0}{k}}$$

For $\frac{x_0}{\frac{F_0}{k}} = 1.00 + 0.01 = 1.01$ we have $\left(\frac{w_1}{w_n}\right)^4 - 0.04\left(\frac{w_1}{w_n}\right)^2 + 1 - \left(\frac{1}{1.01}\right)^2 = 0$ and

$w_1 \rightarrow$ imaginary.

For $\frac{x_0}{\frac{F_0}{k}} = 1.00 - 0.01 = 0.99$ we have $\left(\frac{w_1}{w_n}\right)^4 - 0.04\left(\frac{w_1}{w_n}\right)^2 + 1 - \left(\frac{1}{0.99}\right)^2 = 0$

which gives $\frac{w_1}{w_n} = 0.306$.

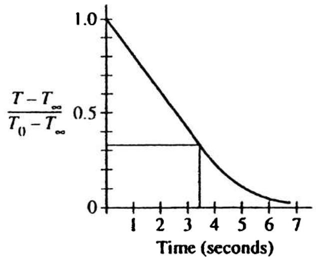
$w_n = (100)(2\pi) = 628$ rad/sec

$w_1 = (0.306)(628)$

$w_1 = 192.1$ rad/sec $= 30.6$ Hz

2-5

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-\left(\frac{1}{RC}\right)t}$$



At $t = 3\,\text{sec}, T = 200°\text{F}$

$$\frac{T - T_\infty}{T_0 - T_\infty} = 0.435. \text{ At } t = 5\,\text{sec}, T = 270°\text{F}$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = 0.1304$$

$1 - 0.632 = 0.328$

$RC \approx 3.4\,\text{sec}$

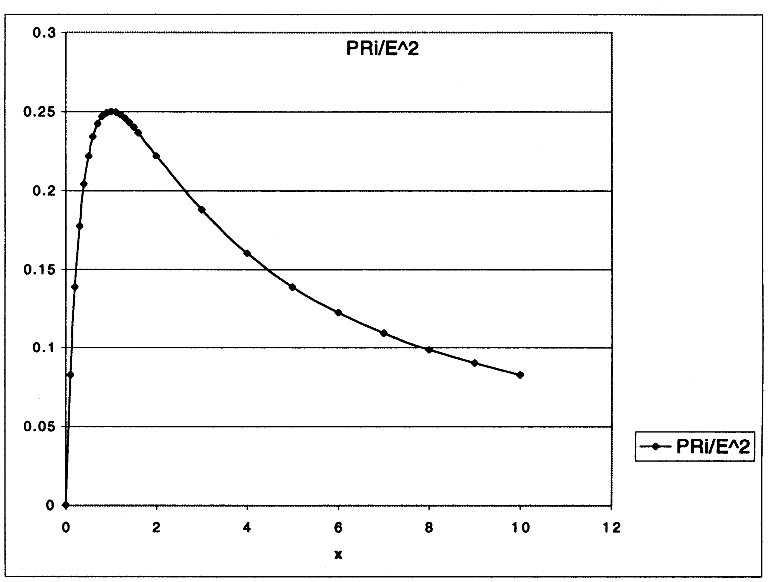
2-6

$$P = \frac{E_{AB}^2}{R}$$

$$E_{AB} = E\left(\frac{R}{R + R_i}\right)$$

$$P = \frac{1}{\frac{R}{R_i}}\left(\frac{\frac{R}{R_i}}{\frac{R}{R_i}+1}\right)^2$$

R/Ri	PRi/E^2
0	0
0.1	0.082645
0.2	0.138889
0.3	0.177515
0.4	0.204082
0.5	0.222222
0.6	0.234375
0.7	0.242215
0.8	0.246914
0.9	0.249307
1	0.25
1.1	0.249433
1.2	0.247934
1.3	0.245747
1.4	0.243056
1.5	0.24
1.6	0.236686
2	0.222222
3	0.1875
4	0.16
5	0.138889
6	0.122449
7	0.109375
8	0.098765
9	0.09
10	0.082645



2-7

Readability $\rightarrow \frac{1}{64}$ inch

Least count $\rightarrow \frac{1}{32}$ inch

2-8

$t = RC =$ time constant

$t = (10^6 \text{ohms})(10^{-5}\text{f}) = 10 \text{ sec}$

$t = 10 \text{ sec}$

2-9

$$\% \text{ error } = \left[\frac{(R + R_i) - R}{(R + R_i)}\right] \times 100$$

$$= \frac{5000}{25{,}000} \times 100$$

% error $= 20\%$

2-10

$$P = \frac{E_{AB}^2}{R}; E_{AB} = E\frac{R}{R + R_i}$$

$E = 100$ v

$R = 20{,}000$ ohms

$R_i = 5000$ ohms

$$P = \frac{E^2}{R}\left(\frac{R}{R + R_i}\right)^2 = \frac{10^4}{2(10)^4}\left[\frac{2 \times 10^4}{2.5 \times 10^4}\right] = 0.32 \text{ Watts}$$

Maximum power occurs when $\frac{dP}{dR} = 0 \rightarrow R = R_i \quad \therefore R = 5000$ ohms

$$P_{\max} = \frac{E^2}{R}\left(\frac{P}{2R}\right)^2 = \frac{10^4}{2.0 \times 10^4} = 5000 \text{ Watts}$$

When $R = 1000$ ohms and $R_i = 5000$ ohms:

$$P = \frac{10^4}{10^3}\left[\frac{10^3}{6 \times 10^3}\right]^2 = \frac{10}{36}\frac{\text{volts}^2}{\text{ohm}} = 0.278 \text{ Watts}$$

2-11

$m\ddot{x} + kx = 0$

$$\ddot{x} + \frac{k}{m}x = 0 \text{ where } w_n^2 = \frac{k}{m} \rightarrow w_n = \sqrt{\frac{k}{m}}$$

From the static deflection: $k\Delta = mg$ where $\Delta =$ deflection $= 0.5$ cm

$$\frac{k}{m} = \frac{g}{\Delta} \rightarrow w_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{980 \frac{\text{cm}}{\text{sec}^2}}{0.5 \text{ cm}}}$$

$w_n = 44.3$ rad/sec

2-12

$\Delta = 0.25$ inch; $g = 386$ in/sec^2

$$w_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{986 \frac{\text{in.}}{\text{sec}^2}}{0.25 \text{ in.}}} = 39.4 \text{ rad/sec}$$

2-14

$w_n = 39.4$ rad/sec $= 6.27$ Hz

w	$\frac{w}{w_n}$	$\frac{x_0}{\frac{F_0}{k}}$ for $\frac{c}{c_c} = 0$
20	3.19	0.108
40	6.38	0.025
60	9.57	0.011

2-15

$$\frac{dV}{d\tau} = -cV \qquad \frac{V}{V_0} = e^{-c\tau}$$

At $\tau = 0$, $V = 10$ liters, $\frac{dV}{d\tau} = -6$

$c = 0.6$ hr^{-1}

2-16

(1 lbf/in^2)(4.448 N/lbf)(144 in^2/ft^2)(3.28^2 ft^2/m^2) $= 6890$ N/m^2

1 kgf $= 9.806$ N

1 lbf/in^2 $=$ (6890)(9.806) $= 67570$ kgf/m^2 $= 6.757$ kp/cm^2

2-17

$$(\text{mi/gal})(5280 \text{ ft/mi})\left(\frac{1}{231} \text{ gal/in}^3\right)(1728 \text{ in}^3/\text{ft}^3)$$
$$\times (35.313 \text{ ft}^3/\text{m}^3)\left(\frac{1}{1000} \text{ m}^3/\text{l}\right) \times (3.2808 \times 10^{-3} \text{ km/ft}) = 4.576 \text{ km/l}$$

2-18

$$(\text{lbf-s/ft}^2)\left(32.17 \frac{\text{lbm}}{\text{lbf}} \frac{\text{ft}}{\text{s}^2}\right) = 32.17 \text{ lbm/s·ft} \times (0.454 \text{ kg/lbm})(3.2808 \text{ ft/m})$$
$$= 47.92 \text{ kg/m·s}$$

2-19

$$(\text{kJ/kg·°C})\left(\frac{1}{1.055} \frac{\text{Btu}}{\text{kJ}}\right)(0.454 \text{ kg/lbm}) \times \left(\frac{5}{9} \text{°C/°F}\right) = 0.2391 \text{ Btu/lbm·°F}$$

$$(\text{kJ/kg·°C})\left(\frac{1}{4.182} \frac{\text{kcal}}{\text{kJ}}\right)\left(\frac{1}{1000} \frac{\text{kg}}{\text{g}}\right) = 2.391 \times 10^{-4} \text{ kcal/g-°C}$$

2-20

$$(\text{g/m}^3)(0.02832 \text{ m}^3/\text{ft}^3)\left(\frac{1}{454} \text{ lbm/g}\right) \times \left(\frac{1}{32.17} \text{ slug/lbm}\right)$$
$$= 1.939 \times 10^{-6} \text{ slug/ft}^3$$

2-21

$$(\text{Btu/h-ft-°F})(1055\ \text{J/Btu})\left(\frac{1}{3600}\ \text{sec/h}\right) \times (10^7\ \text{erg/J})\left(\frac{9}{5}\text{°F/°C}\right)$$

$$= 5.275 \times 10^6\ \text{erg/s}\cdot\text{ft}\cdot\text{°C} \times \left(\frac{1}{12 \times 2.54}\ \text{ft/cm}\right)$$

$$= 1.731 \times 10^5\ \text{erg/s}\cdot\text{cm}\cdot\text{°C}$$

2-22

$$(\text{cm}^2/\text{s})\left(\frac{1}{2.54 \times 12}\frac{\text{ft}}{\text{cm}}\right)^2 = 1.076\ \text{ft}^2/s$$

2-23

$$(\text{W}/\text{m}^3)\left(3.413\frac{\text{Btu}}{\text{W}\cdot\text{h}}\right)\left(\frac{1}{3.2808}\frac{\text{m}}{\text{ft}}\right)^3 = 0.09664\ \text{Btu/h}\cdot\text{ft}^3$$

2-24

$$(\text{dyn}\cdot\text{s/cm}^2)(10^{-5}\ \text{N/dyn})(0.2248\ \text{lbf/N}) \times (2.54 \times 12\ \text{cm/ft})^2\left(32.17\frac{\text{lbm}}{\text{lbf}}\frac{\text{ft}}{\text{s}^2}\right)$$

$$= 0.0672\ \text{lbm/s}\cdot\text{ft} \times 3600\ \text{s/h}$$

$$= 241.8\ \frac{\text{lhm}}{\text{h}\cdot\text{ft}}$$

2-25

$$\frac{W}{\text{cm}^3} \times \frac{3.413\ \text{Btu/W}\cdot\text{h}}{\left(\frac{1}{2.54}\right)^2 \frac{\text{in}^2}{\text{cm}^2} \times \frac{1}{144}\frac{\text{ft}^2}{\text{in}^2}}$$

$$\frac{W}{\text{cm}^2} \times 3170 = \text{Btu/hr-ft}^2$$

2-26

$$R = 1545\frac{\text{ft-lbf}}{\text{lbm}\cdot\text{mol}\cdot\text{°R}} \times \frac{0.3048\ \frac{\text{m}}{\text{ft}} \times 4.448\ \frac{\text{N}}{\text{lbf}}}{0.454\ \frac{\text{kg}}{\text{lbm}} \times \frac{5}{9}\ \frac{\text{°K}}{\text{°R}}} = 8305\ \frac{\text{J}}{\text{kg mol}\cdot\text{°K}}$$

2-27

$$\frac{\text{cm}^3}{\text{s}} \times \left(\frac{1}{2.54}\right)^3 \frac{\text{in}^3}{\text{cm}^3} \times \frac{1}{231}\frac{\text{gal}}{\text{in}^2}$$

$$\frac{\text{cm}^3}{\text{s}} \times 0.01585 = \text{gal/min}$$

2-28

$$\text{°R} = \frac{9}{5}\text{°K}$$

2-29

$\tau = 105,\ T_0 = 30°\text{C},\ T_\infty = 100°\text{C}$

Rise time $\quad 90\% = 2.303\tau = 23.03\,\text{s}$

$$0.01 = e^{-t/\tau}$$

$$\frac{t}{\tau} = 4.605$$

$$t(99\%) = 46.05 \text{ sec}$$

2-30

$\text{A} = 20°\ \text{C} \qquad \omega = 0.01\,\text{Hz} = 0.0628\ \text{rad/s}$

$$\begin{aligned}\Phi(\omega) &= -\tan^{-1}\omega T \\ &= -\tan^{-1}[(0.0628)(10)] \\ &= -32.14\ \text{deg} \\ &= -0.561\ \text{rad}\end{aligned}$$

$$\Delta t = \frac{\Phi(\omega)}{\omega} = \frac{-0.561}{0.0628} = 8.93\ \text{sec}$$

2-31

$\omega_n = 10{,}000\ \text{Hz} \qquad \dfrac{c}{c_c} = 0.3, 0.4$

For $\dfrac{c}{c_c} = 0.3$, resonance at $\dfrac{\omega}{\omega_n} = 0.9,\ \omega = 9000\ \text{Hz}$

For $\dfrac{c}{c_c} = 0.4$, resonance at $\dfrac{\omega}{\omega_n} = 0.8,\ \omega = 8000\ \text{Hz}$

2-32

$\dfrac{\omega}{\omega_n} = 0.2$ and 0.4 for $\dfrac{c}{c_c} = 0.3$

At 2000 Hz

$$\frac{x_0}{\frac{F_0}{k}} = \frac{1}{\{(1 - 0.2^2)^2 + [(2)(0.3)(0.2)]^2\}^{1/2}} = 1.034$$

$$\Phi = \tan^{-1}\left[\frac{(2)(0.3)(0.2)}{1 - 0.2^2}\right] = 7.13\,\text{deg}$$

At 4000 Hz

$$\frac{x_0}{\frac{F_0}{k}} = \frac{1}{\{[1 - 0.4^2]^2 + [(2)(3)(0.4)]^2\}^{1/2}} = 1.145$$

$$\Phi = \tan^{-1}\left[\frac{(2)(0.3)(0.4)}{1 - 0.4^2}\right] = 15.9\,\text{deg}$$

2-33

$\frac{x_0}{\frac{F_0}{k}} = 0.4$ $\omega_1 = 10$ Hz

$\omega_2 = 50$ Hz

From Fig. 2-6,

For $\frac{c}{c_c} = 1.0$ $\frac{\omega}{\omega_n} > \sim 0.3$

$\omega_n < \frac{10}{0.3} = 33$ Hz

2-34

$\Phi(\omega_1) = -50° = -\tan^{-1}(\omega_1 \tau)$

$\omega_1 \tau = 1.1918$

$\omega_2 \tau = (2)(1.1918) = 2.3835$

$\Phi(\omega_2) = 67.3°$

At ω_1 amp response $\sim \frac{1}{[1 + 1.1918^2]^{1/2}} = 0.643$

At ω_2 amp response $\sim \frac{1}{[1 + 2.384^2]^{1/2}} = 0.387$

2-35

$\omega = 3$ Hz $= 18.85$ rad/s $\tau = 0.5$ sec

$\Phi(\omega) = -\tan - 1[(18.85)(0.5)] = -8.39°$

$\frac{1}{[1 + (\omega\tau)^2]^{1/2}} = 0.1055$

2-36

$T_0 = 35°$ C $T_\infty = 110°$ C $T(8 \text{ sec}) = 75°$ C

$\frac{75 - 110}{35 - 110} = e^{-t/\tau}$

$\frac{t}{\tau} = 0.7621$

$\tau = 810.7621 = 10.497$ sec

90% rise time $= 2.303\tau = 24.174$ sec

2-38

static sens $= 1.0$ V/kgf

output $= (10)(1.0) = 10.0$ V

2-39

rise time $= 0.003$ ms

$e^{-t/\tau} = e^{-\frac{1}{Rc}t} = 0.1$

$\frac{1}{RC} = 7.86 \times 10^5$

$RC = 1.303 \times 10^{-6}$

R in ohm, C in farads

2-42

$\tau = 0.1\text{sec} \qquad T_0 = 100°\text{ C} \qquad T_\infty = 15°\text{ C}$

$T(t) = 17°\text{ C}$

$$\frac{17 - 15}{100 - 15} = e^{-t/0.1}$$

$$\frac{t}{0.1} = 3.75$$

$$t = 0.375\text{ sec}$$

2-43

$$\frac{1}{[1 + (\omega\tau)^2]^{1/2}} = 0.9$$

$\omega\tau = 0.4843 \qquad \omega$ to 4.84 rad/s

$$\Phi(\omega) = -\tan^{-1}(0.4843) = -25.84 = 0.451\text{ rad}$$

$$\Delta t = \frac{0.451}{4.84} = 0.093\text{ sec}$$

2-44

$$\omega = 500\text{ Hz} \qquad \omega_n = 1500\text{ Hz} \qquad \frac{\omega}{\omega_n} = \frac{1}{3}$$

$$0.98 = \frac{1}{\left\{\left[1 - \left(\frac{1}{3}\right)^2\right]^2 + \left[(2)\left(\frac{c}{c_c}\right)\left(\frac{1}{3}\right)\right]^2\right\}^{1/2}}$$

$$\frac{c}{c_c} = 0.619$$

2-45

$t = 1 \times 10^{-6}\text{ sec} = 90\%$ rise time

$$0.1 = e^{-\frac{1}{RC}(1\times10-6)}$$

$RC = 4.34 \times 10^{-7}$

R in ohm, C in farads

2-46

$\Delta t = 2\text{ hr}$

$$\omega = \frac{1}{24}\text{ cyc/hr} = \frac{2\pi}{(24)(3600)} = 9.27 \times 10^{-5}\text{rad/sec}$$

$$\Delta t = 2\text{ hr} = 3600\text{sec} = \frac{\Phi(\omega)}{\omega}$$

$$\Phi(\omega) = 0.2618\text{ rad} = -\tan(\omega\tau)$$

$$\omega\tau = 0.2679$$

$$\tau = \frac{0.2679}{7.27 \times 10^{-5}} = 3685\text{sec} = 1.024\text{ hr}$$

$$\text{Amp. response} = \frac{1}{[1 + (\omega\tau)^2]^{1/2}} = 0.966$$

2-47

$m = 1.3 \text{ kg} \qquad k = 100 \text{ N/m}$

$$\omega_n = \sqrt{\frac{k}{m}} = \left(\frac{100}{1.3}\right)^{1/2} = 8.77 \text{ rad/s}$$

$$c_c = 2\sqrt{mk} = 2[(1.3)(100)]^{1/2} = 72.11$$

$$\frac{c}{c_c} = 1.0$$

From Figure 2-8, $\omega_n t = 3.6$ for 90%.

$$t = \frac{3.6}{8.77} = 0.41 \text{ sec}$$

2-48

$$\frac{c}{c_c} = 0.1$$

From Figure 2-9, $\omega_n t = 3.1$.

$$t = \frac{3.1}{8.77} = 0.353 \text{ sec}$$

2-49

$$\frac{x(t)}{x_0} = 0.9 \qquad \frac{c}{c_c} = 1.5$$

From Figure 2-9 $\omega_n t = 6.2, \quad t = \frac{6.2}{8.77} = 0.71 \text{ sec}.$

2-50

$t = 1 \text{ sec}, \quad \omega_n t = 8.77$

$$c = 5.7 \qquad \frac{c}{c_c} = \frac{5.7}{72.11} = 0.79$$

From Figure 2-9, $\frac{x}{x_0} \approx 1.8$.

2-51

Rise time $= 10^{-12}$ s

At $\frac{c}{c_c} = 1.0$ and $\frac{x}{x_0} = 0.9$, $\omega_n t \sim 3.6$, $\omega_n = 3.7 \times 10^{12}$ rad/s

$$f = 5.7 \times 10^{11} \text{ Hz} = 570 \text{ GHz}$$

2-52

$m = 1000 \text{ lbm} = 2203 \text{ kg}$

$$k = \frac{1000 \text{ lbf}}{\frac{1.5}{12}} = 8000 \text{ lbf/ft} = 117{,}000 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \left(\frac{117{,}000}{2203}\right)^{1/2} = 7.29 \text{ rad/s}$$

At $\frac{x}{x_0} = 0.9 \qquad \frac{c}{c_c} = 1.0 \qquad \omega_n t \approx 3.6$

$$t = \frac{3.6}{7.29} = 0.495 \text{ s}$$

2-53

$T_0 = 20°C \quad T_\infty = 125°C \quad t = 0.05 \text{ sec}$

$e^{-t/\tau} = 0.1$

$\tau = 34.54 \text{ sec}$

$$\frac{T(t) - 125}{20 - 125} = \exp\left(-\frac{t}{\tau}\right) = e^{-\left(\frac{0.05}{34.54}\right)}$$

$T(t) = 20.15°C$

2-54

$$1\frac{\text{kg}}{\text{m}\cdot\text{s}} = 0.02088 \text{ lbf}\cdot\text{sec/ft}^2$$

2-55

English units

$\rho = \text{lbm/ft}^3, u = \text{ ft/sec}$

$x = \text{ft}, \mu = \text{lbm/s-ft}$

SI units

$\rho = \text{kg/m}^3, u = \text{m/s}$

$x = \text{m}, \mu = \text{kg/m-s}$

2-56

SI system

$g = \text{m/s}^2,\ \beta = 1\ /\ °\text{C},\ \rho = \text{kg/m}^3$

$\Delta T = °\text{C},\ x = \text{m},\ \mu = \text{kg/m-s}$

English system

$g = \text{ft/s}^2,\ \beta = 1/°\text{F},\ \rho = \text{lbm/ft}^3$

$\Delta T = °\text{F},\ x = \text{ft}, \mu = \text{lbm/ft-s}$

2-57

$$\frac{W\text{-cm}}{\text{in}^2\text{-°F}} \times \frac{0.01 \text{ m/cm}}{(2.54 \text{ cm/in.})^2(0.01 \text{ m/cm})^2\left(\frac{5}{9}°\text{C/°F}\right)}$$

$$\frac{W\text{-cm}}{\text{in}^2\text{-° F}} \times 27.9 = \text{W/m-°C}$$

2-58

$T_0 = 45,\ T_\infty = 100$ rise time $= 0.2$ s $T(0.1\text{s}) = ?$

$0.2 = 2.303\tau,\ \tau = 0.0868 \text{ s}$

$(T - 100)/(45 - 100) = \exp(-0.1/0.0868) = 0.316$

$T = 82.6°\text{C}$

2-59

$m = 6000/4 = 1500 \text{ lbm} = 3303 \text{ kg}$

$k = 1500/(1/12) = 18{,}000 \text{ lb/ft} = 263{,}250 \text{ N/m}$

$\omega_n = (263250/3303)^{1/2} = 8.93 \text{ rad/s}$

For critically damped system:

$0.9 = 1 - (1 + \omega_n t)\exp(-\omega_n t)$

Solution is $\omega_n t = 3.8901$

$t = (3.8901)(8.93) = 0.435$ s

2-60

$\omega = 400$ Hz, $\omega_n = 1200$ Hz $\omega/\omega_n = 400/1200 = 1/3$

$0.98 = 1/\{[1 - (1/3)^2]^2 + [(2)(c/c_c)(1/3)]^2\}^{1/2}$

$c/c_c = 0.619$

2-61

Insert function in Equations (2-38) and (2-39), manipulate algebra and the indicated result will be given.

2-62

$\Delta t = 1.5$ h, $\omega = 1/24$ cyc/h $= 2\pi/(24)(3600) = 7.27 \times 10^{-5}$ rad/sec

$(1.5)(3600) = 5400$ s $= \phi(\omega)/\omega$

$\phi(\omega) = (5400)(0.0000727) = 0.3926 = -\tan^{-1}(\omega\tau)$

$\omega\tau = 0.414$

$\tau = 0.414/7.27 \times 10^{-5} = 5695$ s $= 1.58$ h

2-63

$T_0 = 45 \quad T_\infty = 100 \quad T(6\text{s}) = 70$

$(70 - 45)/(100 - 45) = \exp(-6/\tau)$

$\tau = 7.61$ s

For 90% rise time $\exp(-\tau/7.61) = 0.1$

Rise time $= 17.52$ s

2-64

Insert function in Equations (2-38) and (2-39), manipulate algebra to give the indicated result.

2-65

$\tau = 8$ s, $T_0 = 40$, $T_\infty = 100$

Rise time $= 2.303\tau = 18.424$ s

$T(99\%) = 99°$ C

$(99 - 100)/(40 - 100) = \exp(-t/8)$

$t = 32.75$ s

2-66

$\omega = 5$ Hz, $\tau = 0.6$ s

$\omega = (2\pi)(5) = 31.4$ rad/s

$\phi(\omega) = -\tan^{-1}(\omega\tau) = -8.7°$

$1/[1 + (\omega\tau)^2]^{1/2} = 0.053$

2-67

$A = 15$, $\omega = 0.01$ Hz $= 0.0628$ rad/s

$\phi(\omega) = -\tan^{-1}\omega\tau = -\tan^{-1}(0.0628)(8) = -26.7°$

Attenuation $= 1/[1 + (\omega\tau)^2]^{1/2} = 0.894$

Chapter 3

3-1

READING	x_i	$d_i = x_i - x_m$	$(x_i - x_m)^2 \times 10^2$
1	100.0	–0.09	0.81
2	100.9	0.81	65.61
3	99.3	–0.79	62.41
4	99.9	–0.19	3.61
5	100.1	0.01	0.01
6	100.2	0.11	1.21
7	99.9	–0.19	3.61
8	100.1	0.01	0.01
9	100.0	–0.09	0.81
10	100.5	0.41	16.81
	1000.9		154.90

$$x_m = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{10}(1000.9)$$

$$x_m = 100.09$$

$$\sigma = \left[\frac{\sum_{i=1}^{n}(x_i - x_m)^2}{n-1}\right]^{1/2} = \left[\frac{1}{10-1}(1.549)\right]^{1/2}$$

$$\sigma = 0.414;\ \text{all } \frac{d_{\max}}{\sigma} < 1.96 \text{ (Table 3.4)}$$

$$\therefore \sigma_m = \frac{\sigma}{\sqrt{n}} = \frac{0.414}{\sqrt{10}} = 0.131$$

By Table 3.3, we have:

$$\begin{aligned} x_m &= 100.09 - 0.131 \text{ (2.15 to 1)} \\ &= 100.09 \pm 0.262 \text{ (21 to 1)} \\ &= 100.09 \pm 0.393 \text{ (356 to 1)} \end{aligned}$$

3-2

$$P = VI = (110.2)(5.3) = 584.06 \text{ Watts}$$

$$P = P(V, I)$$

$$W_P = \left[\left(\frac{\partial P}{\partial V} W_r\right)^2 + \left(\frac{\partial P}{\partial I} W_I\right)^2\right]^{1/2}$$

$$W_P = \{[(5.3)(0.2)]^2 + [(110.2)(0.06)]^2\}^{1/2}$$

$$W_P = 6.7 \text{ Watts or } 1.149\%$$

3-3

$A = WL \qquad \frac{\partial A}{\partial W} = L \qquad \frac{\partial A}{\partial L} = W$

$$W_A = \left[\left(\frac{\partial A}{\partial W} W_w\right)^2 + \left(\frac{\partial A}{\partial L} W_L\right)^2\right]^{1/2}$$

$W_A = \{[(150)(0.01)]^2 + [(50)(0)]^2\}^{1/2}$

$W_A = 1.5$

We want to find W_L with $W_A = 150\%$ or 1.5

$\therefore 2.25 = \{[(150)(0.01)]^2 + [(50)(W_L)]^2\}^{1/2}$

$W_L = \pm 0.0336$ ft

3-4

Series

$R_T = R_1 + R_2 \qquad \frac{\partial R_T}{\partial R_1} = 1; \frac{\partial R_T}{\partial R_2} = 1$

$$W_{RT} = \left[\left(\frac{\partial R_T}{\partial R_1} W_{R_1}\right)^2 + \left(\frac{\partial R_T}{\partial R_2} WR_2\right)^2\right]^{1/2}$$

$W_{RT} = \{[(1)(0.1)]^2 + [(1.0)(0.03)]^2\}^{1/2} = 0.1044$ ohms

Parallel

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{\partial R_T}{\partial R_1} = \frac{(R_1 + R_2)(R_2) - R_1 R_2}{(R_1 + R_2)^2}; \frac{\partial R_T}{\partial R_2} = \frac{(R_1 + R_2)R_1 - R_1 R_2}{(R_1 + R_2)^2}$$

$$W_{R_T} = \left[\left(\frac{R_2^2}{(R_1 + R_2)^2} W_{R_1}\right)^2 + \left(\frac{R_1^2}{(R_1 + R_2)^2} WR_2\right)^2\right]^{1/2}$$

$$= \left\{\left[\frac{2500}{22,500}(0.01)\right]^2 + \left[\frac{10,000}{22,500}(0.03)\right]^2\right\}^{1/2}$$

$W_{RT} = 0.01338$ ohms

3-5

25 ohm series arrangement

$W_R = \{2[(1)(0.002)]^2\}^{1/2}$

$W_R = 0.0283 \leftarrow \therefore$ lowest uncertainty should be used.

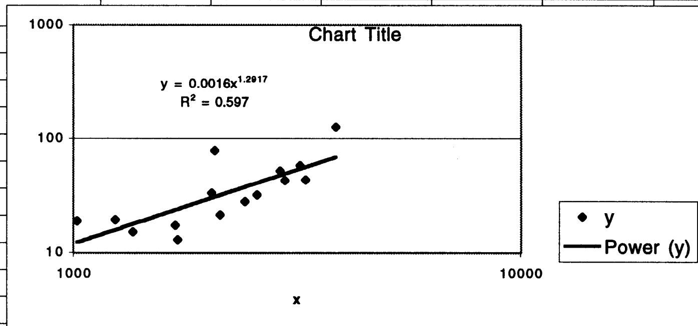
100 ohm parallel arrangement

$R_{||} = \frac{R_1 R_2}{R_1 + R_2}$ but $R_1 = R_2 \therefore R_{||} = \frac{R}{2}$

$$\frac{\partial R_{||}}{\partial R_1} = \frac{\partial R_{||}}{\partial R_2} = \frac{(R_1 + R_2)R_2 - R_1 R_2}{(R_1 + R_2)^2} = \frac{20,000 - 10,000}{40,000} = \frac{1}{4}$$

$$W_{R_{||}} = \left\{2\left[\left(\frac{1}{4}\right)(0.1)\right]^2\right\}^{1/2} = 0.0354 \text{ ohms}$$

x	y
2040	33.2
2580	32
2980	42.7
3220	57.8
3870	126
1690	17.4
2130	21.4
2420	27.8
2900	52.1
3310	43.1
1020	18.8
1240	19.2
1360	15.1
1710	12.9
2070	78.5



From (2)

$$2ab\sum_{i=1}^{n}\ln x_i + a\sum_{i=1}^{n}\ln(\ln x_i) = \sum_{i=1}^{n}\ln y_i + b\sum_{i=1}^{n}\ln x_i + \sum_{i=1}^{n}\ln(\ln x_i)$$

By solving simultaneously:

$$a = 1;\ b = \frac{\sum_{i=1}^{n}\ln y_i}{\sum_{i=1}^{n}\ln x_i} = \frac{52.07}{115.13} = 0.452$$

$$y_c = x^{0.452}$$

y	x	$\ln y$	$\ln x$	y_c	d_i	$\lvert d_i \rvert$
33.2	2040	3.50	7.63	31.2	2.0	2.0
32.0	2580	3.46	7.85	35.0	−3.0	3.0
42.7	2980	3.74	7.99	37.0	5.7	5.7
57.8	3220	4.05	8.08	38.5	19.3	19.3
126.0	3870	4.84	8.26	42.0	84.0	84.0
17.4	1690	2.86	7.42	28.6	−11.2	11.2
21.4	2130	3.06	7.66	32.0	−10.6	10.6
27.8	2420	3.32	7.80	34.0	−6.2	6.2
52.1	2900	3.95	7.97	36.8	15.3	15.3
43.1	3310	3.76	8.10	38.8	4.3	4.3
18.8	1020	2.94	6.94	23.0	−4.2	4.2
19.2	1240	2.95	7.13	25.0	−5.8	5.8
15.1	1360	2.72	7.21	26.0	−10.9	10.9
12.9	1710	2.56	7.45	29.0	−16.1	16.1
78.5	2070	4.36	7.64	31.8	46.7	46.7
		52.07	115.13			245.3

$$d_i = y_i - y_c$$

$$|\overline{d_i}| = \frac{1}{n}\sum_{i=1}^{n}|\overline{d_i}| = \frac{1}{n}\sum_{i=1}^{n}|y_i - y_c| = \frac{1}{15}[245.3]$$

$$|\overline{d_i}| = 16.4$$

3-7

$$\sigma = \left[\frac{\sum_{i=1}^{n}(x_i - x_m)^2}{n-1}\right]^{1/2} = \left(\frac{61.08}{9}\right)^{1/2}$$

$$\sigma = 2.6$$

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-x_m)^2/2\sigma^2}$$

$$P(x_m) = \frac{1}{\sigma\sqrt{2\pi}} = \frac{1}{2.6(2.5)}$$

$$P(x_m) = 0.154 \leftarrow \text{not a very good player}$$

$$x_m = \frac{1}{n}\sum x_i = \frac{1}{10}\sum(30 + d_i)$$

$$x_m = 30.6 \leftarrow \text{the player should toss easier}$$

Toss	Deviation	$(x_i - x_m)^2$
1	0	0
2	+3	9.00
3	−4.2	17.70
4	0	0
5	+1.5	2.25
6	+2.4	5.78
7	−2.6	6.80
8	+3.5	12.25
9	+2.7	7.30
10	0	0
		61.08

3-10

$$P = \frac{1}{\sqrt{2\pi}}\int_{-\eta_1}^{+\eta_1} e^{-\eta^2/2}d\eta \text{ where } \eta_1 = \frac{x_1}{\sigma}$$

$$0.20 = \frac{1}{\sqrt{2\pi}}\int_0^{\eta_1} e^{-\eta^2/2}d\eta \leftarrow \text{interpolating in Table 3-2:} \frac{153}{347} = 0.441$$

$$\therefore \eta_1 = 0.5244$$

$$\sigma = \frac{x_1}{\eta_1} = 0.955$$

$\eta = \dfrac{0.75}{0.955} = 0.785 \leftarrow$ using Table 3.2 we find

$P = (2)(0.284) = 0.568$

$\therefore$ Probability of error of 0.75 volt $= 43.2\%$

3-11

For 5% F's and 5% A's

$P = 2\left(\dfrac{1}{\sqrt{2\pi}}\right)\displaystyle\int_0^{\eta_1} e^{-\eta^2/2}\,d\eta = 0.90 \leftarrow$ Table 3.2: $\eta_1 = 1.645$

$\sigma = \dfrac{x_1}{\eta_1}$ where $x_1 = 15, \sigma = 9.12$

if $x_1 = 5, \eta_1 = 0.548 \rightarrow$ Table 3.2

$P = 2(0.20797) = 0.416$

$\therefore$ 5% A's and F's; 24.2% B's and D's; 41.6% C's

For 10% A's and F's

$P = 0.80 \rightarrow$ Table 3.2 $\rightarrow \eta_1 = 1.282$

$\sigma = \dfrac{15}{1.282} = 11.7$

if $x_1 = 5, \eta_1 = 0.428 \rightarrow$ Table 3.2

$P = 2(0.16567) = 0.3314$

$\therefore$ 10% A's and F's; 23.43: B's or D's; 33.14% C's

For 15% A's and F's

$P = 0.70 \rightarrow$ Table 3.2 $\rightarrow \eta_1 = 1.036$

$\sigma = \dfrac{15}{1.036} = 14.5$

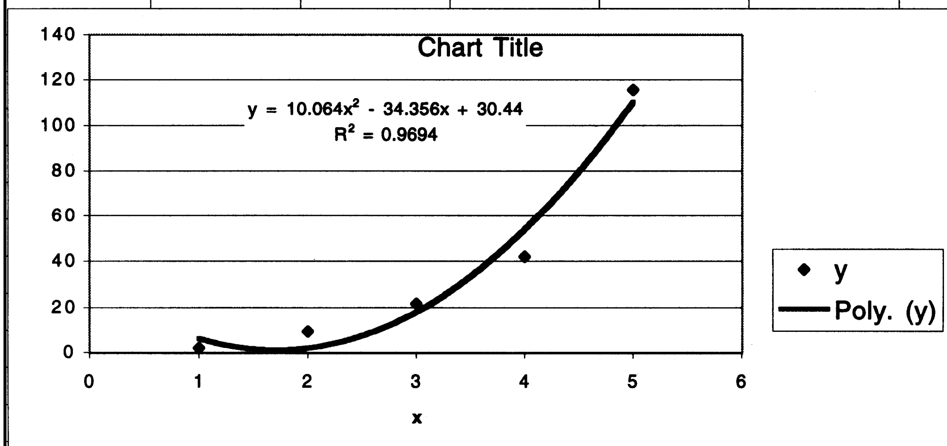
if $x_1 = 5; \eta_1 = 0.345 \rightarrow$ Table 3.2

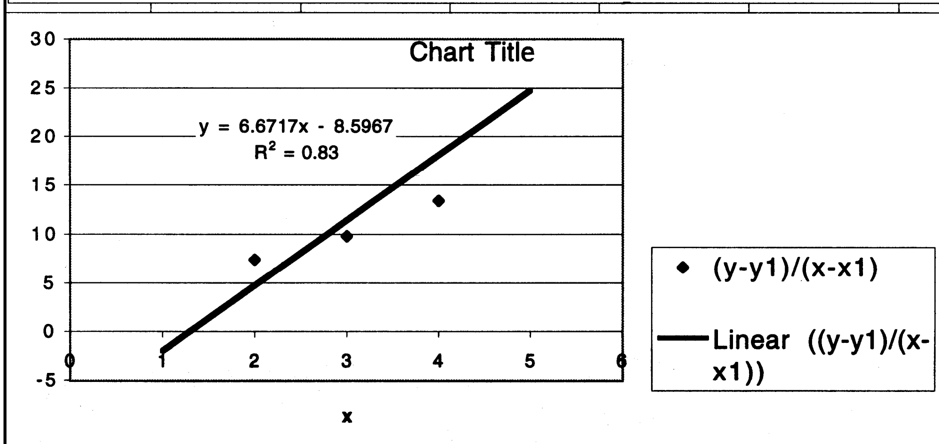
$P = 2(0.13495) = 0.270$

$\therefore$ 15% A's and F's; 21.5% B's and D's; 27% C's

3-12

x	y	x-x1	y-y1	(y-y1)/(x-x1)
1	1.9	0	0	
2	9.3	1	7.4	7.4
3	21.5	2	19.6	9.8
4	42	3	40.1	13.36667
5	115.7	4	113.8	28.45





3-13

Temperature	Total production	Number rejected	Number non-rejected
T_1	150	12	138
T_2	75	8	67
T_3	120	10	110
T_4	200	13	187
	545	43	502

Apply Chi-Square Test:

$F = n - k \quad F = 3 \qquad n = 8$ (number of restrictions from number of observations)

$k = 5$

$$\text{expected rejection rate} = \frac{43}{545} = 0.079$$

$$\text{expected non-rejection rate} = \frac{502}{545} = 0.921$$

$$\chi^2 = \frac{\left(\frac{12}{150} - 0.079\right)^2}{0.079} + \frac{\left(\frac{8}{75} - 0.079\right)^2}{0.079} + \frac{\left(\frac{10}{120} - 0.079\right)^2}{0.079} + \frac{\left(\frac{13}{200} - 0.079\right)^2}{0.079}$$

$$+ \frac{\left(\frac{138}{150} - 0.921\right)^2}{0.921} + \frac{\left(\frac{67}{75} - 0.921\right)^2}{0.921} + \frac{\left(\frac{110}{120} - 0.921\right)^2}{0.921} + \frac{\left(\frac{187}{200} - 0.921\right)^2}{0.921}$$

$$= [0.1266 + 96.89 + 2.38 + 24.81 + 0.0109 + 8.3110 + 0.2039 + 2.13] \times 10^{-4}$$

$$\chi^2 = 134.8624 \times 10^{-4} = 0.0135$$

From Table 3-5 $\rightarrow$ $p > 0.995$

We conclude that the rejection rate is dependent on the temperature at which the cups are molded.

3-16

	A	B	C	D
1				
2				
3				
4	**Read**	**x**	**Dev**	**Dev/sigma**
5	1	49.36	=B5-$C$17	=C5/$C$18
6	2	50.12	=B6-$C$17	=C6/$C$18
7	3	48.98	=B7-$C$17	=C7/$C$18
8	4	49.24	=B8-$C$17	=C8/$C$18
9	5	49.26	=B9-$C$17	=C9/$C$18
10	6	50.56	=B10-$C$17	=C10/$C$18
11	7	49.18	=B11-$C$17	=C11/$C$18
12	8	49.89	=B12-$C$17	=C12/$C$18
13	9	49.33	=B13-$C$17	=C13/$C$18
14	10	49.39	=B14-$C$17	=C14/$C$18
15				
16				
17		MEAN=	=AVERAGE(B5:B14)	
18		STDEV=	=STDEV(B5:B14)	

	A	B	C	D
1				
2				
3				
4	**Read**	**x**	**Dev**	**Dev/sigma**
5	1	49.36	-0.171	-0.34515
6	2	50.12	0.589	1.188858
7	3	48.98	-0.551	-1.11216
8	4	49.24	-0.291	-0.58736
9	5	49.26	-0.271	-0.547
10	6	50.56	1.029	2.076969
11	7	49.18	-0.351	-0.70847
12	8	49.89	0.359	0.724618
13	9	49.33	-0.201	-0.40571
14	10	49.39	-0.141	-0.2846
15				
16				
17		MEAN=	49.531	
18		STDEV=	0.495434	

	A	B	C	D	E
1			Eliminate		
2			point 6		
3					
4	**Read**	**x**	**Dev**	**Dev/sigma**	
5	1	49.36	-0.05667	-0.15773	
6	2	50.12	0.703333	1.957673	
7	3	48.98	-0.43667	-1.21543	
8	4	49.24	-0.17667	-0.49174	
9	5	49.26	-0.15667	-0.43607	
10	6				
11	7	49.18	-0.23667	-0.65874	
12	8	49.89	0.473333	1.317486	
13	9	49.33	-0.08667	-0.24123	
14	10	49.39	-0.02667	-0.07422	
15					
16					
17		MEAN=	49.41667		
18		STDEV=	0.35927		
19					
20		Uncertainty = 0.35927/9^0.5 =			
21		0.11976			

3-20

$n = 100 \qquad x_m = 6.826 \text{ ft} \qquad \sigma = 0.01 \text{ ft}$

a. $\pm 0.005 \text{ ft} = \pm \frac{1}{2}\sigma$

$P\left(\frac{1}{2}\right) = (2)(0.19146) = 0.38292$

38.92 out of 100 readings.

b. $\pm 0.02 \text{ ft} = \pm 2\sigma$

$P(2) = (2)(0.47725) = 0.9545$

95.45 out of 100 readings

c. $\pm 0.05 \text{ ft} = \pm 5\sigma$

$P(5) = (2)(0.49999) = 0.99998$

100 out of 100 readings

d. $\pm 0.001 \text{ ft} = \pm 0.1\sigma$

$P(0.1) = (1)(0.0398) = 0.0398$

3.98 out of 100 readings

3-22

$$R_{\text{Total}} = R_1 + R_2 = (10^4 + 10^6) \text{ ohms} = 1.01 \times 10^6 \text{ ohms}$$

$$W_{R_{\text{Total}}} = \left[\left(\frac{\partial R_{\text{Total}}}{\partial R_1} W_{R_1}\right)^2 + \left(\frac{\partial R_{\text{Total}}}{\partial R_2} W_{R_2}\right)^2\right]^{1/2}$$

$$\frac{\partial R_{\text{Total}}}{\partial R_1} = 1 \qquad \frac{\partial R_{\text{Total}}}{\partial R_2} = 1 \qquad W_{R_1} = (10^4)(0.05) = 500 \text{ ohms}$$

$$W_{R_2} = (10^6)(0.10) = 10^5 \text{ ohms} \qquad W_{R_{\text{Total}}} = 100{,}000 \text{ ohms}$$

$$W_{R_{\text{Total}}} = \frac{1.0 \times 10^5}{1.01 \times 10^6} \times 100 = 9.9\%$$

3-23

$$x_m = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{10}(46.45) = 4.645 \text{ cm}$$

Reading	x, cm	$d_i = x_i - x_m$	$(x_n - x_m)^2 \times 10^4$
1	4.62	−0.025	6.25
2	4.69	0.045	20.25
3	4.86	0.215	462.25
4	4.53	−0.115	132.25
5	4.60	−0.045	20.25
6	4.65	0.005	0.25
7	4.59	−0.055	30.25
8	4.70	0.055	30.25
9	4.58	−0.065	42.25
10	4.65	−0.015	2.25
Σ	46.45		746.50

$$\sigma = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - x_m)^2\right] = \left[\frac{1}{9}(746.50 \times 10^{-4})\right]^{1/2} = 0.0911 \text{ cm}$$

The acceptable maximum value of $\dfrac{d_i}{\sigma} = 1.96$ (Table 3.4)

At reading $3\dfrac{d_i}{\sigma} = 2.49 \rightarrow \therefore$ reading 3 can be eliminated.

With the elimination of reading 3: $x_m = \dfrac{1}{9}(41.59) = 4.621$ cm.

Reading	$d_i = x_i - x_m$	$(x_n - x_m)^2 \times 10^4$
1	−0.001	0.01
2	0.069	47.61
3	−0.091	82.81
4	−0.091	82.81
5	−0.021	4.41
6	0.029	8.41
7	−0.031	9.61
8	0.079	62.41
9	−0.041	16.81
10	0.009	0.81
Σ		232.89

$$\sigma = \left[\frac{1}{8}(232.89 \times 10^{-4})\right]^{1/2}$$

$$\sigma = 0.0539 \text{ cm}$$

3-25

Group	Service Calls	Expected Calls
A	10	8
B	6	8
	16	16

$$X^2 = \sum_{i=1}^{n} \frac{[(\text{observed value})_i - (\text{expected value})_i]^2}{(\text{expected value})_i}$$

$F = n - k \qquad n = 2 =$ number of observations $\qquad K = 1 =$ number of restrications

$$X^2 = \frac{(10-8)^2}{8} + \frac{(6-8)^2}{9} = \frac{1}{2} + \frac{1}{2} = 1.0$$

From Table 3.5, $P = 0.3425$

$\therefore$ There is a 34.25% change that Group A was harder on the equipment than Group B.

3-26

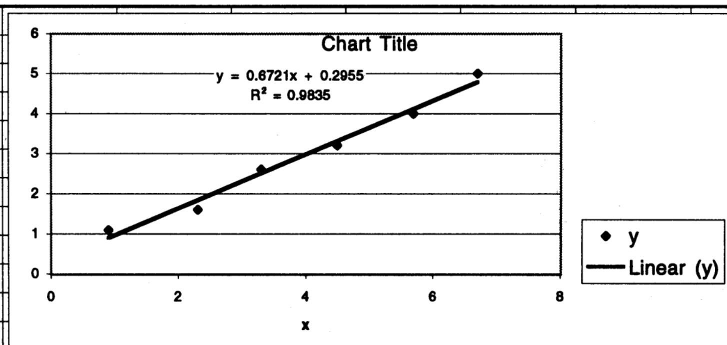
Read	Viscos
1	0.04
2	0.041
3	0.041
4	0.042
5	0.039
6	0.04
7	0.043
8	0.041
9	0.039

MEAN =	0.040667
STDEV =	0.001323
VAR =	1.75E-06

3-27

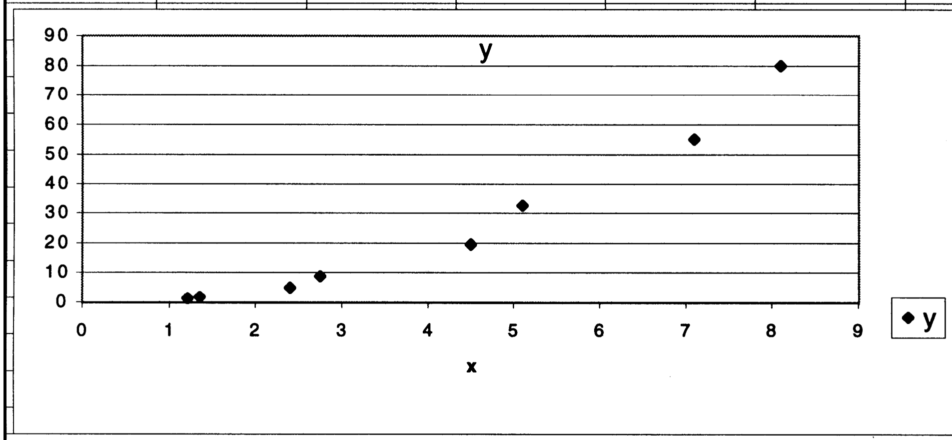
Least square bu spreadsheet

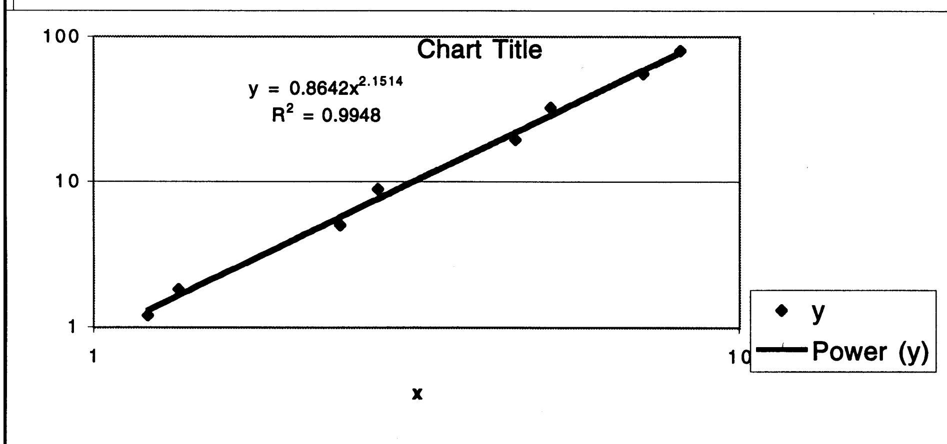
x	y
0.9	1.1
2.3	1.6
3.3	2.6
4.5	3.2
5.7	4
6.7	5



3-28

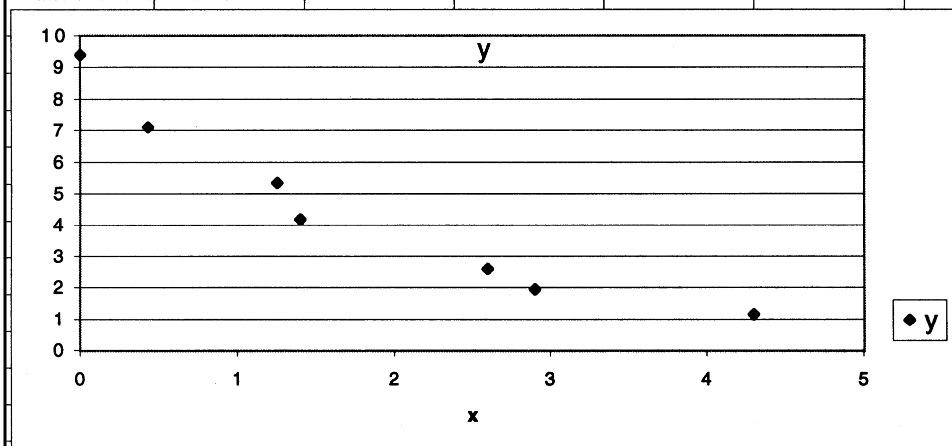
x	y
1.21	1.2
1.35	1.82
2.4	5
2.75	8.8
4.5	19.5
5.1	32.5
7.1	55
8.1	80

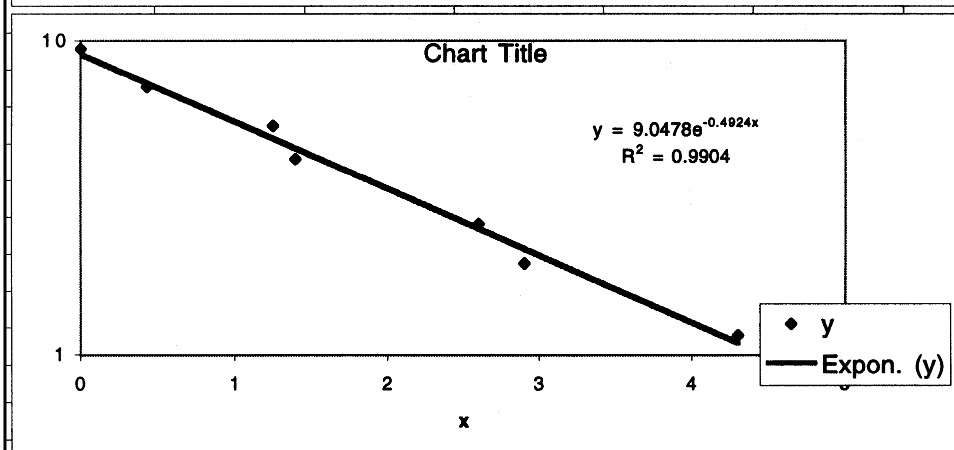




3-29

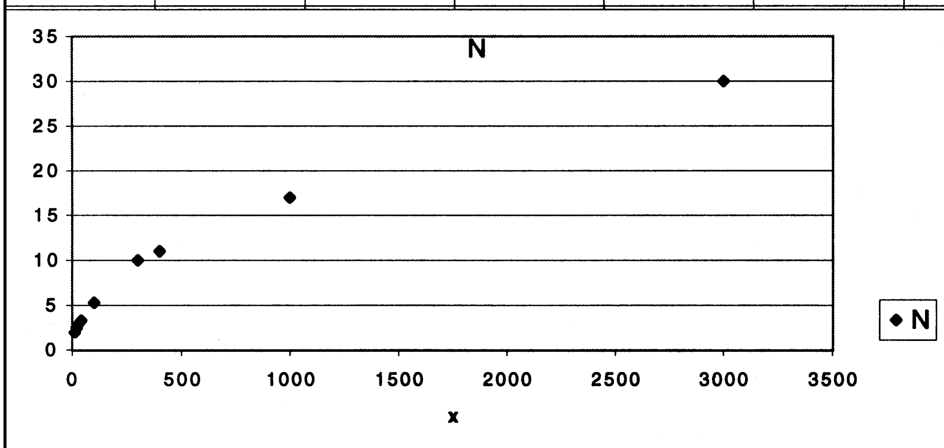
x	y
0	9.4
0.43	7.1
1.25	5.35
1.4	4.2
2.6	2.6
2.9	1.95
4.3	1.15

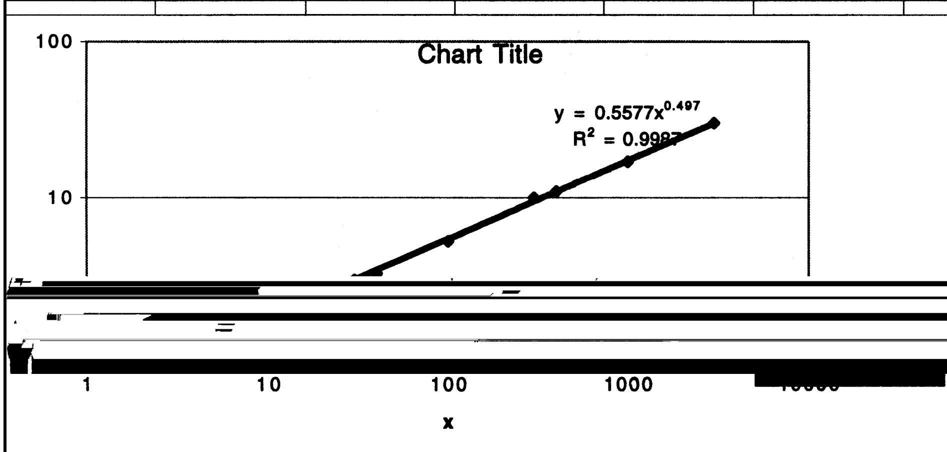




3-30

R	N
12	2
20	2.5
30	3
40	3.3
100	5.3
300	10
400	11
1000	17
3000	30





3-31

	A	B	C	D
1				
2				
3	**Rd No**	**Res**	**Dev**	**Dev/sigma**
4	1	12	=B4-$C$18	=C4/$C$19
5	=A4+1	12.1	=B5-$C$18	=C5/$C$19
6	=A5+1	12.5	=B6-$C$18	=C6/$C$19
7	=A6+1	12.8	=B7-$C$18	=C7/$C$19
8	=A7+1	13.6	=B8-$C$18	=C8/$C$19
9	=A8+1	13.9	=B9-$C$18	=C9/$C$19
10	=A9+1	12.2	=B10-$C$18	=C10/$C$19
11	=A10+1	11.9	=B11-$C$18	=C11/$C$19
12	=A11+1	12	=B12-$C$18	=C12/$C$19
13	=A12+1	12.3	=B13-$C$18	=C13/$C$19
14	=A13+1	12.1	=B14-$C$18	=C14/$C$19
15	=A14+1	11.85	=B15-$C$18	=C15/$C$19
16				
17				
18		MEAN=	=AVERAGE(B4:B15)	
19		STDEV=	=STDEV(B4:B15)	

	A	B	C	D
1				
2				
3	**Rd No**	**Res**	**Dev**	**Dev/sigma**
4	1	12	-0.4375	-0.65247
5	2	12.1	-0.3375	-0.50334
6	3	12.5	0.0625	0.093211
7	4	12.8	0.3625	0.540622
8	5	13.6	1.1625	1.733719
9	6	13.9	1.4625	2.18113
10	7	12.2	-0.2375	-0.3542
11	8	11.9	-0.5375	-0.80161
12	9	12	-0.4375	-0.65247
13	10	12.3	-0.1375	-0.20506
14	11	12.1	-0.3375	-0.50334
15	12	11.85	-0.5875	-0.87618
16				
17				
18		MEAN=	12.4375	
19		STDEV=	0.670524	

	A	B	C	D	E
1		Eliminate			
2		point 6			
3	**Rd No**	**Res**	**Dev**	**Dev/sigma**	
4	1	12	-0.30455	-0.59586	
5	2	12.1	-0.20455	-0.4002	
6	3	12.5	0.195455	0.382416	
7	4	12.8	0.495455	0.969381	
8	5	13.6	1.295455	2.53462	
9	6				
10	7	12.2	-0.10455	-0.20455	
11	8	11.9	-0.40455	-0.79151	
12	9	12	-0.30455	-0.59586	
13	10	12.3	-0.00455	-0.00889	
14	11	12.1	-0.20455	-0.4002	
15	12	11.85	-0.45455	-0.88934	
16					
17					
18		MEAN=	12.30455		
19		STDEV=	0.511104		
20					
21		Uncertainty = 0.511104/11^.5 =			
22		0.1541			

3-35

$$\frac{W_P}{P} = \left[4\left(\frac{W_E}{E}\right)^2 + \left(\frac{W_R}{R}\right)^2\right]^{1/2}$$

$$0.01414 = \left[(4)(0.01)^2 + \left(\frac{W_R}{R}\right)^2\right]^{1/2}$$

$$\frac{W_R}{R} = [-2.0006 \times 10^{-4}]^{1/2}$$

Impossible to achieve the 1.4% with this method.

3-36

Nominal value

$$\dot{m} = (0.92)(1.0\ \text{in}^2)\left[\frac{(2)\left(32.17\,\frac{\text{lbm}}{\text{inf}}\cdot\frac{\text{ft}}{\text{s}^2}\right)\left(25\,\frac{\text{Ibf}}{\text{in}^2}\right)}{\left(53.35\,\frac{\text{lbf}\cdot\text{ft}}{\text{lbm}\cdot{}^\circ\text{R}}\right)(530^\circ\text{R})} \times 1.4\,\frac{\text{lbf}}{\text{in}^2}\right]^{1/2}$$

$$= 0.25963\ \text{lbm/s}$$

$$\dot{m}(c + 0.001) = \left(\frac{0.921}{0.92}\right)\dot{m} = 1.001086\dot{m}$$

$$\frac{\partial \dot{m}}{\partial c} = \frac{0.001086\dot{m}}{0.201} = 1.08696\dot{m}$$

$$\dot{m}(A + 0.001) = 1.001\dot{m}$$

$$\frac{\partial \dot{m}}{\partial A} = \frac{0.001\dot{m}}{0.001} = 1.0\dot{m}$$

$$\dot{m}(p + 0.01) = \dot{m}\left[\frac{25.01}{25}\right]^{1/2} = 1.000199\dot{m}$$

$$\frac{\partial \dot{m}}{\partial p} = \frac{0.000199}{0.01}\dot{m} = 0.02\dot{m}$$

$$\dot{m}(\Delta p + 0.001) = \dot{m}\left[\frac{1.401}{1.4}\right]^{1/2} = 1.000357\dot{m}$$

$$\frac{\partial \dot{m}}{\partial \Delta p} = \frac{0.000357\dot{m}}{0.001} = 0.357\dot{m}$$

$$\dot{m}(T + 0.1) = \dot{m}\left[\frac{530}{530.1}\right]^{1/2} = 0.999906\dot{m}$$

$$\frac{\partial \dot{m}}{\partial T} = -9.4326 \times 10^{-4}$$

$$W\dot{m} = \dot{m}[(1.08696)^2(0.005)^2 + (1)^2(0.001)^2 + (0.02)^2(0.5)^2 + (0.357)^2(0.005)^2 + (-9.433 \times 10^{-4})^2(2)^2]^{1/2}$$

$$= 0.01172\dot{m}$$

$$= 1.172\%$$

3-37

Example 3.2

$W_E = 1.0 \qquad W_I = 0.1 \qquad W_R = 0.1$

a. $P = \dfrac{E^2}{R}$

$$\text{Nominal } P = \frac{(100)^2}{10} = 1000 \text{ W}$$

$$P(E + 0.1) = P\left(\frac{100.1}{100}\right)^2 = 1.002001P$$

$$\frac{\partial P}{\partial E} = \frac{0.002001}{0.1} = 0.02001P$$

$$P(R + 0.01) = P\left(\frac{10}{10.01}\right) = 0.9990009P$$

$$\frac{\partial P}{\partial R} = \frac{0.99P - P}{0.01} = -0.0999P$$

$$W_P = P[(0.02001)^2(1.0)^2 + (-0.0999)^2(0.1)^2]^{1/2}$$

$$= 0.02236P$$

$$= 2.236\%$$

b. $P = EI$

$$P(E + 0.1) = P\left(\frac{100.1}{100}\right) = 1.001P$$

$$\frac{\partial P}{\partial E} = 0.01$$

$$P(I + 0.01) = P\left(\frac{10.01}{10}\right) = 1.001P$$

$$\frac{\partial P}{\partial I} = 0.1P$$

$$W_P = P[(0.01)^2(1)^2 + (0.1)^2(0.1)^2]^{1/2}$$
$$= 0.01414P$$
$$= 1.414\%$$

Example 3.3

$$P = EI - \frac{E^2}{R_m} \qquad W_E = 5, W_I = 0.05, W_{RM} = 50$$

$$P(E+1) = P\left[\frac{(501)(5) - \frac{(501)^2}{1000}}{2250}\right]$$
$$= 1.001777P$$
$$\frac{\partial P}{\partial E} = 0.001777P$$

$$P(I+0.01) = P\left[\frac{(500)(5.01) - \frac{(500)^2}{1000}}{2250}\right]$$
$$= 1.002222P$$
$$\frac{\partial P}{\partial I} = 0.2222P$$

$$P(R_m+1) = P\left[\frac{(500)(5) - \frac{500^2}{1001}}{2250}\right]$$
$$= 1.000111P$$
$$\frac{\partial P}{\partial R_m} = 0.000111P$$

$$W_P = P[(0.001777)^2(5)^2 + (0.2222)^2(0.05)^2 + (0.000111)^2(50)^2]^{1/2}$$
$$= 0.01527$$
$$= 1.53\%$$

3-38

$$y_m = \frac{\sum y_i}{n} = \frac{17.5}{6} = 2.917$$

$$\sum (y_i - y_m)^2 = 10.728$$

$$\sigma_y = \left[\frac{10.728}{5}\right]^{1/2} = 1.465$$

$$\sigma_{y,x} = \left[\frac{17.264 \times 10^{-2}}{4}\right]^{1/2} = 0.2099$$

$$r = \left[1 - \frac{0.004406}{2.146}\right]^{1/2} = 0.999$$

3-40

$$\log N = -0.253 + 0.497 \log R$$

$$y_m = \sum \frac{\log N}{9} = \frac{7.167}{9} = 0.7963$$

$$\sum (y_i - y_m)^2 = 0.6514 \qquad \sigma_y = \left(\frac{0.6514}{9-1}\right)^{1/2} = 0.2853$$

$$\sum (\log N_i - \log N_{ic})^2 = 0.0017717$$

$$\sigma_{y,x} = \left[\frac{0.0017717}{9-2}\right]^{1/2} = 0.01591$$

$$r = \left[1 - \left(\frac{0.01591}{0.2853}\right)^2\right]^{1/2} = 0.998$$

3-42

$$\Delta T_m = \frac{(T_{h_1} - T_{c_1}) - (T_{h_2} - T_{c_2})}{h_2\left(\frac{T_{h_1} - T_{c_1}}{T_{h_2} - T_{c_2}}\right)}$$

$$T_{h_1} = 100,\ T_{h_2} = 80,\ T_{c_1} = 75,\ T_{c_2} = 55$$

$$W_T = \pm 1°\text{C}$$

$$\Delta T_m(\text{nominal}) = \frac{25 - 25}{\ln\left(\frac{25}{25}\right)} = 25$$

$$\Delta T_m(T_{h_1} + 1) = 25.49673$$

$$\frac{\partial \Delta T_m}{\partial T_{h_1}} = 0.49673$$

$$\Delta T_m(T_{h_2} + 1) = 25.49673 \qquad \frac{\partial \Delta T_m}{\partial T_{h_2}} = 0.49673$$

$$\Delta T_m(T_{c_1} + 1) = 24.49659 \qquad \frac{\partial \Delta T_m}{\partial T_{c_1}} = -0.50341$$

$$\Delta T_m(T_{c_2} + 1) = 24.49659 \qquad \frac{\partial \Delta T_m}{\partial T_{c_2}} = -0.50341$$

$$W_{\Delta T_m} = [(2)(0.49673)^2 + (2)(-0.50341)^2]^{1/2}$$

$$= 1.00016°\text{C}$$

3-44

$$R_1 = 1, R_2 = 1.5, R_3 = 3, R_4 = 2.5\ \text{k}\Omega$$

$$\text{W}_R = 10\% \qquad E = 100\ \text{V} \pm 1\text{V}$$

$$P = \frac{E^2}{R}$$

$$= E^2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)$$

$$= (100)^2\left(1 + \frac{2}{3} + \frac{1}{3} + 0.4\right)$$

$$= 24\ \text{W}$$

$P(R_1 + 0.01) = 23.901 \qquad \dfrac{\partial P}{\partial R_1} = 99$

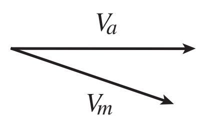
$P(R_2 + 0.01) = 23.956 \qquad \dfrac{\partial P}{\partial R_2} = 44$

$P(R_3 + 0.01) = 23.989 \qquad \dfrac{\partial P}{\partial R_3} = 11$

$P(R_4 + 0.01) = 23.984 \qquad \dfrac{\partial P}{\partial R_4} = 16$

$$W_P = [(99)^2(0.1)^2 + (44)^2(0.15)^2 + (11)^2(0.3)^2 + (16)^2(0.25)^2]^{1/2}$$
$$= 1.298 \text{ W}$$
$$= 5.41\%$$

3-45



V_a = auto speed
V_m = V measured

V_a = auto speed
V_m = V measured

$V_a = V_m \cos\theta \qquad W_\theta = \pm 10^0 \qquad W_m = \pm 4\%$

$\dfrac{\partial V_a}{\partial V_m} = \cos\theta \qquad \dfrac{\partial V_a}{\partial \theta} = -V_m \sin\theta$

$$W_{V_a} = [(\cos\theta W_{V_m})^2 + (-V_m \sin\theta W_\theta)^2]^{1/2}$$

$$\frac{W_{V_a}}{V_a} = \left[\left(\frac{W_{V_m}}{V_m}\right)^2 + (\tan\theta W_\theta)^2\right]^{1/2}$$

$\theta = 10° \qquad \dfrac{W_{V_a}}{V_a} = 0.0505$

$\theta = 20° \qquad \dfrac{W_{V_a}}{V_a} = 0.0751$

$\theta = 30° \qquad \dfrac{W_{V_a}}{V_a} = 0.1083$

Section 3.5

Take $V_m = 25,\ W_{V_m} = 1$

$\theta = 20°,\ W_\theta = 10°,\ V_a = 23.492$

$$V_a(V_m + 0.1) = \frac{25.1}{25}V_a = 1.004V_a$$

$$V_a(\theta + 0.1) = \frac{\cos 20.1}{\cos 20}V_a = 0.99936V_a$$

$$\frac{\partial V_a}{\partial V_m} = \frac{0.004}{0.1} = 0.04$$

$$\frac{\partial V_a}{\partial \theta} = \frac{6.368 \times 10^{-4}}{0.1} = 0.006368$$

$$W_{V_a} = V_a[(0.04)^2(1)^2 + (0.006368)^2(10)^2]^{1/2}$$
$$= 0.0752V_a$$
$$= 7.52\%$$

3-47

n	1	2	3	4	5	6	7	8	9	10
T	101.1	99.8	99.9	100.2	100.5	99.6	100.9	99.7	100.1	100.3

$$\sum T_i = 10002.1 \qquad T_m = 100.21$$

$$\sum (T - T_{\text{act}})^2 = 2.71$$

$$\sigma = \left(\frac{2.71}{10}\right)^{1/2} = 0.521°\text{C}$$

3-48

n	1	2	3	4	5	6	7
T	101.1	99.8	99.9	100.2	100.5	99.6	100.9

$$\sum x = 14.003 \qquad x_3 = \frac{14.003}{7} = 2.00043$$

$$\sum (x - 2.000)^2 = 3.7 \times 10^{-5}$$

$$\sigma = \left(\frac{3.7 \times 10^{-5}}{7}\right)^{1/2} = 2.3 \times 10^{-3}$$

3-49

120 rocks $\qquad x_m = 6.8 \text{ cm}^3$

$\sigma = 0.7 \text{ cm}^3$

$x = 6.5 \text{ cm}^3$ to 7.2 cm^3

$x - x_m \qquad - 0.3$ to 0.4 cm^3

$= 0.43\,\sigma$ to $0.57\,\sigma$

$\eta = 0.43 \quad \int = 0.1664$

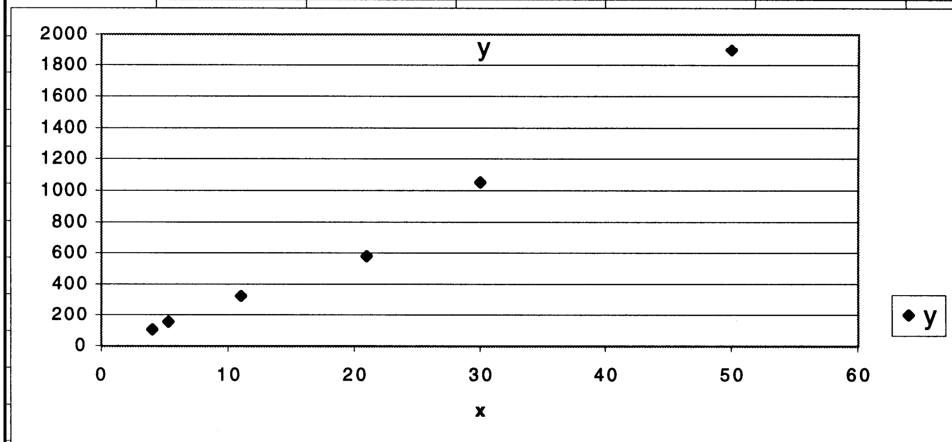
$\eta = 0.57 \quad \int = \underline{0.21566}$

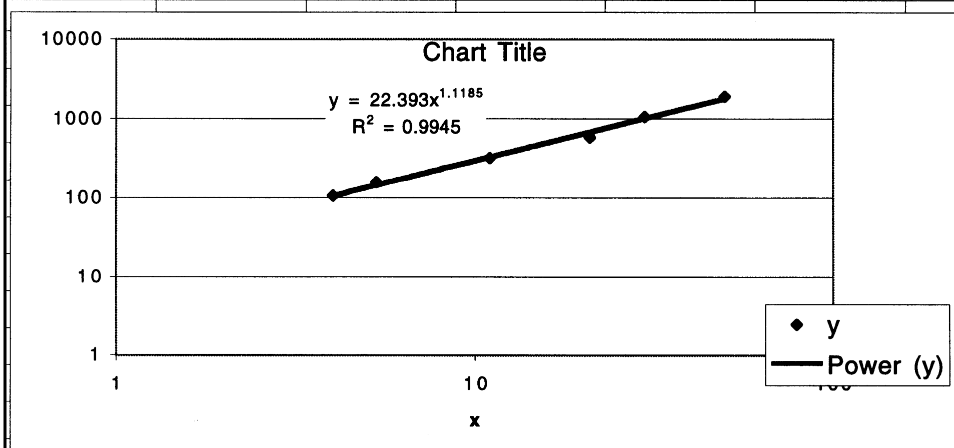
$\qquad\qquad\qquad 0.38206$

Number in this range $= (120)(0.382) = 46$ rocks

3-51

x	y
4	105
5.3	155
11	320
21	580
30	1050
50	1900





3-52

$y = 2 - 0.3x + 0.01x^2$

$\dfrac{\partial y}{\partial x} = -0.3 + (2)(0.01)x$

At $x = 2,\ y = 1.44$

at $W_x = 1\%$ $W_y = [-0.3 + (2)(0.01)(2)](0.01)$

$= -0.0026$

$\dfrac{W_y}{y} = -\dfrac{0.0026}{1.44} = -0.0018$

At $x = 0,\ y = 2$

$W_x = 1\%\ W_y = -(0.03)(0.01) = -0.003$

$$\frac{W_y}{y} = -\frac{0.003}{2} = -0.0015$$

3-53

$0.01x^2 - 0.3x + 2 = y$

$$(2)(0.01)x\frac{\partial x}{\partial y} - 0.3\frac{\partial x}{\partial y} = 1$$

$$\text{At } x = 2,\ \frac{\partial x}{\partial y} = \frac{1}{(0.04 - 0.3)} = -3.846$$

$y = (0.01)(2)^2 - (0.3)(2) + 2 = 1.44$

$$W_x = \frac{\partial x}{\partial y}W_y = (-3.846)(0.01)(1.44)$$

$$= 0.0554$$

$$\frac{W_x}{x} = 0.0277$$

3-54

$$\dot{m} = 12 \text{ lbm/min} = 5.448\,\frac{\text{kg}}{\text{min}} = 0.0908\,\frac{\text{kg}}{\text{s}}$$

$d = 0.5 \text{ m} = 0.0127 \text{ m}$

$\mu = 4.64 \times 10^{-2} \text{ lbm/hr} \cdot \text{ft} = 1.92 \times 10^{-5} \text{ kg/m} \cdot \text{s}$

$$\text{Re} = \frac{(4)(0.0908)}{\pi(0.0127)(1.92 \times 10^{-5})} = 474{,}000$$

$$\frac{W_{\text{Re}}}{\text{Re}} = \left[\left(\frac{W_d}{d}\right)^2 + \left(\frac{W_{\dot{m}}}{\dot{m}}\right)^2 + \left(\frac{W_\mu}{\mu}\right)^2\right]^{1/2}$$

$$= \left[\left(\frac{0.005}{0.5}\right)^2 + (0.005)^2 + (0.01)^2\right]^{1/2}$$

$$= 0.015$$

$W_{\text{Re}} = (0.015)(474{,}000) = 7112$

3-56

100 mi/hr $= 146.7$ ft/s $= 44.7$ m/s

$r = 5$ m

$$\text{Nominal values } \omega = \frac{44.7}{2\pi(5)} = 1.423 \text{ lap/s}$$

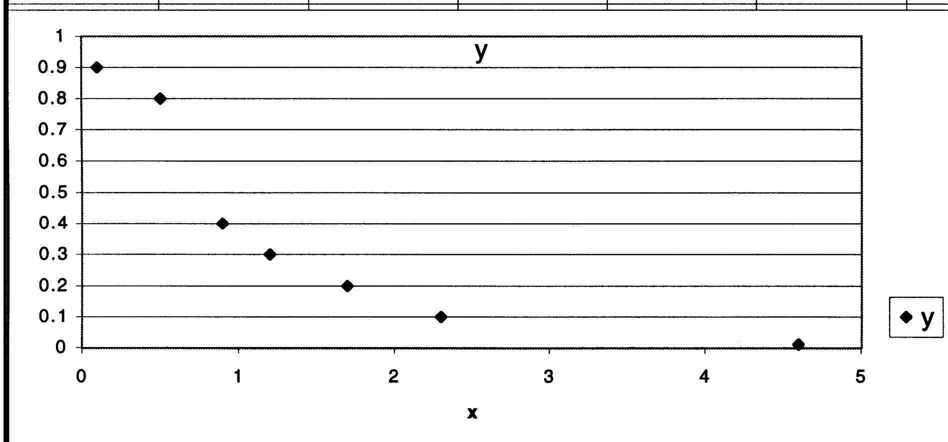
$$\text{4 laps: } \Delta\tau = \frac{4}{1.423} = 2.81\,\text{sec}$$

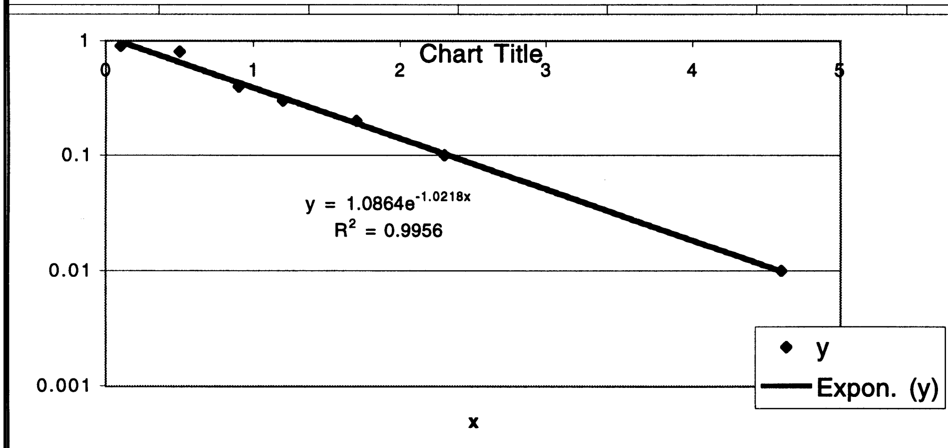
$\Delta\tau = t_1 - t_2 \qquad W_t = 0.2\,\text{sec}$

$$W_{\Delta\tau} = \left[(1)^2 W_{t_1^2} + (-1)^2 W_{t_2^2}\right]^{1/2}$$
$$= 0.2828 \text{ sec}$$
$$\frac{W_{\Delta\tau}}{\Delta\tau} = \frac{0.2828}{2.81} = 0.1006$$

3-57

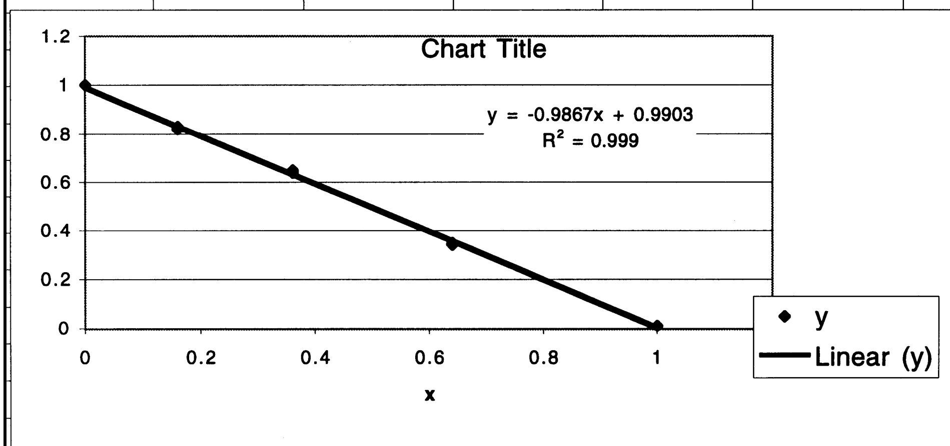
t	y
0.1	0.9
0.5	0.8
0.9	0.4
1.2	0.3
1.7	0.2
2.3	0.1
4.6	0.01

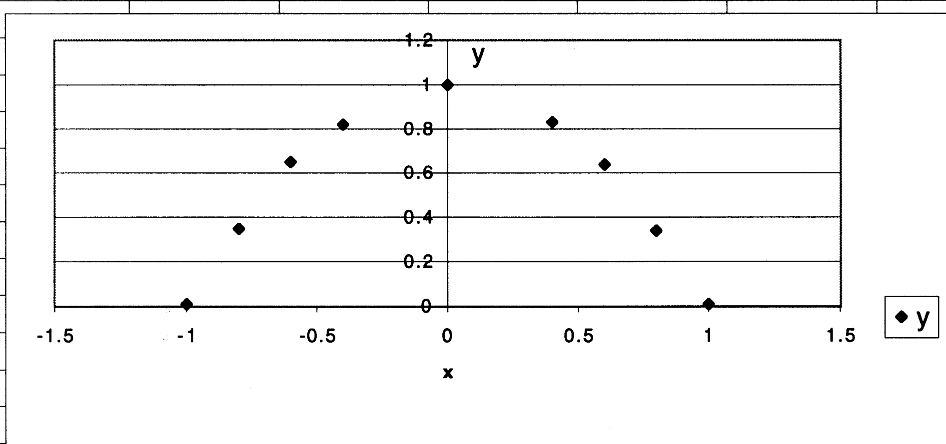




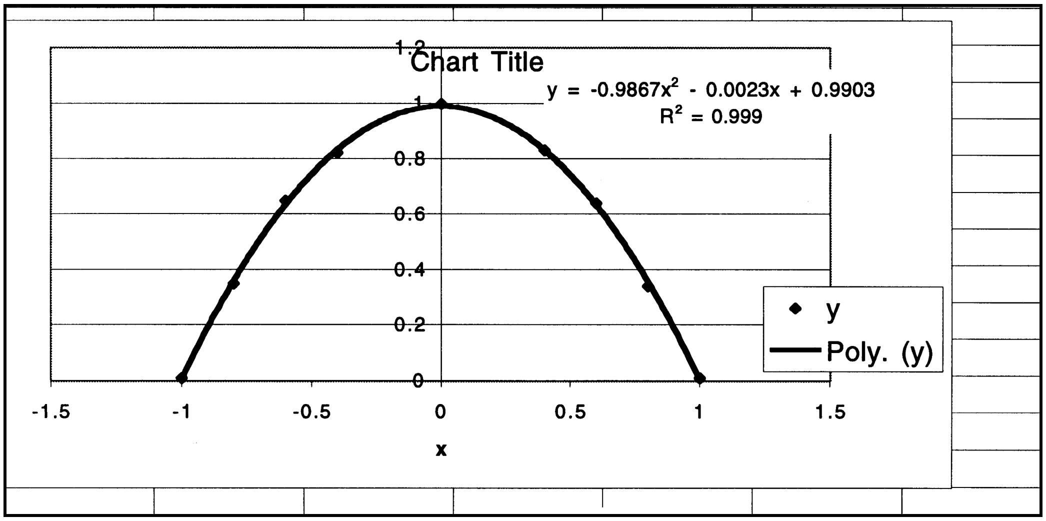
3-58

x	x^2	y
-1	1	0.01
-0.8	0.64	0.35
-0.6	0.36	0.65
-0.4	0.16	0.82
0	0	1
0.4	0.16	0.83
0.6	0.36	0.64
0.8	0.64	0.34
1	1	0.01





3-59



3-60

$$n = 10,\ x_m = 1.226 = \frac{\sum x_i}{n}$$

$$\sigma = \left[\frac{\sum (x_i - x_m)}{n-1}\right]^{1/2} = 0.0143$$

90% conf. $= 1.65\sigma = 0.0236$

90% conf. $= 1.96\sigma = 0.0280$

3-61

$n = 12 \qquad x_m = 125.8333\,\text{kPa}$

$$\sigma = \left[\frac{\sum (x_i - x_m)}{n-1}\right]^{1/2} = 2.3677$$

90% conf. $= 1.65\sigma = 3.907\,\text{kPa}$

95% conf. $= 1.96\sigma = 4.617\,\text{kPa}$

3-63

$x_m = 11.0\ \text{V} \qquad n = 9$

$\sigma = 0.03\ \text{V}$

5% significance $= 1.96\sigma = \pm 0.0588\ \text{V}$

1% significance $= 2.57\sigma = \pm 0.0771\ \text{V}$

3-64

$\sigma = \pm 0.1\ \text{k}\Omega$; 5% significance $= 1.96\sigma$ for large $n = \Delta = \dfrac{1.96\sigma}{\sqrt{n}}$ for small n

$$n = \left[\frac{(1.96)(0.1)}{0.05}\right]^2 = 15.37 \approx 16$$

For $\Delta = 0.1,\ n = [(1.96)(1)]^2 = 3.84 \approx 4$

For t − dist, at 5% significance

$$\Delta = \frac{t\sigma}{\sqrt{n}}, t = 0.5\sqrt{n}, v = n - 1$$

n	t_{95} (Table)	t (calc)
15	2.145	1.936
16	2.131	2.0
17	2.120	2.062
18	2.11	2.121

$n = 18$ required

3-65

$\sigma = 0.2$ $V = 50.0 \pm 0.2$

at $V = 50.2$ 1σ

at $V = 50.4$ 2σ

$$\left.\begin{aligned} P(1\sigma) &= 0.34134 \\ P(2\sigma) &= 0.47725 \end{aligned}\right\}\text{one side of mean}$$

$P(\text{between}) = 0.13591$

3-66

$x_{m_1} = 3.56$ mm $\sigma_1 = 0.06$ $n_1 = 20$

$x_{m_2} = 3.58$ $\sigma_2 = 0.03$ $n_2 = 23$

90% confidence

$$v = \frac{\left[\frac{0.06^2}{20} + \frac{0.03^2}{23}\right]^2}{\frac{\left(\frac{0.062}{20}\right)^2}{19} + \frac{\left(\frac{0.032}{23}\right)^2}{22}} = 27.05 \text{ (Round to 27)}$$

$$t = \frac{0.06 - 0.03}{(2.191 \times 10^{-4})^{1/2}} = 2.03$$

From table, $t_{95} = 2.052$

$2.03 < 2.052$, so expect same conf. level.

3-67

$x_{m_1} = 3.632$ $\sigma_1 = 0.06$ $n_1 = 17$

$x_{m_2} = 3.611$ $\sigma_2 = 0.02$ $n_2 = 24$

conf. = 90% level

$$v = \frac{\left[\frac{0.06^2}{17} + \frac{0.02^2}{24}\right]^2}{\frac{\left(\frac{0.062}{17}\right)^2}{16} + \frac{\left(\frac{0.062}{24}\right)^2}{23}} = 18.54 \text{ (Round to 19)}$$

$t_{95} = 2.093$

$$t = \frac{0.06 - 0.02}{(2.284 \times 10^{-4})^{1/2}} = 2.647$$

$2.647 > 2.093$, so may not yield same results.

3-68

$\sigma = \pm 0.1 \qquad \Delta = 0.05$

For 5% significance $z = 1.96$

10% significance $z = 1.65$

$$5\%: \ 0.05 = \frac{1.96(0.1)}{\sqrt{n}} \qquad n = 15.37 \approx 16$$

$$10\%: \ 0.05 = \frac{1.65(0.1)}{\sqrt{n}} \qquad n = 10.89 \approx 11$$

$C =$ cost/part reject

$M =$ cost of meas./part

$T =$ total cost

$T(5\%) = 0.05C + 16M \qquad (n = 16)$

$T(10\%) = 0.1C + 11M \qquad (n = 11)$

$0.05C + 16M = 0.1C + 11M$

$C = 100M$

3-69

1% sig. $z = 2.57$

$$0.05 = \frac{2.57(0.1)}{\sqrt{n}} \qquad n = 26.42 \approx 27$$

$T(1\%) = 0.01C + 27M \qquad (n = 27)$

$T(5\%) = 0.05C + 16M$

$C = 225M$

3-70

$$\rho = \frac{P}{RT} = \frac{125 \times 10^3}{(287.1)(328.15)} = 1.327 \text{ kg/m}^3$$

$$\frac{W_\rho}{\rho} = \left[\left(\frac{W_T}{T}\right)^2 + \left(\frac{W_\rho}{\rho}\right)^2 + 0\right]^{1/2}$$

$$= \left[\left(\frac{0.4}{328.15}\right)^2 + \left(\frac{0.5}{1.25}\right)^2\right]^{1/2}$$

$$= 0.004182$$

$W_\rho = 0.00555 \text{ kg/m}^3$

3-72

$x = 3 \pm 0.1 \qquad W_x = 3.33\%$

$y = 5x^2 = 45$

$W_y = (10)(3)(0.1) = 3 = 6.67\%$

$y = 5x^3 = 135$

$W_y = (3)(5)(3)^2(0.1) = 13.5 = 10\%$

$y = 5x^4 = 405$

$W_y = (4)(5)(3)^3(0.1) = 54 = 13.33\%$

$y = 5x^2 + 3x + 2 = 56$

$= [(2)(5)(3) + 3](0.1) = 3.3 = 5.89\%$

3-73

$y = 3x^2 - 2x + 5 = 13$

$x = 2, -\frac{8}{6}$

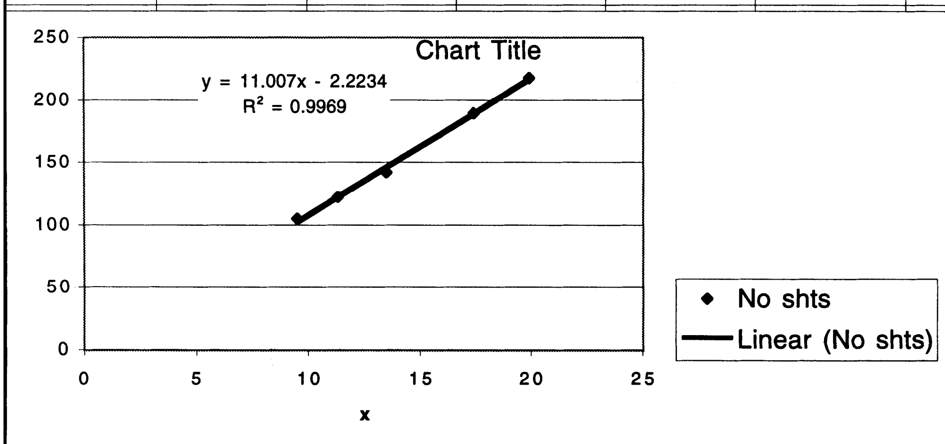
$W_y = (6x - 2)W_x = (0.01)(13)$

$W_x = \frac{(0.01)(13)}{12 - 2} = 0.013$

$\frac{W_x}{x} = \frac{0.013}{2} = 0.0065 = 0.65\%$

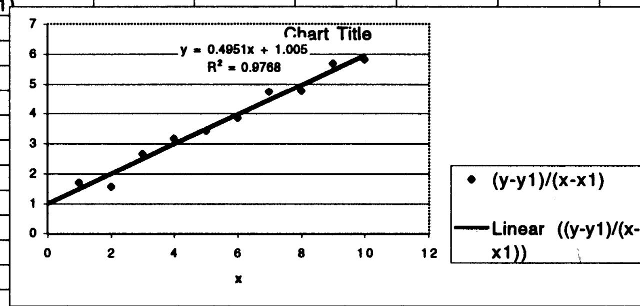
3-74

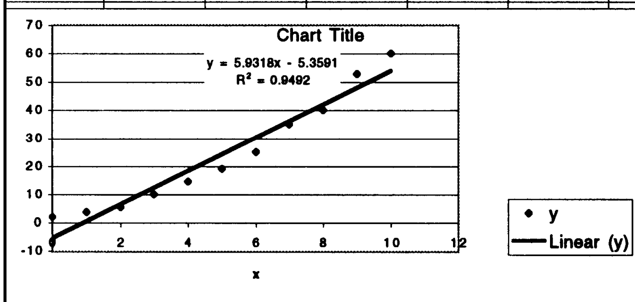
ht	No shts
9.5	105
11.3	122
13.5	142
17.4	190
19.9	218

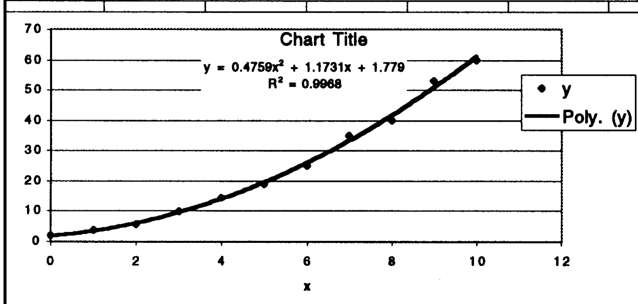


3-75

x	y		x-x1	y-y1	(y-y1)/(x-x1)
0	1.9		0	0	
1	3.6		1	1.7	1.7
2	5.5		2	3.1	1.55
3	9.8		3	7.9	2.63333
4	14.5		4	12.6	3.15
5	19		5	17.1	3.42
6	25		6	23.1	3.85
7	35		7	33.1	4.72857
8	40		8	38.1	4.7625
9	53		9	51.1	5.67778
10	60		10	58.1	5.81

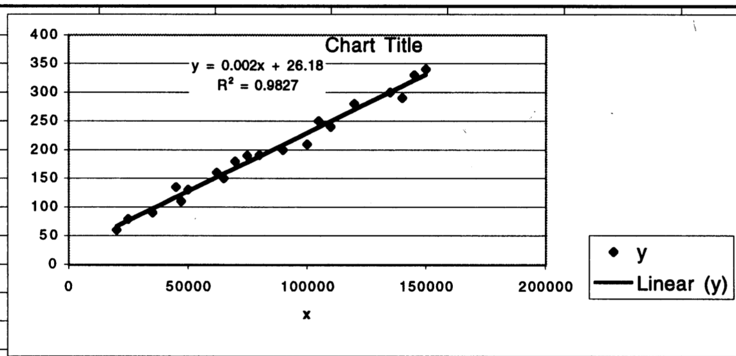


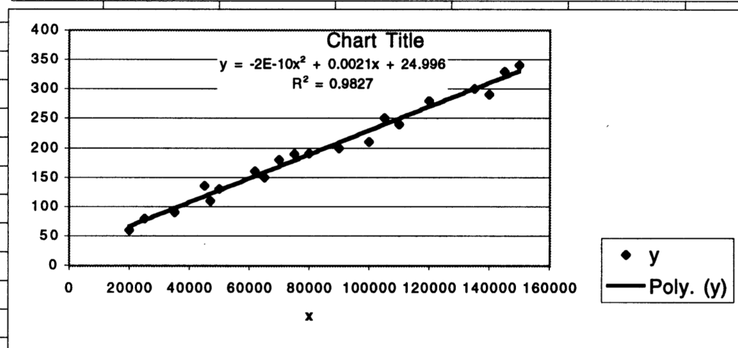


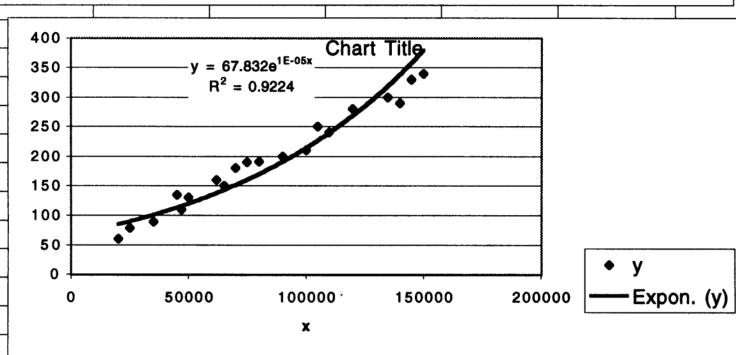


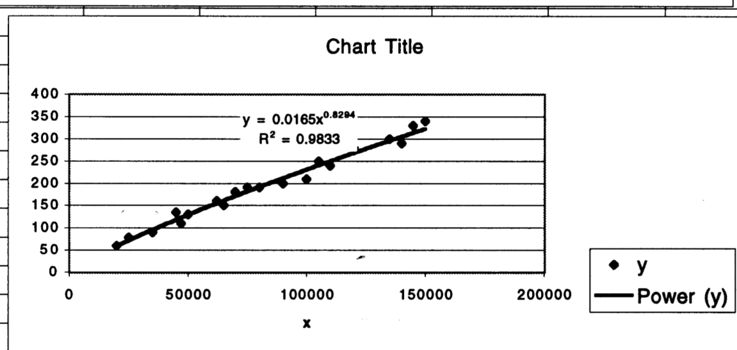
3-76

x	y
20000	60
25000	79
35000	90
45000	135
47000	110
50000	130
62000	160
65000	150
70000	180
75000	190
80000	191
90000	200
100000	210
105000	250
110000	240
120000	280
135000	300
140000	290
145000	330
150000	340



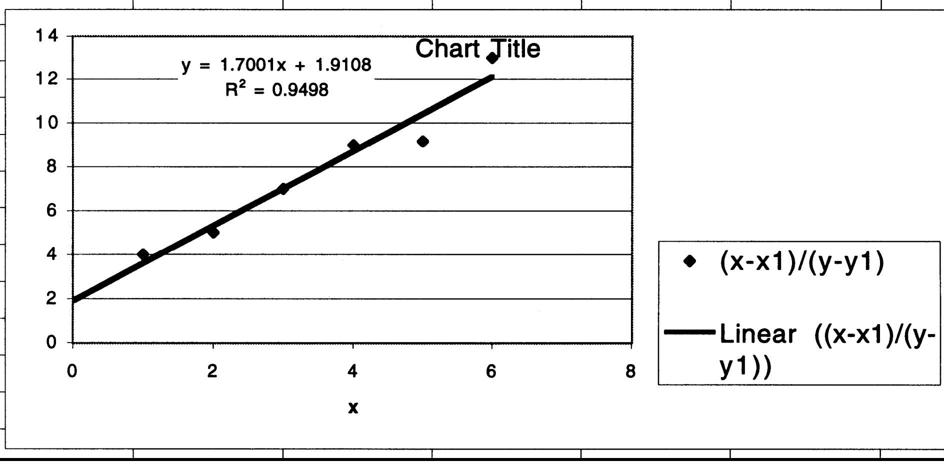






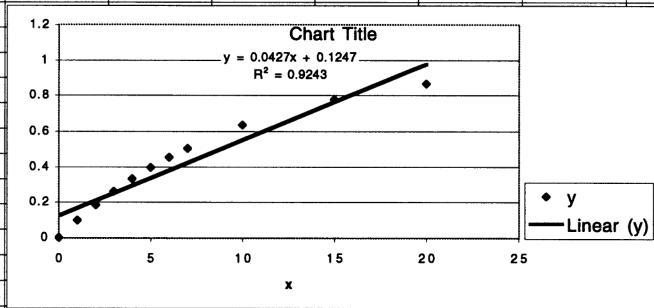
3-77

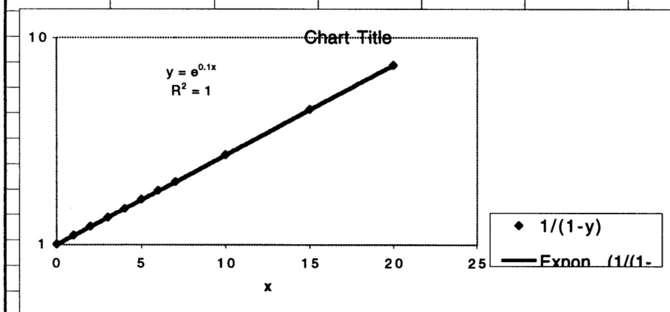
x	y		x-x1	y-y1	(x-x1)/(y-y1)
0	3		0	0	
1	3.25		1	0.25	4
2	3.4		2	0.4	5
3	3.4286		3	0.4286	6.999533
4	3.4444		4	0.4444	9.0009
5	3.5455		5	0.5455	9.165903
6	3.4615		6	0.4615	13.00108

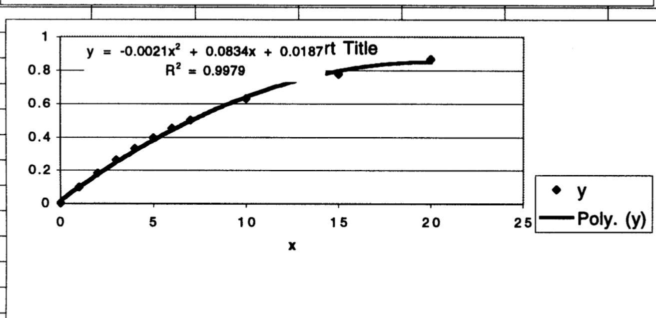


3-78

x	y	1/(1-y)
0	0	1
1	0.09516	1.10517
2	0.1813	1.22145
3	0.2592	1.34989
4	0.3297	1.49187
5	0.3935	1.6488
6	0.4512	1.82216
7	0.5034	2.01369
10	0.6321	2.71813
15	0.7769	4.48229
20	0.8647	7.39098

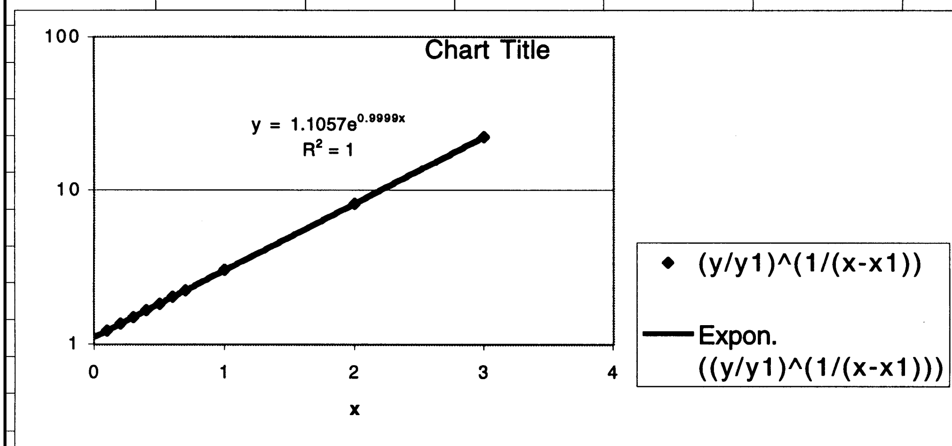






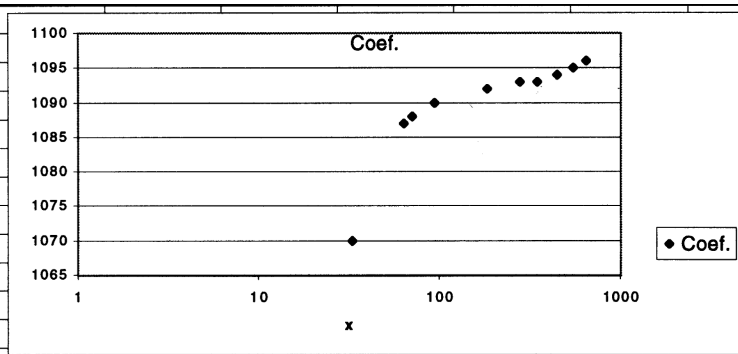
3-79

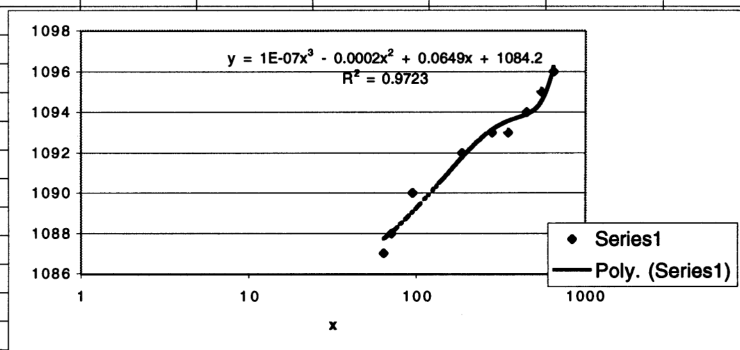
x	y		x-x1	y/y1	(y/y1)^(1/(x-x1))
0	2		0	1	
0.1	2.0404		0.1	1.0202	1.221387
0.2	2.1237		0.2	1.06185	1.349944
0.3	2.255		0.3	1.1275	1.491839
0.4	2.4428		0.4	1.2214	1.648712
0.5	2.6997		0.5	1.34985	1.822095
0.6	3.0439		0.6	1.52195	2.013727
0.7	3.5103		0.7	1.75515	2.233677
1	6.0083		1	3.00415	3.00415
2	133.372		2	66.686	8.16615
3	21876.04		3	10938.02	22.19795

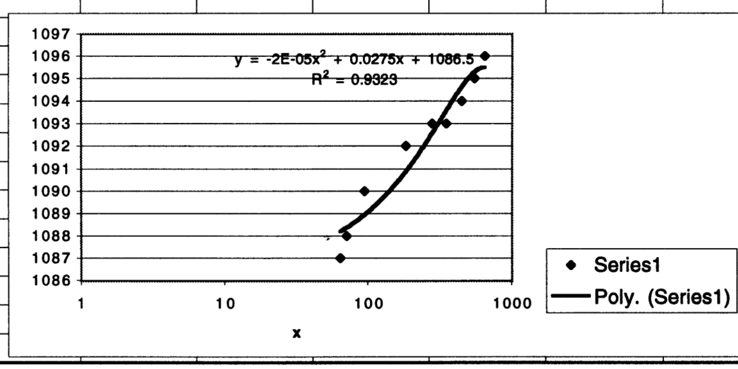


3-80

Freqpara	Coef.
33	1070
64	1087
71	1088
95	1090
185	1092
280	1093
350	1093
450	1094
550	1095
650	1096

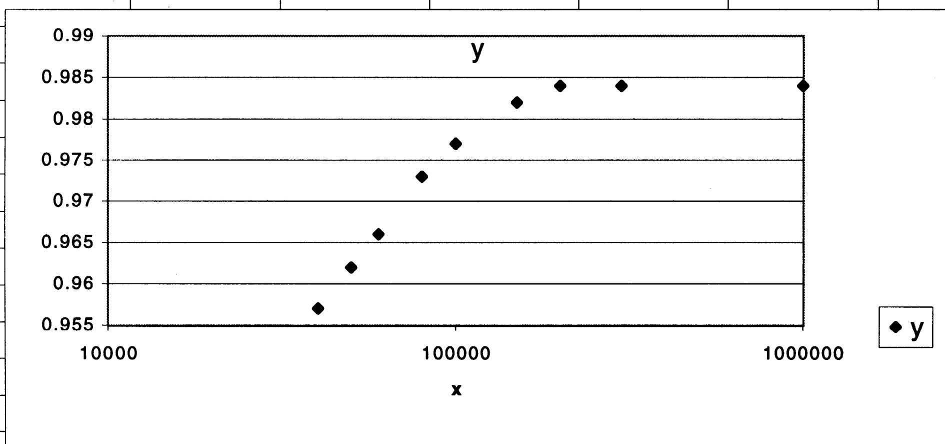


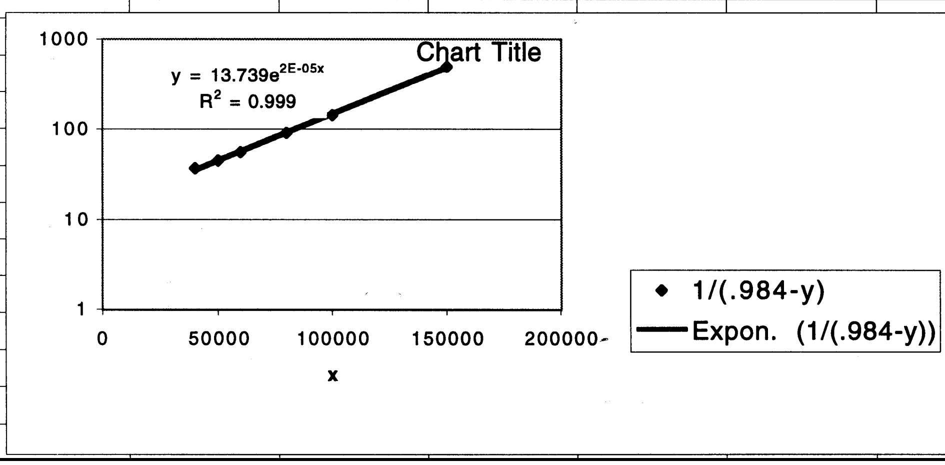




3-81

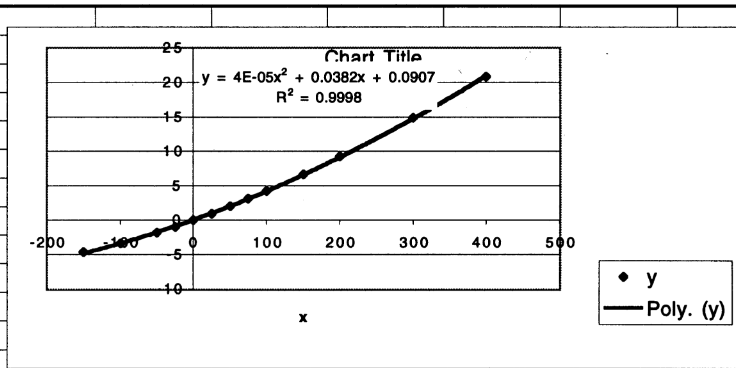
x	y	1-y	ln(x)	.984-y	1/(.984-y)
40000	0.957	0.043	10.59663	0.027	37.03704
50000	0.962	0.038	10.81978	0.022	45.45455
60000	0.966	0.034	11.0021	0.018	55.55556
80000	0.973	0.027	11.28978	0.011	90.90909
100000	0.977	0.023	11.51293	0.007	142.8571
150000	0.982	0.018	11.91839	0.002	500
200000	0.984	0.016	12.20607	0	
300000	0.984	0.016	12.61154	0	
1000000	0.984	0.016	13.81551	0	

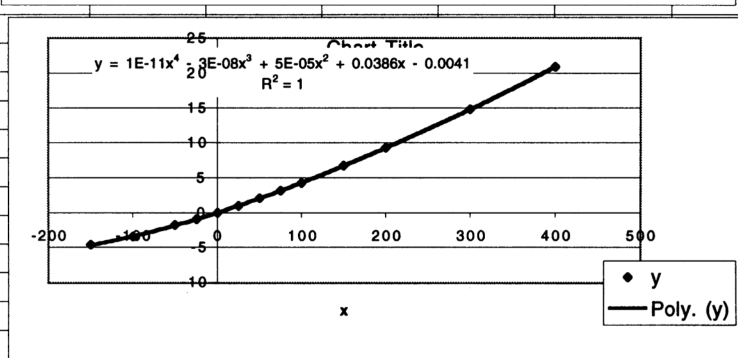


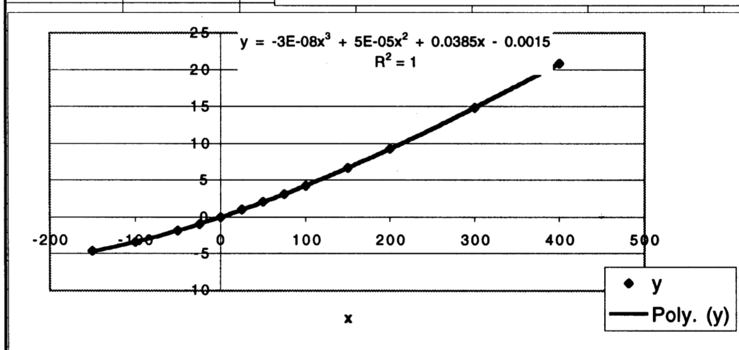


3-82

x	y
-150	-4.648
-100	-3.379
-50	-1.819
-25	-0.94
0	0
25	0.993
50	2.036
75	3.132
100	4.279
150	6.704
200	9.288
300	14.862
400	20.872







3-83

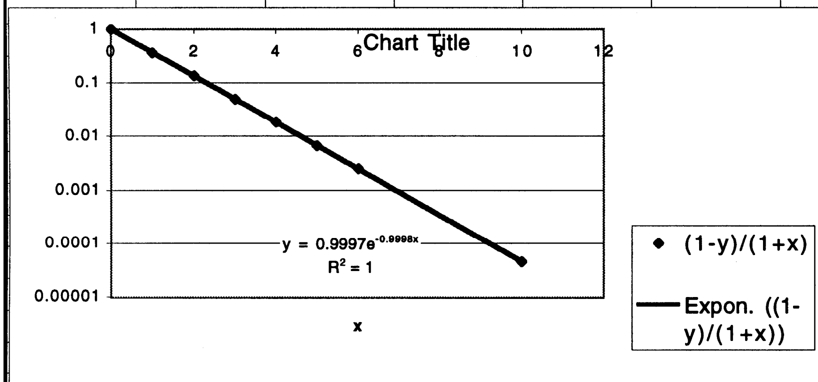
Let $y = ax_3 + bx_1$

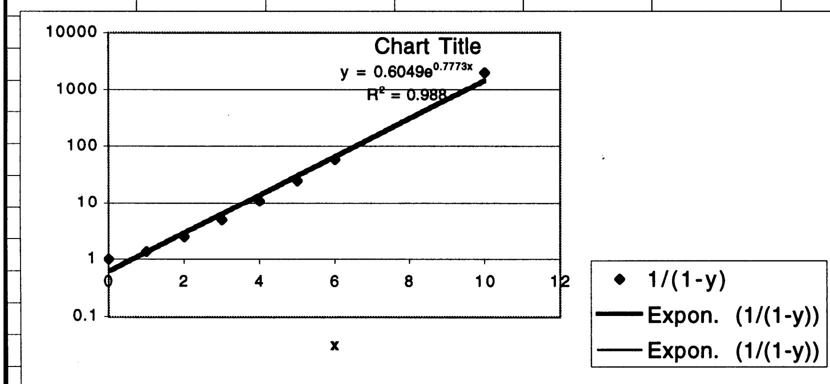
$$w_y = \left[a_2 w_{x_3}^2 + b^2 w_{x_1}^2\right]^{1/2}$$

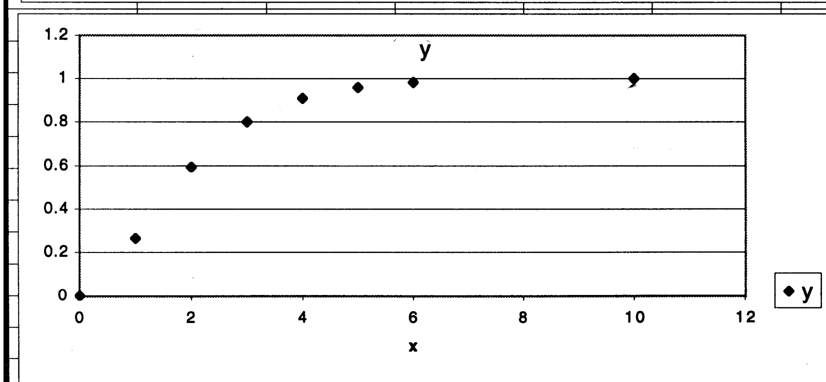
$$\frac{W_R}{R} = \left[\left(\frac{W_y}{y}\right)^2 + \left(0.75\frac{W_{x_2}}{x_2}\right)^2 + \left(0.25\frac{W_{x_4}}{x_4}\right)^2\right]^{1/2}$$

3-86

x	y	1/(1-y)	x+1	(1-y)/(1+x)
0	0	1	1	1
1	0.2642	1.359065	2	0.3679
2	0.594	2.463054	3	0.135333
3	0.8009	5.022602	4	0.049775
4	0.9084	10.91703	5	0.01832
5	0.9596	24.75248	6	0.006733
6	0.9826	57.47126	7	0.002486
10	0.9995	2000	11	4.55E-05

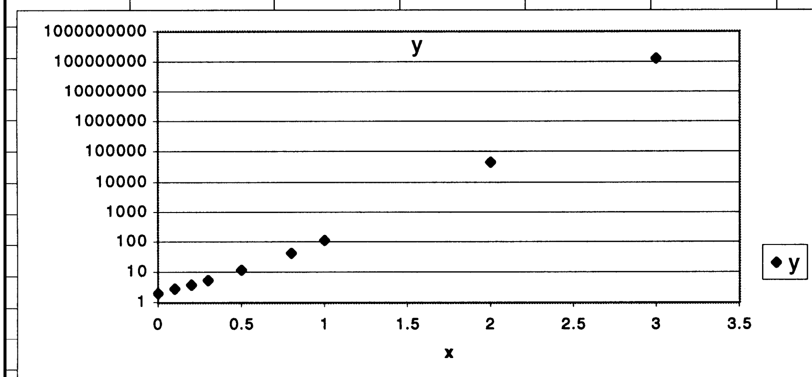


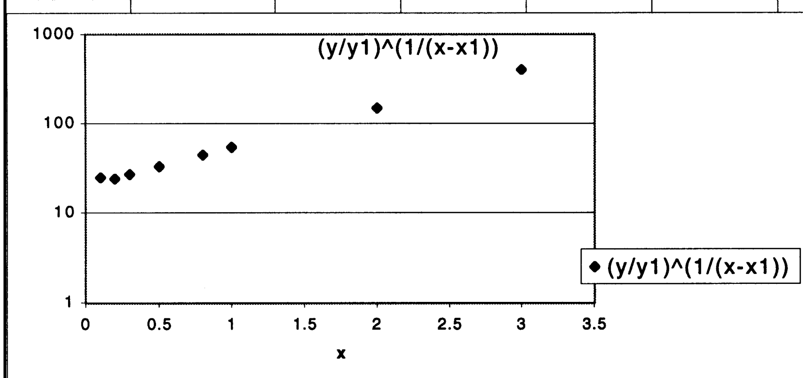


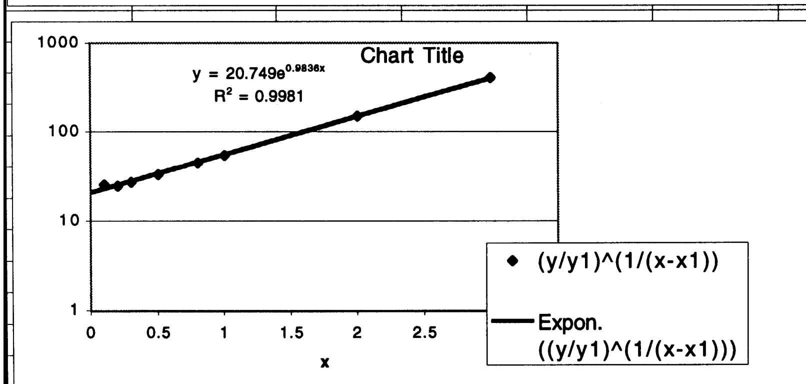


3-87

x	y	y/y1	(y/y1)^(1/(x-x1))
0	2	1	
0.1	2.7629	1.38145	25.31347
0.2	3.793	1.8965	24.53377
0.3	5.3825	2.69125	27.11316
0.5	11.5092	5.7546	33.11542
0.8	41.8105	20.90525	44.7012
1	109.1963	54.59815	54.59815
2	44052.9	22026.45	148.4131
3	131000000	65500000	403.1009







3-88

$$\frac{\partial y}{\partial x} = (-x)e^{-x} + e^{-x}(1)$$

$$= (1 - x)e^{-x}$$

At $x = 5,\ y = 1 + (1 + 5)e^{-5} = 1.040427$

$$W_y = (0.01)(1.040427) = 0.01040427$$

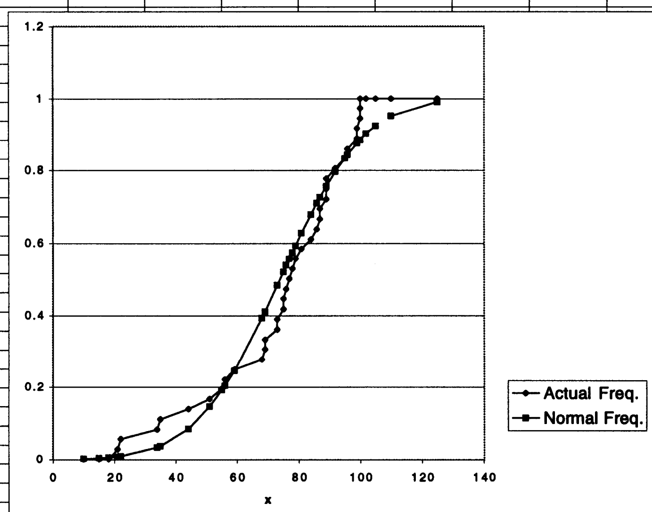
$$W_y = \left[\left(\frac{\partial y}{\partial x}\right)^2 W_x^2\right]^{1/2}$$

$$W_x = 0.38603$$

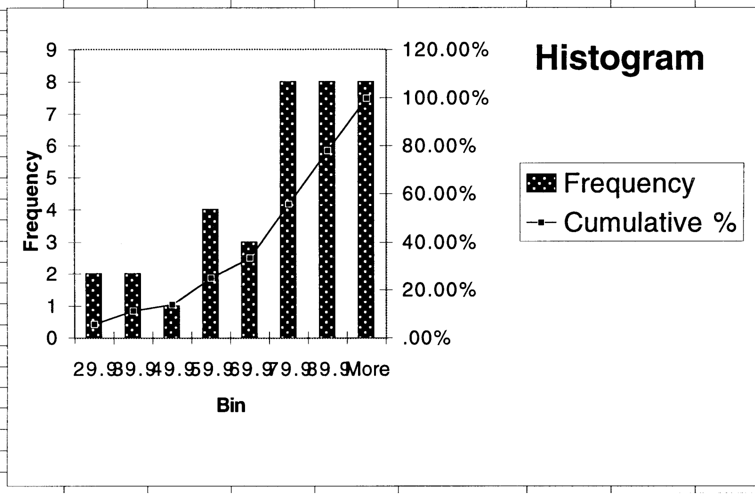
$$\frac{W_x}{x} = \frac{0.3603}{5} = 0.0772\%$$

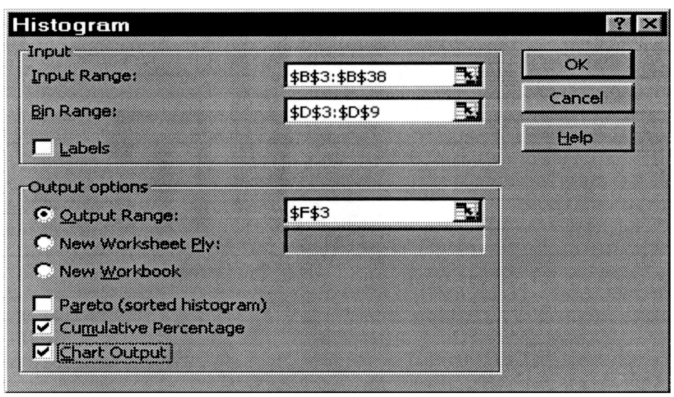
3-89

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	3.89	Student	Actual	Normal									
2	Index	Score	Freq.	Freq.									
3	1	10	0	0.0016									
4	2	15	0	0.0033									
5	3	18	0	0.00497									
6	4	21	0.02778	0.00734									
7	5	22	0.05556	0.00834									
8	6	34	0.08333	0.03282									
9	7	35	0.11111	0.03634									
10	8	44	0.13889	0.08379									
11	9	51	0.16667	0.14516									
12	10	55	0.19444	0.19131									
13	11	56	0.22222	0.20412									
14	12	59	0.25	0.2455									
15	13	68	0.27778	0.39206									
16	14	69	0.30556	0.40987									
17	15	69	0.33333	0.40987									
18	16	73	0.36111	0.48264									
19	17	73	0.38889	0.48264									
20	18	75	0.41667	0.5194									
21	19	75	0.44444	0.5194									
22	20	76	0.47222	0.53774									
23	21	77	0.5	0.55599									
24	22	78	0.52778	0.57413									
25	23	79	0.55556	0.59212									
26	24	81	0.58333	0.62747									
27	25	84	0.61111	0.67847									
28	26	86	0.63889	0.71076									
29	27	87	0.66667	0.72631									
30	28	87	0.69444	0.72631									
31	29	89	0.72222	0.75611									
32	30	89	0.75	0.75611									
33	31	89	0.77778	0.75611									
34	32	92	0.80556	0.79733									
35	33	95	0.83333	0.83407									
36	34	96	0.86111	0.84529									
37	35	99	0.88889	0.8759									
38	36	99	0.91667	0.8759									
39	37	100	0.94444	0.88508									
40	38	100	0.97222	0.88508									
41	39	100	1	0.88508									
42	40	102	1	0.90199									
43	41	105	1	0.92382									
44	42	110	1	0.95171									
45	43	125	1	0.99069									



	A	B	C	D	E	F	G	H	I	J	K
1	**Student**	**Score**	**3.89**	**Bin Upper**			**Output**			**Other**	**Parameters**
2	**Number**			**Bound**						**(calculated)**	
3	1	22		29.9		*Bin*	*Frequency*	*Cumulative %*		Mean=	73.9444444
4	2	35		39.9		29.9	2	5.56%		STDEV=	22.0064204
5	3	99		49.9		39.9	2	11.11%		STDEVP=	21.6986232
6	4	87		59.9		49.9	1	13.89%		Max =	100
7	5	77		69.9		59.9	4	25.00%		Min=	21
8	6	78		79.9		69.9	3	33.33%			
9	7	89		89.9		79.9	8	55.56%			
10	8	100				89.9	8	77.78%			
11	9	68				More	8	100.00%			
12	10	84									
13	11	86									
14	12	75									
15	13	69									
16	14	44									
17	15	56									
18	16	99									
19	17	95									
20	18	34									
21	19	73									
22	20	51									
23	21	100									
24	22	96									
25	23	21									
26	24	79									
27	25	87									
28	26	89									
29	27	69									
30	28	81									
31	29	100									
32	30	55									
33	31	89									
34	32	73									
35	33	75									





3-90

Set up in Excel with the result:

$y = 3.247x_1 + 2.041x_2 + 1.242x_3 - 3.864$

x_1	x_2	x_3	y
1	3.2	3	9.5
2	4.5	4	15.6
3	4	5.6	21
4	5.7	6	31
5	6	7	32
6	7	8	41
7	8	9	45
8	9	10	53
1.241881	2.041483	3.247873	−3.86409
5.329923	2.770111	7.06166	17.03189
0.991044	1.881147	#N/A	#N/A
147.5351	4	#N/A	#N/A
1566.254	14.15486	#N/A	#N/A

3-91

Entering in Excel worksheet the solution is

$Q = 0.526T_i - 0.228T_0 + 34.723$

T_i	T_o	Q
72	85	54.3
72	95	51.8
72	105	49.4
72	115	46.6
67	85	50.4
67	95	48
67	105	45.5
67	115	42.9
62	85	46.5
62	95	44.4
62	105	42.2
62	115	40
57	85	45.7
57	95	43.9
57	105	42
57	115	40

−0.228	0.526	34.723
0.023821	0.0476643	3.897265
0.942605	1.065328	#N/A
106.75	13	#N/A
242.306	14.754	#N/A

3-92

Lower limit

$$w_y/y = [(1/5)^2 + (0.5/5)^2 + (0.2/15)^2]^{1/2}$$
$$= 0.0224 = 2.24\%$$

Upper limit

$$w_y/y = [(1/100)^2 + (0.5/100)^2 + (0.2/100)^2]^{1/2}$$
$$= 0.01136 = 1.136\%$$

3-93

$$w_y = \left[w_1{}^2 + ((1/2)x_3/x_2{}^{1/2})^2 w_2{}^2 + ((1/2)x_2/x_3{}^{1/2})^2 w_3{}^2\right]^{1/2}$$

Lower limit $y = 20$

$w_y = 2.449$

$w_y/y = 0.1225 = 12.25\%$

Upper limit $y = 200$

$w_y = 7.1414$

$w_y/y = 0.0357 = 3.57\%$

3-94

Follow procedure described in Example 3.6. Result will depend on values selected for increments in the variables.

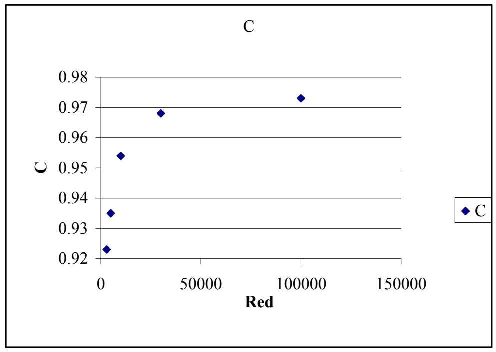
3-95
Excel plot and curve fit shown below

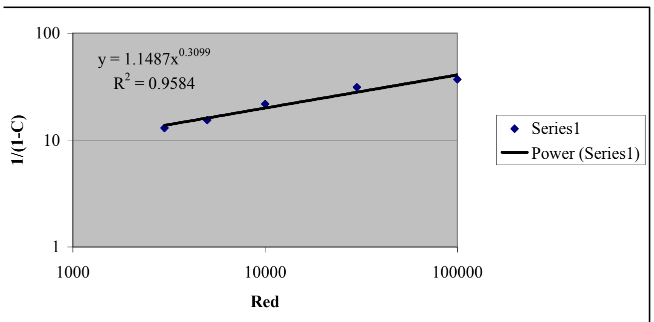
Solution is:

$1/(1 - C) = 1.149\, R_{e_d}^{\;0.3}$

Red	C	1-C	1/(1-C)
3000	0.923	0.077	12.98701
5000	0.935	0.065	15.38462
10000	0.954	0.046	21.73913
30000	0.968	0.032	31.25
100000	0.973	0.027	37.03704

3000	12.98701
5000	15.38462
10000	21.73913
30000	31.25
100000	37.03704



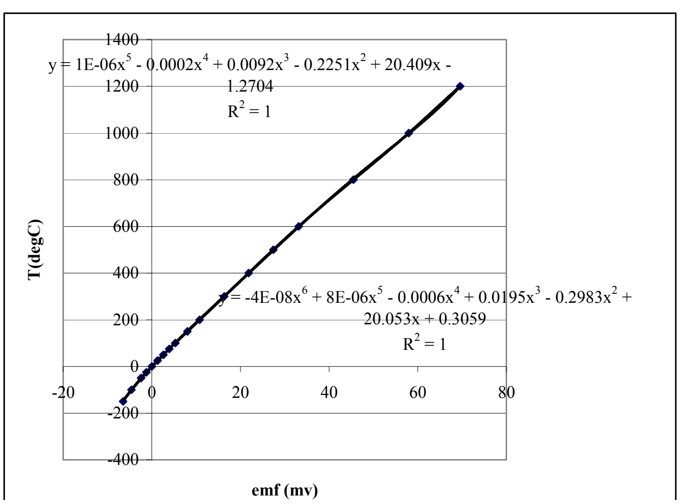


3-96
Follow procedure described in Example 3.6. Result will depend on values selected for increments in the variables.

3-97
Follow procedure described in Example 3.6. Result will depend on values selected for increments in the variables.

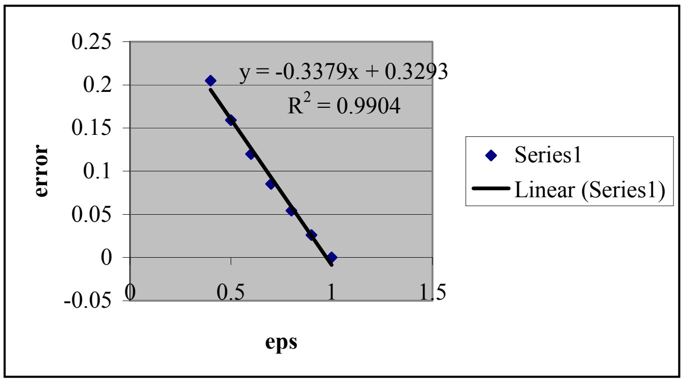
3-98
Excel chart shown below

E	T (degC)
-6.5	-150
-4.633	-100
-2.431	-50
-1.239	-25
0	0
1.277	25
2.585	50
3.918	75
5.269	100
8.01	150
10.779	200
16.327	300
21.848	400
27.393	500
33.102	600
45.494	800
57.953	1000
69.553	1200



3-99
Excel worksheet shown below. Deviation column is the difference between the linear relation and the true value.

eps	error	linear	deviation
0.4	0.204729	0.19414	-0.01059
0.5	0.159104	0.16035	0.001246
0.6	0.119888	0.12656	0.006672
0.7	0.085309	0.09277	0.007461
0.8	0.054258	0.05898	0.004722
0.9	0.025996	0.02519	-0.00081
1	0	-0.0086	-0.0086



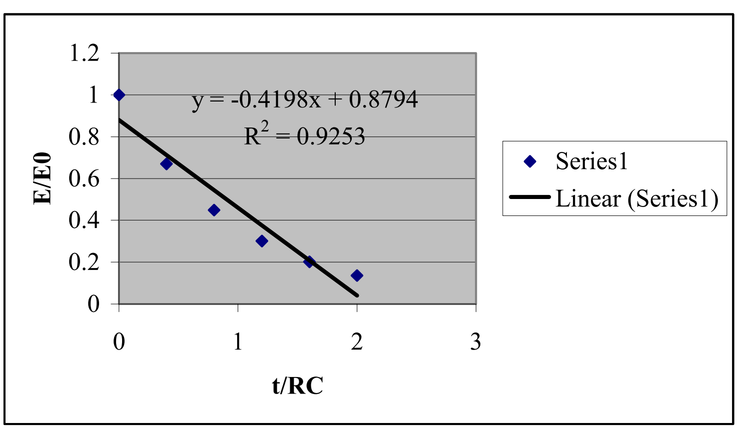
3-100
Excel worksheet shown below. Using the linear relation the value of t/RC to achieve a response of 63.2 percent is given by

$1 - 0.632 = -0.4198(t/RC) + 0.8794$

t/RC $= 1.218.$

This differs from the true value of 1.0 by 21.8%

t/RC	E/E0
0	1
0.4	0.67032
0.8	0.449329
1.2	0.301194
1.6	0.201897
2	0.135335



3-101
Fitting a combined straight line and parabola.
Information needed:
1. Desired slope of straight line, m
2. Intercept of straight line (i.e., y-coordinate for which $x = 0, b$).
3. x-coordinate value where straight line and parabola are to intersect, x_1
4. x and y coordinates of another point on the parabola, x_2, y_2.
This information can be obtained from a hand sketch.

The equation of the combined curve is
$y_c = mx + b$ for $0 < x < x_1$.

$$y_c = a_0 + a_1 x + a_2 x^2 \text{ for } x_1 < x < x_2 \quad (1)$$

Setting the slopes equal at $x = x_1$ and manipulating the algebra gives for the parabola coefficients.

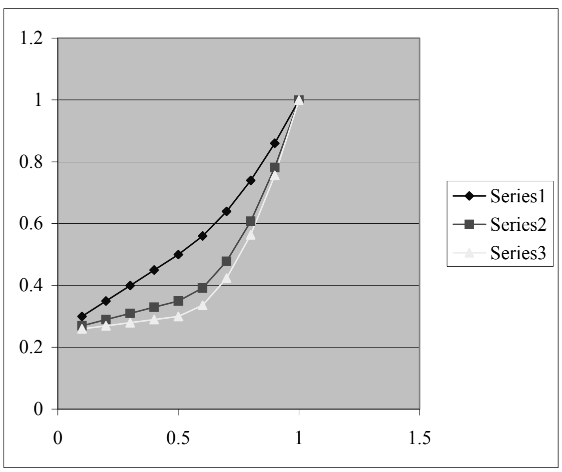
$$a_2 = [(y_2 - y_1) - m(x_2 - x_1)]/[(x_2^2 - x_1^2) - 2x_1(x_2 - x_1)] \quad (2)$$

$$a_1 = m - 2a_2 x_1 \quad (3)$$

$$a_0 = y_2 - a_2 x_2^2 - a_1 x_2 \quad (4)$$

where

$$y_1 = mx_1 + b \quad (5)$$



Note once again the given quantities are m, b, x_1, x_2, and y_2.

3-102
The above formulas may be programmed into an Excel spreadsheet to plot by themselves, or along with a set of data for comparison. The attached figure shows results for the set
$b = 0.25,\ x_1 = 0.5,\ x_2 = 1.0,\ y_2 = 1.0,\ m = 0.1, 0.2, 0.5.$

3-103
Straight line parabola area.
For normalized chart (1 by 1 unit)
Area above curve with initial line segment, connecting to parabola

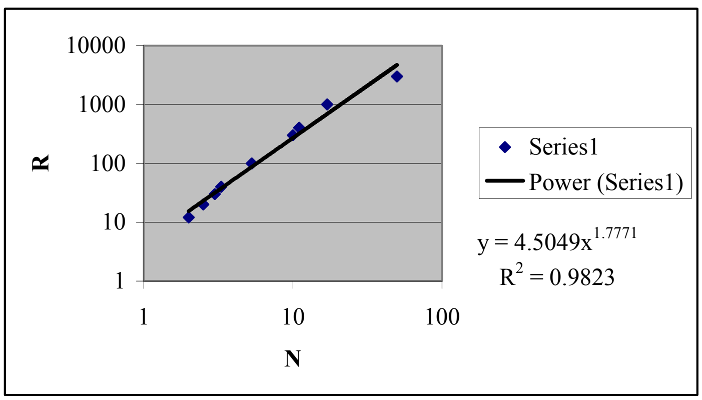
$$\text{Area} = (2 - b - y_1)(x_1/2) + (1 - a_0)(x_2 - x_1) - (a_1/2)(x_2^2 - x_1^2) - (a_2/3)(x_2^3 - x_1^3)$$

For normalized chart on x-coordinate, $x_2 = 1$.
For normalization on both coordinates, the total chart area is 1.0, so the area *under* the curve would be 1 minus above curve.

3-104
From Excel worksheet below, the resulting correlation is:
$R = 4.5049N^{1.7771}$
Written in the form of Problem 3-30 gives
$N = 0.4287R^{0.5627}$ (a)
From Problem 3-30
$N = 0.5577R^{0.487}$ (b)

R	N (Eq. a)	N (Eq. b)
12	1.74	1.87
3000	38.79	27.53

N	R
2	12
2.5	20
3	30
3.3	40
5.3	100
10	300
11	400
17	1000
50	3000



3-105

From Excel worksheet below

$x = 1.0765y^{0.4624}$

In form of Problem 3-28,

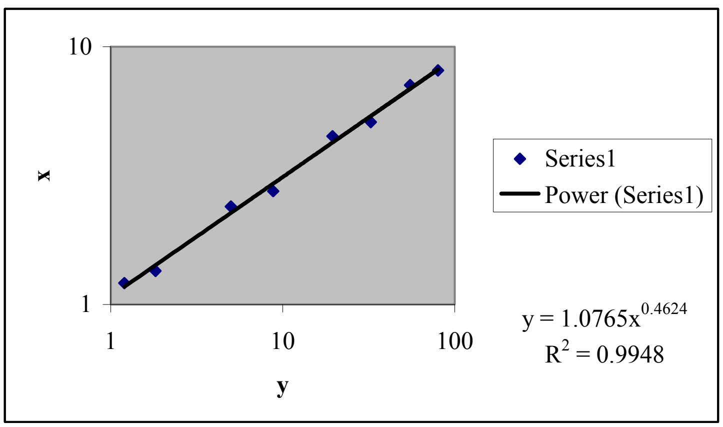
$y = 0.8526x^{2.1626}$ (a)

From Problem 3-28,

$y = 0.8642x^{2.1514}$ (b)

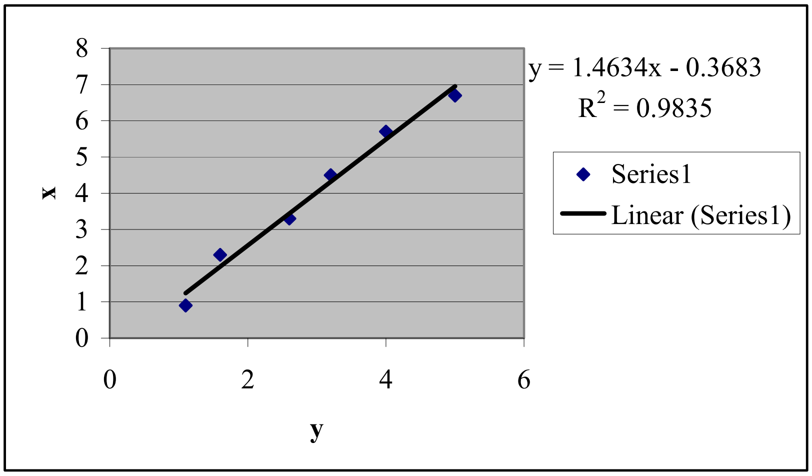
x	y (Eq. a)	y(Eq. b)
1.21	1.2876	1,302
8.1	78.6	77.83

y	x
1.2	1.21
1.82	1.35
5	2.4
8.8	2.75
19.5	4.5
32.5	5.1
55	7.1
80	8.1



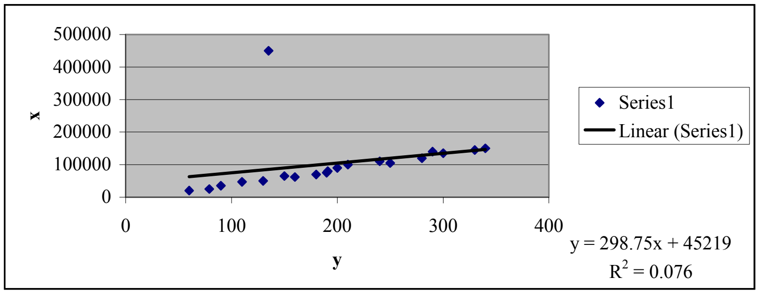
3-106
Worksheet shown below from Problem 3-27
$y = 0.6721x + 0.2955$
From worksheet
$y = 0.6833x + 0.2519$

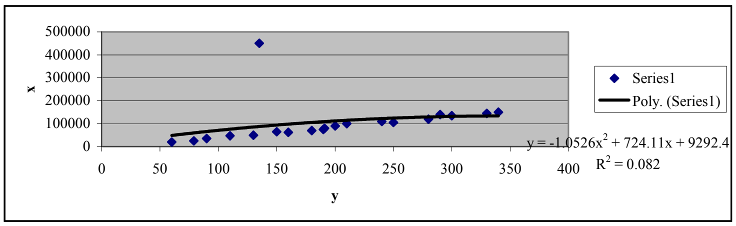
y	x
1.1	0.9
1.6	2.3
2.6	3.3
3.2	4.5
4	5.7
5	6.7

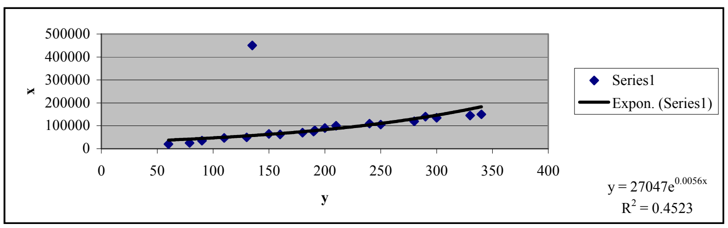


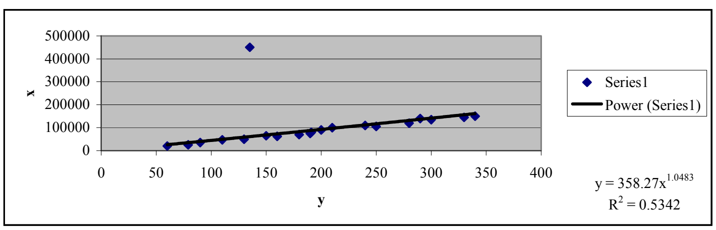
3-107

y	x
60	20000
79	25000
90	35000
135	450000
110	47000
130	50000
160	62000
150	65000
180	70000
190	75000
191	80000
200	90000
210	100000
250	105000
240	110000
280	120000
300	135000
290	140000
330	145000
340	150000









Chapter 4

4-1

Equation (4-11) $\dfrac{i}{\frac{E_i}{R_i}} = \dfrac{1}{\left(\frac{R}{R_m}\right)\left(\frac{R_m}{R_i}\right) + 1}$

By binomial expansion:

$$\frac{i}{\frac{E_i}{R_i}} = 1 - \left[\left(\frac{R}{R_m}\right)\left(\frac{R_m}{R_i}\right)\right] + \left[\left(\frac{R}{R_m}\right)\left(\frac{R_m}{R_i}\right)\right]^2 - \left[\left(\frac{R}{R_m}\right)\left(\frac{R_m}{R_i}\right)\right]^3 + \cdots$$

$$= 1 - \left[\frac{R}{R_i}\right] + \left[\frac{R}{R_i}\right]^2 - \left[\frac{R}{R_i}\right]^3 + \cdots$$

If we select the design variable R_i large compared to R, then $\dfrac{R}{R_i}$ is small and

$$\frac{i}{\frac{E_i}{R_i}} = 1 - \frac{R}{R_i} \qquad i = \frac{E_i}{P_i}\left[1 - \frac{R}{R_i}\right]$$

$$\text{Error} = \frac{E_i}{R + R_i} - \frac{E_i}{R_i}\left[1 - \frac{R}{R_i}\right]$$

$$\text{Error} = \frac{R^2 E_i}{R_i^2[R + R_i]}$$

4-3

$$F = B_i L = (1)(4)(0.1) = 0.4 \text{ N}$$

$$x = \frac{0.4}{1} = 0.4 \text{ m}$$

4-4

$$I_{\text{RMS}} = \left\{\frac{1}{T}\int_0^T i^2(t)\,dt\right\}^{1/21}$$

a. $I_{\text{RMS}} = \left[\dfrac{1}{T}\displaystyle\int_0^T 100\cos^2(t)\,d\tau\right]$

$$= \left\{\frac{1}{T}\left[\frac{t}{2} + \frac{\sin 2t}{4}\right]_0^T\right\}^{1/2}$$

$$= \left\{\frac{100}{T}\left[\frac{T}{2} + \frac{\sin 4T}{4}\right]\right\}^{1/2}$$

$$= 7.071$$

b. $I_{RMS} = \left\{\dfrac{100}{T(377)}\dfrac{T}{2}\right\}^{1/2} = 0.3642$

4-5

Equation (4-13) $\dfrac{E}{E_i} = \dfrac{\left(\frac{R}{R_m}\right)\left(\frac{R_m}{R_i}\right)}{1 + \left(\frac{R}{R_m}\right)\left(\frac{R_m}{R_i}\right)}$

$$\frac{E}{E_i} = \left(\frac{R}{R_m}\right)\left(\frac{R_m}{R_i}\right)\left[1 - \left(\frac{R}{R_i}\right) + \left(\frac{R}{R_i}\right)^2 - \left(\frac{R}{R_i}\right)^3 + \cdots\right]$$

If we select the design variable R_i large compared to R then $\frac{R}{R_i}$ is small and:

$$\frac{E}{E_i} = \frac{R}{R_i}\left[1 - \frac{R}{R_i}\right]$$

$$\text{Error} = \frac{\left(\frac{R}{R_m}\right)\left(\frac{R_m}{R_i}\right)}{1 + \left(\frac{R}{R_m}\right)\left(\frac{R_m}{R_i}\right)} - \frac{R}{R_i}\left[1 - \frac{R}{R_i}\right]$$

$$= \frac{\frac{R}{R_i}}{1 + \frac{R}{R_i}} - \left[\frac{R}{R_i} - \frac{R^2}{R_i^2}\right]$$

$$= \frac{R}{R_i + R} - \frac{RR_i - R^2}{R_i^2}$$

$$= \frac{R_i^2 R - R_i^2 R + R^3 + R^2 R_i - R^2 R_i}{R_i^2[R_i + R]}$$

$$\text{Error} = \frac{R^3}{R_i^2[R_i + R]}$$

4-6

$$i = \frac{E_i}{R + R_i} \rightarrow S = \frac{d_i}{dR} = \frac{-E_i}{(R + R_i)^2}$$

For maximum sensitivity, $\dfrac{ds}{dR_i} = 0$

$$\frac{ds}{dR_i} = \frac{2E_i}{(R + R_i)^3}$$

$\therefore$ The circuit has max sensitivity for $R_i \rightarrow \infty$.

4-7

Equation (4-14) $S = \dfrac{dE}{dR} = \dfrac{E_i R_i}{(R_i + R)^2} = \dfrac{E_i}{R}\left[1 + \dfrac{R}{R_i}\right]^{-2}$

$$S = \frac{E_i}{R}\left[1 - 2\frac{R}{R_i} + 3\left(\frac{R}{R_i}\right)^2 - 4\left(\frac{R}{R_i}\right)^3 + \cdots\right] \text{ where } \left(\frac{R}{R_i}\right)^2 < 1$$

If $\dfrac{R}{R_i}$ is very small compared to 1, then we can neglect higher order terms, and

$$S = \frac{E_i R_i}{(R_i + R)^2} - \frac{E_i}{R_i}\left[1 - 2\frac{R}{R_i}\right]$$

$$= \frac{E_i}{R_i}\left[\frac{R_i^3 - R_i R^2 - 2R_i^2 R - R_i^3 + 2R^3 + 4R_i R^2 + 2RR_i^2}{R_i(R + R_i)^2}\right]$$

$$\text{Error} = \frac{E_i R^2}{R_i^2}\left[\frac{3R_i + 2R}{(R + R_i)^2}\right]$$

4-11

Equation (4-26) $E_g = E\left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3}\right)$

$$E_g = 3\left(\frac{400}{400 + 6000} - \frac{40}{40 + 602}\right) = 0.584 \text{ millivolts}$$

Equation (4-24) $E_g = I_g(R + R_g)$ where $R = \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_3}{R_2 + R_3}$

$$R = \frac{(400)(6000)}{400 + 6000} + \frac{(40)(602)}{40 + 602} = 412.5 \text{ ohms}$$

$$I_g = \frac{0.584 \times 10^{-3}}{412.5 + 110} = 1.118\ \mu\text{amp}$$

4-12

From Example (4-4) it was found that:

$$R_4 = \frac{ER_1R_3 - I_g[R_gR_1(R_2 + R_3) + R_3R_2R_1]}{I_g(1 + R_1 + R_g)(R_2 + R_3) + ER_2}$$

$$= \frac{(4)(800)^2 - 0.08 \times 10^{-6}[(100)(800)(1600) + (800)^3]}{0.08 \times 10^{-6}(1 + 800 + 100)(1600) + (4)(800)}$$

$$= \frac{2.56 \times 10^6 - 51.20}{0.115 + 3200}$$

$$R_4 = 799.955 \text{ ohms}$$

4-14

At balance conditions: $R_1 = \frac{R_2 R_4}{R_3}$ $\therefore R_1 = \frac{(100)(4000)}{400} = 1000$ ohms

Assuming a battery voltage of 4 and negligible internal resistance, some values of I_g can be found as R_1 is adjusted to the balance condition. By this means the most current sensitive galvanometer may be observed.

If $R_1 = 900$ ohms: $E_g = E\left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3}\right) = 4\left(\frac{900}{1000} - \frac{4000}{4400}\right)$

$$E_g = 0.04 \text{ volts}$$

$$I_g = \frac{E_g}{R + R_g} \quad \text{but } R = \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_3}{R_2 + R_3} = \frac{(900)(100)}{(900 + 100)} + \frac{(4000)(400)}{4000 + 400}$$

$$I_g = \frac{0.04}{454 + R_g} \qquad R = 454 \text{ ohms}$$

For gal #1

$$I_{g_1} = \frac{.04}{454 + 50}$$

$$I_{g_1} = 79.5\ \mu\text{amps}$$

$$\text{deflection}_1 = \frac{I_{g_1}}{s_1} = \frac{79.5 \times 10^{-6}}{0.05 \times 10^{-6}}$$

$$\text{deflection}_1 = 1590 \text{ mm}$$

For gal #2

$$I_{g_2} = \frac{0.04}{454 + 500}$$

$$I_{g_2} = 42.0\ \mu\text{amps}$$

$$\text{deflection}_2 = \frac{I_{g_2}}{s_2} = \frac{42.0 \times 10^{-6}}{0.2 \times 10^{-6}}$$

$$\text{deflection}_2 = 210 \text{ mm}$$

If $R_1 = 975$ ohms

$$E_g = 4\left(\frac{975}{1075} - \frac{4000}{4400}\right) = 0.016 \qquad R = \frac{(975)(100)}{1075} + \frac{(4000)(400)}{4400}$$

$$I_g = \frac{0.016}{454.3 + R_g} \qquad = 454.3 \text{ ohms}$$

For gal #1

$$I_{g_1} = \frac{0.016}{454.3 + 50}$$

$$I_{g_1} = 317\ \mu\text{amps}$$

$$\text{deflection}_1 = \frac{31.7 \times 10^{-6}}{0.05 \times 10^{-6}} = 634 \text{ mm}$$

For gal #2

$$I_{g_2} = \frac{0.016}{454.3 + 500}$$

$$I_{g_2} = 16.78\ \mu\text{amps}$$

$$\text{deflection}_2 = \frac{16.78 \times 10^{-6}}{0.2 \times 10^{-6}} = 83.9 \text{ mm}$$

$\therefore$ Use galvanometer #1 because as R_1 is adjusted closer and closer to the balance condition it is more difficult to detect movement of the needle, and with galvanometer #1 the deflection is greater and therefore, easier to detect.

4-15

$$I_g = (30)(0.05 \times 10^{-6}) = 1.50 \times 10^{-6} \text{ amp}$$

$$R_4 = \frac{ER_1R_3 - I_g[R_gR_1(R_2 + R_3) + R_2R_3R_1]}{I_g[1 + R_i + R_g](R_2 + R_3)ER_2}$$

$$R_4 = \frac{4(12{,}000) - 1.5 \times 10^{-6}[3000(800) + 7{,}200{,}000]}{1.5 \times 10^{-6}(111)(800) + (4)(600)}$$

$$R_4 = \frac{47{,}985.6}{2400.1332} = 19.993 \text{ ohms}$$

4-16

$$\text{Deflection} = \frac{I_g}{S}$$

$$I_g = \frac{E_g}{R_e + R_g} \text{ where } R_i = \frac{R_{\text{shunt}}R}{R_{\text{shunt}} + R}$$

$$R = \frac{R_1R_4}{R_1 + R_4} + \frac{R_2R_3}{R_2 + R_3} = \frac{(290)(600)}{890} + \frac{(500)(1000)}{1500} = 528.5 \text{ ohms}$$

$$R_e = \frac{(30)(528.5)}{558.5} = 28.4 \text{ ohms}$$

$$E_g = E\left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3}\right) = 3\left(\frac{290}{890} - \frac{500}{1500}\right) = 0.021 \text{ volt}$$

$$I_g = \frac{21\times 10^{-3}}{28.4 + 50} = 0.268\times 10^{-3} \text{ amp}$$

$$\text{Deflection} = \frac{0.268\times 10^{-3}}{0.05\times 10^{-6}} = 5.36\times 10^{3} \text{ mm shunted circuit}$$

$$I_g = \frac{E_g}{R + R_g + R_{\text{series}}}$$

$$= \frac{21\times 10^{-3}}{528.5 + 50 + 30}$$

$$= 0.0345\times 10^{-3} \text{ amp}$$

$$\text{Deflection} = \frac{0.0345\times 10^{-3}}{0.05\times 10^{-6}} = 690 \text{ mm}$$

4-18

Prototype T-section.
Refer to Figure 4-43, constant-*k* *T* section.
Assume load resistance of 16 ohms.

$$L_{1k} = \frac{R}{\pi(f_2 - f_1)} = \frac{16}{3.14(2000 - 500)} = 3.39\times 10^{-3} \text{ henrys}$$

$$L_{2k} = \frac{(f_2 - f_1)R}{4\pi f_1 f_2} = \frac{(2000 - 500)16}{4(3.14)(2000)(500)} = 1.91\times 10^{-3} \text{ henrys}$$

$$C_{1k} = \frac{f_2 - f_1}{4\pi f_1 f_2 R} = \frac{2000 - 500}{4(3.14)(2000)(500)(16)} = 7.46\times 10^{-6} \text{ farads}$$

$$C_{2k} = \frac{1}{\pi(f_2 - f_1)R} = \frac{1}{(3.14)(2000 - 500)(16)} = 13.25\times 10^{-6} \text{ farads}$$

4-19

Prototype T-section
Refer to Figure 4-43, constant-*k* T section. Assume load resistance of 1000 ohms.

$$L_k = \frac{R}{\pi f_c} = \frac{1000}{(3.14)(500)}\frac{L_k}{2} = 0.3185 \text{ henrys}$$

$$C_k = \frac{1}{\pi f_c R} = \frac{1}{(3.14)(500)(1000)} = 0.637\ \mu\text{farads}$$

4-20

Prototype T-section
Refer to Figure 4-43, constant-*k* T section. Assume load resistance of 1000 ohms.

$$L_k = \frac{R}{4\pi f_c} = \frac{1000}{4(3.14)(1000)} = 0.0796 \text{ henrys}$$

$$C_k = \frac{1}{4\pi f_c R} = \frac{1}{4(3.14)(1000)(1000)} 2C_k = 0.1592\ \mu\text{farads}$$

4-21

Volt-ohmmeter:

$$i = \frac{E}{R_i + R} = \frac{5000}{160,000} = 3.12 \times 10^{-3} \text{ amp}$$

$$E_{v-0} = iR = (3.12 \times 10^{-3})(1.5 \times 10^5) = 468 \text{ v actual}$$

$$E_{v-0} = E\frac{\frac{RR_m}{R+R_m}}{R_i + \frac{RR_m}{R+R_m}} = 500\left[\frac{\frac{(150)(468)}{618}}{10 + \frac{(150)(468)}{618}}\right] = 460 \text{ volts}$$

VTVM:

$$E_{\text{VTVM}} = 500\left[\frac{\frac{(150)(11,000)}{11,150}}{10 + \frac{(150)(11,000)}{11,150}}\right] = 500\left[\frac{148}{158}\right] = 468 \text{ volts}$$

4-23

$$C = 4(0.225)\varepsilon\frac{A}{d} \qquad A = 1.0x \qquad d = 0.01$$

$$S = \frac{dC}{dx} = 4(0.225)\ \varepsilon\frac{(1.0)}{d} = 400(0.225)\ \varepsilon = 90 \text{ pf/in.}$$

4-24

$$S = \frac{\partial v}{\partial d} = \frac{\Delta v}{\Delta d} = \frac{1.0 - 0}{0.064 - 0} = 15.61\frac{\text{volts}}{\text{unit disp}}$$

4-25

$$p = (100)(6.895 \times 10^3) = 6.895 \times 10^5 \text{ newton/m}^2$$

$$t = 2 \times 10^{-3} \text{ m}; E = gtp = 0.055(2 \times 10^{-3})(6.895 \times 10^5) = 75.8 \text{ v}$$

$$p = \frac{E}{gt}; \frac{\partial p}{\partial E} = \frac{1}{gt}; \frac{\partial p}{\partial E} = -\frac{E}{gt^2} \text{ From Equation (3-2)}$$

$$Wp = \left[\left(\frac{\partial p}{\partial E}W_E\right)^2 + \left(\frac{\partial p}{\partial t}W_t\right)^2\right]^{1/2}$$

$$= \left\{\left[\frac{1}{(0.055)(2 \times\ 10^{-3})}(0.5)\right]^2 + \left[\frac{75.8}{(0.055)(2 \times\ 10^{-3})^2}(7.62 \times 10^{-6})\right]^2\right\}^{1/2}$$

$$W_p = \{[4550]^2 + [2630^2\}^{1/2} = 5255 \text{ newtons/m}^2 \text{ or } 0.7625\%$$

4-26

$$E_{\text{RMS}} = \frac{1}{\sqrt{2}}NABW = \frac{1}{\sqrt{2}}(50)(10^{-4})(1)(180)\left(\frac{2\pi}{60}\right)$$

$$E_{\text{RMS}} = 66.7 \times 10^{-3} \text{ volts}$$

4-27

$$\text{db} = 10\log\frac{P_2}{P_1} \rightarrow -1 = 10\log\frac{P_2}{35}$$

$$-0.1 = \log\frac{P}{35} \rightarrow \log(0.7932) = -0.1$$

$$\frac{P_2}{35} = 0.7932 \rightarrow 27.76 \text{ watts}$$

4-28

$$E_{\text{IN}} = 2.2\times10^{-3} \text{ volts};\ P_{\text{OUT}} = 35 \text{ watts}; R = 8 \text{ ohms}$$

$$P_{\text{OUT}} = \frac{E_{\text{OUT}}^2}{R} = 35 \text{ watts} \rightarrow E_{\text{OUT}} = 16.7 \text{ volts}$$

$$E_{\text{INPUT}} = E_{\text{SIGNAL}} + E_{\text{NOISE}} \qquad 10 \text{ mv} = 1.0\times10^{-2} \text{ volts}$$

$$-65 = 20\log\frac{(E_{\text{NOISE}})_{\text{IN}}}{1.0\times10^{-2}} \therefore (E_{\text{NOISE}})_{\text{IN}} = 5.62\ \mu\text{volts}$$

$$\frac{E_{\text{IN}}}{E_{\text{OUT}}} = \frac{(E_{\text{NOISE}})_{\text{IN}}}{(E_{\text{NOISE}})_{\text{OUT}}} \rightarrow (E_{\text{NOISE}})_{\text{OUT}} = \frac{(16.7)(5.62\times10^{-6})}{2.2\times10^{-3}}$$

$$(E_{\text{NOISE}})_{\text{OUT}} = 42.8 \text{ mv}$$

4-29

$$E_{\text{OUT}} = (P_{\text{OUT}}R)^{1/2} = 16.7 \text{ volts}$$

$$-75 = 20\log\frac{(E_{\text{NOISE}})_{\text{IN}}}{2.5\times10^{-1}} \rightarrow (E_{\text{NOISE}})_{\text{IN}} = 44.5\ \mu\text{volts}$$

$$\frac{(E_{\text{NOISE}})_{\text{OUT}}}{(E_{\text{NOISE}})_{\text{IN}}} = \frac{E_{\text{OUT}}}{E_{\text{IN}}} \rightarrow (E_{\text{NOISE}})_{\text{OUT}} = \frac{(16.7)(44.5\times10^{-6})}{180\times10^{-3}}$$

$$(E_{\text{NOISE}})_{\text{OUT}} = 4.125 \text{ mv}$$

$$\text{db} = 20\log\left(\frac{4.125\times10^{-3}}{42.8\times10^{-3}}\right) = 20\log 0.0964 = -20.32$$

4-32

$$R_m = 10 \text{ kilohms}, R_i = 100 \text{ kilohms} \rightarrow \frac{R_i}{R_m} = 0.10$$

$$\text{Loading error} = \frac{-\left(\frac{R}{R_m}\right)^2\left(1-\frac{R}{R_m}\right)}{\left(\frac{R}{R_m}\right)\left(1-\frac{R}{R_m}\right)+\left(\frac{R_i}{R_m}\right)}\times 100$$

$$\text{At } 10\% \rightarrow \frac{R}{R_m} = 0.10 \therefore \text{Loading error} = -4.74\%$$

$$\text{At } 90\% \rightarrow \frac{R}{R_m} = 0.90 \therefore \text{Loading error} = -42.6\%$$

4-33

$E_i = .100$ volts $\qquad \dfrac{R_m}{R_i} = \dfrac{1}{6} = 0.1667$

$$\frac{E}{E_i} = \frac{\left(\frac{R}{R_m}\right)\left(\frac{R_m}{R_i}\right)}{1 + \left(\frac{R}{R_m}\right)\left(\frac{R_m}{R_i}\right)}$$

$\frac{R}{R_m}$	E_{ACTUAL} (volts)	E_{LINEAR} (volts)	% Error $= \frac{E_{LIN} - E_{ACT}}{E_{ACT}} \times 100$
0	0	0	0
0.25	4.00	3.60	−10%
0.50	7.70	7.20	−6.5%
0.60	9.10	8.64	−5.05%
0.80	11.76	11.50	−2.21%
1.00	14.40	14.40	0

4-34

$R_1 = 121$ ohms; $R_2 = 119$ ohms; $R_3 = 121$ ohms

$$R_4 = \frac{R_3 R_1}{R_2} = \frac{(121)^2}{119} = 123.0336 \text{ ohms}$$

When $R_4 = 122$ ohms and $E_b = 100$ volts $\rightarrow E = ?$ and $E_g = ?$

$$E = E_b \frac{R_0}{R_0 + R_b} \text{ where } R_b = 0 \therefore E = E_b = 100 \text{ volts}$$

$$E_g = E\left[\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3}\right] = 100\left[\frac{121}{121 + 122} - \frac{119}{119 + 121}\right]$$

$E_g = 100[0.4979 - 0.4958]$ volts $= 0.21$ volts

4-35

For PbS at 3.5 μ $\rightarrow D_1^* = 1.0 \times 10^8$ (cm)Hz$^{1/2}$/watt

For PbS at − 196° C and 3.5 μ $\rightarrow D_2^* = 8.0 \times 10^{10}$(cm)Hz$^{1/2}$/watt

$$D^* = [A(\Delta f)]^{1/2} \frac{E_{OUT}}{E_{NOISE}} \frac{1}{P_{INCIDENT}} \rightarrow \text{when } \frac{S}{N} = \text{CONSTANT}$$

$$\frac{E_{OUT}}{E_{NOISE}} = \text{CONSTANT}$$

$$\left.\frac{P_2}{P_1}\right)_{INCIDENT} = 1.25 \times 10^{-3}; \text{db} = 10 \log \frac{P_2}{P_1} = 10 \log(1.25 \times 10^{-3}) = -29.03$$

4-36

In Sb at 5 $\mu \rightarrow D^* = 1.0 \times 10^8$ cm(Hz)$^{1/2}$/watt $\qquad A = 4.0 \times 10^{-2}$ cm^2

$$\frac{S}{N} = 45 \text{ db} \qquad 45 = 20 \log \frac{E_{OUT}}{E_{NOISE}} \rightarrow \frac{E_{OUT}}{E_{NOISE}} = 178$$

$$\frac{P_{INCIDENT}}{(\Delta f)^{1/2}} = (A^{1/2})\left(\frac{E_{OUT}}{E_{NOISE}}\right)\left(\frac{1}{D^*}\right) = \frac{(0.2)(178)}{1.0 \times 10^8} = \frac{35.6}{10^8}$$

$P_{INCIDENT} = 35.6 \times 10^{-8}$ watts

4-37

$R_m = 500 \text{ ohms}, \ R_i = 7000 \text{ ohms} \Rightarrow \dfrac{R_m}{R_i} = 7.16 \times 10^{-2}$

$E_i = 12 \text{ volts} \qquad i = ?$

$\frac{R}{R_m}$	$I_{\text{ACTUAL}} \times 10^3$ amps	$i_{\text{LINEAR}} \times 10^3$ amps	$\%\,\text{Error} = \dfrac{i_{\text{LIN}} - i_{\text{ACT}}}{i_{\text{ACT}}} \times 100$
0	1.716	1.716	0
0.25	1.690	1.687	−0.01775%
0.50	1.656	1.658	0.01210%
0.75	1.628	1.629	0.00615%
1.00	1.600	1.600	0

where $i = \dfrac{E_i}{R_i}\left[\dfrac{1}{\left(\frac{R}{R_m}\right)\left(\frac{R_m}{R_i}\right) + 1}\right]$

4-38

$$\frac{E}{E_0} = \frac{\left(\frac{R}{R_m}\right)}{\left(\frac{R}{R_m}\right)\left[1 - \left(\frac{R}{R_m}\right)\right]\left(\frac{R_m}{R_i}\right) + 1}$$

$E_0 = 24 \text{ volts}$

$\dfrac{R_m}{R_i} = 0.1$

at E volts	$\frac{R}{R_m}$
6	0.225
12	0.501
18	0.780
24	1.00

4-40

0.1 W at 0 dB 20 to 100 kHz

+0.5, −1.0 db

$0.5 = 10 \log \dfrac{P_2}{0.1} \qquad P_2 = 0.112 \text{ W}$

$-1.0 = 10 \log \dfrac{P}{0.1} \qquad P_2 = 0.079 \text{ W}$

4-41

$R_i = 15000\,\Omega \pm 10\%$

$R_m = 800\,\Omega \pm 2\%$

$E_0 = 30 \text{ V} \pm 0.1 \text{ V}$

$\dfrac{R_m}{R_i} = 0.0533$

At 5 V $\dfrac{R}{R_m} = 0.168$

At 10 V $\dfrac{R}{R_m} = 0.341$

At 15 V $\dfrac{R}{R_m} = 0.521$

4-42

$$-80 = 10 \log \frac{P_N}{100}$$

$$P_N = 10^{-6}\ \text{W}$$

$$-80 = 20 \log \frac{E_N}{1.0\ \text{mV}}$$

$$E_N = 10^{-4}\ \text{mV} = 0.1\ \mu\text{V}$$

4-43

$$1\ \text{Rev} = \frac{60}{1800} = 0.0333\,\text{sec}$$

$$\begin{aligned} \text{Displacement in 1 Rev} &= (20)(0.0333) \\ &= 0.6667\ \text{m} \\ &= 666.7\ \text{mm} \end{aligned}$$

$$V = \frac{s}{t_1 - t_2} \qquad \frac{\partial v}{\partial s} = \frac{1}{t_1 - t_2} \qquad \frac{\partial v}{\partial t_1} = \frac{-5}{(t_1 - t_2)^2}$$

$$\frac{\partial v}{\partial t_2} = \frac{+s}{(t_1 - t_2)^2}$$

$$W_v = \left[\left(\frac{1}{t_1 - t_2}\right) W_s^2 + \left(\frac{s}{(t_1 - t_2)^2}\right)^2 W_{t_1}^2 + \left(\frac{-s}{(t_1 - t_2)^2}\right)^2 W_{t_2}^2\right]^{1/2}$$

$$\begin{aligned} \frac{W_v}{v} &= \left[\left(\frac{1}{666.7}\right)^2 + (0.001)^2 + (0.001)^2\right]^{1/2} \\ &= 0.00206 \end{aligned}$$

4-44

$$\text{Gain} = -\frac{R_f}{R_1}$$

$$R_f = (25)(5) = 125\ \text{M}\Omega$$

4-45

$$R_2 = \frac{R_1 R_f}{R_1 + R_f}$$

$$\text{Gain} = 1 + \frac{R_f}{R_1}$$

$$R_f = (25 - 1)(5) = 120\ \text{M}\Omega$$

$$R_2 = \frac{(5)(120)}{5 + 120} = 4.8\ \text{M}\Omega$$

4-46

$a = 10,\ b = 14,\ c = 1$

4-49

$$E_0 = -R_f\left(\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3}\right) = E_1 + 2E_2 + 3E_3$$

$$\frac{R_f}{R_1} = 1 \qquad \frac{R_f}{R_2} = 2 \qquad \frac{R_f}{R_3} = 3$$

4-50

$\text{Gain} = 15 + 1 + n \qquad n = 14$

4-51

Choose values of $R_i C$ to match integration times. At $\tau = 1$ ms could use

$R_i = 1\ \text{k}\Omega$

$C = 1\ \mu\text{F}$

$R_i C = (1000)(1 \times 10^{-6}) = 0.001\ \text{sec}$

4-58

$$\text{Voltage sensitivity} = \frac{\Delta V}{\Delta x} = \frac{0.94}{0.06} = 15.67\ \text{V/in.}$$

In null region, sensitivity $= \dfrac{0.01}{0.0007} = 14.29$ V/in. or very nearly the same.

4-59

PbS detector

$\lambda = 1.0\ \mu\text{m}$

$$\frac{S}{N} = 30\ \text{dB}$$

$A = 1$ mm square $= 10^{-2}\ \text{cm}^2$

From Example 4-14, $D^* = 1.5 \times 10^{11}$ cm $\text{Hz}^{1/2}$/W

$$30 = 20\log\left(\frac{E}{E_N}\right); \frac{E}{E_N} = 31.26$$

$$1.5 \times 10^{11} = \frac{(10^{-2})^{1/2}(31.26)}{P}$$

$P = 2.11 \times 10^{-11}\ \text{W}$

4-60

Open loop gain $= 2000\,\dfrac{S}{N} = 2000$

Neg. feedback $k = 0.05$

For $E_i = 1.0$ $\quad E_0 = 2000 \pm 1 \quad (N = \pm 1)$

$$A_f = \frac{1}{k} = 20;\ E_i = \frac{E_0}{20} = \frac{2000}{20} = 10$$

From Equation (4-40)

$$E_0 = \frac{100}{0.05} \pm \frac{1}{(0.05)(2000)} = 2000 \pm 0.01$$

$$SN \text{ Ratio} = \frac{2000}{0.01} = 200{,}000 = 106 \text{ dB}$$

Initial dB for $SN = 2000$ is only 66 dB

4-61

Non-inverting op-amp

$R_f = 1\ \text{M}\Omega,\ R_1 = 100\ \text{k}\Omega$

Case (d) Figure 4-32

$$\text{Gain} = 1 + \frac{R_f}{R_1} = 1 + \frac{106}{105} = 11$$

4-62

Inverting, differential input

Case (c) Figure 4-32

$$\text{Gain} = -\frac{R_f}{R_1} = -\frac{10^6}{10^5} = -10$$

4-63

Figure 4-38, constant k T section, $R = 8,\ f_c = 40$ Hz

$L_K = 8/\pi(40) = 0.064$ henrys

$C_K = 1/(8)(40)\pi = 995\ \mu$Farads

4-64

Figure 4-38, constant k T section, $R = 8,\ f_c = 450$ Hz

$L_K = 8/4\pi(450) = 1.41$ millihenrys

$C_K = 1/4\pi(8)(450) = 22.1\ \mu$farads

4-65

Figure 4-57, PbS detector, $S/N =$ constant

At $\lambda = 3.0\ \mu$m $D^* = 5 \times 10^9$ at room temp

$= 1.5 \times 10^{11}$ at 76K

$P_2/P_1 = 5 \times 10^9/1.5 \times 10^{11} = 0.0333$

dB $= 10 \log(0.0333) = -14.77$

4-66

$P = 100$ W at 0 dB, 10 to 50 kHz

$+1.0 = 10\log(P_2/100)$

$P_2 = 126$ W upper limit

$-1.0 = 10\log(P_2/100)$

$P_2 = 79.4$ lower limit

4-67

$-90 = 20\log(E_N / 0.005)$

$E_N = 0.16\ \mu$V

4-68

From Figure 4-51

Voltage sensitivity $= \Delta V/\Delta x = 0.62/0.04 = 15.5$ V/im

4-69

$S/N = 25$ dB

$A = 4.0\ \text{mm}^2$

$\lambda = 2.0\ \mu\text{m}$

From Figure 4-57, $D^* = 1.6 \times 10^{11}$

Noise $= 25$ dB $= 20\log(E/E_N)$

$E/E_N = 1.778$

$D^* = (Af)^{1/2} E/E_N)/P$

Bandwidth f for Figure 4-57 is $f = 1$ Hz

Solving for P,

$P = (0.04)^{1/2}(1.778)/1.6 \times 10^{11}$

$= 9.86 \times 10^{-12}$ W

Chapter 5

5-1

$L(1 + \alpha\Delta T) = 11[1 + (6.47 \times 10^{-6})(40)] = 11.003$ in.

$\therefore$ Error $= 11.003 - 11.0 = 0.003$ in. One should not be concerned with this error because it is fixed and may be calculated.

5-2

$L(1 + \alpha\Delta T) = 76[1 + (6.47 \times 10^{-6})(-70)]$

True measurement $= 75.966$ feet

5-3

For no reflected light:

$$2d = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$$

$2d = n\lambda - \frac{\lambda}{2}$ where $m = 1, 2, 3, \ldots$ is the number of fringe lines.

$$\therefore d = \frac{n\lambda}{2} - \frac{\lambda}{4} = \frac{2n-1}{4}\lambda$$

5-5

x	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
y	3	3.84	3.67	4.293	7.32	14.13	26.07	44.0	68.92	102.2	141.75

Trapezoidal method: $\Delta x = 1$

$$A = \left[\frac{y_0 + y_n}{2} + \sum_{i=1}^{n-1} y_i\right]\Delta x$$

$$= \left[\frac{3 + 141.75}{2} + 3.67 + 7.32 + 26.07468.92\right](1)$$

$= 178.36$ sq units

Simpson's Rule:

$$A = \frac{\Delta x}{3}\left\{y_0 + y_n + \sum_{i=1}^{n-1} y_i[3 + (-1)^{i+1}]\right\}.$$

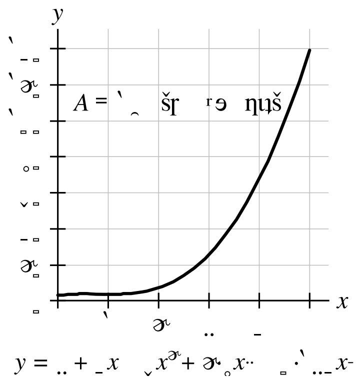
$$= \frac{0.5}{3}\left\{\begin{array}{c} 3 + 141.75 + 4(3.84) + 2(3.67) + 4(4.293) + 2(7.32) + 4(14.13) + 2(26.07) \\ + 4(44.0) + 2(68.92) + 4(102.2) \end{array}\right\}$$

$A = 172.10$ square units

Direct Integration:

$$A = \int_0^5 [3 + 4x - 6x^2 + 2.8x^3 - 0.13x^4]\,dx$$

$= 171.25$ sq units



Graphical method:
Error: Graphical

$E = 171.25 - 175 = -3.75$ sq units

Trapezoidal:

$E = 171.25 - 178.36 = -7.11$ sq units

Simpson's

$\text{Error} = 171.25 - 172.10$

$E = 0.85$ sq units

5-6

Actual area:

$$A = \int_0^{\pi} \sin x\, dx = 2 \text{ sq units}$$

Trapezoidal rule: for $\Delta x = \dfrac{\pi}{4}$

$$A = \left(\frac{y_0 + y_n}{2} + \sum_{i=1}^{n-1} y_i \right) \Delta x = \left[\sin\frac{\pi}{4} + \sin\frac{\pi}{2} + \sin\frac{3\pi}{4} \right] \frac{\pi}{4}$$

$A = 1.895$ sq units

$\text{Error} = 2.0 - 1.895 = 0.105$ sq units

For $\Delta x = \dfrac{\pi}{8}$:

$$A = \left[\sin\frac{\pi}{8} + \sin\frac{\pi}{4} + \sin\frac{3\pi}{4} + \sin\frac{\pi}{2} + \sin\frac{5\pi}{8} + \sin\frac{3\pi}{4} + \sin\frac{7\pi}{8} \right] \frac{\pi}{8}$$

$A = 1.973$ sq units

$\text{Error} = 0.027$ sq units

For $\Delta x = \dfrac{\pi}{12}$:

$$A = \left[\begin{array}{l} \sin\frac{\pi}{12} + \sin\frac{\pi}{6} + \sin\frac{\pi}{4} + \sin\frac{\pi}{3} + \sin\frac{5\pi}{12} + \sin\frac{\pi}{2} + \sin\frac{7\pi}{12} + \sin\frac{2\pi}{3} + \sin\frac{3\pi}{4} \\ + \sin\frac{5\pi}{6} + \sin\frac{11\pi}{12} \end{array} \right] \frac{\pi}{12}$$

$A = 1.995$ sq units

$\text{Error} = 0.005$ sq units

Simpson's Rule

$$A = \frac{4x}{3}\left\{ y_0 + y_n + \sum_{i=1}^{n-1} y_i[3 + (-1)^{i+1}] \right\}$$

For $\Delta x = \frac{\pi}{4}$:

$$A = \frac{\pi}{12}\{0 + 0 + 4(0.707) + 2(1) + 4(0.707)\} = 2.005 \text{ sq units}$$

Error $= -0.005$ sq units

For $\Delta x = \frac{\pi}{8}$:

$$A = \frac{\pi}{24}\left\{ \begin{array}{l} 0 + 0 + 4(0.2585) + 2(0.5) + 4(0.707) + 2(0.886) + 4(0.965) + 2(1.0) \\ \qquad + 4(0.965) + 2(0.886) + 4(0.707) + 2(0.5) + 4(0.2585) \end{array} \right\}$$

$$= 2.005 \text{ sq units}$$

Error $= -0.005$ sq units

5-7

$$x^2 + y^2 + z^2 = 25$$

$$A = \sum_p \sum_n \left[\left(\frac{z_{n+1}^p - z_n^p}{\Delta x} \right)^2 + \left(\frac{z_n^{p+1} - z_n^p}{\Delta y} \right) + 1 \right]^{1/2} \Delta x \Delta y$$

$$z = \sqrt{25 - x^2 - y^2}; \Delta x \Delta y = 0.25$$

$$A = 4\left\{ \left[\left(\frac{z_1^0 - z_0^0}{\Delta x} \right) + \left(\frac{z_0^1 - z_0^0}{\Delta y} \right)^2 + 1 \right]^{1/2} \Delta x \Delta y \right.$$

$$+ \left[\left(\frac{z_2^0 - z_1^0}{\Delta x} \right) + \left(\frac{z_1^1 - z_1^0}{\Delta y} \right) + 1 \right]^{1/2} \Delta x \Delta y$$

$$+ \left[\left(\frac{z_1^1 - z_0^1}{\Delta x} \right) + \left(\frac{z_0^2 - z_0^1}{\Delta y} \right) + 1 \right]^{1/2} \Delta x \Delta y$$

$$\left. + \left[\left(\frac{z_2^1 - z_1^1}{\Delta x} \right)^2 + \left(\frac{z_1^2 - z_1^1}{\Delta y} \right)^2 + 1 \right]^{1/2} \Delta x \Delta y \right\}$$

$$z_1^0 = \sqrt{25 - (0.5)^2 - 0} = 4.97$$

$$z_0^0 = \sqrt{25} = 5$$

$$z_0^1 = \sqrt{25 - 0 - (0.5)^2} = 4.97$$

$$z_2^0 = \sqrt{25 - (1)^2 - 0} = 4.9$$

$$z_1^1 = \sqrt{25 - (0.5)^2 - (0.5)^2} = 4.95$$

$$z_0^2 = \sqrt{24} = 4.9$$

$$z_2^1 = \sqrt{25 - (1)^2 - (0.5)^2} = 4.87$$

$$z_1^2 = 4.87$$

$$A = 4\left\{\left[\left(\frac{4.97-5}{0.5}\right)^2 + \left(\frac{4.97-5}{0.5}\right)^2 + 1\right]^{1/2}(0.25)\right.$$
$$+\left[\left(\frac{4.9-4.97}{0.5}\right)^2 + \left(\frac{4.95-4.97}{0.5}\right)^2 + 1\right]^{1/2}(0.25)$$
$$+\left[\left(\frac{4.95-4.97}{0.5}\right)^2 + \left(\frac{4.9-4.97}{0.5}\right)^2 + 1\right]^{1/2}(0.25)$$
$$\left.+\left[\left(\frac{4.87-4.95}{0.5}\right)^2 + \left(\frac{4.87-4.95}{0.5}\right)^2 + 1\right]^{1/2}(0.25)\right\}$$

$A = 4.046$ sq units

5-9

$\alpha = 11.65\times 10^{-6}/°\,\text{C}$ True reading at 20° C

At 10°C

$20 = L[1 + (11.65\times 10^{-6})(10-20)] = 20.00233$ m

At 40°C

$20 = L[1 + (11.65\times 10^{-6})(40-20)] = 19.99534$ m

5-10

$\lambda = 2.15\times 10^{-5}$ in. $n = 9$ $d = \left(\frac{2n-1}{4}\right)\lambda$ (from Problem 5-3)

$d = 9.14\times 10^{-5}$ in.

5-12

$\Delta p = p_2 - p_a$: $w_{\Delta p} = 1.805\times 10^{-3}$ psig; $W_{p_1} = 0.07$ psig

$$r = \frac{\Delta p}{p_1 - p_2} = 1.10 - 2.00\left(\frac{d_2}{d_1^2}\right)x \rightarrow x = \frac{1.10}{2.00}\left(\frac{d_1^2}{d_2}\right) - \frac{1}{2}\left(\frac{d_1^2}{d_2}\right)\frac{\Delta p}{p_1 - p_a}$$

$$W_x = \left[\left(\frac{\partial x}{\partial p_1}W_{p_1}\right)^2 + \left(\frac{\partial x}{\partial \Delta p}W_{\Delta p}\right)^2\right]^{1/2} \qquad r = \frac{p_2 - p_a}{p_1 - p_a} \qquad p_1 = 10 \text{ psig}$$

$$\frac{\partial x}{\partial \Delta p} = -\frac{1}{2}\left(\frac{d_1^2}{d_2}\right)\frac{1}{p_1 - p_a} = -7.25\times 10^{-4}$$

At $r = 0.4 \rightarrow p_2 = 4$ psig

At $r = 0.9 \rightarrow p_2 = 9$ psig

$$\frac{\partial x}{\partial p_1} = \frac{1}{2}\left(\frac{d_1^2}{d_2}\right)\frac{\Delta p}{(p_1 - p_a)^2} = \begin{Bmatrix}\text{at } r = 0.4 \rightarrow 2.90\times 10^{-4}\\ \text{at } r = 0.9 \rightarrow 6.53\times 10^{-4}\end{Bmatrix}$$

2.035×10^{-5} in. $< W_x < 4.56\times 10^{-5}$ in.

5-13

$y = 1 + 3x + 4x^2$

$$\int_0^4 y\,dx = \left(x + 1.5x^2 + \frac{4}{3}x^3 \right)_0^4 = 113.333$$

x	0	1	2	3	4
y	1	8	23	46	77

Trapezoid Rule

$$A = \frac{1 + 77}{2} + 8 + 23 + 46 = 116$$

Error $= \pm 1.61667$

Simpson rule

$$A = \left(\frac{1}{3}\right)[1 + 77 + (-4)(8) + (2)(28) + (4)(46)]$$

$$= 113.333 \text{ Exact}$$

5-24

$$A = \sum y_i (x_{i+1} - x_i)$$

$$= (70)(14 - 0) + (100)(58 - 14) + (106)(96 - 58) + (64)(74 - 96)$$
$$+ (20)(30 - 74) + (10)(0 - 30)$$
$$= 980 + 4400 + 4028 - 1408 - 880 - 300$$
$$= 6820$$

$$\text{Error} = \frac{6820 - 6168}{6168} = 10.56\%$$

5-25

x_1	y_i	ΔA (rect)	ΔA (trap)
0	50	500	
10	72	720	610
20	84	840	780
30	93	930	885
40	60	−600	765
30	33	−330	−465
20	41	−410	−370
10	45	−450	−430
0	50	0	−475
		1200	1300

$$A \text{ (Simpson)} = \frac{10}{3}[50 + 60 + 72(4) + 84(2) + 93(4)]$$
$$- \frac{10}{3}[60 + 50 + 33(4) + 41(2) + 45(4)]$$
$$= 1447$$

A (Trap) $=$ exact $= 1300$

$$\text{Error (rect)} = \frac{1200 - 1300}{1300} = -7.7\%$$

$$\text{Error (Simpson)} = \frac{1447 - 1300}{1300} = +11.3\%$$

5-26

$\alpha = 11.65 \times 10^{-6}/\,^\circ\text{C at } 15^\circ\text{ C}$

$$L(1 + \alpha\Delta T) = (30)[1 + (11.65 \times 10^{-6})(50 - 15)]$$
$$= 30.017475 \text{ m}$$

Error $= 0.0017475 \text{ m} = 0.05825\%$

5-28

x_1	y_i	ΔA_i (rect)	ΔA_i (trap)
0	0.5	0	0
0	1	0.5	0.5
0.5	1	0.5	0.5
1	1	0.5	0.5
1.5	1	0.5	0.5
2	1	0	0
2	0.5	0	0
2	0	0	0
1.5	0	0	0
1	0	0	0
0.5	0	0	0
0	0	0	0
0	0.5	0	0
A (total)		2.0	2.0

Both give the correct area.

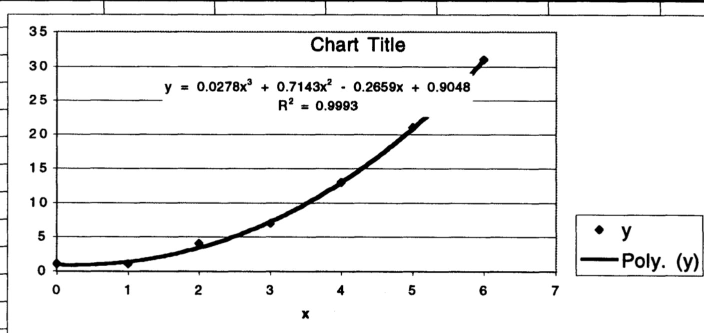
5-29

See chart below.

$$y = 0.0278x^3 + 0.7143x^2 - 0.2659x + 0.9048$$

$$A = \int_0^6 y\,dx = 61.0794$$

	A	B	C	D	E	F	G	H	I
1									
2									
3									
4	x	y							
5	0	1							
6	1	1							
7	2	4							
8	3	7							
9	4	13							
10	5	21							
11	6	31							
12									
13									



5-30

The values of the integrals are:

$0 \text{ to } 10\mu\text{m T} = 1000\text{K Integral} = 51{,}835$

$0 \text{ to } 10\mu\text{m T} = 2000\text{K Integral} = 893{,}933$

$0 \text{ to } 20\mu\text{m T} = 1000\text{K Integral} = 55{,}871$

$0 \text{ to } 20\mu\text{m T} = 2000\text{K Integral} = 905{,}153$

Chapter 6

6-1

$$\% \text{ Error } = \frac{h^1\left[\frac{A_2}{A_1} - h^1\right] - h^1}{h^1\left[\frac{A_2}{A_1} - 1\right]} \times 100$$

$$= \frac{A_2}{A_2 + A_1} \times 100$$

$$= 5.59\%$$

6-2

$$y_{\max} = \frac{1}{3}t = 100wy = \frac{3\Delta p a^4(1-\mu^2)}{16Et^3}$$

$$a^4 = \frac{16t^4E}{9\Delta p(1-\mu^2)} = \frac{16(300wy)^4E}{9\Delta p(1-\mu^2)}$$

$$a = 600wy\left[\frac{E}{9\Delta p(1-\mu^2)}\right]$$

6-3

$$\Delta p = \gamma h \qquad h = \frac{\Delta p}{\gamma} = \frac{2116 \text{ lb/ft}^2}{62.4 \text{ lb/ft}^3} \times \frac{12 \text{ in.}}{\text{ft}}$$

$$h = 407 \text{ in. } H_2O$$

$$\text{factor} = \frac{2116 \text{ lb/ft}^2}{407 \text{ in. } H_2O} = 5.2$$

6-4

$$\lambda = \frac{\sqrt{2}}{8\pi r^2 n}; n = \frac{p}{kT}$$

for $p = 1$ atm

$n = 0.214 \times 10^{20}$ molecule/cm^3

$\lambda = 0.770 \times 10^{-5}$ cm

for $p = 1$ torr

$n = 0.28 \times 10^{17}$ molecule/cm^3

$\lambda = 0.586 \times 10^{-2}$ cm

for $p = 1\,\mu$

$n = 0.28 \times 10^{14}$ molecule/cm^3

$\lambda = 5.86$ cm

for $p = 1$ in. H_2O

$n = 0.524 \times 10^{17}$ molecule/cm^3

$\lambda = 0.313 \times 10^{-2}$ cm

for $p = 10^{-3}\ \mu$

$n = 0.28 \times 10^{11}$ molecule/cm^3

$\lambda = 5860$ cm

6-5

$$\lambda = \frac{\sqrt{2}}{8\pi r^2 n}$$

$$n = \frac{3p}{mv_{\text{rms}}^2} \rightarrow v_{\text{rms}} = \frac{3kT}{m}$$

$$\therefore n = \frac{p}{kT}$$

$$\dot{\lambda} = \frac{kT\sqrt{2}}{8\pi \frac{D^2}{4} P}$$

$$\lambda = 2.34 \times 10^{-17} \frac{T}{D^2 p}$$

T in °K
p in microns
D in cm

6-6

By Equation (6-4)

$$\left|\frac{p}{p_0}\right| = \frac{1}{\left\{\left[1 - \left(\frac{w}{w_n}\right)^2\right]^2 + 4h^2\left(\frac{w}{w_n}\right)^2\right\}^{1/2}}$$

$$w_n = \sqrt{\frac{3\pi r^2 C^2}{4LV}}; C = 1128 \text{ ft/sec}$$

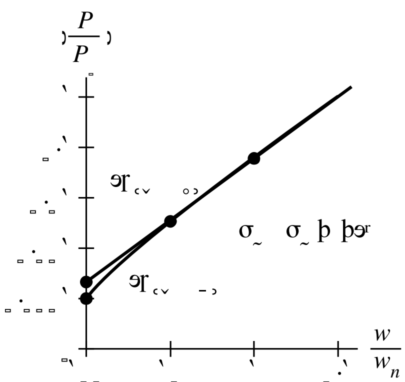
$$w_n = 207.5 \frac{1}{\text{sec}}$$

$$h = \frac{2\mu}{\rho t r^3}\sqrt{\frac{3LV}{\pi}}; \rho = 0.075 \text{ lbm/ft}^3; h = 13.8$$

$$\left|\frac{p}{p_0}\right| = \frac{1}{\left\{\left[1 - \frac{w^2}{4.31\times 10^4}\right]^2 + 7.64 \times 10^2\left(\frac{w^2}{4.31\times 10^4}\right)\right\}^{1/2}}$$

$$\left|\frac{p}{p_0}\right| = \frac{1}{\left[1 + 7.64 \times 10^2\left(\frac{w}{207.5}\right)^2\right]^{1/2}}$$

Eq (6-4)			Eq (6-8)		
w	$\frac{w}{w_n}$	$\left\lvert \frac{p}{p_0} \right\rvert$	w	$\frac{w}{w_n}$	$\left\lvert \frac{p}{p_0} \right\rvert$
0	0	1	0	0	1
103.75	0.5	0.0725	103.25	0.5	0.0723
155.625	0.75	0.0482	155.625	0.75	0.0482
207.5	1.0	0.0362	207.5	1.0	0.0362
1037.5	5.0	0.00714	1037.5	5.0	0.00725
2075.0	10.0	0.00341	2075.0	10.0	0.00362
20,750.0	100.0	0.0000965	20,750.0	100.0	0.000362
↓	↓	↓	↓	↓	↓
∞	∞	∞	∞	∞	∞



6-8

$$w = \left[\frac{\pi r^2 c^2}{V\left(L + \frac{1}{2}\sqrt{\pi^2 r^2}\right)} \right]^{1/2} = 239\ \frac{1}{\text{sec}}$$

6-9

$$\Delta p = p_1 - p_2 = \frac{g}{g_c} h^1 \left(\frac{A_2}{A_1} + 1 \right) (\rho_m - \rho_f)$$

$A_2 = 0.0314\ \text{in}^2;\ A_1 = 7.06\ \text{in}^2;\ \gamma_m = 184\ \text{lb/ft}^3$

$\gamma_f = 62.4\ \text{lb/ft}^3;\ \Delta p = h^1 (0.0705)\ \text{lb/in}^2$

factor $= 0.0705\ \text{lb/in}^3$

6-10

$E = 2 \times 10^{11}\ \text{N}/m^2$ $\quad \mu = 0.3$ $\quad r = 0.075$ $\quad m = a$

$\frac{1}{3} t = \frac{3\Delta p}{16 E t^3} a^4 (1 - \mu^2)$ $\quad \Delta p_{\max} = 1\ \text{atm} = 1.0132 \times 10^5\ \text{N}/m^2$

$$t^4 = \frac{(9)(1.0132 \times 10^5)(0.025)^4 (1 - 0.09)}{(16)(2 \times 10^{11})}$$

$$t = 1.692 \times 10^{-3}\ \text{m} = 1.692\ \text{mm}$$

Because sensitivity of *LVDT* is 2.5 nm, this is the minimum change in deflection which can be measured.
Max deflection at 1 atm $= \frac{1}{3}t = 0.564$ mm

$$\Delta p \text{ for } 2.5 \text{ nm} = \frac{2.5 \times 10^{-9}}{0.564 \times 10^{-6}} \times 1 \text{ atm}$$
$$= 4.431 \times 10^{-6} \text{ atm}$$
$$= 0.449 \text{ N/m}^2$$

6-12

$$R = R_1(1 + b\Delta p) \qquad W_R = \left[\left(\frac{\partial R}{\partial \Delta p} w_{\Delta p}\right)^2\right]^{1/2}$$

$$W_R = 1.7 \times 10^{-5} \text{ ohms}$$

6-14

$$p = \frac{ay^2}{V_B - ay}; \quad V_C = 0.9420 \text{ mm}^3$$
$$V_B = 10^5 \text{ mm}^3$$

$p = 2.82 \times 10^{-4}$ mm Hg Error = negligible if Eq. (6-22) is used.

6-15

at $p = 1.0\ \mu$

$$F = \frac{p(T - T_g)}{4T_g} \qquad 1\ \mu = 1.33 \text{ dyne/cm}^2$$

$$F = \frac{(1.333)(50)}{4(293)} = 5.68 \times 10^{-2} \text{ dynes}$$

at $p = 0.01\ \mu$, $F = 5.68 \times 10^{-4}$ dynes

6-16

Assume a steel diaphragm and a deflection equal to $\frac{1}{3}t$.

$$y_{\max} = \frac{t}{3} = \frac{3p}{16Et^3} a^4(1 - \mu^2) \qquad t^4 = 0.0687 \times 10^{-6}$$

$t = 0.01618$ and check to see if $f > 30{,}000$ cps

$$f = 1.934 \times 10^6 \frac{t}{\pi a^2}$$

$f = 159{,}000$ cps $> 30{,}000$ cps

$\therefore t = 0.01618$ in. $a = 0.500$

Assume spacing equal to 0.01 in. Then

$$C = 0.225\varepsilon \frac{A}{d}; \quad A = 0.196 \text{ in}^2$$

$$\frac{\partial C}{\partial y} = -0.225\varepsilon \frac{A}{d^2} \qquad \frac{\partial y}{\partial p} = \frac{3}{16Et^3} a^4(1 - \mu^2)$$

$$\frac{\partial C}{\partial y} = \frac{\partial c}{\partial y}\frac{\partial y}{\partial p} \qquad \frac{\partial c}{\partial p} = -0.0382 \text{ pf/psi}$$

6-18

$V = 0.6 \text{ in}^3 = 3.472 \times 10^{-4} \text{ ft}^3$

air at 14 psia and 70° F; $\rho = 0.0752 \text{ lbm/ft}^3$

$w = 50 \text{ Hz} \quad C = 4.057 \times 10^6 \text{ ft/hr}; \mu = 0.0440 \dfrac{\text{lbm}}{\text{hr} \cdot \text{ft}}$

a. $w_n = 50$ Hz

$$h = \frac{2\mu}{\rho C r^3}\sqrt{\frac{3LV}{\pi}} = 5.3 \times 10^{-9} \frac{L^{1/2}}{r^3} \rightarrow 4L^2 = 1.113 \times 10^{-6} \frac{L}{r^6}$$

$$w_n = \sqrt{\frac{3\pi r^2 c^2}{4LV}} = 50 \rightarrow \frac{r^2}{L} = 2.90 \times 10^{-7}$$

$$\left|\frac{p}{p_0}\right| = 0.5 = \frac{1}{\left\{1 - \left(\frac{w}{w_n}\right)^2 + 4L^2\left(\frac{w}{w_n}\right)^2\right\}^{1/2}} \rightarrow L = r^6[3.63 \times 10^{16}]$$

$$r^2 = r^6(3.63 \times 10^{16})(2.90 \times 10^{-7}) \rightarrow r^4 = 9.54 \times 10^{-12}$$

$$r = 3.12 \times 10^{-3} \text{ ft} \rightarrow d = 6.24 \times 10^{-3} \text{ ft}$$

$$L = (3.12 \times 10^{-3})^6(3.63 \times 10^{16}) = 3260 \times 10^{-2} \rightarrow L = 32.6 \text{ ft}$$

b. $w_n = 100$ Hz

$$8.62 \times 10^9 \frac{r^2}{L} = 100^2 \rightarrow \frac{r^2}{L} = 1.16 \times 10^{-6}$$

$$\frac{L}{r^6} = \frac{3.438}{2.785 \times 10^{-17}} = 1.23 \times 10^{17} \rightarrow r^4 = \frac{1}{(1.16 \times 10^{-6})(1.23 \times 10^{17})}$$

$$r = 1.625 \times 10^{-3} \text{ ft} \rightarrow d = 3.25 \times 10^{-3} \text{ ft}$$

$$L = 2.26 \text{ ft}$$

c. $w_n = 500$ Hz

$$\frac{r^2}{L} = 2.90 \times 10^{-5} \qquad L = r^6(2.71 \times 10^{18}) \rightarrow r^4 = 0.785 \times 10^{-12}$$

$$r = 0.941 \times 10^{-3} \rightarrow d = 1.882 \times 10^{-3} \text{ ft} \qquad L = 1.88 \text{ ft}$$

6-19

The reading would be the same. 10 in. Hg

6-20

$T = 90° \text{ C} = 363° \text{ K} \qquad c = (20.04)(363)^{1/2} = 381.8 \text{ m/s}$

$$w_n = \left[\frac{3\pi r^2 c^2}{4LV}\right]^{1/2} \qquad r = 0.75 \times 10^{-3} \text{ m}$$

$L = 1.5 \text{ m} \qquad V = 5 \times 10^{-6} \text{ m}^3$

$$w_n = \left[\frac{(3)\pi(0.75\times10^{-3})^2(381.8)^2}{4(1.5)(5\times10^{-6})}\right]^{1/2} = 160.5 \ \text{Hz}$$

$$\rho = \frac{p}{RT} = \frac{6.9\times10^5}{(287)(363)} = 6.623 \ \text{kg/m}^3$$

$$\mu = 2.13\times10^{-5}\ \text{kg/m}\cdot\text{s} \qquad h = \frac{2\mu}{pcr^3}\left[\frac{3LV}{\pi}\right]^{1/2}$$

$$h = \frac{(2)(2.13\times10^{-5})}{(6.623)(381.8)(0.75\times10^{-3})^3}\left[\frac{(3)(1.5)(5\times10^{-6})}{\pi}\right]^{1/2}$$

$$h = 0.1069$$

6-21

$$\Delta p = (1.85-1)(5.0) \rightarrow 4.25 \text{ in. } H_2O$$

$$\Delta p = 4.25 \text{ in. } H_2O$$

6-22

$$y_{\max} = \frac{1}{3}t = \frac{3\Delta p a^4}{16Et^3}(1-\mu^2) \qquad E = 29\times10^6 \text{ psi}$$

$$t^4 = 6.87\times10^{-8} = 0.0162 \text{ in.} \qquad \mu = 0.3 \qquad \Delta p = 10^3 \text{ psi}$$

$$a = 0.25 \text{ in.}$$

$$f = 1.934\times10^6\frac{t}{\pi a^2} \text{ (for steel)} \qquad f = 159{,}500 \text{ Hz}$$

6-23

$$R = R_1(1+b\Delta p) \qquad b = (1.7\times10^{-7})\text{ psi}^{-1} \qquad R_1 = 100 \text{ ohms}$$

$$R = 100.17 \text{ ohms} \qquad \Delta p = 10^4 \text{ psi}$$

At $E = 24$ volts and $R_4 = 100.17$ ohms $\qquad R_1 = R_2 = R_3 = 100$ ohms

$$E_g = E\left[\frac{R_1}{R_1+R_4} - \frac{R_2}{R_2+R_3}\right] = 24\left[\frac{(100)(200)-(100)(200.17)}{(200)^2}\right]$$

$$E_g = -1.02\times10^{-2} \text{ volts}$$

6-24

$$ay \ll V_B \qquad V_B = 1.5\times10^5 \text{ mm}^3, d = 0.3 \text{ mm}, p = 0.030 \text{ mmHg}$$

$$p = \frac{\pi d^2}{4}\frac{y^2}{V_B} \rightarrow y^2 = 5.94\times10^4 \text{ mm}^2$$

$$y = 2.44 \text{ cm}$$

6-25

For air: $\lambda = (8.64\times10^{-7})\dfrac{T}{P}$ = mean free path

P is in lbf/ft^2; T in °R

For Knudsen Gage:

$10^{-5}\mu < p < 10\mu$ or

$2.82 \times 10^{-8} \frac{\text{lbf}}{\text{ft}^2} < p < 2.82 \times 10^{-2} \frac{\text{lbf}}{\text{ft}^2}$

$\therefore 3.06 \times 10^{-5} T < \lambda < 30.6T$

6-26

$$\Delta p = \frac{g}{g_c} h(\rho_m - \rho_f) \qquad h = 135 \text{ mm} = 0.135 \text{ m}$$

$$\rho_m = (0.83)(1000) = 830 \text{ kg/m}^3$$

$$\rho_f = \rho_a = \frac{p}{RT} = \frac{13.8 \times 10^6}{(287)(293)} = 164.11 \text{ kg/m}^3$$

$$\Delta p = (9.806)(0.135)(830 - 164.11) = 881.51 \text{ N/m}^2$$

6-27

$$y_{\max} = \frac{3W(1 - \mu^2)}{4\pi E K^3}\left[a^2 - b^2 - \frac{4a^2b^2}{a^2 - b^2}\left(\ln\frac{a}{b}\right)^2\right]$$

$$y_{\max} = 4.064 \times 10^{-2}[0.9844 - 0.635(4.323)]$$

$$y_{\max} = 2.885 \times 10^{-2} \text{ in.}$$

6-28

$$\rho_a = \frac{400 \times 10^3}{(287)(283)} = 4.925 \text{ kg/m}^3$$

$$\rho_w = 999 \text{ kg/m}^3$$

$$\rho_0 = (0.8)(999) = 799 \text{ kg/m}^3$$

$$\begin{aligned}\Delta p &= \frac{g}{g_c} h(\rho_0 - \rho_a) \\ &= (0.12)(799 - 4.9) \\ &= 95.3 \text{ N/m}^2 \\ &= 0.0138 \text{ psia}\end{aligned}$$

6-29

$$\begin{aligned}\lambda &= 2.27 \times 10^{-5} \frac{T}{p} \\ &= \frac{(2.77 \times 10^{-5})(293)}{1.01 \times 10^5} \\ &= 6.56 \times 10^{-8} \text{ m}\end{aligned}$$

6-30

$a = 2.5 \text{ cm}, b = 0.3 \text{ cm}, t = 0.122 \text{ cm}$

Deflection $= 0.04$ cm

$E = 200 \text{ GN/m}^2 = 20 \text{ MN/cm}^2$

Inserting in deflection equation gives $p = 13.786 \text{ N/cm}^2 = 1.379 \times 10^5 \text{Pa}$

6-33

$29.8 \text{ in Hg} = p_b = 1.009 \times 10^5 \text{ Pa}$

$p(\text{gage}) = 10 \text{ kPa}$

$z = 7200 \text{ ft}$

$$p_b = p_0\left(1 - \frac{BZ}{T_0}\right)^{5.26}$$

$$= (29.8)\left[1 - \frac{(0.003566)(7200)}{518.69}\right]^{5.26}$$

$$= 22.82 \text{ in. Hg}$$

$$= 77263 \text{ Pa}$$

$p(\text{abs}) = 87.263 \text{ kPa}$

$$m = \frac{pV}{RT} = \frac{(87{,}263)(100 \times 10^{-3})}{(287)(293)} = 0.1038 \text{ kg}$$

If use weather bureau $p(\text{abs}) = 1.109 \times 10^5$ Pa

$$\text{Error} = \frac{110{,}900 - 87{,}263}{87{,}263} = 27.1\% \text{ too high}$$

6-34

$10 \text{ psia} = 68{,}940 \text{ Pa}$

Using weather bureau, $p(\text{abs}) > 110{,}900 - 68{,}940 = 41{,}960$ Pa

Using correct pressure, $p(\text{abs}) = 87{,}263 - 68{,}940 = 18{,}323$ Pa

$$\text{Error} = \frac{41{,}960}{18{,}323} - 1 = 129\% \text{ too high}$$

6-35

$1'' \text{ fluid} = (0.75)(5.203) = 3.902 \text{ lbf/ft}^2$

$1 \text{ psf} = 47.88 \text{ Pa}$

$$\Delta p = \frac{(3.902)(47.88)}{2.54} = 735 \text{ Pa}$$

6-36

$\rho_a =$ small compared to fluid

$$\Delta p = \left(\frac{1.75}{0.75}\right)(935) = 1716 \text{ Pa}$$

6-37

$\gamma = 0.85 \qquad h' = 15 \sin 30° = 7.5 \text{ cm}$

$\rho_a = \rho_f \sim \text{small} \qquad \frac{A_2}{A_1} = \left(\frac{0.005}{0.05}\right)^2$

$$\Delta p = (7.5)\left[\left(\frac{0.005}{0.05}\right)^2 + 1\right] = 7.575 \text{ cm fluid}$$

$$= (7.575)(0.85)(5.203) = 33.5 \text{ psf}$$

$$= 1604 \text{ Pa}$$

6-38

$$SG = 13.6$$

$$\begin{aligned}\Delta p &= (13.6 - 1)\left(\frac{13}{2.54}\right) = 64.49 \text{ in. } H_2O \\ &= 335.5 \text{ psf} \\ &= 16.06 \text{ kPa}\end{aligned}$$

6-40

$$\begin{aligned}\Delta p &= (13.6 - 1)\left(\frac{13}{2.54}\right)\left[\left(\frac{5}{10}\right)^2 + 1\right] \\ &= 80.61 \text{ in. } H_2O \\ &= 20.08 \text{ kPa}\end{aligned}$$

6-41

$$\Delta p = 10 \text{ kPa at } 20° \text{ C} \qquad \rho = 7800 \text{ kg/m}^3$$

$$a = 1.25 \text{ cm} \qquad E = 200 \times 10^9 \text{ N/m}^2 \qquad \mu = 0.3$$

$$\frac{1}{3}t = \frac{(3)(10^4)(0.0125)^4}{(16)(2 \times 10^{11})t}(1 - 0.3^2)$$

$$t = 1.58 \times 10^{-4} \text{ m} = 0.158 \text{ mm}$$

$$f = \frac{10.21}{(0.0125)^2} = \left[\frac{(1.0)(2 \times 10^{11})(1.58 \times 10^{-4})}{(12)(1 - 0.3^2)(7800)}\right]^{1/2} = 15{,}820 \text{ Hz}$$

6-42

$$p = 65 \text{ atm} \qquad T = 20° \text{ C}$$

$$SG = 0.85 \qquad h = 15.3 \text{ cm} \pm 1.0 \text{ mm}$$

$$\rho_m = (0.85)(996) = 847 \text{ kg/m}^3$$

$$\rho_a = \frac{(65)(1.01 \times 10^5)}{(287)(293)} = 78.3 \text{ kg/m}^3$$

$$\begin{aligned}\Delta p &= \frac{9.8}{1.0}(0.153)(847 - 78.3) \\ &= 11.53 \text{ Pa}\end{aligned}$$

6-43

$$p = 5 \text{ atm} \qquad T = 50° \text{ C} = 323 \text{ K} \qquad r = 0.0005 \text{ m}$$

$$L = 0.8 \text{ m} \qquad V = 3 \text{ cm}^3 = 3 \times 10^{-6} \text{ m}^3$$

$$\rho = \frac{p}{RT} = \frac{(5)(1.01 \times 10^5)}{(287)(323)} = 5.46 \text{ kg/m}^3$$

$$c = 20(323)^{1/2} = 359 \text{ m/s} \qquad \mu = 1.97 \times 10^{-5} \frac{\text{kg}}{\text{m-s}}$$

$$w_n = \left[\frac{\pi(0.0005)^2(359)^2}{(3 \times 10^{-6})(0.8 + 0.5(\pi)(0.005))}\right]^{1/2} = 205 \text{ Hz}$$

$$h = \frac{(2)(1.97 \times 10^{-5})}{(5.46)(359)(0.0005)^3}\left[\frac{(3)(0.8)(3 \times 10^{-6})}{\pi}\right]^{1/2} = 0.243$$

$$\frac{w}{w_n} = 0.5$$

$$\left|\frac{p}{p_0}\right| = \frac{1}{[(1 - 0.5^2)^2 + (4)(0.243)^2(0.5)^2]^{1/2}}$$
$$= 1.27$$

6-44

$$V_B = 125 \text{ cm}^3 = 1.25 \times 10^5 \text{ mm}^3$$

$$V_C = ay = \frac{\pi(0.2)^2}{4} y = 0.0314 \text{ mm}^3$$

$$p = 20\ \mu\text{m} = \frac{ay^2}{V_B} = 0.02 \text{ mm (torr)}$$

$$y = 282 \text{ mm}$$

6-45

$$T = 120°\text{F} = 580°\text{R}$$

$$p = -13 \text{ psig} = -89{,}627 \text{ Pa}$$

$$p_{atm}(\text{weather}) = 29.83 \text{ in. Hg} = 1.01 \times 10^5 \text{ Pa}$$

$$V = 3.0 \text{ m}^3 \qquad z = 600 \text{ ft}$$

$$p_{atm} = (29.83)\left[1 - \frac{(0.003566)(600)}{518.7}\right]^{5.26}$$
$$= 29.188 \text{ in. Hg}$$
$$= 98{,}840 \text{ Pa}$$

$$p = 98{,}840 - 89{,}627 = 9213 \text{ Pa}$$

$$m = \frac{pV}{RT} = \frac{(9213)(3)}{(287)(580)(519)} = 0.2989 \text{ kg}$$

If use weather p_{atm}, $p = 101{,}000 - 89{,}627 = 11{,}373$ Pa

$$\text{Error} = \frac{11{,}373}{9213} - 1 = 0.234 \text{ (23.4\% high)}$$

6-46

$$140 \text{ dB} = 0.029 \text{ psia} = 200 \text{ Pa}$$

$$\omega = 5000 \text{ Hz} \qquad \omega_n = 1000 \text{ Hz} \qquad \rho = 7800$$

$$a = 0.5 \text{ cm} = 0.005 \text{ m} \qquad E = 2 \times 10^{11} \qquad \mu = 0.3$$

$$10{,}000 = \frac{10.21}{(0.005)^2}\left[\frac{(1.0)(2 \times 10^{11})t^2}{12(1 - 0.3^2)(7800)}\right]^{1/2}$$

$$t = 1.6 \times 10^{-5} \text{ m} = 0.016 \text{ mm}$$

6-47

$$p = (1.013 \times 10^5)\left[1 - \frac{(0.003566)(14{,}000)}{518.7}\right]^{5.26}$$
$$= 59{,}500 \text{ Pa}$$

6-48

$h^1 = 25 \text{ cm}$

$$\Delta p = (13.6 - 1)\frac{(25)}{2.54}\left[\left(\frac{4}{50}\right)^2 + 1\right] = 124.9 \text{ in. } H_2O$$

$$= 649 \text{ psf} = 31 \text{ kPa}$$

6-49

$R = R_1(1 + b\Delta p) \qquad \Delta p = 700 - 1 = 699 \text{ atm}$

$R_1 = 90\ \Omega$

$b = 2.5 \times 10^{-11} \text{ Pa}^{-1}$

$$R = (90)[1 + (2.5 \times 10^{-11})(699)(1.013 \times 10^5)]$$
$$= 90.159\ \Omega$$

6-50

$r = 0.0006 \text{ m} \qquad L = 0.1 \text{ m}$

$V = 15 \times 10^{-6} \text{ m}^3$

$p = 500 \text{ kPa}$

$T = 50° \text{ C} = 323 \text{ K}$

$$\rho = \frac{p}{RT} = \frac{500 \times 10^3}{(287)(323)} = 5.394 \text{ kg/m}^3$$

$\mu = 1.97 \times 10^{-5} \text{ kg/m-s}$

$c = 359 \text{ m/s}$

$$\omega_n = \left[\frac{\pi(0.0006)^2(359)^2}{(1.5 \times 10^{-6})(0.1 + 0.5\pi(0.0006))}\right]^{1/2}$$
$$= 981 \text{ Hz}$$

$$h = \frac{(2)(1.97 \times 10^{-5})}{(5.394)(359)(0.0006)^3}\left[\frac{(3)(0.1)(1.5 \times 10^{-6})}{\pi}\right]^{1/2}$$
$$= 0.113$$

6-52

See conversion list at start of chapter.

6-53

$$p_{atm} = \left(\frac{750}{760}\right)(101.32 \text{ kPa})$$
$$= 99.99 \text{ kPa}$$

$$p_{abs} = 825.0 + 99.99 = 925 \text{ kPa}$$

6-54

$V = 5 \text{ L} = 5 \times 10^{-3} \text{ m}^3$

$T = 10°\text{C} = 283 \text{ K} = 509°\text{R}$

Indicated barometer $= 29.85$ mm Hg $= 101.079$ kPa

$$\text{Actual barometer} = (101.079)\left[1 - \frac{0.003566(600)}{518.69}\right]^{5.26}$$
$$= 98.9 \text{ kPa}$$

$p(\text{indicated}) = 101.079 - 80 = 21.079 \text{ kPa}$

$p(\text{actual}) = 98.9 - 80 = 18.9 \text{ kPa}$

$$m(\text{actual}) = \frac{(18{,}900)(0.005)}{(2078)(283)} = 1.61 \times 10^{-4} \text{ kg}$$

$$\text{Error} = \frac{21.079 - 18.9}{18.9} = +11.5\%$$

6-55

$h = 45.2 \text{ cm} \pm 0.1 = 1.483 \text{ ft}$

$\rho(H_2O) = 997 \text{ kg/m}^3 = 62.3 \text{ lhm/ft}^3$

SG of Hg $= 13.595$

$$\Delta p = \frac{g}{g_c} h(\rho_m - \rho_f)$$
$$= \frac{32.2}{32.2}(1.483)(13.6 - 1)(62.3)$$
$$= 1164 \text{ psf}$$
$$= 8.08 \text{ psi}$$

6-56

Reading that is $\frac{1}{12.6} = 7.94\%$ too high

6-57

$\Delta p = 2 \text{ in. } H_2O = 10.41 \text{ psf}$

$$\rho_a = \frac{p}{RT} = \frac{(50)(144)}{(53.35)(530)} = 0.2546 \text{ lbm/ft}^3$$

$\rho_f = (0.82)(62.4) = 51.17 \text{ lbm/ft}^3$

$$\Delta p = \frac{g}{g_c} h(\rho_m - \rho_f)$$

$$h = \frac{10.41}{51.17 - 0.2546} = 0.204 \text{ ft}$$

$$\text{Length displacement} = \frac{0.204}{\sin 20^\circ} = 0.598 \text{ ft}$$

6-60

$(0.01 \times 10^{-2})(1 \times 10^6 \text{ Pa}) = 1000 \text{ Pa}$

At $\Delta p = 40{,}000$ Pa

$$\% \text{ uncertainty} = \frac{1000}{40{,}000} = 2.5\%$$

6-61

for $D \gg d$ the sensitivity for the inclined micromanometer becomes

$S = h/\Delta p = 1/(\rho_2 - \rho_1)\sin\theta\,(g/g_c)$

For water density of 1000kg/m^3, the sensitivity becomes

$S = 1/(1000)(1 - 0.85)(9.8/1.0)(0.5) = 1.36$ mm/Pa

6-62

1 mm $= 0.735$ Pa, density for air at 300 K, $p = 1$ atm is 1.17 kg/m^3

$0.735 = (1.17)u^2/2(1.0)$

and $u = 1.04$ m/s

6-63

For and oil with specific gravity of 0.9, sensitivity becomes

$S = 2.04$ mm/pa

and

$0.49 = (1.17)u^2/2(1.0)$

$u = 0.915$ m/s

6-64

$\Delta p = (9.8/1.0)(1000)(2.95 - 1)(0.15)$

$= 2867$ Pa $= 0.4158$ psi

6-65

$\rho_{air} = (0.5)(1.013 \times 10^5)/(287)(308) = 0.573$

$\Delta p = 2867 = h(9.8/1.0)(2950 - 0.573)$

$h = 0.0992$ m $= 9.92$ cm

6-66

$E = 200$ GN/m^2, $\mu = 0.3$

$t/3 = (3)(200000)(0.02)^4(1 - 0.09)\ /\ (16)(2 \times 10^{11})t^3$

$t = 0.535$ mm

$f = (10.21/0.0004)[(2 \times 10^{11})(0.000535)^2/(12)(1 - 0.09)(7800)]^{0.5}$

$= 20.92$ kHz

6-67

$\rho_{air} = (75)(1.013 \times 10^5)/(287)(308) = 85.95$

$\Delta p = (0.2)(9.8/1.0)(1000 - 85.95)$

$= 1792$ Pa

6-68

For air $\rho = 2.41$, $c = 343$, $\mu = 1.91 \times 10^{-5}$

$L = 0.15$ m, $r = 0.5$ mm, $V = 1.2$ mL $= 1.2 \times 10^{-6}$m^3

Inserting values in Eqs (6-5) and (6-6) gives

$\omega_n = 620$ Hz

$h = 0.306$

6-69

The standard atm pressure at the altitude given is

$$\rho = (1.0132 \times 10^5)[1 - (1520)(0.0065)/288.16]^{5.26}$$
$$= 84,332 \text{ Pa}$$

Vacuum readings greater than this value are impossible.

Chapter 7

7-1

$$\dot{m} = \frac{2g_c}{RT_1} A_2^2 \frac{\gamma}{\gamma - 1} p_1^2 \left[\left(\frac{p_2}{p_1} \right)^{3/\gamma} - \left(\frac{p_2}{p_1} \right)^{\gamma+1/\gamma} \right]$$

Consider:

$$p_1^2 \left[\left(\frac{p_2}{p_1} \right)^{2/\gamma} - \left(\frac{p_2}{p_1} \right)^{\gamma+1/\gamma} \right]; p_1 = p_2 + \Delta p; \frac{p_2}{p_1} = \frac{p_2}{p_2 + \Delta p}$$

$$p_1^2 \left(\frac{p_2}{p_1} \right)^{2/\gamma} = \left(\frac{p_2}{p_2 + \Delta p} \right)^{2/\gamma} (p_2 + \Delta p)^2$$

$$p_1^2 \left(\frac{p_2}{p_1} \right)^{2/\gamma} = p_2^2 \left(1 + \frac{\Delta p}{p} \right)^{2\left(\frac{\gamma-1}{\gamma}\right)}$$

By the binomial theorem:

$$p_1^2 \left(\frac{p_2}{p_1} \right)^{2/\gamma} = p_2^2 \left[1 + \frac{2(\gamma - 1)}{\gamma} \frac{\Delta p}{p_2} + \cdots + \right] \quad (1)$$

also:

$$\left[1 - \left(\frac{p_2}{p_1} \right)^{\gamma-1/\gamma} \right] = \left[1 - \left(1 + \frac{\Delta p}{p_2} \right)^{1-\gamma/\gamma} \right]$$

By the binomial theorem:

$$\left[1 - \left(1 + \frac{\Delta p}{p_2} \right)^{1-\gamma/\gamma} \right] = \frac{\gamma - 1}{\gamma} \left[\frac{\Delta p}{p_2} + \frac{1}{2} \left(\frac{1 - 2\gamma}{\gamma} \right) \left(\frac{\Delta p}{p_2} \right)^2 + \cdots \right] \quad (2)$$

Multiply (1) by (2)

$$\therefore \dot{m} = \sqrt{\frac{2g_c}{RT}} A_2 \left[p_2 \Delta p - \left(\frac{1.5}{\gamma} - 1 \right) (\Delta p)^2 + \cdots \right]^{1/2}$$

if $\Delta p < \frac{p_1}{10}$ in Eq. (7-9)

$$\dot{m} = A_2 \sqrt{\frac{2g_c p_2}{RT_1} (p_1 - p_2)} \text{ in Eq. (7-9)}$$

$$\text{Error} = \sqrt{\frac{2g_c}{RT_1}} A_2 p_1 \left\{ \left(\frac{\gamma}{\gamma - 1} \right)^{1/2} \left[\left(\frac{3}{4} \right)^{2/\gamma} - \left(\frac{3}{4} \right)^{\gamma+1/\gamma} \right]^{1/2} - \frac{1}{4} \left[3 - \left(\frac{1.5}{\gamma} - 1 \right) + \cdots \right]^{1/2} \right\}$$

In Equation (7-10)

$$\text{Error} = A_2 p_1 \sqrt{\frac{2g_c}{RT_1}} \left\{ \left(\frac{\gamma}{\gamma - 1} \right)^{1/2} \left[\left(\frac{9}{10} \right)^{2/\gamma} - \left(\frac{9}{10} \right)^{\gamma+1/\gamma} \right]^{1/2} - \frac{3}{10} \right\}$$

7-3

$\mathrm{Re}_d = 10^5 \rightarrow d = 3.84$ in.

Select a venturi from Figure 7-10.

$C = 0.978$ for $10^5 < \mathrm{Re}_d < 8 \times 10^5$

$\rho_1 = \dfrac{p}{RT} = 1.502\ \mathrm{lbm/ft^3};\ A_2 = 0.02185\ \mathrm{ft^2}$

$M = \dfrac{1}{\sqrt{1 - \frac{1}{16}}} = 1.035$

$\Delta p = (p_1 - p_2) = \dfrac{1}{2 g_c \rho_1}\left[\dfrac{\dot{m}_{\mathrm{act}}}{YCMA_2}\right]^2$

$\Delta p = 21.0\dfrac{\dot{m}^2}{Y^2}$ Using Figure 7-14 and the above equation assuming $Y = 0.9 \rightarrow$

$\dot{m} = 0.3\ \mathrm{lbm/sec} \rightarrow \Delta p = 2.33\ \mathrm{lbf/ft^2}$

$\dot{m} = 0.5\ \mathrm{lbm/sec} \rightarrow \Delta p = 6.47\ \mathrm{lbf/ft^2}$

$\dot{m} = 0.7\ \mathrm{lbm/sec} \rightarrow \Delta p = 12.7\ \mathrm{lbf/ft^2}$

$\dot{m} = 1.0\ \mathrm{lbm/sec} \rightarrow \Delta p = 25.9\ \mathrm{lbf/ft^2}$

7-4

Follow the same procedures as in the above problem.

$\Delta p = 3.53\left[\dfrac{\dot{m}}{Y}\right]^2$ Assume $Y = 0.9$

$\dot{m} = 0.3\ \mathrm{lbm/sec} \rightarrow \Delta p = 0.392\ \mathrm{lbf/ft^2}$

$\dot{m} = 0.5\ \mathrm{lbm/sec} \rightarrow \Delta p = 1.09\ \mathrm{lbf/ft^2}$

$\dot{m} = 0.7\ \mathrm{lbm/sec} \rightarrow \Delta p = 2.14\ \mathrm{lbf/ft^2}$

$\dot{m} = 1.0\ \mathrm{lbm/sec} \rightarrow \Delta p = 4.36\ \mathrm{lbf/ft^2}$

7-5

Follow the above procedure.

$\Delta p = 21.0\dfrac{\dot{m}^2}{Y^2}$ Assume $Y = 0.9$

$\dot{m} = 0.3\ \mathrm{lbm/sec} \rightarrow \Delta p = 2.33\ \mathrm{lbf/ft^2}$

$\dot{m} = 0.5\ \mathrm{lbm/sec} \rightarrow \Delta p = 6.47\ \mathrm{lbf/ft^2}$

$\dot{m} = 0.7\ \mathrm{lbm/sec} \rightarrow \Delta p = 12.7\ \mathrm{lbf/ft^2}$

$\dot{m} = 1.0\ \mathrm{lbm/sec} \rightarrow \Delta p = 25.9\ \mathrm{lbf/ft^2}$

7-6

$$\dot{m} = CA_2 p_1 \sqrt{\frac{2g_c}{RT_1}} \left[\frac{\gamma}{\gamma+1} \left(\frac{2}{\gamma+1} \right)^{2/\gamma-1} \right]^{1/2}$$

$$\frac{W_{\dot{m}}}{\dot{m}} = \left[\left(\frac{W_{p_1}}{p_1} \right)^2 + \left(\frac{W_{T_1}}{2T_1} \right)^2 \right]^{1/2} = 0.01$$

$$\left(\frac{W_{p_1}}{p_1} \right)^2 + \frac{1}{4} \left(\frac{W_{T_1}}{T_1} \right)^2 = 0.0001$$

Pressure is more likely to be the controlling factor.

7-7

$$u_1 = \frac{\dot{m}RT_{1s}}{P_{1s}A_1} = 126.8 \text{ ft/sec}$$

$$p_{10} = p_3 + \frac{1}{2}\rho u_1^2 = 381 \text{ lb/in}^2$$

$$\dot{m} = CA_2 p_{10} \sqrt{\frac{2g_c}{RT_{1s}}} \left[\frac{\gamma}{\gamma+1} \left(\frac{2}{\gamma+1} \right)^{2/\gamma-1} \right]^{1/2}$$

$$1 = A_2(381)(144) \sqrt{\frac{(2)(32.2)}{(53.35)(500)}} \left[\frac{1.4}{2.4} \left(\frac{2}{2.4} \right)^{2/0.4} \right]^{1/2}$$

$$A_2 = 0.1222 \text{ in}^2 \quad \therefore d = 0.496 \text{ in.}$$

7-8

$$Q = \frac{A}{C_d} \sqrt{\frac{2g_c V_b}{A_b} \left(\frac{\rho_b}{\rho_f} - 1 \right)}$$

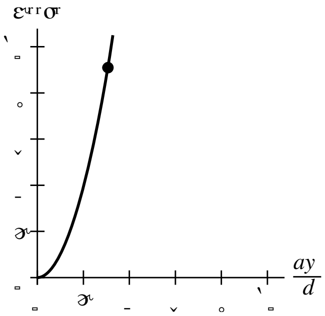
Where $A = \frac{\pi}{4}[(D + ay)^2 - d^2]$ if $d \approx D$ so that

$$1 - \left(\frac{d}{D} \right)^2 \ll \frac{ay}{D}$$

$$Q = \left[\frac{2ay}{D} + \left(\frac{ay}{D} \right)^2 \right] \frac{\pi D}{4C_d} \sqrt{\frac{2g_c V_b}{A_b} \left(\frac{\rho_0}{\rho_f} - 1 \right)}$$

if $\frac{ay}{D}$ is small $\left(\frac{ay}{D} \right)^2$ may be neglected, and $\dot{m} = c_1 y \sqrt{(\rho_b - \rho_f)\rho_f}$

$$\text{Error} = c \left(\frac{ay}{D} \right)^2 \sqrt{(\rho_b - \rho_f)\rho_f}$$



7-9

$\dot{m} = \dfrac{c_1 y \rho_b}{2}$; where $\rho_b = 2\rho_f$

for $\rho_b = 2.1\rho_f$

$$\text{Error} = \frac{c_1 y\sqrt{(\rho_0 - \rho_f)\rho_f} - \frac{c_1 y \rho b}{2}}{\frac{c_1 y \rho_b}{2}} \times 100$$

$$\text{Error} = \frac{1.048 \times 1.05}{1.05} \times 100 = -0.191\%$$

for $\rho_b = 1.90\rho_f$

$\text{Error} = 0.1895\%$

7-10

From Figure 7-19

for $K = 2.5$ GPM $\pm$ 0.25%

$$55 < \frac{f}{v} < 650 \frac{\text{cycles/sec}}{\text{centistoke}}$$

$1{,}085 < K < 1{,}095$ cycles/gal

7-11

$$A = \frac{QC_d}{\sqrt{\dfrac{2g_c V_b}{A_b}\left(\dfrac{\rho_b}{\rho_f} - 1\right)}} \text{ for } C_d = 0.4 \rightarrow$$

$$A = 0.492 \text{ in}^2 = \frac{\pi}{4}[(D + ay)^2 \cdot d^2]$$

$1 + 13a = 1.272$ $\qquad a = 0.0209$

$$c_1 = \frac{a\pi D}{2c_d}\sqrt{\frac{2g_c V_b}{A_b}}$$

$c_1 = 0.01787 \text{ ft}^2/S\text{ec}$

For $C_d = 0.8$

$A_2 = 0.987 \text{ in}^2$ $\qquad a = 0.0386$ $\qquad c_1 = 0.01648 \text{ ft}^2/\text{sec}$

For $C_d = 1.20$

$A_3 = 1.476 \text{ in}^2$ $\qquad a = 0.0454$ $\qquad c_1 = 0.01292 \text{ ft}^2/\text{sec}$

Error $\approx$ 0.00443 ft for $C_d = 0.4$

7-14

$$S = \frac{\frac{\Delta y}{y_1}}{E}; \text{ but } \varepsilon = \frac{\Delta y}{f_2} \quad S = \frac{f_2}{y_1}$$

$$C = \frac{\Delta I}{I} = \frac{\Delta y}{y} = \frac{\varepsilon f_2}{y_1} = \varepsilon S$$

$$\text{but } \varepsilon = \frac{L}{n_1}\left(\frac{dn}{dy}\right)_{y=y_1} = \frac{L\beta}{p_s}\left(\frac{d\rho}{dy}\right)_{y=y_1}$$

$$\therefore C = S\frac{L\beta}{\rho_s}\left(\frac{d\rho}{dy}\right)_{y=y_1}$$

7-15

$$S = \frac{N}{\rho - \rho_0} \text{ but } N = \frac{\Delta L}{\lambda} = \frac{\beta L}{\lambda}\frac{\rho - \rho_0}{p_s} \quad \therefore S = \frac{\beta L}{\lambda \rho_s}$$

7-16

$$N = \frac{\beta L}{\lambda}\left(\frac{T_\infty}{T_w} - 1\right); \frac{T_\infty}{T_w} = 1 - \frac{N\lambda}{\beta L} \qquad T_\infty = T_w\left[1 - \frac{N\lambda}{\beta L}\right]$$

$T_1 = 577.5°\text{R}; T_2 = 575°\text{R}; T_3 = 573°\text{R}; T_4 = 570°\text{R}$

7-18

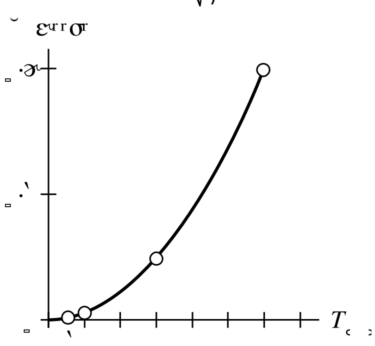
$$p_0 - p_\infty = \frac{1}{2}\rho u_\infty^2 \qquad \text{Water at } 70° \text{ C}$$

$$\rho = 1000 \text{ kg/m}^3$$

$$p_0 - p_\infty = (0.5)(1000)(3)^2 = 4500 \text{ N/m}^2$$

7-20

$$\% \text{ Error} = \frac{\sqrt{\frac{1}{\rho_{\text{ref}}}} - \sqrt{\frac{1}{\rho}}}{\sqrt{\frac{1}{\rho}}}$$

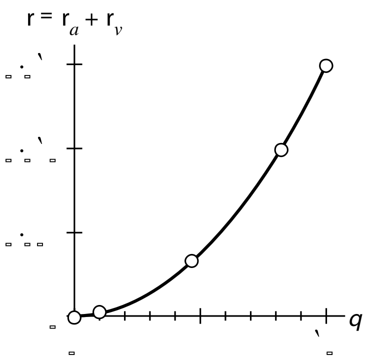


T(°F)	ρ	% Error
70	62.27	0
100	61.99	0.001
300	57.31	0.04
600	42.37	0.212

7-21

$$\rho = \frac{\rho_v}{\rho_g}; \qquad \text{assume } \Delta p = \frac{p_1}{10} \quad \therefore \dot{m} = A_2\sqrt{\frac{2g_c p_2}{RT_1}(p_1 - p_2)}$$

$$\%\ \text{Error} = 1 - \left(\frac{\rho}{\rho_1}\right)^{1/2} \qquad \rho_1 = \frac{p}{RT} = 0.0722\ \text{lbm/ft}^3$$



ϕ	ρ_v	ρ	% Error
0	0	0.0722	0
0.1	0.000214	0.0724	0.001
0.4	0.000855	0.07306	0.005
0.7	0.00150	0.0737	0.01
0.9	0.00193	0.07413	0.013
1.0	0.00214	0.07434	0.015

7-22

From Figure 7-10 at $d_1 = \frac{1''}{2}$ and $d_2 = \frac{1''}{4} \rightarrow R_e = 10^5$

$C = 0.97$ assume H_2O at 70° F

$$u_m = \frac{R_{\text{ed}}\mu}{\rho d_2} = \frac{(10^5)(2.36\ \text{lbm/hr})}{(62.4\ \text{lbm/ft}^3)\left(\frac{1}{48}\ \text{ft}\right)} = 1.815 \times 10^5\ \text{ft/hr}$$

$$Q_{\text{Ideal}} = A_2 u_m = \frac{\pi d_L^2}{4} u_m,\ Q_{\text{actual}} = CQ_{\text{ideal}}$$

$$Q_{\text{Act}} = 60\ \text{ft}^3/\text{hr} = 0.0167\ \text{ft}^3/\text{sec} \rightarrow Q_{\text{Act}})_{\min} = 0.0167\ \text{ft}^3/\text{sec}$$

$$\Delta p = \frac{Q_{\text{Act}}\sqrt{\frac{\rho}{2g_c}}}{CMA_2} \text{ where } M = \frac{1}{\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]^{1/2}} = 1.032$$

$$\Delta p = 2.32 \times 10^3\ \text{lbf/ft}^2 \rightarrow \Delta p = 32.8\ \text{in. Hg}$$

7-23

$$\dot{m} = A_2\sqrt{\frac{2g_c}{RT_1}}\left[P_2\Delta P - \left(\frac{1.5}{8} - 1\right)\Delta p_2 + \cdots\right]^{1/2}$$

$\Delta p = 20 \text{ psia} < \frac{p_1}{4} = 25 \text{ psia}$ $\quad \gamma = 1.4 \quad T_1 = 580° \text{ R}$

$P_2 = 80 \text{ psia} \quad A_2 = \left(\frac{\pi d_2^2}{4}\right) = 3.4 \times 10^{-4} \text{ ft}^2$

$\dot{m} = 0.0884 \text{ lbm/sec}$

7-24

$D = 1.25 \text{ in.}; d = 0.50 \text{ in.}; u_1 = 10 \text{ ft/sec}$

$f = 62.41 \text{ lbm/ft}^3; Q_{act} = A_1 u_1 = \left(\frac{\pi}{4}D^2\right)u_1 = 8.54 \times 10^{-2} \text{ ft}^3\text{/sec}$

$\text{Re}_D = \frac{u_1 D}{\gamma} = \frac{(10)(1.25)}{(1.08 \times 10^{-5})(12)} = 96,300$ (assuming water at 68°F)

From Figure 7-13 $\rightarrow CM = 0.61$

$$Q_{\text{Actual}} = CMA_2\sqrt{\frac{2g_c}{f}}\sqrt{p_1 - p_2} = 8.54 \times 10^{-2} \text{ ft}^3\text{/sec}$$

$$\Delta p = \left[\frac{8.54 \times 10^{-2}}{8.38 \times 10^{-4}}\right]^2 \rightarrow \Delta p = 10,404 \text{ lbf/ft}^2 = 147 \text{ in. Hg}$$

at $\Delta p = 20,808 \text{ lbf/ft}^2$

$$Q_{\text{Actual}} = (0.61)\frac{\pi}{4}\left(\frac{0.5}{12}\right)^2\left(\frac{64.4}{62.4}\right)^{1/2}(20,808)^{1/2}$$

$Q_{\text{Actual}} = 1.21 \times 10^{-1} \text{ ft}^3\text{/sec}$

7-25

Air $\quad \gamma = 1.4$

$$\dot{m} = A_2 p_1\left[\frac{2g_c}{RT_1}\right]^{1/2}\left[\left(\frac{\gamma}{\gamma+1}\right)\left(\frac{2}{\gamma+1}\right)^{2/\gamma-1}\right]^{1/2}$$

$$= \pi(0.4 \times 10^{-3})^2(1 \times 10^6)\left[\frac{(2)(1.0)}{(287)(293)}\right]^{1/2}\left[\left(\frac{1.4}{2.4}\right)\left(\frac{2}{2.4}\right)^{2/0.4}\right]^{1/2}$$

$\dot{m} = 1.187 \times 10^{-3} \text{kg/sec}$

7-26

$p_\infty = 20 \text{ kN/m}^2 \quad \gamma = 1.4 \quad p_{0_2} = 116 \text{ kN/m}^2$

$T_\infty = -40° \text{ C} = 233° \text{ K} \quad \frac{p_\infty}{p_{0_2}} = 0.1724$

$$\frac{p_\infty}{p_{0_2}} = \frac{\left[\frac{2.8}{2.4}M_1^2 - \frac{0.4}{2.4}\right]^{2.5}}{\left(1.2M_1^2\right)^{3.5}}$$

$$0.3263M_1^7 = \left[1.1666M_1^2 - 0.16667\right]^{2.5}$$

$$0.6389M_1^{2.8} - 1.1666M_1^2 + 0.16667 = 0$$

By iteration $M_1 = 2.03$

$$u_1 = a_1M_1 = (20.04)(233)^{1/2}(2.03)$$
$$= 620.97 \text{ m/sec}$$

7.27

$$p_0 - p_\infty = \frac{1}{2}\rho u_\infty^2$$
$$= \frac{\left(\frac{1}{2}\right)(1.0132 \times 10^5)}{(287)(293)}(25)^2$$
$$= 376.5 \text{ N/m}^2$$

$$W_{\Delta p} = 5 \text{ N/m}^2$$

$$U_\infty = \left[\frac{2\Delta p}{\rho}\right]^{1/2}$$

$$\frac{\partial U_\infty}{\partial \Delta p} = \frac{1}{2}\left[\frac{2\Delta p}{\rho}\right]^{-1/2}\left(\frac{2}{\rho}\right) - \frac{1}{\rho}\left[\frac{2\Delta p}{\rho}\right]^{-1/2}$$

$$\rho = \frac{1.0132 \times 10^{-5}}{(287)(293)} = 1.2049 \text{ kg/m}^3$$

$$\frac{\partial U_\infty}{\partial \Delta p} = \frac{1}{1.2049}\left[\frac{(2)(376.75)}{1.2049}\right]^{-1/2} = 0.0332$$

$$W_{U_\infty} = (0.0332)(5) = 0.166 \text{ m/sec}$$

7-28

$D = 2.4$ cm $\quad d = 1.2$ cm

$\text{Re}_d = 10^5 \quad C = 0.97 \quad \rho = 1329 \text{ kg/m}^3$

$v = 0.02 \times 10^{-5} \text{ m}^2/\text{s}$

$$M = \frac{1}{\left[1 - \left(\frac{1}{2}\right)^2\right]^{1/2}} = 1.155$$

$$10^5 = \frac{du^2}{v} \qquad U_2 = 1.667 \text{ m/s}$$

$$U_2 = CM\left(\frac{2g_c}{\rho}\right)\sqrt{\Delta g} \qquad \Delta p = 1.47 \text{ kPa}$$

7-29

$$K \approx 0.63 = CM$$

$$\Delta p = 1.47\left(\frac{0.97}{0.63}\right)^2 = 3.48 \text{ kPa}$$

7-31

$\dot{m} = 1$ kg/s $\quad p = 30$ atm

$T = 20°$ C $= 293$ K

$$1 = A_2(30)(1.013 \times 10^5)\left[\frac{(2)(1)}{(287)(293)}\right]^{1/2} \times \left[\frac{1.4}{2.4}\left(\frac{2}{2.4}\right)^{2/0.4}\right]^{1/2}$$

$$A_2 = 1.393 \times 10^{-4} \text{ m}^2 = \frac{\pi d^2}{4}$$

$d = 0.0133$ m $= 1.33$ m

7-32

$\rho_b = 2\rho_f = (2)(1329) = 2658$ kg/m^3

7-34

$\rho = 983$ kg / m^3

$$\dot{m} = (983)\pi\frac{(0.075)^2(8)}{4} = 34.74 \text{ kg/s}$$

$= 4591$ lbm/min

$$\dot{V} = AV = \frac{\pi(0.075)^2}{4}(8) = 0.03534 \text{ m}^3/\text{s}$$

$= 35.34$ l/s

$= 560$ gal/min

7-35

$\rho = 612$ kg/m^3

$$\dot{m} = \left(\frac{612}{983}\right)(34.74) = 21.63 \text{ kg/s}$$

$= 2858$ lbm/min

Volume flows are the same.

7-36

$T = 40°$ C $= 313$ K $= 563°$ R $\quad \mu = 1.86 \times 10^{-5}$ kg/m·s

$$\rho = \frac{p}{RT} = \frac{4 \times 10^5}{(287)(313)} = 4.453 \text{ kg/m}^3$$

$$50,000 = \frac{(4.453)V(0.05)}{1.86 \times 10^{-5}}, V = 4.18 \text{ m/s}$$

$$\dot{m} = (4.453)(4.18)\pi(0.025)^2 = 0.0365 \text{ kg/s}$$

$= 4.83$ lbm/mm

$$\text{Standard volume } = \frac{RT}{p} = \frac{(53.35)(528)}{(14.7)(144)}$$

$= 13.31$ ft^3/lbm

$$\dot{V} = (4.83)(13.31) = 64.3 \text{ SCFM}$$

$$\dot{V} = AV = \frac{\pi(5)^2}{4}(418) = 8207 \text{ cm}^3/\text{s}$$

$$\text{Standard volume } = \frac{RT}{p} = \frac{(287)(293)}{1.01 \times 10^5} = 0.83 \text{ m}^3/\text{kg}$$

$$= 8.3 \times 10^5 \text{ cm}^3/\text{kg}$$

$$\dot{V} = (8.3) \times 10^5)(0.0365) = 30{,}293 \text{ SCCS}$$

$$= 1.81 \times 10^6 \text{ SCCM}$$

7-37

Re (throat) about 200,000

$$d(\text{throat}) = \frac{1}{2}(5) = 2.5 \text{ cm}$$

About a 2″ × 1″ venturi

From Figure $C \approx 0.972$

7-38

$$0.5 = A_2(1 \times 10^6)\left[\frac{2}{(287)(373)}\right]^{1/2}\left[\frac{1.4}{2.4}\left(\frac{2}{2.4}\right)^{2/0.4}\right]^{1/2}$$

$$A_2 = 2.29 \times 10^{-4} \text{ m}^2 = \frac{\pi d_2^2}{4}$$

$$d_2 = 0.0174 \text{ m}$$

7-39

$$\frac{p_{0\text{ ind}} - p_0}{\frac{1}{2}\rho u_\infty^2} = -0.01$$

$$\rho = \frac{p}{RT} = 1.205 \text{ kg/m}^3$$

$$\frac{1}{2}\rho U_\infty^2 = \frac{(1.205)(20)^2}{2} = 241 \text{ Pa}$$

$$p_{0\text{ ind}} = 1 \text{ atm} + (0.99)(241)$$

$$= 101{,}558 \text{ Pa}$$

7-40

$$\Delta p_{\text{stat}} = 2\%\ p_{\text{dyn}}$$

$$\Delta p_{\text{dyn}} = 1\%\ p_{\text{dyn}}$$

$$\Delta p_{\text{stag}} = -1.8\%\ p_{\text{dyn}}$$

$$p_{\text{dyn}} = 241 \text{ Pa}$$

$$p_{\text{dyn ind}} = (241)(1.01) = 243 \text{ Pa}$$

$$p_{\text{stat ind}} = 1.0132 \times 10^5 + (0.02)(241)$$

$$\approx 1.013 \times 10^5$$

$$p_{\text{stat ind}} = 1.0132 \times 10^5 + (0.98)(241)$$

$$= 1.01556 \times 10^5 \text{ Pa}$$

7-41

$\rho = 998 \text{ kg/m}^3 \qquad \mu = 9.8 \times 10^{-4} \text{ kg/m} \cdot \text{s}$

$$\text{Re} = \frac{(998)(3)(5)(0.0254)}{9.8 \times 10^{-4}} = 3.88 \times 10^5$$

$\beta = 0.5$; Flow coeff $= 0.62$

$$Q = A_1 V_1 = A_1(3) = (0.02)\left(\frac{1}{2}A_1\right)\left[\frac{2}{998}\right]^{1/2} \Delta p^{1/2}$$

$\Delta p = 46,.730 \text{ Pa} = 6.78 \text{ psia}$

7-42

$a = (\gamma g_c RT)^{1/2} = [(1.4)(287)(283)]l^{1/2} = 337 \text{ m/s}$

$$M = \frac{700}{337} = 2.076$$

$$\frac{p_\infty}{p_{0_2}} = \frac{\left\{\left[\frac{(2)(1.4)}{2.4}\right](2.076)^2 - \frac{0.4}{2.4}\right\}^{1/0.4}}{\left[\frac{(2.4)(2.076)}{2}\right]^{1.4/0.4}} = 0.1656$$

$$p_{0_2} = \frac{40}{0.1656} = 241 \text{ kPa}$$

7-43

$$0.88 = \frac{f_s(0.003)}{4}$$

$f_s = 1173 \text{ Hz}$

7-44

$C = 0.97 \qquad \text{Re} > 10^5 \qquad d = 0.5 \text{ in.}$

$$10^5 = \frac{\rho U_m (0.1512)}{2.37}$$

$$\dot{m} = \frac{(10^5)(2.37)\pi\left(\frac{0.5}{12}\right)^2}{\left(\frac{0.5}{12}\right)(4)} = 7756 \text{ lbm/hr}$$

$$M = \frac{1}{\left[1 - \left(\frac{1}{4}\right)^2\right]^{1/2}} = 1.0328$$

For $\text{Re} = 10^6$ flow $\qquad \dot{m} = 77{,}560 \text{ lbm/hr} = 21.5 \text{ lbm/s}$

$$Q = \frac{21.5}{62.5} = 0.345 \text{ ft}^3/\text{sec}$$

$$Q = CMA_2\sqrt{\frac{2g_c}{\rho}}\sqrt{\Delta p}$$

$$0.345 = (0.97)(1.033)\frac{\pi\left(\frac{0.5}{12}\right)^2}{4}\left[\frac{(2)(32.17)}{0.24}\right]^{1/2}\sqrt{\Delta p}$$

$\Delta p = 249 \text{ lbf/ft}^2 = 1.73 \text{ psi}$

7-45

Figure 7-40

$Q = 30°$ $\quad p_{00} = 200 \text{ kPa}$

$T_w = 50° \text{ C} = 323 \text{ K}$

$$\frac{p_{0 \text{ ind}} - p_0}{\frac{1}{2}\rho u_\infty^2} = -7$$

$$\rho = \frac{p}{RT} = \frac{200,000}{(287)(323)} = 2.158 \text{ kg/m}^3$$

$$\frac{1}{2}\rho u_\infty^2 = (0.5)(2.158)(20)^2 = 431 \text{ Pa}$$

$$p_0 = 200,000 + 431 = 200,431$$

$$p_{0,\text{ ind}} = (07)(431) + 200,431 = 197,414 \text{ Pa}$$

7-46

At $\Theta = 8^\circ$

$p_s = +1.1\%$ of p_{dyn}

$p_{\text{dyn}} = +0.5\%$ of p_{dyn}

$p_{\text{stag}} = -0.8\%$ of p_{dyn}

$$p_s = 200,000 + (0.011)(431) = 200,005 \text{ Pa}$$

$$p_{\text{dyn}} = 431 + (0.005)(431) = 433.2 \text{ Pa}$$

$$p_{\text{stag}} = 200,431 - (0.008)(471) = 200,428 \text{ Pa}$$

7-47

$T_\infty = 0° \text{ C} = 273 \text{ K}$ $\quad p = 20 \text{ kPa}$

$C = 20.04\sqrt{273} = 331 \text{ m/s}$ $\quad M = \frac{600}{331} = 1.812$

$$\frac{p_\infty}{p_{02}} = \frac{\left\{\left[\frac{(2)(1.4)}{2.4}\right](1.812)^2 - 0.412.4\right\}^{1/2}}{[1.2(1.812)^2]^{3.5}} = 0.2116$$

$$p_{02} = \frac{20}{0.2116} = 94.5 \text{ kPa}$$

7-49

$T = 20°\text{F} = 480°\text{R} = 267 \text{ K}$ $\quad p = 0.95 \text{ atm}$

CH_4

v at 0.95 atm and 267 K $\sim \frac{267}{0.95} = 281$

v at 1 atm and 293 K $\sim \frac{293}{1} = 293$

Factor to convert to SCFM $= \frac{293}{281} = 1.0425$

7-51

$d = 0.75 \text{ in.} \qquad D = 1.5 \text{ in.} \qquad \dfrac{d}{D} = \beta = 0.5$

$\text{Re} = 10{,}000 \qquad CM = 0.63 \qquad T = 25° \text{ C}$

$\rho = 62.2 \qquad \mu = 2.08 \text{ lbm/hr-ft}$

$$10{,}000 = \frac{(62.2)U_m(1.5112)}{2.08}; U_m = 2675 \frac{\text{ft}}{\text{hr}} = 0.74 \text{ ft/s}$$

$$Q = \frac{(0.74)\pi\left(\frac{1.5}{12}\right)^2}{4} = 0.63\frac{\pi}{4}\left(\frac{0.75}{12}\right)^2\left[\frac{(2)(32.2)}{62.2}\right]^{1/2}\Delta p^{1/2}$$

$\Delta p = 21.3 \text{ psf}$

7-52

$SG = 0.82 \qquad 1 \text{ atm}, 25° \text{ C} = 298 \text{ K} \qquad U = 30 \text{ m/s}$

$W_d = 0.1 \text{ mm} \qquad \rho = 1.18 \text{ kg/m}^2$

$$p_d = \frac{1}{2}\rho u_\infty^2 = (0.5)(1.8)(30)^2 = 533 \text{ Pa}$$

$$= 11.16 \text{ psf} = 2.615 \text{ in fluid} = 66.42 \text{ mm fluid}$$

$$U_\infty = \left(\frac{2p_d}{\rho}\right)^{1/2} = \left[\frac{2p_dRT}{p}\right]^{1/2}$$

$$\frac{W_u}{U_{00}} = \left[\frac{1}{4}\left(\frac{W_d}{p_d}\right)^2 + \frac{1}{4}\left(\frac{W_T}{T}\right)^2\right]^{1/4}$$

$$= \frac{1}{2}\left[\left(\frac{0.1}{66.4}\right)^2 + \left(\frac{1.2}{298}\right)^2\right]^{1/2}$$

$$= (4.63 \times 10^{-6})^{1/2}$$

$$W_U = (30)(4.62 \times 10^6)^{1/2} = 0.065 \text{ m/s}$$

7-53

$$\frac{p_{\text{ind}} - p_\infty}{\frac{1}{2}\rho U_\infty^2} \times 100 = 3 \qquad T = -40°\text{C} = 233 \text{ K}$$

$$p_{\text{ind}} = 22{,}000 \text{ Pa} \qquad M = 0.8 \qquad c = 20\sqrt{233} = 306 \frac{\text{m}}{\text{s}}$$

$$U_{00} = (306)(0.8) = 245 \text{ ml}$$

$$22{,}000 - p_\infty = (0.03)(0.5)\left[\frac{p_\infty}{(287)(233)}\right](245)^2$$

$$= 0.0134p_\infty$$

$p_\infty = 21.7 \text{ kPa}$

7-54

$$\frac{p_i - p_\infty}{\frac{1}{2}pU_\infty^2} = -0.01$$

$$22{,}000 - p_\infty = -0.0045p_{00}$$

$$p_\infty = 22.1 \text{ kPa}$$

7-55

$$2.35 \text{ gpm} = (2.35)\left(\frac{231}{1728}\right) = 0.314 \text{ ft}^3/\text{min}$$
$$= 8.8896 \times 10^{-3} \text{ m}^3/\text{min}$$
$$= 1.483 \times 10^{-4} \text{ m}^3/\text{s}$$
$$\rho = 1329 \text{ kg/m}^3$$
$$\dot{m} = (1.483 \times 10^{-4})(1329) = 0.197 \text{ kg/s}$$

7-56

$p_0 = 800 \text{ kPa} \pm 10 \text{ kPa}$ $\quad T_0 = 50° \text{ C} = 323 \text{ K} \pm 10$

$C = 0.97 \pm 0.5\%$ $\quad d = 2 \text{ cm}$ $\quad p_{\text{exit}} = 100 \text{ kPa}$

$$\dot{m} = \frac{\pi(0.02)^2}{4}(800,000)\left[\frac{(2)}{(287)(323)}\right]^{1/2}[0.2344]^{1/2}$$
$$= 0.565 \text{ kg/s}$$
$$W_{\dot{m}/\dot{m}} = \left[\left(\frac{W_p}{p}\right)^2 + \frac{1}{4}\left(\frac{W_T}{T}\right)^2\right]^{1/2}$$
$$= \left[\left(\frac{10}{800}\right)^2 + \frac{1}{4}\left(\frac{1}{323}\right)^2\right]^{1/2}$$
$$= 0.0126$$
$$W_{\dot{m}} = (0.0126)(0.585) = 0.0071 \text{ kg/s}$$

7-58

$P_\infty = 15 \text{ psia}$ $\quad T_\infty = 120° \text{ F} = 580° \text{ R}$

$U_\infty = 100 \text{ ft/s}$ $\quad \theta = 10°$ $\quad \rho = 0.0698 \text{ lbm/ft}^3$

$$p_d = \frac{1}{2}\rho U_\infty^2 = \frac{(0.5)(0.0698)(100)^2}{32.17} = 10.85 \text{ psf}$$
$$= 0.0753 \text{ psl}$$
$$p_s = 15 + (0.0753)(0.02) = 15.0015 \text{ psi}$$
$$p_d = 0.0753(1 + 0.005) = 0.0757 \text{ psi}$$
$$p_{\text{stag}} = (15 + 0.0753) - (0.0753)(0.015)$$
$$= 15.0742 \text{ psi}$$

7-59

$$\frac{p_\infty}{p_{0_2}} = \frac{\left\{\frac{2\gamma}{\gamma+1} - \frac{\gamma-1}{\gamma+1}\right\}^{1/\gamma-1}}{\left[\frac{\gamma+1}{2}\right]^{\gamma/\gamma+1}}$$

$$= \frac{\left[\frac{\gamma+1}{\gamma+1}\right]^{1/\gamma-1}}{\left[\frac{\gamma+1}{2}\right]^{\gamma/\gamma-1}}$$

$$= \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}$$

$=$ same as Eq. (7-23) as expected

7-60

$d_0 = 2.5$ cm $\qquad d_p = 5.0$ cm

$\beta = 0.5 \qquad \Delta p = 1932$ mm $H_2O = 39.6$ pst $= 1.895$ kPa

$$p_1 = 400 + (101.3)\left(\frac{750}{760}\right) = 500 \text{ kPa}$$

$T_1 = 27° \text{ C} = 300 \text{ K}$

$$\frac{\Delta p}{p_1} = \frac{1.895}{500} = 0.004; Y \approx 1.0$$

$MC = 0.63$

$$A_2 = \frac{\pi(0.025)^2}{4} = 4.9 \times 10^{-4} \text{ m}^2$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{(500)(1000)}{(287)(300)} = 5.81 \text{ kg/m}^3$$

$$\dot{m} = YA_2CM[2g_c\Delta p\rho_1]^{1/2}$$
$$= 0.046 \text{ kg/s}$$

7-61

$$\dot{m} = (0.046)\left[\frac{190}{1932}\right]^{1/2} = 0.014 \text{ kg/s}$$

7-62

$W_{\Delta p} = \pm 12$ mm H_2O

$W_{p_1} = \pm 10$ kPa

$W_{T1} = \pm 2°\text{C} = \pm 2°\text{K}$

$$\frac{Wm}{m} = \left[\left(\frac{W_{\Delta p}}{\Delta p}\right)^2\left(\frac{1}{2}\right)^2 + \left(\frac{W_p}{p}\right)^2\left(\frac{1}{2}\right)^2 + \left(\frac{W_T}{T}\right)^2\left(\frac{1}{2}\right)^2\right]^{1/2}$$

For Problem 7-60 $\frac{W_m}{m} = 0.011 = 1.1\%$

For Problem 7-61 $\frac{W_m}{m} = 0.033 = 3.3\%$

7-63

$Y = 1.0 \qquad C = 0.98$

$$M = \frac{1}{(1-\beta^4)^{1/2}} = 1.033$$

$$\begin{aligned}\dot{m} &= YCMA_2[2g_c\rho_1\Delta p]^{1/2} \\ &= (1.0)(0.98)(1.033)(4.9\times10^{-4})[(2)(5.81)(1895)]^{1/2} \\ &= 0.074\text{ kg/s}\end{aligned}$$

7-64

$$\dot{m} = (0.074)\left[\frac{190}{1932}\right]^{1/2} = 0.023\text{ kg/sec}$$

7-65

From Table 7-1

Problem 7-60 $\Delta p(\text{loss}) = (0.78)(1932) = 1550$ mm H_2O

Problem 7-61 $\Delta p(\text{loss}) = (0.78)(190) = 148$ mm H_2O

Problem 7-63 $\Delta p(\text{loss}) = (0.1)(1932) = 193$ mmH_2O

Problem 7-64 $\Delta p(\text{loss}) = (0.1)(1900) = 19$ mm H_2O

7-74

Rotameter with air

$T = 10°\text{C} = 283\text{ K}$

$p = 400 + 100 = 500\text{ kPa}$

Rating $= 100$ L/min at full scale and 1 atm, 20°C

$$\begin{aligned}\text{Volume flow at meter condition} &= Q_{corr} \\ &= (50)\left(\frac{500}{101}\right)\left(\frac{293}{283}\right) \\ &= 256\text{ std L/min}\end{aligned}$$

At standard conditions

$$\dot{m}_{std} = \frac{(256\times10^{-3})(101{,}000)}{(287)(293)} = 0.307\text{ kg/min}$$

At meter condition

$$\begin{aligned}\dot{m}_{corr} &= (0.307)\left[\left(\frac{283}{293}\right)\left(\frac{101}{500}\right)\right]^{1/2} \\ &= 0.136\text{ kg/min}\end{aligned}$$

7-75

Properties of water

$\rho = 997\text{ kg/m}^3$

$\mu = 9.8\times10^{-4}\text{ kg/m-s}$

$\nu = \mu/\rho = 9.87\times10^{-7}\text{ m}^2/\text{s}$

$= 0.987$ cSt

1 gal $= 3.777$ kg

The approximate range for +/ − 0.4%
is 60 to 600 cyles/s-cSt.
The frequency range is
$f_{\text{low}} = (60)(0.987) = 59.2$ cycles/s
$f_{\text{high}} = (600)(0.987) = 592$ cycles/s
The flow coefficient $K = (1092)/3.777 = 289.1$ cyc/kg
The flow range is
$m_{\text{low}} = 59.2/289.1 = 0.204$ kg/s
$m_{\text{high}} = 592/289.1 = 2.04$ k/s

7-76

$\rho = (2)(1.013 \times 10^5)/(287)(293) = 2.41\ \text{kg/m}^3$

$p_{\text{dynamic}} = (2.41)(10)^2/(2)(1) = 120.5$ Pa

7-77

$d = 0.25$ in $= 6.35$ mm
$D = 0.5$ in $= 12.7$ mm
$\rho_1 = (10)(1.013 \times 10^5)/(287)(313) = 11.28$
From Figure 7-14 $\beta = 0.5$,
$p_2/p_1 = 0.8,\ Y = 0.88$
$M = 1/[1 - (0.5)^4]^{1/2} = 1.033$
$A_2 = \pi(0.00635)^2/4 = 3.167 \times 10^{-5}\ \text{m}^2$
$C = 0.97$

$$\begin{aligned} m &= (0.88)(0.97)(1.033)(3.167 \times 10^{-5}) \\ &\quad \times [(2)(1.0)(0.2)(10^6)(11.28)]^{0.5} \\ &= 0.059\ \text{kg/s} \end{aligned}$$

7-78

$\rho_b = (2)(612) = 1224\ \text{kg/m}^3$

7-79

Using Eq. (7-24) with $C = 0.97$ and $\gamma = 1.4$
with $p_1 = 20$ atm $= 2.026 \times 10^6$ Pa, $T_1 = 323$ K
gives
$A_2 = 3.29 \times 10^{-4}\ \text{m}^2$

7-80

$\rho_1 = (5000000)/(287)(323) = 53.94\ \text{kg/m}^3$
$\mu = 5.5 \times 10^{-4}$ kg/m-s
$Re = 50{,}000 = (53.94)u(0.05)/5.5 \times 10^{-4}$
$u = 10.2$ m/s
$m = (53.94)(10.2)\pi(0.05)^2/4 = 1.08$ kg/s

Take $C = 0.973$ at $Re = 100000$

Using $\beta = 0$ and $Y = 1$

$$1.08 = 0.973[1 - (0.5)^4]\pi(0.0125)^2 \times [(2)(53.94)\Delta p]^{0.5}$$

$\Delta p = 53{,}930$ Pa

$\Delta p/p_1 = 0.0107$ and $y = 1.0$ justified

7-81

$d = 0.004$, $Re = 110{,}000$

$f_s = (0.88)(3.9)/0.004 = 8580$ Hz

7-82

$M = 0.75$, $T_\infty = -35° = 238\,\text{K}$

$p = 33\,\text{kPa}$, $c = (20)(238)^{1/2} = 309\,\text{m/s}$

$u = (0.75)(309) = 231\,\text{m/s}$

$P_0 - \rho_\infty = (1/2)\rho u^2$

$\rho = 33{,}000/(287)(238) = 0.483$

$$p_0 = 33{,}000 + (0.483)(231)^2/2$$
$$= 45{,}887 \text{ Pa}$$

Chapter 8

8-1

$$°F = 32.0 + \frac{9}{5}°C \qquad \text{For } °C = °F$$

$$°F = 32.0 + \frac{9}{5}°F$$

$$°F = °C = -40.0°$$

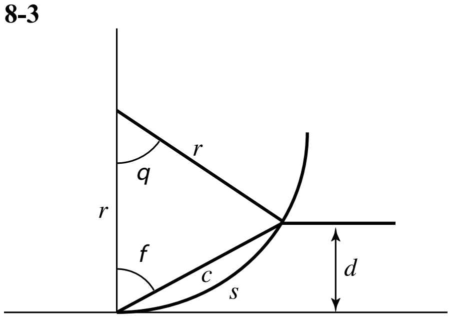
8-2

$$T_{true} = T_{ind} - 0.000088(T_{ind} - T_{amb})D$$

$$\text{Error} = 0.000088(T_{ind} - T_{amb})D$$

$$\text{Error} = 0.123°F$$

8-3



$$\theta = \frac{180s}{\pi r}$$

$$\phi = \frac{180 - \theta}{2}$$

$$C = 2r\sin\frac{\theta}{2}$$

$$d = C\cos\phi$$

$$r = \frac{t\left\{3(1+m)^2 + (1+mn)\left[m^2 + \left(\frac{1}{mn}\right)\right]\right\}}{6(\alpha_2 - \alpha_1)(T - T_0)(1+m)^2}$$

For $T - T_0 = 38.3°C$

$r = 26.7$ in. $\qquad C = 4$ in.

$\theta = 8.6 \qquad d = 0.30$ in.

$\phi = 85.7°$

For $T - T_0 = 37.2°C$

$r = 27.4$ in.

$\theta = 8.35°$

$\phi = 85.82°$

$d = 0.292$ in.

deflection at $\pm 1° = 0.008$ in.

For $T - T_0 = 38.9$°C	For $T - T_0 = 36.6$°C
$r = 26.2$ in.	$r = 27.9$ in.
$\theta = 8.75°$	$\theta = 8.22°$
$\phi = 85.62°$	$\phi = 85.89°$
$d = 0.306$ in.	$d = 0.286$ in.

deflection at $\pm 2° = 0.020$ in.

8-4

$d = C\cos\alpha \qquad C \approx s$

$$d = S\sin\frac{90s}{\pi r}$$

$$W_d = \left[\left(\frac{\partial d}{\partial s}W_s\right)^2 + \left(\frac{\partial d}{\partial t}W_t\right)^2\right]^{1/2}$$

$$W_d = \left\{\left[\left(\sin\frac{90s}{\pi r} + \frac{s}{2r}\cos\frac{90s}{\pi r}\right)(W_s)\right]^2 + \left[\left(S\cos\frac{90s}{\pi r}\right)\left(\frac{S}{\pi r}\right)(W_t)\right]^2\right\}^{1/2}$$

For $T = 150$°F

$$r = \frac{0.0102 \times 10^5}{10} = 102 \text{ in.} \qquad W_d = 0.0001358 \text{ in.}$$

$$\frac{1°\text{F}}{0.008 \text{ in.}} = 125°\text{F/in.} \qquad W_T = 0.01695°\text{F}$$

For $T = 200$°F, $r = 27.0$ in.

$W_d = 0.00149$ in. $\qquad W_T = 0.186$°F

For $T = 300$°F, $r = 10.92$ in.

$W_d = 0.00365$ in. $\qquad W_T = 0.456$°F

8-5

$\alpha_1 = 1.35 \times 10^{-5}\text{C}^{-1}$; $\alpha_2 = 2.02 \times 10^{-5}\text{C}^{-1}$

$$n = \frac{26.0 \times 10^6}{14.0 \times 10^6} = 1.855; \qquad m = \frac{0.010 \pm 0.001}{0.014 \pm 0.0002} = 0.714 \pm 0.017$$

$$d = \delta\sin\frac{s}{2r} = \delta\sin\frac{6(\alpha_2 - \alpha_1)(T - T_0)(1 + m)^2 s}{2t\left\{3(1 + m)^2 + (1 + mn)\left[m^2 + \frac{1}{mn}\right]\right\}}$$

For small deflections $\sin\theta = \theta$

$$\therefore \frac{\partial d}{\partial(T - T_0)} = \frac{6(\alpha_2 - \alpha_1)(1 + m)^2(s)^2}{2t\left\{3(1 + m)^2 + (1 + mn)\left[m^2 + \frac{1}{mn}\right]\right\}}$$

$$\frac{\partial d}{\partial T - T_0} = 0.00292 \text{ in./°F}$$

$$W_{\text{sen}} = \left[\left(\frac{\partial \text{sen}}{\partial t}W_t\right)^2 + \left(\frac{\partial \text{sen}}{\partial m}W_m\right)^2\right]^{1/2}$$

$$W_{\text{sen}} = 3.70 \times 10^{-5} \text{ in./°F} \qquad \frac{3.7 \times 10^{-5}}{2.92 \times 10^{-3}} \times 100 = 1.27\%$$

8-8

Temperature level $= 200°F$ From Table 8-3

At 250°F 5.280 mv

At 150°F $\underline{2.711 \text{ mv}}$ 0.02569 mv/°F

2.569 mv

$$\frac{W_{\Delta T}}{\Delta T} = \frac{W_R}{R} = \frac{0.05}{5} = 0.01$$

$$R = \frac{0.004}{0.01} = 0.4 \text{ mv}$$

$(\Delta T)(0.02569)n = 0.4$ mv

$n = 3t$ use $n = 4$

8-9

Temperature level $= 400°F$

At 350°F 7.20 mv

At 450°F $\underline{9.43 \text{ mv}}$

2.23 mv

$S = 0.0223 \pm 0.0001065$ mv/°F

$R = (\Delta T)(S)(n) = 0.1705 \pm 0.002$ mv

$$W_{\Delta T} = \Delta T\left\{\left(\frac{W_R}{R}\right)^2 + \left(\frac{W_S}{S}\right)^2\right\}^{1/2}$$

$W_{\Delta T} = 0.051°F$ or 1.275%

8-10

$$R = R_0 e^{\left[\beta\left(\frac{1}{T}-\frac{1}{T_0}\right)\right]}$$

$$2{,}315 = 1{,}010 e^{\left[3{,}420\left(\frac{1}{T}-\frac{1}{366.5}\right)\right]} \qquad T = 337°K$$

$$W_T = \left[\left(\frac{\partial T}{\partial R}W_R\right)^2 + \left(\frac{\partial T}{\partial R_0}W_{R_0}\right)^2\right]^{1/2}$$

$$\ln\frac{R}{R_0} = \beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\ln e \qquad [\ln e = 1.0]$$

$$T = \frac{\beta}{\ln\frac{R}{R_0} + \frac{\beta}{T_0}}$$

$$W_T = \left\{\left[\frac{\frac{\beta}{R_0}}{\left(\ln\frac{R}{R_0} + \frac{\beta}{T_0}\right)^2}(W_{R_0})\right]^2 + \left[\frac{-\frac{\beta}{R}}{\left(\ln\frac{R}{R_0} + \frac{\beta}{T_0}\right)^2}(W_R)\right]^2\right\}^{1/2}$$

$W_T = 0.1135°K$

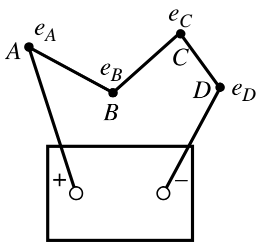
8-12

$E_{83} = 1.133$ mv $T = 1560°F$

$E_T = 35.28$ mv $E_T = E_{83} + E_{POT}$

$E_{POT} = 35.28 - 1.133 = 34.15$ mv

8-13



$$E = e_A - e_B + e_C - e_D = 4.91 - 7.94 + 1.94 - 0$$
$$E = \pm 1.09 \text{ mv}$$

8-14

Iron constantan.
Interpolate in Table 8-3.

$32°\text{F} \le T \le 369.8°\text{F}$

$$\frac{W_T}{T} = \frac{W_R}{R} = 0.0025 \rightarrow W_T = \pm 0.925°\text{F}$$

Copper constantan.

$26.4°\text{F} \le T \le 415.8°\text{F}$

$W_T = \pm 1.04°\text{F}$

Chromel alumel.

$26.85°\text{F} \le T \le 475°\text{F}$

$W_T = \pm 1.188°\text{F}$

8-16

$$E = \sigma\varepsilon T^4$$

$$T = \left(\frac{E}{\sigma\varepsilon}\right)^{1/4} = \left(\frac{28,000}{(5.668 \times 10^{-8})(0.9)}\right)^{1/4} = 860.74°\text{K}$$

$$\frac{W_T}{T} = \frac{1}{4}\left[\left(\frac{W_E}{E}\right)^2 + \left(\frac{W_\varepsilon}{\varepsilon}\right)\right]^{1/2}$$

$$= \frac{1}{4}\left[\left(\frac{0.4}{28}\right)^2 + \left(\frac{0.05}{0.9}\right)^2\right]^{1/2}$$

$$= 0.01434$$

$$W_T = 12.34°\text{C}$$

8-17

$$(W_{q|A})_1 = (W_{q|A})_2 = \pm 225 \text{ W/m}^2$$

$(q|A)_2 = 2.23(q|A)_1$ $\qquad T_1 = 280 \pm 0.5°\text{C} = 553°\text{F}$

$q|A \sim \varepsilon\sigma T^4$ $\qquad \varepsilon_1 = 0.95 \pm 0.03$

$\varepsilon_2 = 0.72 \pm 0.05$

$$\varepsilon_2 \sigma T_2^4 = 2.23\left(\varepsilon_1 \sigma T_1^4\right)$$

$$T_2 = \left[\frac{(2.23)(0.95)}{0.72}\right]^{1/4}(553) = 724.3°\text{K}$$

Uncertainty results either from $(q|A)_2$ and ε_2 or $\varepsilon_1, T_1,$ and ε_2 whichever combination is worst.

For $(q|A)_2$ and ε_2 $\qquad T_2 = \left(\frac{q|A}{\varepsilon_2 \sigma}\right)^{1/4}$

$$\frac{\partial T_2}{\partial q|A} = \frac{1}{4}\left[\frac{q|A}{\varepsilon_2 \sigma}\right]^{-3/4}\left(\frac{1}{\varepsilon_2 \sigma}\right)$$

$$\frac{\partial T_2}{\partial \varepsilon_2} = \frac{1}{4}\left[\frac{q|A}{\varepsilon \sigma}\right]^{-3/4}\left(\frac{1}{\varepsilon_2^2 \sigma}\right)$$

$$\frac{W_{T_2}}{T_2} = \left[\frac{1}{16}\left(\frac{W_{q|A}}{q|A}\right)^2 + \frac{1}{16}\left(\frac{W_\varepsilon}{\varepsilon}\right)^2\right]^{1/2}$$

$$q|A = (0.72)(724.3)^4(5.668 \times 10^{-8}) = 11,231 \text{ W/m}^2$$

$$\frac{W_{T_2}}{T_2} = \frac{1}{4}\left[\left(\frac{225}{11,231}\right)^2 + \left(\frac{0.05}{0.72}\right)^2\right]^{1/2} = 0.01807$$

$W_{T_2} = 13.09°\text{K}$ $\qquad$ For $\varepsilon_1, T_1, \varepsilon_2$:

$$T_2 = \left[\frac{\varepsilon_1(2.23)}{\varepsilon_2}\right]^{1/4} T_1 \qquad \frac{W_{T_2}}{T_2} = \left[\frac{1}{16}\left(\frac{W_{\varepsilon_1}}{\varepsilon_1}\right)^2 + \frac{1}{16}\left(\frac{W_{\varepsilon_2}}{\varepsilon_2}\right)^2 + \left(\frac{W_{T_1}}{T_1}\right)^2\right]^{1/2}$$

$$= 0.01909$$

$W_{T_2} = 13.83°\text{K}$

This is greater so it is taken as the uncertainty.

8-18

$$h_t(T_g - T_t) = \frac{\varepsilon_t}{1 - \varepsilon_t}(E_{bt} - J_t)$$

$$J_t = \frac{E_{bt}\left[\frac{\varepsilon_t}{1-\varepsilon_t}\right] + F_{te}E_{be} + \frac{E_{bs}}{\left[\left(\frac{1}{E_{ts}}\right) + \left(\frac{1}{\varepsilon_s} - 1\right)\frac{A_t}{A_s}\right]}}{F_{te} + -\frac{\varepsilon_t}{(1-\varepsilon_t)} + \frac{1}{\left[\left(\frac{1}{F_{ts}}\right) + \left(\frac{1}{\varepsilon_s} - 1\right)\frac{A_t}{A_s}\right]}}$$

For $A_s \ll A_t$

$$J_t = \frac{2165\left[\frac{0.8}{0.2}\right] + 168}{\frac{0.8}{0.2} + 1} = 1765$$

$$T_g = \frac{\varepsilon_t}{h_t(1 - \varepsilon_t)}(E_{bt} - J_t) + T_t \qquad (1)$$

$T_g = 1400°\text{F}$

For the shield removed:

$$hA(T_g - T_t) = \sigma A\varepsilon\left(T_t^4 - T_e^4\right)$$

$$2hs(T_g - T_s) = \frac{E_{bs} - J_t}{\left(\frac{1}{E_{ts}}\right)\left(\frac{A_s}{A_t}\right) + \frac{1}{\varepsilon_s} - 1} + \frac{E_{bs} - E_{be}}{\frac{1}{\varepsilon_s} - 1 + \frac{1}{F_{se}}}$$

$$E_{be} = E_{bs} - \frac{2hs(T_g - T_s)}{\varepsilon_s} \qquad (2)$$

Solve T_e in equation (2), then use equation (1) to find T_t.

8-19

$$J_t = \frac{86.60 + 0.01E_{be} + 1.46}{0.041 + 0.0087}$$

$$J_t = 1772 + 0.0201E_{be}$$

$$h_t(T_g - T_t) = \frac{\varepsilon_t}{1 - \varepsilon_t}(E_{bt} - J_t)$$

$$T_g + 0.0402E_{be} = 1386 \qquad (1)$$

Also:

$$2h_s(T_g - T_s) = \frac{E_{bs} - J_t}{\left(\frac{1}{F_{ts}}\right)\left(\frac{A_s}{A_t}\right) + \frac{1}{\varepsilon_s} - 1} + \frac{E_{bs} - E_{be}}{\frac{1}{\varepsilon_s} - 1 + \frac{1}{F_{se}}} \text{ gives:}$$

$$T_g + 0.06807\,E_{be} = 106.75 \qquad (2)$$

Solve (1) and (2) simultaneously.

$T_g = 1467°\text{F}$

For the shield removed:

$$h(T_g - T_t) = \sigma\varepsilon T_t^4 - \varepsilon E_{be}$$

$$2(1927 - T_t) = \sigma\varepsilon T_t^4 - (0.8)(19{,}950)$$

$$T_t = 1390°\text{F}$$

8-20

$$hA(T_g - T_t) = \sigma\varepsilon A\left(T_t^4 - T_e^4\right)$$

$U_\infty = 3\text{ m/sec}$	$p_\infty = 1.0\text{ atm}$
$T_t = 115°\text{C} = 388°\text{K}$	$T_e = 193°\text{C} = 466°\text{K}$
$\varepsilon = 0.08 \pm 0.02$	$d = 6\text{ mm}$
$W_{T_t} = \pm 0.03°\text{C}$	$T_f \approx 410°\text{K}$
$v = 27.06 \times 10^{-6}$	$\text{Pr} = 0.68$
$k = 0.0343$	$\text{Re} = \frac{(3)(0.006)}{27.06 \times 10^{-6}} = 665$

$$h = \frac{0.0343}{0.006}(0.683)(665)^{0.466}(0.68)^{1/3} = 103.9\text{ W/m}^2 - °\text{C}$$

$$T_g = \frac{\sigma\varepsilon\left(T_t^4 - T_e^4\right)}{h} + T_t$$

$$= \frac{(5.668 \times 10^{-8})(0.08)(388^4 - 466^4)}{103.9} + 388$$

$$= 386.9°\text{K}$$

$$W_{T_g} = \left[\left(\frac{\partial T_g}{\partial T_t}\right) W_{T_t^2} + \left(\frac{\partial T_g}{\partial h}\right)^2 W_h^2 + \left(\frac{\partial T_g}{\partial \varepsilon}\right)^2 W_\varepsilon^2\right]^{1/2}$$

$$= [(1.01)^2(0.03)^2 + (-0.0103)^2(15.59)^2 + (-13.361)^2(0.02)^2]^{1/2}$$

$$= [9.181 \times 10^{-4} + 0.0258 + 0.0714]^{1/2}$$

$$= 0.313°\text{K} \quad (h \text{ and } \varepsilon \text{ control})$$

8-22

$$T_g = \frac{(0.1714 \times 10^{-8})(0.57)[(1475)^4 - (1660)^4]}{5.0} + 1475°\text{R}$$

$$T_g = 454°\text{F}$$

$$W_{T_g} = \left[\left(\frac{\partial T_g}{\partial h} W_h\right)^2 + \left(\frac{\partial T_g}{\partial T_t} W_{T_t}\right)^2 + \left(\frac{\partial T_g}{\partial \varepsilon_s} W_{\varepsilon_s}\right)^2 + \left(\frac{\partial T_g}{\partial T_e} W_{T_e}\right)^2\right]^{1/2}$$

$$= \{[(112.1)(0.750)]^2 + [(3.51)(1.0)]^2 + [(-985)(0.04)]^2 + [(-2.58)(20)]^2\}^{1/2}$$

$$W_{T_g} = 101.7°\text{F}$$

For chrome plated:

$$T_g = 956°\text{F}$$

$$W_{T_g} = \{[(11.8)(0.750)]^2 + [(1.264)(1.0)]^2 + [(983)(0.021)]^2 + [(0.624)(20)]^2\}^{1/2}$$

$$W_{T_g} = 25°\text{F}$$

8-23

$$\tau = \frac{mC}{hA}; m = \rho v = \rho \pi r^2 L \quad A = 2\pi r L$$

$$\tau = \frac{\rho r C}{2h} = \frac{(555)(0.095)}{2(12.45)(8)(12)} = 79.2 \text{ sec}$$

8-24

$$\bar{h}A(T_g - T_f) = \sigma A \varepsilon\left(T_t^4 - T_e^4\right)$$

$$T_g = 1660°\text{R}; T_e = 1960°\text{R}$$

$$\text{Take } \bar{h} = 0.27\left(\frac{T_g - T_t}{d}\right)^{1/4}$$

$$0.27\frac{(T_t - T_g)^{5/4}}{d^{1/4}} = \sigma\varepsilon\left(T_e^4 - T_t^4\right)$$

$$0.27\frac{(T_t - 1660)^{5/4}}{\left(\frac{1}{96}\right)^{1/4}} = (0.17174 \times 10^{-8})(0.78)\left(1960^4 - T_t^4\right)$$

Use trial and error method to solve for T_t.

$T_t \approx 1940°\text{R}$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{(-hA/mc)\tau} \qquad \frac{T - T_i}{T_t - T_i} = 0.5$$

$T = 0.5(1940 - 530) + 530 = 1235°\text{R}$

$$e^{(-hA/mc)\tau} = \frac{1235 - 1960}{530 - 1960} = 0.507\left(\frac{hA}{mc}\right)\tau = 0.678$$

$$\tau = 0.678\left\{\frac{(555)(0.095)\pi L\left[\left(\frac{1}{96}\right)^2 - \left(\frac{1}{192}\right)^2\right]}{[29.9]\left[\pi\left(\frac{1}{96}\right)L\right]}\right\}$$

$\tau = 34$ seconds

8-25

$\tau = R_cC = 1.2$ sec

$$\left(\frac{E_0}{E_i}\right)_{ss} = \frac{R}{R + R_c} = \alpha = 0.125$$

Take $R_c = 0.12 \times 10^6\ \Omega$ than $C = 10\,\mu\text{f}$

even mult value

$R = 0.125(R + R_c)$

$R = 0.01712$ megaohms

$R_c = 0.12$ megaohms

$c = 10\ \mu\text{f}$

8-26

$r = 0.98 \pm 0.01 \qquad M = 3.0 \qquad T_r = 380 \pm 1.0°\text{C} = 653°\text{K}$

$$\frac{T_0}{T_\infty} = 1 + \frac{\gamma - 1}{2}M^2 = 2.8$$

$$r = \frac{T_r - T_\infty}{T_0 - T_\infty} = \frac{T_r - T_\infty}{1.8T_\infty}$$

$$T_\infty = \frac{T_r}{1.8r + 1} = \frac{653}{(1.8)(0.98) + 1} = 236.25°\text{K}$$

$$\frac{\partial T_\infty}{\partial T_r} = \frac{1}{1.8r + 1} = 0.3618$$

$$\frac{\partial T_\infty}{\partial_r} = -T_r(1.8r + 1)^{-2}(1.8) = -153.85$$

$$W_{T_\infty} = [(0.3618)^2(1.0)^2 + (-153.85)^2(0.01)^2]^{1/2}$$
$$= [0.1309 + 2.367]^{1/2}$$
$$= 1.58°\text{K}$$

8-27

$$\overline{Nu}_L = \frac{\bar{h}L}{k_i};\ Gr_L = \frac{\rho^2 g\beta(T_w - T_p)L^3}{\mu^2} \qquad L = \frac{1}{3}\text{ ft}$$

$$T_f = \frac{T_p + T_w}{2} = \frac{95 + 110}{2} = 102.5°\text{F}$$

From Table A-8 $\rightarrow$ air at 102.5°F

$\mu = 0.046$ lbm/hr $\cdot$ *ft* $\qquad c_p = 0.240$ Btu/lbm-°F

$$\text{Pr} = 0.706 \qquad \beta = \frac{1}{°\text{R}} = \frac{1}{562.5°\text{R}} = 1.78 \times 10^{-3}\ °\text{R}^{-1}$$

$$k_{\text{air}} = 0.0157\ \text{Btu/hr-ft-°F} \qquad \rho = \frac{P}{RT} = \frac{(14.7)(144)}{(53.35)(562.5)} = 0.0705\ \text{lbm/ft}^3$$

$$Gr_L = 9.65 \times 10^5 \rightarrow (Gr_L)(Pr) = 6.82 \times 10^5$$

From Table 8-5 $\rightarrow \overline{Nu_L} = 0.59(Gr_L Pr)^{1/4} = \bar{h}\dfrac{L}{k}$

$$\bar{h} = \frac{(0.0157)(0.59)(68.2 \times 10^4)^{1/4}}{\left(\frac{1}{3}\right)} \rightarrow \bar{h} = 0.806\ \frac{\text{Btu}}{\text{hr-ft}^2\text{-°F}}$$

$$\bar{h}(T_p - T_\infty) = \sigma t\left(T_w^4 - T_p^4\right) \qquad \sigma = 0.1714 \times 10^{-8}\ \text{Btu/hr} \cdot \text{ft}^2\text{-°R}^4$$

$$T_\infty = 79°\text{F} \qquad t = 0.7 \pm 0.05$$

$$W_\infty = \frac{\partial T_\infty}{\partial \varepsilon} W_e \qquad W_\varepsilon = 0.05$$

$$\frac{\partial T_\infty}{\partial \varepsilon} = \frac{\sigma}{\bar{h}}\left(T_w^4 - T_p^4\right) = \frac{(0.1714 \times 10^{-8})(1.0 \times 10^{10})}{8.06 \times 10^{-1}} = 2.125 \times 10^1$$

$$W_{T_\infty} = 1.065°\text{F}$$

8-28

$$\bar{h}(T_p - T_\infty) = \sigma t\left(T_w^4 - T_p^4\right) \text{ where } t = 0.05 \pm 0.01$$

$$T_\infty = 95°\text{F} - \frac{(0.1714 \times 10^{-8})(5.0 \times 10^{-2})(1.0 \times 10^{10})}{8.06 \times 10^{-1}}$$

$$T_\infty = 93.86°\text{F};\ W_{T_\infty} = \frac{\partial T_\infty}{\partial \varepsilon} W_\varepsilon = (21.25)(0.01) = 0.2125°\text{F}$$

8-29

$$(0.9)(5.669 \times 10^{-8})(308^4 - 294^4) = (5.0)(298 - T_a)$$

$$T_a = 286.6°\text{K} = 13.6°\text{C}$$

8-30

$$T_\infty = 50°\text{C} \qquad h = 20\ \text{W/m}^2 - °\text{C}$$

$$d = 3.0\ \text{mm} \qquad T_0 = 20°\text{C}$$

$$\rho = 7817\ \text{kg/m}^3 \qquad c = 0.46\ \text{kJ/kg-°C}$$

$$\frac{hA}{mc} = \frac{hA}{\rho c V} = \frac{(20)(4)\pi(0.0015)^2}{(7817)(460)\left(\frac{4}{3}\right)(\pi)(0.0015)^3}$$

$$= 0.01112\ \text{sec}^{-1}$$

Time constant $= 89.9$ sec

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-0.011112T}$$

8-34

$\varepsilon = 0.3$

$(50)(T_s - 373) = (5.668 \times 10^{-8})(0.3)(373^4 - 293^4)$

$T_s = 377 \text{ K}$

8-35

For polished aluminum $\varepsilon = 0.09$

$T_s = 374.2 \text{ K}$

8-36

$m = 1 \qquad n = 1.52$

$\alpha_1 = 1.7 \times 10^{-6}, \alpha_2 = 2.02 \times 10^{-5}$

$t = 0.6 \times 10^{-3} \text{ m}$

$T_1 = -10°\text{C} \qquad T_1 - T_0 = -40$

$T_2 = 120°\text{C} \qquad T_2 - T_0 = 90$

Inserting in formula

$r_1 = -0.546 \text{ m}$

$r_2 = +0.2427 \text{ m}$

$$\theta = \frac{L}{r}; \theta_1 = \frac{0.025}{-0.546} = -0.0458 \text{ rad} = 2.62°$$

$$\theta_2 = \frac{0.025}{0.2427} = 0.103 \text{ rad} = 5.9°$$

Deflection angle of strip $\approx \frac{\theta}{2}$

$$\text{Deflection} = L \sin\left(\frac{\theta}{2}\right)$$

$\text{Deflec l}_1 = (2.5)\sin 1.31 = -0.057 \text{ cm}$

$\text{Deflec l}_2 = (2.5)\sin 2.95 = 0.128 \text{ cm}$

8-37

$R_0 = 8000\ \Omega \text{ at } 0°\text{C} = 273 \text{ K}$

$\text{at } 200°\text{C} = 473 \text{ K} \qquad R = 10$

$$\frac{10}{8000} = \exp\left[\beta\left(\frac{1}{473} - \frac{1}{273}\right)\right]$$

$\beta = 4316 \text{ K}$

8-39

$\rho = 7894 \text{ kg/m}^2 \qquad c = 452/\text{kg} \cdot °\text{C}$

$$mc = \frac{(7897)(0.5)(0.1)(0.005)(2.54)^3}{10^6}(452)$$

$$= 0.0146$$

$$hA = \frac{(100)(2)(0.5)(0.1)(2.54)^2}{10^4} = 0.00645$$

$$\tau = \frac{mc}{hA} = 2.26 \text{ sec}$$

8-40

$k = 73 \text{ W/m°C}$

$100 \text{ W/cm}^2 = 10^6 \text{ W/m}^2$

$$\frac{q}{A} = k\frac{\Delta T}{\Delta x}$$

$$\Delta T = \frac{(10^6)(0.005)(0.0254)}{73} = 1.74°\text{C}$$

8-41

$T_\infty = 32°\text{C} = 305 \text{ K}$ $\varepsilon = 0.9$

$T_r = 50°\text{C} = 323 \text{ K}$ $h = 7 \text{ W/m}^2\text{-°C}$

$$(5.669 \times 10^{-8})(0.9)\left(323^4 - T_t^4\right) = (7)(T_t - 305)$$

$$T_t = 314 \text{ K} = 41°C$$

8-42

$T_t \sim 32°\text{C}$

8-43

$T_r = 550°\text{C} = 823 \text{ K}$ $h = 30 \text{ W/m}^2\text{-°C}$

$\varepsilon = 0.8$ $T_a = 400°\text{C} = 673 \text{ K}$

$$(5.669 \times 10^{-8})(0.8)\left(823^4 - T_t^4\right) = (30)(T_t - 673)$$

$T_t = 787 \text{ K} = 514°\text{C}$

8-45

$E_{100} = 1.520 \text{ mV}$ $E_{400} = 8.314 \text{ mV}$

$E_{\text{pot}} = (5)(8.314 - 1.520) = 33.97 \text{ mV}$

8-46

At 50°F $s = \dfrac{0.526 + 0.885}{50} = 0.0282 \text{ mV/°F}$

$0.2°\text{F} = 0.00564 \text{ mV}$

Comp. volt based on $32°\text{F} = 0.526 \text{ mV}$

8-47

$E_{\text{ind}} = \sigma T_{\text{ind}}{}^4$ $T_{\text{ind}} = 300°\text{C} = 573 \text{ K}$

$E_{\text{act}} = \varepsilon\sigma T_a^4$ $T_{\text{act}} = 315°\text{C} = 588 \text{ K}$

$$\varepsilon = \left(\frac{T_i}{T_a}\right)^4 = \left(\frac{573}{588}\right)^4 = 0.902$$

$$\frac{W_\varepsilon}{\varepsilon} = \left[16\left(\frac{W_{T_i}}{T_i}\right) + 16\left(\frac{W_{T_a}}{T_a}\right)^2\right]^{1/2}$$

$$= 0.0097$$

$W_\varepsilon = (0.0097)(0.902) = 0.0088$

8-48

$$T_a = \frac{T_i}{\varepsilon^{1/4}} = \frac{450 + 273}{0.902^{1/4}} = 742\text{ K} = 469°\text{C}$$

8-49

$d = 1.0$ mm $\qquad \rho = 2400$ kg/m^3

$h = 275$ W/m^2-°C $\qquad c = 650$ J/hg-°C

$T_f = 50$°C $\qquad k = 0.85$ W/m-°C

$h_5 = 25 \qquad kt = 75$ W/m-°C

L ~ 00 $\qquad T_i = 220$°C

$$R = \frac{1}{h2\pi r_i} = \frac{1}{(275)(2\pi)(0.0005)} = 1.157$$

$$\overline{kA} = (25)\pi(0.0005)^2 = 5.89 \times 10^{-5}$$

$$\tanh(\infty) \rightarrow 1.0$$

$$x = \frac{\sqrt{\frac{\overline{kA}}{R}}}{\pi rk} = 5.344$$

$$Bi = \frac{hsr}{k} = \frac{(25)(0.0005)}{0.85} = 0.0147$$

$$F(Bi) = 1.286$$

$$\frac{T_p - 220}{T_p - 50} = \frac{5.344 - 0.0147}{5.344 + 1.286} = 0.804$$

$$T_p = 917°\text{C}$$

8-51

$T_\infty = 20°\text{C} = 293\text{ K} \qquad r = 0.9$

$$T_0 = (293)[1 + 0.2(3)^2] = 527\text{ K}$$

$$r = 0.9 = \frac{T_r - 293}{527 - 293} \qquad T_r = 504\text{ K}$$

$$(5.669 \times 10^{-8})(0.9)\left(T_t^4 - 293^4\right) = 180(504 - T_t)$$

$$T_t = 490\text{ K} = 217°C$$

8-53

$$(5.669 \times 10^{-8})(0.07)\left(T_t^4 - 293^4\right) = 180(504 - T_t)$$

$$T_t = 502\text{ K} = 229°\text{C}$$

8-54

$\varepsilon = 0.05$ and 0.8

At $T = 210°\text{F} = 670°\text{R}$ $\quad t = 0.05$

$E = \sigma(0.05)(670)^4$

At $\varepsilon = 0.8$

$E = \sigma(0.8)(670)^4 = \sigma(0.05)T_i^4$

$$T_i = \left(\frac{0.8}{0.05}\right)^{1/4}(670) = 1340°\text{R} = 880°\text{F}$$

8-55

$d = 12$ mm $\quad r = 6$ mm

Copper $\quad \rho = 8890 \quad c = 398 \quad h = 97$

$T_0 = 100°\text{C} \quad T_{00} = 20°\text{C} \quad T = 25°\text{C}$

$$\frac{hA}{mc} = \frac{(97)4\pi(0.006)^2}{(8890)\left(\frac{4}{3}\right)\pi(0.0006)^3(398)} = 0.01371$$

$$\tau = \frac{1}{0.01371} = 72.95 \text{ sec}$$

$$\frac{25 - 20}{100 - 20} = e^{-0.01371t}$$

$$t = 202 \text{ sec}$$

8-56

18-8 ss $\quad \rho = 7817 \quad c = 460$

$T_\infty = 0°\text{C} \quad T_0 = 25°\text{C} \quad h = 3800$

$$\frac{hA}{mc} = \frac{h\pi dL}{\rho\pi\frac{d^2}{\pi}Lc} = \frac{(4)(3800)}{(7817)(0.05 \times 10^{-3})(460)}$$

$$= 84.5 \text{ sec}^{-1}$$

$\tau = 0.012$ sec

$$\frac{1 - 0}{25 - 0} = e^{-84.5t}$$

$$t = 0.038 \text{ sec}$$

8-57

T(°C)	E(μV) Ch-AL
30	1203
100	4095
150	6137
200	8136
250	10,151

$\Delta E = 10{,}151 + 8136 + 6137 + 4095 - 1203$

$= 27.32$ mV

8-58

Ni-Al $\quad S = -5 - (-3.5) = 18.5\ \mu\text{V/°C}$

Ni-Cn $\quad S = -35 - 25 = 60\ \mu\text{V/°C}$

8-60

$E_{50} = 0.836 \text{ mV}$

$E_{100} = 1.785 \text{ mV}$

$E(\text{total}) = 10(1.785 - 0.836) = 9.49 \text{ mV}$

8-61

$E_0 = 0$

$E(\text{total}) = 10(1.785) = 17.85 \text{ mV}$

8-62

$E_{600} = 14.37 \text{ mV}$

$E_{500} = 11.603 \text{ mV}$

$E(\text{total}) = 10(14.37 - 1.603) = 27.67 \text{ mV}$

Substantial difference

8-63

$T = 600°\text{C}$ $\quad E(K) = 24{,}906 \text{ mV}$ $\quad E(J) = 33.102 \text{ mV}$

$T(\text{ref}) = 25°\text{C}$ $\quad E(K) = 1.000$ $\quad E(J) = 1.277$

At 600°C $\quad E(K) = 24.906 - 1.000 = 23.906$

$E(J) = 33.102 - 1.277 = 31.825$

If type J installed the temperature readout on a type K instrument would correspond to $31.825 + 1.000 = 32.825$ mV.

$T(K, \text{at } 32.825 \text{ mV}) = 789°\text{C}$ or an error of $+189°\text{C}$

8-64

$hA(T_a - T_t) = \sigma\varepsilon A\left(T_t^4 - T_w^4\right)$

$T_t = 28°\text{C} = 301 \text{ K}, T_w = 0°\text{C} = 273 \text{ K}$

$\varepsilon = 0.9$ $\quad h = 14$

Solution of equation gives $T_a = 310.7 \text{ K} = 37.7°\text{C}$

8-66

$k_t = 70$ $\quad k_s = 5$ $\quad d = 2 \text{ mm}$

$h_s = 15$ $\quad h = 125$ $\quad T_i = 125°\text{C}$ $\quad T_f = 20°\text{C}$

$$R = \frac{1}{h2\pi r_i} = \frac{1}{(125)(2\pi)(0.001)} = 1.273$$

$\overline{kA} = (70)\pi(0.001)^2 = 2.2 \times 10^{-4}$

$L \to \infty$ and $\tanh(\infty) \to 1.0$

$$X = \frac{\left(\frac{2.2\times10^{-4}}{1.273}\right)^{1/2}}{\pi(0.001)(5)} = 0.837$$

$$Bi = \frac{(15)(0.001)}{5} = 0.003$$

$F(Bi) = 1.273$

$$\frac{T_p - 125}{T_p - 20} = \frac{0.837 - 0.003}{0.837 + 1.273} = 0.395$$

$$T_p = 194°\text{C}$$

8-67

$k_s = 70$; all other values the same

$$X = \frac{\left(\frac{2.2\times10^{-4}}{1.273}\right)^{1/2}}{\pi(0.001)(70)} = 0.0598$$

$$Bi = \frac{(15)(0.001)}{70} = 2.14 \times 10^{-4}$$

$$F(Bi) = 1.27$$

$$\frac{T_p - 125}{T_p - 20} = \frac{0.0598 - 0.00021}{0.0598 + 1.27} = 0.0448$$

$$T_p = 130°\text{C}$$

8-69

$\varepsilon = 0.3 \pm 0.03$ $\quad$ $T = 350$ K and 600 K

$W_E = \pm 1\%$ of E_b at 500 K

$E_b(500\ K) = (5.668 \times 10^8)(500)^4 = 3543\ \text{W/m}^2$

$1\% = 35.4\ \text{W/m}^2 = W_E$

$$E = \sigma\varepsilon T^4 \qquad T = \left(\frac{E}{\sigma\varepsilon}\right)^{1/4}$$

$$\frac{W_T}{T} = \left[\left(\frac{W_\varepsilon}{\varepsilon}\right)^2(-1)^2 + \left(\frac{W_E}{E}\right)^2\left(\frac{1}{4}\right)^2\right]^{1/2}$$

At 350 K $\quad E = \sigma\varepsilon(350)^4 = 255\ \text{W/m}^2$

At 600 K $\quad E = \sigma\varepsilon(600)^4 = 2203\ \text{W/m}^2$

At 350 K

$$\frac{W_T}{T} = \left[\left(\frac{0.03}{0.3}\right)^2 + \left(\frac{1}{4}\right)^2\left(\frac{35.4}{255}\right)^2\right]^{1/2}$$

$$= [0.01 + 0.0012]^{1/2}$$

$$= 0.106$$

At 600 K

$$\frac{W_T}{T} = \left[0.01 + \left(\frac{1}{4}\right)^2\left(\frac{35.4}{2203}\right)^2\right]^{1/2}$$

$$= 0.10008$$

Most of the uncertainty results from the uncertainty in ε.

8-70

$\varepsilon = 0.8 \pm 0.04$

At 350 K $\quad \dfrac{W_T}{T} = \left[\left(\dfrac{0.04}{0.8}\right)^2 + 0.0012\right]^{1/2}$

$= 0.061$

At 600 K $\quad \dfrac{W_T}{T} = \left[0.0025 + \left(\dfrac{1}{4}\right)^2\left(\dfrac{35.4}{2203}\right)^2\right]^{1/2}$

$= 0.0502$

Still, most of the uncertainty results from uncertainty in ε.

8-71

Type E thermocouple

$W = \pm 1.0°\text{C}$ or 0.4%

a. $W_{\Delta T} = \left[W_{T_1}^2 + W_{T_2}^2\right]^{1/2}$

$= \pm 1.41°\text{C}$

b. Sensitivity (mV/°C) should be same for all junctions and give better determination of ΔT.

8-73

$T_\infty = 65,\ T_0 = 20°\text{C}$

$\tau = \text{mc/hA}$

$= (2675)(0.004)^3(921)/(24)(0.004)^2(6)$

$= 68.4$ s

At $t = 34.2$ s $(T - 65)/(20 - 65) = e^{-0.5}$

$T - 37.7$ °C

From Table 8.3a $E = 1942$ mV

8-74

$h(T_p - T\infty) = \sigma\varepsilon\left(T_e{}^4 - T_p{}^4\right)$

$\varepsilon = 0.25$

$(40)(86 - 30) = (5.669 \times 10^{-8})(0.25)(Te^4 - 359^4)$

$T_e = 646\text{ K} = 373°\text{ C}$

8-75

$\varepsilon = 0.09$

$T_e = 821\text{ K} = 548°\text{C}$

8-76

$R_0 = 1000$ at $T = 250°\text{C} = 523\text{ K}, \quad R = 1$

$1 = (1000)\exp[\beta(1/523 - 1/273)]$

$\beta = 3945$ K

8-77

$\varepsilon = 0.7, \quad T_w = 600°C = 873\text{ K}, \quad T_\infty = 350°C$

$h = 40$

$(40)(T_t - 623) = 5.669 \times 10^{-8}(0.7)\left(873^4 - T_t^4\right)$

$T_t = 798\text{ K} = 525°C$

8-78

$T_a = T_i/\varepsilon^{1/4} = 673/0.902^{0.25} = 690.6\text{ K} = 417.6°C$

8-79

Let T_i be indicated temperature.

True temperature will be

$T_{\text{true}} = T_i(0.7/0.08)^{1/4}$

$1.72\ T_i$

8-80

Platinum-constantan $= 0 - (-35) = 35$ mV/° C

Platinum-nickel $= 0 - (-15) = 15$ mV/° C

8-81

Chromel-constantan at 100°C $= 6.319$ mV/junction

E(15 junctions) $= (15)(6.319) = 94.785$ mV

8-82

$\varepsilon = 0.85, T_t = 17°\text{ C} = 290\text{ K}$

$h = 10, Tw = -10°\text{ C} = 263\text{ K}$

$(10)(T_a - 290) = (5.669 \times 10^{-8})(0.85)(290^4 - 263^4)$

$T_a = 301\ K = 28°C$

Chapter 9

9-1

$k = 0.352$ W/cm-°C at 0°C

$$\frac{q}{A} = 0.352\left(\frac{dT}{dx}\right)_{T=0} = k_T\left(\frac{dT}{dx}\right)_T$$

$T - 277.3 = 0.0551x^2 - 14.787x + 0.0457$

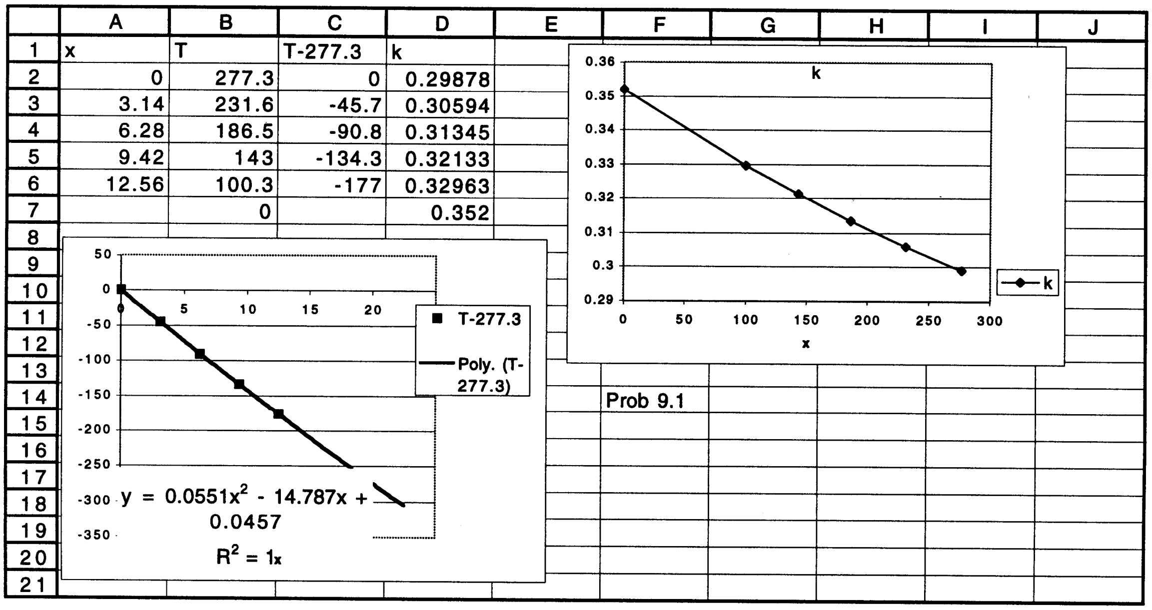
from Excel quadratic fit as shown

$$\frac{dT}{dx} = 0.1102x - 14.787$$

$T = 0°$ at $x = 20.29$ $\qquad \left(\frac{dT}{dx}\right)_{0°\text{C}} = -12.551$

$$k_T = \frac{4.418}{(14.787 - 0.1102x)}$$

T and *k* table as function of *x*. Then plot of *k* as function of *T*. Excel spreadsheets are shown.



	A	B	C	D
1	x	T	T-277.3	k
2	0	277.3	0	0.29878
3	3.14	231.6	-45.7	0.30594
4	6.28	186.5	-90.8	0.31345
5	9.42	143	-134.3	0.32133
6	12.56	100.3	-177	0.32963
7		0		0.352

	A	B	C	D
1	x	T	T-277.3	k
2	0	277.3	=B2-277.3	=4.418/(14.787-0.1102*A2)
3	3.14	231.6	=B3-277.3	=4.418/(14.787-0.1102*A3)
4	6.28	186.5	=B4-277.3	=4.418/(14.787-0.1102*A4)
5	9.42	143	=B5-277.3	=4.418/(14.787-0.1102*A5)
6	12.56	100.3	=B6-277.3	=4.418/(14.787-0.1102*A6)
7		0		0.352

9-5

$$T = \mu\pi w r_1^2\left(\frac{r^2}{2a} + \frac{2L\sqrt{2}}{b}\right)$$

For water at 100°F

$\mu = 1.65$ lbm/ft-hr $\pm$ 5%

$T = 0.01705$ in.-lbf

Using the simplified expression

$$T = \frac{2\mu\pi r_1^2 r_2 L w}{b}$$

$$W_T = \left\{\left[\left(\frac{\partial T}{\partial \mu}\right)(W_\mu)\right]^2 + \left[\left(\frac{\partial T}{\partial r_1}\right)(W_{T_1})\right]^2 + \left[\left(\frac{\partial T}{\partial r_2}\right)(W_{r_2})\right]^2 + \left[\left(\frac{\partial T}{\partial L}\right)(W_L)\right]^2 + \left[\left(\frac{\partial T}{\partial W}\right)(W_w)\right]^2 + \left[\left(\frac{\partial T}{\partial b}\right)(W_b)^2\right]\right\}^{1/2}$$

$W_T = 0.000916$ in - lbf

For Glycerine at 100°F, same procedure

$T = 0.0254$ in.-lbf

$W_T = 0.00137$ in - lbf

9-8

$$v = \left(0.00237t - \frac{1.93}{t}\right) \times 10^{-3}\ \text{ft}^2/\text{sec}$$

$$v = \frac{v}{\rho} = 0.4654 \times 10^{-3}\ \text{ft}^2/\text{sec}$$

$\mu = 87.0$ lbm/hr-ft *or* $\mu = 35.9$ centipoise

9-9

$t_{opt} \geqslant 10$ min $\therefore$ use series solution

For $t = 600$ sec

$$F = \frac{8}{\pi^2}\left[e^{-\frac{600}{16{,}600}} + \frac{1}{9}e^{\frac{-(9)(600)}{16{,}600}} + \frac{1}{25}e^{-\frac{(25)(600)}{16{,}600}} + \cdots\right]$$

$F = 0.856$

$x_{1A} + x_{1B} = 1.0$ $\qquad$ $x_{1A} - x_{1B} = 0.856$

$x_{1A} = .928$ $\qquad$ $x_{1B} = 0.072$

For $t = 10{,}800$ sec

Use two terms of series: $\rightarrow F = 0.422$

$x_{1A} = 0.711$; $x_{1B} = 0.289$

9-10

$x_{1A} + x_{1B} = 1.0$ $\qquad$ $x_{1A} = 1.0 - 0.0912 - 0.9088$

$F = x_{1A} - x_{1B} = 0.8176$

$$F = 0.808\left[e^{-\frac{391.2}{t_{opt}}} + \frac{1}{9}e^{-\frac{3520}{t_{opt}}} + \frac{1}{25}e^{-\frac{9780}{t_{opt}}}\right]$$

$t_{opt} = 6250$ sec $\qquad$ $D_{12} = \dfrac{4L^2}{\pi^2 t_{opt}}$

$D_{12} = 0.1645\ \text{cm}^2/\text{sec}$

9-11

$t = t_{opt}$

$$F = \frac{8}{\pi^2}\left[e^{-1} + \frac{1}{9}e^{-9} + \frac{1}{25}e^{-25} + \frac{1}{49}e^{-49} + \cdots\right]$$

$$F = 0.297 + 1.11 \times 10^{-5} + 4.5 \times 10^{-13} + 8.65 \times 10^{-24}$$

$$t = \frac{1}{2}t_{opt}$$

$$F = \frac{8}{\pi^2}\left[e^{-1/2} + \frac{1}{9}e^{-9/2} + \frac{1}{25}e^{-25/2} + \frac{1}{49}e^{-49/2} + \cdots\right]$$

$$F = 0.490 + 0.993 \times 10^{-3} + 1.2 \times 10^{-7} + 3.78 \times 10^{-13}$$

$$t = 2t_{opt}$$

$$F = \frac{8}{\pi^2}\left[e^{-2} + \frac{1}{9}e^{-18} + \frac{1}{25}e^{-50} + \frac{1}{49}e^{-98} + \cdots\right]$$

$$F = 0.109 + 1.375 \times 10^{-9} + 6.13 \times 10^{-24} + 4.4 \times 10^{-45}$$

9-15

Cu properties

$\rho = 8890 \text{ kg/m}^3 \qquad c = 398 \text{ J/kg-°C}$

$r = 0.0125 \text{ m}$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left(-\frac{hA\tau}{\rho c V}\right)$$

$T_0 = 95°\text{C} \qquad T_\infty = 35°\text{C}$

$h = 570 \text{ W/m}^2\text{-°C}$

$$\frac{hA}{\rho c V} = \frac{(570)(4)\pi(0.0125)^2}{(8890)(389)\frac{4}{3}\pi(0.0125)^3} = 0.03867 \text{ sec}^{-1}$$

$$\frac{\rho c V}{hA} = 25.86 \text{ sec}$$

τ(sec)	T(°C)
5	84.45
10	75.76
20	62.69
40	47.78
60	40.90

9-16

$$\frac{q}{A} = -k\frac{\Delta T}{\Delta x} = 10^5 \frac{\text{Btu}}{\text{hr-ft}^2\text{-°F}}, \Delta x = 6.66 \times 10^{-5} \text{ ft}$$

$$k = 0.4 \text{ Btu/hr-ft-°F}$$

$$\Delta T = -\frac{(10^5)(6.66 \times 10^{-5})}{0.4} \rightarrow \Delta T = -16.6°\text{F}$$

$$S = S_{cu} - S_{constantan} = 6.5 \text{ mv/°C} - (-35\ \mu V/°\text{C}) = 41.5\ \mu\text{V/°C}$$

$$E = S\Delta T = (41.5 \times 10^{-6})\left(\frac{5}{9}\right)(16.6) = 383 \times 10^{-6} \text{ volts/ junction pair}$$

$$E_{Tot} = \frac{E}{\text{pairs}} \times 5 \text{ pairs} = 383\ \mu\text{v} \times 5 = 1.915 \text{ mvolts}$$

9-17

$\phi = \dfrac{P_v}{P_g}$ $\quad P = 14.7$ psia $\quad T_{db} = 95°F$ $\quad T_{wb} = 75°F$

P_{gw} at 75°F $= 0.435$ psia

$$P_v = P_{g_w} - \frac{(P - P_{gw})(T_{db} - T_{wb})}{2800 - T_{wb}} = 0.435 - \frac{(14.265)(20)}{2725}$$

$P_v = 0.330$ psia at 95°F $\quad P_g = 0.823$ psia

$$\phi = \frac{P_v}{P_g} = \frac{0.330}{0.823} \rightarrow \phi = 40\%$$

at $P_v = 0.330$ psia

Dew point temp $\rightarrow T_{dp} = 66.8°F$

9-18

$p = 2.06 \times 10^5$ N/m^2 $\quad T_{DB} = 32°C$ $\quad \phi = 50\%$

$$p_v = pgw - \frac{(p - pgw)(T_{DB} - T_{WB})}{1537.8 - T_{WB}}$$

p_{sat} at 32°C $= 4759$ N/m^2

$p_v = (0.5)(4759) = 2380$ N/m^2

Solve equation by iteration using Steam Tables to obtain p_{gw} as a function of T_{WB}.

Solution yields $T_{WB} = 25.46°C$.

9-19

$$k = \left(\frac{q}{A}\right)\frac{\Delta x}{\Delta T} = \frac{(5000)(0.002)}{(0.3)^2(55)} = 2.0 \text{ W/m-°C}$$

$$\frac{W_k}{k} = \left[\left(\frac{W_{\Delta T}}{\Delta T}\right)^2 + \left(\frac{W_{q/A}}{\frac{q}{A}}\right)^2\right]^{1/2}$$

$$= \left[\left(\frac{0.3}{55}\right)^2 + (0.01)^2\right]^{1/2}$$

$$= 0.01139$$

$W_k = 0.2278$ W/m-°C

9-20

$T_{WB} = 20°C$ $\quad T_{DB} = 35°C \pm 0.5°C$

$p_{gw} = 2339$ Pa $\quad p_g = 5628$ Pa

$$p_v = 2339 - \frac{(101{,}300 - 2339)(35 - 20)}{1538 - 20}$$

$$= 1361 \text{ Pa}$$

$T_{DP} = 11.4°C$ $\quad \phi = \dfrac{1361}{5628} = 0.242$

Perturb by 1°C $\quad T_{WB} = 21, T_{DB} = 36$

$p_{gw} = 2487$

$(W_{WB} + 1) \qquad p_v = 2487 - \dfrac{(101{,}300 - 2487)(35 - 21)}{1538 - 21}$

$= 1575$

$\phi = \dfrac{1595}{5628} = 0.28$

$T_{DP} = 13.8°\text{C}$

$\dfrac{\partial T_{DP}}{\partial T_{WB}} = \dfrac{11.4 - 13.8}{1} = -2.4$

$\dfrac{\partial \phi}{\partial T_{WB}} = \dfrac{0.242 - 0.280}{1} = -0.038$

$(T_{DB} + 1) \qquad P_v = 2339 - \dfrac{(101{,}300 - 2339)(36 - 20)}{1538 - 20}$

$= 1296$

$T_{DP} = 10.8°\text{C} \qquad \phi = \dfrac{1296}{5947} = 0.218$

$\dfrac{\partial T_{DP}}{\partial T_{DB}} = \dfrac{11.4 - 10.8}{1} = 0.6 \qquad \dfrac{\partial \phi}{\partial T_{DB}} = \dfrac{0.242 - 0.218}{1}$

$= 0.024$

$W_{DP} = [(-24)^2(0.5)^2 + (0.6)^2(0.5)^2]^{1/2}$

$= 1.24°\text{C}$

$W\phi = [(-0.038)^2)(0.5)^2 + (0.024)^2(0.5)^2]^{1/2}$

$= 0.0225$

9-23

$T_{DB} = 40°\text{C} \qquad T_{DP} = 25°\text{C} \pm 0.5°\text{C}$

$pg = 7384 \text{ Pa} \qquad pv = 3169 \text{ Pa}$

$\phi = \dfrac{3169}{9384} = 0.429$

$p_v = p_{gw} - \dfrac{(p - p_{gw})(T_{DB} - T_{WB})}{1538 - T_{WB}}$

$T_{WB} = 28.4°\text{C}$ by iteration.

Assume *DB* exact and perturb *DP* by 1°C

to 26°C, $p_v = 3363$

$\phi = \dfrac{3363}{7384} = 0.455$

$T_{WB} = 29.2°\text{C}$

$\dfrac{\partial \phi}{\partial T_{DP}} = \dfrac{0.429 - 0.455}{1} = -0.026$

$\dfrac{\partial T_{WB}}{\partial T_{DP}} = \dfrac{28.4 - 29.2}{1} = -0.8$

$W_\phi = (0.026)(0.5) = 0.013$

$W_{WB} = (0.8)(0.5) = 0.4°\text{C}$

9-24

$$\left(\frac{\text{lbf}\cdot\text{s}}{\text{ft}^2}\right)\left(32.2\,\frac{\text{lbm ft}}{\text{lbf s}^2}\right)\left(3600\,\frac{\text{s}}{\text{hr}}\right) = 1.158\times10^5\ \text{lbm/hr-ft}$$

9-25

$$T_f = \frac{200+20}{2} = 110°\text{C} = 383\ K$$

$$k = 0.032\ \text{W/m}\cdot°\text{C} \qquad v = 24.8\times10^{-6}\ \text{m}^2/\text{s}$$

$$\text{Pr} = 0.7$$

$$\text{GrPr} = \frac{(9.8)\left(\frac{1}{383}\right)(200-20)(0.01)^3(0.7)}{(24.8\times10^{-6})^2}$$

$$= 5242$$

$$h = \frac{0.032}{0.01}[2+(0.43)(5242)^{1/4}] = 18.1\ \text{W/m}^2\text{-}°\text{C}$$

Copper

$$\rho = 8890\ \text{kg/m}^3 \qquad c = 398\ \text{J/kg}\cdot°\text{C}$$

$$\frac{hA}{pcV} = \frac{(18.1)4\pi(0.005)^2}{(8890)(398)\frac{4}{3}\pi(0.005)^3}$$

$$= 0.00307$$

$$\frac{T-T_\infty}{T_0-T_\infty} = e^{-0.00307\tau}$$

9-30

$$\frac{q}{A} = 100\ \text{W/cm}^2 = 10^6\ \text{W/m}^2$$

k for 18–8 SS = 167.3 W/m · °C

$$\Delta T = \frac{\left(\frac{q}{A}\right)\Delta x}{k} = \frac{(10)^6(0.002)}{16.3} = 122.7°\text{C} = 220.89°\text{F}$$

For copper-constantan thermocouple emf generated is 5.361 mV.

9-32

$$v = 50\ \text{cSt} \qquad W_v = \pm1\%$$

$$\frac{\partial v}{\partial t} = \left[0.00237 - \frac{1.93(-1)}{t^2}\right]\times10^{-3}$$

$$v = (50)(0.1076\times10^{-4}) = \left[0.00237-\frac{1.93}{t}\right]\times10^{-3}\ \text{ft}^2/s = 5.38\times10^{-4}$$

$$t = 230.5\ \text{sec}$$

$$\frac{\partial v}{\partial t} = 1.074\times10^{-5}$$

$$W_t = \frac{(0.01)(5.38\times10^{-4})}{1.074\times10^{-5}} = 0.5\ \text{sec}$$

9-33

$v = 0.001\ \text{m}^2/s \qquad \rho = 890\ \text{kg/m}^2$

$\text{Re} = 5 \qquad d = 1\ \text{cm}$

$\Delta p = 40\ \text{kPa}$

$$U = \frac{(5)(0.001)}{0.01} = 0.5\ \text{m/s}$$

$$Q = (0.5)\pi\frac{(0.01)^2}{4} = 3.93 \times 10^{-5}\ \text{m}^3/\text{s}$$

$$L = \frac{\pi r^4 \Delta\beta}{8\mu Q} = \frac{\pi(0.005)^4(40,000)}{(8)(0.001)(890)(3.93 \times 10^{-5})}$$

$$= 0.281\ \text{m}$$

$$\frac{L}{d} = 28.1 > \frac{5}{8} = \frac{\text{Re}}{8}$$

9-34

$T = 400K \qquad \Delta T = 10°\text{C}$

$k = 0.03365\ \text{W/m-°C}$

$d_i = 6\ \text{mm} \qquad L_i = 50\ \text{mm}$

$d_0 = 10\ \text{mm} \qquad L_0 = 125\ \text{mm}$

$$q = \frac{2\pi Lk\Delta T}{\ln\left(\frac{d_0}{d_i}\right)} = \frac{2\pi(0.03365)(10)}{\ln\left(\frac{10}{6}\right)}$$

$$= 4.14\ \text{W}$$

9-35

$$\frac{q}{A} = \frac{\Delta T}{\left(\frac{\Delta x}{k}\right)_s + \left(T\frac{\Delta x}{k}\right)_{\text{std}}} = \frac{25}{\left(\frac{0.1}{15}\right) + \left(\frac{0.1}{69}\right)}$$

$$= 3080\ \text{W/m}^2$$

9-36

$k \sim 0.05\ \text{W/m-°C}$

$\Delta x = 10\ \text{cm} \qquad \Delta T(\text{sample}) = 15 \pm 0.2°\text{C}$

$W\left(\frac{q}{A}\right) = \pm 1\% \qquad W_h = \pm 1\%$

$$\frac{q}{A} = k\frac{\Delta T}{\Delta x} \qquad \Delta x = \frac{k\Delta T}{\frac{q}{A}}$$

$$\frac{W_{\Delta x}}{\Delta x} = \left[\left(\frac{W_k}{k}\right)^2 + \left(\frac{W_{\Delta T}}{\Delta T}\right)^2 + \left(\frac{Wq/A}{\frac{q}{A}}\right)^2\right]^{1/2}$$

$$= \left[(0.05)^2 + \left(\frac{0.2}{15}\right)^2 + (0.01)^2\right]^{1/2}$$

$$= 0.0527$$

$$W_{\Delta x} = (10)(0.0527) = 0.527\ \text{cm}$$

9-37

$t = 140 \pm 1\,\text{sec} \qquad \rho = 880\ \text{kg/m}^3$

$$v = \left[(0.00237)(140) - \frac{1.93}{140}\right] \times 10^{-3} = 3.19 \times 10^{-4}\ \text{ft}^2/\text{s}$$

$$= 2.95 \times 10^{-5}\text{m}^2/\text{s}$$

$$\mu = \rho v = (2.95 \times 10^{-5})(880)$$
$$= 0.026\ \text{kg/m} \cdot \text{s}$$

9-39

$\sqcup = 0.1\ \text{W/m}^2\text{-°C} \qquad$ cube 2 mm on side

$\rho = 8890\ \text{kg/m}^3 \qquad c = 398\ \text{J/kg-°C} \qquad \tau = 20\ \text{sec}$

$$\frac{q}{A} = \frac{mc}{A}\frac{dT_s}{d\tau} - W(T_s - T_w)$$

Solution: $T_s - T_w = \dfrac{\frac{q}{A}}{W}\left[\exp\left(\dfrac{WA}{mc}\tau\right) - 1\right]$

$$A = (0.002)^2 = 4 \times 10^{-6}$$

$$\frac{\sqcup A}{mc} = \frac{(0.1)(4 \times 10^{-6})}{(8890)(0.002)^3(398)} = 1.413 \times 10^{-5}$$

$$50 = \frac{\frac{q}{A}}{0.1}[\exp(1.413 \times 10^{-5} \times 20) - 1]$$

$$\frac{q}{A} = 17{,}690\ \text{W/m}^2$$

9-42

$$\frac{\sqcup A}{mc} = (1.413 \times 10^{-5})\left(\frac{100}{10.1}\right) = 0.01413$$

$$50 = \frac{\frac{q}{A}}{100}[\exp(0.01413 \times 20) - 1]$$

$$\frac{q}{A} = 15{,}310\ \text{W/m}^2$$

9-43

$T_{DB} = 40°\ \text{C}, \ T_{WB} = 20°\ \text{C}$

Figure 9-20: $p_g = 7500$ Pa, $p_{gw} = 2400$ Pa

$$p_v = 2400 - (101300 - 2400)(40 - 20)/(1537.8 - 20)$$
$$= 1097\ \text{Pa}$$

Figure 9-20 $\quad T_{\text{dew point}} = 8°$ C

$\phi = 1097/7500 = 14.6\ \%$

9-44

$v = 60\text{cSt} = 6.458 \times 10^{-4}\ \text{ft}^2/\text{s}$

$0.6458 = (0.00237t - 1.93/t)$

$t = 275\ \text{s}$

9-45

$v = 0.0015\ \text{m}^2/\text{s},\ \rho = 750\ \text{kg/m}^3$

$Re = 600, d = 0.001$

$u = (600)(0.0015)/(0.001)(750) = 1.2\ \text{m/s}$

$Q = \text{Au} = \pi(1.2)(.001)^2/4 = 9.42 \times 10^{-7}$

$45{,}000 = (9.42 \times 10^{-7})(8)(0.0015)L/\pi(0.0005)^4$

$L = 0.782\ \text{m}$

9-46

1 mol CH_4 produces 1 mol or 44 kg of CO_2

1 mol C_3H_8 produces 3 mol or 132 kg of CO_2

At 1 atm and 293 K:

1 mol at 1 atm and 293 K

$= (8314)(293)/101{,}320 = 24.04\ \text{m}^3/\text{kg-mol}$

$\text{HHV}(CH_4) = (24.04)(1000)(37.3) = 897000\ \text{kJ/kg-mol}$

$\text{HHV}(C_3H_8) = 24.04(1000)(93.1) = 2238120\ \text{kJ/kg-mol}$

kg CO_2/kJ HHV

$CH_4 = 44/897000 = 4.905 \times 10^{-5}$

$C_3H_8 = 132/2238000 = 5.9 \times 10^{-5}$

9-47

$\Delta x = 0.09$

$q/A = 30/(0.09/12 + 0.09/65) = 3377\ \text{W/m}^2$

9-48

$t = 150\ \text{s}, \rho = 0.8(999) = 800$

$v = [0.00237(150) - 1.93/150] \times 10^{-3}$

$= 3{,}43 \times 10^{-4}\ \text{ft}^2/\text{s} = 3.18 \times 10^{-6}\ \text{m}^2/\text{s}$

$\mu = v\rho = 0.00254\ \text{kg/m-s}$

9-49

$\rho = 8890,\ c = 398,\ A = (0.0025)^2$

$U = 0.15\ \ T_s - T_w = 40,\ \ \tau = 15\text{s}$

$Ts - Tw = (q/UA)[\exp(UA\tau/mc) - 1]$

$40 = [(q/A)/0.15]$

$\times \{\exp[(0.15)(0.025)^2(15)/(8890)(0.0025)^3(398)] - 1\}$

$q/A = 23{,}590\ \text{W/m}^2$

9-50

Repeating the calculation with $U = 50$ but all other parameters the same yields

$q/A = 22{,}600\ \text{W/m}^2$

Chapter 10

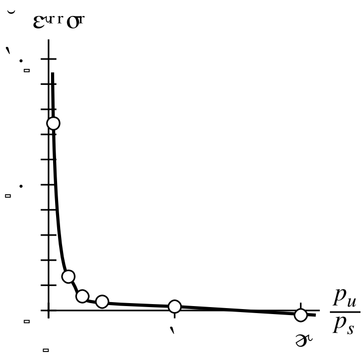
10-1

$$\% \text{ indicated Error } = \frac{W_u - W_s}{W_s} \times 100$$

$$= \frac{\rho_a}{\rho_s}\left(\frac{\rho_s - \rho_u}{\rho_u - \rho_a}\right)$$

$$= 0.1415\left(\frac{530 - \rho_u}{\rho_u - \rho_a}\right)$$

$$\% \text{ indicated Error } = 0.01415\left(\frac{1 - \frac{\rho_u}{\rho_s}}{\frac{\rho_u}{\rho_s} - \frac{0.075}{530}}\right)$$



$\frac{\rho_u}{\rho_s}$	% Error
0	−100
0.00189	8.09
0.0189	0.755
0.0944	0.136
0.189	0.0608
0.377	0.0234
0.944	0.0006
1.89	−0.00665
↓	↓
∞	−0.01415

10-2

$$\frac{\Delta W}{W} = \frac{1}{1 \times 10^8} \qquad \Delta W = \frac{\phi}{s} = 1 \times 10^{-5} g$$

$$\therefore S = \frac{0.25\pi}{(180)(1 \times 10^{-5})} = 436.0 \text{ rad/g}$$

10-3

$$F = \frac{AE}{L} y \qquad E = 28.3 \times 10^6 \text{ psi}$$

$$= 1.95 \times 10^{11} \text{ N/m}^2$$

$$\frac{\partial F}{\partial y} = \frac{AE}{L}$$

$$\frac{W_F}{F} = \frac{\partial F}{\partial y}\frac{W_y}{F} = \frac{W_y}{y}$$

$$y = (100)(0.025\text{ mm}) = 2.5\text{ mm}$$

$$\frac{A}{L} = \frac{F}{Ey} = \frac{1100}{(1.95 \times 10^{11})(2.5 \times 10^{-3})} = 2.255 \times 10^{-6}$$

Choose $L = 0.1\text{ m}$ then

$$A = \frac{\pi d^2}{4} = (0.1)(2.255 \times 10^{-6})$$

$$d = 5.36 \times 10^{-4}\text{ m} = 0.536\text{ mm}$$

10-4

$$\frac{W_{y_{\max}}}{y_{\max}} = \frac{W_F}{F}$$

$$y_{\max} = (100)(2.5 \times 10^{-6}) = 2.5 \times 10^{-4}\text{m} = \frac{1}{3}t$$

$$t = 7.5 \times 10^{-4}\text{m} \qquad E = 1.95 \times 10^{11}\text{ N/m}^2$$

$$\mu = 0.3 \qquad \text{Take } \frac{a}{b} = 2.0$$

$$2.5 \times 10^{-4} = \frac{(3)(4500)[1-(0.3)^2]}{4\pi(1.95 \times 10^{11})(7.5 \times 10^{-4})^3}\left[(36)^2 - \frac{(166)^4}{(36)^2}(\ln 2)^2\right]$$

$$b^2 = 4.808 \times 10^{-5}$$

$$b = 6.934 \times 10^{-3}\text{m}$$

$$a = 0.01387\text{ m}$$

10-5

$$F = \frac{3EI}{L^3}y \qquad I = \frac{\pi(od)^4}{64} - \frac{\pi(Id)^4}{64} = 0.1039\text{ in}^4$$

Say we can read as closely as 1% $y_{\max}$

$$\frac{W_y}{y} = 0.01 \rightarrow y_{\max} = 6.25\text{ in.}$$

$$L^3 = \frac{3(28.3 \times 10^6)(0.1039)}{150}(6.25)$$

$$L^3 = 368{,}000\text{ in}^3$$

$$L = 71.6\text{ in.}$$

10-6

$$I = \frac{bh^3}{12} = \frac{1}{48}\text{ in}^4$$

$$F = \frac{16EI}{\left[\frac{\pi}{2} - \frac{4}{\pi}\right]d^3}y$$

$$W_F = \left\{\left[\frac{\partial F}{\partial y} W_y\right]^2\right\}^{1/2} = \frac{16EI}{\left(\frac{\pi}{2} - \frac{4}{\pi}\right)d^3} W_y$$

$W_F = 157.0\ \text{lb}_\text{f}$ $\qquad F = 15{,}000\ \text{lb}_\text{f}$

For $y = 0.01$ in.

$$\frac{W_F}{F} = \left\{\left[\frac{W_y}{y}\right]^2 + \left[3\frac{W_d}{d}\right]^2 + \left[\frac{W_b}{b}\right]^2 + \left[3\frac{W_h}{h}\right]^2\right\}^{1/2}$$

$$\frac{W_F}{F} = 1.121\%$$

10-7

$$M = \frac{\pi G\left(r_0{}^4 - r_i{}^4\right)}{2L}\phi = 8.85 \times 10^3\ \text{in - lb}_\text{f}$$

$$W_m = \left[\left(\frac{\partial M}{\partial \phi} W\phi\right)^2 + \left(\frac{\partial M}{\partial r_i} W_{T_i}\right)^2 + \left(\frac{\partial M}{\partial T_0} WT_0\right)^2 + \left(\frac{\partial M}{\partial L} WL\right)^2\right]^{1/2}$$

$$= [87{,}200 + 216.0 + 831.0 + 3.14]^{1/2}$$

$W_m = 297.0\ \text{in-lb}_\text{f}$

$$\varepsilon_{45^\circ} = \pm\frac{Mr_0}{\pi G\left(r_0{}^4 - r_i{}^4\right)} = \pm 0.164 \times 10^{-2}\ \text{in./in.}$$

10-8

$$C = R_1R_2R_4 + R_1R_3R_4 + R_1R_2R_3 + R_2R_3R_4 + R_G(R_1 + R_4)(R_2 + R_3)$$

$C = 7.68 \times 10^6$

$$\Delta Ig = \frac{E}{C} R_eR_1F_\varepsilon$$

$\Delta Ig = 5.00\ \mu\ \text{amp/in.}$

10-9

$$\frac{\Delta E_D}{E} = \frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_4} - \frac{R_2}{R_2 + R_3}$$

$$\frac{\Delta E_D}{E} = \frac{1 + \frac{\Delta R_1}{R_1}}{1 + \frac{\Delta R_1}{R_1} + \frac{R_4}{R_1}} - \frac{R_2}{R_2 + R_3}$$

$$\frac{\Delta R_1}{R1} = F\varepsilon = (2.0)(400 \times 10^{-6}) = 800 \times 10^{-6}$$

$$\frac{\Delta E_D}{E} = 200 \times 10^{-6}$$

$\Delta E_D = 800\ \mu\text{v}$

10-10

$$\varepsilon_{\max}, \varepsilon_{\min} = \frac{\varepsilon_1 + \varepsilon_3}{2} \pm \frac{1}{\sqrt{2}}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{1/2}$$

For $\theta = 0$; $\tan\theta = 0$

$$\varepsilon_2 = \frac{\varepsilon_1 + \varepsilon_3}{2}$$

$$\varepsilon_{max} = \frac{\varepsilon_1 + \varepsilon_3}{2} + \frac{1}{\sqrt{2}}\left[\left(\varepsilon_1 - \frac{\varepsilon_1 + \varepsilon_3}{2}\right)^2 + \left(\frac{\varepsilon_1 + \varepsilon_3}{2} - \varepsilon_3\right)^2\right]^{1/2}$$

$$= \frac{\varepsilon_1 + \varepsilon_3}{2} + \frac{\varepsilon_1 - \varepsilon_3}{2}$$

$$\varepsilon_{max} = \varepsilon_1$$

$$\varepsilon_{min} = \frac{\varepsilon_1 + \varepsilon_3}{2} - \frac{1}{\sqrt{2}}\left[\frac{(\varepsilon_1 - \varepsilon_3)^2}{2}\right]^{1/2}$$

$$= \frac{\varepsilon_1 + \varepsilon_3}{2} - \frac{\varepsilon_1 - \varepsilon_3}{2}$$

$$\varepsilon_{min} = \varepsilon_3$$

$$\sigma_{max}, \sigma_{min} = \frac{E(\varepsilon_1 + \varepsilon_3)}{2(1-\mu)} \pm \frac{E}{\sqrt{2}(1+\mu)}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{1/2}$$

$$\sigma_{max} = \frac{E[\varepsilon_{max} + \mu\varepsilon_{min}]}{(1-\mu^2)}$$

$$\sigma_{min} = \frac{E[\varepsilon_{min} + \mu\varepsilon_{max}]}{(1-\mu^2)}$$

$$\tau_{max} = \frac{E}{\sqrt{2}(1+\mu)}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{1/2}$$

$$\tau_{max} = \frac{E(\varepsilon_{max} - \varepsilon_{min})}{\sqrt{2}(1+\mu)}$$

10-11

$$\varepsilon_{max} = \frac{400 + 84 - 250}{3} + \frac{\sqrt{2}}{3}[(400-84)^2 + (84+250)^2 + (-250-400)^2]^{1/2}$$

$$= 78 + 375.3$$

$$\varepsilon_{max} = 453.3\mu \text{ in./in.}$$

$$\varepsilon_{min} = 78 - 375.3 = -297.3\mu \text{ in./in.}$$

$$\sigma_{max} = \frac{(29\times10^6)(234\times10^{-6})}{3(1-0.3)} + \frac{1.412(29\times10^6)}{3(1+0.3)}[795\times10^{-6}]$$

$$= 3230 + 8347$$

$$\sigma_{max} = 11{,}580 \text{ psi}$$

$$\sigma_{min} = -5120 \text{ psi}$$

$$\tau_{max} = 8350 \text{ psi}$$

$$\tan 2\theta = \frac{\sqrt{3}(\varepsilon_3 - \varepsilon_2)}{2\varepsilon_1 - \varepsilon_2 - \varepsilon_3} = \frac{1.73(-250-84)}{2(400) - 84 + 250} = -\frac{577}{966}$$

$$\tan 2\theta = -0.598$$

$$\theta = 164.55^\circ$$

10-12

If $\varepsilon_1 = \varepsilon_{\max}$ $\qquad \theta = 0$

Similar to problem 10-10

$$\varepsilon_{\max} = \varepsilon_1$$

$$\varepsilon_{\min} = \frac{4\varepsilon_2 - \varepsilon_1}{3}$$

10-13

$$\tan 2\theta = -0.598 = \frac{2\varepsilon_2 - \varepsilon_1 - \varepsilon_3}{\varepsilon_1 - \varepsilon_3}$$

$$\varepsilon_2 = 0.201\varepsilon_1 + 0.799\varepsilon_3$$

$$\varepsilon_{1\,\text{rect}} = \varepsilon_{1\,\text{delta}} = 400\mu \text{ in./in.}$$

$$\tau_{\max} = \frac{E}{\sqrt{2}(1+\mu)}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{1/2}$$

$$8350 = \frac{29 \times 10^6}{(1.412)(1.3)}[(319.5 \times 10^{-6} - 0.799\varepsilon_3)^2 + (80.5 \times 10^{-6} - 0.201\varepsilon_3)^2]^{1/2}$$

$$\varepsilon_3^2 - 797 \times 10^{-6}\varepsilon_3 - 252{,}000 \times 10^{-12} = 0$$

$$\varepsilon_3 = -241.5\mu \text{ in./in.}$$

$$\varepsilon_2 = -112.5\mu \text{ in./in.}$$

10-16

$$\varepsilon_{\max}, \varepsilon = \frac{563 - 480}{2} \pm \frac{1}{1.412}[(563 + 155)^2 + (-155 + 480)^2]^{1/2}$$

$$= 41.5 \pm 560$$

$$\varepsilon_{\max} = 601.5\mu \text{ in./in.}$$

$$\varepsilon_{\min} = -518.5\mu \text{ in./in.}$$

$$\sigma_{\max}, \sigma_{\min} = \frac{(29 \times 10^6)(41.5 \times 10^{-6})}{0.7} \pm \frac{(29 \times 10^6)(560 \times 10^{-6})}{1.3}$$

$$\sigma_{\max} = 14{,}220 \text{ psi}$$

$$\sigma_{\min} = -10{,}780 \text{ psi}$$

$\tau_{\max} = 12{,}500$ psi $\qquad \theta = 169.7^\circ$

10-17

$F = 1.90 \qquad \mu = \dfrac{1.90 - 1}{2} = 0.45$

$\varepsilon = \dfrac{\frac{F}{A}}{E} \qquad E = 1.95 \times 10^{11}\ \text{N/m}^2$

$$F_{\max} = (2000 \times 10^{-6})(1.95 \times 10^{11})(1.6)(50)(10^{-6})$$

$$= 31{,}200 \text{ N}$$

Lower limit depends on accuracy desired.

10-18

$R_1 = 110$ ohms, $R_2 = R_3 = 100$ ohms, $R_g = 750$ ohms

$F = 2.0, \varepsilon = 300\mu$ in./in. $\quad E = 6$ volts

$$\Delta Ig = \frac{E}{C} R_3 R_1 F \varepsilon$$

$$C = R_1R_2R_4 + R_1R_3R_4 + R_1R_2R_3 + R_2R_3R_4 + R_g(R_1 + R_4)(R_2 + R_3)$$

$$R_4 = \frac{R_1R_3}{R_2} = 110 \text{ ohms} \qquad C = 4.95 \times 10^6 \text{ ohms}$$

$$\Delta Ig = 8.0\mu \text{ amps}$$

$$\frac{\Delta E_D}{E} = \frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_4} - \frac{R_2}{R_2 + R_3} = \frac{1 + A\frac{R_1}{R_1}}{1 + \frac{\Delta R_1}{R_1} + \frac{R_4}{R_1}} - \frac{R_2}{R_2 + R_3}$$

$$\frac{\Delta R_1}{R_1} = F\varepsilon = (2.0)(300 \times 10^{-6}) = 6.0 \times 10^{-4}$$

$$\Delta E_D = E(1.5 \times 10^{-4}) \rightarrow \Delta E_D = 900\mu \text{ volts}$$

10-19

$\varepsilon = 400\mu$ in./in.; $E = 29 \times 10^6$ psi, σ_a = axial stress

$$\sigma_a = \varepsilon E = (29 \times 10^6)(400 \times 10^{-6}) \rightarrow \sigma_a = 1.16 \times 10^4 \text{ psi}$$

10-20

$$G = 11.5 \times 10^6 \text{ psi} = 7.93 \times 10^{10} \text{ N/m}^2$$

a.
$$\phi = \frac{2LM}{\pi G\left(r_0^4 - r_i^4\right)} = \frac{(2)(0.15)(22.6)}{\pi(7.93 \times 10^{10})(0.032^4 - 0.025^4)}$$
$$= 4.136 \times 10^{-5} \text{ rad}$$

b.
$$E_{45^\circ} = \pm\frac{Mr_0}{\pi G\left(r_0^4 - r_i^4\right)} = 4.412\mu \text{ m/m}$$

10-21

$T = 540$ N-m $\qquad N = 3000$ rpm

$$P = 2\pi TN = (2\pi)(540)(3000)$$
$$= 1.0179 \times 10^7 \text{ Nm/min}$$
$$= 169{,}649 \text{ N} \cdot \text{m/s} = \text{Watts}$$
$$= 227.4 \text{ HP}$$

10-24

$R_2 = R_3 = 110\,\Omega \qquad R_g = 70\,\Omega$

$F = 1.8 \qquad \mu = 350\,\mu\text{m/m} \qquad E = 4.0\,\text{V}$

$R_1 = 120\,\Omega = R_x$

$c = 9.768 \times 10^6$

$$\frac{\Delta Ig}{\varepsilon} = \frac{(4.0)(110)(120)(1.8)}{9.768 \times 10^c} = 9.73 \times 10^{-3} \text{ A/in.}$$

$$\Delta Ig = (9.73 \times 10^{-3})(350 \times 10^{-6}) = 3.41 \times 10^{-6}\text{A}$$

$$\frac{\Delta R_1}{R_1} = F\varepsilon = (1.8)(350 \times 10^{-6}) = 6.3 \times 10^{-4}$$

$$\frac{\Delta E_D}{E} = \frac{\frac{\Delta R_1}{R_1}}{4 + 2\left(\frac{\Delta R_1}{R_1}\right)} = 1.575 \times 10^{-4}$$

$$\Delta E_D = 6.298 \times 10^{-4}\text{V}$$

$$\sigma = (350 \times 10^{-6})(30 \times 10^{6}) = 10{,}500 \text{ psi}$$

10-25

$N = 5000$ rpm $\quad P = 5$ hp $\quad E = 28.6 \times 10^6$

$\mu = 0.3 \quad d = 10° \quad G = \frac{28.3 \times 10^6}{2(1.3)} = 1.088 \times 10^7$

$$5 \text{ hp} = \frac{2\pi T(5000)}{33{,}000}; \qquad T = 5.25 \text{ ft-lb}_f = 63.03 \text{ in lb}_f$$

$$63.03 = \frac{\pi(1.08 \times 10^7) r_0^4 \left(\frac{10}{360}\right)(2\pi)}{2L}$$

$$\frac{r_0^4}{L} = 2.11 \times 10^{-5}$$

For $L = 1$ in. $\quad r_0 = 0.0678$ in.

$L = 3$ in. $\quad r_0 = 0.0892$ in.

10-26

$r_0 = 6.25 \text{ mm} = 0.246$ in.

$r_0 - r_i = 0.8 \text{ mm} = 0.0315$ in.

$r_i = 0.2145$ in.

$$63.03 = \frac{\pi(1.088 \times 10^7)(0.246^4 - 0.2145^4)\left(\frac{10}{360}\right)(2\pi)}{2L}$$

$L = 73.1$ in.

10-27

$m = 150\ g \quad \rho_s = 8490 \quad \rho_u = 100 \quad \rho_a = 1.17$

$$1 + \frac{(1.17)(8490 - 100)}{(8490)(100) - 1.17} = 1.01562$$

1.562% high

10-28

$R = 120\,\Omega \quad F = 2.0$

$R_g = 75\,\Omega \quad E = 3.7\,V$

$$\frac{\Delta I_g}{\varepsilon} = \frac{(3.7)(120)^2(2.0)}{(4)(120)^3 + (75)(120)^2}$$

$$= 0.133 \times 10^{-3}\ \mu\text{A}/\mu\text{in.}$$

10-29

$\varepsilon = 2.0$ μin./in.

$$\frac{\Delta E_D}{E} = \frac{\frac{\Delta R_1}{R_1}}{4 + 2\left(\frac{\Delta R_1}{R_1}\right)}$$

$$\frac{\Delta R_1}{R_1} = F\varepsilon = (2.0)(2.0 \times 10^{-6}) = 4.0 \times 10^{-6}$$

$$\frac{\Delta ED}{E} = \frac{4 \times 10^{-6}}{4 + 8 \times 10^{-6}} = 1 \times 10^{-6}$$

$$\Delta E_D = (3.7)(1 \times 10^{-6}) = 3.7\ \mu\text{V}$$

10-30

$d = 1.6$ mm $\qquad r = 0.8$ mm $= 0.0315$ in.

$E = 28.3 \times 10^6$

$y = 1.0$ cm $= 0.394$ in.

$W_F = 1\%$

$$I = \frac{\pi r^4}{4} = \frac{\pi(0.0315)^4}{4} = 7.73 \times 10^{-7}$$

$$FL^3 = (3)(28.3 \times 10^6)(7.73 \times 10^{-7})(0.394)$$

$$= 25.85\ \text{lb}_\text{f}\text{-in}^3$$

For $L = 1$ in. $\quad F = 25.8\ \text{lb}_\text{f}$

$L = 2$ in. $\quad F = 2.35\ \text{lb}_\text{f}$

$L = 5$ in. $\quad F = 1.73\ \text{lb}_\text{f}$

10-31

$F = 200\ \text{lb}_\text{f} \qquad L = 70$ cm $= 27.56$ in.

$d = \frac{1}{16}$ in.

$$200 = \frac{\pi\left(\frac{1}{16}\right)^2(28.3 \times 10^6)}{(4)(27.66)}y$$

$y = 0.0635$ in.

10-32

$F = 0.47 \qquad R = 120\ \Omega$

$\varepsilon = 2 \times 10^{-6}$

$$\frac{\Delta R_1}{R_1} = (0.47)(2 \times 10^{-6}) = 0.94 \times 10^{-6}$$

$$\frac{\Delta E_D}{E} = \frac{0.94 \times 10^{-6}}{4 + 2(0.94 \times 10^{-6})} = 2.34 \times 10^{-7}$$

$$\Delta E_D = (3.7)(2.34 \times 10^{-7}) = 0.869\ \mu\text{V}$$

10-33

$\varepsilon = 300\ \mu\text{m/m} = 300\ \mu\ \text{in./in.}$

$$\sigma = \varepsilon E = (300 \times 10^{-6})(28.3 \times 10^{-6})$$
$$= 8490 \text{ psi}$$

10-34

$$1 = \frac{2\pi T(10,000)}{33,000} \qquad T = 0.525 \text{ lb}_f\text{-ft}$$
$$= 6.303\text{E lb}_f\text{-in.}$$

For $L = 15 \text{ cm} = 5.91 \text{ in.}, F = 1.067 \text{ lb}_f$

10-36

$R_1 = R_2 = R_3 = R_4 = 120\ \Omega \qquad F = 1.9$

$E = 4.5\text{ V} \qquad R_D = 350\ \Omega$

Denominator of Equation (10.36)

$$c = (4)(120)^3 + (350)(240)^2 = 2.707 \times 10^7$$

$$\frac{\Delta I_D}{\varepsilon} = \frac{E}{c} R_1 R_3 F = \frac{(4.5)(120)^2(29)}{2.707 \times 10^7} = 4.548 \times 10^{-3} = 4.548 \times 10^{-3}\ \mu A/\mu\text{in.}$$

10-37

$R_1 = R_2 = R_3 = R_4 = 120\ \Omega$

$$\frac{\Delta E_D}{E} = \frac{\frac{\Delta R_1}{R_1}}{4 + 2\left(\frac{\Delta R_1}{R_1}\right)}$$

$$\frac{\Delta R_1}{R_1} = F\varepsilon = (1.9)(3.1 \times 10^{-6}) = 5.89 \times 10^{-6}$$

$$\frac{\Delta E_D}{E} = \frac{5.89 \times 10^{-6}}{4 + (2)(5.89 \times 10^{-6})^2} = 1.472 \times 10^{-6}$$

$$\Delta E_D = (4.5)(1.472 \times 10^{-6}) = 5.89\ \mu\text{V}$$

10-38

$\Delta L = 10\mu\text{m} \qquad L = 50 \text{ cm} \qquad d = 2 \text{ cm}$

$$\varepsilon = \frac{10\ \mu\text{m}}{0.5} = 20\ \mu\text{m/m} = 20\ \mu\text{in./in.}$$

$$\sigma = \varepsilon E = (20 \times 10^{-6})(28.3 \times 10^6) = 566 \text{ psi}$$

$$T = (566)\frac{\pi}{4}\left(\frac{2}{2.54}\right)^2 = 276 \text{ lb}_f = 1228 \text{ N}$$

$$\frac{\Delta R}{R} = (1.9)(20 \times 10^{-6}) = 3.8 \times 10^{-5}$$

$$\frac{\Delta E_D}{E} = \frac{3.8 \times 10^{-5}}{4 + (2)(3.8 \times 10^{-5})} = 9.5 \times 10^{-6}$$

$$\Delta E_D = (4.5)(9.5 \times 10^{-6}) = 4.27 \times 10^{-5} \text{ V}$$

10-39

$r_i = 0.005$ m $\quad r_0 = 0.0065$ m $\quad L = 0.05$ m

$\phi = 10° = 0.1745$ rad

$$M = \frac{\pi G\left(r_0^4 - r_i^4\right)\phi}{2L}$$

$$\frac{M}{G} = 9.46 \times 10^{-5}$$

10-40

$\rho_a = 1.205\ \text{kg/m}^3 \quad \rho_u = 21{,}380\ \text{kg/m}^3$

$\rho_5 = 5490\ \text{kg/m}^3$

$$\text{Error (troy oz)} = (15)\left[\frac{(1.205)(8490 - 21{,}380)}{(8490)(21{,}380 - 1.205)}\right]$$

$$= 0.00128 \text{ oz}$$

$$\text{Dollar value} = (0.00128)(400)$$

$$= \$0.51$$

10-41

$L = 0.18,\ M = 30,\ r_i = 0.026$

$r_o = 0.028 \quad G = 79\ \text{GN/m}^2$

Eq. (10.l8):

$$30 = \pi(79 \times 10^9)(0.028^4 - 0.026^4)\phi/(2)(0.18)$$

$$\phi = 2.76 \times 10^{-4} \text{ rad}$$

10-42

$$375 = 2\pi T(4000)/33{,}000$$

$$T = 492 \text{ ft-lb}$$

10-43

$F = 3EIy/L^3$

$I = \pi r^4/4$

$E = 28.3 \times 10^6 \text{ psi} = 195\ \text{GN/m}^2$

$L = 0.059$ m

$y = 0.01$

$$F = (3)(195 \times 10^9)\pi(0.01)(0.001)^4/(4)(0.059)^3$$

$$= 22.4 \text{ N}$$

10-44

$F = AEy/L$

$$800 = \pi(0.0004)^2 y(195 \times 10^9)/1.0$$

$$y = 0.00816 \text{ m} = 8.16 \text{ mm}$$

10-45

$\rho_u = 1.5 \text{ lbm/ft}^3 = 24.03 \text{ kg/m}^3$

$\rho_s = 8490 \text{ kg/m}^3$

$\rho_a = 1.205 \text{ kg/m}^3$

$W_{\text{ind}}/W_{\text{true}} = 1 + (1.205)(8490 - 24)/(8490)(24 - 1.205)$
$= 1.0527 = 5.27\ \%$ error

$W_{\text{true}} = (24.03)(200 \times 10^{-6}) = 0.0048 \text{ kg}$

10-46

$\rho_s = 2675 \text{ kg/m}^3$

$W_{\text{ind}}/W_{\text{true}} = 1 + (1.205)(2675 - 24)/(2675)(24 - 1.205)$
$= 1.0524 = 5.24\ \%$ error

Chapter 11

11-1

$L = 15 \text{ cm} \pm 0.5 \text{ mm} \qquad x = 5.6 \text{ cm} \pm 1.3 \text{ mm}$

$t = 2.5 \text{ cm} \pm 0.2 \text{ mm}$

$$a = x\tan\left[\tan^{-1}\left(\frac{t}{2L}\right)\right] = 5.6\tan\left[\tan^{-1}\left(\frac{1.25}{15}\right)\right]$$

$a = 0.4667 \text{ cm}$

$$W_a = \left\{\left[\frac{\partial a}{\partial x}W_x\right]^2 + \left[\frac{\partial a}{\partial t}W_t\right]^2 + \left[\frac{\partial a}{\partial L}W_L\right]\right\}^{1/2}$$

$= 0.0117 \text{ cm}$

$= 2.5\%$

11-2

$$\omega_n = 11.0\sqrt{\frac{EI}{mL^4}} \text{ Hz} = 300 \pm 2$$

at $L = 0.056 \text{ m} \pm 0.2 \text{ mm}$

$$\frac{EI}{m} = \left(\frac{300}{11}\right)^2 (0.056)^4 = 7.315 \times 10^{-3} \text{ m}^4$$

at $L = 0.10 \text{ m} \pm 0.5 \text{ mm}$

$$\omega_n = 11.0\left[\frac{7.315 \times 10^{-3}}{(0.1)^4}\right]^{1/2} = 94.08 \text{ Hz}$$

$$W_{EI/m} = \left[\left(\frac{\partial C}{\partial \omega_n}W_{\omega_n}\right)^2 + \left(\frac{\partial c}{\partial L}W_L\right)^2\right]^{1/2}$$

$$= \left[\left(\frac{2\omega_n L^4}{11^2}W_{\omega_n}\right)^2 + \left(\frac{4L^3\omega_n^2}{11^2}W_L\right)^2\right]^{1/2}$$

$$= \left\{\left[\frac{(2)(300)(0.056)^4}{11^2}(2)\right]^2 + \left[\frac{(4)(0.056)^3(300)^2}{11^2}(0.0002)\right]^2\right\}^{1/2}$$

$= 1.429 \times 10^{-4}$

at $L = 10 \text{ cm}$

$$W_{\omega_n} = \left[\left(\frac{\partial \omega_m}{\partial c} W_c\right)^2 + \left(\frac{\partial \omega_n}{\partial L} W_L\right)^2\right]^{1/2}$$

$$= \left[\left(\frac{(11.0)\left(\frac{1}{2}\right)c^{-1/2}}{L^2} W_c\right)^2 + (11.0(-2)L^{-3}c^{1/2}W_L)^2\right]^{1/2}$$

$$= \left\{\left[\frac{(11.0)\left(\frac{1}{2}\right)(1.429 \times 10^{-4})}{(0.1)^2(7.315 \times 10^{-3})^{1/2}}\right] + \left[\frac{(11.0)(-2)(7.315 \times 10^{-3})^{1/2}(0.0005)}{(0.1)^3}\right]^2\right\}^{1/2}$$

$= 1.315$ Hz

$=1.4\%$

11-3

$$\omega_n = \frac{11 \cdot 0}{rL^2}\sqrt{\frac{EI}{\rho\pi}} \qquad \text{For } L = 1.0 \rightarrow$$

$$\frac{W_{\omega_n}}{\omega_n} = \left[\left(\frac{\partial \omega_n}{\partial E} W_E\right)^2 + \left(\frac{\partial \omega_n}{\partial r} W_r\right)^2 + \left(\frac{\partial \omega_n}{\partial \rho} W_\rho\right)^2 + \left(\frac{\partial W_n}{\partial L} W_L\right)^2\right]^{1/2}$$

$W_{\omega_n} = 48.10$ Hz or 2.80%

For $L = 4.0$

$W_{\omega_n} = 2.17$ Hz or 2.01%

11-4

$$\omega_n = 11.0\sqrt{\frac{Er^2}{4\rho L^4}} \rightarrow r = \frac{2L^2\omega_n}{11.0\sqrt{\frac{E}{\rho}}}$$

$r = 0.0143$ in. $\rightarrow d = 0.0286$ in.

$$\frac{W_1}{W_2} = \frac{L_2^2}{L_1^2} \rightarrow L_2 = \left[\left(\frac{L_1^2 W_1}{W_2}\right)\right]^{1/2} = 1.58 \text{ in.}$$

$$\frac{W_{\omega_n}}{\omega_n} = \left[\left(\frac{W_E}{2E}\right)^2 + \left(\frac{W_r}{r}\right)^2 + \left(\frac{W_\rho}{2\rho}\right)^2 + \left(\frac{2W_L}{L}\right)^2\right]^{1/2}$$

For $\omega_n = 100$ Hz

$W_{\omega_n} = 23.2$ Hz or 3.69%

For $\omega_n = 1000$ Hz

$W_{\omega_n} = 424.0$ Hz or 6.75%

11-5

$$\frac{\left[(x_2 - x_1)_0\omega_n^2\right]}{a_0} = 0.95$$

$$\Delta x_0 = (x_2 - x_1)_0 = \frac{0.95a_0}{\omega_n^2}$$

$$\omega_{\Delta_{x_0}} = \left\{ \left(\frac{\partial \Delta x_0}{\partial \omega_n} W_{\omega_n} \right)^2 \right\}^{1/2} = \frac{1.90 a_0}{\omega_n^3} W_{\omega_n}$$

$$= \frac{(1.9)(30)}{(200)^3}(2) = 1.425 \times 10^{-5} \text{ m}$$

11-6

Assume $x_1 = x_0 \sin \omega_1 t$

$$v_0 = \left(\frac{dx}{dt} \right)_0 = x_0 \omega_1$$

Solution to D.E.

$$x_2 = x_1 = e - \left(\frac{c}{2m} \right)^t (A \cos \omega t + B \sin \omega t) + \frac{m x_0 \omega_1^2 \cos(\omega_1 t - \phi)}{[(K - m\omega_1^2)^2 + c^2 \omega_1^2]^{1/2}}$$

↓ Transient term dies out ↓ steady state term

$$x_2 - x_1 = \frac{x_0 \left(\frac{\omega_1}{\omega_n} \right)^2 \cos(\omega_1 t - \phi)}{\left\{ \left[1 - \left(\frac{\omega_1}{\omega_n} \right)^2 \right]^2 + \left[2 \left(\frac{c}{c_e} \right) \left(\frac{\omega_1}{\omega_n} \right) \right]^2 \right\}^{1/2}}$$

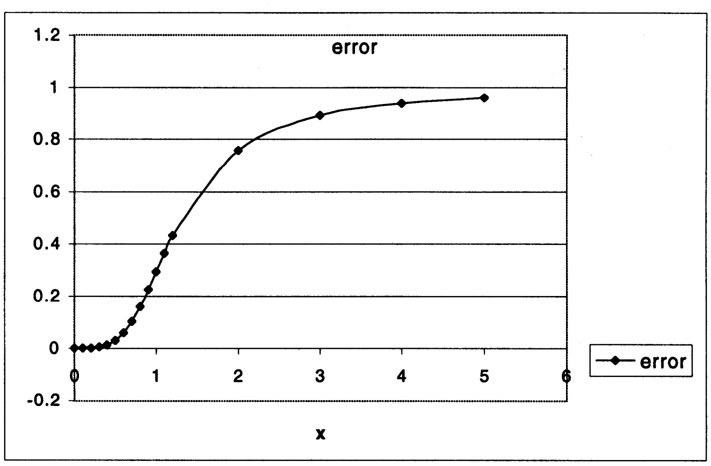
$$(x_2 - x_1)_0 = \frac{v_0 \left(\frac{\omega_1}{\omega_n} \right)}{\omega_n \left\{ \left[1 - \left(\frac{\omega_1}{\omega_n} \right)^2 \right]^2 + \left[2 \left(\frac{c}{c_c} \right) \left(\frac{\omega_1}{\omega_n} \right) \right]^2 \right\}^{1/2}}$$

11-7

$$T = \frac{1}{\omega \left(\frac{c}{c_c} \right)} = 0.001225 \text{ sec}$$

11-8

w/wn	error
0	0
0.1	4.99963E-05
0.2	0.000799041
0.3	0.004025561
0.4	0.012559368
0.5	0.0298575
0.6	0.059112588
0.7	0.102009698
0.8	0.157728599
0.9	0.222936122
1	0.292893219
1.1	0.362953938
1.2	0.429604191
2	0.757464375
3	0.889568474
4	0.937621714
5	0.960031962



11-9

$$(x_2 - x_1)_0 = \frac{a_0}{\omega_n^2 \left\{ \left[1 - \left(\frac{\omega_1}{\omega_n} \right)^2 \right]^2 + \left[2 \left(\frac{c}{c_c} \right) \left(\frac{\omega_1}{\omega_n} \right) \right]^2 \right\}^{1/2}}$$

For $\frac{(x_2 - x_1)_0}{x_0} = 0.99, \frac{c}{c_c} = 0.707$

$\frac{\omega_1}{\omega_n} = 3.16 \qquad k = 2.9\ \text{kN/m} \qquad m = 45\ \text{kg}$

$$\omega_n = \sqrt{\frac{k}{m}} = \left(\frac{2900}{45} \right)^{1/2} = 8.028\ \text{rad/sec}$$

$$a_0 = (2.5 \times 10^{-3})(8.028)^2 \{[1 - 3.16^2]^2 + [2(0.707)(3.16)]^2\}^{1/2}$$
$$= 1.617\ \text{m/sec}^2$$

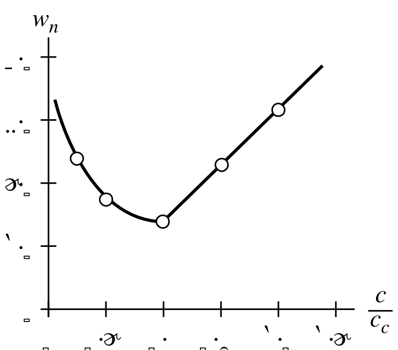
11-10

$T = \frac{1}{\omega_n \left(\frac{c}{C_c} \right)} = \omega_1$ since $T = \omega_1$ for $\frac{(x_2 - x_1)_0 \omega_n^2}{a_0} = 0.99$

$$0.99 = \frac{1}{\left\{ \left[1 - \left(\frac{\omega_1}{\omega_n} \right)^2 \right]^2 + \left[2 \left(\frac{c}{c_c} \right) \left(\frac{\omega_1}{\omega_n} \right) \right]^2 \right\}^{1/2}}$$

$$\left(\frac{\omega_1}{\omega_n} \right)^2 = \frac{-2 \left[2 \left(\frac{c}{c_c} \right)^2 - 1 \right] \pm \sqrt{4 \left[2 \left(\frac{c}{c_c} \right)^2 - 1 \right]^2 + 0.08}}{2}$$

$$\omega_n = \left\{ \frac{1}{\left(\frac{c}{c_c} \right)^2 \left[1 - 2 \left(\frac{c}{c_c} \right)^2 \pm 2 \sqrt{\left(\frac{c}{c_c} \right)^4 - \left(\frac{c}{c_c} \right)^2 + 0.255} \right]} \right\}^{1/4}$$



$\frac{c}{c_c}$	ω_n
0	∞
1	3.16
2	imaginary
$\frac{1}{4}$	1.739
$\frac{1}{2}$	1.407
$\frac{3}{4}$	2.29
$\frac{1}{8}$	2.39
$\frac{5}{4}$	$\to\infty$

11-11

$$\phi = \tan^{-1}\frac{2\left(\frac{c}{c_c}\right)\left(\frac{\omega_1}{\omega_n}\right)}{1-\left(\frac{\omega_1}{\omega_n}\right)}$$

$$= \tan^{-1}\frac{2(0.7)(2.47)}{1-(2.47)^2}$$

$$\phi = 145.8°$$

11-12

For maximum acceleration $\frac{\omega_1}{\omega_n} = 0$

$$\phi = \tan^{-1}\frac{2\left(\frac{c}{c_c}\right)\left(\frac{\omega_1}{\omega_n}\right)}{1-\left(\frac{\omega_1}{\omega_n}\right)^2}$$

$$= \tan^{-1}0$$

$$\phi = 0$$

11-13

For 100 db

$$100 = 10\log_{10}\frac{I}{10^{-16}}$$

$$I = 10^{-6}\ \text{watt/cm}^2$$

$$100 = 20\log_{10}\frac{p}{2.9\times10^{-9}}$$

$$p = 2.9\times10^{-4}\ \text{psi}$$

11-14

$$I_{db} = 10\log_{10}\frac{I}{10^{-16}} \to I = I_0 10^{Idb/10}$$

$$W_I = \frac{\partial I}{\partial I_{db}}W_{Idb} = I_0(10^{Idb/10})(0.1)W_{Idb}$$

$$\%W_I = 10(\ln 10)W_{Idb}\%$$

W_{Idb} ↓	I_{db} →	50	80	120	$\%W_I$
± 1		2.3×10^{-12}	2.3×10^{-9}	2.3×10^{-5}	23.0%
± 2		4.6×10^{-12}	4.6×10^{-9}	4.6×10^{-5}	46.0%
± 3		6.9×10^{-12}	6.9×10^{-9}	6.9×10^{-5}	69.0%

11-16
From Figure 11-10:

a. Two 50 db levels
Difference $= 0$ db
Amount added $= 3$ db
Total sound level $= 53$ db

b. Three 50 db sources
53 and 50 db combination
Difference $= 3$ db
Amount added $= 1.8$ db
Total sound level $= 54.8$ db

c. Four 50 db sources, 54.8 and 50 db combination
Difference $= 4.8$ db
Amount added $= 1.3$ db
Total sound level $= 56.1$ db

11-17
From Figure 11-10:
63 db an d73 db combination
Difference $= 10$ db
Amount added $= 0.4$ db
Total sound intensity $= 73.4$ db

11-18

I. 58 db and 62 db levels
Difference $= 4$ db
Amount added $= 1.5$ db
Equivalent level $= 63.5$ db

63.5 db and 65 db combination
Difference $= 1.5$ db
Amount added $= 2.3$ db
Equivalent level $= 67.3$ db

67.3 db and 70 db combination
Difference $= 2.7$ db
Amount added $= 1.9$ db
Total sound intensity $= 71.9$ db

II. 58 db and 65 db levels
Difference $= 7$ db
Amount added $= 0.8$ db
Equivalent level $= 65.8$ db

62 db and 65.8 db combination
Difference $= 3.8$ db
Amount added $= 1.5$ db

Equivalent level $= 67.3$ db

67.3 and 70 db combination
Difference $= 2.7$ db
Amount added $= 1.9$ db
Total sound intensity $= 71.9$ db

11-19

a. $60 = 20\log\dfrac{P}{2.0\times 10^{-4}} \rightarrow P = 0.20 \text{ dyne/cm}^2$

$-55 = 20\log\dfrac{E}{1} \rightarrow E = 1.775\times 10^{-3}$

$E = (0.20)(1.775\times 10^{-3}) = 3.55\times 10^{-4}$ volts

$E_{\text{output}} = 0.355$ mv

b. $80 = 20\log\dfrac{P}{2.0\times 10^{-4}} \rightarrow P = 2.0 \text{ dyne/cm}^2$

$-55 = 20\log\dfrac{E}{1} \rightarrow E = 1.775\times 10^{-3}$

$E_{\text{output}} = 3.55$ mv

11-20

$-55 \text{ db} - (-70 \text{ db}) = 15$ db

amplification factor $= 15$ db

11-21

a. Figure 11-16:

1-in. material $\rightarrow NRC = \dfrac{(28 + 59 + 80 + 78)}{4} = 61.25$

2-in. material $\rightarrow NRCl = \dfrac{(50 + 87 + 92 + 87)}{4} = 79.00$

b. Figure 11-17:

Material A $\rightarrow NRC = \dfrac{(28 + 57 + 80 + 79)}{4} = 61.00$

Material B $\rightarrow NRC = \dfrac{(29 + 25 + 22 + 19)}{4} = 23.75$

Material C $\rightarrow NRC = \dfrac{(18 + 12 + 9 + 8)}{4} = 11.75$

11-23

Figure 11-12: {1000 Hz $\rightarrow$ Intensity $= 70$ db} $\rightarrow$ at 70 phon

a. {at 70 phon and 50 Hz} $\rightarrow$ intensity $= 86$ db

b. {at 70 phon and 100 Hz} $\rightarrow$ intensity $= 76$ db

c. {at 70 phon and 10,000 Hz} $\rightarrow$ intensity $= 74$ db

11-24

Figure 11-12:

$\langle 80 \text{ db and } 1000 \text{ Hz}\rangle \rightarrow 80$ phon

$\langle 45 \text{ db and } 1000 \text{ Hz}\rangle \rightarrow 45$ phon

a. at 60 Hz and 45 phon $\rightarrow$ 65 db
at 60 Hz and 80 phon $\rightarrow$ 91 db
$[(65 - 45) - (91 - 80)]$ db $= +9$ db over what it was at 80.

b. at 100 Hz and 45 phon $\rightarrow$ 55 db
at 100 Hz and 80 phon $\rightarrow$ 85 db
$[(55 - 45) - (85 - 80)]$ db $= +5$ db over what it was at 80.

11-27

63, 78, 89, 92 dB

$63 + 78 = 78 + 0.1 = 78.1$

$89 + 92 = 92 + 1.8 = 93.8$

$78.1 + 93.8 = 93.8 + 0.1 = 93.9$ dB

$63 + 89 = 89$

$78 + 92 = 92 + 0.2 = 92.2$

$89 + 92.2 = 92.2 + 1.7 = 93.9$

11-29

$$100 = 20\log\frac{p}{2\times10^{-4}} \qquad p = 20 \text{ dyn/cm}^2$$

$$120 = 20\log\frac{p}{2\times10^{-4}} \qquad p = 200 \text{ dyn/cm}^2$$

$$-40 \text{ db} = 20\log\frac{E}{1.0} \qquad E = 10^{-2} \text{ V}$$

at 100 dB SPL $\quad E = (10^{-2})(20) = 0.2$ V

at 120 dB SPL $\quad E = (10^{-2})(200) = 2.0$ V

11-30

$$\text{dB atten} = 10\log\frac{I}{I_0} = 10\log(1-\alpha)$$

Hz	α, %	dB
75–150	0.35	–1.87
150–300	0.32	–1.61
300–600	0.25	–1.25
600–1200	0.21	–1.02
1200–2400	0.19	–0.92
2400–4800	0.18	–0.86
>48000	0.18	–0.86

11-31

$75 + 89$ dB

Diff $= 14$ dB

Amount added + larger $= 0.2$

Total $= 89.2$ dB

$89.2 + 100$ dB

Diff $= 10.8$

Amount added $= 0.4$

Total $= 100.4$ dB

11-32

$$100 \text{ dB} = 20 \log \frac{p}{2 \times 10^{-5}}$$
$$p = 2.0 \text{ Pa}$$
$$110 \text{ dB} = 20 \log \frac{p}{2 \times 10^{-5}}$$
$$p = 6.32 \text{ Pa}$$
$$130 \text{ dB} = 20 \log \frac{p}{2 \times 10^{-5}}$$
$$p = 63.2 \text{ Pa}$$

11-34

One de-emphasizes low frequencies

11-35

$r = 1.0$ m $\quad P = 1$ W

$$A = \frac{1}{2}(4\pi r^2) = 6.283 \text{ m}^2$$
$$I = \frac{1 \text{ W}}{6.283} = 0.159 \text{ W/m}^2$$
$$\text{dB} = 10 \log\left(\frac{I}{10^{-12}}\right)$$
$$= 10 \log\left(\frac{0.159}{10^{-12}}\right)$$
$$= 112 \text{ dB}$$

11-36

$\omega_n = 500$ Hz $\quad d = 0.8$ mm

$$I = \frac{\pi r^4}{4} = \frac{\pi\left[\frac{0.8}{25.4}\right]^4}{4} = 9.08 \times 10^{-7}$$
$$m = \rho \pi r^2 = \frac{(489)\pi\left(\frac{0.8}{25.4}\right)^2}{144}\left(\frac{1}{12}\right)$$
$$= 8.82 \times 10^{-4}$$

$$500 = \left[\frac{(28.3 \times 10^6)(9.08 \times 10^{-7})}{(8.82 \times 10^{-4})L^4}\right]^{1/2} = \frac{170.7}{L^2}$$

$L = 0.584 \text{ in.} = 1.48 \text{ cm}$

11-37

dB = 90 $\quad P = 1$ W

$$90 = 10\log\left(\frac{I}{10^{-12}}\right) \qquad I = 0.00 \text{ W/m}^2$$

$$A = \frac{1}{2}(4\pi r^2) = 6.283 \text{ m}^2$$

$P(\text{sound}) = (6.283)(0.001) = 0.00628$ W

$$\frac{\text{sound power}}{\text{elec power}} = \frac{0.00628}{1} = 0.00628$$

11-38

Measure at 90 dB

Sens. = −60 dV $\quad$ ref 1 V

$$90 = 20\log\frac{P}{2 \times 10^{-4}}$$

$$P = 6.234 \text{ dyn/cm}^2$$

$$-60 \text{ dB} = 20\log\frac{E}{1\text{ V}}$$

$$E = 0.001 \text{ V} = 1 \text{ mV}$$

11-40

90 dB

$$I = \frac{1}{2}\rho a c \omega^2 \xi_m^2$$

$$90 = 20\log\frac{I}{10^{-12}}$$

$$I = 3.16 \times 10^{-8} \text{ W/m}^2$$

$\rho_a = 1.204 \text{ kg/m}^3 \qquad c = 20\sqrt{293} = 342 \text{ m/s}$

at $\omega = 100$ Hz $= 628$ rad/s

$$3.16 \times 10^{-8} = (0.5)(1.204)(342)(628)^2 \xi_m^2$$

$$\xi m = 1.97 \times 10^{-8}\ m$$

At 1000 Hz $\xi_m = 1.972 \times 10^{-9}$ m

At 10,000 Hz $\xi_m = 1.97 \times 10^{-10}$ m

11-41

$$3.16 \times 10^{-8} = 0.5(1.204)(342)\xi_m^2$$

$$\xi_m = 1.24 \times 10^{-5} \text{ m/s}$$

11-44

90 + 100 dB

dB(tot) − 90 = 10.3

dB(tot) = 10.3 + 90 = 100.3 dB

11-46

160 dB $\omega = 300$ and 1000 Hz

$160 = 10\log\frac{I}{10^{-12}}$ $I = 10^4 \text{ W/m}^2$

$\rho_a = 1.204 \text{ kg/m}^3$ $c = 342 \text{ m/s}$

At $\omega = 300 \text{ Hz} = 1884 \text{ rad/s}$

$10^4 = (1.204)(342)\xi_{\text{rms}}^2$ $\xi_{\text{rm}} = 4.93 \text{ m/s}$

$10^4 = \frac{p^2_{\text{rms}}}{(1.204)(342)}$ $p_{\text{rms}} = 2029 \text{ Pa}$

$10^4 = (0.5)(1.204)(342)(1884)^2\xi_m^2$

$\xi_m = 3.7 \times 10^{-3} \text{ m}$

$10^4 = (0.5)(1.204)(342)\xi_m^2$ $\xi_m = 6.97 \text{ m/s}$

At 1000 Hz

$p_{\text{rms}} = 2029 \text{ Pa}$

$\xi_m = 1.11 \times 10^{-3} \text{ m}$

$\xi_m = 6.97 \text{ m/s}$

11-47

1000 Hz 100 dB reduce to 90 dB

$\alpha = \frac{\text{sound absorbed}}{\text{incident sound}}$

At 100dB $I = 10^{-2}$

90 dB $I = 10^{-3}$

$\alpha = 1 - \frac{10^{-3}}{10^{-2}} = 0.9$

Thickness ~1.8 in.

11-48

$\alpha_A = 80\%$ $\alpha_B = 22\%$ $\alpha_c = 9\%$

dB incident = 90 dB $I = 10^{-3} \text{ W/m}^2$

(A) $0.8 = 1 - \frac{I_2}{10^{-3}}; I_2 = 2 \times 10^{-4} = 83.01 \text{ dB}$

$\text{Atten}_A = 90 - 83.01 = 6.99 \text{ dB}$

(B) $I_2 = 88.921 \text{ dB}$ Atten = 90 − 88.921 = 1.079 dB

(C) $I_2 = 89.59 \text{ dB}$ Atten = 0.41 dB

11-49

From Figure 11.6, max overshoot at $\frac{c}{c_c} = 0.2$ is 155% at $\frac{\omega_1}{\omega_n} = 1.0$ or $\omega_1 = 60$ Hz.

At $\frac{c}{c_c} = 0.3,$ overshoot is 75% at $\frac{\omega_1}{\omega_n} = 0.93$ or $\omega_1 = 55$ Hz.

11-50

From Figure 11.6 at $\frac{c}{c_c} = 0.707$ attenuation at $\frac{\omega_1}{\omega_n} = 1.0$ is 0.875

$\text{dB} = 10 \log (0.875) = -0.58 \text{ dB}$

11-51

$\pm 1 \text{ dB} = 10 \log (\text{ratio})$

Acceleration ratio $= 0.794$ to 1.259

$\frac{c}{c_c}$ from 0.2 to 0.3

11-52

$\frac{c}{c_c}$ from 0.47 to 0.7

11-53

$$\log\left(\frac{I_1}{I_0}\right) = \frac{82}{10} = 8.2$$

$$\log\left(\frac{I_2}{I_0}\right) = 7.6$$

$$\log\left(\frac{I_3}{I_0}\right) = 9.5$$

$$\log\left(\frac{I_4}{I_0}\right) = 11.1$$

$$I = I_1 + I_2 + I_3 + I_4 = I_0(1.2926 \times 10^{11})$$

$$\log\left(\frac{I}{I_0}\right) = 11.1114$$

$$\text{dB} = 10 \log\left(\frac{I}{I_0}\right) = 111.14$$

11-54

$I_0 = 10^{-16} \text{ W/cm}^2 = 10^{-12} = \text{W/m}^2$

At 80 dB $I = (10^{-12})(10^8) = 10^{-4} \text{ W/m}^2$

At 77 dB $I = (10^{-12})(5.01 \times 10^7) = 5.01 \times 10^{-5} \text{ W/m}^2$

Intensity differs by factor of two.

11-55

SPL $= 110$ dB at 16 Hz

$$110 = 10\log\left(\frac{I}{I_0}\right) \qquad I_0 = 10^{-12}\ \text{W/m}^2$$

$$I = 0.1\ \text{W/m}^2 = \frac{p^2_{\text{rms}}}{\rho_a c} = p_{\text{rms}}\xi_{\text{rms}}\omega$$

$$c = 345\ \text{m/s} \qquad \rho_a = 1.2\ \text{kg/m}^3$$

$$0.1 = \frac{p^2_{\text{rms}}}{(1.2)(345)}$$

$$p_{\text{rms}} = 6.43\ \text{Pa}$$

$$\xi_{\text{rms}} = \frac{0.1}{(6.43)(16)} = 9.72 \times 10^{-4}\ \text{m}$$

11-56

$$I_1 = I_0 \times 10^{9.5} = 3.162 \times 10^9 I_0 = 73.7\%$$

$$I_2 = I_0 \times 10^9 = 1 \times 10^9 I_0 = 23.3\%$$

$$I_3 = I_0 \times 10^{7.8} = 6.31 \times 10^7 I_0 = 1.5\%$$

$$I_5 = I_4 = I_0 \times 10^{7.5} = 3.16 \times 10^7 I_0 = \underline{1.5\%}$$

$$\text{Total} = 4.288 \times 10^9 I_0 = 96.3\ \text{dB}$$

11-57

Output voltage at SPL $= 25$ dB

$$25\ \text{dB} = 20\log\left(\frac{p}{2 \times 10^{-4}}\right)$$

$$p = 3.557 \times 10^{-4}\ \text{dyn/cm}^2$$

$$E = (3.557 \times 10^{-4})(10^{-4}) = 3.557 \times 10^{-8}\ \text{V}$$

$$\frac{S}{N} = -30\ \text{dB} = 20\log\left(\frac{E_N}{E}\right)$$

$$E_N = (3.557 \times 10^{-8})(0.03162) = 1.12 \times 10^{-9}\ \text{V}$$

At SPL $= 100$ dB, $E = 20 \times 10^{-4}$ V from microphone

$E = 0.1$ V from amplifier

$$\frac{E_N}{E} = \frac{1.12 \times 10^{-9}}{0.1} = 1.12 \times 10^{-8}$$

$$\frac{S}{N}\ \text{dB} = 20\log\left(\frac{1}{1.12 \times 10^{-8}}\right) = 159\ \text{dB}$$

11-58

From Exercise 11.2, $\dfrac{\omega_1}{\omega_n} > 2.47$ for $\pm 1\%$ condition. Therefore, for

$\omega_1 > 100$ Hz, ω_n must be $< \dfrac{\omega_1}{2.47} = 40.5$ Hz.

11-59

$-37 = 20\log((E/1.0)$

$E = 0.0141$ V

At SPL $= 70 = 20\log(p/0.0002)$

$p = 0.632$

$E = (0.0141)(0.632) = 0.0089$ V

At SPL $= 90 = 20\log(p/0.0002)$

$p = 6.32$

$E = 0.089$ V

11-60

Loudness of 100 Hz tone at SPL = 80 dB is 75 phon

At Loudness = 75 phon, and 10 kHz, SPL = 79 dB

11-61

60 and 70 dB diff = 10

amt add to larger level = 0.43

combination = 70.43

combination of 80 and 90 = 90.43

combination of 70.43 and 90.43 diff = 20

amt added to lager = 0.1

total = 90.53 db

11-62

$-60 = 20\log(E/1.0)$

$E = 0.001$ V

$85 = 20\log(p/0.0002)$

$p = 3.56$

$E = 0.00356$

$30 = 20\log(p/0.0002)$

$p_{\text{noise}} = 0.00632$

At 85 dB referred to noise level

dB $= 20\log(3.56/0.00632) = 55$ dB

11-63

$E = 28.3 \times 10^6$ psi, $I = \pi r^4/4$

$\rho = 489$ lb/ft^3 $\quad r = 0.5$ mm $= 0.0197$ in

$m = (489)\pi(0.0197)^2/12 = 0.0497$ lbm/in

$I = \pi(0.0197)^4/4 = 1.18 \times 10^{-7}$ in^4

Eq. (11.2)

$600 = (11)[28.3 \times 10^6)(1.18 \times 10^{-7})/(0.0497)L^4]^{1/2}$

$L = 0.387$ in

11-64

Mat A α at 500 Hz $=$ 57%

Mat B α $=$ 25%

Mat C α $=$ 12%

$dB_A = 10\log(1 - 0.8) = -6.99$

$dB_B = 10\log(1 - 0.25) = -1.25$

$dB_C = 10\log(1 - 0.12) = -0.56$

11-65

$0.98 = R^2/[(1 - R^2)^2 + ((2)(0.7)R)^2]^{1/2}$

$R = 4.463464 = (\omega/\omega_n)^2$

$\omega/\omega_n = 2.112$

$\omega_n = 120/2.112 = 56.8$ Hz

Chapter 12

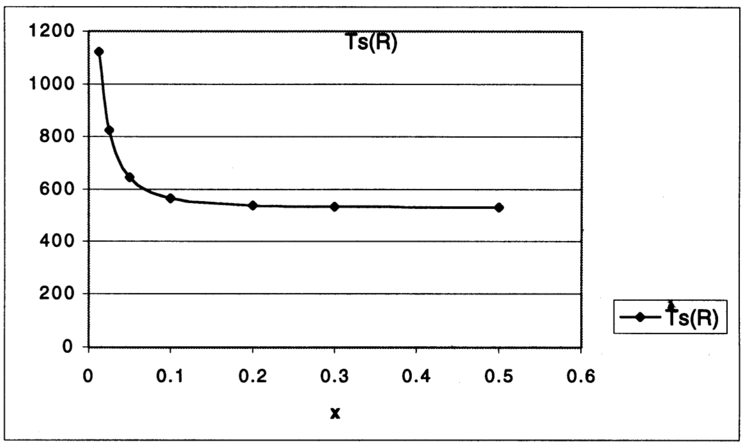
12-1

$$T_s^4 - T_d^4 = \frac{gs^2}{\sigma A}$$

$$T_s^4 - T_d^4 = \frac{\left(6.826 \times 10^{-6}\ \frac{\text{BTU}}{\text{hr}}\right)(36^2)}{\left(0.1714 \times 10^{-8}\ \frac{\text{BTU}}{\text{hr-ft}^2\text{-°R}^4}\right)\left(\pi r_d^2\right)}$$

$$T_S = \left[780 \times 10^8 + (2.370 \times 10^8)\left(\frac{1}{r_d{}^2}\right)\right]^{1/4}$$

rd	Ts(R)
0.0125	1123.768
0.025	822.2929
0.05	644.742
0.1	564.7162
0.2	538.2361
0.3	532.8791
0.5	530.0725



12-2
For $S = 6$ ft

$$T_S = \left[780 \times 10^8 + (9.480 \times 10^8)\left(\frac{1}{r_d^2}\right)\right]^{1/4}$$

For $S = 12$ ft

$$T_S = \left[780 \times 10^8 + (37.95 \times 10^8)\left(\frac{1}{r_d^2}\right)\right]^{1/4}$$

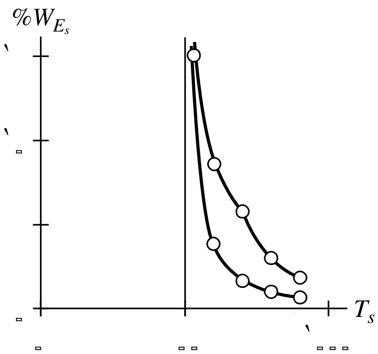
Plot looks like the one in Problem 12-1

For $S = 6$ ft		For $S = 12$ ft	
T_d (in.)	T_S (°R)	T_d (in.)	T_S (°R)
0	∞	0	∞
0.05	820	0.05	1120
01	648	0.1	820
0.2	565	0.2	648
0.3	545	0.3	590
↓	↓	↓	↓
∞	530	∞	530

12-4

$$\varepsilon_S = \frac{7.84\,E}{F_{ts}\sigma\left(T_S^4 - T_R^4\right)}$$

$$\%\,W_{E_s} = \frac{100}{T_s^4 - 530^4}\left\{1.082 \times 10^{20} + 16T_S^6 + 25 \times 10^{16}\right\}^{1/2}$$



For $\varepsilon_s = 0.2$		For $\varepsilon_s = 0.8$	
T_S	$\%\,W_{es}$	T_S	$\%\,W_{es}$
600	14.45	600	3.86
700	5.76	700	1.64
800	3.0	800	0.95
900	1.79	900	0.65

12-5

For $r_T = 100$ and $r_b = 22$

$T_s = 100 - 22 = 78$ counts/min $\qquad \sigma_s^2 = [(0.02)(78)^2] = 2.45$

$$t_T = \frac{1}{2.45}[(22)^{1/2}(100)^{1/2} + 100] = 59.9 \text{ min}$$

$$t_6 = 59.9\left(\frac{22}{100}\right)^{1/2} = 28 \text{ min}$$

$t_{\text{Total}} = 87.9$ min

For $r_T = 500$ and $r_0 = 100$

$r_s = 500 - 100 = 400$

$\sigma_S^2 = [(0.02)(400)]^2 = 64.0$

$$t_T = \frac{1}{64}[(100)^{1/2}(500)^{1/2} + 500] = 11.3 \text{ min}$$

$$t_b = 11.3\left(\frac{1}{5}\right)^{1/2} = 5.07$$

$t_{\text{Total}} = 16.37$ min

12-6

by Chauvenet's criterion we find that no points may be eliminated.

$r_s = r_T - r_b = 503.75 - 23 = 480.75$ counts/min

$$W_s = \left(W_T^2 + W_b^2\right)^{1/2} = \sigma_S = \eta_s^{\ 1/2}$$

$W_S = (480.75)^{1/2} = 22.1$ counts/min

12-8

$$\sigma_S^2 = W_S^2 = \frac{r_S}{t_b} + \frac{r_T}{t_T}; r_b = r_S \qquad t_b + t_T = \text{const.}$$

Optimize W_S by setting

$$\frac{dW_S}{dt_b} = 0; 2W_s\frac{dW_S}{dt_b} = -\frac{-r_S}{t_b^2} - \frac{r_T}{t_T^2}\frac{dt_T}{dt_b} = 0$$

$$\frac{dt_T}{dt_b} = -1 \quad \therefore \frac{r_S}{t_b^2} = \frac{r_T}{t_T^2} - t_b = t_T\left(\frac{r_S}{r_T}\right)^{1/2}$$

$$\sigma_S^2 = \frac{r_S}{t_T\left(\frac{r_S}{r_T}\right)^{1/2}} + \frac{r_T}{t_T}$$

$$t_T = \frac{1}{\sigma_S^2}\left(r_S^{1/2}r_T^{1/2} + r_T\right)$$

$$r_T = 2r_S \quad \therefore t_T = \frac{3.414r_S}{\sigma_S^2}$$

12-11

$T = 300°\text{C} = 573\text{ K} \pm 1°\text{C}$

$T_a = 231°\text{C} = 504\text{ K} \pm 3°\text{C}$

$$\varepsilon_a = \left(\frac{T_a}{T}\right)^4 = \left(\frac{504}{573}\right)^4 = 0.599$$

$$\frac{W_\varepsilon}{\varepsilon} = \left[16\left(\frac{W_{T_a}}{T_a}\right)^2 + 16\left(\frac{W_T}{T}\right)^2\right]^{1/2}$$

$$= 4\left[\left(\frac{3}{504}\right)^2 + \left(\frac{1}{573}\right)^2\right]^{1/2}$$

$$= 0.0248$$

$W_\varepsilon = (0.0248)(0.599) = \pm 0.015$

12-12

$\varepsilon = 0.599$

$T_S = T\varepsilon^{1/4} = 573(0.599)^{1/4} = 504\text{ K}$

12-13

$T_a = 151°\text{C} = 424 \text{ K}$

$\varepsilon_a \left(\frac{424}{573}\right)^4 = 3$

$\frac{W_t}{\varepsilon} = 4\left[\left(\frac{3}{424}\right)^2 + \left(\frac{1}{573}\right)^2\right]^{1/2}$

$= 0.0291$

$W_\varepsilon = (0.0291)(0.3) = 0.0087$

12-17

$r_T = 600; \;\; r_b = 100$

$r_s = 600 - 100 = 500$ counts/min

$\sigma_s^2 = [(0.03)(500)]^2 = 225$

$= r_b/t_b + r_T/t_T$

$t_b/t_T = (r_b/r_T)^{1/2}$

$t_T = (1/\sigma_s{}^2)[(r_b r_T)^{1/2} + r_T]$

$= (60{,}000^{1/2} + 600)/225$

$= 3.76$ min

$= (3.76)(100/600)^{1/2} = 1.54$ min

$t_{\text{total}} = 3.76 + 1.54 = 5.3$ min

12-18

$T = 300°\text{ C} = 573 \text{ K} \pm 1°\text{ C}$

$T_a = 200°\text{ C} = 473 \text{ K} \pm 3°\text{ C}$

$\varepsilon_a = (473/573)^4 = 0.464$

$w_\varepsilon/\varepsilon = 4[(3/473)^2 + (1/573)^2]^{1/2}$

$= 0.0263$

$w_\varepsilon = 0.0263(0.464) = 0.0122$

12-19

$T = (573)\varepsilon^{1/4}$

$= (573)(0.464)^{1/4}$

$= 473 \text{ K}$

Chapter 13

13-1

a. CO

$$\text{at } 0°\text{C} \frac{m_p}{V} = (ppm)\frac{M_p p}{RT} \times 10^{-6}$$

$$= \frac{(1)(28)(1.0132 \times 10^5)}{(8315)(273)}(10^{-6})$$

$$= 1.25 \times 10^{-6} \text{ kg/m}^3$$

$$= 1250 \ \mu\text{g/m}^3$$

$$\text{at } 25°\text{C} \frac{m_p}{V} = 1250\left(\frac{273}{298}\right) = 1145 \ \mu\text{g/m}^3$$

b. SO_2

$$\text{at } 0°\text{C} = \frac{m_p}{V} = \frac{64}{28}(1250) = 2857 \ \mu\text{g/m}^3$$

$$\text{at } 25°\text{C} = \frac{m_p}{V} = 2857\left(\frac{273}{298}\right) = 2617 \ \mu\text{g/m}^3$$

13-2

$V = 100 \text{ cm}^3$ 90% purge

$Q = 125 \text{ cm}^3/\text{min}$ $\frac{C}{C_i} = 1 - e^{-Q\tau/V}$

$0.9 = 1 - e^{-Q\tau V}$ $\frac{Q\tau}{V} = 2.3026$

$$\tau = \frac{(2.3026)(100)}{125} = 1.842 \text{ min}$$

13-3

$Q = 10 \text{ ft}^3/\text{min}$ $\tau = 1 \text{ hr}$ $d = 1.4 \text{ in.}$

80% to 40% reduction in transmission

$$A = \frac{\pi(1.4)^2}{4(144)} = 0.01069 \text{ ft}^2$$

$$T_{\text{frac}} = \frac{0.4}{0.8} = 0.5$$

$$U_g = \frac{10}{0.01069} = 935.44 \text{ ft/min} = 15.591 \text{ ft/sec}$$

$$\frac{C_{\text{oh}}}{1000 \text{ ft}} = \frac{10^5 A}{V}\log_{10}[(T_{\text{frac}})^{-1}]$$

$$= \frac{(10)^5(0.01069)}{(10)(60)}\log_{10}\left(\frac{1}{0.5}\right)$$

$$= 0.536$$

13-4

$a\mathrm{CH_4} + x\mathrm{O_2} + 3.76 \times \mathrm{N_2} \rightarrow 10\,\mathrm{CO_2} + 0.7\,\mathrm{CO} + 2\,\mathrm{O_2} + 6\,\mathrm{H_2O} + 3.76x\,\mathrm{N_2}$

$a = 10 + 0.7 = 10.7$

$4(10.7) = 2b \qquad b = 21.4$

$2x(2)(10) + 0.7 + (2)(2) + 21.4 \qquad x = 23.05$

$m_a = 23.05[32 + (3.76)(28)] = 3164$

$m_f = (10.7)(16) = 171.2 \qquad \dfrac{m_a}{m_f} = \dfrac{3164}{171.2} = 18.48$

For stoichiometric

$4(10.7) = 2b \qquad b = 21.4$

$2x = (2)(10.7) + 21.4 \;\; x = 21.4$

$$\% \text{ excess air} = \frac{23.05 - 20.04}{21.4} \times 100 = 7.71\%$$

13-5

$\mathrm{C_4H_{10}} + x\mathrm{O_2} + 3.70x\,\mathrm{N_2} \rightarrow a\mathrm{CO_2} + b\mathrm{H_2O} + 3.26x\,\mathrm{N_2}$

$a = 4, 2b = 10, b = 5$

$2x = 8 + 5 \qquad x = 6.5$

25% excess: $x = 6.5 + 1.625 = 8.125$

Dry products: $4\,\mathrm{CO_2} + 1.625\,\mathrm{O_2} + 30.55\,\mathrm{N_2}$

$n_{\text{tot}} = 36.175$

$$\%\mathrm{CO_2} = \frac{4}{36.175} = 11.06\%$$

$$\%\mathrm{O_2} = \frac{1.625}{36.175} = 4.49\%$$

$$\%\mathrm{N_2} = \frac{30.55}{36.175} = 84.45\%$$

13-6

$$\frac{m_p}{V} = (ppm)\frac{M_p p}{RT} \times 10^{-6}$$

butane

$$= \frac{(1)(58)(1.013 \times 10^5)}{(8314)(293)} \times 10^{-6}$$

$$= 2.412 \times 10^{-6}\ \mathrm{kg/m^3}$$

$$= 2412\ \mu\mathrm{g/m^3}$$

propane

$M = 44$

$$\frac{m_p}{V} = (2412)\frac{(44)}{58} = 1830\ \mu\mathrm{g/m^3}$$

13-7

At 80°F = 26.7°C = 299.7 K

$p_{\text{sat}} = 0.5073$ psia

$p_v = (0.5)(0.5073) = 0.2537$ psia $= 1749$ Pa

$$\frac{m_v}{V} = \frac{p_v}{R_v T} = \frac{(1749)(18)}{(8314)(299.7)} = 0.0126\ \text{kg/m}^3 = 1.26 \times 10^7\ \mu\text{g/m}^3$$

$$0.0126 = \frac{(ppm)(1.013 \times 10^5)(18)}{(8314)(299.7)} \times 10^{-6}$$

$$ppm = 17{,}214$$

13-9

$$C_8H_{18} + x\,O_2 + 3.76x\,N_2 \rightarrow a\,CO_2 + b\,H_2O + 3.76x\,N_2$$

$$a = 8, 18 = 2b, b = 9$$

$$2x = 16 + 9 \qquad x = 12.5$$

30% excess: $x = 12.5 + 3.75 = 16.25$

Dry products $= 8\,CO_2 + 3.75\,O_2 + 61.1\,N_2$

$n_{\text{tot}} = 72.85$

$\%CO_2 = 8172.85 = 10.98\%$

$$\%O_2 = \frac{3.75}{72.85} = 5.15\%$$

$$\%N_2 = \frac{61.1}{72.85} = 83.87\%$$

100% excess: $x = 12.5 + 12.5 = 25$

Dry products $= 8\,CO_2 + 12.5\,O_2 + 94\,N_2$

$n_{\text{tot}} = 114.5$

$\%CO_2 = 6.99\%$

$\%O_2 = 10.92\%$

$\%N_2 = 82.09\%$

13-10

$V = 125\ \text{cm}^3$

$Q = 100\ \text{cm}^3/\text{min}$

$$\frac{Q}{V} = 0.8\ \text{m}^{-1}$$

$$\frac{C}{C_i} = 1 - e^{-0.8\tau}$$

$\frac{C}{C_i}$	τ, min
0.8	2.012
0.9	2.878
0.95	3.745
0.99	5.756

13-11

$$\int_{C_0}^{C} \frac{dC}{C_i - C} = \frac{Q}{C}\int_0^{\tau} d\tau$$

$$-\ln\left(\frac{C_i - C}{C_i - C_0}\right) = \frac{Q\tau}{V}$$

$$\frac{C_i - C}{C_i - C_0} = e^{-Q\tau N}$$

13-12

$C_0 = 0.1 \qquad C_i = 0.5 \qquad V = 125\ \text{cm}^3 \qquad Q = 100\ \text{cm}^3/\text{min}$

for $C = 0.2 \quad \dfrac{-0.5 - 0.2}{0.5 - 0.1} = \exp\left(\dfrac{-100\tau}{125}\right); \tau = 0.36\ \text{s}$

for $C = 0.4 \quad \dfrac{0.5 - 0.4}{0.5 - 0.1} = \exp\left(\dfrac{-100\tau}{125}\right); \tau = 0.866\ \text{s}$

for $C = 0.49 \quad \dfrac{0.5 - 0.49}{0.5 - 0.4} = \exp\left(-\dfrac{100\tau}{125}\right); \tau = 4.61\ \text{s}$

13-13

$C_0 = 0.5 \qquad C_i = 1.0 \qquad C = 0.99$

$$\frac{1.0 - 0.99}{1.0 - 0.5} = \exp\left(-\frac{100\tau}{125}\right)$$

$$\tau = 4.89\ \text{s}$$

13-14

$a\text{CH}_4 + x\text{O}_2 + 3.76x\text{N}_2 >$

$12\text{CO}_2 + 0.5\text{CO} + 3\text{O}_2 + \text{bH}_2\text{O} + 3.76x\text{N}_2$

$a = 12.5$

$b = 25$

$x = (24 + 0.5 + 25 + 6)/2 = 27.75$

$m_a = 27.75[32 + (3.76)(28)] = 3810$

$m_f = (12.5)(16) = 200$

AF ratio $= 3810/200 = 19.05$

For theoretical air

$a = 12.5,\ b = 25, x = (25 + 25)/2 = 25$

Excess air $= 27.75/25 - 1 = 0.11$ 11%

13-15

Eq. (6-14)

$p = (1.0132 \times 10^5)(1 - 0.003566(5000)/518.69)^{5.26}$

$= 84,290\ Pa$

$mp/V = (\text{ppm})M_p p \times 10^{-6}/\Re(298)$

Butane $M = 58$

$m_p/V = (1.5)(58)(0.08429)/(8314)(298)$

$= 2960\ \mu g/m^3$

Propane $M = 44$

$m_p/V = (2690)(44/58) = 2246\ \mu g/m^3$

13-16

Assume no CO

$n(CO_2) = 3$

$n(H_2O) = 4$

$x_{\text{theo}} = 5$

x (30% excess) $= 6.5$

$n(O_2 \text{ products}) = 1.5$

$n(N_2 \text{ products}) = (3.76)(6.5)$

$= 24.44$

n(total dry products) $= 24.44 + 1.5 + 3 = 28.94$

$\%\ CO_2 = 3/28.94 = 10.37$

$\%\ O_2 = 1.5/28.94 = 5.18$

$\%\ N_2 = 24.44/29.94 = 84.45$

13-16

$C_0 = 0.1$

$C_i = 1.0$

$C = 0.95$

$Q = 100\ \text{cm}^3/\text{min}$

$V = 125\ \text{cm}^3$

$Q/V = 0.8$

$(1 - 0.95)/(1 - 0.1) = \text{EXP}(-0.8\tau)$

$\tau = 3.61$ min

13-18

$T = 27°\text{C} = 300$ K

$d = 0.016$ mm

$U_0 = 0.9$ m/s

$U = 0.75$ m/s

$U/U_0 = 0.833$

From Fig. 13.4

$C/C_0 = 0.92$

Chapter 14

14-4

$932_{10} = 1110100100$

$10721_{10} = 10100111100001$

$36_{10} = 100100$

$9_{10} = 1001$

14-5

BCD

$932_{10} = 1001\,0011\,0010$

$10721_{10} = 0001\,0000\,0111\,0010\,0001$

$36_{10} = 0011\,0110$

$9_{10} = 1001$

14-6

$32°\text{F} < T < 250°\text{F}$ At $T = 250°\text{F}, E = 6.425 \text{ mV}$

$$\text{gain} = \frac{10}{6.425 \times 10^{-3}} = 1556$$

$$\text{Resolution} = \frac{E_{FS}}{2M} = \frac{10}{28} = 0.0391 \text{ V}$$

$$\text{Reduce to therm voltage} = 2.51 \times 10^{-5} \text{ V}$$
$$= 0.0251 \text{ mV}$$

$$\text{Over temp range 1°F} = \frac{6.425}{250 - 32} = 0.0295 \frac{\text{mV}}{°\text{F}}$$

$$\text{Resolution in temp} = \frac{0.0251}{0.0295} = 0.85°\text{F}$$

14-7

$$\text{Resolution} = \frac{E_{FS}}{2M}$$

$$\text{dB} = 20 \log \frac{E}{E_{FS}} = 20 \log\left(\frac{1}{2M}\right)$$

8 bit dB $= -48.2$

12 bit dB $= -72.2$

16 bit dB $= -96.3$

14-8

$$0.001\% = 0.00001 = \frac{1}{2M}$$

$M = 16.6 \sim 17$ bits

14-9

$2M = 10^{15}$

$$M = \frac{15}{\log 2} = 49.8 \sim 50 \text{ bits}$$

14-10

Table 8.3(a)

$E_{1000} = 26.046$ mV

$E_{25} = 0.402$ mV

$$\text{Sensitivity} = \frac{0.402}{25} = 0.01605\,\frac{\text{mV}}{°\text{C}}$$

Resolution of $0.5°\text{C} = 0.00802$ mV

$$0.00802 = \frac{E_{FS}}{2M}$$

$M = 11.67 = 12$ bits

14-11

Type S $E_{1000} = 9.587$ mV

$E_{25} = 0.143$ mV

$$\text{Sensitivity at } 25°\text{C} = \frac{0.143}{25} = 0.00572 \text{ mV}/°\text{C}$$

$$\text{Resolution} = \frac{0.00802}{0.00572} = 1.402°\text{C}$$

14-12

Type E $\quad E_{1000} = 76.373 \text{ mV} = E_{FS}$

$E_{25} = 1.495$ mV

$$\text{Resolution} = \frac{E_{FS}}{2M} = \frac{76.373}{2^{16}} = 0.001165 \text{ mV}$$

$$\text{Type E sensitivity at } 25°\text{C} = \frac{1.495}{25} = 0.0598 \text{ mV}/°\text{C}$$

Temperature resolution at 25°C

Type N $\quad \dfrac{0.001165}{0.01608} = 0.072°\text{C}$

Type S $\quad \dfrac{0.001165}{0.00572} = 0.204°\text{C}$

Type E $\quad \dfrac{0.001165}{0.0558} = 0.021°\text{C}$

14-13

$$\begin{aligned} 0.05°\text{C resolution} &= (0.05)(0.01605) \\ &= 0.000802 \text{ mV} \\ &= \frac{26.046}{2M} \end{aligned}$$

$M = 14.98 = 15$ bits

14-14

$$50 = 20\log\left(\frac{100}{E_0}\right) \qquad E_0 = 0.316228 \text{ mV}$$

$$0.1 = 20\log\left(\frac{E}{E_0}\right)$$

$$\text{Resolution} = E - E_0 = 0.00366\ \text{mV} = \frac{100}{2M}$$

$$M = 14.73 = 15\ \text{bits}$$

14-15

$$80 = 20\log\left(\frac{100}{E_0}\right) \qquad E_0 = 0.01\ \text{mV}$$

$$0.1\ \text{dB} = 20\log\left(\frac{E}{E_0}\right)$$

$$E - E_0 = 1.158 \times 10^{-4}\ \text{mV} = E_{\text{resolution}} = \frac{E_{FS}}{2^M} = \frac{100}{2^M}$$

$$M = 19.7 = 20\ \text{bits}$$

14-16

At 20 phon, minimum SPL = 12 dB

at SPL = 80 dB $\qquad E = 1.0\ \text{V}$

$$80 - 12 = 20\log\left(\frac{1.0}{E_{12}}\right)$$

$$E_{12} = 3.981 \times 10^{-4}\ \text{V}$$

$$+1\ \text{dB} = E_{13} = 1.122E_{12} = 0.122E_{12}$$

$$E_{\text{resolution}} = 4.858 \times 10^{-5} = \frac{E_{FS}}{2^M} = \frac{1}{2^M}$$

$$M = 14.33 = 15\ \text{bits}$$

14-17

A curve

At 30 Hz, SPL = −42 dB

at maximum point, SPL = +2 dB

Range = 2 − (−42) = 44 dB

$$44 = 20\log\left(\frac{1}{E_{30}}\right)$$

$$E_{30} = 0.0063096\ \text{V}$$

For resolution of 0.5 dB

$$0.5 = 20\log\left(\frac{E}{E_0}\right)$$

$$E - E_0 = 3.739 \times 10^{-4}\ \text{V} = E_{\text{resolution}} = \frac{1.0}{2^M}$$

$$M = 11.39 = 12\ \text{bits}$$

14-18

$-45 = 20\log(E/90)$

$E = 0.506$ mV

± 0.2 dB $= 20\log(E./0.506)$

$E = 0.0517$ mV

$= E_{FS}/2^M$

$= 90/2^M$

$M = 10.77 = 11$ bits

14-19

From Table 8.3a E @ 800° C $=$ 20.094 mV

$E_{\text{resolution}} = 20.094/2^{20} = 1.92 \times 10^{-5}$mV

@ 25° C S $=$ 0.402/25 $=$ 0.016 mV/°C

resolution $= 1.92 \times 10^{-5}/0.016 = 0.012°$ C

14-20

At 40 phon min SPL $=$ 12 dB

At SPL $=$ 80 dB, $E = 0.4$V

$80 - 12 = 20\log(0.4/E)$

$E = 0.000159$ V

± 1 dB $= 1.122$E $= 0.000179$

$= 0.4/2^M$

$M = 11.12 = 12$ bits

14-21

$E_{1000} = 26.046$

Sensitivity $=$ 0.406/25 $=$ 0.01605 mV/°C

Resolution of 0.1°C $=$ 0.001605 mV

$0.001605 = 26.046/2^M$

$M = 14$ bits