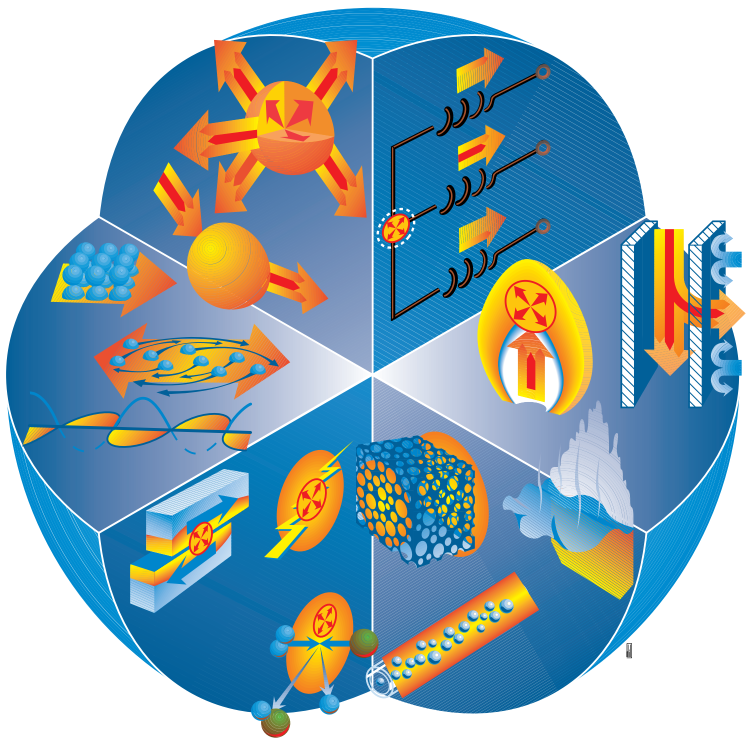
Solutions Manual to Accompany

Principles of Heat Transfer



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Outline of Solutions Manual Objectives

Literature is strewn with the wreckage of men who have minded beyond reason the opinions of others.
V. Woolf

The structure of problem statements and problem solutions, the major instructional objectives for each chapter, and a typical syllabus for a 15-week term, are given here. The syllabus can change to emphasize and de-emphasize topics, per instructors discretion.

1 Problem Statement

The format used in the problem statement is as follows.

1.1 Problem Label

The problem label follows the format: *Chapter Number.Problem Number.Purpose.Software*. The *purpose* is categorized as follows.
(i) *Familiarity* (FAM) introduces the use of available relations.
(ii) *Fundamental* (FUN) gives further insights into the principles and requires combining some concepts and relations.
(iii) *Design* (DES) uses the available relations and searches for an optimum engineering solution.
(iv) *Solver* option (S) indicates if the problem is intended for use with a solver.

For example, PROBLEM 3.5.DES.S indicates an end of chapter problem (as compared to EXAMPLE which is a solved example problem). The problem is in Chapter 3; the problem number is 5; it is in the Design category; and it is intended to be solved using a solver.

1.2 Problem Statement

The problem statement gives the following.
(i) The thermal problem considered and the knowns.
(ii) The questions and the unknowns.
(iii) Any hints on needed simplifications and assumptions for the analysis.

1.3 Sketch

The sketch provides the following.
(i) The heat transfer media.
(ii) The significant variables properly labeled.

2 Problem Solution

The format used for the problem solution is as follows.

2.1 Re-State Problem Statement (GIVEN)

Re-word and re-state the problem knowns, assumptions, and simplifications.

2.2 Re-Draw the Physical Problem (SKETCH)

Draw the sketch of the thermal problem considered. Show the direction of the heat (and when appropriate the fluid) flow. Identify the mechanisms of heat transfer and mechanisms of energy conversion. Make any other needed additions to the sketch.

2.3 Re-State Questions (OBJECTIVE)

Write the objectives of the problem and state the questions asked and the unknowns.

2.4 Solve the Problem (SOLUTION)

The solution of the problem includes some or all of the following steps.
(i) *Control Volume and Control Surface*: Mark the appropriate bounding surfaces and define the control volumes and control surfaces.
(ii) *Thermal Circuit Diagram*: Draw the thermal circuit diagram for the problem, when appropriate.
(iii) *Energy Equation*: Write the appropriate form of the energy equation.
(iv) *Energy Conversion*: Write the appropriate relations for each of the energy conversion mechanisms.
(v) *Heat Transfer Rates and Thermal Resistances*: Write the appropriate relations for the heat transfer rates and the thermal resistances.
(vi) *Numerical Values*: Determine the thermophysical and thermochemical properties from the tables, graphs, and relations. When using tables, make the needed, appropriate interpolations. Always check the units of each parameter and variable.
(vii) *Solver*: When needed, solve algebraic or differental equations using a solver such as SOPHT.
(viii) *Final Numerical Solutions*: Determine the magnitude of the unknowns.

2.5 Make Additional Comments (COMMENT)

Examine the numerical values and compare them to what is avaliable or what is initially expected. State what insights have been gained from the exercise.

3 Major Instructional Objectives

3.1 Chapter 1: Introduction and Preliminaries

(i) Mechanisms of Heat Transfer
(ii) Qualitative Heat Flux Vector Tracking
(iii) Qualitative Analysis of Energy Conservation Equation
(iv) Quantitative Analysis of Energy Conservation Equation

3.2 Chapter 2: Energy Equation

(i) Finite- and Differential-Length Energy Equation
(ii) Divergence of Heat Flux Vector
(iii) Energy Conversion Mechanisms (to and from Thermal Energy)
(iv) Chemical and Physical Bonds Energy Conversion
(v) Electromagnetic Energy Conversion
(vi) Mechanical Energy Conversion
(vii) Bounding-Surface Thermal Conditions

3.3 Chapter 3: Conduction

(i) Physics of Specific Heat Capacity
(ii) Physics of Thermal Conductivity
(iii) Thermal Conduction Resistance and Thermal Circuit Analysis
(iv) Conduction and Energy Conversion
(v) Thermoelectric Cooling
(vi) Multidimensional Conduction
(vii) Distributed Transient Conduction and Penetration Depth
(viii) Lumped-Capacitance Transient Conduction
(ix) Multinodal Systems and Finite-Small Volume Analysis
(x) Conduction and Solid-Liquid Phase Change
(xi) Thermal Expansion and Thermal Stress

3.4 Chapter 4: Radiation

(i) Surface Emission
(ii) View Factor for Diffuse Gray Enclosures
(iii) Enclosure Radiation for Diffuse Gray Surfaces
(iv) Two-Surface Enclosures
(v) Three-Surface Enclosures with One Surface Re-Radiating
(vi) Enclosures with Large Number of Surfaces
(vii) Prescribed Irradiation and Nongray Surfaces
(viii) Inclusion of Substrate

3.5 Chapter 5: Convection: Unbounded Fluid Streams

(i) Conduction-Convection Resistance and Péclet Number
(ii) Evaporation Cooling of Gaseous Streams
(iii) Combustion Heating of Gaseous Streams
(iv) Joule Heating of Gaseous Streams
(v) Gas-Stream Radiation Losses

3.6 Chapter 6: Convection: Semi-Bounded Fluid Streams

(i) Laminar Parallel Flow and Heat Transfer: Nusselt, Péclet, Reynolds, and Prandtl Numbers
(ii) Average Surface-Convection Resistance
(iii) Turbulent, Parallel Flow and Heat Transfer
(iv) Impinging Jets
(v) Thermobuoyant Flows
(vi) Liquid-Gas Phase Change
(vii) Nusselt Number and Heat Transfer for Other Geometries
(viii) Inclusion of Substrate
(ix) Surface-Convection Evaporation Cooling

3.7 Chapter 7: Convection: Bounded Fluid Streams

(i) Average Convection Resistance, NTU, and Effectiveness
(ii) Nusselt Number and Heat Transfer for Tubes
(iii) Nusselt Number and Heat Transfer for Other Geometries
(iv) Inclusion of Bounding Surface
(v) Heat Exchanger Analysis
(vi) Overall Thermal Resistance

3.8 Chapter 8: Heat Transfer and Thermal Systems

(i) Combined Mechanisms of Heat Transfer
(ii) Various Energy Conversion Mechanisms
(iii) Innovation Applications

4 Typical Syllabus

WEEK	SUBJECT	READING	PROBLEMS
1	Introduction: Control Volume and Surface, Heat Flux Vector and Mechanisms of Heat Transfer, Conservation Equations	1.1 - 1.9	1.1, 1.4, 1.6, 1.15, 1.18
2	Energy Equation for Differential Volume, Integral Volume, and Combined Integral- and Differential-Length Volume	2.1 - 2.2	2.1, 2.5, 2.7, 2.9, 2.11
3	Work and Energy Conversion: Mechanisms of Energy Conversion, Bounding-Surface Thermal Conditions, Methodology for Heat Transfer Analysis	2.3 - 2.6	2.14, 2.17, 2.18, 2.32, 2.35
4	Conduction: Specific Heat and Thermal Conductivity of Matter; Steady-State Conduction: Conduction Thermal Resistance	3.1 - 3.3	3.1, 3.3, 3.9, 3.12, 3.13
5	Steady-State Conduction: Composites, Thermal Circuit Analysis, Contact Resistance, Energy Conversion, Thermoelectric Cooling	3.3	3.15, 3.26, 3.27, 3.30, 3.32
6	Transient Conduction: Distributed Capacitance, Lumped Capacitance, Discretization of Medium into Small-Finite Volumes	3.4 - 3.7, 3.10	3.53, 3.55, 3.63, 3.67, 3.70
7	Radiation: Surface Emission, Interaction of Irradiation and Surface, Thermal Radiometry, Review	4.1 - 4.3	4.1, 4.4, 4.8, 4.9
	EXAM I (Covering Energy Equation and Conduction)		
8	Radiation: Gray-Diffuse-Opaque Surface Enclosures, View-Factor and Grayness Radiation Resistances, Thermal Circuit Analysis, Prescribed Irradiation and Nongray Surfaces, Inclusion of Substrate	4.4 - 4.7	4.10, 4.19, 4.24, 4.43, 4.49
9	Convection (Unbounded Fluid Streams): Conduction-Convection Resistance, Péclet Number, Combustion Heating of Gaseous Streams	5.1 - 5.2, 5.4, 5.7	5.1, 5.3, 5.5, 5.19, 5.20
10	Surface Convection (Semi-Bounded Fluid Streams): Flow and Surface Characteristics, Laminar Parallel Flow over Semi-Infinite Plate, Péclet, Prandtl, Reynolds, and Nusselt Numbers, Surface-Convection Resistance	6.1 - 6.2	6.1, 6.2, 6.3, 6.4
11	Convection (Semi-Bounded Fluid Streams): Turbulent Parallel Flow, Perpendicular Flow, Thermobuoyant Flows	6.3 - 6.5	6.7, 6.9, 6.10, 6.14, 6.18
12	Convection (Semi-Bounded Fluid Streams): Liquid-Vapor Phase-Change, Nusselt Number Correlations for Other Geometries, Inclusion of Substrate	6.6 - 6.8, 6.10	6.19, 6.21, 6.25, 6.40, 6.45
13	Convection (Bounded Fluid Streams): Flow and Surface Characteristics, Tube Flow and Heat Transfer, Average Convection Resistance, Review	7.1 - 7.2	7.1, 7.3, 7.4
	EXAM II (Covering Surface Radiation and Convection: Semi-Bounded Fluid Streams)		
14	Convection (Bounded Fluid Streams): Tube and Ducts, High Specific Surface Areas, Nusselt Number Correlations, Inclusion of Bounding Surface, Heat Exchangers	7.3 - 7.7	7.7, 7.12, 7.20, 7.25, 7.33
15	Heat Transfer in Thermal Systems: Thermal Functions, Analysis, and Examples	8.1 - 8.3	8.1
	FINAL EXAM (Comprehensive)		

Answers to Problems

1.13	(b)	$t = 247$ s
1.14	(b)	$dT/dt = 1.889 \times 10^{-3}$ °C/s
1.15	(b)	$T_g(t_f) = 5{,}200$°C, (c) $\eta = 2.5\%$
1.16		$Q_{ku} + Q_r = 12$ kW
1.17	(c)	$u_F = 0.5904$ mm/hr
1.18	(b)	$\langle \overline{T} \rangle_A = 13.55$°C
1.19		$\tau = 744$ s $= 0.207$ hr
1.20	(b)	$dT_r/dt = 8.696$°C/s
2.1	(a)	$\lim_{\Delta V \to 0} \int_A \mathbf{q} \cdot \mathbf{s}_n dA/\Delta V \equiv \nabla \cdot \mathbf{q} = 8a$
2.3	(a)	$\frac{2}{R_o^2 - R_i^2}(q_{ku,o}R_o + q_{ku,i}R_i) + \frac{dq_{k,z}}{dz} = 0$
2.4	(b)	$L = 7.362$ m
2.5		brake pad region: $T(t = 4 \text{ s}) = 93.50$°C; entire rotor: $T(t = 4 \text{ s}) = 66.94$°C
2.6	(a)	$\dot{m}_{ph} = 5.353 \times 10^{35}$ photon/m^2-s, (b) $\int_{-\infty}^{\infty} \dot{S}_{e,\sigma} dt = 0.1296$ J
	(c)	$\dot{S}_{e,\sigma}/V = 2.972 \times 10^{12}$ J/m^3
2.7	(a)	$\dot{s}_{e,\text{J}} = 2.576 \times 10^8$ W/m^3, (b) $\rho_e(T) = 1.610 \times 10^{-5}$ ohm-m
2.8	(a)	$\frac{\partial T}{\partial t} = 0.5580$°C/s, (b) $\frac{\partial T}{\partial t} = 0.7259$°C/s
2.10	(a)	$\nabla \cdot \mathbf{q} = 10^3$ W/m^3
2.11	(a)	$\mathbf{q}_k = -[1.883 \times 10^3(\text{°C/m}) \times k(\text{W/m-°C})]\mathbf{s}_x$
	(c)	$q_{ku}\frac{2(L_y + L_z)}{L_y L_z} - k\frac{d^2T}{dx^2} = 0$
2.12		$\dot{s}_{m,\mu} = 1.66 \times 10^{13}$ W/m^3
2.13	(a)	$\rho_{\text{CH}_4} = 8.427 \times 10^{-5}$ g/cm^3, $\rho_{\text{O}_2} = 3.362 \times 10^{-4}$ g/cm^3
	(b)	$\dot{m}_{r,\text{CH}_4}$(without Pt) $= 1.598 \times 10^{-7}$ g/cm^2-s, $\dot{m}_{r,\text{CH}_4}$(with Pt) $= 1.754 \times 10^{-3}$ g/cm^2-s
2.14		$T_s = 1{,}454$ K
2.15	(a)	$\alpha_e(\text{Ta}) = 1.035 \times 10^{-3}$ 1/K, $\alpha_e(\text{W}) = 1.349 \times 10^{-3}$ 1/K
	(b)	$\rho_e(\text{Ta}) = 1.178 \times 10^{-6}$ ohm-m, $\rho_e(\text{W}) = 7.582 \times 10^{-7}$ ohm-m
	(c)	$R_e(\text{Ta}) = 7.500$ ohm, $R_e(\text{W}) = 4.827$ ohm
	(d)	$J_e(\text{Ta}) = 3.651$ A, $J_e(\text{W}) = 4.551$ A
	(e)	$\Delta\varphi(\text{Ta}) = 27.38$ V, $\Delta\varphi(\text{W}) = 21.97$ V
2.16	(a)	$J_e = 3.520$ A, (b) $P_e = 0.2478$ W
2.17	(c)	$T_2 = 2{,}195$ K
2.18	(a)	$\Delta t_1 = 0.4577$ hr, (b) $\Delta t_2 = 9.928$ hr
2.19	(a)	$\dot{s}_{m,p} = -2 \times 10^8$ W/m^3, (c) $T_o - T_i = -42.27$°C
2.20	(a)	$\langle \dot{s}_{m,\mu} \rangle_A = 3.660 \times 10^7$ W/m^3

2.21	(a)	$\dot{S}_{m,F}\|_{\rm peak} = 0.65\dfrac{Mu_o^2}{\tau}(1-\frac{t}{\tau})$ (each of the front brakes)
		$\dot{S}_{m,F}\|_{\rm peak} = 0.35\dfrac{Mu_o^2}{\tau}(1-\frac{t}{\tau})$ (each of the rear brakes)
	(b)	$\dot{S}_{m,F}\|_{\rm peak} = 120.3$ kW
2.22	(d)	$T_s = 40.49$°C
2.23	(a)	$\dot{n}_{r,{\rm CH_4}} = -0.9004$ kg/m^3-s,
	(b)	$\dot{n}_{r,{\rm CH_4}} = -0.1280$ kg/m^3-s,
	(c)	$\dot{n}_{r,{\rm CH_4}} = -0.3150$ kg/m^3-s
2.24	(a)	$\dot{M}_{lg} = 0.8593$ g/s, (b) $\dot{M}_{r,{\rm CH_4}} = 0.2683$ g/s, (c) $\dot{S}_{e,{\rm J}} = 1.811 \times 10^5$ W
2.25	(a)	$\dot{s}_{e,{\rm J}} = 5.62 \times 10^8$ W/m^3, (b) $\Delta\varphi = 2.18$ V, (c) $J_e = 1.8$ A
	(d)	$\dot{s}_{e,{\rm J}} = 2.81 \times 10^8$ W/m^3, $\Delta\varphi = 3.08$ V, $J_e = 1.3$ A
2.26	(a)	$q_c = 49{,}808$ W/m^2, (b) $q_h = 56{,}848$ W/m^2
2.27	(c)	$T_{max} = 45.03$°C at $t = 594$ s
2.30	(a)	wet alumina: $\dot{s}_{e,m} = 3.338$ W/m^3, (b) dry alumina: $\dot{s}_{e,m} = 16.69$ W/m^3
	(c)	dry sandy soil: $\dot{s}_{e,m} = 1{,}446$ W/m^3
2.33	(b)	$q_x(x = x_2) = 1.378 \times 10^6$ W/m^2
2.36	(b)	$\dot{S}_{e,\alpha}/A = 637.0$ W/m^2, $\dot{S}_{e,\epsilon}/A = -157.8$ W/m^2, (c) $\dot{S}/A = 479.2$ W/m^2
2.37	(b)	$\dot{m}_{lg} = 5.074 \times 10^{-2}$ kg/m^2-s, (c) $D(t) = D(t=0) - \dfrac{2\dot{m}_{lg}}{\rho_l}t$, (d) $t = 36.57$ s
2.38	(b)	$q_{k,e} = -8 \times 10^4$ W/m^2
2.39	(b)	$q_{k,t} = -140$ W/m^2
2.40	(b)	$q_{k,s} = 8.989 \times 10^9$ W/m^2
3.1	(a)	for $T = 300$ K, $k_{\rm pr} = 424.7$ W/m-K, (b) for $T = 300$ K, $\Delta k(\%) = 6\%$
3.2	(a)	$k = 0.02392$ W/m-K, (b) $\Delta k(\%) = 18.20$ %
3.3	(c)	$(\rho c_p k)^{1/2}$ (argon) = 4.210, (air) = 5.620, (helium) = 11.22,
		(hydrogen) = 15.42 W-s$^{1/2}$/m^2-K
3.9		$L = 0.6$ nm: $k = 0.5914$ W/m-K, $L = 6$ nm: $k = 1.218$ W/m-K
3.12	(a)	$Q_{k,2\text{-}1} = -100$ W, (b) $Q_{k,2\text{-}1} = -83.3$ W, (c) $Q_{k,2\text{-}1} = -82.3$ W
3.13	(a)	$A_k R_{k,1\text{-}2} = 2.5 \times 10^{-5}$°C/(W/m^2), (b) $A_k R_{k,1\text{-}2} = 7.4 \times 10^{-1}$°C/(W/m^2),
	(c)	$T_1 = 60.02$°C, (d) R_k-value (copper) = 1.4×10^{-4}°F/(Btu/hr),
		R_k-value(silica aerogel) = 4.2°F/(Btu/hr)
3.14		$\Delta Q\% = 63.3$ %
3.15	(b)	parallel: $\langle k \rangle = 14.4$ W/m-K, series: $\langle k \rangle = 0.044$ W/m-K,
		random: $\langle k \rangle = 0.19$ W/m-K
3.16	(b)	(i) $Q_{1\text{-}2} = 4.703$ W, (ii) $Q_{1\text{-}2} = 4.492$ W
3.17	(b)	$Q_{k,2-1} = 8.408 \times 10^2$ W, (c) $\dot{M}_{lg} = 3.960$ g/s, (d) $T_{2'} = -10.45$°C
3.18	(b)	(i) $T_s = 43.35$°C, (ii) $T_s = 76.92$°C
3.19	(b)	$\langle k \rangle = 0.373$ W/m-K, (c) $T_g = 1{,}643$ K, (d) $Q_{g,1}/\dot{S}_{r,c} = 0.854$, $Q_{g,2}/\dot{S}_{r,c} = 0.146$
3.20	(b)	$\dot{M}_{lg} = 0.051$ kg/s, (c) $\Delta t = 0.01$ s
3.23	(c)	$Q_{k,1\text{-}2} = -7.423 \times 10^{-2}$ W, (d) $Q_{k,1\text{-}2} = -7.635 \times 10^{-2}$ W
3.24		for $p = 10^5$ Pa, $\Delta T_c = 4.9$°C, for $p = 10^6$ Pa, $\Delta T_c = 2.7$°C
3.25	(b)	for $A_k R_{k,c} = 10^{-4}$ K/(W/m^2), $T_h = 105$°C,
		for $A_k R_{k,c} = 4 \times 10^{-2}$ K/(W/m^2), $T_h = 1{,}469$°C
3.26		(i) $Q_{k,2-1}$(no blanket) = 1.006×10^3 W, (ii) $Q_{k,2-1}$(with blanket) = 6.662×10^2 W
3.27	(b)	$L = 5.984$ mm
3.29	(b)	$A_p = 2.59 \times 10^{-5}$ m^2, (c) $T_{\rm max} = 41.8$°C, (d) $q_c = -5{,}009$ W/m^2,
		$q_h = 17{,}069$ W/m^2
3.30	(a)	Q_c(2.11 A) = -0.079 W, (b) Q_c(1.06 A) = -0.047 W,
	(c)	Q_c(4.22 A) = 0.048 W
3.31	(a)	$T_c = 231$ K, (b) $J_e = 2.245$ A, (c) $T_c = 250.2$ K
3.33	(c)	$T(t \rightarrow \infty) = -18.40$°C
3.34	(c)	$T_1 = 71.36$°C
3.35	(b)	$R_{k,1\text{-}2} = 106.1$°C/W, (c) $T_1 = 60.34$°C
3.36	(b)	$e_{e,o} = 160$ V/m
3.40	(b)	$T_1 = (\dfrac{T_h + T_c}{R_k} + R_e J_e^2)/(2/R_k + \alpha_{\rm S} J_e)$, (c) $T_c(Q_c = 0) = 205.4$ K

3.41 (b) $J_e = 1.210 \times 10^{-2}$ A, (c) $T_{c,min} = 223.8$ K, (d) $Q_{c,max} = -5.731 \times 10^{-4}$ W
3.42 (c) $Q_{k,1\text{-}2} = 21.81$ W
3.44 (b) $T_1 = -19.63$℃, $Q_{k,1\text{-}2} = 61.69$ W, (c) $\dot{M}_{ls} = 665.7$ g/hr
3.45 (b) $L(t = \Delta t) = 6.127$ μm, $R_{k,1\text{-}2} = 145.6$ kW
3.46 (a) $t = 0.8378$ s
3.47 $\delta_\alpha/[2(\alpha\tau)^{1/2}] = 0.4310$
3.48 (a) (i) $T(x = 0, t = 10^{-6}$ s$) = 1.594 \times 10^5$℃, (ii) $T(x = 0, t = 10^{-6}$ s$) = 1.594 \times 10^4$℃
(b) (i) $\delta_\alpha(t = 10^{-6}$ s$) = 6.609$ μm, (ii) $\delta_\alpha(t = 10^{-4}$ s$) = 66.09$ μm
3.49 (a) $t = 3.872$ s, (b) $t = 31.53$ s
3.50 (i) $t =$ 2,970 s, (ii) $t =$ 1,458 s, (iii) $t = 972.1$ s
3.51 (a) $t = 5.6$ min, (b) $t = 35$ min
3.52 $t = 7.7$ min
3.53 $t = 24$ s
3.54 (b) first-degree burn: $x = 8.6 \pm 0.4$ mm; second-degree burn: $x = 5.8 \pm 0.4$ mm;
third-degree burn: $x = 4.75 \pm 0.4$ mm
3.55 (a) $T(x = 1$ mm$) = 215.8$ K, (b) $T(x = 3$ mm$) = 291.9$ K, (c) $q_{\rho ck} = 20,627$ W
3.56 (b) $T_s(x = L,\ t = 1.5$ s$) = 71$℃
3.57 $T_{12} \simeq T_1(t = 0)$
3.58 $t = 7.8$ μs
3.59 (a) $T(x = 4$ mm$, t = 600$ s$) = 42.15$℃
(b) $T(x = 4$ mm$, t = 600$ s$) = 64.30$℃
3.60 $t = 7.87$ min
3.61 (b) $t = 5.1$ ms
3.62 $u_b = 51.5$ cm/min
3.63 (b) $t = 177.8$ s, (c) $t = 675.7$ s, (d) $T_1(t \to \infty) = 230.2$℃
3.64 (b) $t = 1.402$ μs
3.65 (b) $T(x = 0, t = 2$ s$) = 91.41$℃
3.66 $T_1(t) - T_2 = [T_1(t = 0) - T_2]e^{-t/\tau_1} + a_1\tau_1(1 - e^{-t/\tau_1})$
3.67 (b) (i) $T_1(t) = 2.6$℃, (ii) $T_1(t) = 66.9$℃
3.68 (b) $t = 6.236$ s, (c) $\text{Fo}_R = 150.3$, (d) $N_{k,1} = 1.803 \times 10^{-3}$
3.70 (c) $L = 8.427$ m
3.71 $T_e(t \to \infty) =$ 1,672 K
3.72 (b) (i) $T_1(t = 4$ s$) = 66.97$℃, (ii) $T_1(t = 4$ s$) = 254.8$℃
3.73 (a) $\langle k \rangle_{yy} = 0.413$ W/m-K, (b) $\langle k \rangle_{yy} = 0.8375$ W/m-K
3.74 $T^*(x^* = 0.125, y^* = 0.125) = 0.03044$ for $N = 21$
3.75 (b) $T_1 =$ 4,804℃, $T_2 = 409.8$℃, $T_3 = 142.9$℃, $T_4 = 121.3$℃, $T_5 = 114.4$℃
3.76 (b) $Q_{k,h\text{-}c} = 4.283$ W, (c) $\langle k \rangle = 42.83$ W/m-K
3.77 $\delta_\alpha = 47$ μm
3.79 (a) $t = 74$ s, $\int q_k dt = 3.341 \times 10^5$ J/m^2, (b) $t = 21$ s, $\int q_k dt = 3.341 \times 10^5$ J/m^2
3.80 $t_1 = 286.6$ s, $t_2 = 40.31$ s
3.81 (b) $t = 31.8$ s
3.82 (b) $\tau_{rr}(r = 0) = \tau_{\theta\theta}(r = 0) = 1.725 \times 10^8$ Pa
3.83 (b) $T_R = 550.6$℃

4.1 (a) $E_b =$ 201,584 W/m^2, (b) $Q_{r,\epsilon} =$ 82,649 W
(c) $F_{0.39T-0.77T} = 0.13\%$, $F_{0.77T-25T} = 99.55\%$, $F_{25T-1000T} = 0.32\%$
4.2 (b) aluminum: $(Q_{r,\rho})_1 =$ 18,437 W, $(Q_{r,\alpha})_1 =$ 1,823 W, nickel: $(Q_{r,\rho})_2 =$ 13,270 W,
$(Q_{r,\alpha})_2 =$ 6,990 W, paper: $(Q_{r,\rho})_3 =$ 1,103 W, $(Q_{r,\alpha})_3 =$ 19,247 W
(c) aluminum: $(Q_{r,\epsilon})_1 = 98.94$ W, $(Q_{r,o})_1 =$ 18,536 W, nickel: $(Q_{r,\epsilon})_2 = 379.3$ W,
$(Q_{r,o})_2 =$ 13,649 W, paper: $(Q_{r,\epsilon})_3 =$ 1,044 W, $(Q_{r,o})_3 =$ 2,057 W
(d) aluminum: $Q_{r,1} = -$1,724 W, nickel: $Q_{r,2} = -$6,611 W,
paper: $Q_{r,3} = -$18,203 W
4.3 (a) $(\epsilon_r)_1 = 0.09$, $(\epsilon_r)_2 = 0.29$, $(\epsilon_r)_3 = 0.65$
4.4 (a) $q_{r,\epsilon}$(visible) $= 3.350 \times 10^5$ W/m^2, (b) $q_{r,\epsilon}$(near infrared) $= 2.864 \times 10^6$ W/m^2,
(c) $q_{r,\epsilon}$(remaining) $=$ 8,874 W/m^2
4.7 (a) $T = 477.7$ K, (b) $\lambda_{max} = 6.067$ μm, (c) $T_{lg} = 373.2$ K

4.8 (a) $\epsilon_r(\text{SiC}) = 0.8301$, $\epsilon_r(\text{Al}) = 0.008324$
4.9 (i) white potassium zirconium silicate, (ii) black-oxidized copper,
(iii) aluminum foil
4.10 (a) $F_{1\text{-}2} = 1/9$ (b) $F_{1\text{-}1} = 0.7067$, $F_{2\text{-}3} = 0.12$
(c) $F_{1\text{-}2} = 0.08$, $F_{2\text{-}1} = 0.32$, (d) $F_{1\text{-}3} = 0.085$, $F_{2\text{-}1} = 0.415$
(e) $F_{1\text{-}2} = 0.003861$, $F_{2\text{-}3} = 0.7529$
4.11 (a) $F_{1\text{-}2} = 0.2$, $x^* = w^* = 1$, $w^* = a^* = 1$, $1/R_1 = 1/R_2^* = 1.70$,
(b) $l = 0.9591a$ for the discs, $l = a$ for the plates
4.12 (a) $\epsilon_{r,1}{}' = 1/\{[D/(4L+D) \times (1-\epsilon_{r,1})/\epsilon_{r,1}] + 1\}$
4.13 (b) (i) oxygen: $q_{r,1\text{-}2} = 1.17$ W/m^2, (i) hydrogen: $q_{r,1\text{-}2} = 0.14$ W/m^2
(ii) oxygen: $q_{r,1\text{-}2} = 0.596$ W/m^2, (ii) hydrogen: $q_{r,1\text{-}2} = 0.0730$ W/m^2
4.14 (a) $Q_{r,1}/\dot{S}_{lg} = 4.7\%$, (b) $\Delta Q_{r,1} = 4.5\%$
4.15 (b) $Q_{r,1} = 82$ W
4.16 $R_{r,Sigma} = 2/(A_r\epsilon_r) - 1/A_r$
4.17 (b) $\dot{S}_{e,\text{J}} = 1{,}041$ W
4.18 (b) $T_1 = 808.6$℃, (c) $T_1 = 806.6$℃
4.20 (b) $q_{r,1-2} = (E_{b,1} - E_{b,2})/(2[\dfrac{1-\epsilon_r}{\epsilon_r} + \dfrac{1}{2\epsilon + \epsilon_r(1-\epsilon)}])$.
4.21 (b) (i) $q_{r,2\text{-}1} = -245.7$ W/m^2, (ii) $q_{r,2\text{-}1} = -210.6$ W/m^2,
(iii) $q_{r,2\text{-}1} = -179.8$ W/m^2, (iv) $q_{r,2\text{-}1} = -134.0$ W/m^2
4.22 (b) $Q_{r,1\text{-}2} = 1180$ W, (c) $Q_{r,1\text{-}2} = 182.8$ W
4.23 (b) $Q_{r,1-2} = 10{,}765$ W, (c) $Q_{r,2} = 19{,}300$ W
4.24 (b) $F_{1\text{-}2} = 0.125$, $F_{1\text{-}3} = 0.875$, $F_{2\text{-}3} = 0.8958$; (c) $T_1 = 1{,}200$ K
4.25 (a) $Q_{r,1\text{-}2} = -54.13$ W, (c) $Q_{r,1\text{-}2} = 114.1$ W, (d) $T_3 = 400$ K
4.26 $Q_{r,2} = -201{,}322$ W
4.27 (b) $Q_{r,1-2} = 3.860$ kW, (c) $Q_{r,1-2} = 0.7018$ kW,
(d) $Q_{r,1-2} = (A_{r,1}/2)(E_{b,1} - E_{b,2})$ for $F_{1-2} \rightarrow 0$
4.28 (a) $t = 208.3$ s, (b) $T(x=0,t) = 4{,}741$℃
4.29 (b) $\dot{M}_l = 0.5992$ g/s
4.30 (b) $\Delta t = 15.36$ ns
4.31 (c) $T_2 = \dfrac{1}{\sigma_{\text{SB}}}\{\dfrac{\alpha_{r,2}}{2\epsilon_{r,2}}[(q_{r,o})_a + (q_{r,o})_b]\}^{1/4}$
4.32 (b) $(q_{r,i})_f = 9.52 \times 10^4$ W/m^2, (c) $\dot{S}_{e,\sigma} = 2.989 \times 10^5$ W, (d) $\Delta t = 71.14$ s
4.33 (a) $(q_{r,i})_f = 1.826 \times 10^4$ W/m^2, (b) $q_{r,i} = 319.6$ W/m^2
4.34 (a) $q_{ku,3} = 6{,}576$ W/m^2, (b) $Q_{r,1}(\text{IR} + \text{visible}) = 2{,}835$ W,
$Q_{r,1}(\text{UV}) = 148{,}960$ W, (c) $T_{3,max} = 529.9$ K
4.35 (a) (i) $\langle Q_u \rangle_{L\text{-}0} = 500$ W, (ii) $\langle Q_u \rangle_{L\text{-}0} = 362$ W, (b) (i) $\eta = 31.22\%$, (ii) $\eta = 22.60\%$
4.36 (a) $\dot{S}_{e,\sigma}/A = 738$ W/m^2, (b) $d\langle T \rangle_L/dt = 40$ K/day
4.37 (c) (i) $Q_{r,1,t}/A_1 = -1{,}242$ W/m^2, (ii) $Q_{r,1,b}/A = -209$ W/m^2
(d) (i) $(dV_1/dt)/A_1 = -0.478$ μm/s, (ii) $(dV_1/dt)/A_1 = -0.0805$ μm/s
4.38 (a) $\langle Q_u \rangle_{L\text{-}0} = 466.4$ W, (b) $\eta = 19.42\%$
4.39 (a) $\langle Q_u \rangle_{L\text{-}0} = 1{,}376$ W, (b) $Q_1 = -34.15$ W
4.40 (a) $Q_1 = 827.7$ W
4.41 (a) $Q_{r,1\text{-}2} = -3.008$ W
4.42 (b) $T_1 = 860.5$ K
4.43 (b) $Q_{2\text{-}1} = 3{,}235$ W, (c) $Q_{2\text{-}1} = 25.65$ W
4.44 (b) $R_k + R_{r,\Sigma} = \dfrac{(1-\epsilon)(l_1 + l_2)}{A_r k_s} + \dfrac{(2-\epsilon_r)\epsilon(l_1 + l_2)}{4A_r\epsilon_r\sigma_{\text{SB}}T^3 l_2}$
4.45 $\epsilon = 0.8028$, $\langle k_r \rangle = 0.0004399$ W/m-K at $T = 1{,}000$ K
4.46 (b) $t(T_1 = 600 \text{ K}) = 162$ s
4.47 (a) $R_{r,\Sigma} = \dfrac{l_1}{A_r(1-\epsilon)^{2/3}k_s} + \dfrac{1}{4A_r(1-\epsilon)^{2/3}\sigma_{\text{SB}}T^3}\dfrac{2-\epsilon_r}{\epsilon_r}$
4.48 (a) (i) $\sigma_{ex} = 86.75$ 1/m, (ii) $\sigma_{ex} = 8.551 \times 10^5$ 1/m
(b) (i) $\sigma_{ex} = 191.0$ 1/m, (ii) $\sigma_{ex} = 1.586 \times 10^5$ 1/m
4.49 (a) $u_p = 0.48$ m/s, (b) $N_r = 1.2 \times 10^{-6} < 0.1$
4.50 (i) $q_1 = 2.305$ W/m^2, (ii) $q_1 = 2.305$ W/m^2
4.51 (i) $Q_{1-2} = 127.8$ W, (ii) $Q_{1-2} = 71.93$ W

4.52 (b) $t(T_1 = 500℃) = 5$ ms
4.53 (a) (i) $Q_{k,1-2}/L = -25.7$ W/m, (ii) $Q_{r,1-2}/L = -1.743$ W, (b) $R_2 = 2.042 \times 10^8$ m
4.54 (b) $t = 181.1$ s
4.57 (b) $\dot{m}_{ls} = 8.293$ g/m^2-s

5.1 (b) $Q(x=0) = 767.3$ W
5.2 (a) $R_{k,u}/R_{u\,L} = 0.2586$, (b) $R_{k,u}/R_{u\,L} = 6.535 \times 10^{-3}$
5.3 (b) $T_{f,2} = 367.9℃$, (c) $T_{f,2} = 2520℃$
5.5 $(Q_{k,u})_{1-2} = 140.3$ W
5.7 (b) (i) $(Q_{k,u})_{1\text{-}2} = -3.214$ W, $(q_{k,u})_{1\text{-}2} = -4.092 \times 10^6$ W/m^2
(ii) $(Q_{k,u})_{1\text{-}2} = -7.855 \times 10^{-2}$ W, $(q_{k,u})_{1\text{-}2} = -1.000 \times 10^5$ W/m^2
5.8 (b) $\dot{M}_l = 1.160 \times 10^{-7}$ kg/s, $\dot{m}_l = 0.1477$ kg/s-m^2
5.9 (b) $\dot{M}_{lg} = 2.220$ g/s, $T_{f,2} = 186.7℃$
5.10 (b) $T_{f,2} = 23.18℃$
5.11 $\dot{M}_l = 0.5476$ kg/s
5.12 (a) $\langle k \rangle = 0.63$ W/m-K, (b) $\langle k_r \rangle = 0.19$ W/m-K, (c) $u_{f,1} = 1.30$ m/s
5.13 $Tu = 0.2162$
5.15 (a) Ze $= 8.684$, (b) $u_f = 1.037$ m/s
5.16 (a) $T_{f,2} = 2{,}944℃$, (b) $u_{f,1} = 3.744$ m/s
5.17 (b) $T_s = T_{f,2} = 1{,}476$ K, $Q_{r,2-p} = 40,259$ W, (c) $\eta = 37.62\%$
5.18 (b) $T_{f,2} = 197.0℃$, (c) $T_{f,3} = 2{,}747℃$, (d) $T_{f,4} = 2{,}564℃$, (e) $\Delta T_{\text{excess}} = 177℃$
5.19 $T_{f,2} = 2{,}039$ K (for $q_{loss} = 10^5$ W/m^2)
5.21 (b) $T_s = 1{,}040$ K, (c) $\eta = 60.90\%$
5.22 (b) $\dot{M}_f = 1.800$ g/s
5.23 (b) $T_{f,2} = 2{,}472℃$
5.24 (b) $T_{f,2} = 3{,}134℃$
5.25 (b) $T_{f,2} = 203.7℃$

6.2 (a) (i) $q_{ku,L} = -50{,}703$ W/m^2, (ii) $q_{ku,L} = -2{,}269$ W/m^2, (iii) $q_{ku,L} = -39.59$ W/m^2
(b) (i) $\delta_{\alpha,L} = 5.801$ mm, (ii) $\delta_{\alpha,L} = 3.680$ mm, (iii) $\delta_{\alpha,L} = 22.39$ mm
(c) $(q_{ku})_{(i),\text{Pr}\rightarrow 0} = -46{,}115$ W/m^2, $\delta_{\alpha,L,\text{Pr}\rightarrow 0} = 7.805$ mm.
6.3 (a) (i) $\langle Q_{ku} \rangle_L = 1.502$ W, (ii) $\langle Q_{ku} \rangle_L = 65.23$ W, (b) (i) $\delta_\alpha = 14.16$ mm, (ii) $\delta_\alpha = 3.515$ mm
6.7 (a) (i) $\langle \text{Nu} \rangle_L = 119.8$, (ii) $\langle \text{Nu} \rangle_L = 378.8$, (iii) $\langle \text{Nu} \rangle_L = 2{,}335$
(b) (i) $A_{ku}\langle R_{ku} \rangle_L = 3.326 \times 10^{-1}$ ℃/(W/m^2),
(ii) $A_{ku}\langle R_{ku} \rangle_L = 1.052 \times 10^{-1}$ ℃/(W/m^2),
(iii) $A_{ku}\langle R_{ku} \rangle_L = 1.711 \times 10^{-2}$ ℃/(W/m^2)
(c) (i) $\langle Q_{ku} \rangle_L = 150.3$ W, (ii) $\langle Q_{ku} \rangle_L = 475.4$ W, (iii) $\langle Q_{ku} \rangle_L = 2{,}930$ W
6.8 (b) $u_{f,\infty} = 3.78$ m/s
6.9 (b) $T_{f,\infty} = 277.20$ K, (c) $\langle Q_{ku} \rangle_L = -598.4$ W, $Q_{r,1} = 99.57$ W, ice would melt.
6.10 (a) single nozzle: $\langle \text{Nu} \rangle_L = 46.43$, $A_{ku}\langle R_{ku} \rangle_L = 8.413 \times 10^{-2}$ ℃/(W/m^2),
$\langle Q_{ku} \rangle_L = 406.5$ W
(b) multiple nozzles: $\langle \text{Nu} \rangle_L = 28.35$, $A_{ku}\langle R_{ku} \rangle_L = 4.593 \times 10^{-2}$ ℃/(W/m^2),
$\langle Q_{ku} \rangle_L = 744.6$ W
6.11 (c) $\dot{S}_{m,F} = 0.01131$ W, $L_n = 0.5425$ cm
6.12 (b) $t = 21.27$ s
6.13 (b) (i) parallel flow: $t = 2.465$ s (ii) perpendicular flow: $t = 1.123$ s
6.14 (a) vertical: $\langle \text{Nu} \rangle_L = 43.08$, $A_{ku}\langle R_{ku} \rangle_L = 2.232 \times 10^{-1}$ ℃/(W/m^2), $\langle Q_{ku} \rangle_L = -7.390$ W
(b) horizontal: $\langle \text{Nu} \rangle_D = 18.55$, $A_{ku}\langle R_{ku} \rangle_D = 2.073 \times 10^{-1}$ ℃/(W/m^2),
$\langle Q_{ku} \rangle_D = -7.956$ W
6.15 (b) $\langle Q_{ku} \rangle_L = 411.9$ W, (c) $Q_{r,s\text{-}w} = 691.0$ W, (d) $\eta = 5.656\%$
6.18 (b) (i) $\langle Q_{ku} \rangle_L = 938.5$ W, (ii) $\langle Q_{ku} \rangle_L = 1{,}163$ W
6.19 (a) $\dot{S}_{e,\text{J}} = 2{,}045$ W, (b) $Q_{ku,CHF} = 2{,}160$ W, (c) $T_s = 108.9℃$
(d) $A_{ku}\langle R_{ku} \rangle_D = 8.575 \times 10^{-6}$ ℃/(W/m^2), $\langle \text{Nu} \rangle_D = 8{,}587$
6.20 $T_{s,1} = 103.5℃$
6.21 (b) $\langle Q_{ku} \rangle_L = -9{,}058 \times 10^3$ W, (c) $\dot{M}_{lg} = 4.013$ g/s

6.22 (b) $\langle Q_{ku}\rangle_L = 595.2$ W, (c) $\Delta\varphi = 109.1$ V, $J_e = 5.455$ A
6.23 (b) $\langle Q_{ku}\rangle_L = 8.718 \times 10^6$ W
6.24 (a) (i) $\langle q_{ku}\rangle_L = 1.099 \times 10^6$ W/m^2, (ii) $\langle q_{ku}\rangle_L = 2 \times 10^6$ W/m^2
(b) (i) $\langle q_{ku}\rangle_L = 1.896 \times 10^4$ W/m^2, (ii) $\langle q_{ku}\rangle_L = 9.407 \times 10^5$ W/m^2
6.25 (a) $\langle Q_{ku}\rangle_{D,s} = 195.9$ W, (b) $\langle Q_{ku}\rangle_{D,c} = 630.5$ W, (c) $L = 1.08$ cm, (d) $T_2 = 0.97$℃
6.26 (b) $T_2 = 817.3$ K.
6.27 (b) $T_s = 1{,}094$ K
6.28 $T_{f,\infty} - T_{s,L} = 8.71$℃
6.29 (b) $t = 17.77$ s < 20 s
6.30 (b) (i) $\langle Q_{ku}\rangle_D = 378.6$ W, (ii) $\langle Q_{ku}\rangle_L = 91.61$ W, (c) $\delta_\nu/D = 0.6438 < 1.0$
6.31 (b) $r_{tr} = 21.70$ cm, (c) $\langle Q_{ku}\rangle_L/(T_s - T_{f,\infty}) = 19.33$ W/℃, (d) $T_s = 1{,}055$℃
6.32 (a) $t = 71.4$ min, (b) $t = 6.7$ min
6.33 (a) $T_s = 1{,}146$ K, (b) $T_s = 722.6$ K
6.34 (d) $T_s^*(r = 0, t) = 0.2779$
6.35 (b) $\mathrm{Bi}_D = 4.121 \times 10^{-4} < 0.1$, (c) $t = 2.602$ ms
6.36 (a) $\langle Q_{ku}\rangle_w = 47.62$ W, (b) $\langle Q_{ku}\rangle_w = 357.1$ W
6.37 (a) $T_1(t = 4\ \mathrm{s}) = 346.2$ K, (b) $t = 17.0$ min, (c) $\mathrm{Bi}_l = 3.89 \times 10^{-3}$
6.38 (b) $R_{k,sl-b} = 1.25$℃/W, $R_{k,sl-s} = 6.25$℃/W, $\langle R_{ku}\rangle_D = 2.54$℃/W
(c) $Q_{k,sl\text{-}b} = 5.76$ W, (d) $Q_{k,sl\text{-}\infty} = 4.23$ W, (e) $\dot{S}_{sl} = 9.99$ W
6.39 (b) $\langle \mathrm{Nu}\rangle_D = 413.5$, $T_{r,max} = 318$ K
6.40 (b) $T_p = 511$℃, (c) $T_p = 64.5$℃
6.41 (b) $\mathrm{Bi}_l = 0.0658$, (c) $T_1 = 288.15$ K, (d) $T_1 = 273.76$ K
6.42 (b) $\mathrm{Bi}_D = 3.042 \times 10^{-3}$, (c) $t = 6.685 \times 10^{-3}$ s
6.43 (b) $T_s = 352.9$℃, $l/k_s < 1.36 \times 10^{-3}$℃/(W/m^2)
6.44 (b) $\eta_f = 0.9426$, (c) $T_s = 82.57$℃, (d) $\Gamma_f = 8.287$
6.45 (b) $T_1(t = t_0 = 1\ \mathrm{hr}) = 18.41$℃
6.46 (b) $R_c = 5.2$ mm, (c) $R_{k,1\text{-}2} = 0.3369$℃/W, $(R_{ku})_{D,2} = 1.319$℃/W,
(d) $R_c = 5.241$ mm
6.48 (b) $T_1 = 54.45$℃, (c) $T_1 = 50.24$℃
6.49 (b) $\dot{S}_{r,c} = 9{,}082$ W, (c) $\dot{M}_{O_2} = 0.5327$ g/s, (d) $\dot{M}_{O_2} = 0.002639$ g/s
6.50 (d) $T_s = 282.9$ K, (e) $\langle Q_{ku}\rangle_L = -1{,}369.30$ W, $Q_{k,u} = -67.39$ W
6.51 (b) t(droplet vanishes) = 382 s, (c) $L = 38.2$ m
6.52 (b) $\dot{M}_{lg} = 3.120 \times 10^{-3}$ g/s, $T_s = 285.6$ K, (c) $\Delta t = 578.2$ s
6.53 (b) $\langle Q_{ku}\rangle_L = 2.186$ W, (c) $\dot{M}_{lg} = 8.808 \times 10^{-7}$ kg/s, $\dot{S}_{lg} = -2.034$ W
(d) $dT_c/dt = -6.029 \times 10^{-3}$℃/s
6.54 (a) $\langle \mathrm{Nu}_L\rangle = [0.664\mathrm{Re}_{L,t}^{1/2} + 0.037(\mathrm{Re}_L^{4/5} - \mathrm{Re}_{L,t}^{4/5})]\mathrm{Pr}^{1/3}$

7.1 (b) $\langle \mathrm{Nu}\rangle_{D,H} = 38.22$, (c) $NTU = 0.6901$, (d) $\epsilon_{he} = 0.4985$
(e) $\langle R_u\rangle_L = 0.1080$℃/W, (f) $\langle Q_{ku}\rangle_{L-0} = 1{,}019$ W, (g) $\langle T_f\rangle_L = 74.84$℃
7.2 (b) laminar flow: $\langle u_f\rangle = 0.061$ m/s, turbulent flow: $\langle u_f\rangle = 1.42$ m/s
7.4 (a) (i) $NTU = 115.5$, (ii) $NTU = 2.957 \times 10^4$
7.6 (b) $\langle T_f\rangle_L = -43.73$℃, (c) $\langle Q_u\rangle_{L\text{-}0} = 224.4$ W
7.7 (b) $\langle Q_u\rangle_{L\text{-}0} = 1.276$ W, (c) $\langle Q_u\rangle_{L\text{-}0} = 1.276$ W
7.8 (b) $T_s = 92.74$℃ $< T_{lg}$, (c) $T_s = 399.0$℃ $> T_{lg}$
7.9 (b) $\langle Q_u\rangle_{L\text{-}0} = 926.5$ W, (c) $\langle T_f\rangle_L = 129.5$℃
7.10 (b) $\langle Q_{ku}\rangle_{D,h} = -61.80$ W, (c) $\dot{M}_{lg} = 4.459 \times 10^{-4}$ kg/s, $x_L = 0.05410$
7.11 (b) $\langle Q_{ku}\rangle_D = 28.93$ W, (c) $\dot{M}_{lg} = 2.298 \times 10^{-4}$ kg/s, $x_L = 0.6298$
7.12 (b) $T_s = 560.8$℃
7.13 (b) $NTU = 21.57$, $\langle T_f\rangle_L = 30$℃ for $N = 400$
$NTU = 32.36$, $\langle T_f\rangle_L = 30$℃ for $N = 600$
7.14 (b) $\langle Q_{ku}\rangle_L = 3{,}022$ W, $\langle Q_u\rangle_{L-0} = 3{,}158$ W
7.15 (b) $\langle \mathrm{Nu}\rangle_{D,p} = 53.10$, (c) $NTU = 2.770$, (d) $T_s = 20$(℃) + $1.049\dot{S}_{e,\mathrm{J}}$
(e) $\langle T_f\rangle_L = 20$(℃) + $0.9832\dot{S}_{e,\mathrm{J}}$, (f) $\langle T_f\rangle_L = 870.3$℃
7.16 (b) $\langle k\rangle = 0.6521$ W/m-K, (c) $NTU = 3.471$, (d) $\mathrm{Bi}_L = 1.025 \times 10^4$, (e) $\tau_s = 1.159$ hr,
(f) $\int_0^{4\tau_s} Q_{ku} dt = 2.562 \times 10^9$ J

7.17	(b)	$T_s = 484.6$℃, (c) $Q_{r,s\text{-}\infty} = 822.0$ W
7.18	(b)	$\dot{S}_{e,\mathrm{J}} = 10.80$ W
7.19	(b)	$Q_c/A_c = 310.2$ W/m^2
7.20	(b)	$\langle \mathrm{Nu} \rangle_D = 85.39$, (c) $\langle R_{ku} \rangle_D = 2.527 \times 10^{-5}$ K/W, (d) $R_{k,s\text{-}c} = 2.036 \times 10^{-3}$ K/W
	(e)	$\langle Q_u \rangle_{L\text{-}0} = -1.209 \times 10^4$ W, $\dot{M}_{sl} = 36.24$ g/s
7.21	(a)	(i) $\langle \mathrm{Nu} \rangle_D = 14{,}778$ and $\langle q_{ku} \rangle_D = 9{,}946$ kW/m^2
		(ii) $\langle \mathrm{Nu} \rangle_D = 2{,}246$ and $\langle q_{ku} \rangle_D = -237.9$ kW/m^2
7.22	(b)	(i) $Q_{1-2} = 681.7$ W, (ii) $Q_{1-2} = 545.2$ W, (iii) $Q_{1-2} = 567.1$ W, (c) $l_a = 2$ cm
7.24	(b)	$\langle T_{f,c} \rangle_0 = 623.6$℃, (c) $N = 5.81 \simeq 6$
7.25	(b)	$\langle T_{f,h} \rangle_L = 1{,}767$ K, (c) $\langle T_{f,h} \rangle_0 = 450.0$ K and $\langle T_{f,c} \rangle_L = 343.6$ K
	(d)	$\langle Q_u \rangle_{L\text{-}0} = 1{,}161$ W and $\eta = 89.36\%$
7.26	(b)	$\langle T_{f,c} \rangle_L = 27.35$℃, $\langle T_{f,h} \rangle_L = 27.12$℃, (c) $\langle Q_u \rangle_{L\text{-}0} = 171.7$ W
7.27	(b)	$R_{ku,c} = 0.0250$ K/W, $R_{ku,h} = 0.2250$ K/W, (c) $\langle T_{f,c} \rangle_L = 31.43$℃
7.28	(b)	$\langle Q_u \rangle_{L\text{-}0} = 6{,}623$ W, (c) $\eta = 90.73\%$
7.29	(b)	$NTU = 1.106$, (c) $C_r = 0.07212$, (d) $\epsilon_{he} = 0.655$, (e) $\langle T_{f,c} \rangle_L = 34.13$℃,
	(f)	$\langle T_{f,h} \rangle_L = 47.8$℃, (g) $\langle Q_u \rangle_{L\text{-}0} = 908.4$ W
7.30	(b)	$NTU = 0.07679$, (c) $\epsilon_{eh} = 0.07391$, (d) $\langle T_{f,c} \rangle_L = 307.4$ K, (e) $\langle Q_u \rangle_{L\text{-}0} = 32.09$ kW
7.31	(b)	$\eta = 95.0\%$
7.32	(i)	(b) $NTU = 0.8501$, (c) $\langle Q_u \rangle_{L\text{-}0} = 23{,}970$ W, (d) $\dot{M}_{lg} = 0.01090$ kg/s
	(ii)	(b) $NTU = 1.422$, (c) $\langle Q_u \rangle_{L\text{-}0} = 31{,}823$ W, (d) $\dot{M}_{lg} = 0.01446$ kg/s
7.33	(b)	$L = 12.45$ m
7.34	(a)	$\langle Q_{u,c} \rangle_{L\text{-}0} = 33.01$ W, (b)$L = 2.410$ m, (c) $[\Delta \langle T_{f,c} \rangle]_{max} = 66.10$℃
8.1	(b)	$J_e^2 R_{e,o} = 22.18$ W
8.2	(b)	$\langle T_f \rangle_0 = 3.559$℃, (c) $\dot{S}_{e,\mathrm{J}} = 5.683$ W, (d) $T_s = 69.75$℃
8.3	(c)	$\langle Q_u \rangle_{L\text{-}0} = -6.47$ W, $J_e = 0.8$ A, $T_c = 290.5$ K, $T_h = 333.3$ K, $\langle T_f \rangle_L = 293.0$ K,
		$\langle R_{ku} \rangle_w = 1.515$ K/W, $R_{k,h\text{-}c} = 1.235$ K/W, $\langle R_u \rangle_L = 0.4183$ K/W,
		$Q_{k,h\text{-}c} = 36.65$ W, $\dot{S}_{e,\mathrm{P}} = -46.02$ W, $(\dot{S}_{e,\mathrm{J}})_c = 5.04$ W
8.4	(b)	$T_{s,1}(t) = T_{f,1}(t) = 400$℃ for $t > 10$ s

Chapter 1

Introduction and Preliminaries

PROBLEM 1.1.FAM

GIVEN:

Introductory materials, definitions for various quantities, and the concepts related to heat transfer are given in Chapter 1.

OBJECTIVE:

Define the following terms (use words, schematics, and mathematical relations as needed).

(a) Control Volume V.
(b) Control Surface A.
(c) Heat Flux Vector $\mathbf{q}$(W/m^2).
(d) Conduction Heat Flux Vector $\mathbf{q}_k$(W/m^2).
(e) Convection Heat Flux Vector $\mathbf{q}_u$(W/m^2).
(f) Surface-Convection Heat Flux Vector $\mathbf{q}_{ku}$(W/m^2).
(g) Radiation Heat Flux Vector $\mathbf{q}_r$(W/m^2).
(h) Net Rate of Surface Heat Transfer $Q|_A$(W).
(i) Conservation of Energy.

SOLUTION:

Define the following terms (use words as well as schematics and mathematical relations when needed).

(a) Control Volume V: A control volume is a specified enclosed region of space, selected based on the information sought, on which the heat transfer analysis is applied. See Figure Ex.1.5 for examples of control volumes.

(b) Control Surface A: There are two types of control surfaces. One type of control surface is the closed surface which forms the boundary of the control volume, i.e., separates the interior of the control volume from its surroundings. These are used in volumetric energy conservation analysis. The other type of control surface is the surface containing only the bounding surface of a heat transfer medium (or interface between two media). These control surfaces enclose no mass and are used in surface energy conservation analysis. See Figure 1.2 for an example of a control surface.

(c) Heat Flux Vector $\mathbf{q}$(W/m^2): The heat flux vector is a vector whose magnitude gives the heat flow per unit time and per unit area and whose direction indicates the direction of the heat flow at a given point in space $\mathbf{x}$ and instant of time t. The mechanisms contributing to the heat flux vector are the conduction heat flux vector $\mathbf{q}_k$(W/m^2), the convection heat flux vector $\mathbf{q}_u$(W/m^2), and the radiation heat flux vector $\mathbf{q}_r$(W/m^2), i.e.,

$$\mathbf{q} = \mathbf{q}_k + \mathbf{q}_u + \mathbf{q}_r \ .$$

(d) Conduction Heat Flux Vector $\mathbf{q}_k$(W/m^2): The conduction heat flux vector is a vector whose magnitude gives the heat flow rate per unit area due to the presence of temperature nonuniformity inside the heat transfer medium and molecular conduction (molecular interaction, electron motion, or phonon motion).

(e) Convection Heat Flux Vector $\mathbf{q}_u$(W/m^2): The convection heat flux vector is a vector whose magnitude gives the heat flow rate per unit area due to bulk motion of the heat transfer medium.

(f) Surface-Convection Heat Flux vector $\mathbf{q}_{ku}$(W/m^2): This is a special case of conduction heat transfer from the surface of a stationary solid in contact with a moving fluid. The solid and fluid have different temperatures (i.e., are in local thermal nonequilibrium). Since the fluid is stationary at the solid surface (i.e., fluid does not slip on the surface), the heat transfer between the solid and the fluid is by convection, but this heat transfer depends on the fluid velocity (and other fluid properties). Therefore the subscripts k and u are used to emphasize fluid conduction and convection respectively.

(g) Radiation Heat Flux Vector $\mathbf{q}_r$(W/m^2): The radiation heat flux vector is a vector whose magnitude gives the heat flow rate per unit area in the form of thermal radiation (a part of the electromagnetic radiation spectrum).

(h) Net Rate Of Surface Heat Transfer $Q|_A$(W): The net rate of surface heat transfer is the net heat transfer rate entering or leaving a control volume. The relation to the heat flux vector is

$$Q|_A = \int_A (\mathbf{q} \cdot \mathbf{s}_n) dA,$$

where $\mathbf{s}_n$ is the control surface normal vector (pointing outward) and the integration is done over the entire control surface A. See (1.9) for the sign convention for the net rate of surface heat transfer.

(i) Conservation Of Energy: The conservation of energy equation is the first law of thermodynamics and states that the variation of the total energy of a system (which includes kinetic, potential, and internal energy) is equal to the sum of the net heat flow crossing the boundaries of the system and the net work performed inside the system or at its boundaries. The integral-volume energy equation can be written as

$$Q|_A = -\left.\frac{\partial E}{\partial t}\right|_V - \dot{E}_u|_A + \dot{W}_p|_A + \dot{W}_\mu|_A + \dot{W}_{g,e}|_V + \dot{S}_e|_V.$$

See (1.22) for the description of the various terms.

COMMENT:

The mechanisms of heat transfer (conduction, convection, and radiation), and the energy equation (including various energy conversion mechanisms) are the central theme of heat transfer analysis. In Chapter 2, a simplified form of the energy equation will be introduced.

PROBLEM 1.2.FUN

GIVEN:

An automobile radiator is a cross-flow heat exchanger (which will be discussed in Chapter 7) used to cool the hot water leaving the engine block. In the radiator, the hot water flows through a series of interconnected tubes and loses heat to an air stream flowing over the tubes (i.e., air is in cross flow over the tubes), as shown in Figure Pr.1.2(a). The air-side heat transfer is augmented using extended surfaces (i.e., fins) attached to the outside surface of the tubes. Figure Pr.1.2(b) shows a two-dimensional close up of the tube wall and the fins. The hot water convects heat $\mathbf{q}_u(\mathrm{W/m^2})$ as it flows through the tube. A portion of this heat is transferred to the internal surface of the tube wall by surface convection $\mathbf{q}_{ku}(\mathrm{W/m^2})$. This heat flows by conduction $\mathbf{q}_k(\mathrm{W/m^2})$ through the tube wall, reaching the external tube surface, and through the fins, reaching the external surface of the fins. At this surface, heat is transferred to the air stream by surface convection $\mathbf{q}_{ku}(\mathrm{W/m^2})$ and to the surroundings (which include all the surfaces that surround the external surface) by surface radiation $\mathbf{q}_r(\mathrm{W/m^2})$. The heat transferred to the air stream by surface convection is carried away by convection $\mathbf{q}_u(\mathrm{W/m^2})$.

SKETCH:

Figures Pr.1.2(a) to (c) shows an automobile radiator and its various parts.

OBJECTIVE:

On Figure Pr.1.2(c), track the heat flux vector, identifying various mechanisms, as heat flows from the hot water to the air. Assume that the radiator is operating in steady state.

SOLUTION:

Figure Pr.1.2(d) presents the heat flux vector tracking for the control volume shown in Figure Pr.1.2(c). The mechanisms of heat transfer are identified and the thickness of the heat flux vector is proportional to the magnitude of the heat transfer rate.

COMMENT:

(i) Note that at each interface the heat flux vectors entering and leaving are represented. This facilitates the application of the energy equation for control surfaces and control volumes enclosing interfaces or parts of the system.

(ii) The temperature along the fin is not axially uniform; in general, it is also not laterally uniform. The temperature at the base is higher than that at the tip. Thus, the conduction heat transfer rate is larger near the base and decreases toward the tip. As the temperature field is two-dimensional, the conduction heat flux vector is not normal to the surface. For fins of highly conducting materials or small aspect ratios (small thickness to length ratio), the lateral variation in temperature is generally neglected. Finally, for sufficiently long fins, the temperature at the tip approaches that of the air flow and under this condition there is no heat transfer through the fin tip. However, for weight and cost reductions, fin lengths are generally chosen shorter than this limit.

(iii) The direction of the convection heat flux vector depends on the direction of the complicated flow field around the fin and tube. The flow field is usually three-dimensional and the convection heat flux vector is usually not normal to the surface.

(iv) The direction of the radiation heat flux vector depends on the position and temperature of the surfaces surrounding the fin and tube surfaces. These include the other surfaces in the radiator, external surfaces, etc. In this problem such surfaces have not been directly identified.

(v) The conduction heat flux along the tube wall can be neglected for sufficiently thin tube walls.

(vi) The use of fins is justifiable when the heat flux through the fins is larger than the heat flux through the bare surface.

(vii) Heat transfer with extended surfaces (i.e., fins) is studied in Chapter 6. In that chapter, the surface convection for semi-bounded flows (i.e., flows over the exterior of solid bodies) is presented. In Chapter 7, the surface convection for bounded flows (i.e., tube flow) is studied. Surface radiation is studied in Chapter 4.

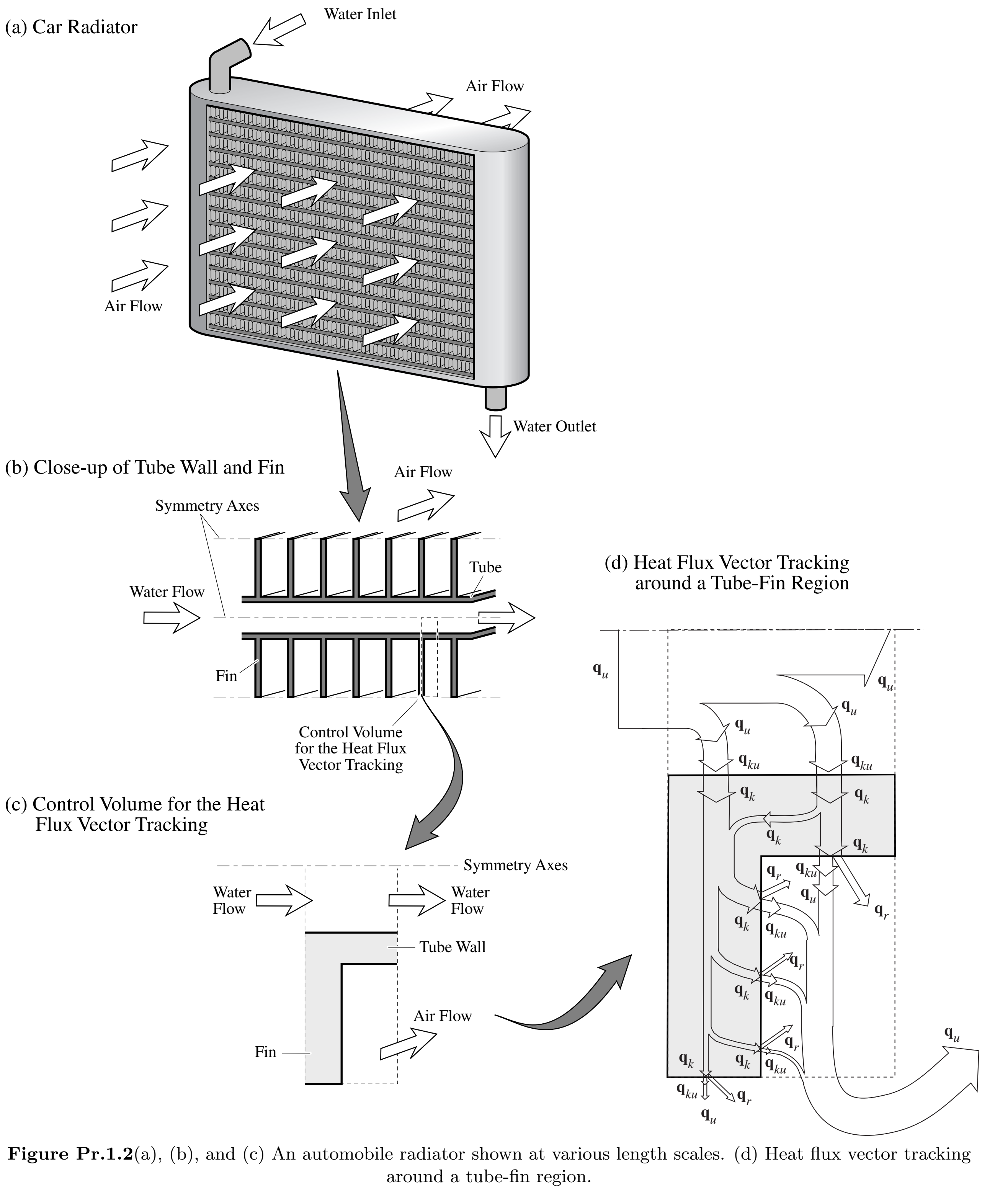


Figure Pr.1.2(a), (b), and (c) An automobile radiator shown at various length scales. (d) Heat flux vector tracking around a tube-fin region.

PROBLEM 1.3.FUN

GIVEN:

A flat-plate solar collector [Figure Pr.1.3(a)] is used to convert thermal radiation (solar energy) into sensible heat. It uses the sun as the radiation source and water for storage of energy as sensible heat. Figure Pr.1.3(b) shows a cross section of a flat-plate solar collector. The space between the tubes and the glass plate is occupied by air. Underneath the tubes, a thermal insulation layer is placed. Assume that the glass absorbs a fraction of the irradiation and designate this heat absorbed per unit volume as $\dot{s}_{e,\sigma}$(W/m^3). Although this fraction is small when compared to the fraction transmitted, the glass temperature is raised relative to the temperature of the air outside the solar collector. The remaining irradiation reaches the tube and fin surfaces, raising their temperatures. The temperature of the air inside the solar collector is higher than the glass temperature and lower than the tube and fin surface temperatures. Then the thermobuoyant flow (i.e., movement of the air due to density differences caused by temperature differences) causes a heat transfer by surface-convection at the glass and tube surfaces. The net heat transfer at the tube surface is then conducted through the tube wall and transferred to the flowing water by surface convection. Finally, the water flow carries this heat away by convection. Assume that the ambient air can flow underneath the solar collector.

SKETCH:

Figures Pr.1.3(a) to (c) show a flat-plate solar collector and its various components.

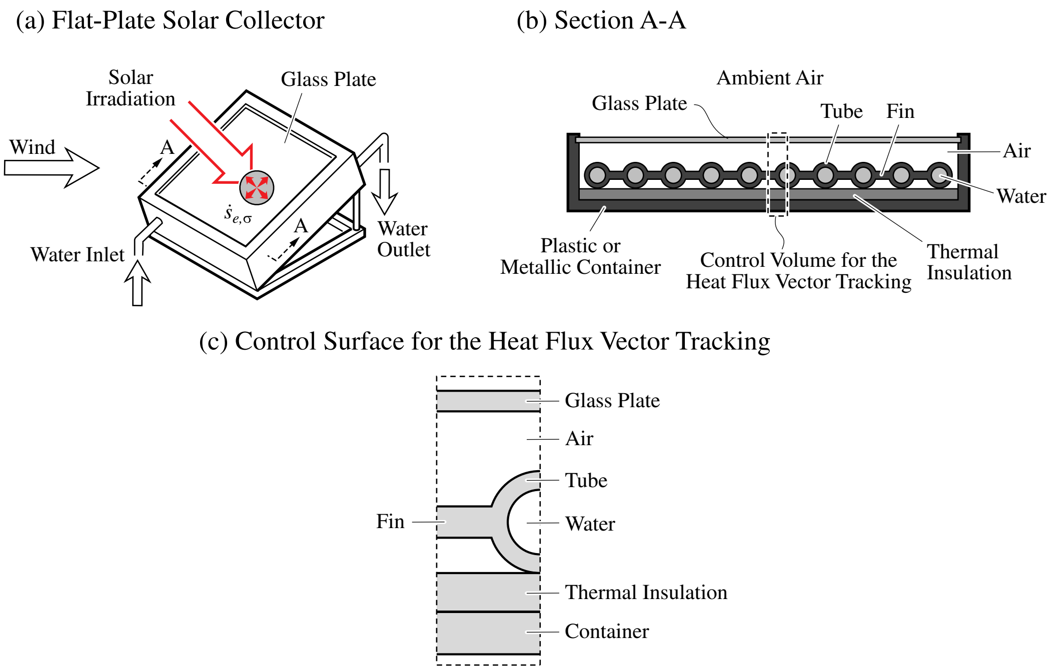


Figure Pr.1.3(a), (b), and (c) A flat-plate solar collector shown at various length scales.

OBJECTIVE:

Track the heat flux vector for this thermal system. Note that the tubes are arranged in a periodic structure and assume a two-dimensional heat transfer. Then, it is sufficient to track the heat flux vector for a control volume that includes half of a tube and half of a connecting fin, as shown on Figure Pr.1.3(c).

SOLUTION:

Figure Pr.1.3(d) shows the heat flux vector path for the cross section of the flat-plate collector.

COMMENT:

The radiation absorption in the glass plate depends on the radiation properties of the glass and on the wavelength of the thermal radiation. The tube and fin surfaces also reflect part of the incident radiation. The diagram in Figure Pr.1.3(d) represents the net radiation heat transfer between the tube and fin surfaces and the surroundings. Radiation heat transfer will be studied in detail in Chapter 4.

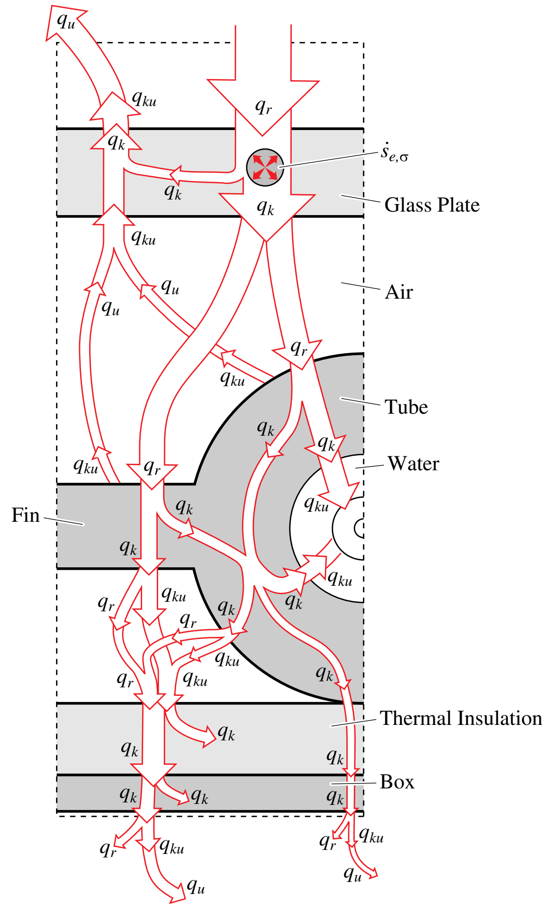


Figure Pr.1.3(d) Heat flux vector tracking around glass, tube, and insulator.

The objective in the flat-plate solar collector is to convert all the available thermal irradiation to sensible heat in the water flow. The heat flux vector tracking allows the identification of the heat losses, the heat transfer mechanisms associated with the heat losses, and the heat transfer media in which heat loss occurs. Minimizing heat loss is usually done through the suppression or minimization of the dominant undesirable heat transfer mechanisms. This can be achieved by a proper selection of heat transfer media, an active control of the heat flux vectors, or a redesign of the system. Economic factors will finally dictate the actions to be taken.

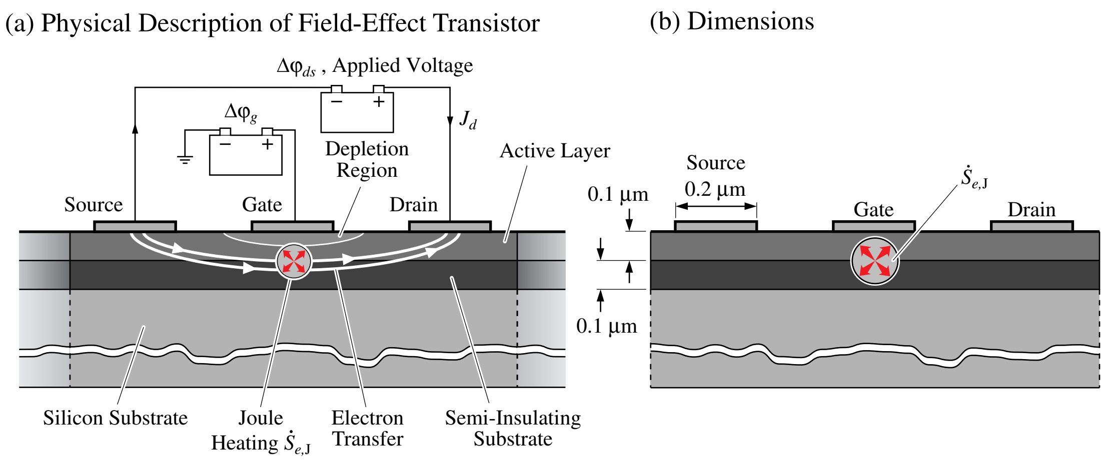
PROBLEM 1.4.FAM

GIVEN:

In printed-circuit field-effect transistors, conversion of electromagnetic energy to thermal energy occurs in the form of Joule heating. The applied electric field is time periodic and the heat generated is stored and transferred within the composite layers. This is shown in Figure Pr.1.4(a). The dimensions of the various layers are rather small (measured in submicrons). Therefore, large electrical fields and the corresponding large heat generation can elevate the local temperature beyond the threshold for damage.

SKETCH:

Figures Pr.1.4(a) and (b) show the field-effect transistor.



Figures Pr.1.4(a) and (b) Field-effect transistor.

OBJECTIVE:

On Figure Pr.1.4(b), track the heat flux vector. Note that the electric field is transient.

SOLUTION:

The electromagnetic energy converted to thermal energy by Joule heating is stored in the device, thus raising its temperature, and is transferred by conduction toward the surface. At the surface the heat is removed by surface convection and radiation. These are shown in Figure Pr.1.4(c).

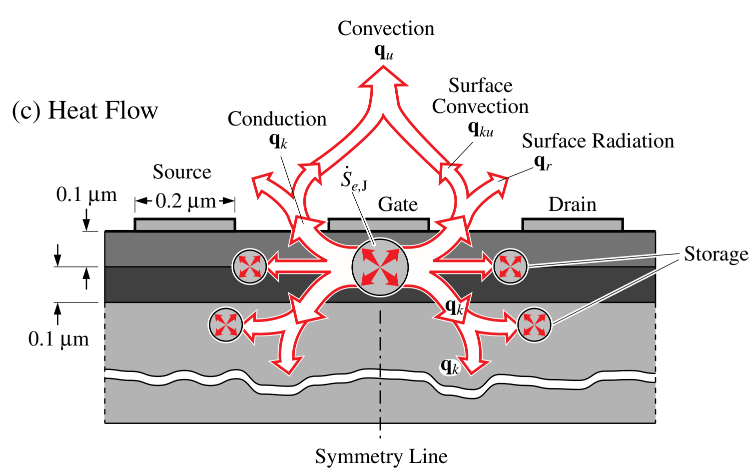


Figure Pr.1.4(c) Heat flux vector tracking in a field-effect transistor.

COMMENT:

For a time periodic electric field with very high frequency the heat is mostly stored, resulting in large local temperatures. This limits the frequency range for the operation of transistors (because at higher temperature the dopants migrate and the transistor fails).

The search for semi-conductors that can safely operate at higher temperatures aims at overcoming this limitation.

The Joule heating is caused as an electric field is applied and the electrons are accelerated and collide with the lattice atoms and other electrons. Since the electrons are at a much higher temperature than the lattice, these collisions result in a loss of momentum and this is the Joule heating.

PROBLEM 1.5.FAM

GIVEN:

The attachment of a microprocessor to a printed circuit board uses many designs. In one, solder balls are used for better heat transfer from the heat generating (Joule heating) microprocessor to the printed circuit board. This is shown in Figure Pr.1.5(a).

SKETCH:

Figure 1.5(a) shows a solder-ball attachment of a microprocessor to a printed circuit board.

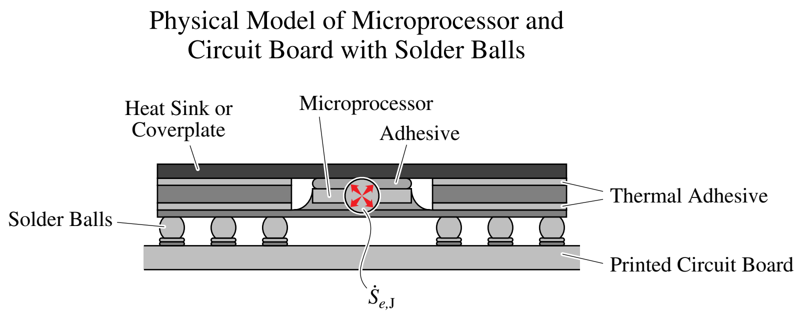


Figure Pr.1.5(a) Solder-ball connection of microprocessor to the printed circuit board.

OBJECTIVE:

Track the heat flux vector from the microprocessor to the heat sink (i.e., bare or finned surface exposed to moving, cold fluid) and the printed circuit board.

SOLUTION:

Figure Pr.1.5(b) shows the heat flux vector starting from the microprocessor. Within the solid phase, the heat transfer is by conduction. From the solid surface to the gas (i.e., air), the heat transfer mechanism is surface radiation. If the gas is in motion, heat is also transferred by surface convection.

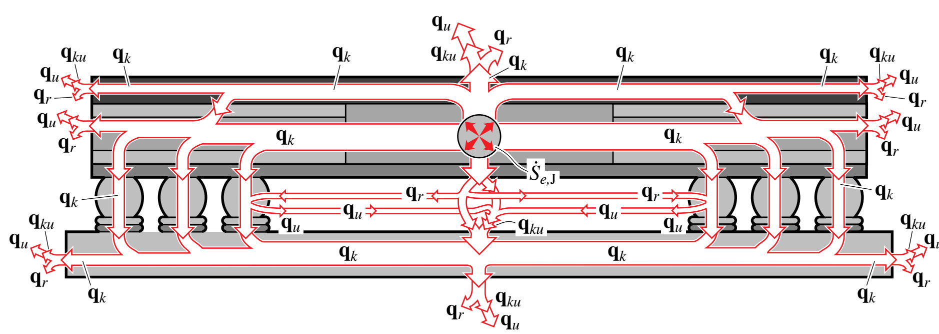


Figure Pr.1.5(b) Heat flux vector tracking in microprocessor and its substrate.

COMMENT:

If the heat generation $\dot{S}_{e,\mathrm{J}}$ is large, which is the case for high performance microprocessors, then a heat sink (e.g., a finned surface) is needed. We will address this in Section 6.8.

PROBLEM 1.6.FAM

GIVEN:

As part of stem-cell transplantation (in cancer treatment), the donor stem cells (bone marrow, peripheral blood, and umbilical cord blood stem cells) are stored for later transplants. Cryopreservation is the rapid directional freezing of these cells to temperatures below −130℃. Cryopreservative agents are added to lower freezing point and enhance dehydration. Cooling rates as high as $-dT/dt = 500$℃/s are used (called rapid vitrification). The cells are frozen and kept in special leak-proof vials inside a liquid nitrogen storage system, shown in Figure Pr.1.6(a). At one atmosphere pressure, from Table C.4, $T_{lg}(p = 1 \text{ atm}) = 77.3$ K $= -195.9$℃. The storage temperature affects the length of time after which a cell can be removed (thawed and able to establish a cell population). The lower the storage temperature, the longer the viable storage period. In one protocol, the liquid nitrogen level in the storage unit is adjusted such that $T = -150$℃ just above the stored material. Then there is a temperature stratification (i.e., fluid layer formation with heavier fluid at the bottom and lighter fluid on top) with the temperature varying from $T = -196$℃ at the bottom to $T = -150$℃ at the top of the unit, as shown in Figure Pr.1.6(a).

SKETCH:

Figure Pr.1.6(a) shows the storage container and the temperature stratification within the container.

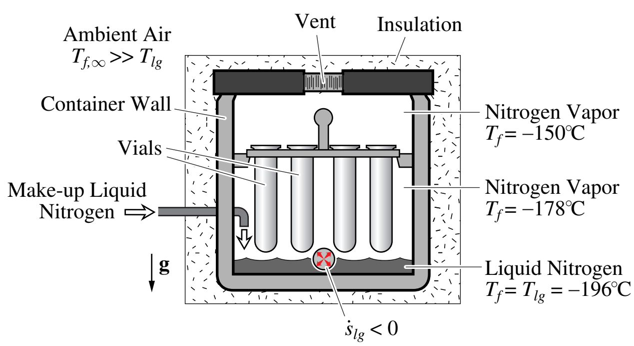


Figure Pr.1.6(a) An insulated container used for storage of cryopreserved stem cells.

OBJECTIVE:

Draw the steady-state heat flux vector tracking for the storage container showing how heat transfer by surface convection and then conduction flows through the container wall toward the liquid nitrogen surface. Also show how heat is conducted along the container wall to the liquid nitrogen surface. Note that $\dot{S}_{lg} < 0$ since heat is absorbed during evaporation. In order to maintain a constant pressure the vapor is vented and make-up liquid nitrogen is added.

SOLUTION:

Figure Pr.1.6(b) shows the heat flux vector tracking, starting from the ambient air convection $\mathbf{q}_u$, surface radiation $\mathbf{q}_r$, and surface convection $\mathbf{q}_{ku}$, and then leading to conduction $\mathbf{q}_k$ through the insulation. Heat is

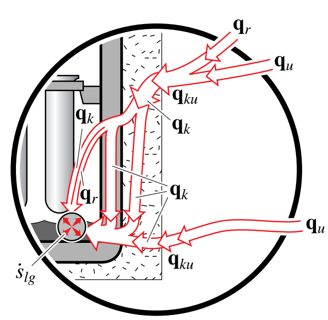


Figure Pr.1.6(b) Tracking of the heat flux vector.

conducted through the container wall and, due to the higher temperature at the top of the container, heat is also conducted along the wall and toward the liquid nitrogen surface. All the conducted heat is converted into the liquid-vapor phase change $\dot{S}_{lg}$ (which is negative). The upper portion of the container also transfers heat to the liquid surface by surface radiation $\mathbf{q}_r$.

COMMENT:

Note that between the top and bottom portions of the container there is a difference in temperature $\Delta T = 46$°C. Since the heavier gas is at the bottom, no thermobuoyant motion will occur. A special insulation is needed to minimize the heat leakage into the container (and thus reduce the needed liquid nitrogen make-up flow rate).

PROBLEM 1.7.FAM

GIVEN:

Induction-coupling (i.e., electrodeless) Joule heating $\dot{S}_{e,J}$ of thermal plasmas, which are high temperature (greater than 10,000 K) ionized-gas streams, is used for particle melting and deposition into substrates. Figure Pr.1.7(a) shows a plasma spray-coating system. The powder flow rate strongly influences particle temperature history $T_p(t)$, i.e., the speed in reaching the melting temperature (note that some evaporation of the particles also occurs). This is called in-flight plasma heating of particles. To protect the plasma torch wall, a high-velocity sheath-gas stream is used, along with liquid-coolant carrying tubes embedded in the wall. These are also shown in Figure Pr.1.7(a).

SKETCH:

Figure Pr.1.7(a) shows the torch and the (i) plasma-gas-particle stream, and (ii) the sheath-gas stream. Also shown is (iii) a single particle.

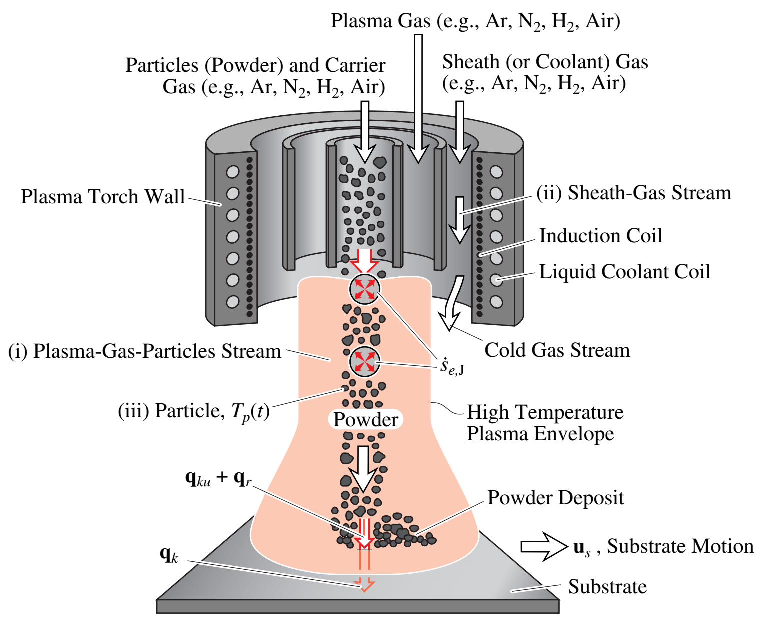


Figure Pr.1.7(a) A plasma spray-coating torch showing various streams, Joule heating, and wall cooling.

OBJECTIVE:

(a) Draw the heat flux vector tracking for the (i) plasma-gases-particles, and (ii) sheath-gas streams. Allow for conduction-convection-radiation heat transfer between these two streams. Follow the plasma gas stream to the substrate.

(b) Draw the heat flux vector tracking for (iii) a single particle, as shown in Figure Pr.1.7(a). Allow for surface convection and radiation and heat storage as $-\partial E/\partial t$ (this is sensible and phase-change heat storage).

SOLUTION:

(a) Figure Pr.1.7(b) shows the heat flux vector tracking. We start with the plasma gas-particle stream and, since it is cold at the torch entrance, its convection heat flux $\mathbf{q}_u$ is shown. Upon Joule heating, its temperature increases and radiation heat transfer also becomes significant. As the stream proceeds, it transfers heat (by conduction, convection, and radiation) to the sheath-gas stream. The sheath-gas stream in turn transfers heat by surface convection to the cold wall. The plasma-gas-particle stream reaches the substrate and transfers heat to the substrate by surface convection (this is similar to an impinging jet).

(b) Figure Pr.1.7(b) shows the heat flux vector tracking for the particles. Heat is transferred to the particles by surface radiation and surface convection. This heat is stored in the particles as sensible heat (resulting in a rise in its temperature), and as heat of phase change (melting and evaporation). Using (1.22), this is shown as $-\partial E/\partial t$.

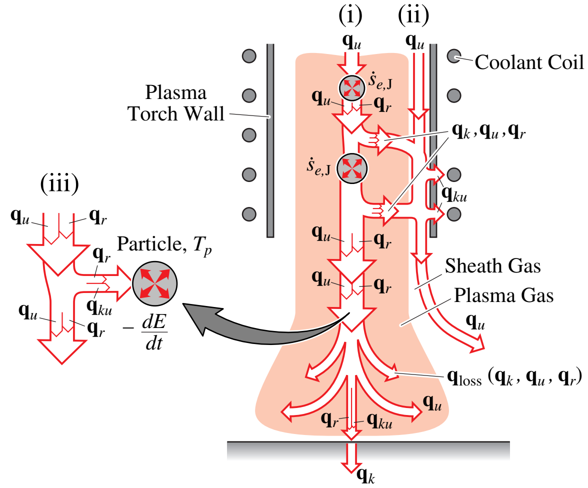


Figure Pr.1.7(b) Tracking of the heat flux vector.

COMMENT:

In Chapter 5, we will discuss Joule heating of gas streams and plasma generators. In Chapters 4 and 7, we will discuss surface-radiation and surface-convection heating of objects.

PROBLEM 1.8.FAM

GIVEN:

A bounded cold air stream is heated, while flowing in a tube, by electric resistance (i.e., Joule heating). This is shown in Figure Pr.1.8(a). The heater is a solid cylinder (ceramics with the thin, resistive wire encapsulated in it) placed centrally in the tube. The heat transfer from the heater is by surface convection and by surface-radiation emission (shown as $\dot{S}_{e,\epsilon}$). This emitted radiation is absorbed on the inside surface of the tube (shown as $\dot{S}_{e,\alpha}$) and then leaves this surface by surface convection. The outside of the tube is ideally insulated. Assume that no heat flows through the tube wall.

SKETCH:

Figure Pr.1.8(a) shows the tube, the air stream, and the Joule heater. The surface radiation emission and absorption are shown as $\dot{S}_{e,\epsilon}$ and $\dot{S}_{e,\alpha}$, respectively.

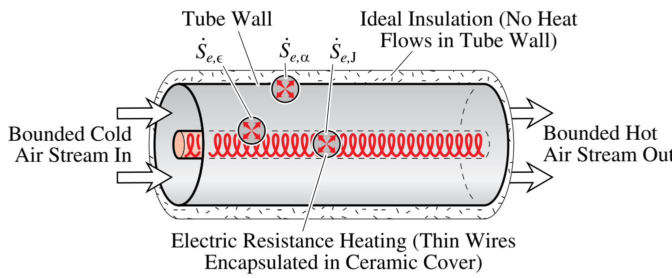


Figure Pr.1.8(a) A bounded air stream flowing through a tube is heated by a Joule heater placed at the center of the tube.

OBJECTIVE:

Draw the steady-state heat flux tracking showing the change in fluid convection heat flux vector $\mathbf{q}_u$, as it flows through the tube.

SOLUTION:

Figure Pr.1.8(b) shows the inlet fluid convection heat flux vector $\mathbf{q}_u$ entering the tube. The heat transfer by surface convection $\mathbf{q}_{ku}$ from the heater contributes to this convection heat flux vector. The surface radiation emission from the heater is absorbed by the inner surface of the tube. Since no heat flows in the tube wall, this heat leaves by surface convection and further contributes to the air stream convection heat flux vector. These are shown in Figure Pr.1.8(b).

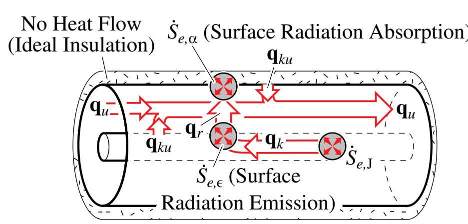


Figure Pr.1.8(b) Tracking of heat flux vector.

COMMENT:

In practice, the heater may not be at a uniform surface temperature and therefore, heat flows along the heater. The same may be true about the tube wall. Although the outer surface is assumed to be ideally insulated, resulting in no radial heat flow at this surface, heat may still flow (by conduction) along the tube wall.

PROBLEM 1.9.FAM

GIVEN:

Water is bounded over surfaces by raising the substrate surface temperature T_s above the saturation temperature $T_{lg}(p)$. Consider heat supplied for boiling by electrical resistance heating (called Joule heating) $\dot{S}_{e,\mathrm{J}}$ in the substrate. This is shown in Figure Pr.1.9(a). This heat will result in evaporation in the form of bubble nucleation, growth, and departure. The evaporation site begins as a bubble nucleation site. Then surface-convection heat transfer q_{ku} is supplied to this growing bubble (i) directly through the vapor (called vapor heating), (ii) through a thin liquid film under the bubble (called micro layer evaporation), and (iii) through the rest of the liquid surrounding the vapor. Surface-convection heat transfer is also supplied (iv) to the liquid (resulting in slightly superheated liquid) and is moved away by liquid motion induced by bubble motion and by thermobuoyancy.

SKETCH:

Figure Pr.1.9(a) shows the heat supplied by Joule heating within the substrate and a site for bubble nucleation, growth, and departure.

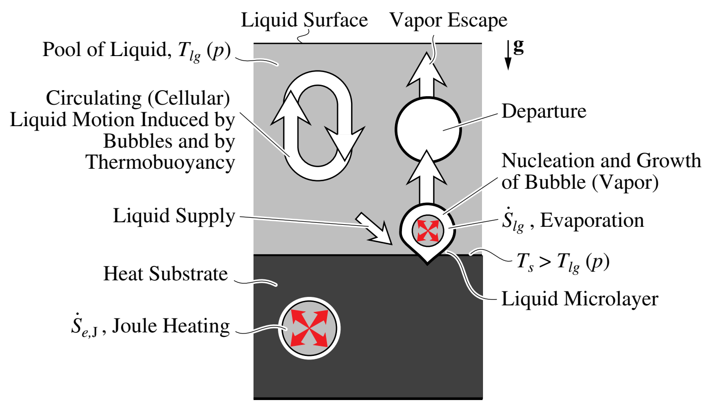


Figure Pr.1.9(a) The nucleate pool boiling on a horizontal surface. The Joule heating results in raising the surface temperature above the saturation temperature T_{lg}, and bubble nucleation, growth, and departure.

OBJECTIVE:

Track the heat flux vector starting from the Joule heating site $\dot{S}_{e,\mathrm{J}}$ within the substrate and show the surface-convection heat transfer, (i) to q_{ku} (iv). Also follow the heat-flux vector to the liquid surface. Assume a time-averaged heat transfer in which the bubbles are formed and depart continuously.

SOLUTION:

In Figure Pr.1.9(b), starting from $\dot{S}_{e,\mathrm{J}}$, the heat flows by conduction to the substrate surface. There is also conduction away from the nucleate pool boiling surface and this is labeled as the heat loss. Heat is transferred from the solid by surface convection $\mathbf{q}_{ku}$ to the vapor, to the thin liquid microlayer, to the liquid surrounding the bubble, and to the bulk liquid phase. These are shown in Figure Pr.1.9(b). The heat is in turn removed by the departing bubbles and by liquid convection $\mathbf{q_u}$ to the surface resulting in vapor escape and further evaporation.

COMMENT:

The relative magnitudes of $\mathbf{q}_{ku}$(i) to $\mathbf{q}_{ku}$(iv) are discussed in Section 6.6.1 and Figure 6.17 gives additional descriptions of the nucleate pool boiling.

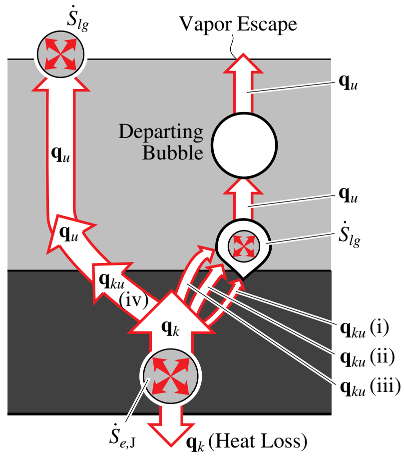


Figure Pr.1.9(b) Tracking of the heat flux vector, starting from the Joule heating location.

PROBLEM 1.10.FAM

GIVEN:

Deep heat mining refers to harvesting of the geothermal energy generated locally by radioactive decay $\dot{S}_{r,\tau}$ and transferred by conduction Q_k from the earth mantle [shown in Figure Ex.1.2(a)]. Mining is done by the injection of cold water into fractured rocks (geothermal reservoir) followed by the recovery of this water, after it has been heated (and pressurized) by surface-convection q_{ku} in the fractures, through the production wells. These are shown in Figure Pr.1.10(a). The heated water passes through a heat exchanger and the heat is used for energy conversion or for process heat transfer.

SKETCH:

Figure Pr.1.10(a) shows a schematic of deep heat mining including cold water injection into hot, fractured rocks and the recovery of heated (and pressurized) water. The heat generation by local radioactive decay $\dot{S}_{r,\tau}$ and by conduction from the earth mantle are also shown.

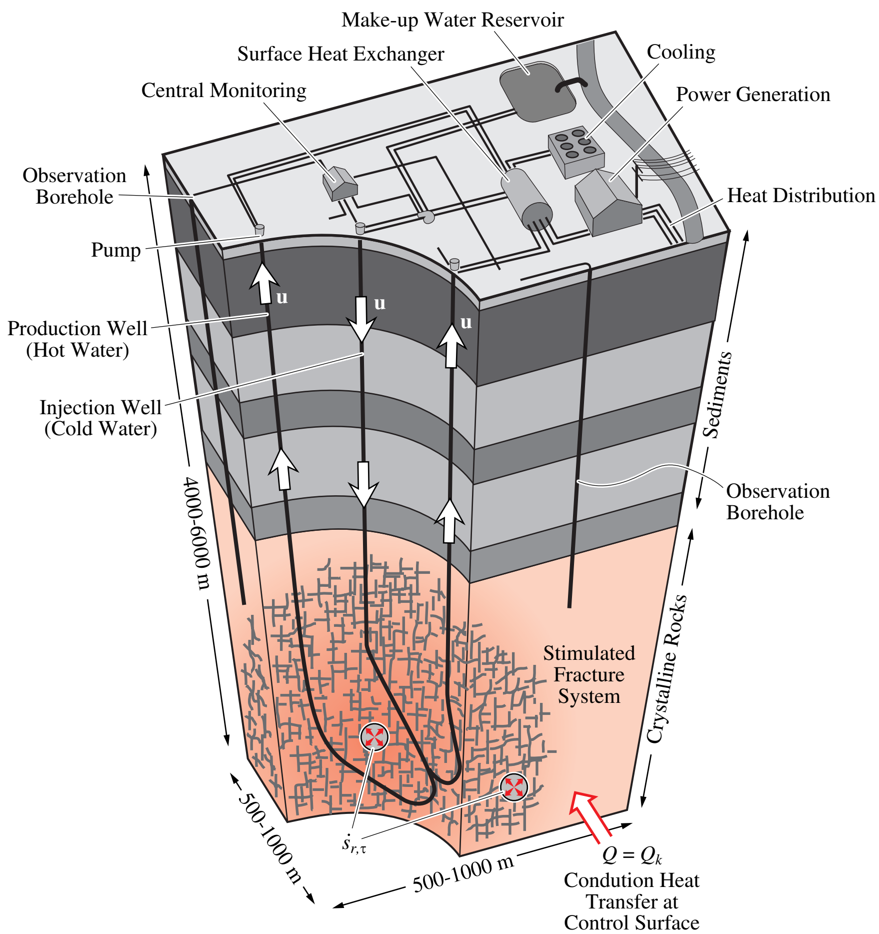


Figure Pr.1.10(a) Deep heat mining by injection of cold water into hot rocks and recovery of heated water.

OBJECTIVE:

Starting from the energy conversion sources $\dot{S}_{r,\tau}$ and the heat conduction from lower section $Q = Q_k$, draw the steady-state heat flux vector tracking and show the heat transfer to the cold stream by surface convection q_{ku}. Note that heat is first conducted through the rock before it reaches the water stream. Follow the returning warm water stream to the surface heat exchanger.

SOLUTION:

Figure Pr.1.10(b) shows the heat flux vector tracking. The heat flux emanating from $\dot{S}_{r,\tau}$ and boundary Q_k flows into the rock by conduction $\mathbf{q}_k$. This heat is then transferred to the water stream by surface convection $\mathbf{q}_{ku}$. The heated stream convects heat $\mathbf{q}_u$ to the surface heat exchanger. The direction of $\mathbf{q}_u$ is the same as $\mathbf{u}$ (fluid velocity).

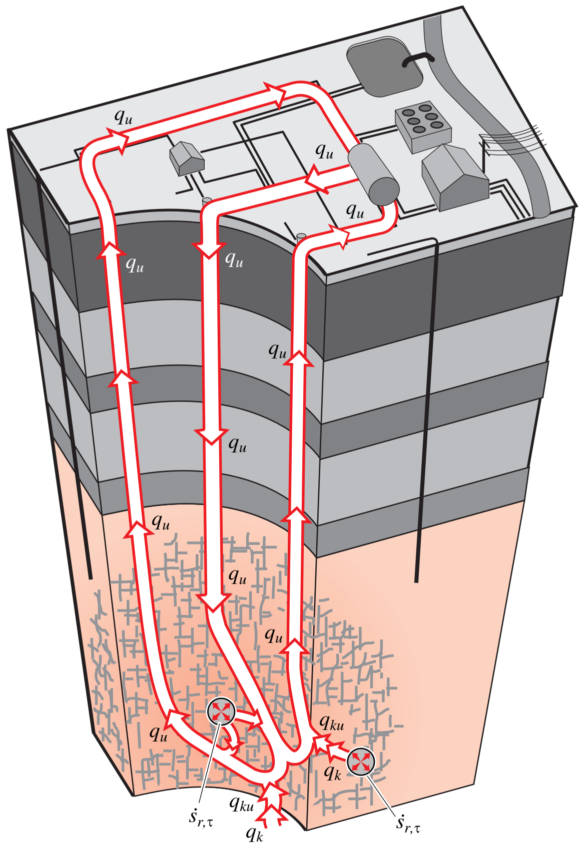


Figure Pr.1.10(b) Tracking of the heat flux vector.

COMMENT:

The surface-convection heat transfer of bounded fluid streams (such as the water stream in fractured rocks) will be discussed in Chapter 7. There we will show that when a large surface area (per unit volume) exists for surface-convection heat transfer, the stream reaches the local bounding solid temperature. Here this temperature can be high, which can cause volumetric expansion (and pressurization) of water.

PROBLEM 1.11.FAM

GIVEN:

In a seabed hydrothermal vent system, shown in Figure Pr.1.11(a), cold seawater flows into the seabed through permeable tissues (fractures) and is heated by the body of magma. The motion is caused by a density difference, which is due to the temperature variations, and is called a thermobuoyant motion (it will be described in Chapter 6). Minerals in the surrounding rock dissolve in the hot water, and the temperature-tolerant bacteria release additional metals and minerals. These chemical reactions are represented by $\dot{S}_{r,c}$ (which can be both endo- and exothermic). Eventually, the superheated water rises through the vent, its plume forming a "black smoker." As the hot water cools, its metal content precipitate, forming concentrated bodies of ore on the seabed.

SKETCH:

Figure Pr.1.11(a) shows the temperature at several locations around the vent and thermobuoyant water flow.

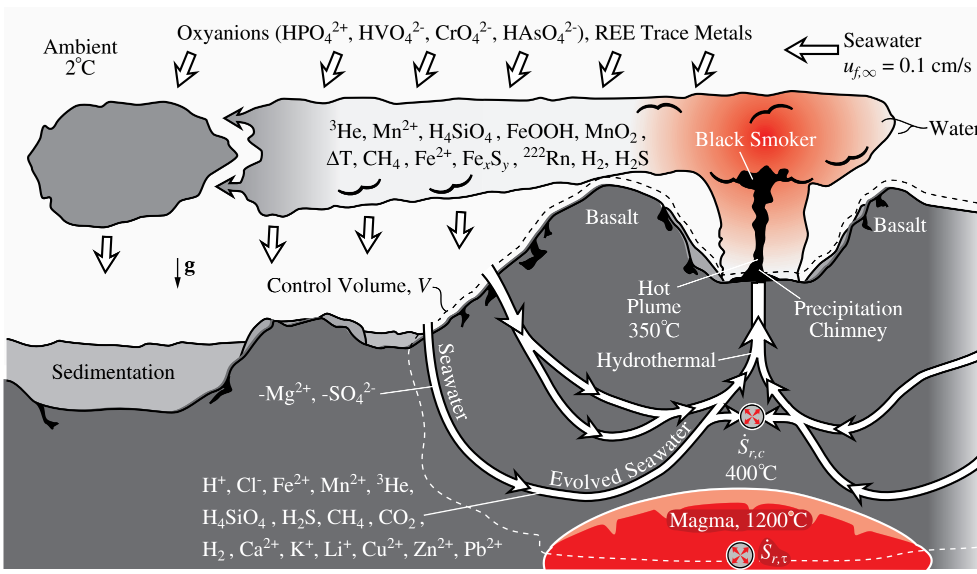


Figure Pr.1.11(a) A hydrothermal vent system showing the temperature at several locations and the thermobuoyant flow.

OBJECTIVE:

Draw the heat flux vector tracking for the volume marked as V. Note that water flows in the permeable seabed, and therefore, convection should be included (in addition to conduction). This is called the intramedium convection (as compared to surface convection) and will be discussed in Chapter 5.

SOLUTION:

Figure Pr.1.11(b) shows the heat flux vector tracking for the hydrothermal vent.

This tracking starts at the location with the highest temperature, which is the magma. The liquid flows toward the high temperature location to be heated and then it rises. Therefore, there is convection toward the magma from the periphery and convection away from the magma toward the vent. The conduction heat transfer is from the magma toward the periphery and the vent. Therefore, the conduction heat flow opposes the convection for the heat flow toward the periphery, but assists it toward the vent. These will be discussed in Chapter 5, where we consider the intramedium conduction-convection.

COMMENT:

The radiation heat transfer is negligible. Although the visible portion of the thermal radiation will penetrate through the water, the infrared portion will be absorbed over a short distance. Therefore, the mean-free path of photon λ_{ph} is very short in water.

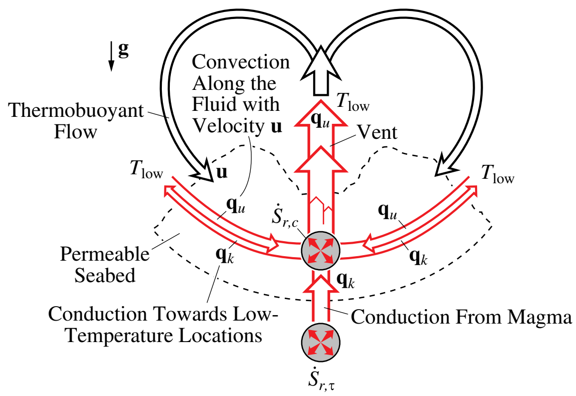


Figure Pr.1.11(b) Tracking of the heat flux vector.

PROBLEM 1.12.FAM

GIVEN:

Electric current-carrying wires are electrically insulated using dielectric material. For low temperature, a polymeric solid (such as Teflon) is used, and for high temperature application (such as in top range electrical oven), an oxide ceramic is used. Figure Pr.1.12(a) shows such a wire covered by a layer of Teflon. The Joule heating $\dot{S}_{e,\mathrm{J}}$ produced in the wire is removed by a cross flow of air, with air far-field temperature $T_{f,\infty}$ being lower than the wire temperature T_w.

SKETCH:

Figure Pr.1.12(a) shows the wire and the cross flow of air.

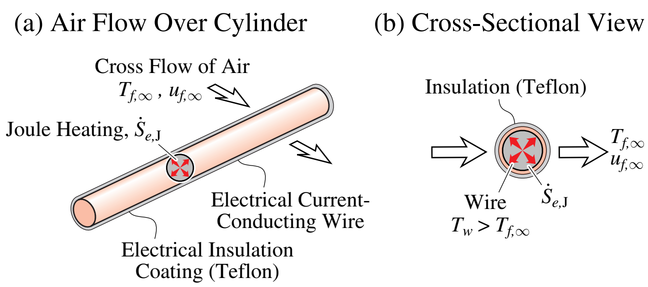


Figure Pr.1.12(a) and (b) An electrical current-carrying wire is covered with a layer of electrical insulation, and Joule heating is removed by surface convection and surface-radiation heat transfer.

OBJECTIVE:

Draw the steady-state heat flux vector tracking, starting from the heating source, for this heat transfer problem. Allow for surface radiation (in addition to surface convection).

SOLUTION:

Figure Pr.1.12(c) shows the heat flux vector tracking, starting from the heat source $\dot{S}_{e,\mathrm{J}}$. Heat is conducted through the wire and electrical insulation (since both media attenuate radiation significantly and therefore, radiation heat transfer is neglected). This heat is removed by surface convection and surface radiation. The surface-radiation heat transfer is to the surroundings (air can be treated as not attenuating the radiation). The surface convection (on the surface, which is by fluid convection but is influenced by fluid motion) leads to convection-conduction adjacent to the surface and then to convection away from the surface. As the hot air flows downstream from the wire, it loses heat by conduction and eventually returns to its upstream temperature.

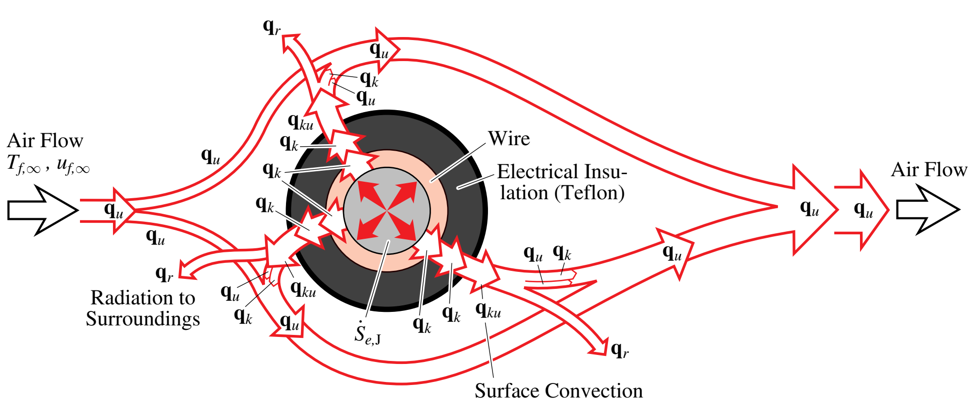


Figure Pr.1.12(c) Tracking of heat flux vector.

COMMENT:

Note that surface convection that occurs on the solid surface is conduction through the contact of the surface with the fluid molecules, which are nonmoving (from a statistical average view point). But this conduction heat transfer rate is influenced by the fluid motion (near and far from the surface).

PROBLEM 1.13.FAM.S

GIVEN:

Popcorn can be prepared in a microwave oven. The corn kernels are heated to make the popcorn by an energy conversion from oscillating electromagnetic waves (in the microwave frequency range) to thermal energy designated as $\dot{s}_{e,m}$(W/m^3). With justifiable assumptions for this problem, (1.23) can be simplified to

$$Q\,|_A = -\rho c_v V \frac{dT}{dt} + \dot{s}_{e,m} V, \qquad \text{integral-volume energy equation,}$$

where the corn kernel temperature T is assumed to be uniform, but time dependent. The control volume for a corn kernel and the associated energy equation terms are shown in Figure Pr.1.13(a).

The surface heat transfer rate is represented by

$$Q\,|_A = \frac{T(t) - T_\infty}{R_t},$$

where T_∞ is the far-field ambient temperature and R_t(K/W) is the constant heat transfer resistance between the surface of the corn kernel and the far-field ambient temperature.

$\rho = 1{,}000$ kg/m^3, $c_v = 1{,}000$ J/kg-K, $V = 1.13 \times 10^{-7}$ m^3, $\dot{s}_{e,m} = 4 \times 10^5$ W/m^3, $T(t=0) = 20$℃, $T_\infty = 20$℃, $R_t = 5 \times 10^3$ K/W.

SKETCH:

Figure Pr.1.13(a) shows the corn kernel and the thermal circuit diagram.

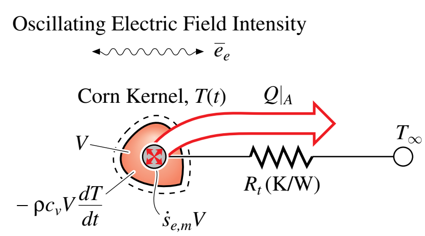


Figure Pr.1.13(a) Thermal circuit model for a corn kernel heated by microwave energy conversion.

OBJECTIVE:

(a) For the conditions given below, determine the rise in the temperature of the corn kernel for elapsed time t up to 5 min. Use a software for the time integration.

(b) At what elapsed time does the temperature reach 100℃?

SOLUTION:

The energy equation

$$Q|_A = \frac{T - T_\infty}{R_t} = -\rho c_v V \frac{dT}{dt} + \dot{s}_{e,m} V$$

is an ordinary differential equation with T as the dependent variable and t as the dependent variable. The solution requires the specification of the initial condition. This initial condition is $T(t=0)$. This energy equation has a steady-state solution (i.e., when the temperature no longer changes). The solution for the steady temperature is found by setting $dT/dt = 0$ in the above energy equation. Then, we have

$$\frac{T - T_\infty}{R_t} = \dot{s}_{e,m} V \qquad \text{or} \qquad T = T_\infty + \dot{s}_{e,m} V R_t.$$

Here we are interested in the transient temperature distribution up to $t = 5$ min.

The solver (such as SOPHT) requires specification of the initial condition and the constants (i.e., ρ, c_v, V, $\dot{s}_{e,m}$ and R_t) and a numerical integration of the transient energy equation.

(a) The solution for $T = T(t)$, up to $t = 1{,}000$ s, is plotted in Figure Pr.1.13(b). Examination shows that initially $(T - T_\infty)/R_t$ is small (it is zero at $t = 0$) and the increase in T is nearly linear. Later the time rate of increase in T begins to decrease. At steady-state (not shown), the time rate of increase is zero.

(b) The time at which $T = 100$℃ is $t = 247$ s and is marked in Figure Pr.1.13(b).

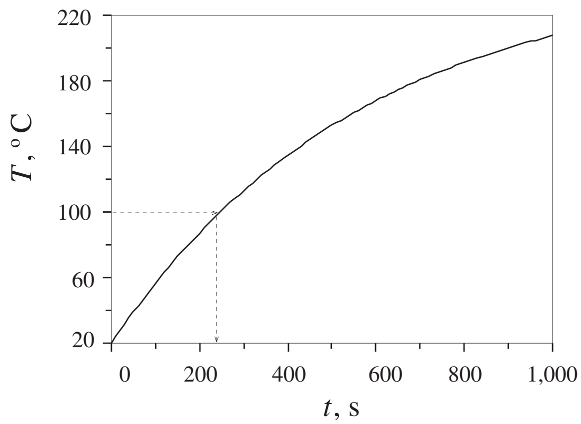


Figure Pr.1.13(b) Variation of corn kernel temperature with respect to time.

COMMENT:

The pressure rise inside the sealed corn kernel is due to the evaporation of the trapped water. This water absorbs most of the electromagnetic energy. Once a threshold pressure is reached inside the corn kernel, the sealing membrane bursts.

PROBLEM 1.14.FAM

GIVEN:

In severely cold weathers, an automobile engine block is kept warm heated prior to startup, using a block Joule heater at a rate $\dot{S}_{e,\mathrm{J}}$ with the electrical power provided through the household electrical circuit. This is shown in Figure Pr.1.14(a). The heat generated conducts through the block of mass M and then is either stored within the volume V or lost through the surface A. The energy equation (1.22) applies to the control surface A.

Consider that there is no heat transfer by convection across the surface A_1, i.e., $Q_u = 0$. The conduction heat transfer rate (through the fasteners and to the chassis) is Q_k, the surface-convection heat transfer rate (to the ambient air) is Q_{ku}, and the surface-radiation heat transfer rate (to the surrounding surface) is Q_r. In addition, there is a prescribed heat transfer rate Q (not related to any heat transfer mechanism)

$Q = 20$ W, $Q_{ku} = 80$ W, $Q_k = 30$ W, $Q_r = 15$ W, $\dot{S}_{e,\mathrm{J}} = 400$ W, $c_v = 900$ J/kg-K, $M = 150$ kg.

SKETCH:

Figure Pr.1.14(a) shows the heated engine block with the Joule heater shown separately.

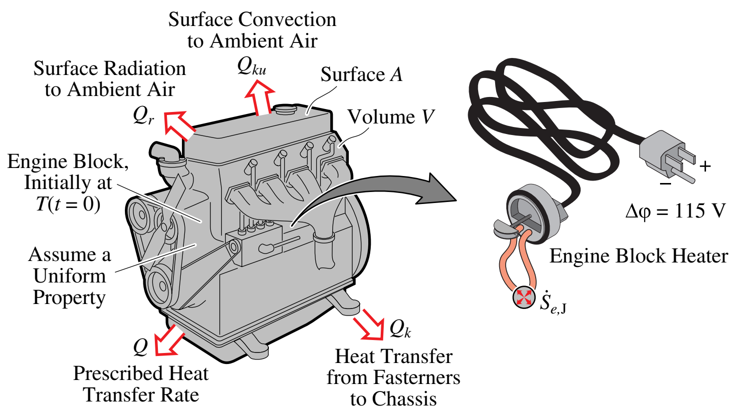


Figure Pr.1.14(a) A block Joule heater inserted in an automobile engine block.

OBJECTIVE:

(a) Draw the heat flux vector tracking starting from the Joule heating site.

(b) By applying the energy conservation equation to the control volume surface, determine the rate of change of the block temperature dT/dt, for the following condition. Use (1.22) and set all terms on the right-hand side except the first and last terms equal to zero. Use $\partial E/\partial t = Mc_v dT/dt$ and the conditions given below. The last term is equal to $\dot{S}_{e,\mathrm{J}}$.

SOLUTION:

(a) Figure Pr.1.14(b) shows the heat flux vector tracking starting from the Joule heater. The heat is conducted $\mathbf{q}_k$ through the block and is either started, $-\partial E/\partial t$ or conducted $\mathbf{q}_k$ to the surface. Then it is transferred through surface-convection $\mathbf{q}_{ku}$, surface radiation $\mathbf{q}_r$, conduction $\mathbf{q}_k$, or through a prescribed (but not explicitly associated with any heat transfer mechanism) rate $\mathbf{q}$ to the surroundings.

(b) The energy equation (1.22), applied to the control volume shown in Figure Pr.1.14(b) becomes

$$\begin{aligned} Q|_A &= Q + Q_k + Q_u + Q_{ku} + Q_r \\ &= -\frac{\partial E}{\partial t} + \dot{S}_{e,\mathrm{J}} \\ &= -Mc_v\frac{dT}{dt} + \dot{S}_{e,\mathrm{J}}. \end{aligned}$$

Noting that $Q_u = 0$, and solving for dT/dt, we have

$$\frac{dT}{dt} = \frac{\dot{S}_{e,\mathrm{J}} - Q - Q_k - Q_{ku} - Q_r}{Mc_v}.$$

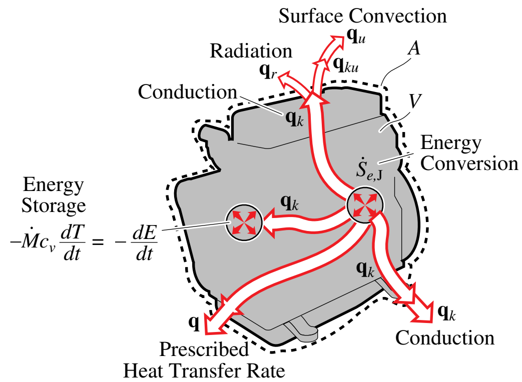


Figure Pr.1.14(b) Tracking of heat flux vector.

Now using the numerical values, we have

$$\begin{aligned}\frac{dT}{dt} &= \frac{(400-20-30-80-15)(\text{W})}{150(\text{kg}) \times 900(\text{J/kg-K})} \\ &= 1.889 \times 10^{-3}\,^\circ\text{C/s}.\end{aligned}$$

COMMENT:

At this rate, to increase the engine block temperature by 10°C, an elapsed time of

$$\begin{aligned}\frac{\Delta T}{\Delta t} &= 1.889 \times 10^{-3}\,^\circ\text{C/s} \\ \Delta t &= \frac{\Delta T(^\circ\text{C})}{1.889 \times 10^{-3}(^\circ\text{C/s})} = 5{,}294 \text{ s} = 1.471 \text{ hr},\end{aligned}$$

is needed.

In general, Q, Q_k, Q_{ku} and Q_r all change with the engine block temperature T. In Chapters 3 to 7 these surface heat transfer rates are related to heat transfer resistances R_t, which in turn depend on the various heat transfer parameters.

PROBLEM 1.15.FAM

GIVEN:

In spark-ignition engines, the electrical discharge produced between the spark plug electrodes by the ignition system produces thermal energy at a rate $\dot{S}_{e,\mathrm{J}}$(W). This is called the Joule heating and will be discussed in Section 2.3. This energy conversion results in a rise in the temperature of the electrodes and the gas surrounding the electrodes. This high-temperature gas volume V, which is called the plasma kernel, is a mixture of air and fuel vapor. This plasma kernel develops into a self-sustaining and propagating flame front.

About $\int_A Q\,|_A \; dt = -1$ mJ is needed to ignite a stagnant, stoichiometric fuel-air mixture of a small surface area A and small volume V, at normal engine conditions. The conventional ignition system delivers 40 mJ to the spark.

$(\rho c_v V)_g = 2 \times 10^{-7}$ J/℃, $T_g(t=0) = 200$℃.

SKETCH:

Figure 1.15(a) shows the spark plug and the small gas volume V being heated.

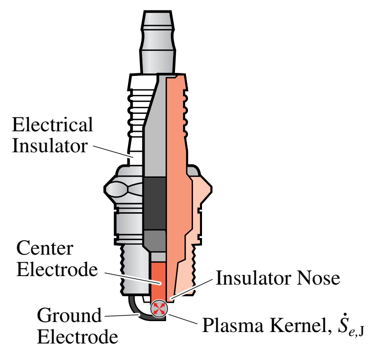


Figure Pr.1.15(a) Ignition of a fuel-air mixture by a spark plug in a spark-ignition engine. The plasma kernel is also shown.

OBJECTIVE:

(a) Draw the heat flux vector tracking for the region around the electrodes marked in Figure Pr.1.15(a). Start from the energy conversion source $\dot{S}_{e,\mathrm{J}}$.

(b) Assume a uniform temperature within the gas volume V. Assume that all terms on the right-hand side of (1.22) are negligible, except for the first term. Represent this term with

$$\left.\frac{\partial E}{\partial t}\right|_V = (\rho c_v V)_g \frac{dT_g}{dt}.$$

Then for the conditions given below, determine the final gas temperature $T_g(t_f)$, where the initial gas temperature is $T_g(t=0)$.

(c) What is the efficiency of this transient heating process?

SOLUTION:

(a) Figure Pr.1.15(b) shows the heat flux vector tracking for conduction and heat storage in the electrodes. The gas kernel, where the energy conversion occurs, also stores and conducts heat.

(b) Using (1.22), with all the right-hand side terms set to zero except for energy storage, we have

$$Q|_A = -\left.\frac{\partial E}{\partial t}\right|_V = -(\rho c_v V)_g \frac{dT_g}{dt}.$$

Integrating this with respect to time, from $t = 0$ to a final time where $t = t_f$, we have

$$\int_0^{t_f} Q|_A dt = -(\rho c_v V)_g [T_g(t_f) - T_g(t=0)].$$

Plasma Kernal Heated by Joule Heating

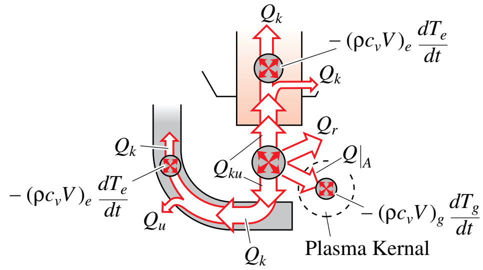


Figure Pr.1.15(b) Thermal circuit diagram.

Solving for $T_g(t_f)$, we have

$$T_g(t_f) = T_g(t=0) - \frac{\int_0^{t_f} Q|_A dt}{(\rho c_v V)_g}.$$

Using the numerical values, we have

$$\begin{aligned} T_g(t_f) &= 200(℃) - \frac{-10^{-3}(\text{J})}{2 \times 10^{-7}(\text{J}/℃)} \\ &= 200(℃) + 5{,}000(℃) = 5{,}200℃. \end{aligned}$$

(c) The efficiency is

$$\eta = \frac{\int_0^{t_f} Q|_A dt}{\int_0^{t_f} \dot{S}_{e,\text{J}} dt} = \frac{10^{-3}(\text{J})}{40 \times 10^{-3}(\text{J})} = 2.5\%.$$

COMMENT:

Very high temperatures are reached for the plasma kernel for a short time. The efficiency may even be smaller than 2.5%. There are several regimes in the short sparking period (order of milliseconds). These are breakdown, arc, and glow-discharge regimes. The gas heat up occurs during the glow-discharge regime. Most of the energy is dissipated during the first two regimes and does not lead to the gas heat up.

PROBLEM 1.16.FAM

GIVEN:

The temperature distributions for the exhaust gas and the exhaust pipe wall of an automotive exhaust system are shown in Figures Pr.1.16(a) and (b). The exhaust gas undergoes a temperature difference $\langle T_f \rangle_0 - \langle T_f \rangle_L$ over the upper-pipe region (between the exhaust manifold and the catalytic converter). It can be shown that when the energy equation (1.23) is written for this upper-pipe region, as shown in the figure, and under steady-state conditions, the right-hand side of this equation is zero. Then the energy equation becomes

$$Q\mid_A = 0 \qquad \text{integral-volume energy equation.}$$

The surface heat flows are convection on the left and right surfaces and surface convection and radiation from the other sides, i.e.,

$$Q\mid_A = Q_{u,L} - Q_{u,0} + Q_{ku} + Q_r = 0.$$

The convection heat flow rates are written (as will be shown in Chapter 2 and Appendix B) as

$$Q_{u,L} - Q_{u,0} = \dot{M}_f c_{p,f}(\langle T_f \rangle_L - \langle T_f \rangle_0),$$

where $\dot{M}_f$ (kg/s) is the gas flow rate and $c_{p,f}$ (J/kg-K) is the specific heat capacity at constant pressure.

SKETCH:

Figures Pr.1.16(a) and (b) show the exhaust pipe and its upper portion and temperature distribution along the pipe.

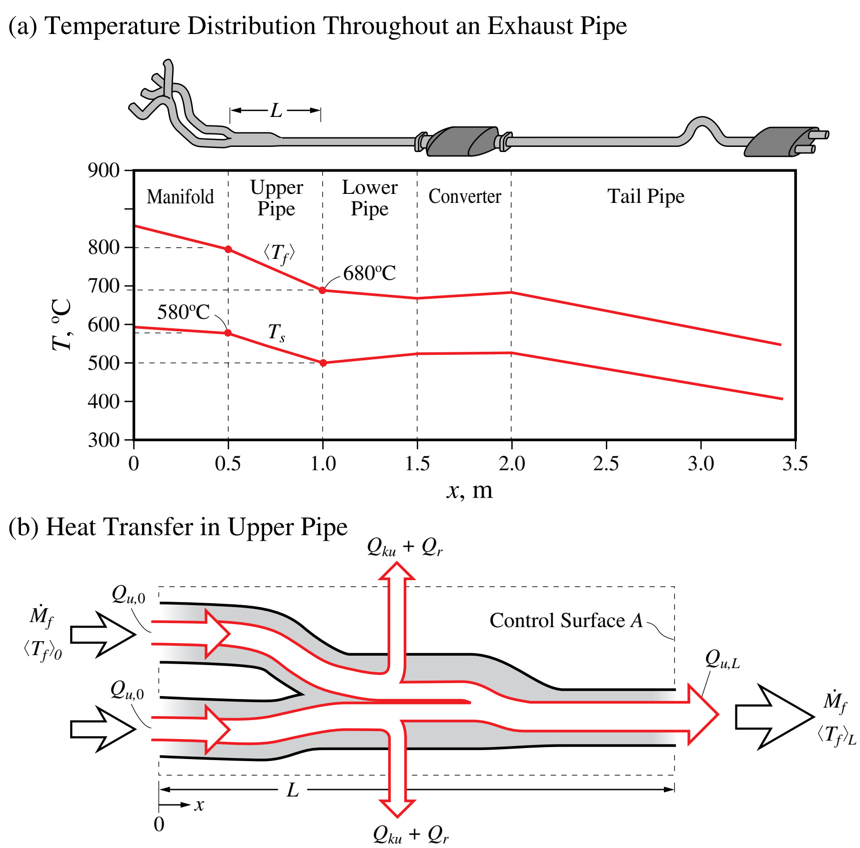


Figure Pr.1.16(a) and (b) Temperature distribution along a exhaust pipe.

OBJECTIVE:

For $\dot{M}_f = 0.10$ kg/s and $c_{p,f} = 1{,}000$ J/kg-K, and using the temperatures given in Figure Pr.1.16(a), determine the sum of the surface convection and radiation heat transfer rates.

SOLUTION:

The energy equation is

$$Q_{ku} + Q_r = -Q_{u,L} - Q_{u,0} = (\dot{M}c_p)_f(\langle T_f\rangle_0 - \langle T_f\rangle_L).$$

From Figure Pr.1.16(a), we have

$$\langle T_f\rangle_0 = 800^\circ\text{C} \qquad \text{and} \quad \langle T_f\rangle_L = 680^\circ\text{C}.$$

Then

$$Q_{ku} + Q_r = 0.10(\text{kg/s}) \times 1{,}000(\text{J/kg-K}) \times (800 - 680)(^\circ\text{C}) = 1.2 \times 10^4 \text{ W} = 12 \text{ kW}.$$

This heat flows out of the control volume (positive).

COMMENT:

The exhaust-gas temperature is most severe in the manifold and upper-pipe region of the exhaust line. Most catalytic converters require this temperature drop for safe and effective operation.

PROBLEM 1.17.FUN

GIVEN:

On a clear night with a calm wind, the surface of a pond can freeze even when the ambient air temperature is above the water freezing point ($T_{sl} = 0$°C). This occurs due to heat transfer by surface radiation $\mathbf{q}_r$(W/m^2) between the water surface and the deep night sky. These are shown in Figure Pr.1.17(a).

In order for freezing to occur and continue, the net heat flow rate from the ice surface must be enough to cool both the liquid water and the ice layer and also allow for the phase change of the water from liquid to solid. Assume that the ambient temperature is $T_\infty = 3$°C, the temperature of the deep sky is $T_{sky} = 0$ K, the earth atmosphere has an average temperature around $T_{atm} = 230$ K, and the temperature of the water at the bottom of the pool is $T_l = 4$°C. Then for this transient heat transfer problem between the deep sky, the ambient air, and the water pool:

SKETCH:

Figure Pr.1.17(a) to (c) show a pond and ice-layer growth resulting from the heat losses.

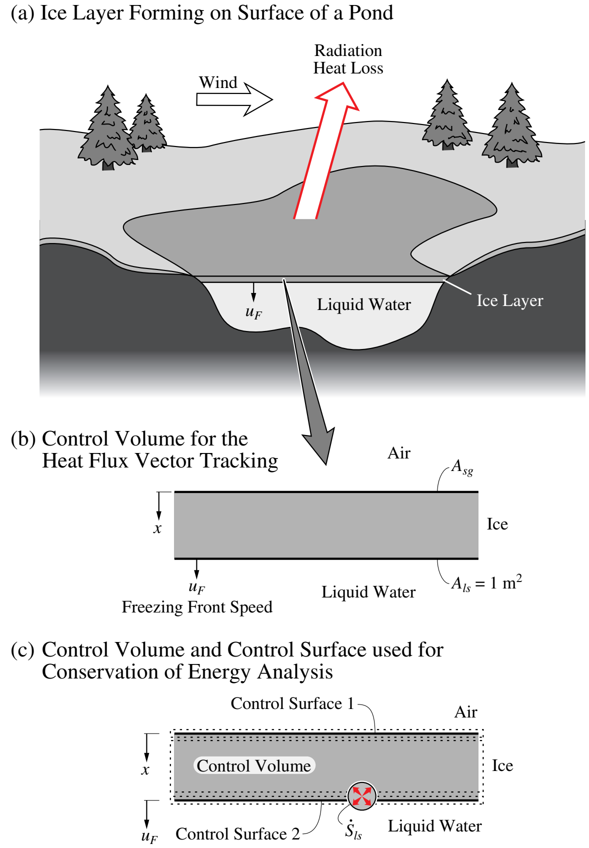


Figure Pr.1.17(a), (b), and (c) Ice formation at the surface of a pond and the control volume and control surfaces selected for heat transfer analysis.

OBJECTIVE:

(a) Track the heat flux vector for the section shown on Figure Pr.1.17(b),

(b) For the control volume and control surfaces shown in Figures Pr.1.17(b) and (c) apply the energy conservation equation (1.22). Note that the control volume and surfaces are for only the ice layer, i.e., the control surface 1 includes only the interface between the ice and the ambient air, and control surface 2 includes only the interface between the ice and the liquid water. For this problem the kinetic energy flux and all the work terms in (1.22) are negligible. For control surface 2 (water/ice interface), due to its zero mass (the control surface is wrapped around the interface), the sensible energy storage is zero but there is a latent heat generated due to the phase change from liquid to solid. (Later in Chapter 2, the latent heat will be separated from the sensible heat and treated as an energy conversion mechanism.) Therefore, for control surface 2, (1.22) becomes

$$Q|_A = \dot{S}_{ls} = -A_{ls}\dot{m}_{ls}\Delta h_{ls}.$$

To evaluate $Q|_A$, use (1.8).
(c) For control surface 2 (water/ice interface), at some elapsed time, the following data applies. The conduction heat flux in the ice is $q_{k,x} = +250$ W/m^2, the surface convection heat flux on the water side is $q_{ku,x} = -200$ W/m^2, the heat absorbed by the interface solidifying is $\dot{S}_{ls}/A_{ls} = -\dot{m}_{ls}\Delta h_{ls}$ where the heat of solidification $\Delta h_{ls} = -3.34 \times 10^5$ J/kg and $\dot{m}_{ls}$(kg/s-m^2) is the rate of solidification. For the density of ice use $\rho_s = 913$ kg/m^3. Then determine the speed of the ice/water interface movements u_F(m/s). Assume that the heat flux is one dimensional and $A_{ls} = 1$ m^2.

SOLUTION:
(a) The heat flux vector tracking is shown in Figure Pr.1.17(d).

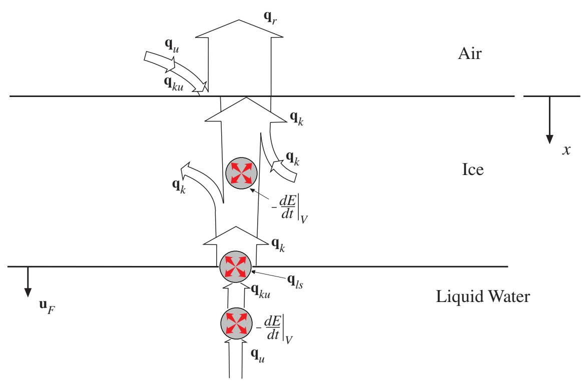


Figure Pr.1.17(d) Heat flux vector in the ice layer.

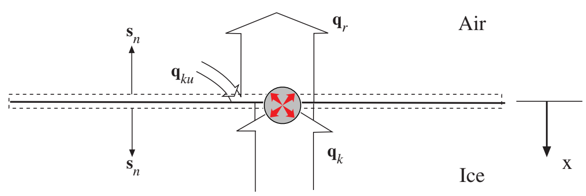


Figure Pr.1.17(e) Air-ice control surface.

(b) Application of the integral-volume energy conservation equation (1.22) to control surface 1, the control surface wrapped around the ice/air interface [Figure Pr.1.17(e)], and application of (1.8) gives

$$Q|_A = \int_A \mathbf{q} \cdot \mathbf{s}_n dA = q_{r,x=0}A_{sg} - q_{ku,x=0}A_{sg} - q_{k,x=0}A_{sg}.$$

Other terms in (1.22) are

$$-\left.\frac{dE}{dt}\right|_V = -\dot{E}_u|_A = \dot{W}_p|_A = \dot{W}_\mu|_A = \dot{W}_{g,e}|_V = \dot{S}_e|_V = 0.$$

Note that the energy storage term is zero because the control surface does not have any mass and no phase change occurs. Therefore, (1.22) becomes

$$A_{sg}(q_{r,x=0} - q_{ku,x=0} - q_{k,x=0}) = 0.$$

(c) For the control volume enclosing the ice layer [Figure Pr.1.17(f)], application of (1.8) gives

$$Q|_A = \int_A \mathbf{q} \cdot \mathbf{s}_n dA = q_{k,x=0}A_{sg} - q_{k,x=\delta_\alpha}A_{ls}.$$

Other terms in (1.22) are

$$-\dot{E}_u|_A = \dot{W}_p|_A = \dot{W}_\mu|_A = \dot{W}_{g,e}|_V = \dot{S}_e|_V = 0.$$

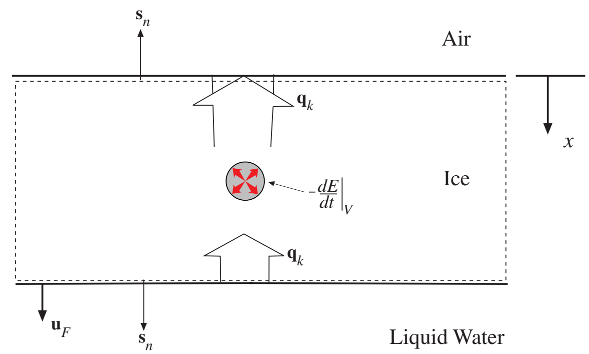


Figure Pr.1.17(f) Control volume for the ice layer.

The energy storage term is not zero because there is some mass inside the control volume (ice) and the temperature within the control volume changes with respect to time as the ice layer thickens. Therefore, (1.22) becomes

$$q_{k,x=0}A_{sg} - q_{k,x=\delta_\alpha}A_{ls} = -\left.\frac{\partial E}{\partial t}\right|_V .$$

Note that for this control volume, $A_{sg} = A_{ls}$.

For the control surface wrapped around the ice/water interface [Figure Pr.1.17(g)], application of (1.8) gives

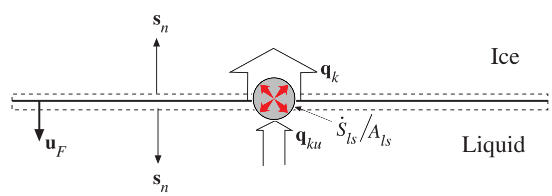


Figure Pr.1.17(g) Ice water control surface.

$$Q|_A = \int_A \mathbf{q}\cdot\mathbf{s}_n dA = q_{k,x=\delta_\alpha}A_{ls} - q_{ku,x=\delta_\alpha}A_{ls}.$$

Other terms in (1.22) are

$$\begin{aligned} -\dot{E}_u|_A &= \dot{W}_p|_A = \dot{W}_\mu|_A = \dot{W}_{g,e}|_V = \dot{S}_e|_V = 0 \\ -\left.\frac{dE}{dt}\right|_V &= \dot{S}_{ls}. \end{aligned}$$

The energy required for phase change at the interface is given by the variation of the internal energy of the water as it is transformed from liquid to solid. This is shown in Appendix B. Then, (1.22) becomes

$$q_{k,x=\delta_\alpha} - q_{ku,x=\delta_\alpha} = \dot{S}_{ls}/A_{ls}.$$

Noting that $\dot{S}_{ls}/A_{ls} = -\dot{m}_{ls}\Delta h_{ls}$, the solidification mass flux $\dot{m}_{ls}$ is determined from

$$q_{k,x=\delta_\alpha} - q_{ku,x=\delta_\alpha} = -\dot{m}_{ls}\Delta h_{ls}.$$

Solving for $\dot{m}_{ls}$ using the values given

$$-\dot{m}_{ls} = \frac{-250(\text{W/m}^2) - (-200)(\text{W/m}^2)}{-3.34\times 10^5(\text{J/kg})} = 1.497\times 10^{-4}\ \text{kg/m}^2\text{-s}.$$

The solidification mass flux is related to the velocity of the solidification front through

$$\dot{m}_{ls} = \rho_s u_F.$$

Solving for u_F and using the numerical values

$$u_F = \frac{1.497 \times 10^{-4}(\text{kg/m}^2\text{-s})}{913(\text{kg/m}^3)} = 1.640 \times 10^{-7}\text{m/s} = 0.5904 \text{ mm/hr}.$$

COMMENT:
(i) Phase change can be viewed as a form of energy conversion associated with the breaking or formation of physical bonds. Solidification involves formation of physical bonds and therefore is associated with a generation of thermal energy. Phase change will be explored in Chapters 2, 3, 6, and 7.
(ii) This transient problem also illustrates the role of the sensible heat as an energy storage mechanism. The energy equation for the control volume around the ice layer shows that the conduction heat flow vector at the ice surface includes contributions from the conduction heat flux vector at the bottom of the ice layer and from the sensible heat of the ice layer. Since the temperature of the ice layer decreases with time (i.e., cooling occurs), the energy storage term is positive.
(iii) Note that we have assumed that the ice layer is opaque to the thermal radiation. In general, this assumption holds for many solids. Further explanations are given in Chapters 2 and 4.

PROBLEM 1.18.FAM

GIVEN:

The temperature of the earth's surface and its atmosphere are determined by various electromagnetic energy conversions and, to a smaller extent, by the radioactive decay (within the earth) $\dot{S}_{r,\tau}$. These are shown in Figure Pr.1.18(a) [which is based on the materials presented in Figures Ex.1.2(a) and (b)]. Starting with solar irradiation $(q_{r,i})_s$, this irradiation is partly absorbed by the atmospheric gases $(\dot{S}_{e,\tau})_s$, partly reflected $(q_{r,i})_\rho$, and the remainder is absorbed by the earth's surface $(\dot{S}_{e,\alpha})_s$. The earth's surface also emits radiation $\dot{S}_{e,\epsilon}$ and this mostly infrared radiation is partly absorbed (mostly by the greenhouse gases, such as CO_2) in the atmosphere $(\dot{S}_{Pe,\tau})_i$ and this is in turn re-emitted $(\dot{S}_{e,\tau})_i = (\dot{S}_{e,\epsilon})_i$.

SKETCH:

Figure Pr.1.18(a) shows the various energy conversions.

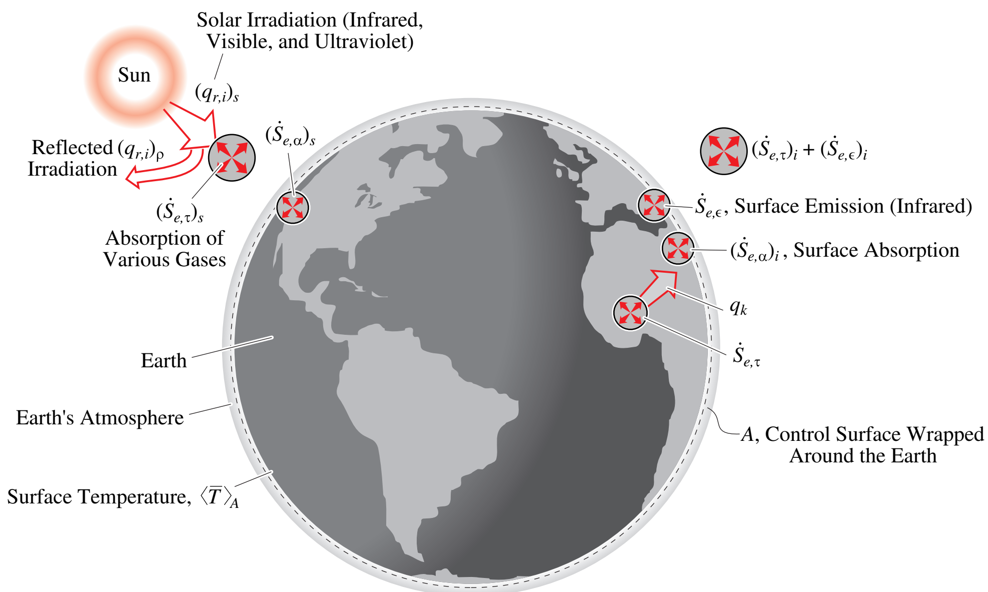


Figure Pr.1.18(a) Solar irradiation and internal radiation heating of the earth and its surface and infrared, radiation emission (part of this is absorbed and emitted by the earth atmosphere).

OBJECTIVE:

(a) Compute the heat flux vector tracking by drawing the radiation q_r and conduction q_k heat flux vectors arriving and leaving the earth control surface A, also shown in Figure Pr.1.18(a). Assume a steady-state heat transfer.
(b) Starting from (1.22) and assuming a steady state with the left-hand side approximated as $Q_k = -Aq_k$, $A = 4\pi R^2$, $q_k = 0.078$ W/m^2, and the right-hand side approximated by

$$(\dot{S}_{e,\alpha})_s + \dot{S}_{e,\epsilon} + (\dot{S}_{e,\alpha})_i,$$

where

$$\begin{aligned}(\dot{S}_{e,\alpha})_s &= A \times 172.4(\text{W/m}^2) \quad \text{time and space average solar irradiation}\\ \dot{S}_{e,\epsilon} + (\dot{S}_{e,\alpha})_i &= -A(1-\alpha_{r,i})\sigma_{\text{SB}}\langle \overline{T} \rangle_A^4, \quad \sigma_{\text{SB}} = 5.67 \times 10^{-8} \text{ W/m}^2\text{-K}^4,\end{aligned}$$

determine the time-space averaged earth surface temperature $\langle \overline{T} \rangle_A$ for $\alpha_{r,i} = 0.55$.

SOLUTION:

(a) The heat flux vector tracking is shown in Figure Pr.1.18(b). Starting from the solar irradiation, this is partly absorbed by the earth's atmosphere, partly reflected, and the remainder is absorbed by the earth's surface.
The earth's surface emits radiation (in the infrared wavelength range, which will be discussed in Chapter 4),

which is partly absorbed in the earth's atmosphere and then re-emitted and part of this is absorbed by the earth's surface.
The radioactive decay releases heat and this will make it turn into the earth's surface by conduction.

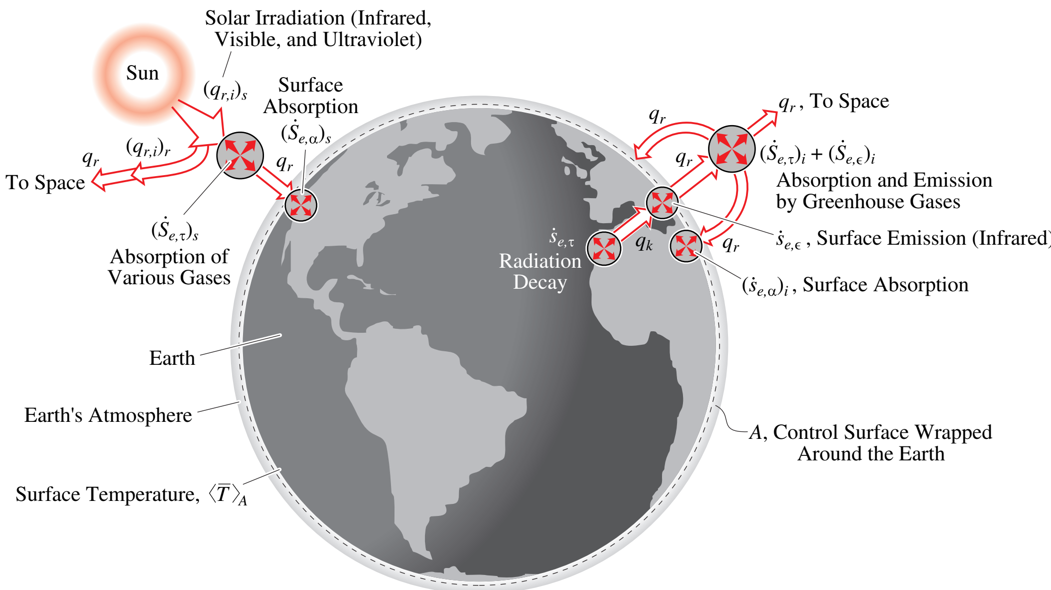


Figure Pr.1.18(b) Tracking of heat flux vector.

(b) The energy equation (1.22) becomes

$$\begin{aligned} Q|_A &= -Aq_k = -A \times (0.078)(\text{W/m}^2) \\ &= (\dot{S}_{e,\alpha})_s + \dot{S}_{e,\epsilon} + (\dot{S}_{e,\alpha})_i \\ &= A \times (172.4)(\text{W/m}^2) - A \times (1-\alpha_{r,i})\sigma_{\text{SB}}\langle \overline{T} \rangle_A^4 . \end{aligned}$$

Solving for $\langle \overline{T} \rangle_A$, we have

$$\begin{aligned} \langle \overline{T} \rangle_A &= \left[\frac{A \times (172.4)(\text{W/m}^2) + A \times (0.078)(\text{W/m}^2)}{A(1-\alpha_{r,i})\sigma_{\text{SB}}} \right]^{1/4} \\ &= \left[\frac{(172.4 + 0.078)(\text{W/m}^2)}{(1-0.55) \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4)} \right]^{1/4} = 286.7 \text{ K} = 13.55^\circ\text{C}. \end{aligned}$$

COMMENT:

The numerical value $\alpha_{r,i} = 0.55$ is used here as an approximate representation of the greenhouse effect. As $\alpha_{r,i}$ increases, due to an increase in the concentration of CO_2 and other greenhouse gases, $\langle \overline{T} \rangle_A$ increases, leading to global warming.

PROBLEM 1.19.FAM

GIVEN:

Sodium acetate (trihydrate) is used as a liquid-solid phase-change heater. It has a heat of melting of $\Delta h_{sl} = 1.86 \times 10^5$ J/kg and melts/freezes at $T_{ls} = 58$℃ [Table C.5(a)]. It can be kept in a sealed container (generally a plastic bag) as liquid in a metastable state down to temperatures as low as -5℃. Upon flexing a metallic disk within the liquid, nucleation sites are created at the disk surface, crystallization begins, heat is released, and the temperature rises. Consider a bag containing a mass $M = 100$ g of sodium acetate. Assume that the liquid is initially at $T = 58$℃ and that during the phase change the transient surface heat transfer rate (i.e., heat loss) is given by

$$Q|_A = Q_o(1 - t/\tau),$$

where $Q_o = 50$ W. This is shown in Figure Pr.1.19.

SKETCH:

Figure Pr.1.19 shows the liquid-solid phase-change hand warmer.

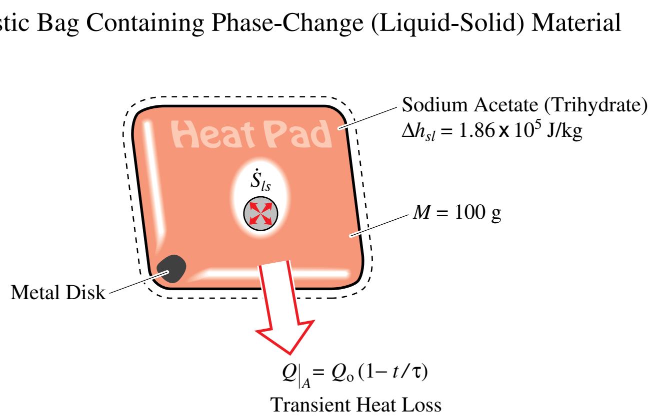


Figure Pr.1.19 Surface heat transfer from a plastic bag containing phase-change material.

OBJECTIVE:

Determine τ, the elapsed time during which all the heat released by phase change will be removed by this surface heat transfer.Start from (1.22) and replace the time rate of change of the internal energy with $-\dot{S}_{ls} = -\dot{M}_{ls}\Delta h_{ls} = \dot{M}_{ls}\Delta h_{sl}$. This represents isothermal phase change. Then in the absence of any other work and energy conversion, this change in internal energy balances with surface heat losses.

SOLUTION:

The variation of the total internal energy of the bag is due to phase change only. Then,

$$\left.\frac{\partial E}{\partial t}\right|_V \equiv -\dot{S}_{ls}$$

from (1.22) and neglecting all other energy and work form, we have

$$Q|_A = \dot{S}_{ls}.$$

Now, substituting for $Q|_A$, we have

$$Q_o\left(1 - \frac{t}{\tau}\right) = \dot{S}_{ls}.$$

Next, substituting for

$$\dot{S}_{ls} = -\dot{M}_{ls}\Delta h_{ls} = \dot{M}_{ls}\Delta h_{sl}$$

and integrating over the time interval of interest, we have

$$\int_0^\tau Q_o\left(1-\frac{t}{\tau}\right)dt = \int_0^\tau \dot{M}_{ls}\Delta h_{sl} dt$$
$$Q_o\left(t-\frac{t^2}{2\tau}\right)\Big|_0^\tau = \dot{M}_{ls}\Delta h_{sl} t\Big|_0^\tau .$$

Noting that $\dot{M}_{ls}\tau = M_{ls}$,

$$Q_o\frac{\tau}{2} = M_{ls}\Delta h_{sl}.$$

Solving for τ,

$$\tau = \frac{2M_{ls}\Delta h_{sl}}{Q_o}.$$

Using the numerical values given,

$$\tau = \frac{2\times 0.1(\text{kg})\times 1.86\times 10^5(\text{J/kg})}{50(\text{W})} = 744 \text{ s} = 0.207 \text{ hr}.$$

COMMENT:
In general, since the liquid is critically in a subcooled state, part of the heat released will be used to raise the temperature of the solid formed at the freezing temperature.

PROBLEM 1.20.FAM

GIVEN:

Nearly all of the kinetic energy of the automobile is converted into friction heating $\dot{S}_{m,F}$ during braking. The front wheels absorb the majority of this energy. Figure Pr.1.20(a) shows a disc brake. This energy conversion raises the rotor temperature T_r and then heat flows from the rotor by conduction (to axle and wheel), by surface radiation to the surroundings and by surface convection to the air. The air flows over the rotor in two parts; one is over the inboard and outboard surfaces, and the other is through the vanes (passages). The air flow is due mostly to rotation of the rotor (similar to a turbomachinary flow).

Assume that the rotor is at a uniform temperature (this may not be justifiable during rapid braking).

$M_r = 15$ kg, and $c_v = 460$ J/kg-K.

SKETCH:

Figure Pr.1.20(a) shows the disc brake and the air streams. An automobile disc brake is heated by friction heating $\dot{S}_{m,F}$, and cooled by various heat transfer mechanisms, and is able to store/release heat.

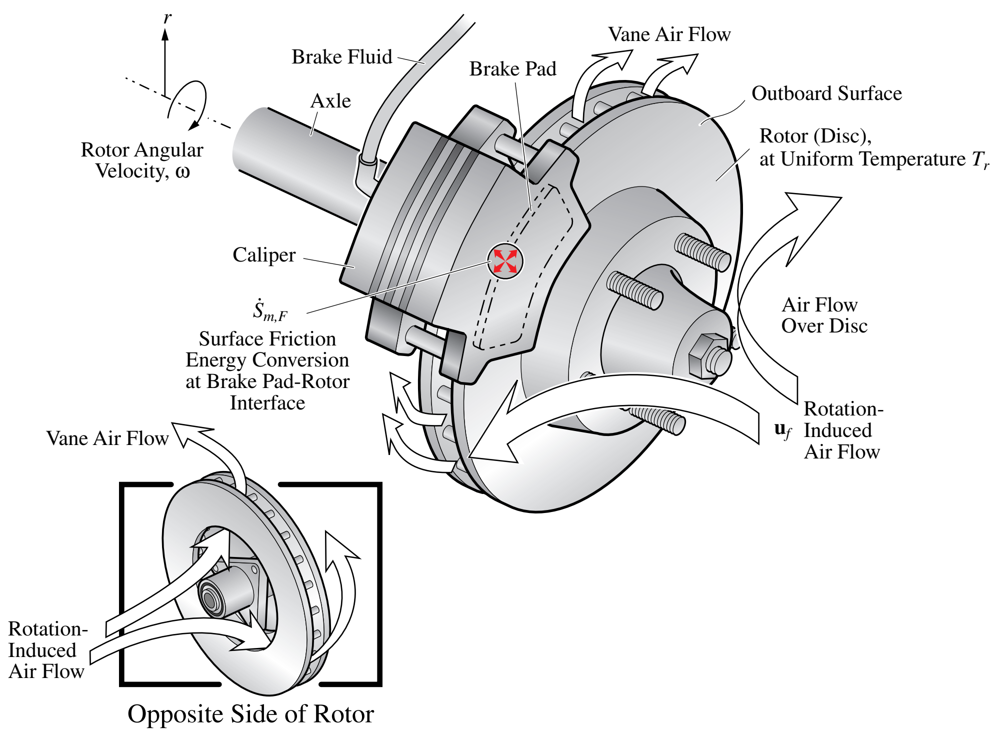


Figure Pr.1.20(a) An automobile disc brake showing the air flow over the disc and through the rotor vanes.

OBJECTIVE:

(a) Draw the heat flux vector tracking for the rotor, by allowing for the heat transfer mechanisms mentioned above.

(b) Now consider the heat storage/release mechanism represented by $-\partial E/\partial t$, in (1.22). During quick brakes, the rate of heat transfer $Q_{A,r}$ is much smaller than $\partial E/\partial t$ and $\dot{S}_{m,F}$. Assume all other terms on the right-hand side of (1.22) are negligible. With no heat transfer, determine the rate of rise in the rotor temperature dT/dt, using

$$\frac{\partial E}{\partial t} = M_r c_v \frac{dT_r}{dt},$$

$M_r = 15$ kg, and $c_v = 460$ J/kg-K.

SOLUTION:

(a) Figure Pr.1.20(b) shows the various heat transfer from the rotor and the tracking of the heat flux vector.

The conduction $\mathbf{q}_k$ is to the lower temperature axle and wheel (there are various materials, areas, and contacts through which the heat flows). The surface radiation heat transfer $\mathbf{q}_r$ is to various close and distant surfaces. The surface convection $\mathbf{q}_{ku}$ is to the air flowing over the rotor (semi-bounded air streams) and to air flowing through the vanes (bounded fluid stream). The heat is added to these convection streams $\mathbf{q}_u$ by surface convection $\mathbf{q}_{ku}$.

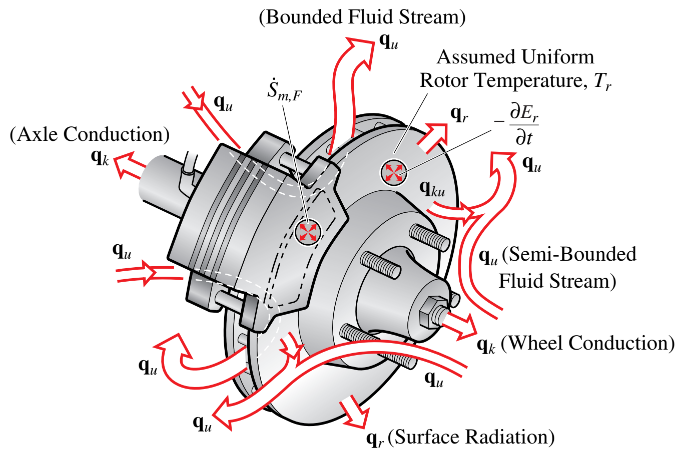


Figure Pr.1.20(b) Tracking of the heat flux vector.

(b) From (1.22), we have

$$
\begin{aligned}
Q|_{A,r} &= 0 = -\frac{\partial E}{\partial t} + \dot{S}_{m,F} \\
\frac{\partial E}{\partial t} &= \dot{M}_r c_{v,r} \frac{dT_r}{dt} = \dot{S}_{m,F} \\
\frac{dT_r}{dt} &= \frac{\dot{S}_{m,F}}{\dot{M}_r c_{v,r}} \\
&= \frac{6 \times 10^4 (\mathrm{W})}{15(\mathrm{kg}) \times 460(\mathrm{J/kg\text{-}K})} = 8.696℃/\mathrm{s}.
\end{aligned}
$$

COMMENT:

The heat transfer through the vanes is the most effective during the cooling period. Note that when multiple brakes are applied (as in the down-hill driving) the temperature of the rotor can become very large (and damaging to the brake pad). In Chapters 3, 4, 6, and 7, we will address conduction, radiation, and semi-bounded and bounded fluid stream surface convection.

Chapter 2

Energy Equation

PROBLEM 2.1.FAM

GIVEN:
Consider a steady-state, two-dimensional heat flux vector field given by

$$\mathbf{q} = 3x^2 \ \mathbf{s_x} + 2xy \ \mathbf{s_y}.$$

The control volume is centered at $x = a$ and $y = b$, with sides $2\Delta x$ and $2\Delta y$ (Figure Pr.2.1).

SKETCH:
Figure Pr.2.1 shows a control volume centered at $x = a$ and $y = b$ with side widths of $2\Delta x$ and $2\Delta y$.

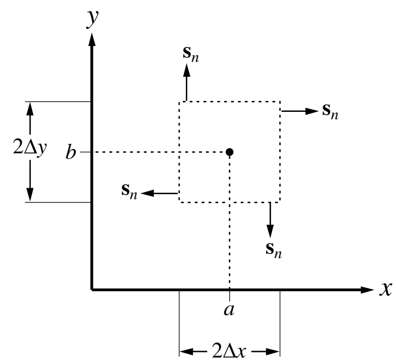


Figure Pr.2.1 A finite control volume in a two-dimensional heat transfer medium.

The depth (along z direction) is w.

OBJECTIVE:
(a) Using the above expression for $\mathbf{q}$ show that

$$\lim_{\Delta V \to 0} \frac{\int_A \mathbf{q} \cdot \mathbf{s}_n \ dA}{\Delta V} = \nabla \cdot \mathbf{q} \,,$$

where the divergence of the heat flux vector is to be evaluated at $x = a$ and $y = b$.
Use a length along z of w (this will not appear in the final answers). (Hint: Show that you can obtain the same final answer starting from both sides.)
(b) If the divergence of the heat flux vector is nonzero, what is the physical cause?
(c) In the energy equation (2.1), for this net heat flow (described by this heat flux vector field), is the sum of the volumetric terms on the right, causing the nonzero divergence of $\mathbf{q}$, a heat source or a heat sink? Also is this a uniform or nonuniform volumetric source or sink? Discuss the behavior of the heat flux field for both positive and negative values of x and y.

SOLUTION:
(a) To prove the validity of $\mathbf{q}$ above for the region shown in Figure Pr.2.1, we calculate separately the left-hand side and the right-hand side. For the right-hand side we have

$$\nabla \cdot \mathbf{q} = \left(\mathbf{s}_x \frac{\partial}{\partial x} + \mathbf{s}_y \frac{\partial}{\partial y} \right) \cdot \left(3x^2 \ \mathbf{s}_x + 2xy \ \mathbf{s}_y \right).$$

Performing the dot product we have

$$\nabla \cdot \mathbf{q} = \frac{\partial \left(3x^2\right)}{\partial x} + \frac{\partial \left(2xy\right)}{\partial y},$$

which results in

$$\nabla \cdot \mathbf{q} = 6x + 2x = 8x.$$

Applying the coordinates of the center of the control volume, we have finally

$$\nabla \cdot \mathbf{q}|_{(x=a,\ y=b)} = 8a.$$

The left-hand side can be divided into four integrals, one for each of the control surfaces:
(i) Control Surface at $x = a - \Delta x$:
The heat flux vector across this control surface and the normal vector are

$$\mathbf{q}_1 = 3(a-\Delta x)^2\ \mathbf{s}_x + 2(a-\Delta x)y\ \mathbf{s}_y$$
$$\mathbf{s}_{n_1} = -\mathbf{s}_x.$$

The dot product between $\mathbf{q}$ and $\mathbf{s}_n$ is

$$\mathbf{q}_1 \cdot \mathbf{s}_{n_1} = 3(a-\Delta x)^2(\mathbf{s}_x \cdot -\mathbf{s}_x) + 2(a-\Delta x)y(\mathbf{s}_y \cdot 0) = -3(a-\Delta x)^2.$$

The net heat flow over this control surface is

$$Q|_{A_1} = \int_{A_1} (\mathbf{q}_1 \cdot \mathbf{s}_{n_1})\ dA = \int_{b-\Delta y}^{b+\Delta y} -3(a-\Delta x)^2 dyw = -6(a-\Delta x)^2 \Delta y w.$$

(ii) Control Surface at $x = a + \Delta x$:
The heat flux vector across this control surface and the normal vector are

$$\mathbf{q}_2 = 3(a+\Delta x)^2\ \mathbf{s}_x + 2(a+\Delta x)y\ \mathbf{s}_y$$
$$\mathbf{s}_{n_2} = \mathbf{s}_x.$$

The net heat flow over this control surface is

$$Q|_{A_2} = \int_{A_2} (\mathbf{q}_2 \cdot \mathbf{s}_{n_2})\ dA = \int_{b-\Delta y}^{b+\Delta y} 3(a+\Delta x)^2 dyw = 6(a+\Delta x)^2 \Delta y w.$$

(iii) Control Surface at $y = b - \Delta y$:
The heat flux vector across this control surface and the normal vector are

$$\mathbf{q}_3 = 3x^2\ \mathbf{s}_x + 2x(b-\Delta y)\ \mathbf{s}_y$$
$$\mathbf{s}_{n_3} = -\mathbf{s}_y.$$

The net heat flow over this control surface is

$$\begin{aligned} Q|_{A_3} &= \int_{A_3} (\mathbf{q}_3 \cdot \mathbf{s}_{n_3})\ dA = \int_{a-\Delta x}^{a+\Delta x} -2x(b-\Delta y)dxw \\ &= -2(b-\Delta y)\frac{(a+\Delta x)^2 - (a-\Delta x)^2}{2}w = -4a\Delta x\,(b-\Delta y)\,w. \end{aligned}$$

(iv) Control Surface at $y = b + \Delta y$:
The heat flux vector across this control surface and the normal vector are

$$\mathbf{q}_4 = 3x^2\ \mathbf{s}_x + 2x(b+\Delta y)\ \mathbf{s}_y$$
$$\mathbf{s}_{n_4} = \mathbf{s}_y.$$

The net heat flow over this control surface is

$$\begin{aligned} Q|_{A_4} &= \int_{A_4} (\mathbf{q}_4 \cdot \mathbf{s}_{n_4})\ dA = \int_{a-\Delta x}^{a+\Delta x} 2x(b+\Delta y)dxw \\ &= 2(b+\Delta y)\frac{(a+\Delta x)^2 - (a-\Delta x)^2}{2}w = 4a\Delta x\,(b+\Delta y)\,w. \end{aligned}$$

Adding up the heat flow across all the surfaces, we have

$$\begin{aligned} Q|_A &= Q|_{A_1} + Q|_{A_2} + Q|_{A_3} + Q|_{A_4} \\ &= \int_{A_1} \mathbf{q}_1 \cdot \mathbf{s}_{n_1}\ dA + \int_{A_2} \mathbf{q}_2 \cdot \mathbf{s}_{n_2}\ dA + \int_{A_3} \mathbf{q}_3 \cdot \mathbf{s}_{n_3}\ dA + \int_{A_4} \mathbf{q}_4 \cdot \mathbf{s}_{n_4}\ dA \\ &= [-6(a-\Delta x)^2\Delta y + 6(a+\Delta x)^2\Delta y - 4a\Delta x\,(b-\Delta y) + 4a\Delta x\,(b+\Delta y)]w \\ &= (24a\Delta x\Delta y + 8a\Delta x\Delta y)w = 32a\Delta x\Delta y w. \end{aligned}$$

Now, applying the limit

$$\lim_{\Delta V \to 0} \frac{\int_A \mathbf{q} \cdot \mathbf{s}_n \, dA}{\Delta V} = \lim_{\Delta x, \Delta y \to 0} \frac{32a\Delta x \Delta y w}{(2\Delta x)(2\Delta y)w} = \lim_{\Delta x, \Delta y \to 0} 8a = 8a,$$

which is identical to the result found before. These are the two methods of determining the divergence of the heat flux vector for a given location in the heat transfer medium.

(b) Since this is a steady-state heat flux vector field (i.e., $\mathbf{q}$ is not a function of time t), the only reason not to have a divergence-free field would be the presence of a heat generation or sink. In this case, the differential-volume energy equation is

$$\nabla \cdot \mathbf{q} = \sum_i \dot{s}_i.$$

The heat generation or sink is caused by the conversion of work or other forms of energy to thermal energy. In the energy equation, these energy conversions are called source terms. The source terms $\dot{s}_i$ could be due to
(i) conversion from physical or chemical bond to thermal energy
(ii) conversion from electromagnetic to thermal energy
(iii) conversion from mechanical to thermal energy

(c) The divergence of the heat flux vector $\mathbf{q}$ given above is $8x$. For $x > 0$, this is a positive source term indicating a heat generation. For $x < 0$, the source term becomes negative indicating a heat sink. Also, since the source term is a function of x, it is a nonuniform source term in the x direction and a uniform source term in the y direction.

COMMENT:

The application of the divergence operator on the heat flux vector (as in the differential-volume energy equation) results in an expression valid for any position x and y while the application of the area-integral (as in the integral-volume energy equation) results on a number which is valid only for that specific point in space $x = a$ and $y = b$. The integral form of the energy equation gives an integral or overall energy balance over a specified closed region within the medium, while the differential form is pointwise valid, i.e., is satisfied for any point within the medium.

For the control surfaces parallel to the x axis, the dot product between the heat flux vector and the surface normal was a function of x (variable). That required the integration along x. The integration is simplified in the case of a constant heat flux vector normal to the control surface, as obtained for the control surfaces parallel to the y axis. Although the first case is more general, here we will mainly deal with situations in which the heat flux normal to the control surface is constant along the control surface. This will allow the use of the thermal resistance concept and the construction of thermal resistance network models, as it will be discussed starting in Chapter 3.

PROBLEM 2.2.FUN

GIVEN:

Figure Pr.2.2(a) shows a flame at the mouth of a cylinder containing a liquid fuel. The heat released within the flame (through chemical reaction) is transferred to the liquid surface by conduction and radiation and used to evaporate the fuel (note that a flame also radiates heat). The flame stabilizes in the gas phase at a location determined by the local temperature and the fuel and oxygen concentrations. The remaining heat at the flame is transferred to the surroundings by convection and by radiation and is transferred to the container wall by conduction and radiation. This heat then conducts through the container wall and is transferred to the ambient, by surface convection and radiation, and to the liquid fuel, by surface convection. The container wall and the liquid fuel also lose some heat through the lower surface by conduction. Figure Pr.2.2(b) shows a cross section of the container and the temperature profiles within the gas and liquid and within the container wall. In small- and medium-scale pool fires, the heat recirculated through the container wall accounts for most of the heating of the liquid pool. Assume that the liquid pool has a make-up fuel line that keeps the fuel level constant and assume that the system has been operating under steady state (long enough time has elapsed).

SKETCH:

Figures Pr.2.2(b) and (c) show a cross-sectional view of the container and the temperature distribution along the gas and liquid and along the container wall.

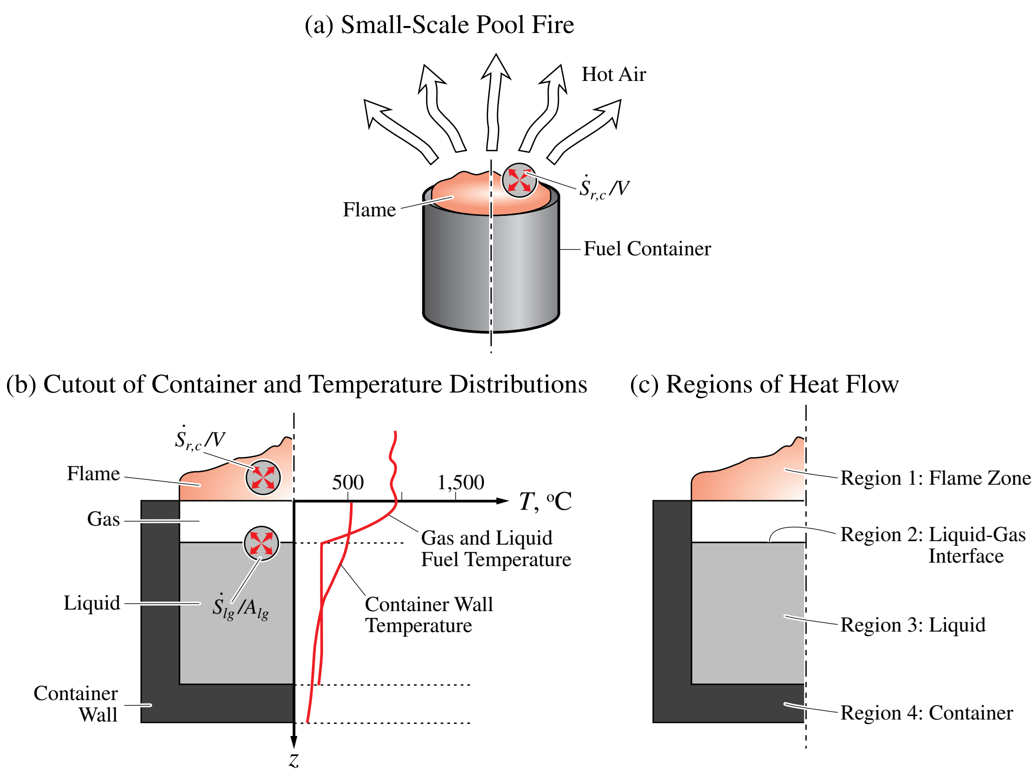


Figure Pr.2.2(a), (b), and (c) A small-scale pool fire showing the various regions.

OBJECTIVE:

(a) On Figure Pr.2.2(b) track the heat flux vector, identifying the various mechanisms.
(b) For the regions shown in Figure Pr.2.2(c), apply the integral-volume energy equation. Note that region 1 encloses the flame and it is assumed that the fuel vapor burns completely. Region 2 surrounds the liquid/gas interface, region 3 encloses the liquid, and region 4 is the container wall.
(c)For each of the regions state whether the area-integral of the heat flux vector $Q|_A$ is equal to zero or not.

SOLUTION:

(a) The heat flux vector tracking is shown in Figure Pr.2.2(d).

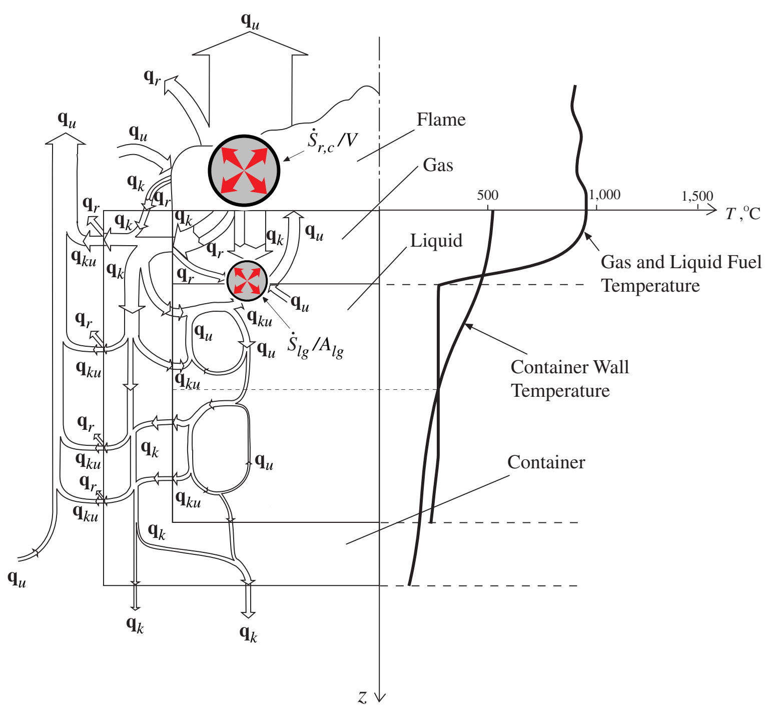


Figure Pr.2.2(d) Heat flux vector tracking around the container wall.

(b) For each of the regions shown on Figure Pr.2.2(c), the integral energy balances are given below.

(i) Region 1: Flame region [Figure Pr.2.2(e)]

We assume that the heat flux vectors normal to surfaces 1 and 2 are uniform along those surfaces and then using the notations in Figures Pr.2.2(b) and (c), referencing the products of combustion as (p), and noting that q is positive when pointing away from the surface, we have,

$$(q_{r,\text{F-}a} + q_{u,p} - q_{u,a})A_1 + (q_{k,\text{F-}c} + q_{r,\text{F-}c} + q_{k,\text{F-}l} + q_{r,\text{F-}l} + q_{k,\text{F-}t} + q_{r,\text{F-}t} - q_{u,fg})A_2 = \dot{S}_{r,c}.$$

Region 1: Flame region

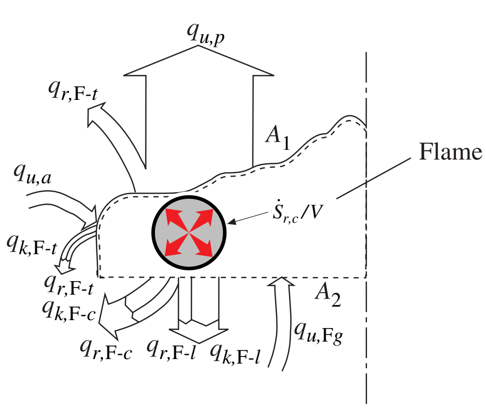


Figure Pr.2.2(e) Heat flux vector tracking in the flame region.

(ii) Region 2: Liquid/gas interface [Figure Pr.2.2(f)]

Assuming that the heat flux vectors normal to surfaces 1 and 2 are uniform along those surfaces,

$$(-q_{r,c\text{-}l} - q_{k,\text{F-}l} - q_{r,\text{F-}l} + q_{u,fg})A_1 + (-q_{ku,l\text{-}i} - q_{u,\text{F}l})A_2 = \dot{S}_{lg}.$$

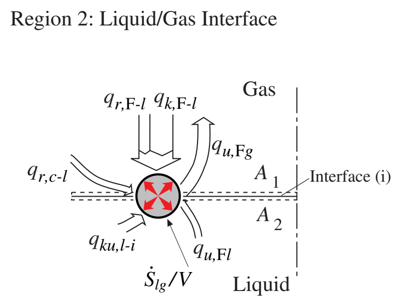


Figure Pr.2.2(f) Heat flux vector tracking at liquid-gas interface.

(iii) Region 3: Liquid [Figure Pr.2.2(g)]
The heat flux vector normal to surfaces 1 and 2 will be assumed uniform, while for surface 3 it will be assumed

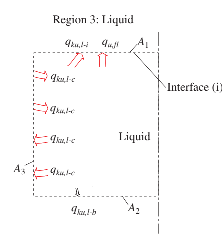


Figure Pr.2.2(g) Heat flowing in and out of the liquid.

nonuniform (i.e., distributed along the height), i.e.,

$$(q_{ku,l\text{-}i} + q_{u,\mathrm{F}l})A_1 + (q_{ku,l\text{-}b})A_2 + \int_{A_3} (\mathbf{q}_{ku,l\text{-}c} \cdot \mathbf{s}_{n,3})dA = 0.$$

(iv) Region 4: Container wall [Figure Pr.2.2(h)]
The heat flux vector leaving surfaces 1 and 2 will be assumed uniform while the heat flux vectors at surfaces 3,4, and 5 will be assumed nonuniform (i.e., distributed along the wall height), i.e.,

$$(-q_{k,c\text{-}t})A_1 + (q_{k,c\text{-}b})A_2 + \int_{A_3} (\mathbf{q}_{k,\mathrm{F}\text{-}c} \cdot \mathbf{s}_{n,3})dA +$$
$$\int_{A_4} (\mathbf{q}_{k,l\text{-}c} \cdot \mathbf{s}_{n,4})dA + \int_{A_5} (\mathbf{q}_{k,c\text{-}a} \cdot \mathbf{s}_{n,5})dA = 0.$$

(c) The divergence of the heat flux vector is zero everywhere inside regions 3 and 4 because no heat sources or sinks are present within these regions. It is greater than zero in region 1, due to the volumetric chemical reaction. At the liquid-gas interface (region 2) the integral of the surface heat flow is less than zero, due to surface phase change from liquid to gas.

COMMENT:
(i) In small-scale pool fires, the heat recirculation from the container wall to the liquid pool accounts for most of the heating and evaporation of the liquid. In large-scale pool fires (large diameter containers), the effect of this heat recirculation is small.

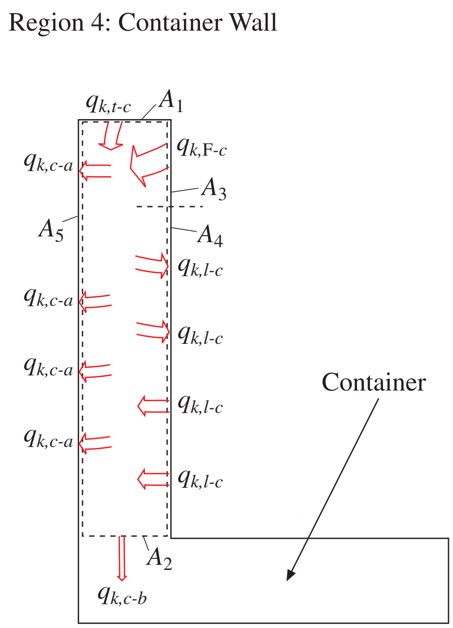


Figure Pr.2.2(h) Heat flowing in and out of the container wall.

(ii) The chemical reaction inside the flame generates heat and it is a positive source term in the energy equation. The liquid to gas phase change at the liquid surface absorbs heat and is a negative source term for the interfacial, integral energy conservation equation.

(iii) For no heat loss from the flame to the ambient or to the liquid surface, there would be a balance between the energy entering the flame by convection carried by the air and the vapor fuel (reactants), the energy leaving the flame by convection carried away by the hot combustion gases (products), and the energy generated inside the flame by the exothermic chemical reaction (combustion). This energy balance determines the adiabatic flame temperature (maximum temperature the combustion gases can reach). This will be discussed in Chapter 5.

PROBLEM 2.3.FUN

GIVEN:

The wall of the burning fuel container is made of a metal, its thickness is small compared to its length, and the surface-convection heat fluxes at the inner and outer surfaces of the container wall are designated by $q_{ku,o}$ and $q_{ku,i}$. Under these conditions, the temperature variation across the wall thickness is negligibly small, when compared to the axial temperature variation. Also, assume that the heat transfer from within the container is axisymmetric (no angular variation of temperature). Figure Pr.2.3(a) shows the differential control volume (with thickness Δz), the inner radius R_i, and outer radius R_o of the container.

SKETCH:

Figures Pr.2.3(a) shows a control volume with a differential length along the z direction.

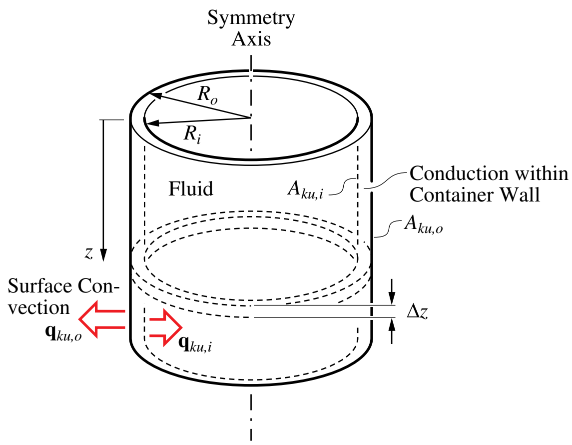


Figure Pr.2.3(a) A cylindrical container with a control volume having a differential length along the z direction.

OBJECTIVE:

(a) Apply a combined integral- and differential-length analysis for the container wall (integral along the radius and polar angle and differential along the z axis) and derive the corresponding combined integral- and differential-length energy conservation equation.

(b) Sketch the anticipated variations of the conduction heat flux $q_{k,z}$ and the wall temperature T, along the container wall (as a function of z).

SOLUTION:

(a) The integral- and differential-length analysis starts with the differential form of the energy conservation equation (2.9) written as

$$\lim_{\Delta V \to 0} \frac{\int_A \mathbf{q} \cdot \mathbf{s}_n dA}{\Delta V} = -\frac{\partial}{\partial t}\rho c_p T + \sum_i \dot{s}_i.$$

For the container walls there is no energy conversion and, as this is a steady-state process, the differential-volume energy conservation equation becomes

$$\lim_{\Delta V \to 0} \frac{\int_A \mathbf{q} \cdot \mathbf{s}_n dA}{\Delta V} = 0.$$

Figure Pr.2.3(b) shows a cross section of the control volume shown in Figure Pr.2.3(a) with the heat flux vectors crossing the control surfaces. For the four control surfaces labeled, the area integral above becomes

$$\frac{\int_A \mathbf{q} \cdot \mathbf{s}_n dA}{\Delta V} = \frac{\int_{A_o} \mathbf{q}_{ku} \cdot \mathbf{s}_n dA}{\Delta V} + \frac{\int_{A_i} \mathbf{q}_{ku} \cdot \mathbf{s}_n dA}{\Delta V} + \frac{\int_{A_z} \mathbf{q}_{k,z} \cdot \mathbf{s}_n dA}{\Delta V} + \frac{\int_{A_{z+\Delta_z}} \mathbf{q}_{k,z+\Delta z} \cdot \mathbf{s}_n dA}{\Delta V}$$

For this infinitesimal control volume, the heat flux vectors normal to the control surfaces are uniform over each control surface and we have

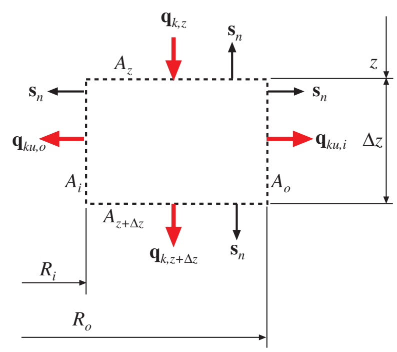


Figure Pr.2.3(b) Cross section of the control volume.

$$
\begin{aligned}
\frac{\int_A \mathbf{q}\cdot\mathbf{s}_n dA}{\Delta V} &= \frac{q_{ku,o}A_o}{\Delta V} + \frac{q_{ku,i}A_i}{\Delta V} + \frac{-q_{k,z}A_z}{\Delta V} + \frac{q_{k,z+\Delta z}A_{z+\Delta z}}{\Delta V} \\
&= \frac{q_{ku,o}(2\pi R_o \Delta z)}{\pi(R_o^2 - R_i^2)\Delta z} + \frac{q_{ku,i}(2\pi R_i \Delta z)}{\pi(R_o^2 - R_i^2)\Delta z} + \\
&\quad \frac{-q_{k,z}\pi(R_o^2 - R_i^2)}{\pi(R_o^2 - R_i^2)\Delta z} + \frac{q_{k,z+\Delta z}\pi(R_o^2 - R_i^2)}{\pi(R_o^2 - R_i^2)\Delta z} \\
&= \frac{q_{ku,o}2R_o}{R_o^2 - R_i^2} + \frac{q_{ku,i}2R_i}{R_o^2 - R_i^2} + \frac{q_{k,z+\Delta z} - q_{k,z}}{\Delta z}.
\end{aligned}
$$

Taking the limit as $\Delta V \rightarrow 0$, this becomes

$$
\begin{aligned}
\lim_{\Delta V\rightarrow 0}\frac{\int_A \mathbf{q}\cdot\mathbf{s}_n dA}{\Delta V} &= \lim_{\Delta z\rightarrow 0}\left(\frac{q_{ku,o}2R_o}{R_o^2 - R_i^2} + \frac{q_{ku,i}2R_i}{R_o^2 - R_i^2} + \frac{q_{k,z+\Delta z} - q_{k,z}}{\Delta z}\right) \\
&= \frac{q_{ku,o}2R_o}{R_o^2 - R_i^2} + \frac{q_{ku,i}2R_i}{R_o^2 - R_i^2} + \frac{dq_{k,z}}{d_z}
\end{aligned}
$$

Finally, rearranging the right-hand side, the combined integral- and differential-length energy equation becomes

$$
\frac{dq_{k,z}}{dz} = -\frac{2}{R_o^2 - R_i^2}(q_{ku,o}R_o + q_{ku,i}R_i).
$$

(b) The anticipated variation of the axial conduction heat flux vector along the container wall $q_{k,z}$ as a function of z is given in Figure Pr.2.3(c).

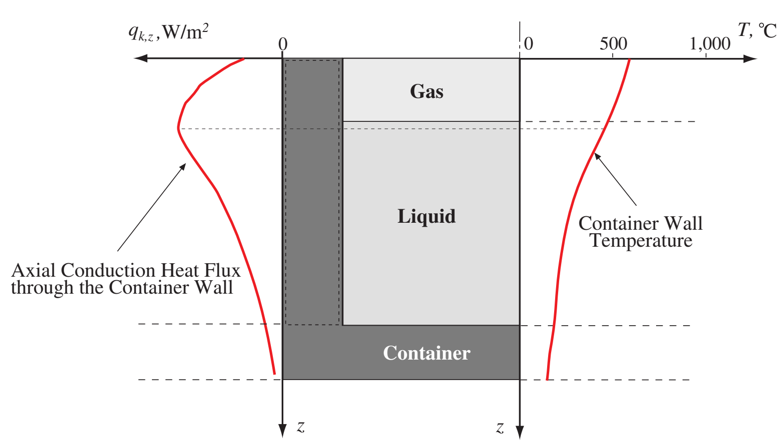


Figure Pr.2.3(c) Distribution of temperature and heat flux along the container wall.

COMMENT:

The direction of the axial conduction heat flux vectors $q_{k,z}$ and $q_{k,z+\Delta z}$ are taken along the direction of the z axis. The direction for the heat flux vectors along the integral length $\mathbf{q}_{ku,o}$ and $\mathbf{q}_{ku,i}$ are arbitrary and are conventionally taken as pointing outward from the control surface.

The axial conduction heat flux vector along the container wall may be obtained from the solution to the above energy equation, once $q_{ku,o}(z)$ and $q_{ku,i}(z)$ are known.

The axial conduction heat flux vector along the container wall is maximum at some point near the interface level. Above the interface, the wall receives heat from the flame. Below the interface, the wall loses heat to the liquid and the maximum heat loss occurs at the interface location.

In Figure Pr.2.3(c), a positive value for $q_{k,z}$ indicates that heat is flowing in the direction of the z-axis. This is in accordance with the reference directions assumed in Figure Pr.2.3(b). Note that the conduction heat flux vector is related to temperature variation through $\mathbf{q}_k = -k\nabla T$. From the temperature distribution shown in Figure Pr.2.3(c), ∇T is negative (T decreases as z increases). Therefore, $q_{k,z}$ is positive everywhere for this temperature distribution, as shown in Figure Pr.2.3(c).

PROBLEM 2.4.FUN

GIVEN:

A nitrogen meat freezer uses nitrogen gas from a pressurized liquid nitrogen tank to freeze meat patties as they move carried by a conveyor belt. The nitrogen flows inside a chamber in direct contact with the meat patties, which move in the opposite direction. The heat transfer mechanism between the nitrogen gas and the meat patties is surface convection. Meat patties are to be cooled down from their processing (initial) temperature of $T_i = 10$℃ to the storage (final) temperature of $T_o = -15$℃. Each meat patty has a mass $M = 80$ g, diameter $D = 10$ cm, and thickness $l = 1$ cm. Assume for the meat the thermophysical properties of water, i.e., specific heat in the solid state $c_{p,s} = 1{,}930$ J/kg-K , specific heat in the liquid state $c_{p,l} = 4{,}200$ J/kg-K, heat of solidification $\Delta h_{ls} = -3.34 \times 10^5$ J/kg, and freezing temperature $T_{ls} = 0$℃. The average surface-convection heat transfer between the nitrogen and the meat patties is estimated as $q_{ku} = 4{,}000$ W/m^2 and the conveyor belt moves with a speed of $u_c = 0.01$ m/s.

OBJECTIVE:

(a) Sketch the temperature variation of a meat patty as it move along the freezing chamber.
(b) Neglecting the heat transfer between the conveyor belt and the meat patties, find the length of the freezing chamber. Use the simplifying assumption that the temperature is uniform within the meat patties. This allows the use of a zeroth-order analysis (lumped-capacitance analysis).

SOLUTION:

(a) The temperature variation of the meat patties as they move along the freezing chamber is given in Figure Pr.2.4.

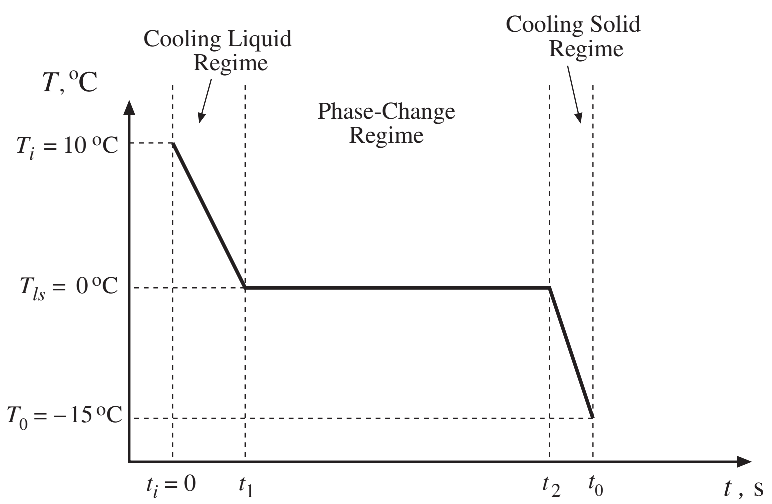


Figure Pr.2.4 Variation of meat patty temperature with respect to time.

(b) To calculate the necessary length for the freezing chamber, the cooling process is divided into three regimes (shown in Figure Pr.2.4).
(i) Regime 1: Cooling of Liquid
During this period of time, the meat patties are cooled from their initial temperature down to the solidification temperature. Application of the integral-volume energy equation for a control volume enclosing the meat gives

$$\int_{A_{ku}} \mathbf{q}_{ku} \cdot \mathbf{s}_n dA = \int_V \left(-\frac{d}{dt}\rho c_p T\right) dV.$$

Assuming that $\mathbf{q}_{ku}$ is constant and normal to the surface and that the meat temperature and properties are constant throughout the meat patty (lumped-capacitance analysis), the energy equation becomes

$$q_{ku} A_{ku} = -\rho c_p V \frac{dT}{dt}.$$

Integrating the equation above from $t = t_i = 0$ to $t = t_1$, for a constant q_{ku}, gives

$$\int_{t_i=0}^{t_1} q_{ku} A_{ku} dt = -\int_{T_i}^{T_{ls}} \rho c_p V dT$$

$$t_1 = \frac{\rho c_p V (T_i - T_{ls})}{q_{ku} A_{ku}} = \frac{M c_p (T_i - T_{ls})}{q_{ku} A_{ku}}$$

From the data given $A_{ku} = \frac{\pi D^2}{4} + \pi D l = 0.011$ m^2 and

$$t_1 = \frac{0.08(\text{kg})\ 4{,}200(\text{J/kg-K}) \times [10(℃) - 0(℃)]}{4{,}000(\text{W/m}^2)\ 0.011(\text{m}^2)} = 76.36 \text{ s} = 1.273 \text{ min.}$$

(ii) Regime 2: Solidification
During this regime the meat patties change phase from liquid to solid. Application of the integral-volume energy equation gives

$$\int_{A_s} \mathbf{q}_{ku} \cdot \mathbf{s}_n dA = \int_V \dot{s}_{ls} dV.$$

Again, assuming that $\mathbf{q}_{ku}$ is uniform and normal to the surface and that the meat properties are constant throughout the meat patty (lumped-capacitance analysis), the energy equation becomes

$$q_{ku} A_{ku} = \dot{s}_{ls} V,$$

where the volumetric heat consumption due to phase change $\dot{s}_{ls}$ is obtained from Table 2.1,

$$\dot{s}_{ls} = -\dot{n}_{ls} \Delta h_{\ell s}.$$

The volumetric solidification rate $\dot{n}_{ls}$(kg/m^3-s) is given by

$$\dot{n}_{ls} = \frac{m}{V(t_2 - t_1)}.$$

Using the relations above, the energy equation becomes

$$q_{ku} A_{ku} = -\frac{m \Delta h_{ls}}{(t_2 - t_1)},$$

and solving for $t_2 - t_1$,

$$t_2 - t_1 = -\frac{m \Delta h_{ls}}{q_{ku} A_{ku}}.$$

From the values given,

$$t_2 - t_1 = -\frac{0.08(\text{kg})\ (-3.34 \times 10^5)(\text{J/kg})}{4{,}000(\text{W/m}^2)\ 0.011(\text{m}^2)} = 607.3 \text{ s} = 10.12 \text{ min.}$$

(iii) Regime 3: Cooling of Solid
During this period of time, the meat patties are cooled from the melting temperature down to the final temperature. Application of the lumped-capacitance analysis for a control volume enclosing the meat results in an equation similar to $t_1 = \rho c_p V (T_i - T_{ls}) / q_{ku} A_{ku} = M c_p (T_i - T_{ls}) / q_{ku} A_{ku}$, i.e.,

$$t_o - t_2 = \frac{\rho c_p V (T_{ls} - T_o)}{q_{ku} A_{ku}} = \frac{m c_p (T_{ls} - T_o)}{q_{ku} A_{ku}}.$$

From the data given,

$$t_o - t_2 = \frac{0.08(\text{kg}) \times 1{,}930(\text{J/kg-K}) \times [0(℃) - (-15)(℃)]}{4{,}000(\text{W/m}^2)\ 0.011(\text{m}^2)} = 52.64 \text{ s} = 0.8773 \text{ min}.$$

The total time for cooling of the meat patties is therefore

$$t_o = t_1 + (t_2 - t_1) + (t_o - t_2) = 1.273 + 10.12 + 0.8773 = 12.27 \text{ min}.$$

For the velocity of the conveyor belt $u_c = 0.01$ m/s, the total length necessary is

$$L = u_c t_o = 0.01(\text{m/s}) \ \times 12.27(\text{min}) \ \times 60(\text{s/min}) \ = 7.362 \text{ m}.$$

COMMENT:

Phase change at constant pressure for a pure substance occurs at constant temperature.

The temperature evolution for regimes 1 and 3 are linear because q_{ku} has been assumed constant (note that all the properties are treated as constants). In practice, the heat loss by surface convection depends on the surface temperature and therefore is not constant with time when this surface temperature is changing. This will be discussed in Chapter 6.

The freezing regime accounts for more than 80 s of the total time, while the cooling of solid accounts for only 7 s of the total time. This is a result of the high heat of solidification exhibited by water and the relatively smaller specific heat capacity of ice compared to liquid water. Liquid water has one of the largest specific heat capacities among the pure substances. The specific heat capacity of substances will be discussed in Chapter 3.

PROBLEM 2.5.FUN

GIVEN:

While the integral-volume energy equation (2.9) assumes a uniform temperature and is applicable to many heat transfer media in which the assumption of negligible internal resistance to heat flow is reasonably justifiable, the differential-volume energy equation (2.1) requires no such assumption and justification. However, (2.1) is a differential equation in space and time and requires an analytical solution. The finite-small volume energy equation (2.13) allows for a middle ground between these two limits and divides the medium into small volumes within each of which a uniform temperature is assumed. For a single such volume (2.9) is recovered and for a very large number of such volumes the results of (2.1) are recovered.

Consider friction heating of a disk-brake rotor, as shown in Figure Pr.2.5. The energy conversion rate is $\dot{S}_{m,F}$. The brake friction pad is in contact, while braking, with only a fraction of the rotor surface (marked by R). During quick brakes (i.e., over less than $t = 5$ s), the heat losses from the rotor can be neglected.

Note that $\dot{S}_{m,F}$ remains constant, while ΔV changes.

SKETCH:

Figure Pr.2.5 shows the rotor and the area under the pad undergoing friction heating.

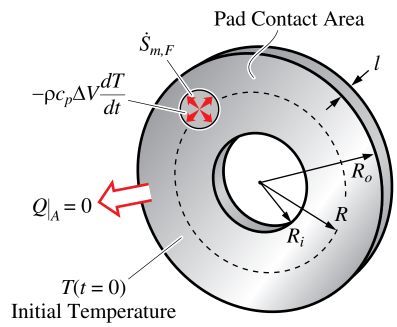


Figure Pr.2.5 A disc-brake rotor heated by friction heating. The region under the brake pad contact is also shown.

OBJECTIVE:

Apply (2.13), with (i) the volume marked as the pad contact region, and (ii) the entire volume in Figure Pr.2.5, and determine the temperature T after $t = 4$ s for cases (i) and (ii) and the conditions given above. Note that the resulting energy equation, which is an ordinary differential equation, can be readily integrated.

SOLUTION:

Starting from (2.13), we have

$$\begin{aligned} Q|_A &= -\frac{d}{dt}(\rho c_p T)_{\Delta V}\Delta V + \dot{S}_{m,F} \\ 0 &= -\rho c_p \Delta V \frac{dT}{dt} + \dot{S}_{m,F} \end{aligned}$$

or by separating the variables, we have

$$dT = \frac{\dot{S}_{m,F}}{\rho c_p \Delta V} dt.$$

Using $T|_{t=0} = T(t = 0)$ and integrating from 0 to t, we have

$$T(t) - T(t = 0) = \frac{\dot{S}_{m,F}}{\rho c_p \Delta V}(t - 0)$$

or

$$T(t) = T(t = 0) + \frac{\dot{S}_{m,F}}{\rho c_p \Delta V} t.$$

Then using the numerical values we have

$$\begin{aligned}
T(t = 4\text{ s}) &= 20(℃) + \frac{3 \times 10^4(\text{W})}{3.5 \times 10^6(\text{J/m}^3\text{-K}) \times \Delta V(\text{m}^3)} \times 4(\text{s}) \\
&= 20(℃) + \frac{3.429 \times 10^{-2}}{\Delta V}(℃).
\end{aligned}$$

(i) The smaller volume gives

$$\begin{aligned}
\Delta V &= \pi(R_o^2 - R^2)l \\
&= \pi(0.18^2 - 0.15^2)(\text{m}^2) \times 0.015(\text{m}) = 4.665 \times 10^{-4}\ \text{m}^3
\end{aligned}$$

$$T(t = 4\text{ s}) = 20(℃) + 73.50(℃) = 93.50℃.$$

(ii) The larger volume gives

$$\begin{aligned}
\Delta V &= \pi(R_o^2 - R_i^2)l \\
&= \pi(0.18^2 - 0.13^2)(\text{m}^2) \times 0.015(\text{m}) = 7.305 \times 10^{-4}\ \text{m}^3
\end{aligned}$$

$$T(t = 4\text{ s}) = 20(℃) + 46.94(℃) = 66.94℃.$$

COMMENT:

For more accurate results, the radial length as well as the length along l are divided into small-finite volumes and then heat transfer is allowed between them. This is discussed in Section 3.7.

PROBLEM 2.6.FUN

GIVEN:

In laser-induced spark ignition, laser irradiation $q_{r,i}$ is used to cause ionization of the fuel-oxidant mixture at the end of the laser pulse. The ionization is caused by multiphoton ionization. In multiphoton ionization, the ionizing gas molecules absorb a large number of photons.

Consider a pulsed laser, emitting a near-infrared radiation, $\lambda = 1.064\ \mu m$, with a time-dependent, focused irradiation flux given by

$$q_{r,i}(t) = (q_{r,i})_o e^{-t^2/\tau^2},$$

where $-\infty < t < \infty$, $(q_{r,i})_o$ is the peak irradiation, and $\tau(s)$ is time constant. Assume that this irradiation flux is uniform over the focal surface.
Note that

$$\int_{-\infty}^{\infty} e^{t^2/\tau^2} dt = \pi^{1/2}\tau.$$

$(q_{r,i})_o = 10^{17}$ W/m^2, $T = 10^6$ K, $a_1 = 0.1645 \times 10^{-42}$ K$^{1/2}$m^5, $n_e = 10^{26}$(1/m^3), $a_2 = 1.35 \times 10^4$ K, $D = 16.92\ \mu m$, $L = 194\ \mu m$, $\tau = 3.3$ ns, $\rho_r = 0$.

SKETCH:

Figure Pr.2.6 shows the laser irradiation focused on the kernel volume V, where it is partly absorbed.

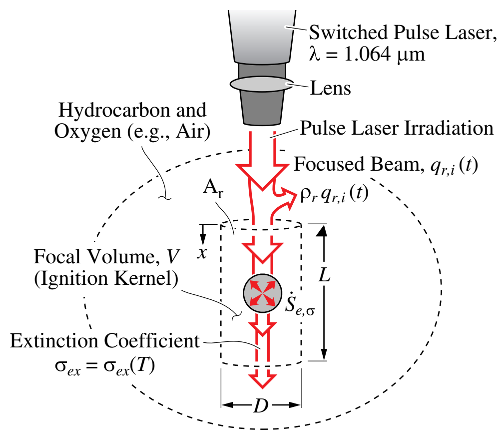


Figure Pr.2.6 Laser-induced spark ignition of a hydrocarbon-oxidizer gaseous mixture.

OBJECTIVE:

(a) Using the maximum photon energy given by

$$h_{\rm P} f = h_{\rm P}\frac{\lambda}{c},$$

where $h_{\rm P}$ is the Planck constant, and f is the frequency, λ is the wavelength, and c is the speed of light, determine the photon flux $\dot{m}_{ph}$(photon/m^2-s). Use the speed of light in vacuum $c = c_o$.

(b) Using a temperature-dependent extinction coefficient

$$\sigma_{ex}(1/\text{m}) = \frac{a_1 n_e^2}{T^{1/2}}\left(1 - e^{-a_2/T}\right),$$

where a_1 and a_2 are constants and T(K) is the kernel temperature, determine the energy absorbed in the focal volume, shown in Figure Pr.2.6, over the time span, $-\infty < t < \infty$, i.e.,

$$\int_{-\infty}^{\infty} \dot{S}_{e,\sigma} dt.$$

(c) Express the results of (b) per kernel volume V.

SOLUTION:
(a) The photon energy flux is $(q_{r,i})_o$. Then

$$(q_{r,i})_o = \dot{m}_{ph} h_{\rm P} c_o / \lambda$$

or

$$\dot{m}_{ph} = \frac{(q_{r,i})_o \lambda}{h_{\rm P} c_o}.$$

Here we use $c = c_o$ and c_o and $h_{\rm P}$ are listed in Table C.1.(b), i.e.,

$$h_{\rm P} = 6.626 \times 10^{-34} \text{ J-s}$$
$$c_o = 3.000 \times 10^8 \text{ m/s}.$$

Then

$$\dot{m}_{ph} = \frac{10^{17}(\text{W/m}^2) \times 1.064 \times 10^{-6}(\text{m})}{6.626 \times 10^{-34}(\text{J-s}) \times 3 \times 10^8(\text{m/s})} = 5.353 \times 10^{35} \text{ photon/m}^2\text{-s}.$$

(b) From (2.43), we have

$$\dot{S}_{e,\sigma} = q_{r,i}(1-\rho_r)\sigma_{ex} e^{-\sigma_{ex} x}.$$

The extinction coefficient is

$$\sigma_{ex} = \frac{0.1645 \times 10^{-42}(\text{K}^{1/2}\text{m}^5) \times (10^{26})^2 (1/\text{m}^3)^2}{(10^6)^{1/2}(\text{K})^{1/2}} \left(1 - e^{-1.35 \times 10^4/10^6}\right)$$
$$= 2.206 \times 10^4 \text{ 1/m}.$$

Here $\rho_r = 0$, and upon the time and volume integration, we have

$$\begin{aligned}
\int_{-\infty}^{\infty} \dot{S}_{e,\sigma} dt &= A_r \sigma_{ex} \int_{-\infty}^{\infty} \int_0^L q_{r,i}(t) \sigma_{ex} e^{-\sigma_{ex} x} dx dt \\
&= -A_r (e^{-\sigma_{ex} L} - 1) \int_{-\infty}^{\infty} q_{r,i}(t) dt \\
&= -A_r (e^{-2.206 \times 10^4 \times 194 \times 10^{-6}} - 1) \int_{-\infty}^{\infty} (q_{r,i})_o e^{-t^2/\tau^2} dt \\
&= 0.9861 A_r (q_{r,i})_o \pi^{1/2} \tau, \quad A_r = \frac{\pi D^2}{4}.
\end{aligned}$$

Using the numerical values, we have $A_r = 2.247 \times 10^{-10}$ m^2, and

$$\begin{aligned}
\int_{-\infty}^{\infty} \dot{S}_{e,\sigma} dt &= 0.9861 \times 2.247 \times 10^{-10}(\text{m}^2) \times 10^{17}(\text{W/m}^2) \times \pi^{1/2} \times 3.3 \times 10^{-9}(\text{s}) \\
&= 0.1296 \text{ J}.
\end{aligned}$$

(c) Using $V = 4.360 \times 10^{-14}$ m^3, we have

$$\begin{aligned}
\frac{\dot{S}_{e,\sigma}}{V} &= \sigma_{ex} (q_{r,i})_o \pi^{1/2} \tau \\
&= 2.972 \times 10^{12} \text{ J/m}^3.
\end{aligned}$$

This is a rather large result.

COMMENT:

Note that, in practice, the focused laser beam will not be uniform and therefore, a radial average should be taken. Also note that the irradiation flux used is for the focused beam. The beam leaving the laser has a much larger diameter, which makes for a smaller irradiation flux.

PROBLEM 2.7.FUN

GIVEN:

In thermoelectric cooling, a pair of p- and n-type semiconductors are jointed at a junction. When an electric current, given as current flux (or current density) $j_e(\text{A/m}^2)$, passes through their junction, heat is absorbed.

This current also produces the undesirable (parasitic) Joule heating. This energy conversion (per unit volume) is given by (2.33) as

$$\dot{s}_{e,\text{J}} = \rho_e(T) j_e^2,$$

where ρ_e(ohm-m) is the electrical resistivity and varies with temperature $\rho_e = \rho_e(T)$. Figure Pr.2.7 shows a semiconductor slab (p- or n-type), which is a part of a pair. The energy equation (2.8) would be simplified by assuming that heat flows only in the x direction, that the heat transfer is in a steady state, and that the energy conversion term is given above. A small length Δx is take along the x direction and the conduction heat flux vectors at x and $x + \delta x$ are given as $\mathbf{q}_k|_x$ and $\mathbf{q}_k|_{x+\Delta x}$.

$\mathbf{q}_k|_x = -1.030 \times 10^4 \mathbf{s}_x$ W/m^2, $\mathbf{q}_k|_{x+\Delta x} = 1.546 \times 10^4 \mathbf{s}_x$ W/m^2, $j_e = 4 \times 10^6$ A/m^2, $\Delta x = 0.1$ mm.

SKETCH:

Figure Pr.2.7 shows the semiconductor slab, the current density j_e, and the conduction heat flux vectors on both sides of a small length Δx.

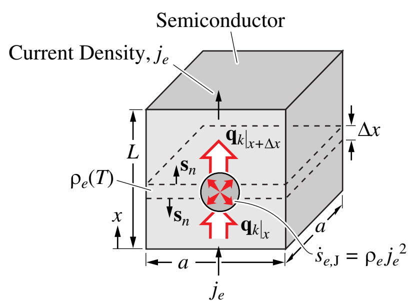


Figure Pr.2.7 A semiconductor slab with a one-dimensional heat conduction and a volumetric energy conversion (Joule heating). The conduction heat flux vectors are prescribed at locations x and $x + \Delta x$.

OBJECTIVE:

(a) Using (2.8), and assuming that the results for the given small Δx are valid for $\Delta x \to 0$, determine the magnitude of $\dot{s}_{e,\text{J}}$.

(b) Using the relationship for $\dot{s}_{e,\text{J}}$ given above, and the value for j_e given below, determine $\rho_e(T)$ and from Tables C.9(a) and (b) find a material with this electrical resistivity $\rho_e(T)$ (ohm-m).

SOLUTION:

(a) From (2.8), we have

$$\frac{\displaystyle\int_{\Delta A} (\mathbf{q} \cdot \mathbf{s}_n) dA}{\Delta V \to 0} = -\frac{\partial}{\partial t} \rho c_p T + \sum_i \dot{s}_i$$

For steady-state, conduction heat transfer only, and Joule heating as the only energy conversion, this becomes

$$\frac{\displaystyle\int_{\Delta A} (\mathbf{q_k} \cdot \mathbf{s}_n) dA}{\Delta V} = \dot{s}_{e,\text{J}},$$

where from Figure Pr.2.7, we have

$$\begin{aligned} \Delta V &= a^2 \Delta x \\ \Delta A &= a^2. \end{aligned}$$

Using the values for $\mathbf{q}_x$ given at locations x and $x + \Delta x$, and noting that for the surface at x we have $\mathbf{s}_n = -\mathbf{s}_x$ and for that at $x = x + \Delta x$, we have $\mathbf{s}_n = \mathbf{s}_x$, we have

$$\begin{aligned}
\frac{(\mathbf{q}_k|_x \cdot \mathbf{s}_n)\Delta A + (\mathbf{q}_k|_{x+\Delta x} \cdot \mathbf{s}_n)\Delta A}{\Delta V} &= \frac{\{-1.030 \times 10^4(\mathrm{W/m^2}) \times [\mathbf{s}_x \cdot (-\mathbf{s}_x)] + 1.546 \times 10^4(\mathrm{W/m^2}) \times [\mathbf{s}_x \cdot (\mathbf{s}_x)]\}a^2}{a^2 \Delta x} \\
&= \frac{(1.030 \times 10^4 + 1.546 \times 10^4)(\mathrm{W/m^2})}{10^{-4}(\mathrm{m})} = 2.576 \times 10^8 \ \mathrm{W/m^3}.
\end{aligned}$$

(b) Noting that

$$\dot{s}_{e,\mathrm{J}} = \rho_e(T) j_e^2,$$

and solving for $\rho_e(T)$, we have

$$\begin{aligned}
\rho_e(T) &= \frac{\dot{s}_{e,\mathrm{J}}}{j_e^2} \\
&= \frac{2.576 \times 10^8(\mathrm{W/m^3})}{(4 \times 10^6)^2(\mathrm{A^2/m^4})} = 1.610 \times 10^{-5} \ \mathrm{W\text{-}m/A^2} = 1.610 \times 10^{-5} \ \mathrm{ohm\text{-}m},
\end{aligned}$$

where we note that ($\mathrm{W}=\mathrm{A}^2$-ohm).

Note that in Table C.9(b) at $T = 700$ K, the n-type silicon-germanium alloy has this electrical resistivity.

COMMENT:

Note that heat leaves both surfaces (x and $x + \Delta x$), as expected from

$$\frac{\int_{\Delta A} (\mathbf{q} \cdot \mathbf{s}_n) dA}{\Delta V \to 0} = \nabla \cdot \mathbf{q} > 0.$$

The products of $(\mathbf{q} \cdot \mathbf{s}_n)$ at x and $x + \Delta x$ are both positive here.

PROBLEM 2.8.FUN

GIVEN:

In some transient heat transfer (i.e., temperature and heat flux vector changing with time) applications, that portion of the heat transfer medium experiencing such a transient behavior is only a small portion of the medium. An example is the seasonal changes of the air temperature near the earth's surface, which only penetrates a very short distance, compared to the earth's radius. Then the medium may be approximated as having an infinite extent in the direction perpendicular to the surface and is referred to a semi-infinite medium. Figure Pr.2.8 shows such a medium for the special case of a sudden change of the surface temperature from the initial (and uniform throughout the semi-infinite medium) temperature of $T(t=0)$ to a temperature T_s. Under these conditions, the solution for the heat flux is given by

$$q_{k,x}(x,t) = \frac{k[T_s - T(t=0)]}{(\pi\alpha t)^{1/2}} e^{-x^2/4\alpha t},$$

where $\alpha = k/\rho c_p$ is called the thermal diffusivity.

$k = 0.25$ W/m-K (for nylon), $\alpha = 1.29 \times 10^{-5}$ m^2/s (for nylon), $T_s = 105$℃, $T(t=0) = 15$℃, $x_o = 1.5$ cm, $t_o = 30$ s.

This conduction heat flux changes with time and in space.

SKETCH:

Figure Pr.2.8 shows the semi-infinite slab, the conduction heat flux, and the local energy storage/release.

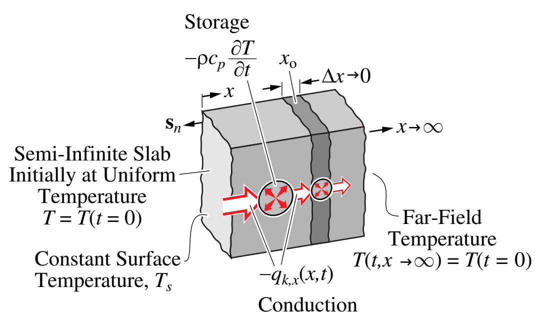


Figure Pr.2.8 A semi-infinite slab with an initial temperature $T(t=0)$ has its surface temperature suddenly changed to T_s.

OBJECTIVE:

(a) Using (2.1), with no energy conversion and conduction as the only heat transfer mechanism, determine the time rate of change of local temperature $\partial T/\partial t$ at location x_o and elapsed time t_o.

(b) Determine the location of largest time rate of change (rise) in the temperature and evaluate this for the elapsed time t_o.

SOLUTION:

(a) Starting from (2.1) and for no energy conversion and a one-dimensional (in the x direction) conduction only, we have

$$\begin{aligned} \nabla \cdot \mathbf{q} &= \left(\frac{\partial}{\partial x}\mathbf{s}_x\right) \cdot (\mathbf{q}_{k,x}\mathbf{s}_x) = -\rho c_p \frac{\partial T}{\partial t} \\ \frac{\partial}{\partial x} q_{k,x} &= -\rho c_p \frac{\partial T}{\partial t} \end{aligned}$$

or

$$-\frac{\partial T}{\partial t} = \frac{1}{\rho c_p} \frac{\partial}{\partial x} q_{k,x}.$$

Using the given expression for $q_{k,x}(x,t)$, we have

$$\begin{aligned}\frac{\partial T}{\partial t} &= -\left(\frac{k}{\rho c_p}\right)\frac{T_s - T(t=0)}{(\pi \alpha t)^{1/2}}\left(\frac{-2x}{4\alpha t}\right)e^{\frac{-x^2}{4\alpha t}} \\ &= \frac{T_s - T(t=0)}{2\pi^{1/2}\alpha^{1/2}t^{3/2}} x e^{\frac{-x^2}{4\alpha t}}.\end{aligned}$$

Evaluating this at x_o and t_o, we have

$$\frac{\partial T}{\partial t} = \frac{T_s - T(t=0)}{2\pi^{1/2}\alpha^{1/2}t_o^{3/2}} x_o e^{\frac{-x_o^2}{4\alpha t_o}}.$$

Using the numerical values, we have

$$\begin{aligned}\frac{\partial T}{\partial t} &= \frac{(105-15)(^\circ\mathrm{C})}{2\pi^{1/2}(1.29\times 10^{-5})^{1/2}(\mathrm{m}^2/\mathrm{s})^{1/2}(30)^{3/2}(\mathrm{s})^{3/2}} \times 1.510^{-2}(\mathrm{m})e^{-\frac{(1.5\times 10^{-2})^2(\mathrm{m})^2}{4\times 1.29\times 10^{-5}(\mathrm{m}^2/\mathrm{s})\times 30(\mathrm{s})}} \\ &= 0.6453(^\circ\mathrm{C/s}) \times e^{-0.1453} = 0.5580^\circ\mathrm{C/s}.\end{aligned}$$

(b) We now differentiate the above expression for $\partial T/\partial t$, with respect to x, at which we find the location of the largest $\partial T/\partial t$ occurs. Then by differentiating and using $t = t_0$, we have

$$\begin{aligned}\frac{\partial}{\partial x}\frac{\partial T}{\partial t} &= \frac{\partial^2 q_{k,x}}{\partial x^2} \\ &= \frac{T_s - T(t=0)}{2\pi^{1/2}\alpha^{1/2}t_o^{3/2}}\left(e^{-\frac{x^2}{4\alpha t_o}} - \frac{2x^2}{4\alpha t_o}e^{-\frac{x^2}{4\alpha t_o}}\right) = 0.\end{aligned}$$

Then

$$1 - \frac{2x^2}{4\alpha t_o} = 0$$

or

$$x = (2\alpha t_o)^{1/2}.$$

Now using the numerical values, we have

$$\begin{aligned}x &= [2\times 1.29\times 10^{-5}(\mathrm{m}^2/\mathrm{s})\times 30(\mathrm{s})]^{1/2} \\ &= 0.02782\ \mathrm{m} = 2.782\ \mathrm{cm}.\end{aligned}$$

From part (a), we have

$$\frac{\partial T}{\partial t} = 1.1968(^\circ\mathrm{C/s}) \times e^{-0.5} = 0.7259^\circ\mathrm{C/s}.$$

COMMENT:

Note that as expected, $\partial T/\partial t = 0$ at $x = 0$ (because T_s is assumed constant). In Section 3.5.1, we will discuss this transient problem and define the penetration front as the location beyond which the effect of the surface temperature change has not yet penetrated and this distance is given as $x \equiv \delta_\alpha = 3.6(\alpha t)^{1/2}$, as compared to $x = 1.414(\alpha t)^{1/2}$ for the location of maximum $\partial T/\partial t$.

PROBLEM 2.9.FUN

GIVEN:

A device that allows for heat transfer between two fluid streams of different temperatures is called a heat exchanger. In most applications, the fluid streams are bounded by flowing through ducts and tubes and are also kept from mixing with each other by using an impermeable solid wall to separate them. This is shown in Figure Pr.2.9. Assume that radiation and conduction are not significant in each stream and that there is steady-state heat transfer and no energy conversion. Then there is only bounded fluid stream convection and surface convection at the separating wall, as shown in Figure Pr.2.9. Assume that the wall has a zero thickness. Also assume convection heat flux q_u a uniform (an average) across the surface area for convection A_u. The surface area for the surface convection over a differential length Δx is ΔA_{ku}.

SKETCH:

Figure Pr.2.9 shows the two streams and the wall separating them.

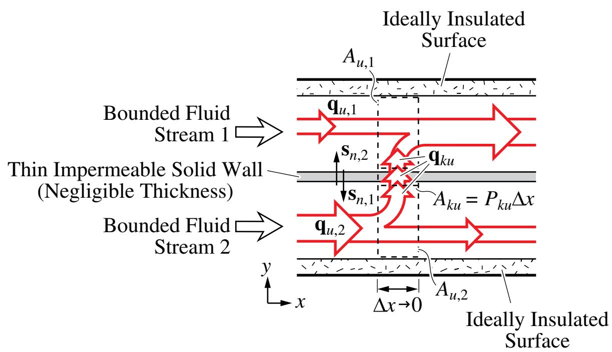


Figure Pr.2.9 Two streams, one having a temperature higher than the other, exchange heat through a wall separating them.

OBJECTIVE:

Starting from (2.8), write the energy equations for the control surfaces ΔA_1 and ΔA_2 shown. These control surfaces include both the convection and the surface convection areas. Show that the energy equations become

$$-\frac{P_{ku}}{A_{u,1}}q_{ku} + \frac{d}{dx}q_{u,1} = 0,$$
$$\frac{P_{ku}}{A_{u,2}}q_{ku} + \frac{d}{dx}q_{u,2} = 0.$$

SOLUTION:

Starting from (2.8), we note that $\partial/\partial t = 0$, $\dot{s} = 0$, and $\mathbf{q} = \mathbf{q}_u$ along the axis and $\mathbf{q} = \mathbf{q}_{ku}$ along the y axis on the wall surface. Then (2.8) becomes, using $\Delta V_1 = A_{u,1}\Delta x$ and $\Delta A_{ku} = P_{ku}\Delta x$,

$$\begin{aligned}
\frac{\int_{\Delta A_1}(\mathbf{q}\cdot\mathbf{s}_n)dA}{\Delta V_1 \rightarrow 0} &= 0 + 0 \\
&= \frac{\int_{\Delta A_{ku}}(\mathbf{q}_{ku}\cdot\mathbf{s}_n)dA + \int_{\Delta A_{u,1}}(\mathbf{q}_u\cdot\mathbf{s}_n)dA}{\Delta V_1 \rightarrow 0} \\
&= \frac{-q_{ku}\Delta A_{ku} + q_{u,1}(x+\Delta x)A_{u,1} - q_{u,1}(x)A_{u,1}}{A_{u,1}\Delta x} \\
&= -\frac{q_{ku}P_{ku}\Delta x}{A_{u,1}\Delta x} + \frac{q_{u,1}(x+\Delta x) - q_{u,1}(x)}{\Delta x} \\
&= -P_{ku}q_{ku} + \frac{d}{dx}q_u = 0 \quad \text{for } \Delta x \rightarrow 0.
\end{aligned}$$

Note that $\mathbf{q}_{ku} \cdot \mathbf{s}_{n,1} = -q_{ku}$, because $\mathbf{s}_n$ is pointing opposite to the assumed direction for $\mathbf{q}_{ku}$. Similarly,

$$\frac{\displaystyle\int_{\Delta A_{u,2}} (\mathbf{q}_u \cdot \mathbf{s}_n) dA}{\Delta V_2 \to 0} = \frac{q_{ku}\Delta A_{ku} + q_{u,2}(x+\Delta x)A_{u,2} - q_{u,2}(x)A_{u,2}}{A_{u,2}\Delta x}$$

$$= P_{ku}q_{ku} + \frac{d}{dx}q_{u,2} = 0 \quad \text{for } \Delta x \to 0.$$

COMMENT:

Note that the two energy equations mathematically state what is rendered in Figure Pr.2.9, i.e., heat is convected along each stream and is exchanged through the wall (by surface convection). In Section 7.6.1, we will use these energy equations to determine the total heat transfer (exchange) rate Q_{ku}.

PROBLEM 2.10.FUN

GIVEN:

A heat transfer medium with a rectangular control volume, shown in Figure Pr.2.10, has the following uniform heat fluxes at its six surfaces:

$$\begin{aligned} q_x|_{x-\Delta x/2} &= -4\ \text{W/m}^2, & q_x|_{x+\Delta x/2} &= -3\ \text{W/m}^2, \\ q_y|_{y-\Delta y/2} &= 6\ \text{W/m}^2, & q_y|_{y+\Delta y/2} &= 8\ \text{W/m}^2, \\ q_z|_{z-\Delta z/2} &= 2\ \text{W/m}^2, & q_z|_{z+\Delta z/2} &= 1\ \text{W/m}^2. \end{aligned}$$

The uniformity of heat flux is justifiable due to the small dimensions $\Delta x = \Delta y = \Delta z = 2$ mm.

SKETCH:

Figure Pr.2.10 A rectangular control volume with the heat flux vector on its six surfaces.

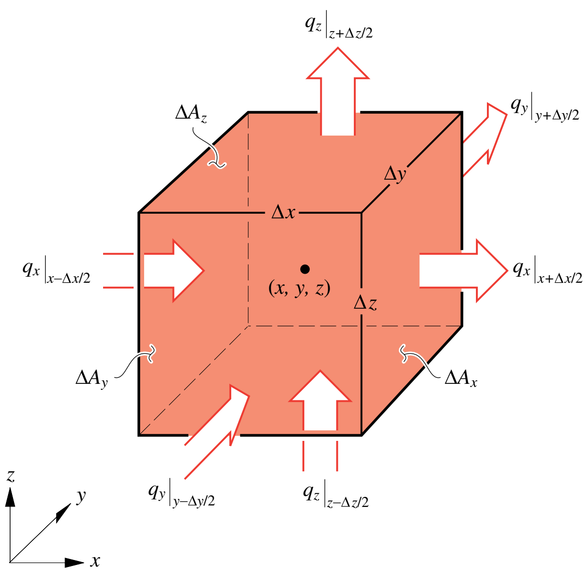


Figure Pr.2.10 A control volume in a heat transfer medium.

OBJECTIVE:

(a) Assume that $\nabla V \rightarrow 0$ is approximately valid for this small, but finite volume and determine the divergence of $\mathbf{q}$ for the center of this control volume, located at (x, y, z).
(b) Is there a sink or a source of heat in this control volume located at (x, y, z)?
(c) What could be the mechanisms for this source or sink of heat?

SOLUTION:

(a) We have assumed that over each of the six surfaces the heat flux is uniform. Then, we can use (2.8) as

$$\lim_{\Delta V \rightarrow 0} \frac{\int_{\Delta A} (\mathbf{q} \cdot \mathbf{s}_n) dA}{\Delta V} \equiv \nabla \cdot \mathbf{q} = -\frac{\partial(\rho c_p T)}{\partial t} + \sum_i \dot{S}_i.$$

The divergence of $\mathbf{q}$ is given above in terms of the surface integral. This surface integral is expanded using the three components of $\mathbf{q}$ and $\mathbf{s}_n$ in the x, y, and z directions. Using that we have

$$\begin{aligned} \int_{\Delta A} (\mathbf{q} \cdot \mathbf{s}_n) dA &= \int_{\Delta A} (\mathbf{q}_x \cdot \mathbf{s}_{n,x} + \mathbf{q}_y \cdot \mathbf{s}_{n,y} + \mathbf{q}_z \cdot \mathbf{s}_{n,z}) dA \\ &= (\mathbf{q}_{x+\Delta x/2} - \mathbf{q}_{x-\Delta x/2}) \Delta A_x + (\mathbf{q}_{y+\Delta y/2} - \mathbf{q}_{y-\Delta y/2}) \Delta A_y + \\ &\quad (\mathbf{q}_{z+\Delta z/2} - \mathbf{q}_{z-\Delta z/2}) \Delta A_z. \end{aligned}$$

Then, for the divergence of $\mathbf{q}$ we use the definition and approximations to obtain

$$\begin{aligned}
\nabla\cdot\mathbf{q} &\equiv \lim_{\Delta V\to 0}\frac{\int_{\Delta A}(\mathbf{q}\cdot\mathbf{s}_n)dA}{\Delta V} \simeq \frac{\int_{\Delta A}(\mathbf{q}\cdot\mathbf{s}_n)dA}{\Delta V} \\
&= \frac{(\mathbf{q}_{x+\Delta x/2}-\mathbf{q}_{x-\Delta x/2})\Delta A_x}{\Delta V} + \frac{(\mathbf{q}_{y+\Delta y/2}-\mathbf{q}_{y-\Delta y/2})\Delta A_y}{\Delta V} + \\
&\quad \frac{(\mathbf{q}_{z+\Delta z/2}-\mathbf{q}_{z-\Delta z/2})\Delta A_z}{\Delta V} \\
&= \frac{(\mathbf{q}_{x+\Delta x/2}-\mathbf{q}_{x-\Delta x/2})\Delta y\Delta z}{\Delta x\Delta y\Delta z} + \frac{(\mathbf{q}_{y+\Delta y/2}-\mathbf{q}_{y-\Delta y/2})\Delta x\Delta z}{\Delta x\Delta y\Delta z} + \\
&\quad \frac{(\mathbf{q}_{z+\Delta z/2}-\mathbf{q}_{z-\Delta z/2})\Delta x\Delta y}{\Delta x\Delta y\Delta z}.
\end{aligned}$$

Since $\Delta x = \Delta y = \Delta z$, we have

$$\nabla\cdot\mathbf{q} \simeq \frac{1}{\Delta x}(\mathbf{q}_{x+\Delta x/2}-\mathbf{q}_{x-\Delta x/2}+\mathbf{q}_{y+\Delta y/2}-\mathbf{q}_{y-\Delta y/2}+\mathbf{q}_{z+\Delta z/2}-\mathbf{q}_{z-\Delta z/2}).$$

Now, using the numerical values we have

$$\nabla\cdot\mathbf{q} = \frac{1}{2\times 10^{-3}(\mathrm{m})}(+4-3-6+8-2+1)(\mathrm{W/m^2}) = 10^3\ \mathrm{W/m^3}.$$

(b) From (2.2), since $\nabla\cdot\mathbf{q}$ is positive, there is a source in the control volume, i.e., there is a source at the location (x, y, z). This is because more heat leaves the control surface than enters it.

(c) From the energy equation (2.8), the mechanism for this heat source is storage or energy conversion. There are many energy conversion mechanisms (to and from thermal energy), for example, those listed in Table 2.1. When the temperature within the control volume increases, i.e., $\partial T/\partial t > 0$, heat is being stored in the control volume as sensible heat. Another example for a sink of heat is when there is an endothermic chemical reaction. Then heat is absorbed in the control volume to move the reaction forward (from reactants to products). When there is a gas flow and the gas undergoes expansion as it flows through the control volume, heat is absorbed as the gas performs work (this is called expansion cooling).

COMMENT:

Here we assumed that $\mathbf{q}$ is uniform over the surfaces. This is strictly true when $\Delta A \to 0$. The assumption of uniform $\mathbf{q}$ over a surface can be justifiably made when the heat flow is unidirectional. When the heat flow over a surface is zero, the surface is called adiabatic.

PROBLEM 2.11.FUN

GIVEN:

Although the temperature variation within a heat transfer medium is generally three dimensional, in many cases there is a dominant direction in which the most significant temperature variation occurs. Then, the use of a one-dimensional treatment results in much simplification in the analysis. Consider the steady-state surface temperatures given in Figure Pr.2.11(a), for selected locations on a solid, rectangular piece. The heat flows through the solid by conduction and from its surface to the ambient by surface convection.

SKETCH:

Figure Pr.2.11(a) shows the rectangular slab and the measured temperature at various locations.

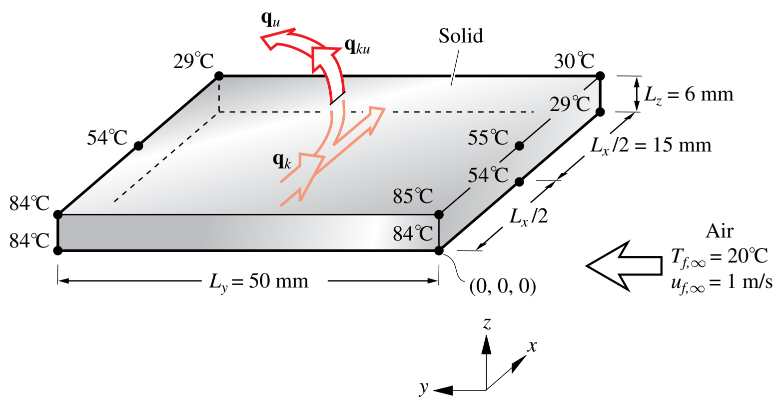


Figure Pr.2.11(a) Temperature at various locations on a rectangular plate.

OBJECTIVE:

(a) By examining the gradient of temperature in each direction, determine the dominant conduction heat flow direction. As an approximation, use

$$\frac{\partial T}{\partial x} \simeq \frac{\Delta T_x}{\Delta x}, \quad \frac{\partial T}{\partial y} \simeq \frac{\Delta T_y}{\Delta y}, \quad \frac{\partial T}{\partial z} \simeq \frac{\Delta T_z}{\Delta z},$$

where Δx is the length over which the temperature change ΔT_x occurs.

(b) Select a control volume that has a differential length in the direction of dominant conduction heat flow and an integral length over the other two directions. Schematically show this integral-differential volume.

(c) Write an energy equation for this control volume.

SOLUTION:

(a) The three principal directions, x, y, and z for the rectangular solid piece are shown in Figure Pr.2.11(b). The heat transfer by conduction is given by Fourier's law (1.11) and using (1.14) we have

$$\mathbf{q}_k = -k\nabla T \equiv -k\left(\frac{\partial T}{\partial x}\mathbf{s}_x + \frac{\partial T}{\partial y}\mathbf{s}_y + \frac{\partial T}{\partial z}\mathbf{s}_z\right).$$

We now use the approximation for the gradient of temperature and write these as

$$\mathbf{q}_k = -k\left(\frac{\Delta T_x}{L_x}\mathbf{s}_x + \frac{\Delta T_y}{L_y}\mathbf{s}_y + \frac{\Delta T_z}{L_z}\mathbf{s}_z\right).$$

Next we substitute for the temperatures and lengths obtaining

$$\begin{aligned} \mathbf{q}_k &= -k\left[\frac{30(℃) - 85(℃)}{0.030(\text{m})}\mathbf{s}_x + \frac{84(℃) - 85(℃)}{0.050(\text{m})}\mathbf{s}_y + \frac{85(℃) - 84(℃)}{0.006(\text{m})}\mathbf{s}_z\right] \\ &= -k[-1.883 \times 10^3(℃/\text{m})\mathbf{s}_x - 2.000 \times 10^1(℃/\text{m})\mathbf{s}_y + 1.667 \times 10^2(℃/\text{m})\mathbf{s}_z]. \end{aligned}$$

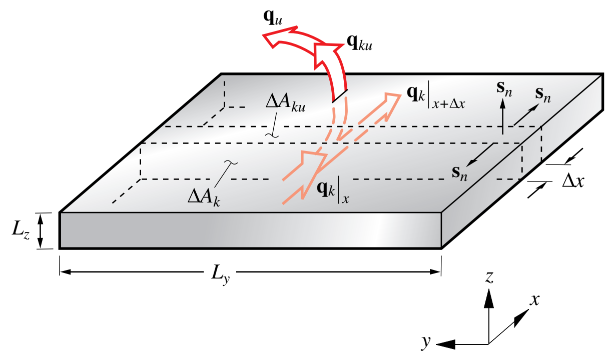


Figure Pr.2.11(b) The heat flow along the x direction, where a differential length Δx is chosen.

We note that the term in x is at least one order of magnitude larger than any of the other two terms. Then, from the order of magnitude of the terms, we can approximate the heat flux by

$$\mathbf{q}_k \simeq [1.883 \times 10^3(°\mathrm{C/m}) \times k(\mathrm{W/m\text{-}°C})]\mathbf{s}_x,$$

which represents a one-dimensional conduction heat flow.

(b) Figure Pr.2.11(b) shows the differential length taken along the x direction.

(c) Since the temperature field is steady and there is no energy conversion, (2.7) becomes

$$\nabla \cdot \mathbf{q} \equiv \lim_{\Delta V \to 0} \frac{\int_{\Delta A}(\mathbf{q}\cdot\mathbf{s}_n)dA}{\Delta V} = 0.$$

As shown in Figure Pr.2.11(b), based on the one-dimensional intramedium conduction and surface convection, one can write the limit above as

$$\lim_{\Delta V \to 0} \frac{\int_{\Delta A}(\mathbf{q}\cdot\mathbf{s}_n)dA}{\Delta V} = \lim_{\Delta V \to 0}\left\{\frac{(\mathbf{q}_{ku}\cdot\mathbf{s}_n)\Delta A_{ku}}{\Delta x L_y L_z} + \frac{[(\mathbf{q}_k\cdot\mathbf{s}_n)A_k]_x + [(\mathbf{q}_k\cdot\mathbf{s}_n)A_k]_{x+\Delta x}}{\Delta x L_y L_z}\right\}.$$

Here we have $A_k = L_y L_z$ and $\Delta A_{ku} = 2(L_y + L_z)\Delta x$. Now, noting that $\mathbf{s}_n$ on the x and $x + \Delta x$ surfaces point in opposite directions, we have

$$\begin{aligned}\lim_{\Delta V \to 0} \frac{\int_{\Delta A}(\mathbf{q}\cdot\mathbf{s}_n)dA}{\Delta V} &= \lim_{\Delta V \to 0}\left[q_{ku}\frac{2(L_y+L_z)\Delta x}{\Delta x L_y L_z} + \frac{(-q_k|_x + q_k|_{x+\Delta x})L_y L_z}{\Delta x L_y L_z}\right] \\ &= q_{ku}\frac{2(L_y+L_z)}{L_y L_z} + \lim_{\Delta V \to 0}\frac{-q_k|_x + q_k|_{x+\Delta x}}{\Delta x}.\end{aligned}$$

Now, the limit in the last term is the definition of a derivative and we have

$$\lim_{\Delta V \to 0} \frac{\int_{\Delta A}(\mathbf{q}\cdot\mathbf{s}_n)dA}{\Delta V} = q_{ku}\frac{2(L_y+L_z)}{L_y L_z} + \frac{dq_k}{dx}$$

and, using Fourier's law of conduction,

$$q_k = -k\frac{dT}{dx}.$$

Assuming that k is constant, we have finally

$$q_{ku}\frac{2(L_y+L_z)}{L_y L_z} - k\frac{d^2T}{dx^2} = 0.$$

COMMENT:

Note that the gradient of temperature along the x direction is not uniform. Over the first half of the length along x, the temperature change is larger than that over the second half. This is a consequence of the local surface-convection heat flux being proportional to the difference between the solid temperature and the far-field fluid temperature. We will discuss this in Section 6.8.

PROBLEM 2.12.FUN

GIVEN:

Many computer disks are read by magnetoresistive transducers. The transducer is located on a thin slider that is situated slightly above the disk, as shown in Figure Pr.2.12. The transistor is developed using the principle that its resistance varies with the variation of the surrounding magnetic field. Since its resistance is also temperature dependent, any temperature change will result in a noise in the readout. When the slider and disk are at the same temperature, the viscous-dissipation heat generation becomes significant in creating this undesired increase in the temperature. Assume the flow of air at $T = 300$ K between the disk and slider is a Newtonian, one-dimensional, Couette flow, as shown in Figure Pr.2.12. The distance between the disk and the slider is $L = 20$ nm, and the relative velocity is $\Delta u_i = 19$ m/s.

Use Table C.22, and the relation $\mu_f = \nu_f/\rho_f$ to determine μ_f for air at $T = 300$ K.

SKETCH:

Figure Pr.2.12 shows the disk and slider, the air flow between them, and the viscous dissipation energy conversion.

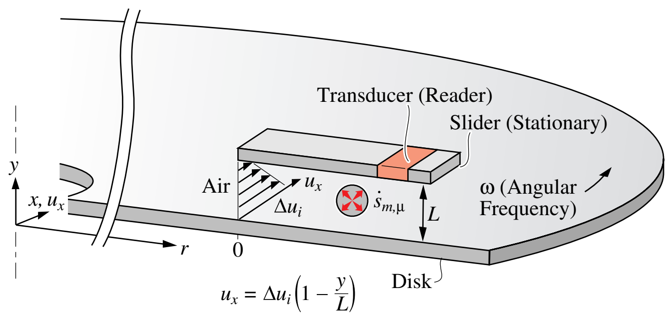


Figure Pr.2.12 A disk read by a magnetoresistive transducer.

OBJECTIVE:

Determine the magnitude of the volumetric viscous-dissipation heat generation $\dot{s}_{m,\mu}$.

SOLUTION:

The properties of air at $T = 300$ K, (Table C.22), are $\nu_f = 15.66 \times 10^{-6}$ m^2/s, and $\rho_f = 1.177$ kg/m^3. For the one-dimensional flow in the x direction, we have from (2.52)

$$\begin{aligned}
\dot{s}_{m,\mu} &= \mu_f \left(\frac{\partial u_x}{\partial y}\right)^2 \\
u_x &= \Delta u_i \left(1 - \frac{y}{L}\right) \\
\frac{\partial u_x}{\partial y} &= \frac{-\Delta u_i}{L} \\
\dot{s}_{m,\mu} &= 15.66 \times 10^{-6}(\mathrm{m}^2/\mathrm{s}) \times 1.177(\mathrm{kg/m}^3) \times \left[\frac{19^2(\mathrm{m/s})^2}{(2 \times 10^{-8})^2(\mathrm{m}^2)}\right] \\
&= 1.66 \times 10^{13}\ \mathrm{W/m}^3.
\end{aligned}$$

COMMENT:

It should be noted that the relation $u_x = \Delta u_i(1 - y/L)$ is derived using the no-slip boundary condition. This condition only holds true if the gap L is greater than the mean-free path of the gas λ_m. For air at STP, we have from Table C.7, $\lambda = 10^{-7}$ m, which is greater than the gap $L = 2 \times 10^{-8}$ m. Thus, the fluid velocity should be given in terms of the slip coefficient used for $\lambda_m > L$.

PROBLEM 2.13.FAM

GIVEN:

Catalytic combustion (and catalytic chemical reaction in general) is an enhancement in the rate of chemical reaction due to the physical-chemical mediation of a solid surface. For example, in the automobile catalytic converter the rate of reaction of exhaust-gas unburned fuel is increased by passing the exhaust gas over the catalytic surface of the converter. The catalytic converter is a solid matrix with a large surface area over which the exhaust gas flows with a relatively small pressure drop. The catalytic effect is produced by a surface impregnated with precious metal particles, such as platinum. For example, consider the following chemical kinetic model for the reaction of methane and oxygen

$$\mathrm{CH_4} + 2\mathrm{O_2} \rightarrow \mathrm{CO_2} + 2\mathrm{H_2O} \quad \text{stoichiometric chemical reaction}$$
$$\dot{m}_{r,\mathrm{CH_4}} = -a_r \rho_{\mathrm{CH_4}} \rho_{\mathrm{O_2}}^{1/2} e^{-\Delta E_a/R_g T} \quad \text{chemical, kinetic model,}$$

where $\dot{m}_{r,\mathrm{CH_4}}$(kg/m^2-s) is the reaction-rate per unit surface area. The pre-exponential factor a_r(cm$^{5/2}$/s-g$^{1/2}$) and the activation energy ΔE_a(J/kmole) are determined empirically. The model is accurate for high oxygen concentrations and for high temperatures. The densities (or concentrations) are in g/cm^3. Consider the following catalytic (in the presence of Pt) and noncatalytic (without Pt) chemical kinetic constants:

$$\text{without Pt} \quad : a_r = 1.5 \times 10^{11}\ \mathrm{cm^{5/2}/s\text{-}g^{1/2}}, \quad \Delta E_a = 1.80 \times 10^8\ \mathrm{J/kmole},$$
$$\text{with Pt} \quad : a_r = 1.5 \times 10^{12}\ \mathrm{cm^{5/2}/s\text{-}g^{1/2}}, \quad \Delta E_a = 1.35 \times 10^8\ \mathrm{J/kmole}.$$

Use the cgs units (cm, g, s).

OBJECTIVE:

For a mixture of methane and oxygen at a pressure of 1 atm and a temperature of 500℃:
(a) Determine the densities of CH_4 and O_2 assuming an ideal-gas behavior,
(b) Determine the rate of reaction per unit surface area $\dot{m}_{r,\mathrm{CH_4}}$.
(c) Comment on the effect of the catalyst.

SOLUTION:

(a) For an ideal gas mixture we have

$$\rho = \rho_{\mathrm{CH_4}} + \rho_{\mathrm{O_2}} = \frac{p}{\frac{R_g}{M} T}$$
$$M = \frac{\nu_{\mathrm{CH_4}} M_{\mathrm{CH_4}} + \nu_{\mathrm{O_2}} M_{\mathrm{O_2}}}{\nu_{\mathrm{CH_4}} + \nu_{\mathrm{O_2}}},$$

where $\nu_{\mathrm{CH_4}}$ and $\nu_{\mathrm{O_2}}$ are the stoichiometric coefficients for CH_4 and O_2 in the chemical reaction.
Also,

$$\frac{\rho_{\mathrm{CH_4}}}{\rho} = \frac{\nu_{\mathrm{CH_4}} M_{\mathrm{CH_4}}}{\nu_{\mathrm{CH_4}} M_{\mathrm{CH_4}} + \nu_{\mathrm{O_2}} M_{\mathrm{O_2}}}, \ \frac{\rho_{\mathrm{O_2}}}{\rho} = \frac{\nu_{\mathrm{O_2}} M_{\mathrm{O_2}}}{\nu_{\mathrm{CH_4}} M_{\mathrm{CH_4}} + \nu_{\mathrm{O_2}} M_{\mathrm{O_2}}}.$$

The molecular weights are found from Table C.4 to be

$$M_{\mathrm{CH_4}} = 12.011 + 4 \times 1.008 = 16.04\ \mathrm{kg/kmole}$$
$$M_{\mathrm{O_2}} = 2 \times 15.999 = 32.00\ \mathrm{kg/kmole}.$$

Then

$$M = \frac{1 \times 16.04 + 2 \times 32.00}{1 + 2} = 26.68\ \mathrm{kg/kmole}.$$

Using the pressure and temperature given

$$
\begin{aligned}
\rho &= \frac{1.013\times 10^5(\text{Pa})}{\frac{8.314\times 10^3(\text{J/kmole-K})}{26.68(\text{kg/kmole})}(500+273.15)(\text{K})} = 0.4205\text{kg/m}^3\\
&= 0.4205(\text{kg/m}^3)\times 1{,}000(\text{g/kg})\times 10^{-6}(\text{m}^3/\text{cm}^3) = 4.205\times 10^{-4}\ \text{g/cm}^3\\
\rho_{\text{CH}_4} &= \left[\frac{1\times 16.04(\text{kg/kmole})}{16.04(\text{kg/kmole})+2\times 32.00(\text{kg/kmole})}\right]4.205\times 10^{-4}(\text{g/cm}^3) = 8.427\times 10^{-5}\ \text{g/cm}^3\\
\rho_{\text{O}_2} &= \left[\frac{2\times 32.00(\text{kg/kmole})}{16.04(\text{kg/kmole})+2\times 32.00(\text{kg/kmole})}\right]4.205\times 10^{-4}(\text{g/cm}^3) = 3.362\times 10^{-4}\ \text{g/cm}^3.
\end{aligned}
$$

(b) Now, using

$$\dot{m}_{r,\text{CH}_4} = \rho_{\text{CH}_4}\rho_{\text{O}_2}^{1/2}a_r e^{-\frac{\Delta E_a}{R_g T}}$$

the reaction rate without catalytic effect is given by

$$
\begin{aligned}
\dot{m}_{r,\text{CH}_4}(\text{without catalyst}) &= 8.427\times 10^{-5}(\text{g/cm}^3)\times[3.362\times 10^{-4}(\text{g/cm}^3)]^{1/2}\times 1.5\times 10^{11}(\text{cm}^{5/2}/\text{g}^{1/2}\text{-s})\times\\
&\quad \exp\left[-\frac{1.80\times 10^8(\text{J/kmole})}{8.314\times 10^3(\text{J/kmole-K})\times(500+273.15)(\text{K})}\right]\\
&= 1.598\times 10^{-7}\ \text{g/cm}^2\text{-s}.
\end{aligned}
$$

Using a catalyst, we have

$$
\begin{aligned}
\dot{m}_{r,\text{CH}_4}(\text{with catalyst}) &= 8.427\times 10^{-5}(\text{g/cm}^3)\times[3.362\times 10^{-4}(\text{ g/cm}^3)]^{1/2}\times 1.5\times 10^{12}(\text{cm}^{5/2}/\text{g}^{1/2}\text{-s})\\
&\quad \exp\left[-\frac{1.35\times 10^8(\text{J/kmole})}{8.314\times 10^3(\text{J/kmole-K})\times(500+273.15)(\text{K})}\right]\\
&= 1.754\times 10^{-3}\ \text{g/cm}^2\text{-s}.
\end{aligned}
$$

(c) Due to the presence of the catalyst, the surface reaction rate is increased by a factor

$$\frac{\dot{m}_{r,\text{CH}_4}(\text{with catalyst})}{\dot{m}_{r,\text{CH}_4}(\text{without catalyst})} = \frac{1.754\times 10^{-3}(\text{g/cm}^2\text{-s})}{1.598\times 10^{-7}(\text{g/cm}^2\text{-s})} = 1.098\times 10^4.$$

There is a substantial increase. Reaction rates that are negligibly small when no catalyst is present can be made substantial with the addition of a catalytic coating.

COMMENT:

Precious metals are deposited on catalyst surfaces as particles. The relevant linear dimensions are on the order of nanometers. The molecules of the gas are adsorbed on the surface, suffer a chemical modification, are desorbed, and then react in the gas phase. The total reaction rate for a catalytic converter with surface area A_{ku} is given by

$$\dot{M}_{r,\text{CH}_4} = A_{ku}\dot{m}_{r,\text{CH}_4}.$$

Using structures such as foams, bundle of tubes, and packed beds, a large surface area per unit volume A_{ku}/V is possible and large reaction rates are achieved.

PROBLEM 2.14.FUN

GIVEN:

In order to produce silicon wafers, single-crystal silicon ingots are formed by the slow solidification of molten silicon at the tip of a cylinder cooled from the base. This was shown in Figure 2.3(c) and is also shown in Figure Pr.2.14. The heat released by solid to fluid phase change $\dot{S}_{ls}$(W) is removed from the solid-liquid interface A_{ls} by conduction through the ingot Q_k. The energy equation for the solid-fluid interface A_{ls} (nonuniform temperature in the liquid), as given by (2.9), is

$$Q_k = \dot{S}_{ls},$$

where the conduction heat flow Q_k is given by

$$Q_k = A_k k \frac{T_{sl} - T_s}{L},$$

where L and T_s are shown in Figure Pr.2.14 and T_{ls} is the melting temperature. The rate of phase-change energy conversion is

$$\dot{S}_{ls} = -\rho_l A_{sl} u_F \Delta h_{ls} = -\dot{M}_{ls}\Delta h_{ls} = \dot{M}_{ls}\Delta h_{sl},$$

where $\Delta h_{sl} > 0$.

Assume that the liquid and solid have the same density. $L = 20$ cm, $u_F = 4$ mm/min.

SKETCH:

Figure Pr.2.14 shows the cooling of molten silicon and the formation of a crystalline silicon.

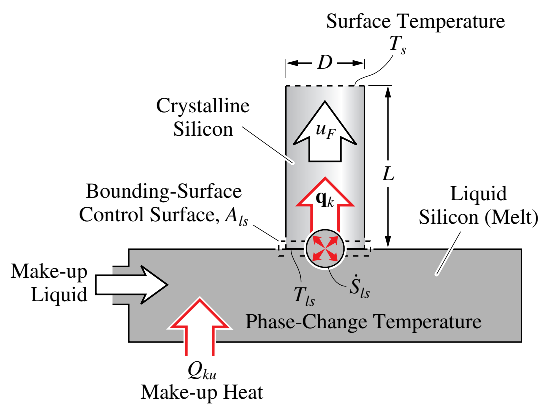


Figure Pr.2.14 Czochralski method for single-crystal growth of silicon.

OBJECTIVE:

Using the thermophysical properties given in Tables C.2 (periodic table for Δh_{sl}) and C.14 (at $T = 1{,}400$ K for k), in Appendix C, determine the temperature T_s (at the top of the ingot).

SOLUTION:

Combining the above equations, we have

$$A_k k \frac{T_{sl} - T_s}{L} = \dot{M}_{ls}\Delta h_{sl}.$$

Solving for T_s and using the equation for $\dot{M}_{ls}$, we have

$$T_s = T_{ls} - \frac{\dot{M}_{ls}\Delta h_{ls} L}{A_k k} = T_{ls} - \frac{\rho_l L u_F \Delta h_{ls} L}{A_k k}.$$

Now, we obtain the properties from Tables C.2 (periodic table) and C.14 as $T_{sl} = 1{,}687$ K, $\rho_l = \rho_s = 2{,}330$ kg/m^3, $\Delta h_{sl} = 1.802 \times 10^6$ J/kg, $k = 24$ W/m-K. Note that from Table C.14, k is obtained for $T = 1{,}400$ K, which is the highest temperature available. Then, using $A_k = A_{sl} = \pi D^2/4$, we have

$$\begin{aligned}
T_s &= T_{ls} - \frac{\rho_l A_{sl} u_F \Delta h_{ls} L}{A_k k} \\
&= 1{,}687(\text{K}) - \frac{2{,}330(\text{kg/m}^3) \times (4 \times 10^{-4}/60)(\text{m/s}) \times 1.802 \times 10^6(\text{J/kg}) \times 0.2(\text{m})}{24(\text{W/m-K})} \\
&= (1{,}687 - 233.3)(\text{K}) \\
&= 1{,}454 \text{ K}
\end{aligned}$$

COMMENT:

Note that the thermal conductivity of solid silicon significantly decreases as the temperature increases. At room temperature, k is about 150 W/m-K, while at T = 1,400 K, it is 24 W/m-K. Also, the melt is not at a uniform temperature and is superheated away from A_{sl}. Then, additional heat flows into A_{sl} from the liquid.

PROBLEM 2.15.FAM

GIVEN:

The electrical resistivity of metals increases with temperature. In a surface radiation emission source made of Joule-heating wire, the desired temperature is 2,500℃. The materials of choice are tantalum Ta and tungsten W. The electrical resistivity for some pure metals, up to T = 900 K, is given in Table C.8. Assume a linear dependence of the electrical resistivity on the temperature, i.e.,

$$\rho_e(T) = \rho_{e,o}[1 + \alpha_e(T - T_o)].$$

OBJECTIVE:

(a) From the data in Table C.8, determine α_e for both metals.
(b) Using the equation above, determine the metals electrical resistivity at T = 2,500℃.
(c) If the wire has a diameter D = 0.1 mm and a length L = 5 cm (coiled), determine the electrical resistance R_e for both metals at T = 25℃ and T = 2,500℃.
(d) If a Joule heating rate of 100 W is needed, what current should be applied at T = 2,500℃?
(e) Determine the voltage needed for this power.

SOLUTION:

(a) From Table C.8, we choose the last set of data for each metal to determine α_e. We also use T_o = 900 K, since we need to extrapolate beyond 900 K. Then we have

$$\begin{aligned}
\text{tantalum:}\quad \alpha_e &= \frac{1}{\rho_{e,o}}\frac{\Delta\rho_e}{\Delta T} = \frac{1}{40.1\times10^{-8}(\text{ohm-m})}\frac{(40.1-31.8)\times10^{-8}(\text{ohm-m})}{200(\text{K})}\\
&= 1.035\times10^{-3}\ 1/\text{K}\\
\text{tungsten:}\quad \alpha_e &= \frac{1}{21.5\times10^{-8}(\text{ ohm-m})}\frac{(21.5-15.7)\times10^{-8}(\text{ohm-m })}{200(\text{K})}\\
&= 1.349\times10^{-3}\ 1/\text{K}.
\end{aligned}$$

(b) Using

$$\rho_e = \rho_{e,o}[1 + \alpha_e(T - T_o)]$$

and choosing T_o = 900 K, for T = 2,773 K we have

$$\begin{aligned}
\text{tantalum:}\quad \rho_e &= 40.1\times10^{-8}(\text{ohm-m})[1+1.035\times10^{-3}(1/\text{K})\times(2{,}773-900)(\text{K})]\\
&= 1.178\times10^{-6}\ \text{ohm-m}\\
\text{tungsten:}\quad \rho_e &= 21.5\times10^{-8}(\text{ohm-m})[1+1.349\times10^{-3}(1/\text{K})\times(2{,}773-900)(\text{K})]\\
&= 7.582\times10^{-7}\ \text{ohm-m}.
\end{aligned}$$

(c) From (2.32), the resistance can be calculated from

$$R_e = \frac{\rho_e L}{A}.$$

For $A = \pi D^2/4$, we have

$$\text{tantalum:}\qquad R_e = \frac{4\times1.178\times10^{-6}(\text{ohm-m})\times0.05(\text{m})}{\pi(10^{-4})^2(\text{m}^2)} = 7.500\ \text{ohm}$$

$$\text{tungsten:}\qquad R_e = \frac{4\times7.582\times10^{-7}(\text{ohm-m})\times0.05(\text{m})}{\pi(10^{-4})^2(\text{m}^2)} = 4.827\ \text{ohm}.$$

(d) Substituting (2.31) into (2.28), the Joule heating rate can be written as

$$\dot{S}_{e,\text{J}} = R_e J_e^2.$$

Solving for J_e we have

$$J_e = \left(\frac{\dot{S}_{e,\text{J}}}{R_e}\right)^{1/2}.$$

Then

$$\begin{aligned}\text{tantalum:} \quad J_e &= [100(\text{W})/7.500(\text{ohm})]^{1/2} = 3.651 \text{ A}\\ \text{tungsten:} \quad J_e &= [100(\text{W})/4.828(\text{ohm})]^{1/2} = 4.551 \text{ A}.\end{aligned}$$

(e) The voltage is given by (2.32), i.e.,

$$\Delta\varphi = R_e J_e.$$

Then

$$\begin{aligned}\text{tantalum:} \quad & \Delta\varphi = 7.500(\text{ohm}) \times 3.651(\text{A}) = 27.38 \text{ V}\\ \text{tungsten:} \quad & \Delta\varphi = 4.828(\text{ohm}) \times 4.551(\text{A}) = 21.97 \text{ V}.\end{aligned}$$

COMMENT:

In order to produce the given Joule heating rate with a given voltage, the diameter and length of the wire are selected accordingly. Care must be taken to keep the wire temperature below the melting point of the wire or insulation. The temperature of the wire will depend on the heat transfer rate (heat losses) at the wire surface. The linear extrapolation of resistivity as a function of temperature, so far from the listed values, is not expected to be very accurate.

PROBLEM 2.16.FAM

GIVEN:

Electrical power is produced from a thermoelectric device. The thermoelectric junctions are heated (heat added) by maintaining the hot junction at $T_h = 400°C$ and cooled (heat removed) by maintaining the cold junction at $T = 80°C$. This is shown in Figure Pr.2.16.

There are 120 p-n pairs. The pairs are p- and n-type bismuth telluride (Bi_2Te_3) alloy with Seebeck coefficients $\alpha_{S,p} = 2.30 \times 10^{-4}$ V/K and $\alpha_{S,n} = -2.10 \times 10^{-4}$ V/K. The resistance (for all 120 pairs) to the electrical current J_e produced is $R_e = 0.02$ ohm for the thermoelectric path. For an optimum performance, the external resistance $R_{e,o}$ is also equal to 0.02 ohm.

SKETCH:

Figure Pr.2.16 shows the p-n junctions in a thermoelectric power generator module. The electrical circuit is also shown.

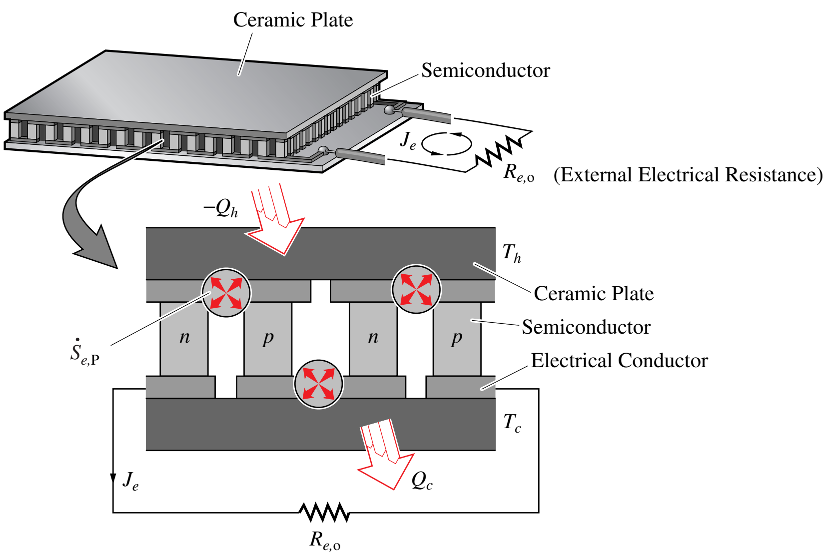


Figure Pr.2.16 A thermoelectric generator.

OBJECTIVE:

(a) Determine the current produced.
(b) Determine the power produced.

SOLUTION:

(a) The electrical circuit diagram for this device is also shown in the text by Figure 2.16(b). The electrical power generated is given by (2.40). The current can also be found from (2.40) and is given by

$$J_e = \frac{(\alpha_{S,p} - \alpha_{S,n})(T_h - T_c)}{R_{e,o} + R_e}.$$

Using the numerical values,

$$J_e = \frac{(2.30 + 2.10) \times 10^{-4}(\text{V/K}) \times (400 - 80)(\text{K})}{0.02(\text{ohm}) + 0.02(\text{ohm})} = 3.520 \text{ A}.$$

(b) The electrical power produced P_e (based on external resistance) is

$$P_e = J_e^2 R_{e,o} = (3.52)^2(\text{A})^2 \times 0.02(\text{ohm}) = 0.2478 \text{ W}.$$

COMMENT:

The Bi-Te alloy is not a high temperature thermoelectric alloy. For higher temperatures (i.e., direct exposure to combustion gases), Si-Ge alloys are used.

PROBLEM 2.17.FUN

GIVEN:

A premixed mixture of methane CH_4 and air burns in a Bunsen-type burner, as shown in Figure Pr.2.17. Assume that the flame can be modeled as a plane flame. The reactants (methane and air) enter the flame zone at a temperature $T_1 = 289$ K. The concentration of methane in the reactant gas mixture is $\rho_{\mathrm{F},1} = 0.0621$ kg$_{\mathrm{CH}_4}$/m^3 and the heat of reaction for the methane/air reaction is $\Delta h_{r,\mathrm{CH}_4} = -5.55 \times 10^7$ J/kg$_{\mathrm{CH}_4}$ (these will be discussed in Chapter 5). For both reactants and products, assume that the average density is $\rho = 1.13$ kg/m^3 and that the average specific heat is $c_p = 1{,}600$ J/kg-K (these are temperature-averaged values between the temperature of the reactants T_1 and the temperature of the products T_2).

SKETCH:

Figure Pr.2.17 shows the modeled flame. The flame is at the opening of a tube and is shown to be thin.

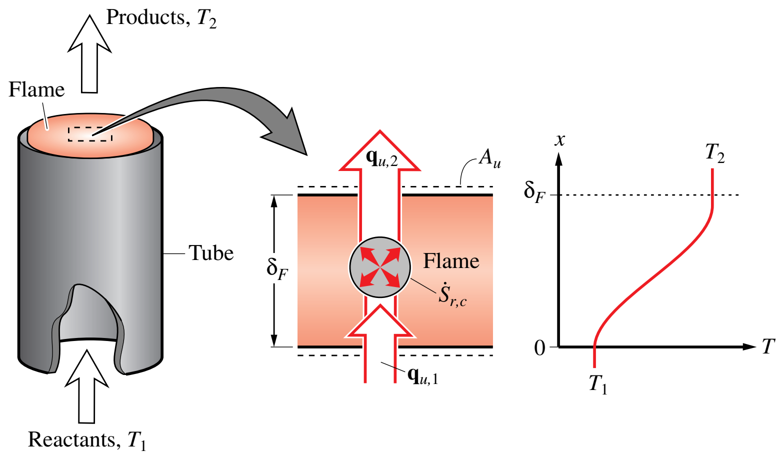


Figure Pr.2.17 A premixed methane-air flame showing the flame and the temperature distribution across δ_F.

OBJECTIVE:

(a) For the control volume enclosing the flame (Figure Pr.2.17) apply the integral-volume energy conservation equation. Neglect the heat loss by radiation and assume a steady-state condition.

(b) Obtain an expression for the heat generation inside the flame $\dot{S}_{r,c}$(W) as a function of the cold flow speed u_f(m/s), the concentration of methane in the reactant gas mixture $\rho_{\mathrm{F},1}$(kg$_{\mathrm{CH}_4}$/m^3), the heat of reaction for the methane/air reaction $\Delta h_{r,\mathrm{CH}_4}$(J/kg$_{\mathrm{CH}_4}$), and the area of the control surface A_u(m^2). Assume a complete combustion of the methane. [Hint: Use the conservation of mass of fuel equation (1.26) to obtain an expression for the volumetric reaction rate $\dot{n}_{r,\mathrm{F}}$(kg/m^3-s).]

(c) Using the integral-volume energy conservation equation obtained in item (a) and the expression for the heat generation obtained in item (b) calculate the temperature of the reacted gases (i.e., the adiabatic flame temperature, T_2).

SOLUTION:

(a) The heat flux vector tracking is shown in Figure Pr.2.17. The following steps complete the solution.
The integral-volume energy equation (2.9) is

$$\int_A \mathbf{q} \cdot \mathbf{s}_n dA = \int_V \left(-\frac{\partial}{\partial t} \rho c_p T \right) dV + \int_V \left(\sum_i \dot{s}_i \right) dV.$$

For this steady-state process the storage term is zero. The only energy conversion present is conversion from chemical bond to thermal energy (chemical reaction). The area integral of the normal component of the heat flux vector over the control surfaces enclosing the flame gives

$$\int_A \mathbf{q} \cdot \mathbf{s}_n dA = -q_{u,1}\ A_u + q_{u,2}\ A_u,$$

where A_u is the surface area of the flame sheet.
Thus, the integral-volume energy conservation equation becomes

$$-q_{u,1}\ A_u + q_{u,2}\ A_u = \int_V \dot{s}_{r,c} dV.$$

The heat flux by convection is given by (2.1) as

$$q_u = \rho c_p u T.$$

Using this, the energy equation becomes

$$-(\rho c_p u_F T)_1\ A_u + (\rho c_p u_F T)_2\ A_u = \int_V \dot{s}_{r,c} dV.$$

Note that the volumetric heat source term depends on the temperature which is not uniform within the flame (i.e., within the control volume).

(b) The volumetric heat source due to chemical reaction $\dot{s}_{r,c}$(W/m^3-s) can be written as

$$\dot{s}_{r,c} = \dot{n}_{r,\mathrm{F}} \Delta h_{r,\mathrm{F}},$$

where $\dot{n}_{r,\mathrm{F}}$(kg/m^3-s) is the volumetric reaction rate which gives the mass of fuel (methane) burned per unit volume and unit time. As $\Delta h_{r,\mathrm{F}}$ is constant for this constant pressure process, the rate of heat generation $\dot{S}_{r,c}$ is

$$\dot{S}_{r,c} = \int_V \dot{s}_{r,\mathrm{F}} dV = \int_V \dot{n}_{r,\mathrm{F}} \Delta h_{r,\mathrm{F}} dV = \Delta h_{r,\mathrm{F}} \int_V \dot{n}_{r,\mathrm{F}} dV.$$

To find the mass consumption rate, we use the conservation of mass equation for methane. The integral-volume species mass equation (1.26) is

$$\int_A \dot{\mathbf{m}}_\mathrm{F} \cdot \mathbf{s}_n dA = -\frac{\partial M_\mathrm{F}}{\partial t} + \int_V \dot{n}_{r,\mathrm{F}} dV.$$

The variation of the species mass with respect to time is zero for the steady-state process. For the control volume shown in Figure Pr.2.17, the net mass flux, in analogy to the net heat flux, is

$$\int_A \dot{\mathbf{m}}_\mathrm{F} \cdot \mathbf{s}_n dA = -\dot{m}_{\mathrm{F},1}\ A_u + \dot{m}_{\mathrm{F},2}\ A_u.$$

The mass flux of fuel F is $\dot{m}_\mathrm{F} = \rho_\mathrm{F} u_F$. The mass flux of methane leaving the control volume is zero because all the methane is burned inside the flame. Therefore, the above equation becomes

$$-\rho_\mathrm{F} u_F A_u = \int_V \dot{n}_{r,\mathrm{F}} dV.$$

The volumetric variation of the methane mass is due to the chemical reaction only. Thus, using the above equation we have

$$\dot{S}_{r,c} = \int_V \dot{s}_{r,\mathrm{F}} dV = -\rho_\mathrm{F} u_F A_u \Delta h_{r,\mathrm{F}}.$$

(c) Using this, the integral-volume energy equation becomes

$$-(\rho c_p u T)_1\ A_u + (\rho c_p u T)_2\ A_u = -\rho_\mathrm{F} u_F A_u \Delta h_{r,\mathrm{F}}.$$

From the conservation of mass of mixture equation (1.25)

$$(\rho u)_1 = (\rho u)_2.$$

By definition, $u_F = (u_g)_1$. Then dividing the energy equation by $(\rho u)_1 A_u$, we have

$$-(c_p T)_1 + (c_p T)_2 = -\frac{\rho_\mathrm{F}}{\rho} \Delta h_{r,\mathrm{F}}.$$

Solving for T_2, assuming that c_p is constant, and using the numerical values given, we have finally

$$T_2 = T_1 - \frac{\rho_{\mathrm{F}} \Delta h_{r,\mathrm{F}}}{\rho c_p} = 289(\mathrm{K}) - \frac{0.0621(\mathrm{kg_{CH_4}/m^3}) \times [-5.55 \times 10^7 (\mathrm{J/kg_{CH_4}})]}{1.13(\mathrm{kg/m^3}) \times 1{,}600(\mathrm{J/kg\text{-}K})} = 2{,}195 \text{ K}.$$

COMMENT:

An expression for the volumetric reaction rate $\dot{n}_{r,\mathrm{F}}$ can also be obtained by looking at the units. The volumetric reaction rate is the mass of methane ($\mathrm{kg_{CH_4}}$) burned per unit volume ($\mathrm{m^3}$) per unit time (s). The mass of methane burned is equal to the mass of methane available in the reactants, as all the methane is burned in the flame zone. This mass per unit volume is equal to the density of methane in the reactant mixture ρ. The time it takes to completely burn this mass of methane is equal to the time it takes for this mixture to travel through the reaction region. If the thickness of the reaction region is δ_{F}, and the velocity of the gas flow is u_F, the time is given by δ_{F}/u_F. Thus, the volumetric reaction rate of methane becomes $\dot{n}_{r,\mathrm{F}} = -\rho u_F/\delta_{\mathrm{F}}$. These are all constant parameters. Therefore, integrating over the volume of the flame (control volume) results in $\int_V \dot{n}_{r,\mathrm{F}} dV = -\rho u_F V/\delta_{\mathrm{F}} = -\rho u_F A_u$ and the rate of heat generation becomes $\dot{S}_{r,c} = -\Delta h_{r,\mathrm{F}} \rho u_F A_u$. Note that the negative sign arises because methane is being consumed (as opposed to produced) in the chemical reaction.

The average ρ and c_p are temperature-averaged values calculated over the temperature range between T_1 and T_2.

Note the high temperatures that are achieved in a flame. The assumption of complete combustion is not true for most combustion processes. Lack of complete mixing of fuel and oxidizer, heat losses, and dissociation of products, all contribute to a lower flame temperature.

The flame temperature can be reduced by diluting the reactant mixture, i.e., by reducing the amount of methane as compared to the amount of air. Then the combustion will occur in non-stoichiometric conditions.

PROBLEM 2.18.FAM

GIVEN:

A moist-powder tablet (pharmaceutical product) is dried before coating. The tablet has a diameter $D = 8$ mm, and a thickness $l = 3$ mm. This is shown in Figure Pr.2.18. The powder is compacted and has a porosity, i.e., void fraction, $V_f/V = 0.4$. This void space is filled with liquid water. The tablet is heated in a microwave oven to remove the water content. The rms of the electric field intensity $(\overline{e^2})^{1/2} = 10^3$ V/m and the frequency $f = 1$ GHz.

SKETCH:

Figure Pr.2.18 shows microwave energy conversion in a moist-powder tablet.

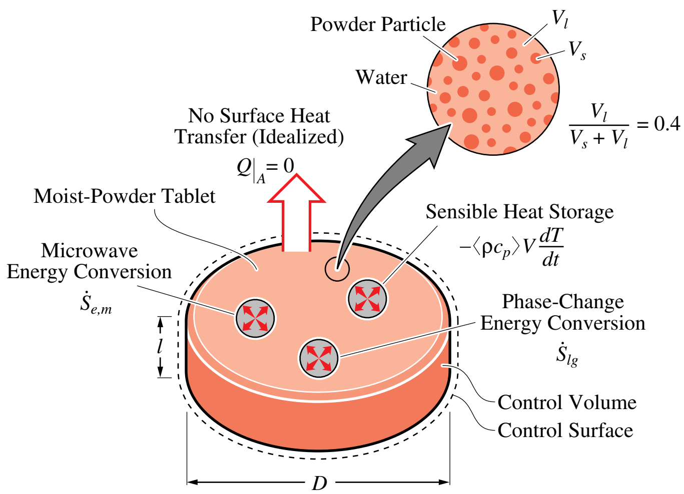


Figure Pr.2.18 Microwave heating of a moist-powder tablet.

OBJECTIVE:

(a) Determine the time it takes to heat the water content from the initial temperature of $T(t = 0) = 18$℃ to the final temperature of $T = 40$℃, assuming no evaporation.

(b) Determine the time it takes to evaporate the water content while the tablet is at a constant temperature $T = 40$℃. For the effective (including the liquid and powder) volumetric heat capacity $\langle\rho c_p\rangle$, which includes both water and powder, use 2×10^6 J/m^3-K. For the water density and heat of evaporation, use Table C.27. Assume that the dielectric loss factor for the powder is negligible compared to that for the water.

SOLUTION:

When the moist-powder tablet is internally heated by microwave electromagnetic energy conversion, the temperature of the tablet increases and the moisture evaporates simultaneously. Here we have assumed that first a rise in the temperature occurs, without any evaporation, and then evaporation occurs.

(a) For the first period, we start with the integral-volume energy equation (2.9) which is

$$Q|_A = -\langle\rho c_p\rangle V \frac{dT}{dt} + \dot{S}.$$

For no heat losses, $Q|_A = 0$. The energy conversion is due to microwave heating only, i.e., $\dot{S} = \dot{S}_{e,m}$. Then we have

$$-\langle\rho c_p\rangle V \frac{dT}{dt} + \dot{S}_{e,m} = 0.$$

For the conversion from microwave to thermal energy, we have, from (2.49),

$$\dot{S}_{e,m} = 2\pi f \epsilon_{ec} \epsilon_o \overline{e_e^2} V_l.$$

The volume of water, which is 40% the total volume of the tablet ($V_l = 0.4V$), and the dielectric loss factor for water $\epsilon_{ec,w}$ will be used. Then we have

$$-\langle\rho c_p\rangle V \frac{dT}{dt} + 2\pi f \epsilon_{ec,w} \epsilon_o \overline{e_e^2}(0.4V) = 0.$$

Integrating the equation above assuming constant properties gives

$$\begin{aligned}\frac{dT}{dt} &= \frac{2(0.4)\pi f \epsilon_{ec,w}\epsilon_o \overline{e_e^2}}{\langle \rho c_p \rangle} \\ T(t) - T(t=0) &= \frac{2(0.4)\pi f \epsilon_{ec,w}\epsilon_o \overline{e_e^2}}{\langle \rho c_p \rangle}\Delta t_1.\end{aligned}$$

Solving for Δt_1 we have

$$\Delta t_1 = \frac{\Delta T \langle \rho c_p \rangle}{2(0.4)\pi f \epsilon_{ec,w}\epsilon_o \overline{e_e^2}}.$$

From Table C.10, for water at $f = 10^9$ Hz, we have $\epsilon_{ec,w} = 1.2$. Using the numerical values we have

$$\begin{aligned}\Delta t_1 &= \frac{(40-18)(\text{K}) \times 2 \times 10^6 (\text{J/m}^3\text{-K})}{\pi(0.8) \times 10^9 (1/\text{s}) \times 1.2 \times 8.8542 \times 10^{-12} (\text{A}^2\text{-s}^2/\text{N-m}^2) \times (10^3)^2 (\text{V/m})^2} \\ &= 1.648 \times 10^3 \text{ s} = 0.4577 \text{ hr}.\end{aligned}$$

(b) For the second period, we start again with the integral energy equation which is

$$Q|_A = -\langle \rho c_p \rangle V \frac{dT}{dt} + \dot{S}.$$

Again, no heat losses are considered, i.e., $Q|_A = 0$. The temperature remains constant while the moisture evaporates and thus $dT/dt = 0$. The energy conversion is due to microwave heating $\dot{S}_{e,m}$ and to phase change only $\dot{S}_{lg}$, i.e., $\dot{S} = \dot{S}_{e,m} + \dot{S}_{lg}$. Thus, the energy equation becomes

$$\dot{S}_{e,m} + \dot{S}_{lg} = 0.$$

Using (2.49) and (2.25) we have

$$\begin{aligned}\dot{S}_{e,m} &= 2\pi f \epsilon_{ec}\epsilon_o \overline{e_e^2} V_l \\ \dot{S}_{lg} &= -\dot{n}_{lg}\Delta h_{lg} V_l.\end{aligned}$$

The evaporation rate has units of (kg/m^3-s). To evaporate all the water we have

$$\frac{\dot{M}_l}{\Delta t_2} = \frac{\dot{n}_{lg} V_l}{\Delta t_2}, \qquad \text{or} \qquad \dot{n}_{lg} = \frac{\rho_l}{\Delta t_2}, \rho_l = \frac{\dot{M}_l}{V_l}.$$

Then, using the equations above, the energy equation becomes

$$2\pi f \epsilon_{ec,w}\epsilon_o \overline{e_e^2} V_l - \frac{\rho_l}{\Delta t_2}\Delta h_{lg} V_l = 0.$$

Solving for Δt_2 we have

$$\Delta t_2 = \frac{\rho_l \Delta h_{lg}}{2\pi f \epsilon_{ec,w}\epsilon_o \overline{e_e^2}}.$$

From Table C.27, at $T = 313.2$ K, we have $\rho_l = 991.7$ kg/m^3 and $\Delta h_{lg} = 2.406 \times 10^6$ J/kg. Then, using the numerical values, we have

$$\begin{aligned}\Delta t_2 &= \frac{991.7 (\text{kg/m}^3) \times 2.406 \times 10^6 (\text{J/kg})}{2\pi \times 10^9 (1/\text{s}) \times 1.2 \times 8.8542 \times 10^{-12} (\text{A}^2\text{-s}^2/\text{N-m}^2) \times (10^3)^2 (\text{V/m})^2} \\ &= 3.574 \times 10^4 \text{ s} = 9.928 \text{ hr}.\end{aligned}$$

COMMENT:

Note that the evaporation period is longer than the sensible heating period.

In the sensible heating period, no evaporation was included. In reality, the evaporation occurs simultaneously with the heating due to the difference in partial pressure of the water inside the powder and outside in the ambient. This evaporation is controlled by the rate of vapor flow out of the powder. The surface heat transfer should also be included if more accurate predictions are required.

We have used a constant amount of water to calculate the microwave heating during the evaporation period. In reality, the amount of liquid decreases as the vapor is removed. This, along with the surface heat transfer, should be considered for more accurate predictions.

Note also that no resistance to the vapor flow out of the moist powder was considered. This may become the limiting transport rate for very fine powders and the increase in the internal pressure could cause the formation of cracks.

PROBLEM 2.19.FUN

GIVEN:

The automobile airbag deploys when the pressure within it is suddenly increased. This pressure increase is a result of the inflow of gaseous products of combustion or pressurized air from an inflater connected to the bag. The airbag fabric may be permeable, and this results in expansion of the gas as it flows through the fabric, as rendered in Figure Pr.2.19(a). Assume that the pressure gradient is approximated by

$$\frac{\partial p}{\partial x} \simeq \frac{p_o - p_i}{D},$$

where p_o and p_i are the external and internal pressure and D is the woven-fiber diameter. Consider the gas flowing with an average gas velocity $\langle u \rangle_{A,x} = 2$ m/s, and pressures of $p_i = 1.5 \times 10^5$ Pa and $p_o = 1.0 \times 10^5$ Pa. The fabric diameter is $D = 0.5$ mm. Use the expression for the volumetric energy conversion $\dot{s}_{m,P}$ for the one-dimensional flow given in Example 2.14.

Use ρc_p for air at 300 K (Table C.22).

SKETCH:

Figure 2.19(a) shows an idealization of airbag fabric and permeation of a gas stream through it.

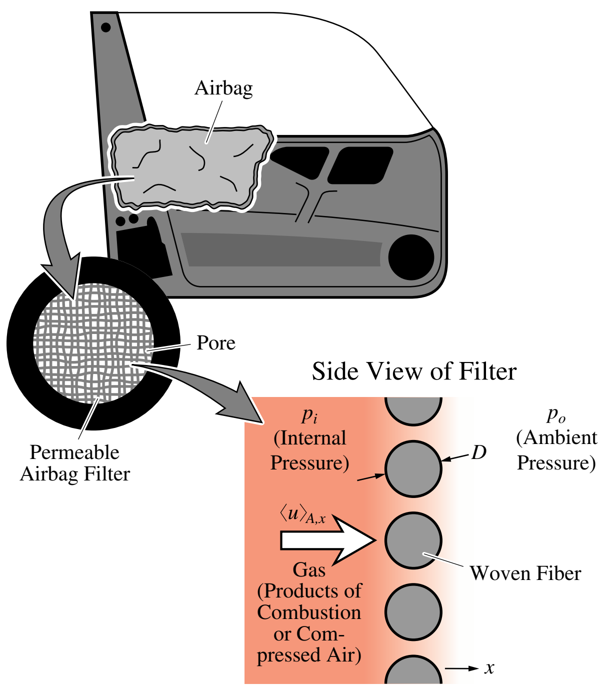


Figure Pr.2.19(a) An automobile airbag system.

OBJECTIVE:

(a) Determine the volumetric expansion cooling rate $\dot{s}_{m,p}$.

(b) Write an integral-volume energy equation for the gas flowing through a pore. Allow for expansion cooling and show the control volume, surface convection, and convection heat flows. Designate the flow area for the pore as A_x, and use $D \times A_x$ for the control volume. For the bounding surfaces, choose the two adjacent woven-fabric and imaginary-pore walls.

(c) For the case of no surface convection heat transfer, determine the drop in temperature $T_i - T_o$.

SOLUTION:

(a) The expression for pressure cooling or heating for a one-dimensional flow is the particular form of (2.50) given in Example 2.14. For an ideal gas, this expression is

$$\begin{aligned}\dot{s}_{m,p} &= \langle u\rangle_{A,x}\frac{\partial p}{\partial x}\\ &= \langle u\rangle_{A,x}\frac{p_o - p_i}{D}.\end{aligned}$$

Using the numerical values, we have

$$\dot{s}_{m,p} = 2(\text{m/s}) \times \frac{(1.0 - 1.5)\times 10^5(\text{Pa})}{5\times 10^{-4}(\text{m})} = -2\times 10^8 \text{ W/m}^3.$$

(b) The control volume and control surface for the gas are shown in Figure Pr.2.19(b). The integral energy equation (2.9) is

$$Q|_A = -\rho c_p V\frac{dT}{dt} + \dot{S}.$$

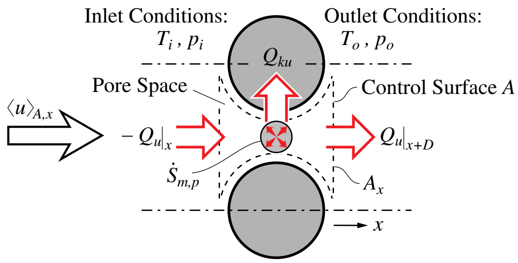


Figure Pr.2.19(b) Energy equation for a unit cell containing two adjacent woven fabrics.

For steady-state conditions, $dT/dt = 0$. Neglecting surface radiation, $Q|_A = -Q_u|_x + Q_u|_{x+D} + Q_{ku}$. The energy conversion is due to expansion cooling only. Then, the integral energy equation becomes

$$-Q_u|_x + Q_u|_{x+D} + Q_{ku} = \dot{s}_{m,p}V.$$

We can write $-Q_u|_x + Q_u|_{x+D}$ in terms of T_i and T_o as

$$-Q_u|_x + Q_u|_{x+D} = \rho c_p\langle u\rangle_{A,x}(-T_i + T_o).$$

Then, we have

$$\rho c_p\langle u\rangle_{A,x}(-T_i + T_o) + Q_{ku} = \dot{s}_{m,p}V.$$

The equation above shows that when solid-phase combustion is used in the inflater to generate the gas, the gas flowing through the fabric cools down due to surface convection heat-transfer ($Q_{ku} > 0$) and expansion cooling ($\dot{s}_{m,p} < 0$).

(c) For $Q_{ku} = 0$ and solving for $(T_o - T_i)$, we have

$$T_o - T_i = \frac{\dot{s}_{m,p}A_x D}{\rho c_p\langle u\rangle_{A,x}} = \frac{p_i - p_o}{\rho c_p}.$$

For air at $T = 300$ K, from Table C.22, we have $\rho = 1.177$ kg/m^3 and $c_p = 1{,}005$ J/kg-K. Then, using these values we have

$$T_o - T_i = \frac{(1.0 - 1.5)\times 10^5(\text{Pa})}{1.177(\text{kg/m}^3)\times 1{,}005(\text{J/kg-K})} = -42.27^\circ\text{C}.$$

COMMENT:

The reduction in temperature of the gas as it flows through the fabric is due to expansion cooling. The gas velocity is not high enough for viscous heating to become important. Also, during an air bag deployment, both the internal pressure and temperature vary. A typical deployment for a passenger air bag is 80 ms. During this time, the internal pressure and temperature vary from a peak pressure of $p = 40$ kPa and a temperature of $T = 500$ K to ambient conditions.

PROBLEM 2.20.FUN

GIVEN:

When high viscosity fluids, such as oils, flow very rapidly through a small tube, large strain rates, i.e., du/dr [where r is the radial location shown in Figure Pr.2.20(a)], are encountered. The high strain rate, combined with large fluid viscosity μ_f, results in noticeable viscous heating. In tube flows, when the Reynolds number $\mathrm{Re}_D = \rho_f \langle u \rangle_A D/\mu_f$ is larger than 2,300, transition from laminar to turbulent flow occurs. In general, high cross-section averaged fluid velocity $\langle u \rangle_A$ results in a turbulent flow. The fluid velocity for a laminar flow is shown in Figure Pr.2.20(a). For laminar flow, the center-line velocity is twice the average velocity, while for turbulent flow the coefficient is less than two. Assume that the cross-section averaged viscous heating rate, (2.51), is approximated as

$$\langle \dot{s}_{m,\mu} \rangle_A = \mu_f \frac{a_1^2 \langle u \rangle_A^2}{D^2},$$

where $a_1 = 1$ is a constant.

SKETCH:

Figure Pr.2.20(a) shows viscous heating in fluid flow through a small tube.

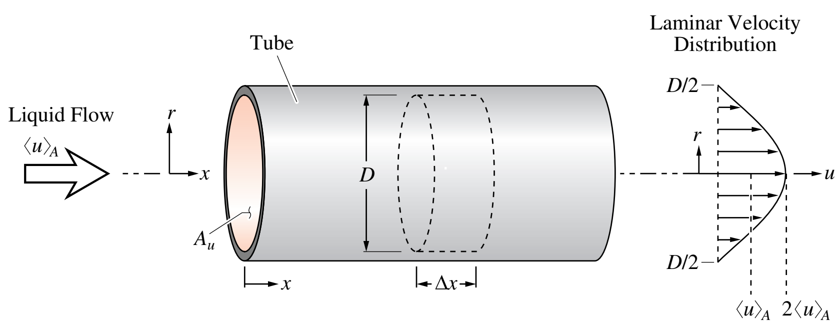


Figure Pr.2.20(a) Viscous heating of fluid flow inside a small tube.

OBJECTIVE:

(a) Determine the volumetric heating rate for engine oil at T = 310 K (Table C.23), noting that $\mu_f = \nu_f \rho_f$, $\langle u \rangle_A$=10 m/s, and D = 1 mm.

(b) Apply (2.8) to a differential length along the tube. Allow only for surface convection, convection along x, and viscous heating, i.e., similar to (2.11), with added energy conversion.

SOLUTION:

(a) The cross-sectional averaged viscous heating rate is

$$\langle \dot{s}_{m,\mu} \rangle_A = \mu_f \left(\frac{a_1 \langle u \rangle_A}{D} \right)^2.$$

The dynamic viscosity of engine oil is given in Table C.23 at T = 310 K as $\mu_f = \rho_f \nu_f = 877.8(\mathrm{kg/m^3}) \times 4.17 \times 10^{-4}(\mathrm{m^2/s}) = 0.3660$ Pa-s. Then, using the values given, we have

$$\langle \dot{s}_{m,\mu} \rangle_A = 0.3660(\text{Pa-s}) \left[\frac{1 \times 10(\mathrm{m/s})}{10^{-3}(\mathrm{m})} \right]^2 = 3.660 \times 10^7 \ \mathrm{W/m^3}.$$

The Reynolds number is

$$\mathrm{Re}_D = \frac{\langle u \rangle_A D}{\nu_f} = \frac{10(\mathrm{m/s}) \times 10^{-3}(\mathrm{m})}{4.17 \times 10^{-4}(\mathrm{m^2/s})} = 23.98.$$

Since $\mathrm{Re}_D < 2300$, the flow is in the laminar regime.

(b) The integral energy equation (2.9) for a steady-state condition and heat conversion due to mechanical friction is

$$\lim_{\Delta V \to 0} \frac{\int_{\Delta A} (\mathbf{q} \cdot \mathbf{s}_n) dA}{\Delta V} = \langle \dot{s}_{m,\mu} \rangle_A.$$

Figure Pr.2.20(b) shows various terms in the energy equation applied to the control volume shown.

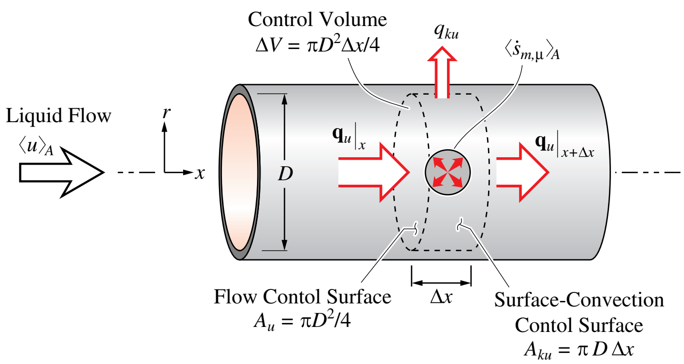


Figure Pr.2.20(b) Energy equation for viscous heating in fluid flow through the inside of a small tube.

As shown in Figure Pr.2.20(b), the heat transfer occurs along the r and x directions. Then the net heat transfer is

$$\begin{aligned}
\int_{\Delta A} (\mathbf{q} \cdot \mathbf{s}_n) dA &= q_{ku} \Delta A_{ku} + (q_u|_{x+\Delta x} - q_u|_x) A_u \\
&= q_{ku} \pi D \Delta x + (q_u|_{x+\Delta x} - q_u|_x) \frac{\pi D^2}{4}.
\end{aligned}$$

Then

$$\begin{aligned}
\lim_{\Delta V \to 0} \frac{\int_{\Delta A} (\mathbf{q} \cdot \mathbf{s}_n) dA}{\Delta V} &= \lim_{\Delta V \to 0} \frac{q_{ku} \pi D \Delta x + (q_u|_{x+\Delta x} - q_u|_x) \frac{\pi D^2}{4}}{\frac{\pi D^2}{4} \Delta x} \\
&= \lim_{\Delta V \to 0} \left[\frac{4 q_{ku}}{D} + \frac{(q_u|_{x+\Delta x} - q_u|_x)}{\Delta x} \right] \\
&= \frac{4 q_{ku}}{D} + \frac{d q_u}{dx}.
\end{aligned}$$

Therefore, the combined integral-and differential- length energy equation becomes

$$\frac{4 q_{ku}}{D} + \frac{d q_u}{dx} = \langle \dot{s}_{m,\mu} \rangle_A.$$

COMMENT:

Note that for very small D and very large ρ_f or $\langle u \rangle_A$, the viscous heating can be significant.

PROBLEM 2.21.FUN

GIVEN:

During braking, nearly all of the kinetic energy of the automobile is converted to frictional heating at the brakes. A small fraction is converted in the tires. The braking time, i.e., the elapsed time for a complete stop, is τ. The automobile mass is M, the initial velocity is u_o, and the stoppage is at a constant deceleration $(du/dt)_o$.

OBJECTIVE:

(a) Determine the rate of friction energy conversion for each brake in terms of M, u_o, and τ. The front brakes convert 65% of the energy and the rear brakes convert the remaining 35%.

(b) Evaluate the peak energy conversion rate for the front brake using $M = 1{,}500$ kg (typical for a mid-size car), $u_o = 80$ km/hr, and $\tau = 4$ s.

SOLUTION:

(a) The total instantaneous friction heating rate $\dot{S}_{m,F}$ is

$$\dot{S}_{m,F} = Fu,$$

where the force F is

$$F = -M\frac{du}{dt}.$$

Now, using a constant deceleration, we have

$$\dot{S}_{m,F} = -M\left(\frac{du}{dt}\right)_o u,$$

where

$$\left(\frac{du}{dt}\right)_o = \frac{\Delta u}{\Delta t} = \frac{0 - u_o}{\tau} = -\frac{u_o}{\tau}.$$

In order to find an expression for u, we integrate the equation above obtaining

$$u = -\frac{u_o}{\tau}t + a_1.$$

For $u(t = 0) = u_o$ we have

$$u = u_o\left(1 - \frac{t}{\tau}\right).$$

Then, using this we have

$$\begin{aligned}\dot{S}_{m,F} &= M\frac{u_o}{\tau}u_o\left(1 - \frac{t}{\tau}\right)\\ &= \frac{Mu_o^2}{\tau}\left(1 - \frac{t}{\tau}\right).\end{aligned}$$

Now, for 65% of the power being dissipated in the front breaks we have for each of the front brakes

$$\dot{S}_{m,F} = 0.65\frac{Mu_o^2}{\tau}\left(1 - \frac{t}{\tau}\right)$$

and for each of the rear brakes

$$\dot{S}_{m,F} = 0.35\frac{Mu_o^2}{\tau}\left(1 - \frac{t}{\tau}\right).$$

(b) Using the numerical values given, the peak heating rate (i.e., heating rate at $t = 0$) is

$$\begin{aligned}\dot{S}_{m,F} &= 0.65\frac{Mu_o^2}{\tau}\\ &= 0.65 \times \frac{1{,}500(\text{kg}) \times (22.22)^2(\text{m/s})^2}{4(\text{s})} = 1.203 \times 10^5 \text{ W} = 120.3 \text{ kW}.\end{aligned}$$

COMMENT:

This is a very large heating rate and its removal from the disc by the heat losses would require a large elapsed time. Therefore, if the brake is applied frequently such that this heat is never removed, overheating of the brake pads occurs.

PROBLEM 2.22.FUN

GIVEN:

In therapeutic heating, biological tissues are heated using electromagnetic (i.e., microwave, and in some cases, Joule heating) or mechanical (i.e., ultrasound heating) energy conversion. In the heated tissue, which may be a sore muscle (e.g., an athletic discomfort or injury), some of this heat is removed through the local blood flow and this is called perfusion heating. Under steady state, the local tissue temperature reaches a temperature where the surface heat transfer from the tissue balances with the energy conversion rate. Consider the therapeutic ultrasound heating shown in Figure Pr.2.22(a).

$I_{ac} = 5 \times 10^4$ W/m^2, σ_{ac} (from Table C.11, for muscle tissue), V (sphere of $R = 3$ cm), $D = 10^{-3}$ m, $A_{ku} = 0.02$ m^2, $\langle \mathrm{Nu} \rangle_D = 3.66$, $k_f = 0.62$ W/m-K (same as water), $T_f = 37$°C.

SKETCH:

Figure Pr.2.22(a) shows the ultrasonic therapeutic heating of a vascular tissue.

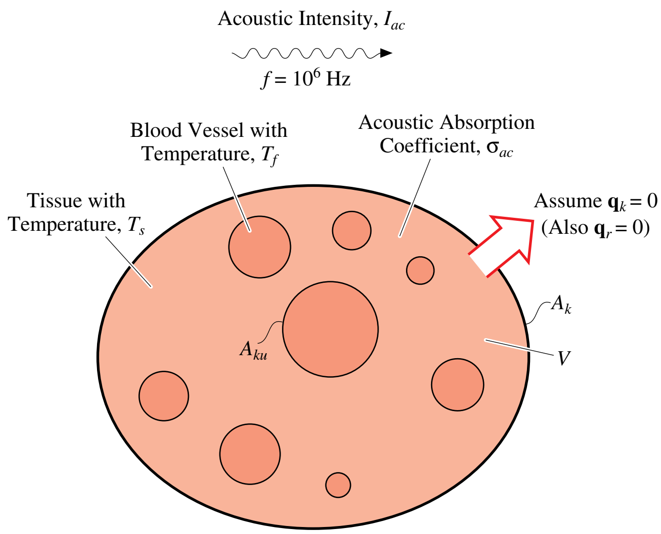


Figure Pr.2.22(a) Therapeutic heating of biological tissue.

OBJECTIVE:

(a) Using (2.9) write the integral-volume energy equation that applies to this steady-state heat transfer. Assume no conduction and radiation heat transfer and allow for surface convection through the blood vessels distributed through the tissue with a surface-convection area A_{ku}. Draw a schematic showing the various terms in the energy equation.

(b) In this energy equation, replace the surface-convection heat transfer with

$$A_{ku} q_{ku} = A_{ku} \langle \mathrm{Nu} \rangle_D \frac{k_f}{D} (T_s - T_f),$$

where $\langle \mathrm{Nu} \rangle_D$ is a dimensionless quantity called the dimensionless surface-convection conductance (or Nusselt number), k_f is the blood thermal conductivity, D is the average blood vessel diameter, T_s is the tissue temperature, and T_f is the blood temperature.

(c) Solve the energy equation for T_s.

(d) Using the following numerical values, determine T_s.

SOLUTION:

(a) The various heat transfer mechanisms and the energy conversion by ultrasound heating are shown in Figure Pr.2.22(b). From (2.9), for steady-state conditions, we have

$$Q|_A = \dot{s}_{m,ac} V,$$

where we have assumed a uniform $\dot{s}_{m,ac}$ throughout the volume. From (2.54), we have

$$\dot{s}_{m,ac} = 2\sigma_{ac} I_{ac}.$$

The surface heat transfer is limited to surface-convection only, i.e.,

$$Q|_A = A_{ku} q_{ku}.$$

Then the energy equation becomes

$$A_{ku} q_{ku} = 2\sigma_{ac} I_{ac} V.$$

Figure Pr.2.22(b) shows the various terms in the energy equation applied to the control volume shown.

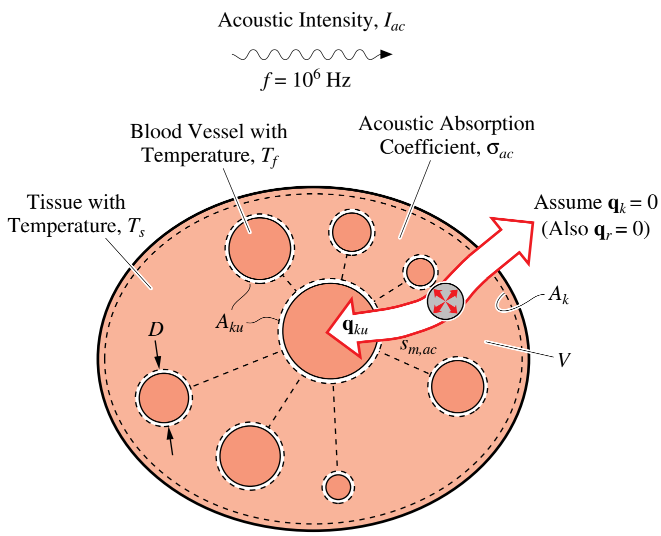


Figure Pr.2.22(b) Various terms in the energy equation for therapeutic ultrasound heating.

(b) The surface-convection heat transfer is given by

$$A_{ku} q_{ku} = A_{ku} \frac{\langle \text{Nu} \rangle_D k_f}{\langle D \rangle} (T_s - T_f).$$

Then, the energy equation becomes

$$A_{ku} \frac{\langle \text{Nu} \rangle_D k_f}{\langle D \rangle} (T_s - T_f) = 2\sigma_{ac} I_{ac} V.$$

The group $\langle \text{Nu} \rangle_D k_f / \langle D \rangle$ is called the (dimensional) surface-convection conductance or the heat transfer coefficient.
(c) Solving the equation above for T_s, we have

$$T_s = T_f + \frac{2\sigma_{ac} I_{ac} V \langle D \rangle}{\langle \text{Nu} \rangle_D k_f A_{ku}}.$$

(d) From Table C.11, $\sigma_{ac} = 14$ m^{-1}. Using the numerical values, we have

$$\begin{aligned} T_s &= 37(^\circ\text{C}) + \frac{2 \times 14(\text{m}^{-1}) \times 5 \times 10^4(\text{W/m}^2) \times (4/3)\pi(3 \times 10^{-2})^3(\text{m}^3) \times 10^{-3}(\text{m})}{3.66 \times 0.62(\text{W/m-K}) \times 0.02(\text{m}^2)} \\ &= 40.49^\circ\text{C}. \end{aligned}$$

COMMENT

The conduction heat losses can be significant and should be included. During heating, the blood vessels dilate causing D to increase. This results in a decrease in T_s. The Nusselt number, $\langle \text{Nu} \rangle_D$, will be discussed in Chapter 7.

PROBLEM 2.23.FAM

GIVEN:

Among the normal paraffins (n-paraffins) are the hydrocarbon fuels, e.g., methane CH_4, propane C_2H_6, and butane C_4H_{10}. Table Pr.2.23 gives two sets of constants for the chemical kinetic model given by

$$\dot{n}_{r,\text{F}} = -a_r \rho_\text{F}^{a_\text{F}} \rho_\text{O}^{a_\text{O}} e^{-\Delta E_a/R_g T},$$

for the CH_4 oxidation represented by a single-step, stoichiometric reaction

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O.$$

These two sets of parameters are found to give a good agreement between predicted and measured flame speeds as a function of methane/oxygen ratio.

OBJECTIVE:

Determine the reaction rates $\dot{n}_{r,\text{F}}$ at T = 1,000℃ using the above model (with the constants from Table Pr.2.23).
(a) Use a reactant-rich condition of $\rho_{O_2} = 0.9307$ kg/m^3, $\rho_{CH_4} = 0.2333$ kg/m^3, and
(b) a product-rich condition of $\rho_{O_2} = 0.1320$ kg/m^3, $\rho_{CH_4} = 0.0328$ kg/m^3,
to represent two locations within the flame. These are characteristics of CH_4 reaction with oxygen (called oxy-fuel reactions as compared to air-fuel reactions) at one atm pressure.
(c) Compare the results with the prediction of the zeroth-order model given in Example 2.6 by (2.21).

Table Pr.2.23 Constants in chemical kinetic model for methane oxidation.

a_r, s^{-1}	ΔE_a, J/kmole	a_F	a_O
1.3×10^8	2.026×10^8	–0.3	1.3
8.3×10^5	1.256×10^8	–0.3	1.3

SOLUTION:

The two chemical kinetic models are

$$\dot{n}_{r,CH_4} = -a_r \exp\left(\frac{-\Delta E_a}{R_g T}\right) \quad \text{zeroth-order-kinetics}$$
$$\dot{n}_{r,CH_4} = -a_r \rho_{CH_4}^{a_F} \rho_\text{O}^{a_O} \exp\left(\frac{-\Delta E_a}{R_g T}\right) \quad \text{first-order-kinetics.}$$

(a) Using the first set of constants, the first-order model and reactant-rich conditions, we have

$$\begin{aligned}\dot{n}_{r,CH_4} &= -1.3 \times 10^8 (1/\text{s})[0.2333(\text{kg/m}^3)]^{-0.3}[0.9307(\text{ kg/m}^3)]^{1.3} \times \\ &\quad \exp\left[\frac{-2.026 \times 10^8 (\text{J/kmole})}{8{,}314(\text{J/kmole-K})(1{,}000 + 273.15)(\text{K})}\right] \\ &= -0.9004 \text{ kg/m}^3\text{-s.}\end{aligned}$$

(b) For the product-rich conditions, we have

$$\begin{aligned}\dot{n}_{r,CH_4} &= -1.3 \times 10^8 (1/\text{s})[0.0328(\text{kg/m}^3)]^{-0.3}[0.1320(\text{ kg/m}^3)]^{1.3} \times \\ &\quad \exp\left[\frac{-2.026 \times 10^8 (\text{J/kmole})}{8{,}314(\text{J/kmole-K})(1{,}000 + 273.15)(\text{K})}\right] = -0.1280 \text{ kg/m}^3\text{-s}\end{aligned}$$

(c) Using the numerical values from Example 2.6 for the zeroth-order model, we have

$$\begin{aligned}\dot{n}_{r,CH_4} &= -1.3 \times 10^8 (\text{kg/m}^3\text{-s}) \exp\left[\frac{-2.10 \times 10^8 (\text{J/kmole})}{8{,}314(\text{J/kmole-K})(1{,}000 + 273.15)(\text{K})}\right] \\ &= -0.3150 \text{ kg/m}^3\text{-s.}\end{aligned}$$

COMMENT:

The zeroth-order chemical kinetic model is a concentration-independent averaged model. Its predictions are comparable with the first-order chemical kinetic model when the predictions for the reactant-rich and the product-rich regions of the flame are averaged. The advantage of using the zeroth-order chemical kinetic model is its relative mathematical simplicity. This will be further explored in Chapter 5. For more accurate predictions, better models are needed. Some of the more complete models account for hundreds of reactions and tenths of species taking part in the reaction.

PROBLEM 2.24.FAM

GIVEN:

In addition to being abundant and readily available, air is a fluid whose temperature can be raised well above and below room temperature, for usage as a hot or cold stream, without undergoing any phase change. In the high temperature limit, the main constituents of air, nitrogen (N_2) and oxygen (O_2), dissociate and ionize at temperatures above $T = 2{,}000$ K. In the low temperature limit, oxygen condenses at $T = 90.0$ K, while nitrogen condenses at $T = 77.3$ K. Consider creating (a) a cold air stream with $T_2 = 250$ K, (b) a hot air stream with $T_2 = 1{,}500$ K, and (c) a hot air stream with $T_2 = 15{,}000$ K.

The air stream is at atmospheric pressure, has a cross-sectional area $A_u = 0.01$ m^2, an inlet temperature $T_1 = 290$ K, and a velocity $u_1 = 1$ m/s, as shown in Figure Pr.2.24(a).

SKETCH:

Figure Pr.2.24(a) gives a general control volume through which an air stream flows, while undergoing energy conversion.

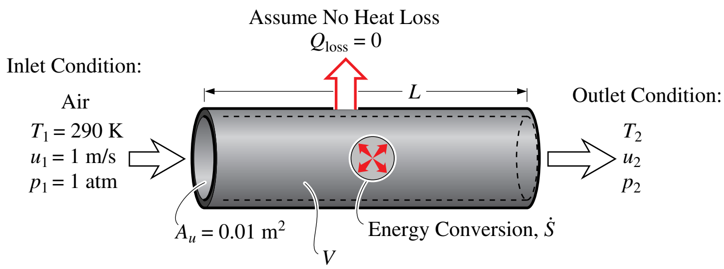


Figure Pr.2.24(a) Heating or cooling of an air stream using various energy conversion mechanisms.

OBJECTIVE:

For each of the cases above, (i) choose an energy conversion mechanism from Table 2.1 that would provide the required energy conversion mechanisms for heating or cooling, (ii) write the integral-volume energy equations (2.9) for a steady-state flow and heat transfer. Give the amount of fluid, electromagnetic energy, etc., that is needed.

SOLUTION:

(a) From Table 2.1, we choose phase-change (evaporation) cooling. This is shown in Figure Pr.2.24(b). The integral-volume energy equation for a steady-state condition is

$$Q|_A = \dot{S}_{lg}.$$

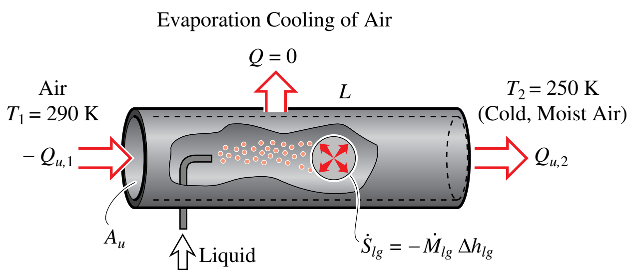


Figure Pr.2.24(b) Evaporation cooling of a gas stream.

The net heat transfer at the control surface is $Q|_A = Q_{u,2} - Q_{u,1}$. For the energy conversion due to phase change, we have $\dot{S}_{lg} = -\dot{M}_{lg}\Delta h_{lg}$. Then, the energy equation becomes

$$Q_{u,2} - Q_{u,1} = -\dot{M}_{lg}\Delta h_{lg}.$$

The convection heat transfer is given by $Q_u = A_u \rho c_p T$. From the conservation of mass equation (1.25) we have

$$(A_u \rho u)_1 = (A_u \rho u)_2.$$

Then, we can write the energy equation as

$$A_u \rho_1 c_{p,1} u_1 (T_2 - T_1) = -\dot{M}_{lg} \Delta h_{lg}.$$

Solving for $\dot{M}_{lg}$ we have

$$\dot{M}_{lg} = -\frac{A_u \rho_1 c_{p,1} u_1 (T_2 - T_1)}{\Delta h_{lg}}.$$

We examine Table C.6 and choose carbon dioxide as the fluid to be evaporated. For this fluid, $T_{lg} = 216.6$ K and $\Delta h_{lg} = 573.2 \times 10^3$ J/kg. From Table C.22, we have at $T = 290$ K, $\rho_1 = 1.224$ kg/m^3. We also assume a constant specific heat of $c_p = 1{,}006$ J/kg-K. Using the values given, we have

$$\begin{aligned}
\dot{M}_{lg} &= -\frac{0.01(\text{m}^2) \times 1.224(\text{kg/m}^3) \times 1{,}006(\text{J/kg-K}) \times 1(\text{m/s}) \times (250 - 290)(\text{K})}{573.2 \times 10^3(\text{J/kg})} \\
&= 8.593 \times 10^{-4} \text{ kg/s} = 0.8593 \text{ g/s}.
\end{aligned}$$

(b) For $T_2 = 1{,}500$ K, we choose combustion and the integral-volume energy equation (2.9) becomes

$$A_u \rho_1 c_{p,1} u_1 (T_2 - T_1) = -\dot{M}_{r,\text{CH}_4} \Delta h_{r,\text{CH}_4}.$$

This is shown in Figure Pr.2.24(c). Solving for $\dot{M}_{r,\text{CH}_4}$ we have

$$\dot{M}_{r,\text{CH}_4} = -\frac{A_u \rho_1 c_{p,1} u_1 (T_2 - T_1)}{\Delta h_{r,\text{CH}_4}}.$$

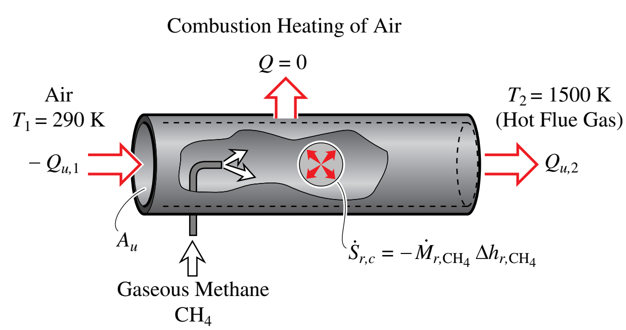


Figure Pr.2.24(c) Combustion heating of a gas stream.

From Table C.21(a), we have $\Delta h_{r,\text{CH}_4} = -5.553 \times 10^7$ J/kg. Then, we have

$$\begin{aligned}
\dot{M}_{r,\text{CH}_4} &= -\frac{0.01(\text{m}^2) \times 1.224(\text{kg/m}^3) \times 1{,}006(\text{J/kg-K}) \times 1(\text{m/s}) \times (1{,}500 - 290)(\text{K})}{-5.553 \times 10^7(\text{J/kg})} \\
&= 2.683 \times 10^{-4} \text{ kg/s} = 0.2683 \text{ g/s}.
\end{aligned}$$

(c) From Table 2.1, for $T_2 = 15{,}000$ K, we choose the Joule heating. After an initial formation of dissociated-ionized air by a combustion torch, induction coils are used to heat the charged gas (i.e., the plasma) stream. This is shown in Figure Pr.2.24(d).

The integral-volume energy equation becomes

$$A_u \rho_1 c_{p,1} u_1 (T_2 - T_1) = \dot{S}_{e,\text{J}}.$$

Solving for $\dot{S}_{e,\text{J}}$ we have

$$\begin{aligned}
\dot{S}_{e,\text{J}} &= 0.01(\text{m}^2) \times 1.224(\text{kg/m}^3) \times 1{,}006(\text{J/kg-K}) \times 1(\text{m/s}) \times (15{,}000 - 290)(\text{K}) \\
&= 1.811 \times 10^5 \text{ W}.
\end{aligned}$$

COMMENT:

In all of these examples, we have neglected any heat losses and assumed complete evaporation and complete reaction. These idealizations can be removed by their proper inclusion in the energy equation. The use of a variable c_p would require a different form of the energy equation, as shown in Appendix B. For most heat transfer analysis, a constant, temperature-averaged c_p is an acceptable approximation. Also, note that other mechanisms of heating/cooling could be used, such as thermoelectric and expansion cooling.

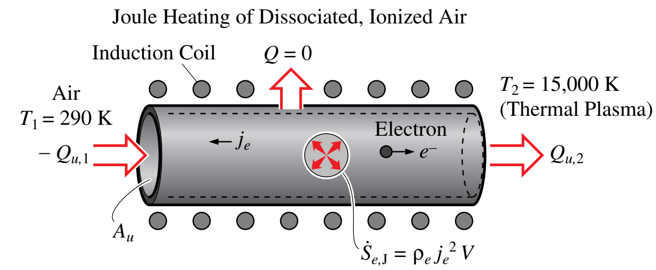


Figure Pr.2.24(d) A charged, gas stream heated by induction coils.

PROBLEM 2.25.FUN

GIVEN:

A transparent thin-foil heater is used to keep a liquid crystal display (LCD) warm under cold weather conditions. The thin foil is sandwiched between the liquid crystal and the backlight. A very thin copper foil, with cross section $w = 0.4$ mm and $l = 0.0254$ mm and a total length $L = 70$ cm, is used as the heating element. The foil is embedded in a thin polyester membrane with dimensions $W = 10$ cm and $H = 2$ cm, which also acts as an electrical insulator [Figure Pr.2.25(a)].

The thin foil heats the liquid crystal by Joule heating. Assume that the amount of heat flowing to the backlight panel is the same as the amount flowing to the liquid crystal and that the system is operating under a steady-state condition. For the electrical resistivity of copper use $\rho_e = 1.725 \times 10^{-8}$ ohm-m.

SKETCH:

Figure Pr.2.25(a) shows the thin foil heater and its dimensions.

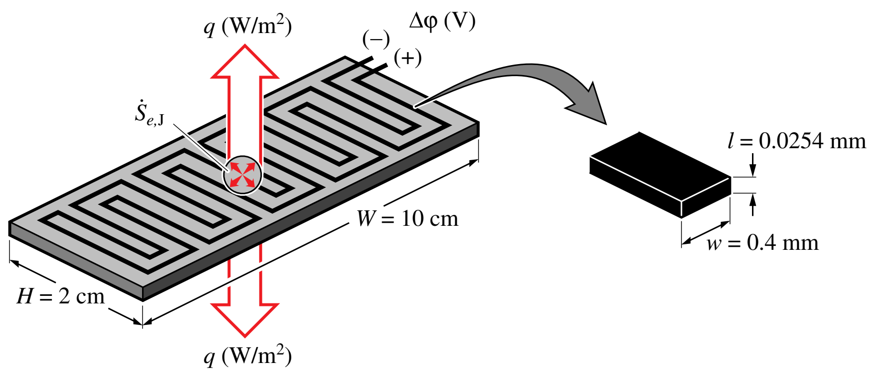


Figure Pr.2.25(a) A transparent thin-foil heater.

OBJECTIVE:

If the thin foil heater is to provide $q = 1{,}000$ W/m^2 to the liquid crystal, calculate:
(a) The volumetric rate of heat generation in the wire $\dot{s}_{e,\mathrm{J}}$(W/m^3),
(b) The electrical potential $\Delta\varphi$(V) needed,
(c) The current flowing in the wire J_e(A), and
(d) Recalculate items (a) to (c) for twice the length L.

SOLUTION:

(a) The volumetric rate of heat generation in the wire $\dot{s}_{e,\mathrm{J}}$(W/m^3) is obtained from the integral-volume energy equation. The steps for the solution are

(i) Draw the heat flux vector. This is shown in Figure Pr.2.25(b).

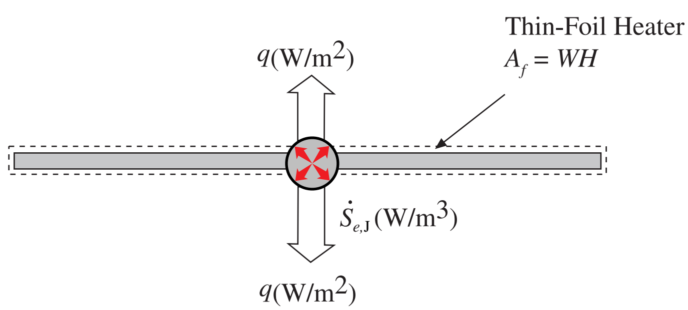


Figure Pr.2.25(b) Energy equation for the heater.

(ii) Apply the conservation of energy equation. The integral-volume energy equation is

$$\int_A \mathbf{q}\cdot\mathbf{s}_n dA = -\frac{d}{dt}\int_V (\rho c_p T)\, dV + \int_V \left(\sum_i \dot{s}_i\right) dV.$$

For this steady-state problem the storage term is zero. Solving for the area integral of the normal component of the heat flux vector over the control surfaces gives [see Figure Pr.2.25(b)]

$$\int_A \mathbf{q}\cdot\mathbf{s}_n dA = q\ A_k + q\ A_k = 2qA_k,$$

where $A_k = WH$ is the surface area of the thin-foil heater.

The only energy conversion taking place inside the control volume is the conversion from electromagnetic to thermal energy by Joule heating. Furthermore, this energy conversion is constant everywhere inside the control volume. Thus, the right-hand side of the energy equation becomes

$$\int_V \left(\sum_i \dot{s}_i\right) dV = \int_V \dot{s}_{e,\mathrm{J}} dV = \dot{s}_{e,\mathrm{J}}\ V_l,$$

where $V_l = wlL$ is the volume of the copper foil (heating element).
Finally, the energy equation becomes

$$2qA_k = \dot{s}_{e,\mathrm{J}}\ V_l$$

or

$$2qWH = \dot{s}_{e,\mathrm{J}} wlL.$$

(iii) Solving the energy equation for $\dot{s}_{e,\mathrm{J}}$, we have

$$\dot{s}_{e,\mathrm{J}} = \frac{2qWH}{wlL}.$$

From the numerical values given, we have

$$\dot{s}_{e,\mathrm{J}} = \frac{2\times 1{,}000(\mathrm{W/m^2})\times 0.02(\mathrm{m})\times 0.1(\mathrm{m})}{4\times 10^{-4}(\mathrm{m})\times 2.54\times 10^{-5}(\mathrm{m})\times 0.7(\mathrm{m})} = 5.62\times 10^{8}\ \mathrm{W/m^3}.$$

(b) The electrical potential $\Delta\varphi$(V) can be determined from the volumetric Joule heating using (2.32),

$$\dot{s}_{e,\mathrm{J}} = \frac{\Delta\varphi^2}{\rho_e L^2}.$$

Solving for $\Delta\varphi$(V) and using the data available, we have

$$\Delta\varphi = L(\dot{s}_{e,\mathrm{J}}\rho_e)^{1/2} = 0.7(\mathrm{m})\times[5.62\times 10^{8}(\mathrm{W/m^3})\times 1.725\times 10^{-8}(\text{ohm-m})]^{1/2} = 2.18\ \mathrm{V}.$$

(c) The current J_e(A) can be calculated from Ohm's law,

$$\Delta\varphi = R_e J_e,$$

where the electrical resistance R_e is given in (2.32) as

$$R_e = \frac{\rho_e L}{A_w}$$

and $A_w = wl$ is the wire cross-sectional area. Solving for J_e and using the data available, we have

$$J_e = \frac{\Delta\varphi wl}{\rho_e L} = \frac{2.18(\mathrm{V})\times 4\times 10^{-4}(\mathrm{m})\times 2.54\times 10^{-5}(\mathrm{m})}{1.725\times 10^{-8}(\text{ohm-m})\times 0.7(\mathrm{m})} = 1.8\ \mathrm{A}.$$

(d) For twice the wire length $2L$, the heat generation, voltage, and current are

$$\begin{aligned}
\dot{s}_{e,\mathrm{J}}(2L) &= \frac{2qWH}{wl2L} = \frac{\dot{s}_{e,\mathrm{J}}(L)}{2} = \frac{5.62\times 10^{8}(\mathrm{W/m^3})}{2} = 2.81\times 10^{8}\ \mathrm{W/m^3} \\
\Delta\varphi(2L) &= 2L\left[\frac{\dot{s}_{e,\mathrm{J}}(2L)}{\rho_e}\right]^{1/2} = 2L\left[\frac{\dot{s}_{e,\mathrm{J}}(L)\rho_e}{2}\right]^{1/2} = (2)^{1/2}L\left[\dot{s}_{e,\mathrm{J}}(L)\rho_e\right]^{1/2} \\
&= (2)^{1/2}\Delta\varphi(L) = (2)^{1/2}2.18(\mathrm{V}) = 3.08\ \mathrm{V} \\
J_e(2L) &= \frac{l\Delta\varphi(2L)wl}{\rho_e 2L} = \frac{(2)^{1/2}l\Delta\varphi(L)wl}{\rho_e 2L} = \frac{l\Delta\varphi(L)w\delta}{(2)^{1/2}\rho_e L} = \frac{J_e(L)}{(2)^{1/2}} = \frac{1.8(\mathrm{A})}{(2)^{1/2}} = 1.3\ \mathrm{A}.
\end{aligned}$$

COMMENT:

Notice the high volumetric energy conversion rate which can be achieved by Joule heating. Doubling the length caused a reduction in power, an increase in voltage, and a reduction in current. A reduction in power leads to a smaller temperature in the heating element. The drawback is the need for a larger voltage.

PROBLEM 2.26.FUN

GIVEN:

A single-stage Peltier cooler/heater is made of Peltier cells electrically connected in series. Each cell is made of p- and n-type bismuth telluride (Bi_2Te_3) alloy with Seebeck coefficients $\alpha_{S,p} = 230 \times 10^{-6}$ V/K and $\alpha_{S,n} = -210 \times 10^{-6}$ V/K. The cells are arranged in an array of 8 by 15 (pairs) cells and they are sandwiched between two square ceramic plates with dimensions $w = L = 3$ cm [see Figure Pr.2.26(a)]. The current flowing through the elements is $J_e = 3$ A.

SKETCH:

Figure Pr.2.26(a) shows a thermoelectric module and its various components.

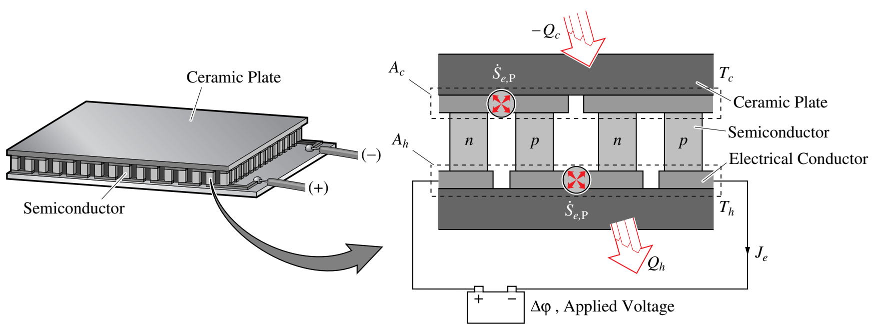


Figure Pr.2.26(a) A single-stage Peltier cooler/heater.

OBJECTIVE:

(a) If the temperature at the cold junction is $T_c = 10$℃, calculate the Peltier heat absorbed at the cold junctions q_c(W/m^2) (per unit area of the ceramic plate).

(b) If the temperature of the hot junctions reach $T_h = 50$℃, calculate the Peltier heat released at the hot junctions q_h(W/m^2) (per unit area of the ceramic plate).

SOLUTION:

(a) To calculate the heat absorbed at the cold junction we again follow the three steps.

(i) Draw the heat flux vector. This is shown in Figure Pr.2.26(b).

(ii) Apply the conservation of energy equation. The integral-surface energy equation (2.9) is

$$\int_A \mathbf{q} \cdot \mathbf{s}_n dA = \sum_i \dot{S}_i.$$

For this steady-state problem, the storage term is zero. The energy conversion term is due to Peltier cooling only. Then the energy equation becomes

$$-q_c A_s = (\dot{S}_{e,\mathrm{P}})_c,$$

where $A_s = wL$ is the surface area of the ceramic plate and $-Q_c$ is the rate of heat absorbed at the Peltier junctions.

(iii) Obtain an expression for the heat absorbed due to Peltier cooling. For 8 × 15 Peltier junctions we have

$$-q_c A_s = 8 \times 15 \times (\dot{S}_{e,\mathrm{P}})_c$$

where $(\dot{S}_{e,\mathrm{P}})_c$ is the heat absorbed at the Peltier cold junction which is given by (2.44)

$$(\dot{S}_{e,\mathrm{P}})_c = -(\alpha_{\mathrm{S},p} - \alpha_{\mathrm{S},n}) T_c J_e.$$

Then, the energy equation becomes

$$-q_c A_s = 15 \times 8 \times [-(\alpha_{\mathrm{S},p} - \alpha_{\mathrm{S},n}) T_c J_e].$$

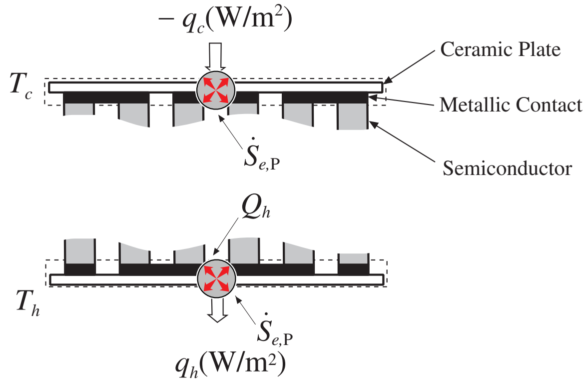


Figure Pr.2.26(b) The cold and hot surfaces of the thermoelectric module.

(iv) Solve for q_c. From the numerical values given, we have

$$
\begin{aligned}
(\dot{S}_{e,\mathrm{P}})_c &= -[230 \times 10^{-6}(\mathrm{V/K}) + 210 \times 10^{-6}(\mathrm{V/K})] \times 283.15(\mathrm{K}) \times 3(\mathrm{A}) = -0.374 \ \mathrm{W} \\
(\dot{S}_{e,\mathrm{P}})_c(15 \times 8) &= -0.374(\mathrm{W}) \times 120 = -44.8 \ \mathrm{W} \\
q_c &= \frac{Q_c}{wL} = \frac{-44.8(\mathrm{W})}{[-0.03(\mathrm{m}) \times 0.03(\mathrm{m})]} = 49{,}808 \ \mathrm{W/m^2}.
\end{aligned}
$$

(b) For the hot junction, a similar approach is used. The heat released is given by (2.41) as

$$
(\dot{S}_{e,\mathrm{P}})_c = (\alpha_{\mathrm{S},p} - \alpha_{\mathrm{S},n}) T_h J_e = [230 \times 10^{-6}(\mathrm{V/K}) + 210 \times 10^{-6}(\mathrm{V/K})] \times 323.15(\mathrm{K}) \times 3(\mathrm{A}) = 0.426 \ \mathrm{W}.
$$

The total heat generated at the hot junction is then

$$
Q_h = (\dot{S}_{e,\mathrm{P}})_h(15 \times 8) = 0.426(\mathrm{W}) \times 120 = 51.2 \ \mathrm{W}.
$$

The heat flux at the ceramic plate is

$$
q_h = \frac{Q_h}{wL} = \frac{51.2(\mathrm{W})}{0.03(\mathrm{m}) \ 0.03(\mathrm{m})} = 56{,}848 \ \mathrm{W/m^2}.
$$

COMMENT:

The values calculated above are ideal values for the Peltier heater/cooler. It will be seen in Chapter 3 that both the Joule heating and the heat conduction through the semiconductor legs of the Peltier cell, reduce the amount of heat that can be absorbed by a Peltier cooler. The analysis will lead to the definition of the figure of merit which express the efficiency of the Peltier cooler.

PROBLEM 2.27.FAM.S

GIVEN:

A pocket combustion heater uses heat released (chemical-bond energy conversion) from the reaction of air with a powder. The powder is a mixture of iron, water, cellulose (a carbohydrate), vermiculite (a clay mineral), activated carbon (made capable of absorbing gases), and salt. Air is introduced by breaking the plastic sealant and exposing the permeable membrane containing the powder to ambient air. Since the air has to diffuse through the powder, and also since the powder is not mixed, the heat release rate is time dependent, decreasing with time. We express this as $\dot{S}_{r,c} = \dot{S}_{r,o}\exp(-t/\tau)$, where $\tau(t)$ is called the time constant. The pocket heater has a mass of $M = 20$ g and a heat capacity of $c_p = 900$ J/kg-K. During the usage, heat leaves the pocket heater surface. This heat is expressed as a resistive-type heat transfer and is given by $Q = (T - T_\infty)/R_t$, where T_∞ is the ambient temperature and R_t(°C/W) is the surface heat transfer resistance. Initially the heater is at the ambient temperature, i.e., $T(t = 0) = T_\infty$. This is shown in Figure Pr.2.27(a).

SKETCH:

Figure Pr.2.27(a) shows the heat transfer model of a combustion pocket heater.

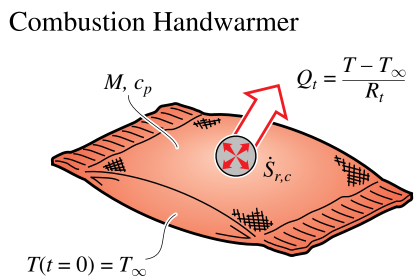


Figure Pr.2.27(a) A pocket combustion heater and its heat transfer model.

OBJECTIVE:

(a) Write the energy equation for the pocket heater.
(b) Using a software, plot the temperature of the pocket heater $T = T(t)$ versus time, up to $t = \tau$.
(c) What is the maximum heater temperature?

SOLUTION:

(a) Since we use a uniform temperature for the heater, the energy equation is the integral-volume energy equation (2.9), i.e.,

$$\begin{aligned} Q \mid_A &= -\rho c_p V \frac{dT}{dt} + \dot{S}_{r,c} \\ &= -M c_p \frac{dT}{dt} + \dot{S}_{r,c}. \end{aligned}$$

Here, the energy conversion term is time dependent, i.e.,

$$\dot{S}_{r,c} = \dot{S}_{r,o} e^{-t/\tau},$$

where $\dot{S}_{r,o}$ is a constant and τ is called the time constant.
The surface heat transfer rate is given by a surface thermal resistance, i.e.,

$$Q \mid_A = Q_t = \frac{T - T_\infty}{R_t},$$

where R_t is the heat transfer resistance and T_∞ is the ambient temperature.
Combining the above equations, we have

$$\frac{T - T_\infty}{R_t} = -M c_p \frac{dT}{dt} + \dot{S}_{r,o} e^{-t/\tau}.$$

The initial temperature is $T(t = 0) = T_\infty$.

(b) The above energy equation can not be readily integrated to give $T = T(t)$. Here, we use software and provide the constants $M, c_p, \dot{S}_{r,o}, \tau,$ and T_∞.
The results are plotted in Figure Pr.2.27(b). We note that initially T increases with time. Then it reaches a maximum. Finally, it begins to decrease. During the increase, the energy conversion rate is larger than the surface heat loss term $Q\mid_A = Q_t$. At the time of maximum temperature, when $dT/dt = 0$, the energy conversion and surface heat loss exactly balance. Due to the time dependence of $\dot{S}_{r,c}$, the temperature begins to decrease after reaching the maximum and, during the decrease, the energy conversion is less than the surface heat loss. These are also shown in Figure Pr.2.27(b).

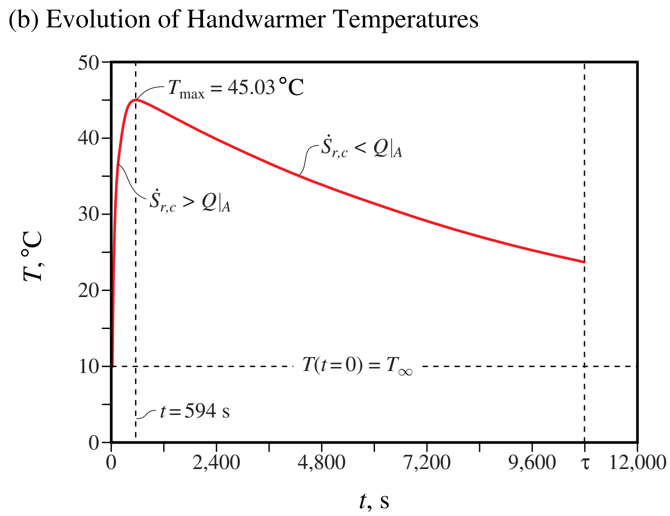


Figure Pr.2.27(b) Variation of the temperature of the pocket heater with respect to time.

(c) The maximum temperature is found to be $T_{max} = 45.03$℃ and occurs at $t = 594$ s. Note that direct contact of the heater with skin will cause damage.

COMMENT:
The model for heat release rate is an approximation. By proper design of the powder and its packaging, a uniform heat release rate may be achieved.

PROBLEM 2.28.FUN

GIVEN:

In electrical power generation using thermoelectric energy conversion, the electrical power can be optimized with respect to the external electrical resistance.

OBJECTIVE:

Starting from (2.40), show that the maximum power generation occurs for $R_{e,o} = R_e$, i.e., when the external electrical resistance is equal to the thermoelectric electrical resistance.

SOLUTION:

The electrical power generation given by (2.40) is maximized with respect to $R_{e,o}$ by taking the derivative of (2.40) and setting the result equal to zero. This gives

$$\frac{\partial}{\partial R_{e,o}}(J_e^2 R_{e,o}) = \frac{\partial}{\partial R_{e,o}}\left[\frac{\alpha_{\rm S}^2 (T_h - T_c)^2}{(R_{e,o} + R_e)^2} R_{e,o}\right] = 0$$

which results in

$$1 - 2\frac{R_{e,o}}{R_{e,o} + R_e} = 0$$

or

$$R_{e,o} = R_e.$$

COMMENT:

To prove that this is minimum, we take the second derivative of (2.40) and evaluate it for $R_{e,o} = R_e$.

$$\frac{d^2}{dR_{e,o}^2}(J_e^2 R_{e,o}) = \frac{\alpha_{\rm S}^2 (T_h - T_c)^2}{(R_{e,o} + R_e)^3}\left(-2 - 2 + 6 \times \frac{1}{2}\right) < 0,$$

and therefore, $R_{e,o} = R_e$ results in a minimum in $J_e^2 R_{e,o}$.

PROBLEM 2.29.FUN

GIVEN:

The volumetric pressure-compressibility heating/cooling energy conversion $\dot{s}_p$ can be represented in an alternative form using c_v instead of c_p in the energy equation.

OBJECTIVE:

Starting from (B.44) in Appendix B, show that for an ideal gas, the volumetric pressure-compressibility energy conversion $\dot{s}_p$ becomes

$$\dot{s}_p = -p\nabla \cdot \mathbf{u} = \left[\left(\frac{c_p}{c_v} - 1\right) c_v \rho T\right] \nabla \cdot \mathbf{u}.$$

Use the following relation, derived from combining (1.4), (1.5), and (1.6),

$$c_p \equiv c_v + T\left(\left.\frac{\partial p}{\partial T}\right|_v \left.\frac{\partial v}{\partial T}\right|_p\right).$$

SOLUTION:

Starting from (B.44), we define $\dot{s}_p$ as

$$\dot{s}_p \equiv -T\left.\frac{\partial p}{\partial T}\right|_v \nabla \cdot \mathbf{u}.$$

From (1.19), for an ideal gas, we have

$$\begin{aligned} p &= \frac{R_g}{M}\rho T = \frac{R_g}{Mv}T \\ T\left.\frac{\partial p}{\partial T}\right|_v &= T\frac{R_g}{M}\rho = p \quad \text{ideal gas.} \end{aligned}$$

Then

$$\dot{s}_p = -p\nabla \cdot \mathbf{u}.$$

Also, for ideal gas we have

$$\begin{aligned} c_p &\equiv c_v + T\left(\left.\frac{\partial p}{\partial T}\right|_v \left.\frac{\partial v}{\partial T}\right|_p\right) \\ &= c_v + T\left(\frac{R_g}{Mv}\frac{R_g}{Mp}\right) = c_v + \frac{R_g}{M}. \end{aligned}$$

Then

$$\begin{aligned} p &= \frac{R_g}{M}\rho T = (c_p - c_v)\rho T \\ &= \left(\frac{c_v}{c_p} - 1\right) c_v \rho T \end{aligned}$$

or

$$\dot{s}_p = -p\nabla \cdot \mathbf{u} = \left[\left(\frac{c_v}{c_p} - 1\right) c_v \rho T\right] \nabla \cdot \mathbf{u}.$$

COMMENT:

Note that for an incompressible fluid flow, from (B.40) we have

$$\nabla \cdot \mathbf{u} = 0 \quad \text{incompressible fluid flow.}$$

Also for an incompressible fluid, we have

$$\left.\frac{\partial v}{\partial T}\right|_p = 0, \quad c_p = c_v \quad \text{incompressible fluid.}$$

In pressure-compressibility cooling/heating, a large c_p/c_v (can be optimized by mixing species), a large $\nabla \cdot \mathbf{u}$ (would require a large pressure gradient), along with a large $c_v \rho T$ (high pressure and temperature) would be needed.

PROBLEM 2.30.FAM

GIVEN:

A microwave heater is used to dry a batch of wet alumina powder. The microwave source is regulated to operate at $f = 10^9$ Hz and to provide an electrical field with a root-mean-square intensity of $(\overline{e^2})^{1/2} = 10^3$ V/m. The effective dielectric loss factor of the alumina powder $\langle \epsilon_{ec} \rangle$ depends on the fluid filling the pores. For a porosity of 0.4, the effective dielectric loss factor of the completely dry alumina powder is $\langle \epsilon_{ec} \rangle = 0.0003$ and the effective dielectric loss factor of the completely wet alumina powder is $\langle \epsilon_{ec} \rangle = 6.0$.
Note that although both ϵ_{ec} and $\langle \epsilon_{ec} \rangle$ are listed, no distinction is made in Table C.10.

OBJECTIVE:

(a) Determine the microwave heating $\dot{s}_{e,m}$(W/m^3) for these two cases.
(b) Discuss the efficiency of the use of microwave heating in drying the alumina powder when the moisture content (i.e., amount of water in the pores) is small.
(c) From Table C.10, would a sandy soil dry faster or slower than the alumina powder?

SOLUTION:

The volumetric energy conversion by microwave heating is given by

$$\dot{s}_{e,m} = 2\pi f \langle \epsilon_{ec} \rangle \epsilon_o \overline{e^2}.$$

(a) For the wet alumina powder, we have

$$\dot{s}_{e,m} = 2\pi \times 10^9 (1/\text{s}) \times 6.0 \times 8.8542 \times 10^{-12} (\text{A}^2\text{-s}^2/\text{N-m}^2) \times 10^6 (\text{V/m})^2 = 3.338 \times 10^5 \text{ W/m}^3.$$

For the dry alumina powder we have

$$\dot{s}_{e,m} = 2\pi \times 10^9 (1/\text{s}) \times 0.0003 \times 8.8542 \times 10^{-12} (\text{A}^2\text{-s}^2/\text{N-m}^2) \times 10^6 (\text{V/m})^2 = 16.69 \text{ W/m}^3.$$

(b) For the same amount of available microwave energy, the wet alumina powder is able to convert 333,795 W/m^3 of that energy into volumetric heating. The dry alumina powder only converts 16.7 W/m^3 of the available energy into thermal energy. Therefore, the wet alumina powder utilizes microwave heating more efficiently in the drying of the powder.

(c) For dry sandy soil, we have

$$\dot{s}_{e,m} = 2\pi \times 10^9 (1/\text{s}) \times 0.026 \times 8.8542 \times 10^{-12} (\text{A}^2\text{-s}^2/\text{N-m}^2) \times 10^6 (\text{V/m})^2 = 1{,}446 \text{ W/m}^3.$$

Assuming that the particle size and porosity of the sandy soil is similar to that of the alumina powder, and since dry sandy soil makes more efficient use of microwave heating than dry alumina powder, we can conclude that wet sandy soil would dry faster.

COMMENT:

Note the ten thousand fold difference in the magnitude of the effective dielectric loss of the dry and the wet alumina powder. The dielectric loss factor for water at 25℃ is $\epsilon_{e,c} = 1.2$ and the dielectric loss for air is $\epsilon_{e,c} = 0$.

The dielectric loss for most dry ceramics is small. This explains the small volumetric heating rates in ceramics under low intensity microwave fields.

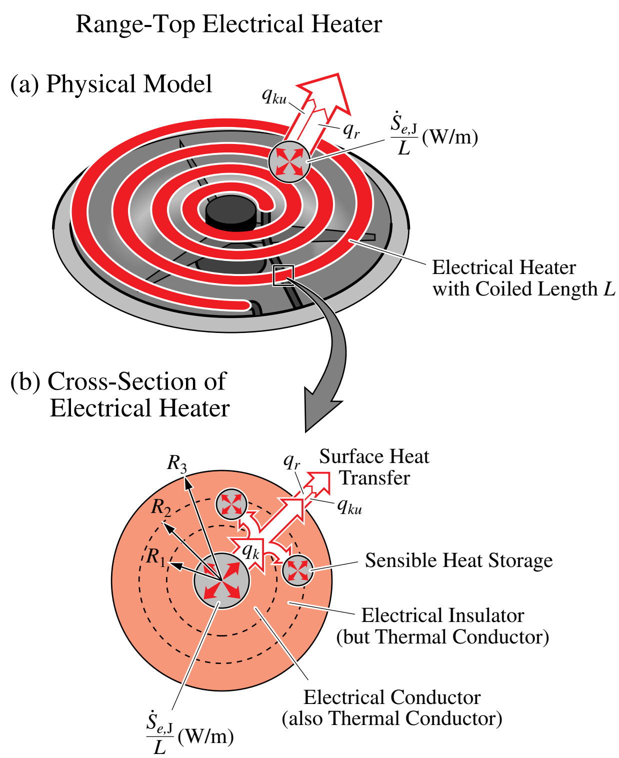
PROBLEM 2.31.FUN

GIVEN:

The range-top electrical heater is shown in Figure Pr.2.31(a). It has electrical elements made of a central electrical conductor (electric current carrying) surrounded by an electrical insulator. The electrical insulator should have a large thermal conductivity to carry the heat generated by Joule heating in the electrical conductor to the surface for surface convection-radiation heat transfer. This is shown in Figure Pr.2.31(b). During the start-up and turn-off, the transient heat transfer in the heater becomes significant. In order to analyze this transient heating, the temperature distribution in the heater is examined. Since the electric conductor also has a high thermal conductivity, it is treated as having a uniform temperature. However, the electrical insulator (generally an oxide ceramic) has a relatively lower thermal conductivity, and this results in a temperature nonuniformity within it.

SKETCH:

Figures Pr.2.31(a) and (b) show a range-top electrical heater and the layers within the heating element.



Figures Pr.2.31(a) A range-top electrical heater.(b) The various layers within the heating element.

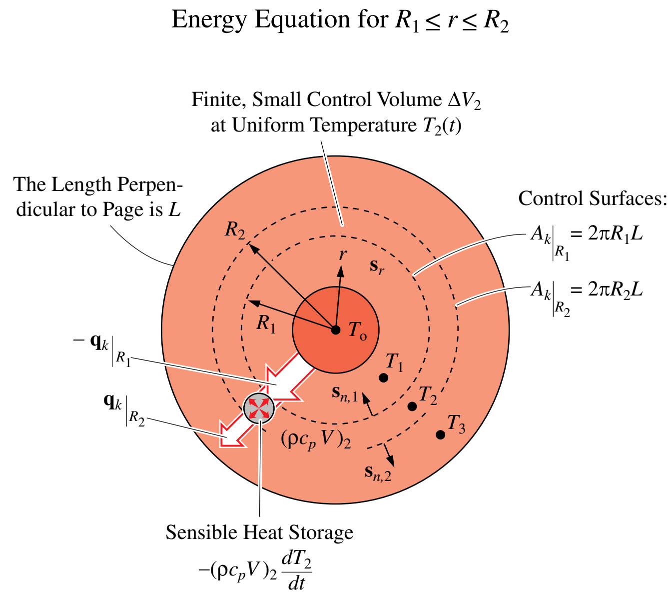
OBJECTIVE:

(a) Divide the volume of the electrical insulator into three regions, as shown in Figure Pr.2.31(b).
(b) Select a control volume in the region between $r = R_1$ and $r = R_2$ and render the heat transfer through this control volume.
(c) Show that the energy equation for this control volume allowing for conduction and sensible heat storage in the electrical insulator is given by

$$k\left.\frac{\partial T}{\partial x}\right|_{R_1} 2R_1 - k\left.\frac{\partial T}{\partial x}\right|_{R_2} 2R_2 = -(\rho c_p)_2(R_2^2 - R_1^2)\frac{\partial T_2}{\partial t}.$$

SOLUTION:

(a) The control volume and control surface for the volume contained in $R_1 \le r \le R_2$ in the electrical insulator is shown in Figure Pr.2.31(c).



Figures Pr.2.31(c) A finite-small control volume in the heater.

(b) The heat transfer is by conduction only. We begin with (2.13) and write

$$Q|_A = -\frac{d}{dt}[(\rho c_p T)_{\Delta V_2} \Delta V_2]$$

and

$$Q|_A = Q|_{A_1} + Q|_{A_2} = [(\mathbf{q}_k \cdot \mathbf{s}_n) A_k]_{R_1} + [(\mathbf{q}_k \cdot \mathbf{s}_n) A_k]_{R_2}.$$

From (1.11), the conduction heat transfer $\mathbf{q}_k$ is related to the temperature gradient. Here we have a one-dimensional conduction heat flow in the r direction, and

$$\mathbf{q}_k = -k \frac{\partial T}{\partial x} \mathbf{s}_r.$$

Also, the geometric parameters are

$$A_k|_{R_1} = 2\pi R_1 L\,, \quad A_k|_{R_2} = 2\pi R_2 L\,, \quad \Delta V_2 = \pi(R_2^2 - R_1^2)L\,.$$

(c) Using these and assuming constant ρc_p, similar to (2.15), we then have

$$\begin{aligned} k\frac{\partial T}{\partial x}\bigg|_{R_1}(2\pi R_1 L) - k\frac{\partial T}{\partial x}\bigg|_{R_2}(2\pi R_2 L) &= -(\rho c_p)_2 \pi (R_2^2 - R_1^2) L \frac{dT_2}{dt} \\ k\frac{\partial T}{\partial x}\bigg|_{R_1} 2R_1 - k\frac{\partial T}{\partial x}\bigg|_{R_2} 2R_2 &= -(\rho c_p)_2 (R_2^2 - R_1^2) \frac{dT_2}{dt}. \end{aligned}$$

COMMENT:

To accurately predict the transient temperature distribution in the electrical insulator, its division (i.e., discretization) into more than three regions is required (as many as twenty regions may be used). This will be discussed in Section 3.7.

PROBLEM 2.32.FUN

GIVEN:

Consider air (fluid) flow parallel to a semi-infinite plate (solid, $0 \le x \le \infty$), as shown in Figure Pr.2.32. The plate surface is at a uniform temperature T_{sf}. The flow is along the x axis. The velocity of air $\mathbf{u}_f$ at the solid surface is zero. Starting from (2.61) show that at a location L along the plate, the surface energy equation becomes

$$k_s \left.\frac{\partial T_s}{\partial y}\right|_{y=0^-} - k_f \left.\frac{\partial T_f}{\partial y}\right|_{y=0^+} = 0 \quad \text{on } A_{sf}.$$

Neglect surface radiation heat transfer, and use $\mathbf{u}_s = 0$, and $\mathbf{u}_f = 0$ on A_{sf}. There is no surface energy conversion.

SKETCH:

Figure Pr.2.32 shows the parallel air flow over a semi-infinite plate.

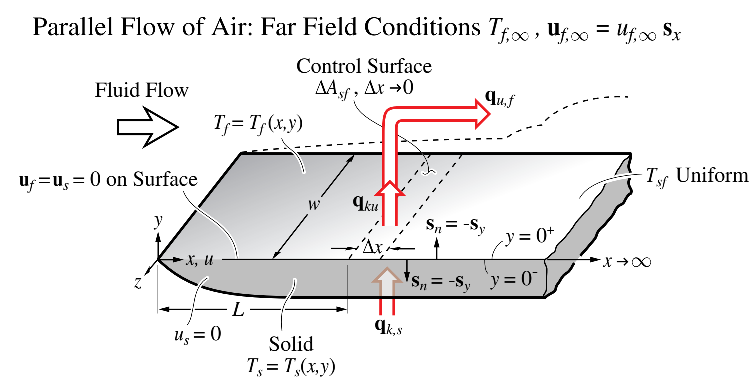


Figure Pr.2.32 A parallel air flow over a semi-infinite plate ($0 \le x \le \infty$), with a uniform surface temperature T_s.

OBJECTIVE:

Use the definition of surface-convection heat flux q_{ku} (positive when leaving the solid toward the gas) given as

$$-k_s \left.\frac{\partial T_s}{\partial y}\right|_{y=0^-} = -k_f \left.\frac{\partial T_f}{\partial y}\right|_{y=0^+} \equiv q_{ku} \quad \text{on } A_{sf}.$$

SOLUTION:

Starting from (2.62), we have

$$\int_{\Delta A_{sf}\rightarrow 0} [-k(\nabla T \cdot \mathbf{s}_n) + \rho c_p T(\mathbf{u} \cdot \mathbf{s}_n) + \mathbf{q}_r \cdot \mathbf{s}_n]_f dA_{sf} + \int_{\Delta A_{sf}\rightarrow 0} [-k(\nabla T \cdot \mathbf{s}_n) + \rho c_p T(\mathbf{u} \cdot \mathbf{s}_n) + \mathbf{q}_r \cdot \mathbf{s}_n]_s dA_{sf} = \dot{S}.$$

Here, $\mathbf{q}_r$ and $\mathbf{u}$ and $\dot{S}$ are all set to zero (no surface radiation heat transfer, no fluid or solid motion on the surface, and no surface energy conversion). Then

$$\int_{\Delta A_{sf}\rightarrow 0} -k_f(\nabla T_f \cdot \mathbf{s}_n) dA_{sf} + \int_{\Delta A_{sg}\rightarrow 0} -k_s(\nabla T_s \cdot \mathbf{s}_n) dA_{fs} = 0.$$

Now using the surface unit normal vectors shown in Figure Pr.2.32, and noting that

$$\nabla T_f \cdot \mathbf{s}_n = \frac{\partial T_f}{\partial y}, \quad \nabla T_s \cdot \mathbf{s}_n = -\frac{\partial T_s}{\partial y},$$

and evaluating these derivatives in their perspective surface (noting that the control surface wraps around the surface), i.e., gas at $y \le 0^+$ and solid at $y \le 0^-$, we have

$$-k_f \left.\frac{\partial T_f}{\partial y}\right|_{y=0^+} + k_s \left.\frac{\partial T_s}{\partial y}\right|_{y=0^-} = 0.$$

Since the surface-convection heat flux is defined as

$$q_{ku} \equiv -k_f \left.\frac{\partial T_f}{\partial y}\right|_{y=0}^{+},$$

we have

$$-k_s \left.\frac{\partial T_s}{\partial y}\right|_{y=0^-} = -k_f \left.\frac{\partial T_f}{\partial y}\right|_{y=0+} \equiv q_{ku}.$$

COMMENT:

Note that, for example for $T_{sf} > T_{f,\infty}$, heat flows, from the surface to the gas stream. Then $\partial T_s/\partial y$ and $\partial T_f/\partial y$ both will be negative. Also note that the relationship between the two derivative is given by this energy equation, i.e.,

$$\frac{\partial T_f/\partial y|_{y=0^+}}{\partial T_s \partial y|_{y=0^-}} = \frac{k_s}{k_f}.$$

PROBLEM 2.33.FUN

GIVEN:

The divergence of the heat flux vector $\nabla \cdot \mathbf{q}$ is indicative of the presence or lack of local heat sources (energy storage/release or conversion). This is stated by (2.2). Consider a gaseous, one-dimensional steady-state fluid flow and heat transfer with a premixed combustion (exothermic chemical reaction) as shown in Figure Pr.2.33(a). For this, (2.2) becomes

$$\nabla \cdot \mathbf{q} = \frac{d}{dx} q_x = \dot{s}_{r,c}(x).$$

Here $q_x = q_{k,x} + q_{u,x}$ (assuming no radiation) is idealized with a distribution and the source terms.

$$\dot{s}_{r,c}(x) = -\rho_{\mathrm{F},1} u_{f,1} \Delta h_{r,\mathrm{F}} \frac{1}{\sigma(2\pi)^{1/2}} e^{-(x-x_o)^2/2\sigma^2},$$

where $\rho_{\mathrm{F},1}$ is the fluid density far upstream of the reaction (or flame) region, $u_{f,1}$ is the fluid velocity there, and $\Delta h_{r,\mathrm{F}}$ is the heat of combustion (per kg of fuel). The exponential expression indicates that the reaction begins to the left of the flame location x_o and ends to its right, with the flame thickness given approximately by 6σ. This is the normal distribution function and represents a chemical reaction that initially increases (as temperature increases) and then decays and vanishes (as products are formed and fuel depletes).

SKETCH:

Figure Pr.2.33(a) shows the variable source term $\dot{s}_{r,c}(x)$. The flame thickness δ is approximated as 6σ.

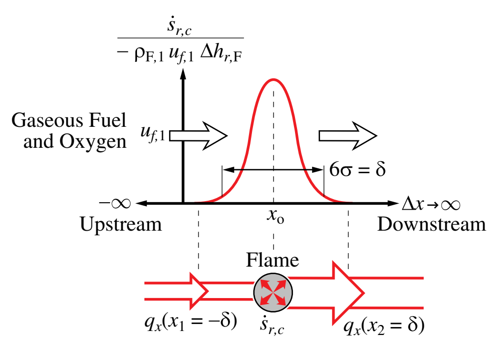


Figure Pr.2.33(a) Variable energy conversion (source) term for combustion in a premixed gaseous flow. The source has a normal distribution around a location x_o. The flame thickness is approximated as $\delta = 6\sigma$.

OBJECTIVE:

(a) For $\sigma = \delta/6 = 0.1$ mm, plot $q_x/(-\rho_{\mathrm{F},1} u_{f,1} \Delta h_{r,\mathrm{F}})$ and $\dot{s}_{r,c}/(-\rho_{\mathrm{F},1} u_{f,1} \Delta h_{r,\mathrm{F}})$, with respect to x (use $x_o = 0$ and $x_1 = -\delta < x < x_L = \delta$). Assume $q_x(x = -\delta) = 0$.

(b) Noting that no temperature gradient is expected at $x = x_1 = -\delta$ and at $x = x_2 = \delta$, i.e., $q_{k,x} = 0$ at $x = x_1$ and $x = x_2$, determine $q_{u,x}$ at $x = x_2$, for $\rho_{\mathrm{F},1} = 0.06041$ kg/m^3, $u_{f,1} = 0.4109$ m/s, and $\Delta h_{r,\mathrm{F}} = -5.553 \times 10^7$ J/kg. These are for a stoichiometric, atmospheric air-methane laminar flame.

SOLUTION:

(a) Using an ordinary differential equation solver such as SOPHT, we integrate

$$\frac{d}{dx} \frac{q_x}{(-\rho_{\mathrm{F},1} u_{f,1} \Delta r, \mathrm{F})} = \frac{1}{\sigma(2\pi)^{1/2}} e^{-\dfrac{x^2}{2\sigma^2}} = \dot{s}_{r,c}(x)$$

for $\sigma = 0.1$ mm.

The result for $-\delta \le x \le \delta$ is plotted in Figure Pr.2.33(b). Also plotted is $\dot{s}_{r,c}(x)$. We note that q_x, which begins as $q_x = 0$ at $x = -\delta$, reaches a maximum value of $1 \times (-\rho_{\mathrm{F},1} u_{f,1} \Delta h_{r,\mathrm{F}})$ at $x = \delta$, while $\dot{s}_{r,c}$ peaks at $x = 0$ and its magnitude is approximately $4 \times (-\rho_{\mathrm{F},1} u_{f,1} \Delta h_{r,\mathrm{F}})$.

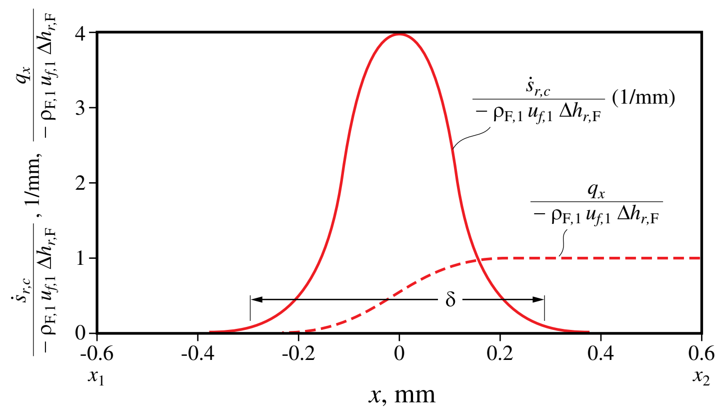


Figure Pr.2.33(b) Variation of convection heat flux and the energy conversion with respect to axial location.

(b) Using the value of q_x at $x = x_2 = \delta$, we have

$$\begin{aligned} q_x(x = x_2) &= 1 \times (-\rho_{\mathrm{F},1} u_{f,1} \Delta h_{r,\mathrm{F}}) \\ &= -0.06041(\mathrm{kg/m^3}) \times 0.4109(\mathrm{m/s}) \times (-5.553 \times 10^7)(\mathrm{J/kg}) \\ &= 1.378 \times 10^6\ \mathrm{W/m^2}. \end{aligned}$$

COMMENT:

Note that $q_x(x) = q_{k,x}(x) + q_{r,x}(x)$ varies over the flame length. In Chapter 5, we will approximate this conduction-convection region and use a more realistic (but still simple) source term representing the chemical reaction. Also note that $q_u = (\rho c_p T u)_f$.

PROBLEM 2.34.FUN

GIVEN:

A p-n junction is shown Figure Pr.2.34(a). The junction (interface) is at temperature T_j. The ends of the two materials are at a lower temperature T_c and a higher temperature T_h.

SKETCH:

Figure Pr.2.34(a) shows the conduction across a slab containing a thermoelectric p-n junction.

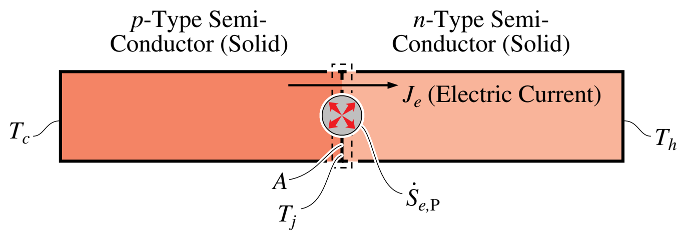


Figure Pr.2.34(a) Conduction heat transfer in a slab containing a thermoelectric p-n junction.

OBJECTIVE:

(a) Starting from (2.62), write the surface energy equation for the interface. Make the appropriate assumptions about the mechanisms of heat transfer expected to be significant.

(b) Express the conduction heat transfer as

$$Q_{k,n} = \frac{T_j - T_h}{R_{k,n}}, \quad Q_{k,p} = \frac{T_j - T_c}{R_{k,p}}.$$

Comment on the signs of $Q_{k,p}$ and $Q_{k,n}$ needed to absorb heat at the junction to produce electrical potential-current.

SOLUTION:

(a) From (2.60), using n and p to designate the two media, the surface energy equation is

$$A[(\mathbf{q}_k \cdot \mathbf{s}_n)_n + (\mathbf{q}_k \cdot \mathbf{s}_n)_p + (\mathbf{q}_u \cdot \mathbf{s}_n)_n + (\mathbf{q}_u \cdot \mathbf{s}_n)_p + (\mathbf{q}_r \cdot \mathbf{s}_n)_n + (\mathbf{q}_r \cdot \mathbf{s}_n)_p] = \dot{S}_{e,\text{P}}.$$

Since both media are not moving, $\mathbf{q}_u = 0$. Also, due to the large optical thickness, the radiation heat transfer with both media is expected to be negligible. Then, using $\dot{S}_{e,\text{P}}$, the surface energy equation becomes

$$A[(\mathbf{q}_k \cdot \mathbf{s}_n)_n + (\mathbf{q}_k \cdot \mathbf{s}_n)_p] = \dot{S}_{e,\text{P}}.$$

This can then be written as

$$Q_{k,n} + Q_{k,p} = \dot{S}_{e,\text{P}}$$

and is shown in Figure Pr.2.34(b).

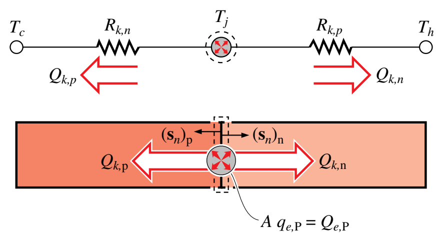


Figure Pr.2.34(b) Energy equation for the slab containing the junction.

(b) Using the equations for the conduction heat transfer,

$$\begin{aligned} Q_{k,n} &= \frac{T_j - T_h}{R_{k,n}} \\ Q_{k,p} &= \frac{T_j - T_c}{R_{k,p}}. \end{aligned}$$

Then, using (2.37) for $\dot{S}_{e,\mathrm{P}}$ (for absorption of energy), the energy equation becomes

$$\frac{T_j - T_h}{R_{k,n}} + \frac{T_j - T_c}{R_{k,p}} = -\alpha_\mathrm{S} J_e T_j.$$

The minus sign is used for the energy absorption.

(c) Since $\alpha_\mathrm{S} > 0$, $J_e > 0$, $T_j > 0$, $(T_j - T_h) < 0$, $R_{k,n} > 0$, $(T_j - T_c) > 0$ and $R_{k,p} > 0$, then $Q_{k,n} < 0$ and $Q_{k,p} > 0$. In order to produce electrical current, we need to have more conduction heat transfer arriving at the junction than leaving the junction. Therefore, we need

$$|Q_{k,n}| > |Q_{k,p}|.$$

COMMENT:

If we assume that $R_{k,p} = R_{k,n}$, then to have energy conversion we need to have $(T_h - T_j) > (T_j - T_c)$. Figure Pr.2.34(b) shows the thermal circuit diagram for this problem.

PROBLEM 2.35.FUN

GIVEN:

Below are described two cases for which there is heat transfer and possibly energy conversion on a bounding surface between two media [Figure Pr.2.35 (a) and (b)].
(a) A hot solid surface is cooled by surface-convection heat transfer to a cold air stream and by surface-radiation heat transfer to its surrounding. Note that the air velocity at the surface is zero, $\mathbf{u}_g = 0$. Also, assume that the radiation is negligible inside the solid (i.e., the solid is opaque).
(b) Two solid surfaces are in contact with each other and there is a relative velocity Δu_i between them. For example, one of the surfaces is a brake pad and the other is a brake drum.

SKETCH:

Figures Pr.2.35(a) and (b) show a gas-solid and a solid-solid interface.

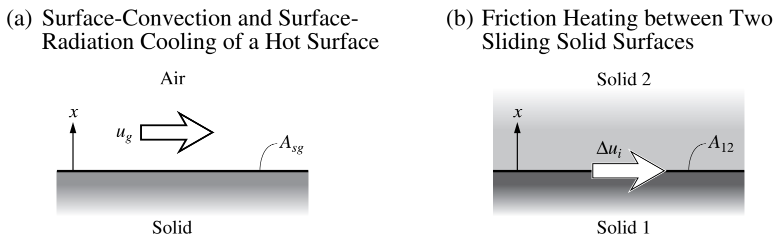


Figure Pr.2.35(a) and (b) Two example of bounding surface between two media.

OBJECTIVE:

For each of the given cases (a) and (b), apply the bounding-surface energy equation (2.62) to the interface separating the two media. Assume that the surfaces are at uniform temperatures. As a consequence, the heat transfer at the interface is one-dimensional and perpendicular to the interface.

SOLUTION:

(a) The solid surface is cooled by surface convection and by surface radiation. For this solid-gas interface, the general bounding-surface energy equation (2.65) is

$$A_{sg}[-k_s(\nabla T \cdot \mathbf{s}_n)_s - k_g(\nabla T \cdot \mathbf{s}_n)_g + (\rho c_p T\mathbf{u} \cdot \mathbf{s}_n)_s + (\rho c_p T\mathbf{u} \cdot \mathbf{s}_n)_g + (\mathbf{q}_r \cdot \mathbf{s}_n)_s$$
$$+(\mathbf{q}_r \cdot \mathbf{s}_n)_g] = \sum_i \dot{S}_i,$$

where A_{sg} is the solid-gas interfacial area. On the left-hand side of the surface energy equation, the first two terms are the conduction heat flux vectors in the solid and in the gas phases normal to the surface, the third and fourth terms are the convection heat flux vectors in the solid and gas phases normal to the surface, and the last two terms are the radiation heat flux vectors in the solid and gas phases normal to the surface. The right-hand side accounts for surface energy conversion to thermal energy. At the solid-gas interface, the convection heat fluxes are zero because the solid is not moving normal to the control surface and the gas phase velocity at the solid surface is zero (the surface is impermeable to the gas molecules). The radiation heat flux in the solid phase is zero because the solid is assumed to be opaque to thermal radiation. At this bounding surface, there is no energy conversion (at low speeds, the energy production due to viscous heating is negligible). Therefore, the bounding-surface energy equation becomes

$$A_{sg}[-k_s(\nabla T \cdot \mathbf{s}_n)_s - k_g(\nabla T \cdot \mathbf{s}_n)_g + (\mathbf{q}_r \cdot \mathbf{s}_n)_g] = 0.$$

For a uniform surface temperature, the conduction and the radiation heat flux vectors are normal to the surface (in the direction of the x axis), i.e.,

$$q_{k,x} = -k(\nabla T \cdot \mathbf{s}_n) = -k\frac{dT}{dx}$$

$$q_{r,x} = \mathbf{q}_r \cdot \mathbf{s}_n = q_r$$

and the bounding-surface energy equation becomes

$$A_{sg}\left(k_s\frac{dT_s}{dx}-k_g\frac{dT_g}{dx}+q_{r,g}\right)=0.$$

Note that the conduction term in the solid is positive as the surface normal in that phase is in the negative x-direction. The conduction heat flux on the gas side causes the surface-convection heat transfer from the solid surface to the gas stream. This surface-convection heat transfer is also influenced by the velocity of the flow. Therefore, the bounding-surface energy equation can be finally written as

$$k_s\frac{dT_s}{dx}+q_{ku,g}+q_{r,g}=0.$$

(b) For the two solid surfaces, the radiation heat flux vectors are zero. The movement of the surfaces creates a convection heat flux vector in the same direction of the velocity vector. Then, the bounding-surface energy equation becomes

$$A_{12}[-k_1(\nabla T\cdot\mathbf{s}_n)_1-k_2(\nabla T\cdot\mathbf{s}_n)_2+(\rho c_pT\mathbf{u}\cdot\mathbf{s}_n)_1+(\rho c_pT\mathbf{u}\cdot\mathbf{s}_n)_2]=\sum_i\dot{S}_i.$$

Due to surface friction, there is energy conversion at the interface between the two solids (conversion from mechanical to thermal energy) and this energy conversion is assumed uniform along the surface. Therefore,

$$\sum_i\dot{S}_i=\dot{S}_{m,F}=\int_{A_{12}}q_{m,F}dA=q_{m,F}A_{12},$$

and, from Table 2.1,

$$q_{m,F}=\mu_Fp_c\Delta u_i.$$

The velocity vectors for both surfaces are normal to the normal vectors. Thus, the dot product of the velocity vectors and the normal vectors is zero. As the interface has a uniform temperature, the conduction heat flux at the surface is one-dimensional and normal to the surface. Therefore, the bounding-surface energy equation becomes

$$A_{12}\left(k_1\frac{dT_1}{dx}-k_2\frac{dT_2}{dx}\right)=\mu_Fp_c\Delta u_iA_{12},$$

or

$$+k_1\frac{dT_1}{dx}-k_2\frac{dT_2}{dx}=\mu_Fp_c\Delta u_i.$$

COMMENT:

The surface convection heat transfer is transferred from the solid to the fluid by fluid conduction. An enhancement in this heat transfer by conduction leads to an enhancement in the surface-convection heat transfer. Consequences and means of enhancing the fluid conduction heat flux at the solid surface will be explored in Chapter 6.

The existence of uniform temperature at the bounding surface results in conduction heat transfer normal to the surface (there is no parallel component).

The convection heat transfer across the interface exists only when there is flow across the interface.

PROBLEM 2.36.FAM

GIVEN:

An opaque (i.e., a medium that does not allow for any transmission of radiation across it) solid surface is called a selective radiation surface when its ability to absorb radiation is different than its ability to emit radiation. This is shown in Figure Pr.2.36. A selective absorber has a higher absorptivity α_r compared to its emissivity ϵ_r.

SKETCH:

Figure Pr.2.36 shows absorption and emission by an opaque surface. **OBJECTIVE**:

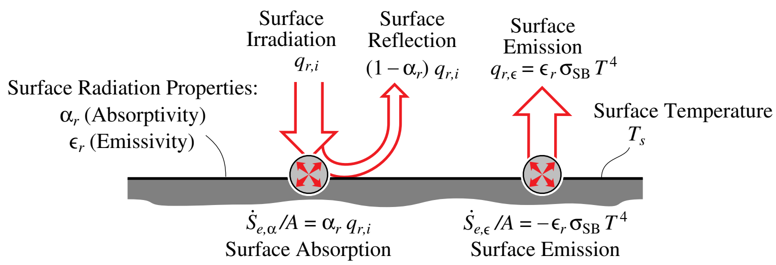


Figure Pr.2.36 A selective thermal radiation absorber.

(a) From Table C.19, choose four surfaces that are selective absorbers and four that are selective emitters. The data in Table C.19 is for absorption of solar irradiation (a high temperature radiation emission).
(b) Using black-oxidized copper, determine the surface-absorption heat flux for a solar irradiation of 700 W/m^2 and surface-emission heat flux at surface temperature of 90℃.
(c) Determine the difference between the heat absorbed and heat emitted.

SOLUTION:

(a) From Table C.19, we have

Table Pr.2.36: Selective absorbers and reflectors.

Selective Absorber (Good Solar Absorber)			Selective Emitter (Good Solar Reflector)		
Material	ϵ_r	α_r	Material	ϵ_r	α_r
Chromium Plate	0.15	0.78	Reflective Aluminum	0.79	0.23
Black-oxidized Copper	0.16	0.91	Glass	0.83	0.13
Nickel (Tabor Solar Absorber)	0.11	0.85	White Epoxy Paint	0.88	0.25
Silicon Solar Cell	0.32	0.94	Inorganic Spacecraft Coating	0.89	0.13

(b) From (2.47) and (2.48), we have

$$\frac{\dot{S}_{e,\alpha}}{A} = \alpha_r q_{r,i}$$
$$\frac{\dot{S}_{e,\epsilon}}{A} = -\epsilon_r \sigma_{\rm SB} T^4.$$

Using the numerical values for black-oxidized copper, we have

$$\dot{S}_{e,\alpha}/A = 0.91 \times 700(\text{W/m}^2) = 637.0\ \text{W/m}^2$$
$$\dot{S}_{e,\epsilon}/A = -0.16 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}) \times (273.15+90)^4(\text{K})^4 = -157.8\ \text{W/m}^2.$$

(c) The net heat generated at the surface is

$$\frac{\dot{S}}{A} = \frac{\dot{S}_{e,\alpha}}{A} + \frac{\dot{S}_{e,\epsilon}}{A} = 637 - 157.8 = 479.2 \text{ W/m}^2.$$

There is a net heat gained by the surface.

COMMENT:

The surfaces that have selective behavior, i.e., $\alpha_r \neq \epsilon_r$, are called nongray surfaces. The gray surfaces are those for which $\alpha_r = \epsilon_r$. We will discuss gray and nongray surfaces in Chapter 4.

PROBLEM 2.37.FUN

GIVEN:

A droplet of refrigeration fluid (refrigerant) R-134a, which is used in automobile air-conditioning systems, is evaporating. The initial droplet diameter is $D(t = 0)$ and the diameter decreases as heat is absorbed on the droplet surface from the gaseous ambient by surface convection and radiation.

SKETCH:

Figure Pr.2.37 shows the surface heating and evaporation of a droplet.

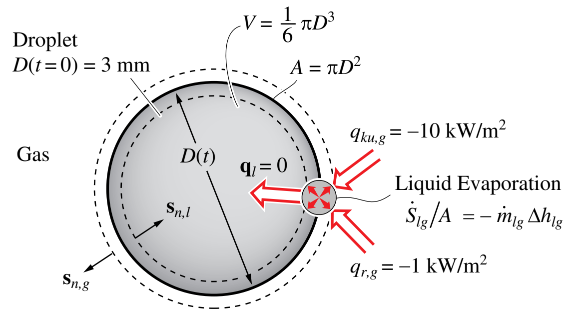


Figure Pr.2.37 Droplet evaporation by surface convection and radiation.

OBJECTIVE:

(a) Starting from (2.62) and by replacing $q_{k,g}$ by $q_{ku,g}$, and noting that the difference between the convection terms is represented by $\dot{S}_{lg}$, write the appropriate surface energy equation. The radiation heat transfer within the droplet can be neglected. Assume a uniform droplet temperature, i.e., assume the liquid conduction can also be neglected.

(b) Using the properties listed in Table C.26 (they are for $p = 1$ atm) and the heat flux rates given in Figure Pr.2.37, determine the evaporation rate per unit area $\dot{m}_{lg}$.

(c) Starting with (1.25) and setting the outgoing mass equal to the evaporation rate, derive an expression giving the instantaneous droplet diameter $D(t)$, as a function of the various parameters.

(d) Determine the time needed for the droplet diameter to decrease by a factor of 10.

SOLUTION:

(a) From (2.62), with $(\dot{S}/A) = -\dot{m}_{lg}\Delta h_{lg}$ from Table 2.1, the surface energy equation becomes

$$A[(\mathbf{q}_{ku} \cdot \mathbf{s}_n)_g + (\mathbf{q}_{ku} \cdot \mathbf{s}_n)_l + (\mathbf{q}_r \cdot \mathbf{s}_n)_g + (\mathbf{q}_r \cdot \mathbf{s}_n)_l] = -A\dot{m}_{lg}\Delta h_{lg}.$$

Since $(\mathbf{q}_{ku})_l$ and $(\mathbf{q}_r)_l$ are assumed zero, we have

$$\begin{aligned} A[(\mathbf{q}_{ku} \cdot \mathbf{s}_n)_g + (\mathbf{q}_r \cdot \mathbf{s}_n)_g] &= -A\dot{m}_{lg}\Delta h_{lg} = 0 \\ (\mathbf{q}_{ku} \cdot \mathbf{s}_n)_g + (\mathbf{q}_r \cdot \mathbf{s}_n)_g &= -\dot{m}_{lg}\Delta h_{lg} = 0. \end{aligned}$$

(b) From Table C.26, for R-134a at $p = 1$ atm, we have $T_{lg} = 246.99$ K, $\rho_l = 1374.3$ kg/m^3 and $\Delta h_{lg} = 2.168 \times 10^5$ J/kg. Solving the energy equation for $\dot{m}_{lg}$, we have

$$\dot{m}_{lg} = -\frac{q_{ku,g} + q_{r,g}}{\Delta h_{lg}}.$$

Using the numerical values, we then have

$$\dot{m}_{lg} = -\frac{(-10{,}000 - 1{,}000)(\mathrm{W/m^2})}{2.168 \times 10^5 (\mathrm{J/kg})} = 5.074 \times 10^{-2}\ \mathrm{kg/m^2\text{-}s}.$$

(c) From (1.25), we have

$$\dot{M}_{lg} = \dot{M}|_A = -\frac{d}{dt}\int_{V(t)} \rho_l dV.$$

For constant ρ_l we then have

$$\dot{M}|_A = -\frac{d}{dt}(\rho_l \int_{V(t)} dV) = -\frac{d}{dt}(\rho_l V) = -\frac{d}{dt}\left(\rho_l \frac{1}{6}\pi D^3\right) = -\frac{\pi}{2}\rho_l D^2 \frac{dD}{dt}.$$

The mass flow rate $\dot{M}|_A$ is related to the mass flux by

$$\dot{M}|_A = A\dot{m}_{lg} = \pi D^2 \dot{m}_{lg}.$$

Then, from the equations above we obtain

$$\frac{dD}{dt} = -\frac{2\dot{m}_{lg}}{\rho_l}.$$

Integrating this equation we have

$$\int_{D(t=o)}^{D(t)} dD = -\int_0^t \frac{2\dot{m}_{lg}}{\rho_l} dt.$$

For constant rate of phase change

$$D(t) = D(t=0) - \frac{2\dot{m}_{lg}}{\rho_l} t.$$

(d) The equation above can be recast as

$$\frac{D(t)}{D(t=0)} = 1 - \frac{2\dot{m}_{lg}}{\rho_l D(t=0)} t.$$

Solving for t, we have

$$t = \left[1 - \frac{D(t)}{D(t=0)}\right] \frac{\rho_l D(t=0)}{2\dot{m}_{lg}}.$$

For $D(t)/D(t=0) = 0.1$ and using the other values, we have

$$t = (1-0.1)\frac{1374.3(\text{kg/m}^3)3\times 10^{-3}(\text{m})}{2\times 5.074\times 10^{-2}(\text{kg/m}^2\text{-s})} = 36.57 \text{ s}.$$

COMMENT:
(i) This rate of heat flow into the droplet is high, but not very high. In order to evaporate the droplet very rapidly, surface heat transfers of the order of 100 kW/m^2 are used. Also, the heat flux changes as the diameter decreases because the area decreases. Thus, the rate of evaporation increases as the diameter decreases.
(ii) The droplet evaporation model above is called a heat transfer controlled evaporation. The evaporation can also be mass transfer controlled if the rate of mass transfer of the vapor from the droplet surface to the ambient is slower than the rate of heat transfer.
(iii) Refrigerant-134a operates under large pressures, both in the evaporator and in the condenser. The heat of evaporation decreases as the critical pressure is approached.

PROBLEM 2.38.FUN

GIVEN:

A thermoelectric element (TE) is exposed at its cold junction surface (at temperature T_c) partly to an electrical connector (e) and partly to the ambient air (a). This is shown in Figure Pr.2.38. Heat is transferred to the surface through the thermoelectric element $\mathbf{q}_{k,TE}$ in addition to a prescribed heat flux, $\mathbf{q}_{TE}$ that combines some parasitic heating. These are over the surface area $A_a + A_e$. Heat is also transferred to the surface from the adjacent air and the connector, over their respective areas A_a and A_e. The area in contact with air undergoes heat transfer by surface convection $q_{ku,a}$ and surface radiation $q_{r,a}$. The connector heat transfer is by conduction $\mathbf{q}_{k,e}$. There is a Peltier energy conversion $(\dot{S}_{e,\mathrm{J}})_c/A_e$ at the surface (and since it occurs where the current passes, it occurs over A_e). The Joule heating $(\dot{S}_{e,\mathrm{J}})_c/A_e$ is also represented as a surface energy conversion (this presentation will be discussed in Section 3.3.6) and is over the entire element area $A_a + A_e$.

$A_a = 10^{-6}$ m^2, $A_e = 10^{-6}$ m^2, $q_{k,TE} = -4 \times 10^4$ W/m^2, $(\dot{S}_{e,\mathrm{J}})_c/A = 2 \times 10^4$ W/m^2, $(\dot{S}_{e,\mathrm{P}})_c/A_e = -2 \times 10^5$ W/m^2, $q_{r,a} = 0$, $q_{ku,a} = 0$, $q_{TE} = 0$.

Assume quantities are uniform over their respective areas.

SKETCH:

Figure Pr.2.38 shows the control surface A and the various surface heat transfer and energy conversions.

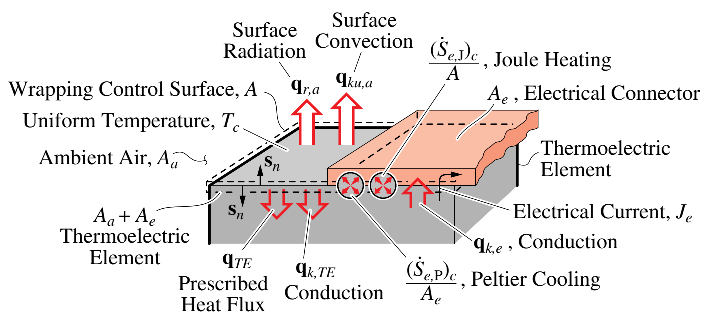


Figure Pr.2.38 The cold-junction surface of a thermoelectric element showing various surface heat transfer and energy conversions.

OBJECTIVE:

(a) Starting from (2.60), write the surface energy equation for the cold junction control surface A.

(b) Determine $q_{k,e}$ for the given conditions.

SOLUTION:

(a) From (2.60), we have

$$\begin{aligned}
\int_A (\mathbf{q} \cdot \mathbf{s}_n) dA &= \sum_i \dot{S}_i \\
\int_A (\mathbf{q} \cdot \mathbf{s}_n) dA &= \int_{A_a + A_e} (\mathbf{q}_{TE} \cdot \mathbf{s}_n) dA + \int_{A_a + A_e} (\mathbf{q}_{k,TE} \cdot \mathbf{s}_n) dA + \int_{A_e} (\mathbf{q}_{k,e} \cdot \mathbf{s}_n) dA + \\
&\quad \int_{A_a} (\mathbf{q}_{r,a} \cdot \mathbf{s}_n) dA + \int_{A_a} (\mathbf{q}_{ku,a} \cdot \mathbf{s}_n) dA \\
&= \sum_i \dot{S}_i = \left[\frac{(\dot{S}_{e,\mathrm{P}})_c}{A_e} \right] A_e + \left[\frac{(\dot{S}_{e,\mathrm{J}})_c}{A} \right] (A_a + A_e).
\end{aligned}$$

Here we have used the appropriate areas for each heat transfer rate and each energy conversion mechanism.
Now, since the various heat flux vectors given in Figure Pr.2.38 are all given as leaving the contact surface, we have

$$(A_a + A_e) q_{TE} + (A_a + A_e) q_{k,TE} - A_e q_{k,e} + A_a q_{r,a} + A_a q_{ku,a} = A_e \left[\frac{(\dot{S}_{e,\mathrm{P}})_c}{A_e} \right] + (A_a + A_e) \left[\frac{(\dot{S}_{e,\mathrm{J}})_c}{A} \right].$$

(b) Using the numerical values, we have

$$
\begin{aligned}
&(10^{-6}+10^{-6})(\mathrm{m}^2)\times 0+(10^{-6}+10^{-6})(\mathrm{m}^2)\times(-4\times 10^4)(\mathrm{W/m}^2)- \\
&10^{-6}(\mathrm{m}^2)\times q_{k,e}+10^{-6}(\mathrm{m}^2)\times 0+10^{-6}(\mathrm{m}^2)\times 0 \\
= \quad & 10^{-6}(\mathrm{m}^2)\times(-2\times 10^5)(\mathrm{W/m}^2)+(10^{-6}+10^{-6})(\mathrm{m}^2)\times 2\times 10^4(\mathrm{W/m}^2)
\end{aligned}
$$

or

$$
\begin{aligned}
q_{k,e} &= (2\times 4\times 10^4-2\times 10^5+4\times 10^4)(\mathrm{W/m}^2) \\
&= -8\times 10^4\ \mathrm{W/m}^2
\end{aligned}
$$

or

$$
\begin{aligned}
Q_{k,e} &= A_e q_{k,e}=10^{-6}(\mathrm{m}^2)\times[-8\times 10^4(\mathrm{W/m}^2)] \\
&= -8\times 10^{-2}\ \mathrm{W}.
\end{aligned}
$$

COMMENT:

Note that heat flows into the electric connecter because $Q_{k,e}<0$. This is the effective cooling heat rate and the object to be cooled is connected to the electric connector (with a thin layer of electrical insulator between them, in case the object is not a dielectric). In practice, many of these junctions are used to produce the desired cooling rate. This is discussed in Section 3.7.

PROBLEM 2.39.FUN

GIVEN:

When the ambient temperature is high or when intensive physical activities results in extra metabolic energy conversion, then the body loses heat by sweating (energy conversion $\dot{S}_{lg}$). Figure Pr.2.39 shows this surface energy exchange, where the heat transfer to the surface from the tissue side is by combined conduction and convection $q_{k,t}$, $q_{u,t}$ and from the ambient air side is by conduction, convection, and surface radiation $q_{k,a}$, $q_{u,a}$, $q_{r,a}$. The surface evaporation is also shown as $\dot{S}_{lg}/A_t$ where A_t is the evaporation surface area. The tissue conduction $q_{k,t}$ is significant for lowering the body temperature or removing extra metabolic heat generation (i.e., when the heat flow is dominantly from the tissue side). Preventing the high ambient temperature from raising the tissue temperature, however, relies only on intercepting the ambient heat transfer on the surface (i.e., when the heat flow is dominated by the ambient air side and the tissue conduction is not significant).

Assume that quantities are uniform over their respective surfaces.

SKETCH:

Figure Pr.2.39 shows the various surface heat transfer mechanisms and the surface energy conversion $\dot{S}_{lg}/A_t$.

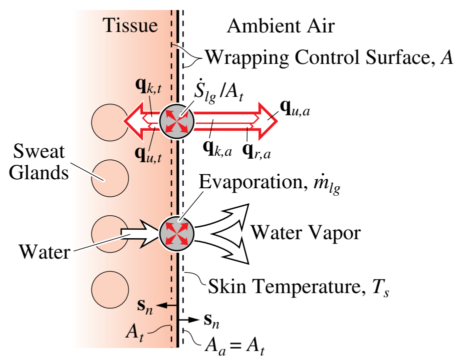


Figure Pr.2.39 The surface heat transfer, and energy conversion by sweat cooling. across human skin.

OBJECTIVE:

(a) Starting from (2.60), write the surface energy equation for the skin control surface A (wrapped around the surface with $A_t = A_a$) in Figure Pr.2.39.

(b) For the conditions given below, determine $q_{k,t}$.

SOLUTION:

(a) From (2.60), we have

$$\begin{aligned} Q|_A &= A_t(\mathbf{q}_k \cdot \mathbf{s}_n)_t + A_t(\mathbf{q}_u \cdot \mathbf{s}_n)_t + A_a(\mathbf{q}_k \cdot \mathbf{s}_n)_a + A_a(\mathbf{q}_u \cdot \mathbf{s}_n)_a + A_a(\mathbf{q}_r \cdot \mathbf{s}_n)_a \\ &= A_t\left(\frac{\dot{S}_{lg}}{A_t}\right). \end{aligned}$$

Since the heat flux vectors are defined in Figure Pr.2.39 to be pointing outward from their respective surfaces (i.e., along $\mathbf{s}_n$), then we have

$$A_t(q_{k,t} + q_{u,t}) + A_a(q_{k,a} + q_{u,a} + q_{r,a}) = A_t\left(\frac{\dot{S}_{lg}}{A_t}\right)$$

and since all the areas are equal, we have

$$q_{k,t} + q_{u,t} + q_{k,a} + q_{u,a} + q_{r,a} = \frac{\dot{S}_{lg}}{A_t}.$$

(b) Now using the numerical values and $q_{k,a} = q_{ku,a}$, we have

$$q_{k,t} + 0 + q_{ku,a} + 0 + q_{r,a} = \frac{\dot{S}_{lg}}{A_t}.$$

Solving for $q_{k,t}$, we have

$$q_{k,t} = -300(\mathrm{W/m^2}) + 150(\mathrm{W/m^2}) + 10(\mathrm{W/m^2}) = -140\ \mathrm{W/m^2}.$$

This corresponds to heat flowing from the tissue to the surface by conduction at this rate.

COMMENT:

The convection heat fluxes are negligible due to the small velocities. Also note that we used the surface convection $q_{ku,a}$ in place of conduction $q_{k,a}$ because as will be shown in Chapter 6, the air velocity at the surface is zero (neglecting the small water vapor velocity leaving the surface due to the evaporation). Then the heat transfer to the air is by conduction, but influenced by the air motion.

PROBLEM 2.40.FUN

GIVEN:

In laser materials processing-manufacturing, high-power, pulsed laser irradiation flux $q_{r,i}$ is used and most of this power is absorbed by the surface. Figure Pr.2.40 shows the laser irradiation absorbed $\dot{S}_{e,\alpha}/A = \alpha_r q_{r,i}$ (where α_r is the surface absorptivity), the surface radiation emission flux $(\dot{S}_{e,\epsilon})/A = \epsilon_r \sigma_{\rm SB} T_s^4$, the gas-side surface convection q_{ku}, and the solid (substrate or working piece) conduction $q_{k,s}$, over a differential control surface $\Delta A \rightarrow 0$. Since the irradiation is time dependent (e.g., pulsed), the heat transfer and energy conversions are all time dependent (and nonuniform over the surface).

$\epsilon_r = 0.8$, $\alpha_r = 0.9$, $q_{r,i} = 10^{10}$ W/m^2, $T_s = 2 \times 10^3$ K, $q_{ku} = 10^7$ W/m^2.

Note that the entire surface radiation is represented as energy conversions $\dot{S}_{e,\alpha}$ and $\dot{S}_{e,\epsilon}$.

SKETCH:

Figure Pr.2.40 shows the laser irradiated surface, the substrate conduction, surface convection, and surface radiation emission represented as an energy conversion.

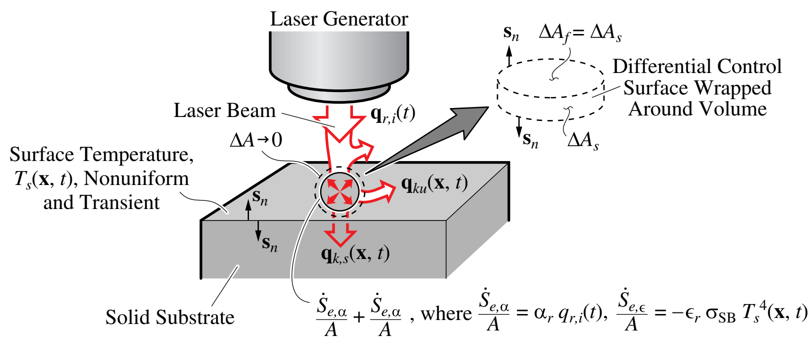


Figure Pr.2.40 Laser irradiation of a substrate and a differential control surface taken in the laser impingement region.

OBJECTIVE:

(a) Starting with (2.58), write the surface energy equation for the differential control surface ΔA.

(b) Determine $q_{k,s}$ for the conditions given.

SOLUTION:

(a) Starting from (2.58), for differential control surface ΔA, we have

$$\int_{\Delta A} (\mathbf{q} \cdot \mathbf{s}_n) dA = \dot{S} = \dot{S}_{e,\alpha} + \dot{S}_{e,\epsilon} = \Delta A_f \alpha_r q_i - \Delta A_f \epsilon_r \sigma_{\rm SB} T_s^4,$$

where we have used the energy conversion terms given in Figure Pr.2.40 for the right-hand side. Then noting that $\mathbf{q}_{ku}$ and $\mathbf{q}_{k,s}$ are along their respective surface normal vectors, we have

$$\int_{\Delta A} (\mathbf{q} \cdot \mathbf{s}_n) dA = \Delta A_f q_{ku} + \Delta A_s q_{k,s} = \Delta A_s \alpha_r q_{r,i} - \Delta A_s \epsilon_r \sigma_{\rm SB} T_s^4.$$

(b) Solving the above equation for $q_{k,s}$, using $\Delta A_s = \Delta A_f$, we have

$$\begin{aligned}
q_{k,s} &= \alpha_r q_{r,i} - \epsilon_r \sigma_{\rm SB} T_s^4 - q_{ku} \\
&= 0.9 \times 10^{10} (\text{W/m}^2) - 0.8 \times 5.67 \times 10^{-8} (\text{ W/m}^2\text{-K}^4) \times (2 \times 10^3)^4 (\text{K}^4) - 10^7 (\text{W/m}^2) \\
&= (9 \times 10^9 - 8.165 \times 10^5 - 10^7)(\text{W/m}^2) = 8.989 \times 10^9 \text{ W/m}^2.
\end{aligned}$$

COMMENT:

Note that during the irradiation, the surface radiation emission and surface convection are rather small and negligible.

Chapter 3

Conduction

PROBLEM 3.1.FUN

GIVEN:

Equation (3.25) relates the thermal conductivity to the electrical resistivity of pure solid metals.

Values for the electrical resistivity as a function of temperature are listed in Table C.8 for different pure metals.

OBJECTIVE:

(a) Using (3.25), calculate the predicted thermal conductivity of copper k_{pr} for T = 200, 300, 500, and 1,000 K. For T = 1,000 K, extrapolate from the values in the table.

(b) Compare the results obtained in (a), for k_{pr}, with the values given in Table C.14, k_{ex}. Calculate the percentage difference from using

$$\Delta k(\%) = \left(\frac{k_{pr} - k_{ex}}{k_{ex}} \right) \times 100.$$

(c) Diamond is an electrical nonconductor ($\sigma_e \simeq 0$). However, Figure 3.9(a) shows that the thermal conductivity of diamond is greater than the thermal conductivity of copper for $T > 40$ K. How can this be explained?

SOLUTION:

(a) Equation (3.25) relates the electronic contribution in the thermal conductivity k^e to the electrical conductivity σ_e (inverse of resistivity ρ_e). Assuming that for copper the electronic contribution is the dominant mechanism for the thermal conductivity ($k \simeq k^e$), (3.25) can be written as

$$\frac{k^e \rho_e}{T} = 2.442 \times 10^{-8} \text{ W-ohm/K}^2.$$

From the values of ρ_e given in Table C.8, the thermal conductivity of copper can be calculated at different temperatures. Table Pr.3.1 lists the results.

Table Pr.3.1 Thermal conductivity of pure copper.

T, K	ρ_e, ohm-m	$k_{\rm pr}$, W/m-K	$k_{\rm ex}$, W/m-K	Δk (%)
200	1.046×10^{-8}	466.9	413	13
300	1.725×10^{-8}	424.7	401	6
500	3.090×10^{-8}	395.2	386	2
1,000	6.804×10^{-8}	358.9	352	2

(b) Table Pr.3.1 shows the data obtained from Table C.14 ($k_{\rm ex}$) and the percentage difference between $k_{\rm pr}$ and $k_{\rm ex}$.

(c) The thermal conduction in diamond occurs dominantly by the mechanism of lattice vibration. The transfer of energy due to lattice vibration is represented by a heat carrier called a phonon and the heat conduction is then said to be due to phonon transport. The phonon transport is more effective at higher temperatures, as shown in Figure 3.7(c). At low temperatures, the heat conduction by electron transport is substantial. Therefore, at low temperatures, copper is a better conductor than diamond. The phonon transport mechanism is also present in copper, but has a relatively smaller contribution.

COMMENT:

The value of ρ_e at T = 1,000 K is extrapolated from the values listed in Table C.8.

PROBLEM 3.2.FAM

GIVEN:

An airplane flies at an altitude of about 10 km (32,808 ft). Use the relation for the polyatomic ideal-gas thermal conductivity given in Example 3.2.

OBJECTIVE:

Using the relation for the polyatomic ideal gas thermal conductivity given in Example 3.2,
(a) Determine the air thermal conductivity at this altitude. Use the thermophysical properties given in Table C.7, and assume that c_v and c_p are constant.
(b) Compare the predicted k with the measured value given in Table C.7. (c) Comment on why k does not change substantially with altitude.

SOLUTION:

(a) From Example 3.2, we have

$$k = \frac{5\pi}{32}\rho\left(c_v + \frac{9}{4}\frac{R_g}{M}\right)a_s\left(\frac{3c_v}{c_p}\right)\lambda.$$

From Table C.7, we have, for $r = 10$ km,

$$\begin{aligned}
\rho &= 0.41351 \text{ kg/m}^3 && \text{Table C.7}\\
M &= 28.965 \text{ kg/kmole} && \text{Table C.7}\\
a_s &= 299.53 \text{ m/s} && \text{Table C.7}\\
\lambda &= 1.97\times 10^{-7} \text{ m} && \text{Table C.7.}
\end{aligned}$$

Also from Example 3.2, we have

$$\begin{aligned}
c_v &= 719 \text{ J/kg-K}\\
c_p &= 1{,}006 \text{ J/kg-K.}
\end{aligned}$$

Using the numerical values, we have

$$\begin{aligned}
k &= \frac{5\pi}{32}\times 0.41351(\text{kg/m}^3)\times\left(719 + \frac{9}{4}\times\frac{8{,}315}{28.964}\right)(\text{J/kg-K})\times 299.53(\text{m/s})\\
&\quad\times\left(\frac{3\times 719}{1{,}006}\right)^{1/2}\times 1.97\times 10^{-7}(\text{m})\\
k &= \frac{5\pi}{32}\times 0.41351\times 1{,}365\times 299.5\times 1.464\times 1.97\times 10^{-7}\\
&= 0.02392 \text{ W/m-K.}
\end{aligned}$$

(b) The measured k from Table C.7 is

$$k = 0.0201 \text{ W/m-K} \quad \text{Table C.7.}$$

The difference, in percentage, is

$$\Delta k(\%) = \frac{0.02392 - 0.0201}{0.0201}\times 100\% = 18.20\%.$$

This is reasonable, considering that we have a mixture of species and the assumptions made in the kinetic theory.

(c) The mean-free path of the air increases with the altitude r, as listed in Table C.7. However, the density decreases with r. These two nearly compensate each other (the changes in the speed of sound is not as substantial as that in λ and ρ), thus making the thermal conductivity not substantially change for $0 < r < 50$ km.

COMMENT:

From Table C.7, note that the air molecular weight does not begin to change substantially until an altitude of about 1,000 km is reached. This is when the air composition begins to change to mostly hydrogen and helium. The temperature at an altitude of 10 km is $T = 233.25$ K $= -39.9$℃, and the pressure is $p = 0.026499$ MPa $= 0.2615$ atm.

PROBLEM 3.3.FUN

GIVEN:

Due to their molecular properties, the elemental, diatomic gases have different thermodynamic properties, e.g., ρ and c_p, and transport properties, e.g., k properties. Consider (i) air, (ii) helium, (iii) hydrogen, and (iv) argon gases at $T = 300$ K and one atmosphere pressure.

OBJECTIVE:

(a) List them in order of the increasing thermal conductivity. Comment on how a gas gap used for insulation may be charged (i.e., filled) with different gases to allow none or less heat transfer.
(b) List them in the order of the increasing thermal diffusivity $\alpha = k/\rho c_p$. Comment on how the penetration speed u_F can be varied by choosing various gases.
(c) List them in order of increasing thermal effusivity $(\rho c_p k)^{1/2}$. Comment on how the transient heat flux $q_{\rho ck}(t)$ can be varied by choosing various gases.

SOLUTION:

The thermal conductivity, density, and specific heat capacity for each of the four gases are listed in Table C.22 for $p = 1$ atm. For $T = 300$ K, we have

$$
\begin{aligned}
\text{(i) air: } k &= 0.0267 \ \text{W/m-K} && \text{Table C.22} \\
\rho &= 1.177 \ \text{kg/m}^3 && \text{Table C.22} \\
c_p &= 1{,}005 \ \text{J/kg-K} && \text{Table C.22} \\
\text{(ii) helium: } k &= 0.1490 \ \text{W/m-K} && \text{Table C.22} \\
\rho &= 0.1624 \ \text{kg/m}^3 && \text{Table C.22} \\
c_p &= 5{,}200 \ \text{J/kg-K} && \text{Table C.22} \\
\text{(iii) hydrogen: } k &= 0.1980 \ \text{W/m-K} && \text{Table C.22} \\
\rho &= 0.0812 \ \text{kg/m}^3 && \text{Table C.22} \\
c_p &= 14{,}780 \ \text{J/kg-K} && \text{Table C.22} \\
\text{(iv) argon: } k &= 0.0176 \ \text{W/m-K} && \text{Table C.22} \\
\rho &= 1.622 \ \text{kg/m}^3 && \text{Table C.22} \\
c_p &= 621 \ \text{J/kg-K} && \text{Table C.22.}
\end{aligned}
$$

(a) Thermal conductivities in order of increasing magnitude are

$$
\begin{aligned}
\text{argon: } k &= 0.0176 \ \text{W/m-K} \\
\text{air: } k &= 0.0267 \ \text{W/m-K} \\
\text{helium: } k &= 0.1490 \ \text{W/m-K} \\
\text{hydrogen: } k &= 0.1980 \ \text{W/m-K.}
\end{aligned}
$$

By changing the gas from argon to hydrogen, the conduction heat transfer rate will be increased by a factor of 11.25.

(b) Thermal diffusivities $\alpha = k/\rho c_p$ in order of increasing magnitude are

$$
\begin{aligned}
\text{argon: } \alpha &= 1.747 \times 10^{-5} \ \text{m}^2/\text{s} \\
\text{air: } \alpha &= 2.257 \times 10^{-5} \ \text{m}^2/\text{s} \\
\text{hydrogen: } \alpha &= 1.650 \times 10^{-4} \ \text{m}^2/\text{s} \\
\text{helium: } \alpha &= 1.764 \times 10^{-4} \ \text{m}^2/\text{s.}
\end{aligned}
$$

From (3.154), u_F is proportional to $\alpha^{1/2}$. Helium has an α that is 10.10 times that of argon. Thus, the penetration speed for helium is 3.178 times larger than that for argon.

(c) Thermal effusivities in order of increasing magnitude are

$$\begin{aligned}
\text{argon: } (\rho c_p k)^{1/2} &= 4.210 \text{ W-s}^{1/2}/\text{m}^2\text{-K} \\
\text{air: } (\rho c_p k)^{1/2} &= 5.620 \text{ W-s}^{1/2}/\text{m}^2\text{-K} \\
\text{helium: } (\rho c_p k)^{1/2} &= 11.22 \text{ W-s}^{1/2}/\text{m}^2\text{-K} \\
\text{hydrogen: } (\rho c_p k)^{1/2} &= 15.42 \text{ W-s}^{1/2}/\text{m}^2\text{-K.}
\end{aligned}$$

From (3.144), we note that $q_{\rho ck}(t)$ is proportional to $(\rho c_p k)^{1/2}$. Hydrogen has an effusivity which is 3.622 times that of argon. Thus, the transient heat flow rate to a semi-infinite stagnant gas layer suddenly heated on its bounded surface is 3.662 larger for hydrogen, compared to argon.

COMMENT:

We have assumed that the gas remains stagnant (i.e., no thermobuoyant motion) while it undergoes heat transfer.

PROBLEM 3.4.FUN

GIVEN:

The bulk (or intrinsic) conductivity refers to the medium property not affected by the size of the medium. In gases, this would indicate that the mean-free path of the gas molecules in thermal motion λ_m is much smaller than the linear dimension of gas volume L. When the linear dimension of the gas volume is nearly the same as or smaller than the mean-free path, then the gas molecules collide with the bounding surface of the gas with a probability comparable to that of the intermolecular collisions. This will occur either at low pressure or for very small L. There are simple, approximation expressions describing this size (or low-dimensionality) effect. These expressions include parameters modeling the gas molecule-bounding surface collision and energy exchange. One of these models that is used to predict the size dependence occurring at low gas pressures is

$$k_f(p,T) = \frac{k_f(p = 1\ \text{atm}, T)}{1 + \dfrac{4a_1(2-\gamma)}{\gamma(c_p/c_v + 1)}\text{Kn}_L},$$

where Kn_L is the Knudsen number defined in (1.20), i.e.,

$$\text{Kn}_L = \frac{\lambda_m}{L},$$

and λ_m is given by (1.19). Here $0 \le \gamma \le 1$ is the accommodation factor and a_1 is another semi-empirical constant. For example, for nitrogen in contact with ceramic surfaces, $a_1 = 1.944$, c_p/c_v=1.401, and $\gamma = 0.8$.

Use Table C.22 for k_f ($p = 1$ atm, $T = 300$ K).

OBJECTIVE:

For nitrogen gas with $L = 10\ \mu$m, use $T = 300$ K, and $d_m = 3 \times 10^{-10}$ m and plot $k_f/k_f(\lambda_m \ll L)$ versus the pressure and the Knudsen number.

SOLUTION:

From (1.19), we have

$$\begin{aligned}\lambda_m &= \frac{1}{2^{1/2}\pi}\frac{k_\text{B}T}{d_m^2 p} = \frac{1}{2^{1/2}\pi}\frac{(1.381\times10^{-23})(\text{J/K})\times300(\text{K})}{(3\times10^{-10})^2(\text{m}^2)\times p(\text{Pa})}\\ &= \frac{1.037\times10^{-2}(\text{Pa-m})}{p(\text{Pa})}.\end{aligned}$$

From Table C.22, for air at $T = 300$ K, we have

$$k_f(p = 1\ \text{atm}, T = 300\ \text{K}) = 0.0267\ \text{W/m-K}.$$

Then

$$\begin{aligned}k_f(p, T = 300\ \text{K}) &= \frac{0.0267(\text{W/m-K})}{1 + \dfrac{4\times1.944\times(2-0.8)}{0.8(1.401+1)}\dfrac{\lambda_m(\text{m})}{10^{-5}(\text{m})}}\\ &= \frac{0.0267(\text{W/m-K})}{1 + 4.858\times10^5(1/\text{m})\times\lambda_m(\text{m})}.\end{aligned}$$

Figures Pr.3.4(a) and (b) show the variations of $k_f(p)$ with respect to p and Kn_L.

COMMENT:

Note that the relation used here for k_f is an approximation. Also note that, as L becomes very large, the asymptotic value $k_f(p = 1$ atm) is recovered.

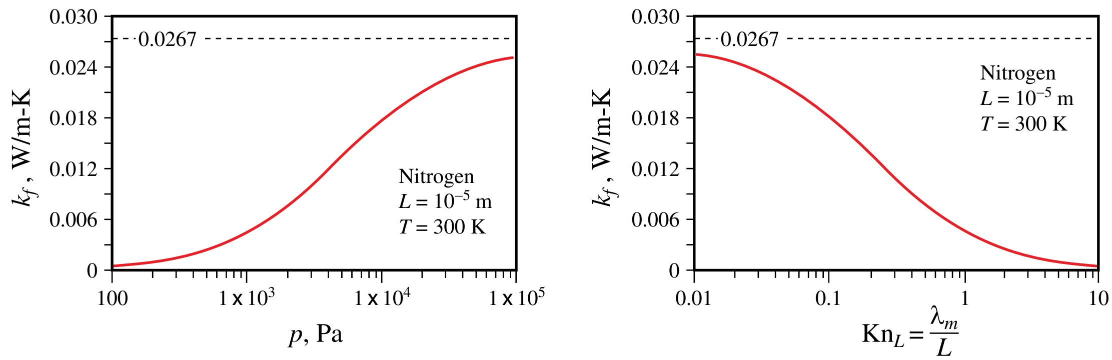


Figure Pr.3.4 Variation of the gas conductivity with respect to (a) pressure and (b) Knudsen number.

PROBLEM 3.5.FUN

GIVEN:

The lattice (phonon) specific heat capacity is related to the internal energy e, which in turn is given by the energy of an ensemble of harmonic oscillators as

$$\begin{aligned} e &= \frac{N_{\rm A}}{M}\sum_i E_i \\ E_i &= \frac{h_{\rm P}}{2\pi} f_i n_{p,i} \qquad n_{p,i} = \frac{1}{e^{x_i}-1} \qquad x_i = \frac{h_{\rm P} f_i}{2\pi k_{\rm B} T}, \end{aligned}$$

where $h_{\rm P}$ is the Planck constant, $k_{\rm B}$ is the Boltzmann constant, $N_{\rm A}$ is the Avogadro number, M is the molecular weight, and E_i is the average energy per vibrational mode i of each oscillator.

This represents the solid as a collection of harmonic oscillators, vibrating over a range of frequencies f, with the number of phonons having a frequency f_i given by $n_{p,i}$.

Note that from (3.4), $R_g \equiv k_{\rm B} N_{\rm A}$.

OBJECTIVE:

Starting from (1.6), and using the above, show that the lattice specific heat capacity is

$$c_v = \frac{R_g}{M}\sum_i \frac{x_i^2 e^{x_i}}{(e^{x_i}-1)^2}.$$

SOLUTION:

The energy per unit mass is

$$\begin{aligned} e &= \frac{N_{\rm A}}{M}\sum_i E_i \\ &= \frac{N_{\rm A}}{M}\sum_i \frac{h_{\rm P}}{2\pi} f_i n_{p,i} \\ &= \frac{N_{\rm A}}{M}\sum_i \frac{h_{\rm P}}{2\pi} f_i \frac{1}{e^{x_i}-1} \\ &= \frac{N_{\rm A}}{M}\sum_i \frac{h_{\rm P}}{2\pi} f_i \frac{1}{\exp\left(\dfrac{h_{\rm P} f_i}{2\pi k_{\rm B} T}\right)-1}. \end{aligned}$$

The specific heat capacity of a solid at constant volume c_v is found by differentiating with respect to temperature T, i.e.,

$$c_v = \left.\frac{\partial e}{\partial T}\right|_v = \frac{N_{\rm A}}{M}\sum_i \frac{h_{\rm P}}{2\pi} f_i \frac{\partial}{\partial T}\left[\frac{1}{\exp\left(\dfrac{h_{\rm P} f_i}{2\pi k_{\rm B} T}\right)-1}\right].$$

Letting $u(x_i) = e^{x_i}-1$, and letting $w(u) = u^{-1}$, we can simplify the differentiation on the right-hand side as

$$\frac{\partial}{\partial T}\left[\frac{1}{\exp\left(\dfrac{h_{\rm P} f_i}{2\pi k_{\rm B} T}\right)-1}\right] = \frac{\partial}{\partial T}\left(\frac{1}{e^{x_i}-1}\right) = \frac{\partial}{\partial T}\left(\frac{1}{u}\right) = \frac{\partial w}{\partial T}.$$

Applying the chain rule, we obtain

$$\begin{aligned} \frac{\partial w}{\partial T} &= \frac{\partial w}{\partial u}\frac{\partial u}{\partial x_i}\frac{\partial x_i}{\partial T} \\ &= (-u^{-2})(e^{x_i})\left(-\frac{h_{\rm P} f_i}{2\pi k_{\rm B} T^2}\right) \\ &= \frac{h_{\rm P} f_i}{2\pi k_{\rm B} T^2}\frac{e^{x_i}}{(e^{x_i}-1)^2}. \end{aligned}$$

Substituting back into the specific heat expression, we have

$$
\begin{aligned}
c_v = \left.\frac{\partial e}{\partial T}\right|_v &= \frac{N_\mathrm{A}}{M} \sum_i \frac{h_\mathrm{P}}{2\pi} f_i \frac{h_\mathrm{P} f_i}{2\pi k_\mathrm{B} T^2} \frac{e^{x_i}}{(e^{x_i} - 1)^2} \\
&= \frac{N_\mathrm{A} k_\mathrm{B}}{M} \sum_i \left(\frac{h_\mathrm{P} f_i}{2\pi k_\mathrm{B} T} \right)^2 \frac{e^{x_i}}{(e^{x_i} - 1)^2} \\
&= \frac{R_g}{M} \sum_i \frac{x_i^2 e^{x_i}}{(e^{x_i} - 1)^2}, \quad x_i = \frac{h_\mathrm{P} f_i}{2\pi k_\mathrm{B} T}.
\end{aligned}
$$

COMMENT:
In practice, the summation is difficult to perform and the Debye approximation given by (3.7) is used instead.

PROBLEM 3.6.FUN

GIVEN:

In the Debye approximation model for the lattice (phonon) specific heat capacity given by (3.7), the number of vibrational modes or density of state (per unit frequency around a frequency f) is given by the distribution function

$$P(f) = \frac{3f^2 V}{2\pi^2 u_p^3}, \quad V = l_m^3 = n^{-1},$$

where V is the volume, l_m is the cubic lattice constant, u_p is the speed of sound (phonon speed), f is the frequency, and n is the number of oscillators (or atoms) per unit volume. The actual lattice may not be cubic and would then be represented by two or more lattice parameters and, if the lattice is tilted, also by a lattice angle. Using this expression, the lattice specific heat capacity is approximated (as an integral approximation of the numerically exact summation) as

$$c_v = \frac{R_g}{M}\sum_i \frac{x_i^2 e^{x_i}}{(e^{x_i}-1)^2} = \frac{R_g}{M}\int_0^{f_\mathrm{D}} \frac{x^2 e^x}{(e^x-1)^2} P(f) df, \qquad x = \frac{h_\mathrm{P} f}{2\pi k_\mathrm{B} T}.$$

The Debye distribution function (or density of state), when integrated over the frequencies, gives the total number of vibrational modes (three per each oscillator)

$$3n = \frac{1}{V}\int_0^{f_\mathrm{D}} P(f) df.$$

OBJECTIVE:

(a) Show that

$$f_\mathrm{D} = (6n\pi^2 u_p^3)^{1/3}.$$

(b) Using this, derive (3.7), i.e., show that

$$c_v = 9\frac{R_g}{M}\left(\frac{T}{T_\mathrm{D}}\right)^3 \int_0^{T_\mathrm{D}/T} \frac{x^4 e^x}{(e^x-1)^2} dx, \qquad T_\mathrm{D} = \frac{h_\mathrm{P} f_\mathrm{D}}{2\pi k_\mathrm{B}}.$$

SOLUTION:

(a) The Debye cut-off frequency is related to the number of oscillators by

$$3 = \int_0^{f_\mathrm{D}} \frac{3f^2}{2n\pi^2 u_p^3} df = \frac{3}{2n\pi^2 u_p^3}\int_0^{f_\mathrm{D}} f^2 df.$$

Evaluating the integral and solving for f_D gives

$$\begin{aligned} 3 &= \frac{3}{2n\pi^2 u_p^3}\frac{f_\mathrm{D}^3}{3} \\ f_\mathrm{D}^3 &= 6n\pi^2 u_p^3 \\ f_\mathrm{D} &= (6n\pi^2 u_p^3)^{1/3}. \end{aligned}$$

(b) Noting that from the definition of the Debye temperature we can write $f_\mathrm{D} = (2\pi k_\mathrm{B} T_\mathrm{D})/(h_\mathrm{P})$, and recalling that $x(T,f) = (h_\mathrm{P} f)/(2\pi k_\mathrm{B} T)$, we can write $x_\mathrm{D} = x(T, f = f_\mathrm{D})$ as

$$x_\mathrm{D} = \frac{h_\mathrm{P} f_\mathrm{D}}{2\pi k_\mathrm{B} T} = \frac{h_\mathrm{P}}{2\pi k_\mathrm{B} T}\frac{2\pi k_\mathrm{B} T_\mathrm{D}}{h_\mathrm{P}} = \frac{T_\mathrm{D}}{T}.$$

Substituting the expressions for f and $P(f)$ into the given integral expression for c_v gives

$$\begin{aligned} c_v &\approx \frac{R_g}{M}\int_0^{f_\mathrm{D}} \frac{x^2 e^x}{(e^x-1)^2} P(f) df \\ &= \frac{R_g}{M}\int_0^{x_\mathrm{D}} \frac{x^2 e^x}{(e^x-1)^2} \frac{3f^2}{2n\pi^2 u_p^3} \frac{2\pi k_\mathrm{B} T}{h_\mathrm{P}} dx \\ &= \frac{R_g}{M}\int_0^{T_\mathrm{D}/T} \frac{3f^2 k_\mathrm{B} T}{n\pi u_p^3 h_\mathrm{P}} \frac{x^2 e^x}{(e^x-1)^2} dx \\ &= \frac{R_g}{M}\int_0^{T_\mathrm{D}/T} \frac{3f^2 k_\mathrm{B} T}{n\pi u_p^3 h_\mathrm{P}} \frac{x^2 e^x}{(e^x-1)^2} dx \times \left(\frac{2\pi k_\mathrm{B} T h_\mathrm{P}}{2\pi k_\mathrm{B} T h_\mathrm{P}}\right)^2 \\ &= \frac{R_g}{M}\int_0^{T_\mathrm{D}/T} \frac{12\pi k_\mathrm{B}^3 T^3}{n u_p^3 h_\mathrm{P}^3} \left(\frac{f h_\mathrm{P}}{2\pi k_\mathrm{B} T}\right)^2 \times \frac{x^2 e^x}{(e^x-1)^2} dx \\ &= \frac{R_g}{M}\int_0^{T_\mathrm{D}/T} \frac{12\pi k_\mathrm{B}^3 T^3}{n u_p^3 h_\mathrm{P}^3} \frac{x^4 e^x}{(e^x-1)^2} dx. \end{aligned}$$

Substituting f_D from part (a) into our expression for T_D gives

$$T_\mathrm{D} = \frac{h_\mathrm{P} f_\mathrm{D}}{2\pi k_\mathrm{B}} = \frac{h_\mathrm{P}}{2\pi k_\mathrm{B}} (6n\pi^2 u_p^3)^{1/3},$$

which, after some manipulation, gives

$$\frac{k_\mathrm{B}^3}{n u_p^3 h_\mathrm{P}^3} = \frac{1}{T_\mathrm{D}^3} \frac{6\pi^2}{(2\pi)^3}.$$

We then have

$$\begin{aligned} c_v &= \frac{R_g}{M}\int_0^{T_\mathrm{D}/T} \frac{12\pi k_\mathrm{B}^3 T^3}{n u_p^3 h_\mathrm{P}^3} \frac{x^4 e^x}{(e^x-1)^2} dx \\ &= \frac{R_g}{M} 12\pi T^3 \frac{k_\mathrm{B}^3}{n u_p^3 h_\mathrm{P}^3} \int_0^{T_\mathrm{D}/T} \frac{x^4 e^x}{(e^x-1)^2} dx \\ &= \frac{R_g}{M} 12\pi T^3 \frac{1}{T_\mathrm{D}^3} \frac{6\pi^2}{(2\pi)^3} \int_0^{T_\mathrm{D}/T} \frac{x^4 e^x}{(e^x-1)^2} dx \\ &= 9\frac{R_g}{M} \left(\frac{T}{T_\mathrm{D}}\right)^3 \int_0^{T_\mathrm{D}/T} \frac{x^4 e^x}{(e^x-1)^2} dx. \end{aligned}$$

COMMENT:

The Debye approximation gives a reasonable prediction of c_v for both metallic and nonmetallic, crystalline solids.

PROBLEM 3.7.FUN

GIVEN:

A simple approximate expression is found for the lattice thermal conductivity by only considering the normal (i.e., momentum conserving) phonon scattering mechanisms. This is done using the expression for c_v, given by (3.7) in the first part of the expression for k^p given by (3.26), i.e.,

$$k^p = \frac{1}{3}\rho c_v u_p \lambda_p,$$

and noting that

$$\lambda_p = u_p \tau_p.$$

OBJECTIVE:

As is done in the Debye approximation, use

$$c_v\lambda_p = \int_0^{T_\mathrm{D}/T} c_v(x)\lambda_p(x)dx, \quad x = \frac{h_\mathrm{P} f}{2\pi k_\mathrm{B} T} \quad \text{and} \quad x_\mathrm{D} = \frac{h_\mathrm{P} f_\mathrm{D}}{2\pi k_\mathrm{B} T} = \frac{T_\mathrm{D}}{T},$$

and

$$T_\mathrm{D} = \frac{h_\mathrm{P} f_\mathrm{D}}{2\pi k_\mathrm{B}} = \frac{h_\mathrm{P}}{2\pi k_\mathrm{B}}(6n\pi^2 u_p^3)^{1/3}$$

to derive an expression for k^p as a function of l_m as

$$k^p = (48\pi^2)^{1/3}\frac{1}{l_m}\frac{k_\mathrm{B}^3}{h_\mathrm{P}^2}\frac{T^3}{T_\mathrm{D}}\int_0^{T_\mathrm{D}/T}\tau_p\frac{x^4 e^x}{(e^x-1)^2}dx,$$

where, for a cubic crystal lattice, l_m is a lattice constant related to the number of atoms per unit volume by $l_m^{-1} = n^{1/3}$. From (1.19), use $\rho R_g/M = nk_\mathrm{B}$.

SOLUTION:

Substituting for $c_v\lambda_p$ into (3.26) and then $u_p\tau_p(x)$ for $\lambda_p(x)$, we obtain

$$\begin{aligned} k^p &= \frac{1}{3}\rho u_p \int_0^{T_\mathrm{D}/T} c_v(x)\lambda_p(x)dx \\ &= \frac{1}{3}\rho u_p^2 \int_0^{T_\mathrm{D}/T} \tau_p(x) c_v(x)dx \\ &= 3\rho\frac{R_g}{M}\left(\frac{T}{T_\mathrm{D}}\right)^3 u_p^2 \int_0^{T_\mathrm{D}/T}\tau_p\frac{x^4 e^x}{(e^x-1)^2}dx. \end{aligned}$$

From the definition of the Debye temperature, we have

$$\frac{u_p^2}{T_\mathrm{D}^2} = \frac{(2\pi)^2 k_\mathrm{B}^2}{h_\mathrm{P}^2(6n\pi^2)^{2/3}}.$$

Upon substitution for $(u_p/T_\mathrm{D})^2$ in k^p, we obtain

$$\begin{aligned} k^p &= 3\rho\frac{R_g}{M}\frac{T^3}{T_\mathrm{D}}\frac{u_p^2}{T_\mathrm{D}^2}\int_0^{T_\mathrm{D}/T}\tau_p\frac{x^4e^x}{(e^x-1)^2}dx \\ &= 3\rho\frac{R_g}{M}\frac{T^3}{T_\mathrm{D}}\frac{(2\pi)^2k_\mathrm{B}^2}{h_\mathrm{P}^2(6n\pi^2)^{2/3}}\int_0^{T_\mathrm{D}/T}\tau_p\frac{x^4e^x}{(e^x-1)^2}dx \\ &= (48\pi^2)^{1/3}\rho\frac{R_g}{M}\frac{T^3}{T_\mathrm{D}}\frac{k_\mathrm{B}^2}{h_\mathrm{P}^2 n^{2/3}}\int_0^{T_\mathrm{D}/T}\tau_p\frac{x^4e^x}{(e^x-1)^2}dx. \end{aligned}$$

Noting that $\rho R_g/M = nk_\mathrm{B}$, and that $l_m^{-1} = n^{1/3}$, this further simplifies to

$$\begin{aligned}
k^p &= (48\pi^2)^{1/3} n k_\mathrm{B} \frac{T^3}{T_\mathrm{D}} \frac{k_\mathrm{B}^2}{h_\mathrm{P}^2 n^{2/3}} \int_0^{T_\mathrm{D}/T} \tau_p \frac{x^4 e^x}{(e^x-1)^2} dx \\
&= (48\pi^2)^{1/3} k_\mathrm{B} \frac{k_\mathrm{B}^2}{h_\mathrm{P}^2} \frac{T^3}{T_\mathrm{D}} n^{1/3} \int_0^{T_\mathrm{D}/T} \tau_p \frac{x^4 e^x}{(e^x-1)^2} dx \\
&= (48\pi^2)^{1/3} \frac{1}{l_m} \frac{k_\mathrm{B}^3}{h_\mathrm{P}^2} \frac{T^3}{T_\mathrm{D}} \int_0^{T_\mathrm{D}/T} \tau_p \frac{x^4 e^x}{(e^x-1)^2} dx.
\end{aligned}$$

COMMENT:

The total phonon time constant is related to the time constants for the normal (momentum conserving, $\tau_{p,n}$) and the resistive (non-momentum conserving, $\tau_{p,r}$) processes that work to restore the phonon distribution to equilibrium (i.e., limit the conduction heat flux by damping the phonon propagation). The determination of the lattice thermal conductivity is highly dependent on the manner in which τ_p, and in turn the various $\tau_{p,n}$ and $\tau_{p,r}$, are evaluated and implemented into the calculation. In the approximate form found here, τ_p can be evaluated as $\tau_p^{-1} = \tau_{p,n}^{-1} + \tau_{p,r}^{-1}$, or for this case in which only normal processes are considered, $\tau_p^{-1} = \tau_{p,n}^{-1}$.

This is a simple form of (3.26). Most of the resistive relaxations neglected above are not very significant at high temperatures (including near room temperature) and therefore, the above simple expression can often be used. The time constant for a normal process can be approximated as

$$\tau_{p,n} = a_n 2\pi f T^4,$$

where a_n is a material constant.

PROBLEM 3.8.FUN.S

GIVEN:

The crystal size influences the phonon thermal conductivity due to phonon scattering caused by variation of phonon propagation properties across the crystal surface (similar to light scattering at the interface of two media of different light propagation properties). This boundary scattering is one of the resistive scattering mechanisms included in (3.26). Consider aluminum oxide (AL_2O_3, also called alumina) single crystals at $T = 300$ K. The effect of crystal size L can be described by a simple relation for the boundary scattering relaxation time constant τ_b as

$$\tau_b = \frac{L}{u_p},$$

where u_p is the average phonon velocity. Using the material constants for alumina and at $T = 300$ K, the lattice conductivity given by (3.26) becomes

$$k^p = b_k T^3 \left[g_1(x, L) + \frac{g_2^2(x, L)}{g_3(x, L)} \right] = b_k \left[h_1(x, L) + \frac{h_2^2(x, L)}{h_3(x, L)} \right],$$

where $b_k = 2.240 \times 10^5$ W/m-K^4 and the g_i's and h_i's represent integrals as defined below.

Some numerical solvers (e.g., SOPHT) have limitations to the size of the numbers which they may use. To avoid this limitation, the T^3 may be taken into the integral by defining $\theta_i = \tau_i T^3$ and then rewriting the integrals in (3.26) as

$$h_1 = g_1 T^3 = \int_0^{T_D/T} \theta_p \frac{x^4 e^x}{(e^x - 1)^2} dx, \qquad h_2 = g_2 = \int_0^{T_D/T} \frac{\theta_p}{\theta_{p,n}} \frac{x^4 e^x}{(1 - e^x)^2} dx,$$

$$h_3 = \frac{g_3}{T^3} = \int_0^{T_D/T} \frac{\theta_p}{\theta_{p,n}\theta_{p,r}} \frac{x^4 e^x}{(1 - e^x)^2} dx,$$

where

$$\frac{1}{\theta_p} = \frac{1}{\theta_{p,n}} + \frac{1}{\theta_{p,r}} = \frac{1}{\theta_{p,n}} + \left(\sum_i \frac{1}{\theta_{p,r,i}} \right)$$

$$= b_n x + \left(2 \times b_p x^4 + b_u x^2 + \frac{b_b}{L} \right),$$

where $T_D = 596$ K, $b_n = 3.181 \times 10^3$ 1/K^3-s, $b_p = 3.596 \times 10^1$ 1/K^3-s, $b_u = 1.079 \times 10^4$ 1/K^3-s, $b_b = 2.596 \times 10^{-4}$ m/K^3-s.

OBJECTIVE:

Use a solver to plot k_p versus grain size, L, for $10^{-9} \le L \le 10^{-4}$ m.

SOLUTION:

Using a solver such as SOPHT, the integrations are performed numerically. SOPHT is a differential solver, and therefore the integrals must be transformed into their associated differential forms. For example, the integral

$$h_1 = g_1 T^3 = \int_0^{T_D/T} \theta_p \frac{x^4 e^x}{(e^x - 1)^2} dx$$

is transformed to the differential form

$$\frac{dh_1}{dx} = \theta_p \frac{x^4 e^x}{(e^x - 1)^2}.$$

Since the lower limit of the integral is zero, the solver can then be used to solve for $h_1(x)$ with the final desired answer being $h_1 = h_1(x = T_D/T)$.

The source code using SOPHT is then

```
h1'=dh1dx
h2'=dh2dx
h3'=dh3dx
x=t
L=1e-2   //This is manually changed
bn=3.181e3
bu=1.079e4
bb=2.596e-4
bpv=3.596e1
bk=2.240e5
kern=(x^4*exp(x))/(exp(x)-1)^2
ithetap=1/theta_p
ithetapn=1/theta_pn
ithetapr=1/theta_pr
ithetap=ithetapn+ithetapr
ithetapn=bn*x
ithetapr=ithetau+ithetab+ithetapv
ithetau=bu*x^2
ithetab=bb/L
ithetapv=2*(bpv*x^4)
dh1dx=theta_p*kern
dh2dx=theta_p/theta_pn*kern
dh3dx=theta_p/theta_pn/theta_pr*kern
k=bk*(h1+h2^2/h3)
```

Note that SOPHT solves initial condition differential equations using t as the independent variable. Here t has been equated to our x. Each execution of SOPHT at different input values of L must be done for a range of $x = 0$ up to $x_\mathrm{D} = T_\mathrm{D}/T$. Note that if the initial conditions for t (i.e., x) or h_3 are equal to zero, there will be a division by zero in the first iterations of the solver execution resulting in an execution error. To avoid this, initial conditions of 1×10^{-10} were used for t (i.e., x) and h_3, initial conditions of zero were used for h_1 and h_2, and the iteration was run for 1,000 steps from a start of $t = 1 \times 10^{-10}$ to an end of $t = x_\mathrm{D} = T_\mathrm{D}/T = 1.987$.

Figure Pr.3.8 shows the results. Note that for $L \leq 1\mu$m, the effect of the boundary scattering becomes noticeable.

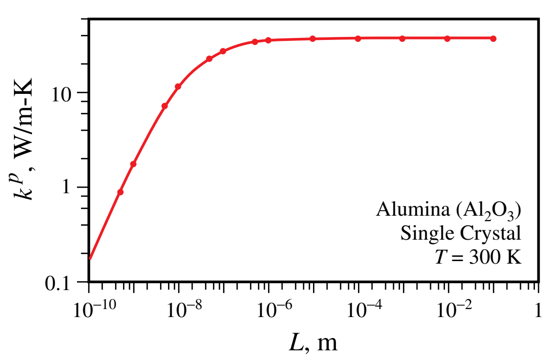


Figure Pr.3.8 Variation of alumina lattice thermal conductivity with respect to crystal dimension.

COMMENT:

Note that 1 Å = 10^{-10} m = 0.1 nm and as the lattice constant l_m = 0.3493 nm (Table 3.1) is reached, this continuum treatment of the lattice vibration will no longer be valid and a direct simulation (e.g., molecular dynamic simulation) is needed.

PROBLEM 3.9.FUN.S

GIVEN:

For thin film deposited on surfaces, the thermal conductivity of the film becomes film-thickness dependent, if the film thickness L is near or smaller than the heat-carrier, mean-free path. Consider a ceramic, amorphous silicon dioxide (SiO_2, also called silica) where the heat carriers are phonons. This film-thickness dependence of the thermal conductivity may be approximated as

$$k = \frac{k(L \gg \lambda_p)}{1 + \dfrac{4}{3}\dfrac{\lambda_p}{L}},$$

where $k(L \gg \lambda_p)$ is the bulk (or size-independent) thermal conductivity, and λ_p is the phonon mean-free path.

The reduction in the thermal conductivity (as λ_p/L increases) is due to the scattering of the phonons at the boundaries of the thin film.

OBJECTIVE:

Using Tables 3.1 and C.17, plot the variation of k for amorphous silica for $0.6 \leq L \leq 6$ nm, for $T = 293$ K.

SOLUTION:

From Table 3.1, we have

$$SiO_2: \quad \lambda_p = 0.6 \text{ nm at } T = 293 \text{ K}, \qquad \text{Table 3.1.}$$

From Table C.17, we have

$$SiO_2: \quad k(L \gg \lambda_p) = 1.38 \text{ W/m-K} \qquad \text{Table C.17.}$$

We note that for $\lambda_p/L = 0.6(\text{nm})/0.6(\text{nm}) = 1$, we have

$$k(\lambda_p/L = 1) = \frac{1.38(\text{W/m-K})}{1 + \dfrac{4}{3} \times 1} = 0.5914 \text{ W/m-K}.$$

For $\lambda_p/L = 0.6(\text{nm})/6(\text{nm}) = 0.1$, we have

$$k(\lambda_p/L = 0.1) = \frac{1.38(\text{W/m-K})}{1 + \dfrac{4}{3} \times 0.1} = 1.218 \text{ W/m-K}.$$

The variation of k as a function of λ_p/L is shown in Figure Pr.3.9.

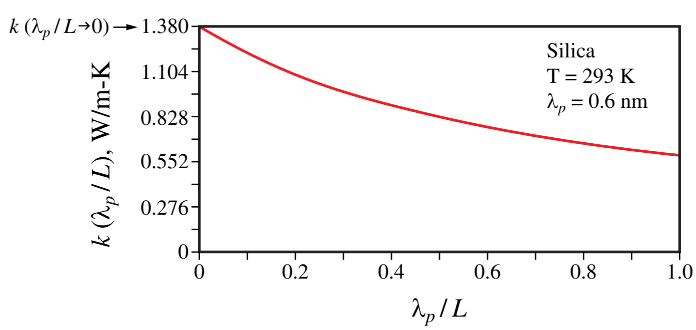


Figure Pr.3.9 Predicted variation of the thermal conductivity of a thin film, amorphous SiO_2 layer as a function of λ_p divided by the film thickness.

COMMENT:

Note that 0.6 nm = 6 Å and this means that the mean-free path of the phonon for silicon is only a few lattice lengths. Also note that this conductivity is along the film thickness. The conductivity along with the film is not the same and it is less affected by the film thickness.

PROBLEM 3.10.FUN

GIVEN:

The effective thermal conductivity $\langle k \rangle$ is used to describe the conductivity of porous solids (a fluid-solid composite). In many applications requiring a large surface area for surface convection A_{ku}, such as in heat storage in solids, packed bed of particles are used. For example, spherical particles are packed randomly or in an ordered arrangement (e.g., simple, body-centered or face-centered, cubic arrangement). Figure Pr.3.10(a) shows a simple (also called square-array) cubic arrangement of particles (porosity $\epsilon = 0.476$). Due to their weight or by a contact pressure p_c, these elastic particles deform and their contact area changes, resulting in a change in the effective thermal conductivity $\langle k \rangle$.

For spheres having a uniform radius R, a Young modulus of elasticity E_s, a Poisson ratio $\nu_{\rm P}$, and a conductivity k_s, the effective conductivity for the case negligible fluid conductivity ($k_f = 0$) and subject to contact pressure p_c is predicted as

$$\frac{\langle k \rangle}{k_s} = 1.36 \left[\frac{(1-\nu_{\rm P}^2)p_c}{E_s} \right]^{1/3}.$$

For aluminum, $E_s = 68$ GPa, $\nu_{\rm P} = 0.25$, $k_s = 237$ W/m-K.
For copper (annealed), $E_s = 110$ GPa, $\nu_{\rm P} = 0.343$, $k_s = 385$ W/m-K.
For magnesium (annealed sheet), $E_s = 44$ GPa, $\nu_{\rm P} = 0.35$, $k_s = 156$ W/m-K.

SKETCH:

Figure Pr.3.10(a) shows the particle arrangements, the contact pressure, the equivalent circuit, and the effective thermal conductivity $\langle k \rangle$.

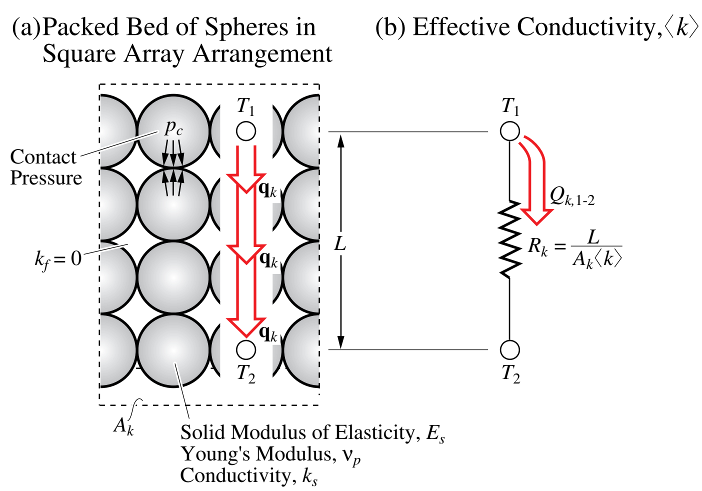


Figure Pr.3.10(a) Packed bed of spherical particles with a simple cubic arrangement and a contact pressure p_c. (b) The effective thermal conductivity $\langle k \rangle$.

OBJECTIVE:

Plot $\langle k \rangle$ versus p_c for $10^5 \le p_c \le 10^9$ Pa for packed beds of (i) aluminum, (ii) copper, and (iii) magnesium spherical particles.

SOLUTION:

Figure Pr.3.10(b) shows the results. Copper has the highest effective conductivity $\langle k \rangle$ at any given pressure.

COMMENT:

Note that relatively high pressures are considered here (10^5 Pa = 14.7 psi = 1 atm). Similar results are obtained by sintering the particles.

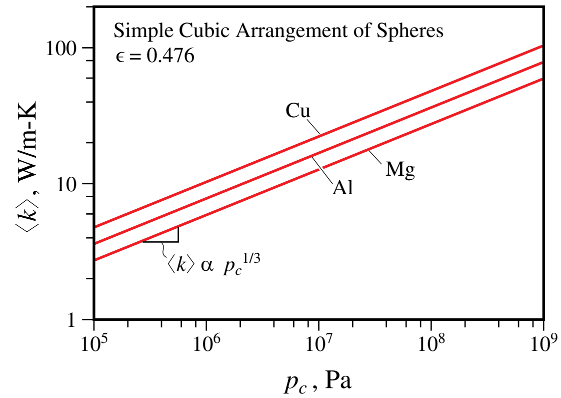


Figure Pr.3.10(b) Variation of the effective conductivity with respect to contact pressure for aluminum, copper, and magnesium.

PROBLEM 3.11.FUN.S

GIVEN:

The crystalline lattice thermal conductivity k_p is given by (3.26) as

$$k^p = (48\pi^2)^{1/3}\frac{1}{l_m}\frac{k_{\rm B}^3}{h_{\rm P}^2}\frac{T^3}{T_{\rm D}}\left[g_1(f,T,\tau_p)+\frac{g_2^2(f,T,\tau_p,\tau_{p,n})}{g_3(f,T,\tau_p,\tau_{p,n},\tau_{p,r})}\right],$$

The integrals g_n, $g_{r,1}$, and $g_{r,2}$ and the relaxation times $\tau_{p,n}$, $\tau_{p,r}$, and τ_p are defined as

$$g_1=\int_0^{T_{\rm D}/T}\tau_p\frac{x^4e^x}{(e^x-1)^2}dx,\quad g_2=\int_0^{T_{\rm D}/T}\frac{\tau_p}{\tau_{p,n}}\frac{x^4e^x}{(e^x-1)^2}dx,\quad g_3=\int_0^{T_{\rm D}/T}\frac{\tau_p}{\tau_{p,n}\tau_{p,r}}\frac{x^4e^x}{(e^x-1)^2}dx,$$

$$\frac{1}{\tau_p}=\frac{1}{\tau_{p,n}}+\frac{1}{\tau_{p,r}},\qquad \frac{1}{\tau_{p,n}}=a_n\frac{2\pi k_{\rm B}}{h_{\rm P}}T^5x,\qquad \frac{1}{\tau_{p,r}}=\sum_i\frac{1}{\tau_{p,r,i}},$$

where $x=(h_{\rm P}f)/(2\pi k_{\rm B}T)$, a_n is a material constant, and the resistive mechanisms in the summation for $tau_{p,r}^{-1}$ include the three-phonon umklapp processes, $\tau_{p,r,u}$, boundary scattering, $\tau_{p,r,b}$, point defect scattering, $\tau_{p,r,p}$, lattice vacancy scattering, $\tau_{p,r,v}$, and phonon-electron scattering, $\tau_{p,r,p\text{-}e}$, among others [6]. For alumina (Al_2O_3), the phonon-electron scattering is negligible compared to the other resistive mechanisms, and for simplicity is not considered here. The overall resistive time constant, due to these resistive mechanisms, is then given by

$$\begin{aligned}\frac{1}{\tau_{p,r}} &= \frac{1}{\tau_{p,r,p}}+\frac{1}{\tau_{p,r,v}}+\frac{1}{\tau_{p,r,u}}+\frac{1}{\tau_{p,r,b}}\\ &= A\left(\frac{2\pi k_{\rm B}}{h_{\rm P}}\right)^4T^4x^4+A\left(\frac{2\pi k_{\rm B}}{h_{\rm P}}\right)^4T^4x^4+a_u\left(\frac{2\pi k_{\rm B}}{h_{\rm P}}\right)^2T^3e^{-T_{\rm D}/(\alpha T)}x^2+\frac{u_p}{L},\end{aligned}$$

where A, a_u, and α are also material constants, u_p is the mean phonon velocity, and L is a characteristic length scale of the crystal or grain boundaries. Note that vacancies and point defects behave identically as resistance mechanisms.

Consider a single alumina crystal with linear dimension $L = 4.12$ mm, and the empirically determined material constants, $a_n = 2.7\times10^{-13}$ K^{-4}, $A = 4.08\times10^{-46}$ s^3, $a_u = 1.7\times10^{-18}$ K^{-1}, and $\alpha = 2$. Also from Table 3.1, we have $T_{\rm D} = 596$ K, $l_m = 0.35$ nm, and $u_p = 7{,}009$ m/s.

Substituting these values into the expression for the total phonon relaxation time constant, we have

$$\begin{aligned}\frac{1}{\tau_p} &= \frac{1}{\tau_{p,n}}+(\frac{1}{\tau_{p,r}})\\ &= b_nT^5x+(2\times b_pT^4x^4+b_uT^3e^{-298/T}x^2+b_b),\end{aligned}$$

where these new b_i constants combine the above a_i constants with the other coefficients and are $b_n = 3.535\times10^{-2}$ 1/K^5-s, $b_p = 1.199\times10^{-1}$ 1/K^4-s, $b_u = 2.914\times10^4$ 1/K^3-s, and $b_b = 1.701\times10^6$ 1/s.

Then the expression for the lattice thermal conductivity becomes

$$k^p = b_kT^3\left[g_1(x,T)+\frac{g_2^2(x,T)}{g_3(x,T)}\right],$$

where $b_k = 2.240\times10^5$ W/m-K^4.

OBJECTIVE:

(a) The integrals in the expression for the crystalline lattice thermal conductivity must be evaluated for a given temperature. For various temperatures, between $T = 1$ and 400 K, use a solver and determine k^p, and then plot k^p versus T.

(b) Compare the result with typical k^p vs. T curves for crystalline nonmetals, as shown in Figure 3.7(c).

Hint: To avoid overflow errors that might occur depending on the solver, factor the T^3 into the brackets containing the g_i integrals (i.e., into the b_i constants) before solving.

SOLUTION:
(a) Using a solver, such as SOPHT, the integrations are performed numerically. SOPHT is a differential solver, and therefore the integrals must be transformed into their associated differential forms. For example, the integral

$$g_1 = \int_0^{T_{\rm D}/T} \tau_p \frac{x^4 e^x}{(e^x - 1)^2} dx$$

is transformed to a differential as

$$\frac{dg_1}{dx} = \tau_p \frac{x^4 e^x}{(e^x - 1)^2}.$$

Since the lower limit of the integral is zero, the solver can then be used to solve for $g_1(x)$ with the final desired answer being $g_1 = g_1(x = T_{\rm D}/T)$.

The source code using SOPHT is then

```
g1'=dg1dx
g2'=dg2dx
g3'=dg3dx
x=t
Temp=1   //This is manually changed
//Factor in Temp^3 from expression for k
//Note b's are in 1/tau's
bn=5.535e-2/Temp^3
bu=2.914e4/Temp^3
bb=1.701e6/Temp^3
bpv=1.199e-1/Temp^3
bk=2.240e5
kern=(x^4*exp(x))/(exp(x)-1)^2
itaup=1/tau_p
itaupn=1/tau_pn
itaupr=1/tau_pr
itaup=itaupn+itaupr
itaupn=bn*Temp^5*x
itaupr=itauu+itaub+itaupv
itauu=bu*Temp^3*exp(-298/Temp)*x^2
itaub=bb
itaupv=2*(bpv*Temp^4*x^4)
dg1dx=tau_p*kern
dg2dx=tau_p/tau_pn*kern
dg3dx=tau_p/tau_pn/tau_pr*kern
//Temp^3 factored in above
k=bk*(g1+g2^2/g3)
```

Note that SOPHT solves initial condition differential equations using t as the independent variable. Here t has been equated to our x. Each execution of SOPHT at different input values of $T =$ Temp must be done for a range of $x = 0$ up to $x_{\rm D} = T_{\rm D}/T$. Note that if the initial conditions for t (i.e., x) or g_3 are equal to zero, there will be a division by zero in the first iterations of the solver execution resulting in an execution error. To avoid this, initial conditions of 1×10^{-10} were used for t (i.e., x) and g_3, initial conditions of zero were used for g_1 and g_2, and the iteration was run for 1,000 steps from a start of $t = 1 \times 10^{-10}$ to and end of $t = x_{\rm D} = T_{\rm D}/T$. For each different input value of T, the correct end value of the iteration of $t = t(T)$ must be entered.

Note that for small T (i.e., $T < 20$ K), x becomes large (i.e., $x = t > 30$) and the numerator and denominator of kern in the SOPHT program both become large and exceed the capability of the software. Plotting the g_i's and the k^p versus x will show that, for small T, the solution for these variables have already converged to near constant values and that the iterations only need be run to an end of $x = t = 25$ or 30, instead of $x = T_{\rm D}/T$, to obtain acceptable predictions.

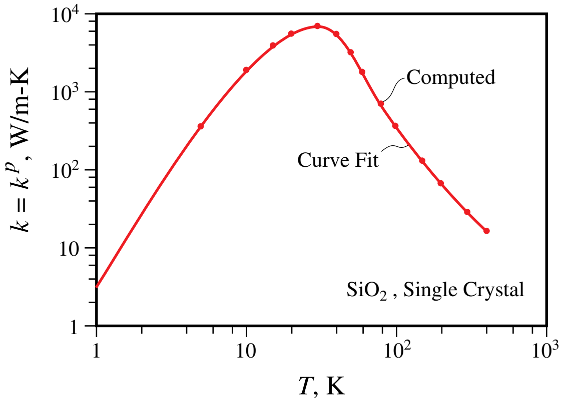


Figure Pr.3.11 Predicted variation of the lattice thermal conductivity with respect to temperature.

The results are plotted in Figure Pr.3.11 for several temperatures.
(b) We note that k^p peaks around $T = 27$ K, where k^p is about 6,543 W/m-K. Comparing to Figure 3.11(c), the results are similar to those for sodium-fluoride, another nonmetal. Note the initial, sharp rise in the low-temperature region were the interphonon scattering is not significant (i.e., $\tau_{p,r,u} \rightarrow 0$). The constants used here slightly underpredict k_p at $T = 300$ K, where the measured value given in Table 3.1 is $k_p = 36$ W/m-K and the predicted value is $k_p = 27.6$ W/m-K.

COMMENT:

Note that the boundary scattering will only be significant when T is small (since all other scattering mechanisms have a strong, slightly nonlinear dependence on T). This boundary scattering is addressed in Problem 3.8.FUN.

PROBLEM 3.12.FUN

GIVEN:

Similar to Example 3.6, consider the internal surface of the three surfaces to be covered with an insulation layer of thickness $l = 5$ cm and thermal conductivity $k = 0.1$ W/m-K. The outside surface is at temperature $T_1 = 90$℃ and the temperature at the inside surface of the insulation is $T_2 = 40$℃. The surfaces have an outside area $A_1 = 1$ m^2 and are of the geometries shown in Figure Pr.3.12, i.e., (a) a planar surface with area $A_1 = L_y L_z$, (b) a cylinder with area $A_1 = 2\pi R_1 L_y$ and length $L_y = 1$ m, and (c) a sphere with surface area $A_1 = 4\pi R_1^2$.

SKETCH:

Figure Pr.3.12 shows the three surfaces to be lined (inside) by insulation.

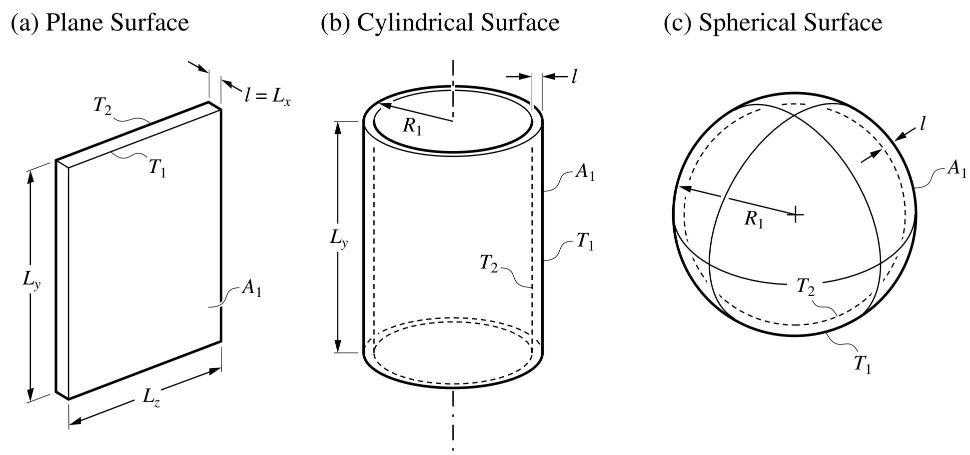


Figure Pr.3.12 (a), (b), and (c) Three geometries to be lined (inside) with insulation.

OBJECTIVE:

For each of these geometries, calculate the rate of heat loss through the vessel surface $Q_{k,2-1}$(W). Compare your results with the results of Example 3.6 and comment on the differences among the answers. Neglect the heat transfer through the ends (i.e., assume a one-dimensional heat transfer).

SOLUTION:

The one-dimensional conduction heat flow rate is given by

$$Q_{k,2-1} = \frac{T_2 - T_1}{R_{k,1\text{-}2}}.$$

(a) For a plane surface with an area $A_1 = L_y L_z$ and thickness l,

$$R_{k,1\text{-}2} = \frac{l}{A_k k} = \frac{0.05(\text{m})}{1(\text{m}^2) \times 0.1(\text{W/m-K})} = 0.5℃/\text{W}$$

$$Q_{k,2\text{-}1} = \frac{T_2 - T_1}{R_{k,1\text{-}2}} = \frac{40(℃) - 90(℃)}{0.5(℃/\text{W})} = -100 \text{ W}.$$

(b) For a cylinder with an external radius $R_1 = A_1/2\pi L_y$, length $L_y = 1$ m, and an internal radius $R_2 = R_1 - l$

$$R_{k,1\text{-}2} = \frac{\ln(R_1/R_2)}{2\pi k L_y} = \frac{\ln[(\frac{1}{2\pi})(\text{m})/(\frac{1}{2\pi} - 0.05)(\text{m})]}{2\pi \times 0.1(\text{W/m-K}) \times 1(\text{m})} = 0.600℃/\text{W}$$

$$Q_{k,2\text{-}1} = \frac{T_2 - T_1}{R_{k,1\text{-}2}} = \frac{40(℃) - 90(℃)}{0.600(℃/\text{W})} = -83.3 \text{ W}.$$

(c) For a sphere with an external radius $R_1 = (A_1/4\pi)^{1/2}$ and an internal radius $R_2 = R_1 - l$

$$R_{k,1\text{-}2} = \frac{1/R_2 - 1/R_1}{4\pi k} = \frac{1/[(4\pi)^{-1/2} - 0.05] - 1/(4\pi)^{-1/2}}{4\pi \times 0.1(\text{W/m-K})} = 0.608℃/\text{W}$$

$$Q_{k,2\text{-}1} = \frac{T_2 - T_1}{R_{k,1\text{-}2}} = \frac{40(℃) - 90(℃)}{0.608(℃/\text{W})} = -82.3 \text{ W}.$$

(d) Contrary to results obtained from placing insulation on the outside surface, the rate of heat transfer decreases as the geometry changed from the flat plate to the cylinder and the sphere. The increase in curvature for a cylinder or a sphere, as compared to a flat plate, decreases the available area on the inside surface, for heat transfer, as the radius decreases. As the heat transfer rate $Q_{k,2\text{-}1}$(W) is proportional to the heat transfer area A_k, a reduction on the available area for heat transfer causes a reduction on the heat transfer rate.

COMMENT:

For a given l, as the axis of a cylinder or the center of a sphere is approached, the area for heat transfer decreases, resulting in a large-resistance to heat flow. The heat transfer rate $Q_{k,2\text{-}1}$ is negative because $T_2 < T_1$. The heat transfer rate $Q_{k,1\text{-}2}$ has the same magnitude but the opposite sign. The negative sign is mostly a matter of convention and the notation $Q_{k,i\text{-}j}$ indicates the heat transfer rate from temperature T_i to temperature T_j.

PROBLEM 3.13.FUN

GIVEN:

Consider an infinite plane wall (called a slab) with thickness $L = 1$ cm, as shown in Figure Pr.3.13. The thermophysical properties of copper and silica aerogel are to be evaluated at 25℃ and 1 atm [Table C.14 and Figure 3.13(a)].

SKETCH:

Figure Pr.3.13 shows the one-dimensional, steady-state conduction across a slab.

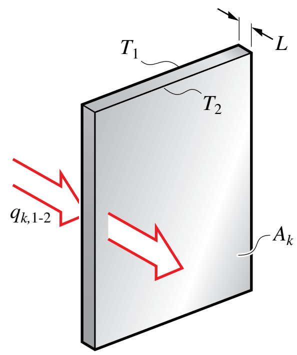


Figure Pr.3.13 One-dimensional conduction across a slab.

OBJECTIVE:

(a) Calculate the conduction thermal resistance $A_k R_{k,1-2}$[℃/(W/m^2)], if the wall is made of copper.
(b) Calculate the conduction thermal resistance $A_k R_{k,1-2}$[℃/(W/m^2)], if the wall is made of silica aerogel.
(c) If the heat flux through the wall is $q_{k,2\text{-}1} = 1{,}000$ W/m^2 and the internal wall temperature is $T_1 = 60$℃, calculate the external wall temperature T_2 for the two materials above.
(d) Express the results for items (a) and (b) in terms of the R_k-value.

SOLUTION:

(a) From Table C.14, the thermal conductivity of pure copper at 300 K is $k = 401$ W/m-K. The conduction thermal resistance is

$$A_k R_{k,1\text{-}2} = \frac{L}{k} = \frac{0.01(\text{m})}{401(\text{W/m-K})} = 2.5 \times 10^{-5}\,℃/(\text{W/m}^2).$$

(b) From Figure 3.13(a), at $p = 1$ atm, the thermal conductivity of silica aerogel at 300 K is $k = 0.0135$ W/m-K. This value is for conditions close to our specified conditions of $p = 1$ atm and $T = 298$ K, therefore this would be a better value to use than one linearly extrapolated from Table C.15. The conduction thermal resistance is

$$A_k R_{k,1\text{-}2} = \frac{L}{k} = \frac{0.01(\text{m})}{0.0135(\text{W/m-K})} = 7.4 \times 10^{-1}\,℃/(\text{W/m}^2).$$

(c) The rate of heat flow per unit area through the wall $q_{k,1\text{-}2}$(W/m^2) is

$$q_{k,1\text{-}2} = \frac{Q_{k,1\text{-}2}}{A_k} = \frac{T_1 - T_2}{A_k R_{k,1\text{-}2}}.$$

For a heat flow per unit area of $q_{k,1\text{-}2} = 1{,}000$ W/m^2, an internal wall temperature of $T_2 = 60$℃, the external wall temperature T_1 for each of the two cases above is
(i) copper

$$T_1 = T_2 + q_{k,1\text{-}2}(A_k R_{k,1\text{-}2}) = 60(℃) + 1{,}000(\text{W/m}^2) \times 2.5 \times 10^{-5}[℃/(\text{W/m}^2)] = 60.02℃,$$

(ii) silica aerogel

$$T_1 = T_2 + q_{k,1\text{-}2}(A_k R_{k,1\text{-}2}) = 60(^\circ\text{C}) + 1{,}000(\text{W/m}^2) \times 7.4 \times 10^{-1}[^\circ\text{C}/(\text{W/m}^2)] = 800^\circ\text{C}.$$

(d) The R_k-value for each of the situations above is
(i) copper

$$R_k\text{-value} = \frac{L}{A_k k} = \frac{0.0328(\text{ft})}{1(\text{ft}^2) \times 231.9(\text{Btu/hr-ft-}^\circ\text{F})} = 1.4 \times 10^{-4}\,^\circ\text{F/(Btu/hr)},$$

(ii) silica aerogel

$$R_k\text{-value} = \frac{L}{A_k k} = \frac{0.0328(\text{ft})}{1(\text{ft}^2) \times 0.00781(\text{Btu/hr-ft-}^\circ\text{F})} = 4.2^\circ\text{F/(Btu/hr)}.$$

COMMENT:
The conversion factor for thermal conductivity from W/m-K to Btu/hr-ft-℉ is obtained from Table C.1(a).

PROBLEM 3.14.FAM

GIVEN:

A furnace wall (slab) is made of asbestos ($\rho = 697$ kg/m^3) and has a thickness $L = 5$ cm [Figure Pr.3.14(i)]. Heat flows through the slab with given inside and outside surface temperatures. In order to reduce the heat transfer (a heat loss), the same thickness of asbestos L is split into two with an air gap of length $L_a = 1$ cm placed between them [Figure Pr.3.14(ii)].

Use Tables C.12 and C.17 to evaluate the conductivity at 273 K or 300 K.

SKETCH:

Figure Pr.3.14 shows the insulating furnace wall with and without an air gap.

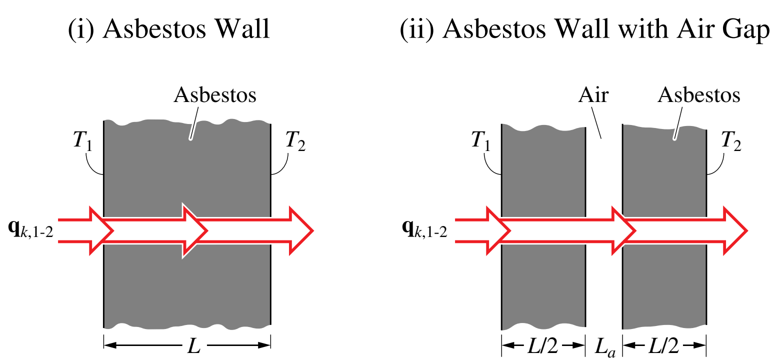


Figure Pr.3.14 A furnace wall. (i) Insulated without an air gap. (ii) With an air gap.

OBJECTIVE:

Determine how much the heat flow out of the wall would decrease (show this as a percentage of the heat flow without the air gap).

SOLUTION:

The conduction heat flux through the wall is

$$A_k q_{k,1\text{-}2} = \frac{T_1 - T_2}{A_k R_{k,1\text{-}2}}.$$

The conduction resistance for a plane wall is

$$A_k R_{k,1\text{-}2} = \frac{L}{k_s}.$$

From Table C.17, for asbestos with $\rho = 697$ kg/m^3 at $T = 273$ K, $k_s = 0.23$ W/m-K, and the conduction resistance becomes

$$A_k R_{k,1\text{-}2} = \frac{0.05(\text{m})}{0.23(\text{W/m-K})} = 0.217 \text{ K/(W/m}^2).$$

For the composite wall, the conduction heat flow through the wall is

$$A_k q_{k,1\text{-}2} = \frac{T_1 - T_2}{A_k (R_{k,\Sigma})_{1\text{-}2}},$$

where

$$A_k (R_{k,\Sigma})_{1\text{-}2} = \frac{L/2}{k_s} + \frac{L_a}{k_a} + \frac{L/2}{k_s} = \frac{L}{k_s} + \frac{L_a}{k_a}.$$

From Table C.12, for air at $T = 300$ K, $k_a = 0.0267$ W/m-K, and the equivalent conduction resistance becomes

$$A_k R_{k,\Sigma,1\text{-}2} = 0.217[\text{K/(W/m}^2)] + \frac{0.01(\text{m})}{0.0267(\text{W/m-K})} = 0.591 \text{ K/(W/m}^2).$$

The percentage of reduction of the heat flow through the wall is

$$\frac{(Q_{k,1\text{-}2})_i - (Q_{k,1\text{-}2})_{ii}}{(Q_{k,1\text{-}2})_i} \times 100 = \frac{\dfrac{1}{A_k R_{k,1\text{-}2}} - \dfrac{1}{A_k R_{k,\Sigma,1\text{-}2}}}{\dfrac{1}{A_k R_{k,1\text{-}2}}}$$

$$= \frac{\dfrac{1}{0.217[\mathrm{K/(W/m^2)}]} - \dfrac{1}{0.591[\mathrm{K/(W/m^2)}]}}{\dfrac{1}{0.217[\mathrm{K/(W/m^2)}]}} \times 100\% = 63.3\%.$$

The heat flow rate through the composite wall is 63.3% lower than the heat flow through the solid wall.

COMMENT:

Here the air-gap resistance is in series with the wall resistance. This is the most effective use of the air gap. When the air-gap resistance is placed in parallel, it is not as effective.

PROBLEM 3.15.FUN

GIVEN:

A low thermal-conductivity composite (solid-air) material is to be designed using alumina as the solid and having the voids occupied by air. There are three geometric arrangements considered for the solid and the fluid. These are shown in Figure Pr.3.15. For all three arrangements , the fraction of volume occupied by the fluid (i.e., porosity) ϵ is the same. In the parallel arrangement, sheets of solid are separated by fluid gaps and are placed parallel to the heat flow direction. In the series arrangement, they are placed perpendicular to the heat flow. In the random arrangement, a nonlayered arrangement is assumed with both solid and fluid phase continuous and the effective conductivity is given by (3.28).

SKETCH:

Figure Pr.3.15 shows the three geometries for solid-fluid arrangements.

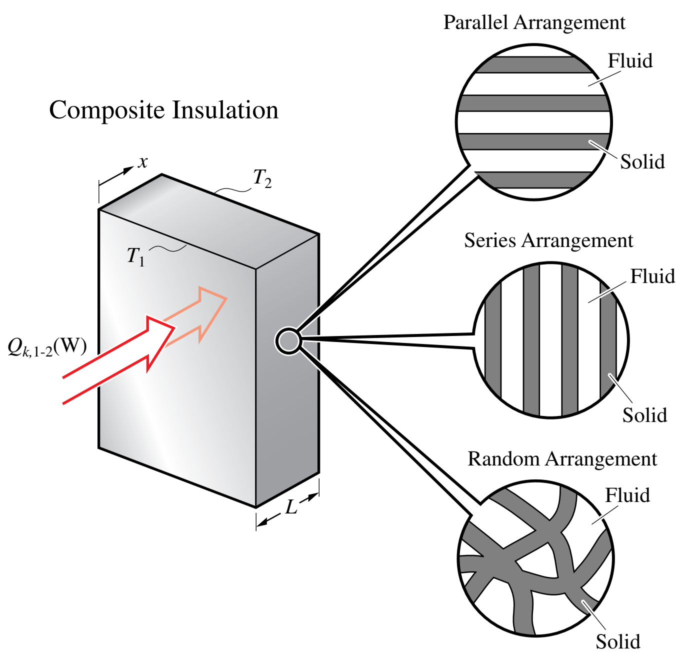


Figure Pr.3.15 Three solid-fluid arrangements for obtaining a low thermal conductivity composite.

OBJECTIVE:

(a) Show that the effective thermal conductivity for the parallel arrangement is given by

$$\langle k \rangle = k_f \epsilon + k_s(1-\epsilon)$$

and that, for the series arrangement, the effective thermal conductivity is given by

$$\frac{1}{\langle k \rangle} = \frac{\epsilon}{k_f} + \frac{(1-\epsilon)}{k_s}.$$

The porosity ϵ is defined as the volume occupied by the fluid divided by the total volume of the medium, i.e.,

$$\epsilon = \frac{V_f}{V_f + V_s}.$$

Also note that $1-\epsilon = V_s/(V_f+V_s)$.

(b) Compare the effective conductivity for the three arrangements for $\epsilon = 0.6$ using the conductivity of alumina (Table C.14) and air (Table C.22) at $T = 300$ K.

(c) Comment regarding the design of low-conductivity composites.

SOLUTION:
(a) (i) Series Arrangement:
The equivalent thermal resistance for resistances arranged in series is given by

$$R_{k,\Sigma} = \sum_{i=1}^{n} R_{k,i}.$$

For n layers of fluid and solid placed in series, the summation becomes

$$R_{k,\Sigma} = n(R_{k,f} + R_{k,s}).$$

The solid and fluid resistances are

$$R_{k,f} = \frac{L_f}{k_f A_f} \quad , \quad R_{k,s} = \frac{L_s}{k_s A_s}.$$

The equivalent resistance can be expressed in terms of the effective thermal conductivity $\langle k \rangle$ as

$$R_{k,\Sigma} = \frac{L}{\langle k \rangle A}.$$

From the equations above and noting that $A_f = A_s = A$ and $L = n(L_f + L_s)$, we obtain

$$\frac{L_f + L_s}{\langle k \rangle} = \frac{L_f}{k_f} + \frac{L_s}{k_s},$$

or rearranging, we have

$$\frac{1}{\langle k \rangle} = \left(\frac{L_f}{L_f + L_s} \right) \frac{1}{k_f} + \left(\frac{L_s}{L_f + L_s} \right) \frac{1}{k_s}.$$

The porosity ϵ is defined as

$$\epsilon = \frac{V_f}{V_f + V_s}.$$

The volumes of the fluid and solid phases can be rewritten as $V_f = nA_f L_f$ and $V_s = nA_s L_s$, respectively. Then noting that $A_f = A_s$, the porosity can be rewritten as

$$\epsilon = \frac{L_f}{L_f + L_s}.$$

Also, note that

$$\frac{V_s}{V_f + V_s} = \frac{V_s + V_f - V_f}{V_f + V_s} = \frac{V_s + V_f}{V_f + V_s} - \frac{V_f}{V_f + V_s} = 1 - \epsilon$$

and using the expressions for the volumes

$$\frac{L_s}{L_f + L_s} = 1 - \epsilon.$$

Therefore, substituting we have

$$\frac{1}{\langle k \rangle} = \frac{\epsilon}{k_f} + \frac{1 - \epsilon}{k_s}.$$

(ii) Parallel Arrangement:
The equivalent thermal resistance for resistances arranged in parallel is

$$\frac{1}{R_{k,\Sigma}} = \sum_{i=1}^{n} \frac{1}{R_{k,i}}.$$

For n layers of fluid and solid arranged in parallel, we have

$$\frac{1}{R_{k,\Sigma}} = n \left(\frac{1}{R_f} + \frac{1}{R_s} \right).$$

As before, the solid and fluid resistances are

$$R_{k,f} = \frac{L_f}{k_f A_f} \quad , \quad R_{k,s} = \frac{L_s}{k_s A_s}$$

and the equivalent resistance can be expressed in terms of the effective thermal conductivity $\langle k \rangle$ as

$$R_{k,\Sigma} = \frac{L}{\langle k \rangle A}.$$

For the parallel arrangement $L_f = L_s = L$ and $A = n(A_f + A_s)$. Then we have

$$\langle k \rangle (A_f + A_s) = k_f A_f + k_s A_s,$$

which can be rearranged as

$$\langle k \rangle = \left(\frac{A_f}{A_f + A_s} \right) k_f + \left(\frac{A_s}{A_f + A_s} \right) k_s.$$

The volumes of the fluid and solid phases, as before, can be rewritten as $V_f = nA_f L_f$ and $V_s = nA_s L_s$. Then noting that $L_f = L_s$, we can write

$$\epsilon = \frac{A_f}{A_f + A_s} \quad , \quad 1 - \epsilon = \frac{A_s}{A_s + A_f}.$$

Therefore, from the equations above, we have

$$\langle k \rangle = \epsilon k_f + (1 - \epsilon) k_s.$$

(b) The thermal conductivities of alumina and air are

alumina:	$T = 300$ K,	$k_s = 36$ W/m-K	Table C.14
air:	$T = 300$ K,	$k_f = 0.0267$ W/m-K	Table C.22.

For each of the arrangements, for $\epsilon = 0.6$, the effective thermal conductivities $\langle k \rangle$ are
Series:

$$\frac{1}{\langle k \rangle} = \frac{\epsilon}{k_f} + \frac{1-\epsilon}{k_s} = \frac{0.6}{0.0267} + \frac{0.4}{36} \Rightarrow \langle k \rangle = 0.044 \quad \text{W/m-K}.$$

Parallel:

$$\langle k \rangle = \epsilon k_f + (1 - \epsilon) k_s = 0.6 \times 0.0267 + 0.4 \times 36 = 14.4 \text{ W/m-K}.$$

Random: Using (3.28)

$$\frac{\langle k \rangle}{k_f} = \left(\frac{k_s}{k_f} \right)^{0.280 - 0.757 \log(\epsilon) - 0.057 \log(k_s/k_f)},$$

we obtain

$$\langle k \rangle = 0.0267 \left(\frac{36}{0.0267} \right)^{0.280 - 0.757 \log(0.6) - 0.057 \log(36/0.0267)} = 0.19 \text{ W/m-K}.$$

(c) The series arrangement leads to the lowest thermal conductivity possible for a medium composed of solid and fluid thermal resistances and for a given porosity. In the design of an insulating material, one should attempt to approach that limit.

COMMENT:
The series resistance allows for the high resistance to dominate the heat flow path.

PROBLEM 3.16.FAM

GIVEN:

During hibernation of warm-blooded animals (homoisotherms), the heart beat and the body temperature are lowered and in some animals the body waste is recycled to reduce energy consumption. Up to 40% of the total weight may be lost during the hibernation period. The nesting chamber of the hibernating animals is at some distance from the ground surface, as shown in Figure Pr.3.16(a)(i). The heat transfer from the body is reduced by the reduction in the body temperature T_1 and by the insulating effects of the body fur and the surrounding air (assumed stagnant). A simple thermal model for the steady-state, one-dimensional heat transfer is given in Figure Pr.3.16(a)(ii). The thermal resistance of the soil can be determined from Table 3.3(a). An average temperature T_2 is used for the ground surrounding the nest. The air gap size $R_a - R_f$ is an average taken around the animal body.

$R_1 = 10$ cm, $R_f = 11$ cm, $R_a = 11.5$ cm, $T_1 = 20$℃, $T_2 = 0$℃.

Evaluate air properties at $T = 300$ K, use soil properties from Table C.15, and for fur use Table C.15 for hair.

SKETCH:

Figure Pr.3.16(a) shows a simple thermal model with conduction heat transfer through the fur, air, and surrounding ground.

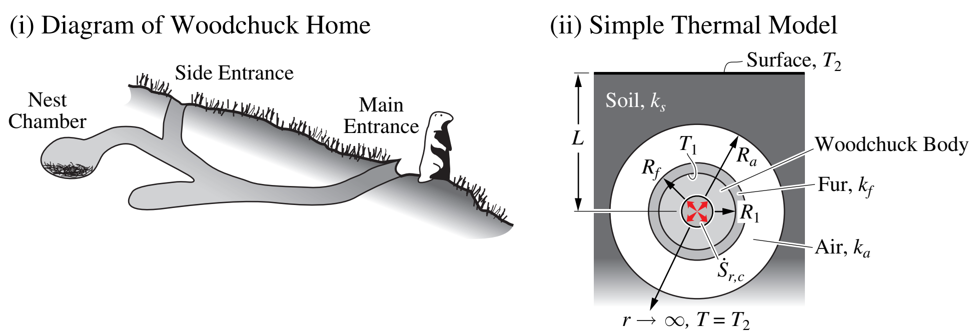


Figure Pr.3.16(a) Conduction heat transfer from a warm-blooded animal during hibernation. (i) Diagram of woodchuck home. (ii) Thermal model.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine $Q_{1\text{-}2}$ for (i) $L = 2.5R_a$, and (ii) $L = 10R_a$.

SOLUTION:

(a) Figure Pr.3.16(b) shows the thermal circuit diagram, starting from the body node T_1 and after encountering the resistances $R_{k,1\text{-}f}$, $R_{k,f\text{-}a}$, $R_{k,a\text{-}2}$, node T_2, which is the far-field thermal condition, is reached.

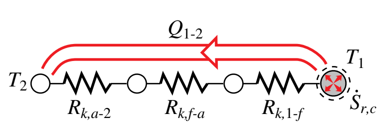


Figure Pr.3.16(b) Thermal circuit diagram.

(b) From Figure Pr.3.16(b), we have

$$Q_{1\text{-}2} = \frac{T_1 - T_2}{R_{k,1\text{-}f} + R_{k,f\text{-}a} + R_{k,a\text{-}2}}.$$

From Table 3.2, we have

$$R_{k,1\text{-}f} = \frac{\dfrac{1}{R_1} - \dfrac{1}{R_f}}{4\pi k_f}$$

$$R_{k,f\text{-}a} = \frac{\dfrac{1}{R_f} - \dfrac{1}{R_a}}{4\pi k_a}.$$

From Table 3.3(a), we have

$$R_{k,a\text{-}2} = \frac{1 - \dfrac{R_a}{2L}}{4\pi k_s R_a} \quad \text{for} \quad L > 2R_a.$$

The thermal conductivities are

$$\begin{aligned} k_f &= 0.036 \text{ W/m-K} && \text{Table C.15} \\ k_a &= 0.0267 \text{ W/m-K} && \text{Table C.22} \\ k_s &= 0.52 \text{ W/m-K} && \text{Table C.15.} \end{aligned}$$

Using the numerical values, we have

$$\begin{aligned} R_{k,1\text{-}f} &= \frac{\dfrac{1}{0.10(\text{m})} - \dfrac{1}{0.11(\text{m})}}{4\pi \times 0.036(\text{W/m-K})} = 2.010 \text{ K/W} \\ R_{k,f\text{-}a} &= \frac{\dfrac{1}{0.11(\text{m})} - \dfrac{1}{0.115(\text{m})}}{4\pi \times 0.0267(\text{W/m-K})} = 1.178 \text{ K/W} \\ \text{(i)} \quad R_{k,a\text{-}2} &= \frac{1 - \dfrac{1}{5}}{4\pi \times 0.52(\text{W/m-K}) \times 0.115(\text{m})} = 1.065 \text{ K/W} \\ \text{(ii)} \quad R_{k,a\text{-}2} &= \frac{1 - \dfrac{1}{20}}{4\pi \times 0.52(\text{W/m-K}) \times 0.115(\text{m})} = 1.264 \text{ K/W.} \end{aligned}$$

Then

$$\begin{aligned} \text{(i)} \quad Q_{1\text{-}2} &= \frac{(20 - 0)(\text{K})}{(2.010 + 1.178 + 1.065)(\text{K/W})} = 4.703 \text{ W} \\ \text{(ii)} \quad Q_{1\text{-}2} &= \frac{(20 - 0)(\text{K})}{(2.010 + 1.178 + 1.264)(\text{K/W})} = 4.492 \text{ W.} \end{aligned}$$

There is only a slightly larger $Q_{1\text{-}2}$ for the nest closer to the surface.

COMMENT:

Note that the conductivity of fur we used is for the direction perpendicular to the fibers and this is lower than what is expected along the fiber (because the fibers have a higher conductivity than the air filling the space between the fibers). Also note that for $L \ll R_a$, the results of Tables 3.2 and 3.3(a), for R_k, are identical (as expected).

PROBLEM 3.17.FAM

GIVEN:

A spherical aluminum tank, inside radius $R_1 = 3$ m, and wall thickness $l_1 = 4$ mm, contains liquid-vapor oxygen at 1 atm pressure (Table C.26 for $T_1 = T_{lg}$). The ambient is at a temperature higher than the liquid-gas mixture. Under steady-state, at the liquid-gas surface, the heat flowing into the tank causes boil off at a rate $\dot{M}_{lg} = \dot{M}_g$. In order to prevent the pressure of the tank from rising, the gas resulting from boil off is vented through a safety valve. This is shown in Figure Pr.3.17(a). Then, to reduce the amount of boil-off vent $\dot{M}_g$(kg/s), insulation is added to the tank. First a low pressure (i.e., evacuated) air gap, extending to location $r = R_2 = 3.1$ m, is placed where the combined conduction-radiation effect for this gap is represented by a conductivity $k_a = 0.004$ W/m-K. Then a layer of low-weight pipe insulation (slag or glass, Table C.15) of thickness $l_2 = 10$ cm is added. The external surface temperature is kept constant at $T_2 = 10$℃.

Evaluate the thermal conductivity of aluminum at $T = 200$ K.

SKETCH:

Figure Pr.3.17(a) shows a tank containing cryogenic liquid oxygen and having heat leaking into the tank from its higher ambient temperature.

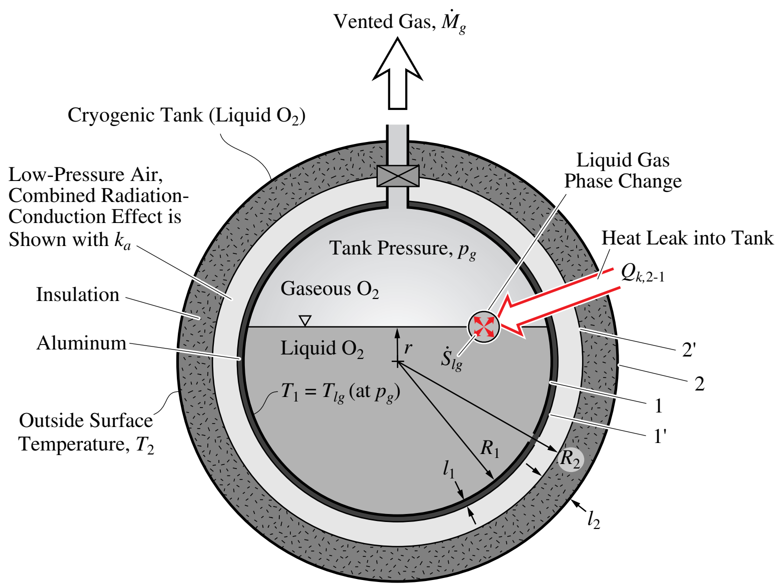


Figure Pr.3.17(a) A tank containing a cryogenic liquid and having heat leak to it from a higher temperature ambient.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine the rate of heat leak $Q_{k,2-1}$.
(c) Determine the amount of boil off $\dot{M}_g$.
(d) Determine the temperature at the inner-surface ($r = R_2$) of the insulation T_2.

SOLUTION:

(a) The thermal circuit diagram for this heat flow is shown in Figure Pr.3.17(b). The temperature at the inner surface of the insulation layer is labeled as $T_{2'}$, and the outer surface of the aluminum shell as $T_{1'}$.

(b) From the diagram, we have

$$Q_{k,2\text{-}1} = \frac{T_2 - T_1}{R_{k,1\text{-}1'} + R_{k,1'\text{-}2'} + R_{k,2'\text{-}2}}.$$

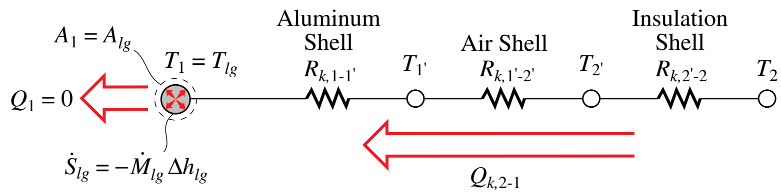


Figure Pr.3.17(b) Thermal circuit diagram.

From Table 3.2, we have for a spherical shell

$$
\begin{aligned}
R_{k,1\text{-}1'} &= \frac{\dfrac{1}{R_1} - \dfrac{1}{R_1 + l_1}}{4\pi k_{Al}} \\
R_{k,1'\text{-}2'} &= \frac{\dfrac{1}{R_1 + l_1} - \dfrac{1}{R_{2'}}}{4\pi k_a} \\
R_{k,2'\text{-}2} &= \frac{\dfrac{1}{R_{2'}} - \dfrac{1}{R_{2'} + l_2}}{4\pi k_i}.
\end{aligned}
$$

From Table C.26, we have

$$
\begin{aligned}
T_1 = T_{lg} &= 90.18 \text{ K} \\
\Delta h_{lg} &= 2.123 \times 10^5 \text{ J/kg} \qquad \text{Table C.26.}
\end{aligned}
$$

From Table C.14, we have (at $T = 200$ K)

$$k_{Al} = 237 \text{ W/m-K} \qquad \text{Table C.14.}$$

From Table C.15, we have (for low weight pipe insulation)

$$k_i = 0.033 \text{ W/m-K} \qquad \text{Table C.15.}$$

Using the numerical values, we have

$$
\begin{aligned}
R_{k,1\text{-}1'} &= \frac{\dfrac{1}{3(\text{m})} - \dfrac{1}{(3 + 0.004)(\text{m})}}{4\pi \times 237(\text{W/m-K})} = 1.490 \times 10^{-7} \text{ K/W} \\
R_{k,1'\text{-}2'} &= \frac{\dfrac{1}{(3 + 0.004)(\text{m})} - \dfrac{1}{3.1(\text{m})}}{4\pi \times 0.004(\text{W/m-K})} = 2.052 \times 10^{-1} \text{ K/W} \\
R_{k,2'\text{-}2} &= \frac{\dfrac{1}{3.1(\text{m})} - \dfrac{1}{(3.1 + 0.1)(\text{m})}}{4\pi \times 0.033(\text{W/m-K})} = 2.432 \times 10^{-2} \text{ K/W}.
\end{aligned}
$$

The largest resistance is that of low-pressure air.
Then

$$
\begin{aligned}
Q_{k,2\text{-}1} &= \frac{[(10 + 273.15) - 90.18](\text{K})}{(1.490 \times 10^{-7} + 2.052 \times 10^{-1} + 2.432 \times 10^{-2})(\text{K/W})} \\
&= \frac{192.97}{2.295 \times 10^{-1}}(\text{W}) = 8.408 \times 10^2 \text{ W}.
\end{aligned}
$$

(c) The boil off is determined from the energy equation for the surface T_1. With no other surface heat transfer for surface node T_1, we have (3.87) as

$$Q_1 = Q_{k,2\text{-}1} = -\dot{S}_{lg} \qquad \text{energy equation for node } T_1,$$

where from Table 2.1,

$$\dot{S}_{lg} = -\dot{m}_{lg} A_{lg} \Delta h_{lg} \qquad \text{energy conversion by phase change.}$$

There is a minus sign because heat is absorbed during evaporation. Then from the above two equations, we have

$$\dot{M}_{lg} = \frac{Q_{k,2\text{-}1}}{\Delta h_{lg}}.$$

Using the numerical values, we have

$$\dot{M}_{lg} = \frac{8.408 \times 10^2 (\text{W})}{2.123 \times 10^5 (\text{J/kg})} = 3.960 \times 10^{-3} \text{ kg/s} = 3.960 \text{ g/s}.$$

(d) The temperature $T_{2'}$, as shown in Figure Pr.3.17(b), is found by the thermal circuit diagram, i.e.,

$$Q_{k,2\text{-}1} = Q_{k,2\text{-}2'} = \frac{T_2 - T_{2'}}{R_{k,2'\text{-}2}}$$

or

$$\begin{aligned} T_{2'} &= T_2 - Q_{k,2\text{-}2'} R_{k,2'\text{-}2} \\ &= 10(^\circ\text{C}) - 8.408 \times 10^2 (\text{W}) \times 2.432 \times 10^{-2} (^\circ\text{C/W}) \\ &= 10(^\circ\text{C}) - 20.45(^\circ\text{C}) = -10.45^\circ\text{C}. \end{aligned}$$

COMMENT:

This heat leak rate, $Q_{k,2\text{-}1} = 840.8$ W, is considered large. Additional insulation is required to reduce the heat leak rate.

PROBLEM 3.18.FAM

GIVEN:

A teacup is filled with water having temperature $T_w = 90$℃. The cup is made of (i) porcelain (Table C.15), or (ii) stainless steel 316. The cup-wall inside diameter is R and its thickness is L. These are shown in Figure Pr.3.18(a). The water is assumed to be well mixed and at a uniform temperature. The ambient air is otherwise quiescent with a far-field temperature of $T_{f,\infty}$, and adjacent to the cup the air undergoes a thermobuoyant motion resulting a surface-convection resistance R_{ku}.

$T_{f,\infty} = 20$℃, $L = 3$ mm, $A_{ku}R_{ku} = 10^{-3}$K/(W/m^2).

Use $L \ll R$ to approximate the wall as a slab and use $A_{ku} = A_k$.

SKETCH:

Figure Pr.3.18(a) shows the cup wall, the uniform water temperature T_w, the far-field temperature $T_{f,\infty}$, and the surface convection resistance R_{ku}.

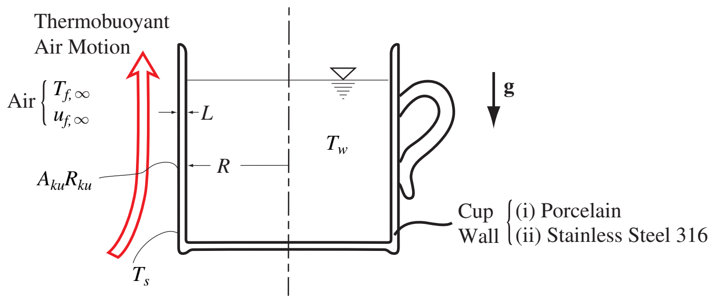


Figure Pr.3.18(a) A cup filled with hot water. The cup is made of (i) porcelain, or (ii) stainless steel 316. The ambient air is otherwise quiescent with a thermobuoyant motion adjacent to the cup wall.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the cup outside surface temperature T_s for cases (i) and (ii).

SOLUTION:

(a) The thermal circuit diagram for the heat flowing through the cup wall is shown in Figure Pr.3.18(b).

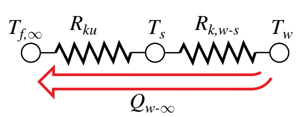


Figure Pr.3.18(b) Thermal circuit diagram.

(b) To determine T_s, we note that the heat flows through the cup wall and the adjacent thermobuoyant-motion resistance, as shown Figure Pr.3.18(b). Then

$$Q_{w\text{-}\infty} = \frac{T_w - T_s}{R_{k,w\text{-}s}} = \frac{T_s - T_{f,\infty}}{R_{ku}},$$

or solving for T_s, we have

$$R_{ku}(T_w - T_s) + R_{k,w\text{-}s}(T_{f,\infty} - T_s) = 0$$

or

$$T_s = \frac{R_{ku}T_w + R_{k,w\text{-}s}T_{f,\infty}}{R_{ku} + R_{k,w\text{-}s}}.$$

Now, from Table 3.2, we have (for $L \ll R$)

$$R_{k,w\text{-}s} = \frac{L}{A_k k_w}.$$

Then using $A_{ku} = A_k$, we have

$$T_s = \frac{A_{ku}R_{ku}T_w + \dfrac{L}{k_w}T_{f,\infty}}{A_{ku}R_{ku} + \dfrac{L}{k_w}}.$$

Now, from Tables C.15 and C.16, we have

$$\begin{aligned} \text{(i) porcelain: } k_w &= 1.5 \text{ W/m-K} && \text{Table C.15} \\ \text{(ii) stainless steel 316: } k_w &= 13 \text{ W/m-K} && \text{Table C.16.} \end{aligned}$$

Determining T_s, we have

$$\begin{aligned}
\text{(i) } T_s &= \frac{10^{-3}[\text{K/(W/m}^2)] \times (90 + 273.15)(\text{K}) + \dfrac{3 \times 10^{-3}(\text{m})}{1.5(\text{W/m-K})} \times (20 + 273.15)(\text{K})}{(10^{-3} + 3 \times 10^{-3}/1.5)[\text{K/(W/m}^2)]} \\
T_s &= \frac{(3.632 \times 10^{-1} + 5.863 \times 10^{-1})}{3.000 \times 10^{-3}} = 316.5 \text{ K} = 43.35^\circ\text{C} \\
\text{(ii) } T_s &= \frac{10^{-3}[\text{K/(W/m}^2)] \times (90 + 273.15)(\text{K}) + \dfrac{3 \times 10^{-3}(\text{m})}{13(\text{W/m-K})} \times (20 + 273.15)(\text{K})}{(10^{-3} + 3 \times 10^{-3}/13)[\text{K/(W/m}^2)]} \\
&= \frac{3.632 \times 10^{-1} + 6.765 \times 10^{-2}}{10^{-3} + 2.308 \times 10^{-4}} = 350.1 \text{ K} = 76.92^\circ\text{C}.
\end{aligned}$$

COMMENT:

The temperature sensor at the surface of the human fingers would sense $T_s = 43.35^\circ\text{C}$ as warm and tolerable and $T_s = 76.92^\circ\text{C}$ as hot and intolerable.

PROBLEM 3.19.FAM

GIVEN:

Gaseous combustion occurs between two plates, as shown in Figure Pr.3.19(a). The energy converted by combustion $\dot{S}_{r,c}$ in the gas flows through the upper and lower bounding plates. The upper plate is used for surface radiation heat transfer and is made of solid alumina (Table C.14). The lower plate is porous and is made of silica (Table C.17, and include the effect of porosity). The porosity $\epsilon = 0.3$ and the randomly distributed pores are filled with air (Table C.22, use $T = T_{s,2}$). Each plate has a length L, a width w, and a thickness l. The outsides of the two plates are at temperatures $T_{s,1}$ and $T_{s,2}$.

$\dot{S}_{r,c} = 10^4$ W, $T_{s,1} = 1{,}050$°C, $T_{s,2} = 500$°C, $L = 0.3$ m, $w = 0.3$ m, $l = 0.02$ m.

SKETCH:

Figure Pr.3.19(a) shows combustion occurring between two plates, one plate is a conductor and the other an insulator.

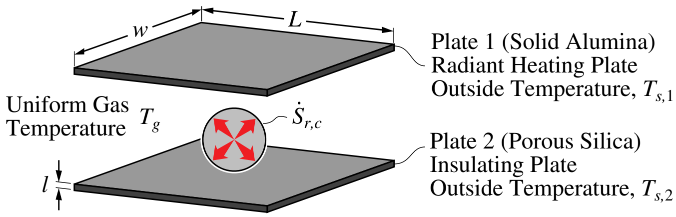


Figure Pr.3.19(a) Combustion between two plates; one plate is a conductor while the other is an insulator.

OBJECTIVE:

(a) Draw the steady-state thermal circuit diagram.
(b) Determine the effective conductivity of the lower plate.
(c) Determine the uniform gas temperature T_g.
(d) Determine the fraction of heat flow through each plate.

SOLUTION:

The gas temperature is spatially uniform and the inner walls of plates 1 and 2 are at T_g. There is 1-D conduction through the plates.

(a) The thermal circuit diagram is shown in Figure Pr.3.19(b).

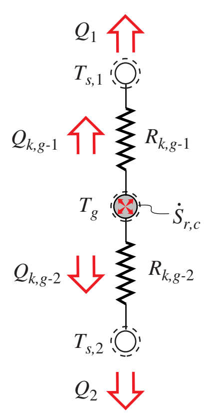


Figure Pr.3.19(b) Thermal circuit diagram.

(b) The lower plate has a $\epsilon = 0.3$ and consists of solid silica with randomly distributed pores filled with air. Since the combustion is our source of heat, we expect $T_{s,2}$ to be the lowest temperature in plate 2. For lack of a more appropriate value, we evaluate all the properties of the plate at this temperature. We also assume that the air occupying the pore space is in thermal equilibrium with the solid and thus evaluate the gas properties at this temperature as well. Therefore, we have at $T_{s,2} = 500℃ = 773.15$ K, (Tables C.17 and C.22)

$$\begin{aligned} k_s &= 1.38 \text{ W/m-K} \quad \text{(at T=293 K, only data available)} && \text{Table C.17} \\ k_g &= 0.0544 \text{ W/m-K} \quad \text{by interpolation} && \text{Table C.22.} \end{aligned}$$

Using Equation 3.28 for the effective conductivity of a random porous medium, we have

$$\begin{aligned} \frac{k_s}{k_g} &= \frac{1.38(\text{W/m-K})}{0.0544(\text{W/m-K})} = 25.37 \\ \frac{\langle k \rangle}{k_g} &= \left(\frac{k_s}{k_g}\right)^{0.280-0.757\ \log\epsilon - 0.057\ \log(k_s/k_f)} \\ \langle k \rangle &= (0.0544\text{W/m-K})(25.37)^{0.280-0.757\ \log(0.3)-0.057\ \log(25.37)} \\ \langle k \rangle &= 0.373 \text{ W/m-K.} \end{aligned}$$

(c) Applying the conservation of energy around the gas node in the figure for steady state conditions, we have

$$\begin{aligned} Q_{k,g\text{-}1} + Q_{k,g\text{-}2} &= -(\rho c)_g V \frac{dT_g}{dt} + \dot{S}_{r,c} \\ \frac{T_g - T_{s,1}}{R_{k,1}} + \frac{T_g - T_{s,2}}{R_{k,2}} &= \dot{S}_{r,c}, \end{aligned}$$

where plate 1 is assumed to be at $T_{s,1} = 1{,}323$ K, at which $k_1 = k_{Al_2O_3} = 5.931$. Then, the thermal conduction resistance is

$$\begin{aligned} R_{k,1} &= \frac{\ell}{k_1 A} = \frac{\ell}{k_1 L w} = \frac{0.02(\text{m})}{5.931(\text{W/m-K}) \times (0.3 \times 0.3)(\text{m})^2} = 0.0375℃/\text{W} \\ R_{k,2} &= \frac{\ell}{k_2 A} = \frac{\ell}{\langle k \rangle L w} = \frac{0.02(\text{m})}{0.373(\text{W/m-K}) \times (0.3 \times 0.3)(\text{m})^2} = 0.596℃/\text{W.} \end{aligned}$$

Substituting these values into the conservation of energy equation and solving for T_g gives

$$\begin{aligned} \frac{T_g - 1{,}323.15(\text{K})}{0.037(℃/\text{W})} + \frac{T_g - 773.15(\text{K})}{0.596(℃/\text{W})} &= 10^4 \text{ W} \\ T_g &= 1{,}643 \text{ K.} \end{aligned}$$

(d) The conservation of energy equation says that the fraction of energy going through the top plate $Q_{k,g\text{-}1}$ plus the fraction of energy going through the bottom plate $Q_{k,g\text{-}2}$ is equal to the amount of energy being generated $\dot{S}_{r,c}$. Therefore,

$$\begin{aligned} \frac{Q_{k,g\text{-}1}}{\dot{S}_{r,c}} &= \frac{\frac{T_g - T_{s,1}}{R_{k,1}}}{\dot{S}_{r,c}} = \frac{8539.7(\text{W})}{10{,}000(\text{W})} = 0.854 \text{ of the energy generated flows through the top plate} \\ \frac{Q_{k,g\text{-}2}}{\dot{S}_{r,c}} &= \frac{\frac{T_g - T_{s,2}}{R_{k,2}}}{\dot{S}_{r,c}} = \frac{1460.13(\text{W})}{10{,}000(\text{W})} = 0.146 \text{ of the energy generated flows through the bottom plate.} \end{aligned}$$

COMMENT:

The ratio of the two heat flow rates is 5.85. This can be further improved by increasing the porosity and thickness of the insulation.

PROBLEM 3.20.FAM

GIVEN:

In IC engines, during injection of liquid fuel into the cylinder, it is possible for the injected fuel droplets to form a thin liquid film over the piston. The heat transferred from the gas above the film and from the piston beneath the film causes surface evaporation. This is shown in Figure Pr.3.20(a). The liquid-gas interface is at the boiling temperature, T_{lg}, corresponding to the vapor pressure. The heat transfer from the piston side is by one-dimensional conduction through the piston and then by one-dimensional conduction through the thin liquid film. The surface-convection heat transfer from the gas side to the surface of the thin liquid film is prescribed as Q_{ku}.

$Q_{ku} = -13{,}500$ W, $\Delta h_{lg} = 3.027 \times 10^5$ J/kg (octane at one atm pressure, Table C.4), $k_l = 0.083$ W/m-K (octane at 360 K, Table C.13), $T_{lg} = 398.9$ K (octane at 1 atm pressure, Table C.4), $\rho_l = 900$ kg/m^3, $k_s = 236$ W/m-K (aluminum at 500 K, Table C.14), $T_1 = 500$ K, $L = 3$ mm, $l = 0.05$ mm, $D = 12$ cm.

SKETCH:

Figure Pr.3.20(a) shows the liquid film being heated by surface convection and by substrate conduction.

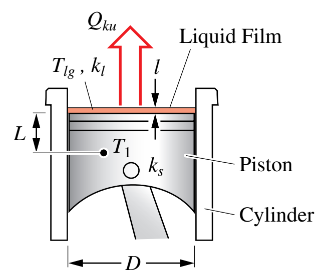


Figure Pr.3.20(a) An IC engine, showing liquid film formation on top of the piston.

OBJECTIVE:

(a) Draw the thermal circuit diagram and write the corresponding energy equation for the liquid-gas interface.
(b) For the conditions given, determine the rate of evaporation of the liquid film, $\dot{M}_{lg}$(kg/s).
(c) Assuming that this evaporation rate remains constant, determine how long it will take for the liquid film to totally evaporate.

SOLUTION:

(a) The thermal circuit is shown in Figure Pr.3.20(b).

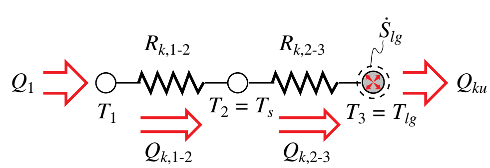


Figure Pr.3.20(b) Thermal circuit diagram.

The energy equation for node T_3 is

$$Q_{ku} - Q_{k,1\text{-}3} = \dot{S}_{lg}$$
$$Q_{ku} - \frac{T_1 - T_3}{R_{k,\Sigma}} = -\dot{M}_{lg}\Delta h_{lg}.$$

(b) $\dot{M}_{lg}$ will increase as the film thickness decreases, since $R_{k,2\text{-}3}$ decreases. For the conditions given, we assume a quasi-steady state. Then we have from Figure Pr.3.20(b) for $R_{k,\Sigma}$

$$\begin{aligned} R_{k,\Sigma} &= R_{k,1\text{-}2} + R_{k,2\text{-}3} \\ &= \frac{L}{k_s A} + \frac{l}{k_l A} = \frac{0.003(\text{m})}{236(\text{W/m-K}) \times \frac{\pi \times 0.12^2}{4}(\text{m}^2)} + \frac{0.00005(\text{m})}{0.083(\text{W/m-K}) \times \frac{\pi \times 0.12^2}{4}(\text{m}^2)} \\ &= 0.001124(\text{K/W}) + 0.05326(\text{K/W}) = 0.0544 \text{ K/W}. \end{aligned}$$

Therefore, from the energy equation we have

$$-13{,}500(\text{W}) - \frac{(500 - 398.9)(\text{K})}{0.0544(\text{K/W})} = -\dot{M}_{lg} \times 3.027 \times 10^5(\text{J/kg}).$$

Then

$$\dot{M}_{lg} = 0.051 \text{ kg/s}.$$

(c) From (b) we noted that $\dot{M}_{lg}$ will increase as the film thickness decreases. If we assume $\dot{M}_{lg}$ to be constant, we can find an upper limit to the amount of time it would take to completely evaporate the liquid film. Then

$$\begin{aligned} \dot{M}_{lg} &= \frac{dM}{dt} = \text{constant} \\ &= \frac{M_i - M_f}{t_i - t_f} = \frac{M_i - 0}{\Delta t} = \frac{\rho_l V}{\Delta t} = \frac{\rho_l A l}{\Delta t} \\ \Delta t &= \frac{\rho_l A l}{\dot{M}_{lg}} = \frac{900(\text{kg/m}^3) \times \frac{\pi \times 0.12^2}{4}(\text{m}^2) \times 0.00005(\text{m})}{0.051(\text{kg/s})} = 0.01 \text{ s}. \end{aligned}$$

COMMENT:

The heat conduction from the piston is only a small fraction of the heat supplied to the liquid film.

PROBLEM 3.21.FUN

GIVEN:

A two-dimensional, periodic porous structure has the solid distribution shown in Figure Pr.3.21(a). This is also called a regular lattice. The steady-state two-dimensional conduction can be shown with a one-dimensional, isotropic resistance for the case of $k_A \ll k_B$.

Use a depth w (length perpendicular to the page).

SKETCH:

Figure Pr.3.21(a) shows the solid geometry which is a continuous zig-zag arm of thickness l in a periodic structure with length L between each arm.

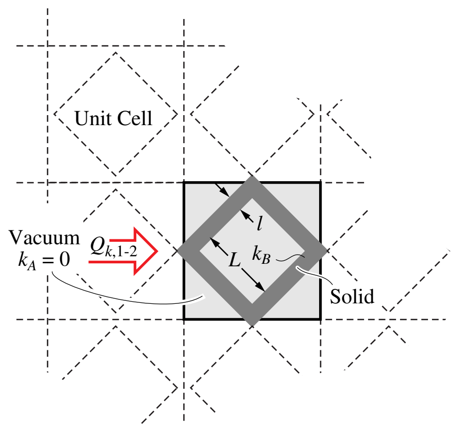


Figure Pr.3.21(a) A two-dimensional, periodic structure composite with material A having a conductivity much smaller than B.

OBJECTIVE:

(a) Draw the thermal circuit model.

(b) Show that for $k_A/k_B \ll 1$, the effective thermal conductivity $\langle k \rangle$ is

$$\frac{\langle k \rangle}{k_B} = 1 - \epsilon^{1/2}, \qquad \frac{k_A}{k_B} \ll 1,$$

where ϵ is the porosity (void fraction) defined by (3.27).

SOLUTION:

(a) Figure Pr.3.21(b) shows the thermal circuit model for the unit cell.

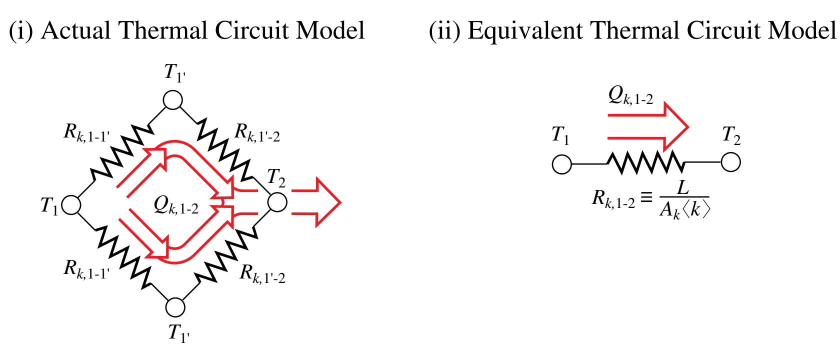


Figure Pr.3.21(b) Thermal circuit diagram and the equivalent circuit.

(b) The overall conduction resistance $R_{k,\Sigma}$ for this circuit is, based on (3.78) and (3.82),

$$\begin{aligned}\frac{1}{R_{k,1\text{-}2}} &= \frac{1}{R_{k,1\text{-}1'}+R_{k,1'\text{-}2}}+\frac{1}{R_{k,1\text{-}1'}+R_{k,1'\text{-}2}}\\ &= \frac{2}{R_{k,1\text{-}1'}+R_{k,1'\text{-}2}},\end{aligned}$$

where

$$\begin{aligned}A_k &= lw\\ R_{k,1\text{-}1\prime} &= R_{k,1'\text{-}2}=\frac{L+l}{lwk_B}\\ \frac{1}{R_{k,1\text{-}1\prime}} &= \frac{lwk_B}{L+l}.\end{aligned}$$

The porosity (3.27) can be shown to be

$$\epsilon=\frac{V_A}{V_A+V_B}=\frac{2L^2}{\left[\dfrac{2(L+l)}{2^{1/2}}\right]^2}=\frac{L^2}{(L+l)^2}.$$

Now using

$$R_{k,1\text{-}2}\equiv\frac{L_1}{L_1w\langle k\rangle}=\frac{1}{w\langle k\rangle}=\frac{L+l}{lwk_B},$$

where L is the arm length.

Finally, we have

$$\frac{\langle k\rangle}{k_B}=\frac{l}{L+l}=1-\epsilon^{1/2}.$$

COMMENT:

Note that for $\epsilon=0$, i.e., $L=0$, we recover $\langle k\rangle=k_B$ and for $\epsilon=1$, i.e., $l=0$, recover $\langle k\rangle=0$ (no heat transfer through vacuum). Also note that this effective resistance is a combination of series and parallel resistance.

PROBLEM 3.22.FUN

GIVEN:

The effective thermal conductivity of two-dimensional, periodic-structure (i.e., regular lattice) composites can be estimated using one-dimensional resistance models. Figure Pr.3.22 shows a simple, two-dimensional unit cell with material B being continuous and material A being the inclusion [similar to the three-dimensional, periodic structure of Section 3.3.2(C)].

Use only the porosity (void fraction) ϵ and the conductivities k_A and k_B. Use a depth w (length perpendicular to the page).

SKETCH:

Figure 3.22(a) shows the composite with material B being continuous and material A being the inclusion.

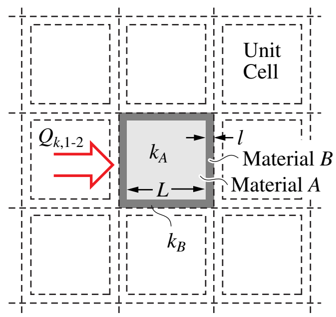


Figure Pr.3.22(a) A two-dimensional, periodic structure with material B being continuous.

OBJECTIVE:

(a) Derive an expression for the effective conductivity $\langle k \rangle$ of this composite using a series-parallel arrangement of resistances.

(b) Derive an expression for the effective conductivity $\langle k \rangle$ of this composite using a parallel-series arrangement of resistances.

(c) Show that for the case of $k_A/k_B \ll 1$, the result for the parallel-series arrangement is

$$\frac{\langle k \rangle}{k_B} = 1 - \epsilon^{1/2},$$

where ϵ is the porosity (this result is also obtained in Problem 3.34).

SOLUTION:

The series-parallel and parallel-series resistance arrangements of the Figure Pr.3.22(a) structure are shown in Figure Pr.3.22(b).

(a) For the circuit shown in Figure Pr.3.22(b)(i), we have

$$R_{k,1\text{-}2} = R_{k,1\text{-}1',B} + \frac{1}{\dfrac{1}{R_{k,1'\text{-}2',B}} + \dfrac{1}{R_{k,1'\text{-}2',A}}} + R_{k,2'\text{-}2,B}.$$

The individual resistances are

$$\begin{aligned} R_{k,1\text{-}1',B} &= R_{k,2'\text{-}2,B} = \frac{l}{(L+2l)wk_B} \\ R_{k,1'\text{-}2',A} &= \frac{L}{Lwk_A} \\ R_{k,1'\text{-}2',B} &= \frac{L}{2lwk_B}. \end{aligned}$$

The porosity of the structure is defined as

$$\epsilon = \frac{V_A}{V_A + V_B} = \frac{L^2}{(L+2l)^2}.$$

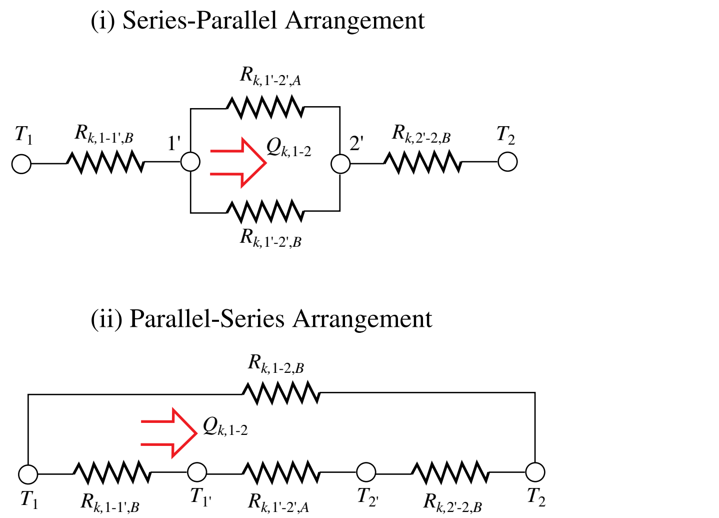


Figure Pr.3.22(b) Thermal circuit diagram for the two resistance arrangements.

Then

$$
\begin{aligned}
R_{k,1\text{-}2} &\equiv \frac{L+2l}{(L+2l)w\langle k\rangle} \\
&= \frac{L}{2lwk_B + Lwk_A} + \frac{2l}{(L+2l)wk_B}.
\end{aligned}
$$

From these, we have

$$
\frac{k_B}{\langle k\rangle} = \frac{\epsilon^{1/2}}{1-\epsilon^{1/2}+\epsilon^{1/2}(k_A/k_B)} + 1 - \epsilon^{1/2}.
$$

Note that this does not lead to $\langle k\rangle/k_B = 1-\epsilon^{1/2}$ for the case of $k_A \ll k_B$.

(b) For the circuit shown in Figure Pr.3.22(b)(ii), we have

$$
R_{k,1\text{-}2} = \frac{1}{R_{k,1\text{-}1',B} + R_{k,1'\text{-}2',A} + R_{k,2'\text{-}2,B}} + \frac{1}{R_{k,1\text{-}2,B}}.
$$

The individual resistances are

$$
\begin{aligned}
R_{k,1\text{-}1',B} &= \frac{l}{Lwk_B} \\
R_{k,1'\text{-}2',A} &= \frac{L}{Lwk_A} \\
R_{k,2'\text{-}2,B} &= \frac{l}{Lwk_B} \\
R_{k,1\text{-}2,B} &= \frac{L+2l}{2lwk_B}.
\end{aligned}
$$

The porosity is the same as determined above. Then

$$
\begin{aligned}
\frac{1}{R_{k,1\text{-}2}} &= \frac{L+2l}{(L+2l)w\langle k\rangle} \\
&= \frac{1}{\dfrac{2l}{Lwk_B} + \dfrac{L}{Lwk_A}} + \frac{2lwk_B}{L+2l} \\
&= \frac{k_B\epsilon^{1/2}}{\epsilon^{1/2} + (1-\epsilon^{1/2})(k_A/k_B)} + (1-\epsilon)^{1/2}.
\end{aligned}
$$

From this, we have

$$\frac{k_B}{\langle k\rangle} = \frac{\epsilon^{1/2} + (1-\epsilon^{1/2})(k_A/k_B)}{(1-\epsilon^{1/2})\epsilon^{1/2} + \epsilon^{1/2}(k_A/k_B) + (1-\epsilon^{1/2})^2(k_A/k_B)}.$$

(c) For the case of the parallel-series arrangement, and for $k_A/k_B \ll 1$, the above equation becomes

$$\frac{k_B}{\langle k\rangle} = \frac{1}{1-\epsilon^{1/2}} \quad \text{or} \quad \frac{\langle k\rangle}{k_B} = 1-\epsilon^{1/2}.$$

COMMENT:

The actual, two-dimensional heat flow results in an effective resistance that is between these two effective resistances. As k_A/k_B becomes closer to unity, the difference between the models decreases and vice versa for k_A/k_B far from unity.

Note that the three-dimensional parallel-series solution given by (3.86) gives, for $k_A \ll k_B$, $\langle k\rangle/k_B = (1-\epsilon^{2/3})/(1-\epsilon^{2/3}+\epsilon)$. This gives a higher $\langle k\rangle/k_B$, compared to the two-dimensional result.

PROBLEM 3.23.FUN

GIVEN:

In the one-dimensional, steady-state conduction treatment of Section 3.3.1, for planar geometries, we assumed a constant cross-sectional area A_k. In some applications, although the conduction is one-dimensional and cross section is planar, the cross-sectional area is not uniform. Figure Pr.3.23 shows a rubber-leg used for the vibration isolation and thermal insulation of a cryogenic liquid container. The rubber stand is in the form of truncated cone [also called a frustum of right cone, a geometry considered in Table C.1(e)].

Note that $\Delta V = \pi R^2(x)\Delta x$, as $\Delta x \rightarrow 0$.

$T_2 = 20$℃, $T_1 = 0$℃, $L_2 = 4$ cm, $L_1 = 10$ cm, $R_1 = 1.5$ cm, $k = 0.15$ W/m-K.

SKETCH:

Figure Pr.3.23 shows the rubber leg, its geometry and parameters, and the one-dimensional heat conduction.

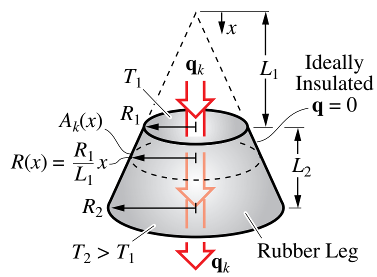


Figure Pr.3.23 One-dimensional, steady-state heat conduction in a variable area rubber leg.

OBJECTIVE:

(a) Starting from (3.29), with $\dot{s} = 0$, use a variable circular conduction area $A_k(x) = \pi R^2(x)$, while $R(x)$ varies linearly along the x axis, i.e.,

$$R(x) = \frac{R_1}{L_1}x,$$

as shown in Figure Pr.3.23. Then derive the expression for the temperature distribution $T = T(x)$.

(b) Using this temperature distribution, determine $Q_{k,1\text{-}2}$ and $R_{k,1\text{-}2}$, by using (3.46) and noting that there are no lateral heat losses.

(c) Evaluate $Q_{k,1\text{-}2}$, for the conditions given below.

(d) Use a constant surface area with $\langle R \rangle = (R_1 + R_2)/2$, and the conduction resistance for a slab, and compare $Q_{k,1\text{-}2}$ with that from part (c).

SOLUTION:

(a) Starting from (3.29) with $\dot{s} = 0$, we use $\Delta V = \pi R^2(x)\Delta x$, and the result is

$$\frac{\displaystyle\int_{\Delta A} (\mathbf{q}_k \cdot \mathbf{s}_n) dA}{\Delta V \rightarrow 0} = \frac{(q_k A_k)_{x+\Delta x} - (q_k A_k)_x}{\pi R^2(x)\Delta x} = 0.$$

Using $A_k = \pi R^2(x)$ and $R(x) = R_1 x/L_1$, we have

$$\frac{\pi \left(\frac{R_1}{L_1}\right)^2 \left[(q_k x^2)_{x+\Delta x} - (q_k x^2)_x\right]}{\pi R^2(x)\Delta x} = 0$$

or

$$\frac{(q_k x^2)_{x+\Delta x} - (q_k x^2)_x}{\Delta x \rightarrow 0} = 0$$

or

$$\frac{d(q_k x^2)}{dx} = 0.$$

Now using (1.11), and noting that $T = T(x)$, we have

$$\frac{d}{dx}\left[\left(-k\frac{dT}{dx}\right)x^2\right] = 0,$$

or

$$\frac{d}{dx}\left(\frac{dT}{dx}x^2\right) = 0.$$

Integrating once gives

$$\frac{dT}{dx}x^2 = a_1.$$

Integrating again gives

$$\begin{aligned} T &= \int \frac{a_1}{x^2}dx + a_2 \\ &= -\frac{a_1}{x} + a_2. \end{aligned}$$

Now using the thermal conditions

$$\begin{aligned} T(x = L_1) &= T_1 \\ T(x = L_1 + L_2) &= T_2, \end{aligned}$$

we have

$$\begin{aligned} T_1 &= -\frac{a_1}{L_1} + a_2 \\ T_2 &= -\frac{a_1}{L_1 + L_2} + a_2. \end{aligned}$$

Solving for a_1 and a_2, we have

$$T = T_1 + (T_2 - T_1)\frac{\dfrac{1}{x} - \dfrac{1}{L_1}}{\dfrac{1}{L_1 + L_2} - \dfrac{1}{L_1}}.$$

(b) Now noting that from (3.46) we have

$$Q_{k,x} = A_k(x)\left(-k\left.\frac{dT}{dx}\right|_x\right),$$

and differentiating $T = T(x)$, we obtain

$$Q_{k,x} = \frac{\pi R^2(x)(-k)(T_1 - T_2)}{\dfrac{1}{L_1} - \dfrac{1}{L_1 + L_2}}\left(-\frac{1}{x^2}\right).$$

Evaluating this at $x = L_2$, we have

$$Q_{k,x} = Q_{k,1\text{-}2} = \pi k\frac{R_1^2}{L_1^2}\frac{1}{\dfrac{1}{L_1} - \dfrac{1}{L_1 + L_2}}(T_1 - T_2)$$

or

$$R_{k,1\text{-}2} = \frac{\dfrac{1}{L_1} + \dfrac{1}{L_1 + L_2}}{\pi k R_1^2 / L_1^2}.$$

(c) Using the numerical values, we have

$$\begin{aligned} Q_{k,1\text{-}2} &= \pi \times 0.15(\text{W/m-K}) \times \frac{(0.015)^2(\text{m}^2)}{(0.10)^2(\text{m}^2)} \frac{1}{\dfrac{1}{0.1(\text{m})} - \dfrac{1}{0.14(\text{m})}}(0 - 20)\text{K} \\ &= -7.423 \times 10^{-2} \text{ W}. \end{aligned}$$

(d) Using a constant area with

$$\begin{aligned} \langle R \rangle &= \frac{R_1 + R_2}{2} = \frac{R_1}{2L_1}[L_1 + (L_1 + L_2)] \\ &= 0.018 \text{ m}. \end{aligned}$$

Then from Table 3.2 for a slab we have

$$Q_{k,1\text{-}2} = \pi \langle R \rangle^2 k(T_1 - T_2)/L_2 = -7.635 \times 10^{-2} \text{ W}.$$

This is very close to the results for the variable area in part (c).

COMMENT:

Note that here $R_1/L_1 = 0.15$ and as this ratio becomes smaller, the role of the variable area becomes more significant.

PROBLEM 3.24.FAM

GIVEN:

A pair of aluminum slabs with a surface roughness of $\langle\delta^2\rangle^{1/2} = 0.25\ \mu$m are placed in contact (with air as the interstitial fluid). A heat flux of $q_k = 4 \times 10^4$ W/m^2 flows across the interface of the two slabs.

OBJECTIVE:

Determine the temperature drop across the interface for contact pressures of 10^5 and 10^6 Pa.

SOLUTION:

The heat flux flowing through the contact between the two solids is related to the temperature difference across the contact by (3.95)

$$q_{k,c} = \frac{Q_{k,c}}{A_k} = \frac{\Delta T_c}{A_k R_{k,c}}.$$

From Figure 3.25, for a pair of soft aluminum surfaces in air, having root-mean-square roughness $\langle\delta^2\rangle^{1/2} = 0.25\ \mu$m, for each of the contact pressures, the contact thermal conductance is

$$p = 10^5 \text{ Pa}, \qquad 1/A_k R_{k,c} \simeq 8.2 \times 10^3 \text{ W/m}^2\text{-°C} \qquad \text{Figure 3.25}$$

$$p = 10^6 \text{ Pa}, \qquad 1/A_k R_{k,c} \simeq 1.5 \times 10^4 \text{ W/m}^2\text{-°C} \qquad \text{Figure 3.25.}$$

From Equation (3.95), for $q_{k,c} = 4 \times 10^4$ W/m^2, the temperature jumps across the contact interface for each contact pressure are

$$p = 10^5 \text{ Pa}, \qquad \Delta T_c = 4.9\text{°C}$$

$$p = 10^6 \text{ Pa}, \qquad \Delta T_c = 2.7\text{°C}.$$

COMMENT:

A high joint pressure is usually necessary to reduce the contact resistance. The use of thermal conductivity pastes and greases also reduces the contact thermal resistance. This occurs because the air present at the contact is replaced by the more conductive paste. These pastes are usually made of a polymer filled with submicron metal particles.

PROBLEM 3.25.FAM

GIVEN:

Thin, flat foil heaters are formed by etching a thin sheet of an electrical conductor such as copper and then electrically insulating it by coating with a nonconductive material. When the maximum heater temperature is not expected to be high, a polymer is used as coating. When high temperatures are expected, thin sheets of mica are used. Mica is a mineral silicate that can be cleaved into very thin layers. However, a disadvantage of the use of mica is that the mica surface offers a much higher thermal-contact resistance, thus requiring a larger joint pressure p_c.

Figure Pr.3.25(a) shows a thin circular heater used to deliver heat to a surface (surface 1). The solid between surface 1 and the heater is aluminum (Table C.14) and has a thickness $L_1 = 5$ mm. In order to direct the heat to this surface, the other side of the heater is thermally well insulated by using a very low conductivity fiber insulating board (Table C.15) with thickness $L_2 = 10$ mm. The temperature of the aluminum surface is maintained at $T_1 = 100$°C, while the outer surface of the thermal insulation is at $T_2 = 30$°C. The heater generates heat by Joule heating at a rate of $\dot{S}_{e,\mathrm{J}}/A_k = 4 \times 10^4$ W/m^2 and is operating under a steady-state condition.

Use the thermal conductivities at the temperatures given in the tables or at 300 K.

SKETCH:

Figure Pr.3.25(a) shows the layered composite with the energy conversion.

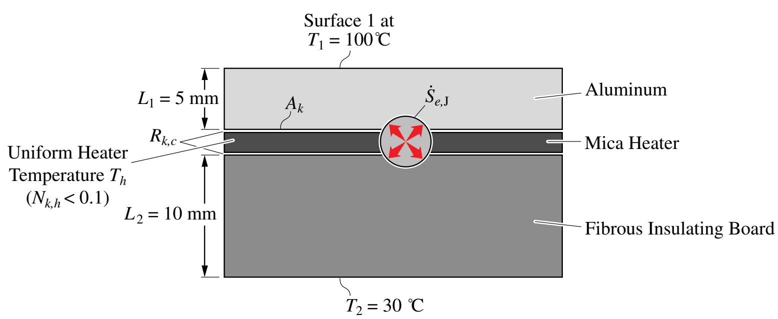


Figure Pr.3.25(a) A thin-foil heater encased in mica and placed between an aluminum and an insulation layer.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the heater temperature T_h for the case of contact resistances of (i) $A_k\ R_{k,c} = 10^{-4}$ [K/(W/m^2)], and (ii) $A_k\ R_{k,c} = 4 \times 10^{-2}$ [K/(W/m^2)].

(c) Comment on the answers obtained above if the heater is expected to fail at $T_{max} = 600$°C.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.3.25(b).

(b) To find the heater temperature, the integral-volume energy equation (3.161) is applied to the heater node T_h. Under steady-state conditions we have

$$Q_h + \sum_j \frac{T_h - T_j}{R_{k,h\text{-}j}} = \dot{S}_{e,\mathrm{J}}.$$

The temperatures T_1 and T_2 are known. Therefore, the heat transfer through the thermal resistances are written as functions of these temperatures. As $Q_h = 0$ (there is no prescribed surface heat transfer), we have

$$\frac{T_h - T_1}{(R_{k,\Sigma})_{h\text{-}1}} + \frac{T_h - T_2}{(R_{k,\Sigma})_{h\text{-}2}} = \dot{S}_{e,\mathrm{J}}.$$

For the resistances arranged in series, the overall thermal resistances $(R_{k,\Sigma})_{h\text{-}1}$ and $(R_{k,\Sigma})_{h\text{-}2}$ are

$$\begin{aligned}(R_{k,\Sigma})_{h\text{-}1} &= R_{k,c} + R_{k,i\text{-}1}\\ (R_{k,\Sigma})_{h\text{-}2} &= R_{k,c} + R_{k,i\text{-}2}.\end{aligned}$$

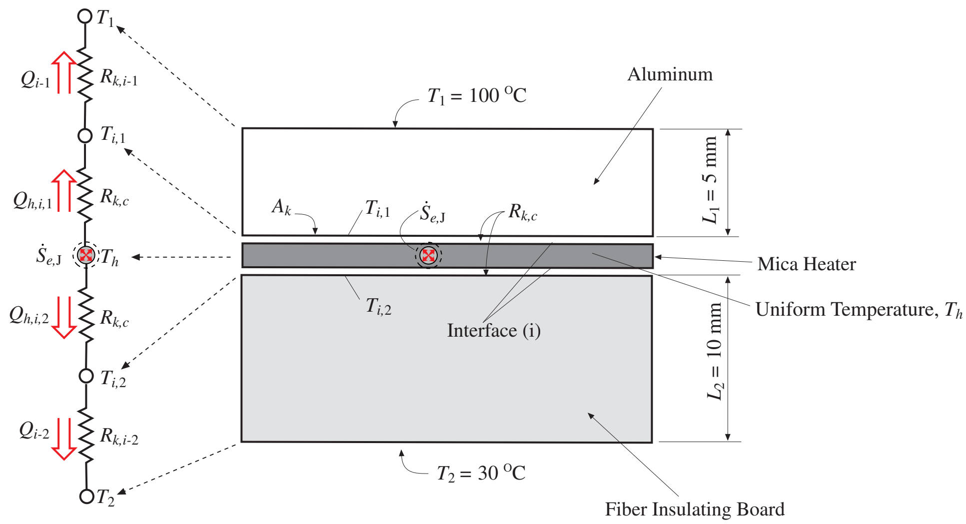


Figure Pr.3.25(b) Thermal circuit diagram.

The conduction resistances $R_{k,i\text{-}1}$ and $R_{k,i\text{-}2}$ for the slabs are

$$R_{k,i\text{-}1} = \frac{L_1}{k_1 A_k}$$
$$R_{k,i\text{-}2} = \frac{L_2}{k_2 A_k},$$

and the contact resistance $R_{k,c}$ is given in the problem statement.
The thermal conductivities are needed to calculate the thermal resistances. For each of the materials we have aluminum (Table C.14, $T = 300$ K) $k = 237$ W/m-K and fiber insulating board (Table C.15, $T = 294$ K) $k = 0.048$ W/m-K .

Solving for T_h, we have

$$T_h = \frac{\dot{S}_{e,\mathrm{J}} + \dfrac{T_1}{(R_{k,\Sigma})_{h\text{-}1}} + \dfrac{T_2}{(R_{k,\Sigma})_{h\text{-}2}}}{\dfrac{1}{(R_{k,\Sigma})_{h\text{-}1}} + \dfrac{1}{(R_{k,\Sigma})_{h\text{-}2}}}.$$

(i) For the first case

$$A_k R_{k,c} = 1 \times 10^{-4} \ ^\circ\mathrm{C}/(\mathrm{W/m^2})$$
$$A_k R_{k,i\text{-}1} = \frac{L_1}{k_1} = \frac{0.005}{237} = 2.110 \times 10^{-5} \ ^\circ\mathrm{C}/(\mathrm{W/m^2})$$
$$A_k R_{k,i\text{-}2} = \frac{L_2}{k_2} = \frac{0.01}{0.048} = 2.083 \times 10^{-1} \ ^\circ\mathrm{C}/(\mathrm{W/m^2}).$$

The equivalent resistances are

$$\begin{aligned}
A_k(R_{k,\Sigma})_{h\text{-}1} &= A_k R_{k,c} + A_k R_{k,i\text{-}1} \\
&= 1 \times 10^{-4}[^\circ\mathrm{C}/(\mathrm{W/m^2})] + 2.110 \times 10^{-5}[^\circ\mathrm{C}/(\mathrm{W/m^2})] \\
&= 1.211 \times 10^{-4} \ ^\circ\mathrm{C}/(\mathrm{W/m^2}) \\
A_k(R_{k,\Sigma})_{h\text{-}2} &= A_k R_{k,c} + A_k R_{k,i\text{-}2} \\
&= 1 \times 10^{-4}[^\circ\mathrm{C}/(\mathrm{W/m^2})] + 2.083 \times 10^{-1}[^\circ\mathrm{C}/(\mathrm{W/m^2})] \\
&= 2.084 \times 10^{-1} \ ^\circ\mathrm{C}/(\mathrm{W/m^2}).
\end{aligned}$$

Dividing all the terms by A_k and solving for T_h we have

$$\begin{aligned}
T_h &= \frac{\dfrac{\dot{S}_{e,\mathrm{J}}}{A_k} + \dfrac{T_1}{A_k(R_{k,\Sigma})_{h\text{-}1}} + \dfrac{T_2}{A_k(R_{k,\Sigma})_{h\text{-}2}}}{\dfrac{1}{A_k(R_{k,\Sigma})_{h\text{-}1}} + \dfrac{1}{A_k(R_{k,\Sigma})_{h\text{-}2}}} \\
&= \frac{4 \times 10^4(\mathrm{W/m^2}) + \dfrac{100(^\circ\mathrm{C})}{1.211 \times 10^{-4}[^\circ\mathrm{C}/(\mathrm{W/m^2})]} + \dfrac{30(^\circ\mathrm{C})}{2.084 \times 10^{-1}[^\circ\mathrm{C}/(\mathrm{W/m^2})]}}{\dfrac{1}{1.211 \times 10^{-4}[^\circ\mathrm{C}/(\mathrm{W/m^2})]} + \dfrac{1}{2.084 \times 10^{-1}[^\circ\mathrm{C}/(\mathrm{W/m^2})]}} \\
&= 105^\circ\mathrm{C}.
\end{aligned}$$

(ii) For the second case

$$A_k R_{k,c} = 4 \times 10^{-2} {}^\circ\mathrm{C}/(\mathrm{W/m^2}).$$

The conduction thermal resistances remain the same. The equivalent resistances now become

$$\begin{aligned}
A_k(R_{k,\Sigma})_{h\text{-}1} &= A_k R_{k,c} + A_k R_{k,i\text{-}1} \\
&= 4 \times 10^{-2}[^\circ\mathrm{C}/(\mathrm{W/m^2})] + 2.110 \times 10^{-5}[^\circ\mathrm{C}/(\mathrm{W/m^2})] \\
&= 4.002 \times 10^{-2}\ ^\circ\mathrm{C}/(\mathrm{W/m^2}) \\
A_k(R_{k,\Sigma})_{h\text{-}2} &= A_k R_{k,c} + A_k R_{k,i\text{-}2} \\
&= 4 \times 10^{-2}[^\circ\mathrm{C}/(\mathrm{W/m^2})] + 2.083 \times 10^{-1}[^\circ\mathrm{C}/(\mathrm{W/m^2})] \\
&= 2.483 \times 10^{-1}\ ^\circ\mathrm{C}/(\mathrm{W/m^2}).
\end{aligned}$$

Solving for T_h we have

$$\begin{aligned}
T_h &= \frac{\dfrac{\dot{S}_{e,\mathrm{J}}}{A_k} + \dfrac{T_1}{A_k(R_{k,\Sigma})_{h\text{-}1}} + \dfrac{T_2}{A_k(R_{k,\Sigma})_{h\text{-}2}}}{\dfrac{1}{A_k(R_{k,\Sigma})_{h\text{-}1}} + \dfrac{1}{A_k(R_{k,\Sigma})_{h\text{-}2}}} \\
&= \frac{4 \times 10^4(\mathrm{W/m^2}) + \dfrac{100(^\circ\mathrm{C})}{4.002 \times 10^{-2}[^\circ\mathrm{C}/(\mathrm{W/m^2})]} + \dfrac{30(^\circ\mathrm{C})}{2.483 \times 10^{-1}[^\circ\mathrm{C}/(\mathrm{W/m^2})]}}{\dfrac{1}{4.002 \times 10^{-2}[^\circ\mathrm{C}/(\mathrm{W/m^2})]} + \dfrac{1}{2.483 \times 10^{-1}[^\circ\mathrm{C}/(\mathrm{W/m^2})]}} \\
&= 1{,}469^\circ\mathrm{C}.
\end{aligned}$$

(c) The heater is rated for 600°C. Therefore, it would operate normally under case (i) (smaller contact thermal resistance), but it would fail under case (ii) (larger contact thermal resistance).

COMMENT:

Note that a small air gap present in series with other low resistance layers causes a large decrease in the heat flow rate and a large increase in the temperature drop.

PROBLEM 3.26.FAM

GIVEN:

The automobile exhaust catalytic converter (for treatment of gaseous pollutants) is generally a large surface area ceramic or metallic monolith that is placed in a stainless steel housing (also called can). Figure Pr.3.26(a) shows a ceramic (cordierite, a mineral consisting of silicate of aluminum, iron, and magnesium) cylindrical monolith that is placed inside the housing with (i) direct ceramic-stainless contact, and (ii) with a blanket of soft ceramic (vermiculite, a micacious mineral, mat) of conductivity k_b placed between them.

The blanket is placed under pressure and prevents the gas from flowing through the gap. The direct contact results in a contact resistance similar to that of stainless steel-stainless steel with $\langle\delta^2\rangle^{1/2} = 1.1$ to $1.5\ \mu$m and $p_c = 10^5$ Pa. The soft blanket (vermiculite mat) has a thickness $l_1 = 3$ mm and $k_b = 0.4$ W/m-K.

Use $T_1 = 500$℃, $T_2 = 450$℃, and stainless steel AISI 316 for thermal conductivity.

SKETCH:

Figure Pr.3.26(a) shows a catalytic converter with and without a ceramic blanket.

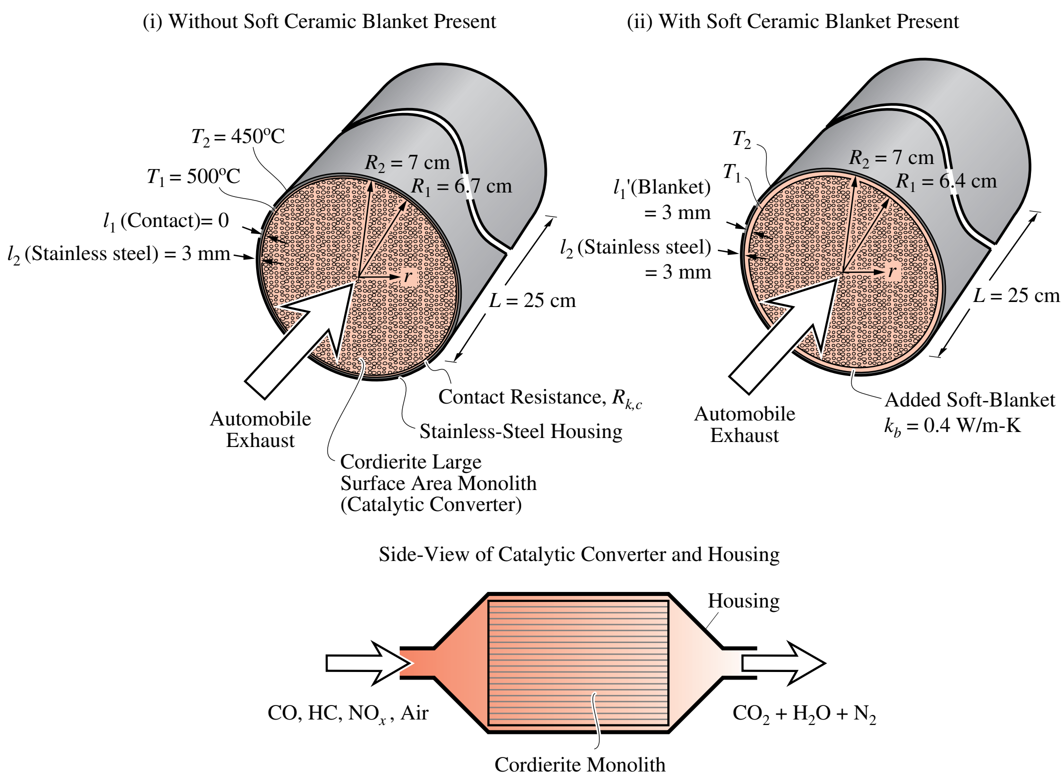


Figure Pr.3.26(a) An automobile catalytic converter (i) without and (i) with a soft ceramic blanket.

OBJECTIVE:

(a) Draw the thermal circuit diagrams.

(b) Determine the heat flow between surface at temperature T_1 and surface at temperature T_2 (i) without and (ii) with the soft ceramic blanket, i.e., $Q_{k,1-2}$.

SOLUTION:

(a) The thermal circuit diagrams for cases (i) and (ii) are shown in Figure Pr.3.26(b).

(b) The heat flow rate is written from the thermal circuit model of Figure Pr.3.26(b) as

$$(Q_{k,1\text{-}2})_{without\ blanket} = \frac{T_1 - T_2}{R_{k,c} + R_{k,1'\text{-}2}}.$$

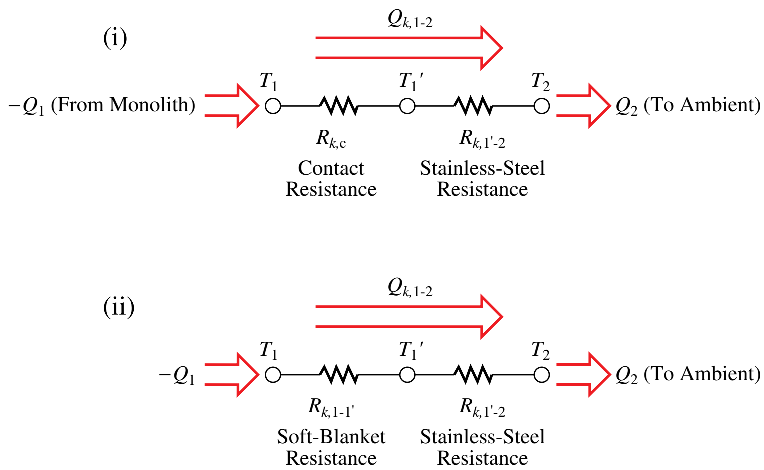


Figure Pr.3.26(b) Thermal circuit diagrams for cases (i) and (ii).

Here $R_1 = R_2 - l_2 = 6.7$ cm. From Figure 3.25, for stainless-steel contact with $\langle \delta^2 \rangle^{1/2} = 1.1$ to 1.5μm and $p_c = 10^5$ Pa, we have

$$\frac{1}{A_k R_{k,c}} = 2 \times 10^2 \ (\text{W/m}^2)/^\circ\text{C} \quad \text{Figure 3.25.}$$

The area is

$$A_k = 2\pi R_1 L.$$

The stainless-steel shell resistance is found from Table 3.2, using the geometrical designations of Figure Pr. 3.26(a), to be

$$R_{k,1'\text{-}2} = \frac{\ln \dfrac{R_2}{R_1}}{2\pi L k_s},$$

where from Table C.16, we have

$$k_s = 13 \ \text{W/m-K.} \qquad \text{Table C.16}$$

(c) From Figure Pr.3.26(b), we have

$$(Q_{k,1\text{-}2})_{with\ blanket} = \frac{T_1 - T_2}{R_{k,1\text{-}1'} + R_{k,1'\text{-}2}}.$$

Here $R_1 = R_2 - l_2 - l_1' = 6.4$ cm, $R_{k,1'\text{-}2}$ remains the same, and

$$R_{k,1\text{-}1'} = \frac{\ln \dfrac{R_{1'}}{R_1}}{2\pi L k_b}.$$

Using the numerical results, we have

$$
\begin{aligned}
R_{k,c} &= \frac{1}{2\times 10^2(\mathrm{W/m^2}-℃)\times 2\pi R_1 L}\\
&= \frac{1}{2\times 10^2(\mathrm{W/m^2}-℃)\times 2\pi\times 0.067(\mathrm{m})\times 0.25(\mathrm{m})}\\
&= 4.750\times 10^{-2}\,℃/\mathrm{W}\\
R_{k,1'\text{-}2} &= \frac{\ln\left(\dfrac{0.07\ \mathrm{m}}{0.067\ \mathrm{m}}\right)}{2\pi\times 0.25\ \mathrm{m}\times 13(\mathrm{W/m\text{-}K})}\\
&= 2.145\times 10^{-3}\,℃/\mathrm{W}\\
R_{k,1\text{-}1'} &= \frac{\ln\left(\dfrac{0.07\ \mathrm{m}}{0.064\ \mathrm{m}}\right)}{2\pi\times 0.25\ \mathrm{m}\times 0.4(\mathrm{W/m\text{-}K})}\\
&= 7.291\times 10^{-2}\,℃/\mathrm{W}.
\end{aligned}
$$

For the heat flow, we have

$$
\begin{aligned}
(Q_{k,1\text{-}2})_{without\ blanket} &= \frac{(500-450)(℃)}{(4.750\times 10^{-2}+2.145\times 10^{-3})(℃/\mathrm{W})} = 1.006\times 10^3\ \mathrm{W}\\
(Q_{k,1\text{-}2})_{with\ blanket} &= \frac{(500-450)(℃)}{(7.291\times 10^{-2}+2.145\times 10^{-3})(℃/\mathrm{W})} = 6.662\times 10^2\ \mathrm{W}.
\end{aligned}
$$

COMMENT:

The soft ceramic blanket prevents flow leaks at the housing contact and reduces the heat loss to the housing ($R_{k,1\text{-}1'}$ is larger than $R_{k,c}$).

PROBLEM 3.27.FAM

GIVEN:

A thermoelectric power generator uses the heat released by gaseous combustion to produce electricity. Since the low temperature thermoelectric materials undergo irreversible damage (such as doping migration) at temperatures above a critical temperature T_{cr}, a relatively low conductivity material (that withstands the high flame temperature; this is referred to as a refractory material) is placed between the flame and the hot junction, as shown in Figure Pr.3.27(a). Additionally, a copper thermal spreader is placed between the refractory material and the hot junction to ensure even distribution of the heat flux into the thermoelectric device. It is desired to generate 20 W of electricity from the thermoelectric module, where this power is 5% of the heat supplied $(-Q_h)$ at the hot junction T_h. The refractory material is amorphous silica with conductivity k_s. In addition, there is a contact resistance $R_{k,c}$ between the copper thermal spreader and the hot junction. The surface area of the hot junction is $a \times a$.

$T_g = 750°C$, $T_{cr} = 250°C$, $k_s = 1.36$ W/m-K, $a = 6$ cm, $A_k R_{k,c} = 10^{-4}$ K/(W/m^2).

SKETCH:

Figure Pr.3.27(a) shows the electrical power generation unit with the thermoelectric cooler and the combustion flue-gas stream. The low-conductivity (refractory) material used to lower T_h (to protect the thermoelectric module from high temperatures) is also shown.

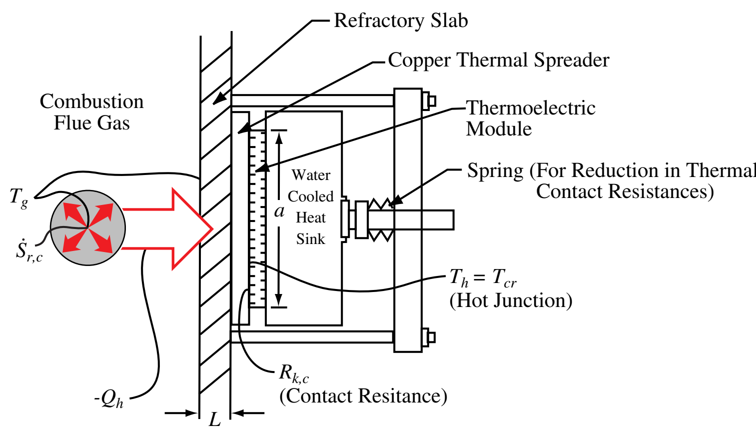


Figure Pr.3.27(a) A thermoelectric module, used for power generation, receives heat from a combustion flue gas stream. To reduce the temperature of the hot junction, a refractory slab is used.

OBJECTIVE:

(a) Draw the thermal circuit diagram for node T_h using the combustion flue gas temperature T_g.

(b) Determine the thickness of the refractory material L, such that $T_h = T_{cr}$.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.3.27(b).

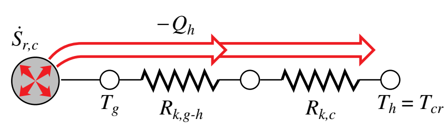


Figure Pr.3.27(b) Thermal circuit diagram.

(b) From Figure Pr.3.27(b), the expression for $-Q_h$ in terms of T_h and T_g is

$$-Q_h = \frac{T_h - T_g}{R_{k,g\text{-}h} + R_{k,c}}.$$

Then from Table 3.2, for a slab, we have

$$R_{k,g\text{-}h} = \frac{L}{A_k k_s}.$$

Using this, we have

$$-Q_h = \frac{T_h - T_g}{\dfrac{L}{A_k k_s} + \dfrac{A_k R_{k,c}}{A_k}}$$

or

$$\frac{L}{A_k k_s} + \frac{A_k R_{k,c}}{A_k} = \frac{T_g - T_h}{Q_h}$$

or

$$L = \frac{T_g - T_h}{Q_h} A_k k_s - A_k R_{k,c} k_s, \quad A_k = a^2.$$

Using the numerical values and noting that

$$Q_h = \frac{P}{\eta} = \frac{20(\text{W})}{0.05} = 400 \text{ W},$$

we have

$$\begin{aligned} L &= \frac{(750-250)(\text{K})}{400(\text{W})} \times (0.06)^2(\text{m}^2) \times 1.36(\text{W/m-K}) - 10^{-4}[\text{K}/(\text{W/m}^2)] \times 1.36(\text{W/m-K}) \\ &= 6.120 \times 10^{-3}(\text{m}) - 1.36 \times 10^{-4}(\text{m}) = 5.984 \times 10^{-3} \text{ m} = 5.984 \text{ mm}. \end{aligned}$$

COMMENT:

Since active cooling of the cold junction is needed to maintain a low T_c, most of the heat arriving at the hot junction is transferred to the cold junction by conduction. This is the reason for the low efficiency.

PROBLEM 3.28.FAM

GIVEN:

In order to reduce the contact resistance, the contacting solids are physically bonded by using high temperatures (as in fusion or sintering) or by using material deposition (as in physical vapor deposition or solidification of melts). This creates a contact layer which has a contact thickness L_c (that is nearly twice the rms roughness) and a contact conductivity k_c. This contact conductivity is intermediate between the conductivity of the joining materials A and B (with $k_A < k_B$), i.e.,

$$k_A \leq k_c \leq k_B \quad \text{for} \quad k_A < k_B.$$

Consider a contact resistance between a bismuth telluride slab (material A) and copper (material B) slab. This pair is used in thermoelectric coolers, where the semiconductor, doped bismuth telluride is the thermoelectric material and copper is the electrical connector. Then use a general relationship $k_c = k_A + a_1(k_B - k_A)$, $0 \leq a_1 \leq 1$, and plot (semilog scales) the temperature drop across the junction ΔT_c, for the following conditions, as a function of a_1. Here a_1 depends on the fabrication method used.

$k_A = 1.6$ W/m-K, $k_B = 385$ W/m-K, $L_c = 2\langle\delta^2\rangle^{1/2} = 0.5$ μm, $q_k = 10^5$ W/m^2.

SKETCH:

Figure Pr.3.28(a) shows the contact region with a physical bonding of materials A and B forming an alloy AB.

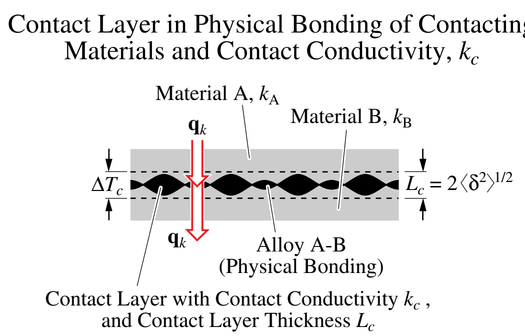


Figure Pr.3.28(a) A contact layer in a physical bounding of contacting materials; also shown is the contact conductivity k_c.

OBJECTIVE:

Use a general relationship $k_c = k_A + a_1(k_B - k_A)$, $0 \leq a_1 \leq 1$, and plot (semilog scales) the temperature drop across the junction ΔT_c, for the following conditions, as a function of a_1. Here a_1 depends on the fabrication method used.

SOLUTION:

From (3.96), we have

$$\Delta T_c = q_k \frac{L_c}{k_c}$$

and using the expression for k_c, we have

$$\begin{aligned} \Delta T_c(\text{K}) &= q_k L \frac{1}{k_A + a_1(k_B - k_A)} \\ &= \frac{10^5(\text{W/m}^2) \times 5 \times 10^{-7}(\text{m})}{[1.6 + a_1(385 - 1.6)](\text{W/m-K})}. \end{aligned}$$

Figure Pr.3.28(b) shows the variation of ΔT_c with respect to a_1. These results show that ΔT_c changes over three orders of magnitude, as a_1 is changed from 0 to 1.

COMMENT:

High k_c becomes essential in obtaining temperatures close to the cold junction temperature, when surfaces are brought in contact with the cold junction for heat transfer and cooling.

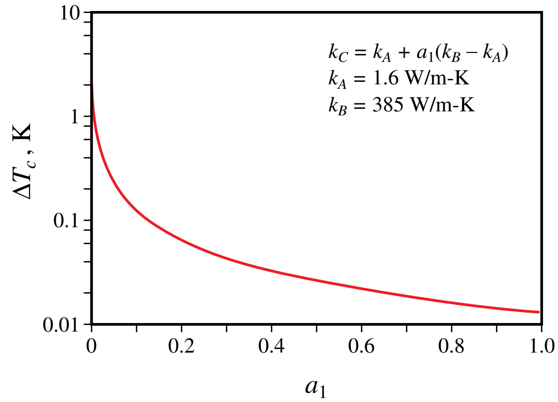


Figure Pr.3.28(b) Variation of the temperature jump across the contact region, as a function of junction conductivity parameter a_1.

PROBLEM 3.29.FUN

GIVEN:

In a thermoelectric cell, the Joule heating that results from the passage of the electric current is removed from the hot and cold ends of the conductor. When the electrical resistivity ρ_e or the current density j_e are large enough, the maximum temperature can be larger than the temperature at the hot-end surface T_h. This is shown in Figure 3.27(b).

OBJECTIVE:

(a) From the temperature distribution given by (3.104), determine the expression for the location of the maximum temperature in the conductor.
(b) Both the *p*- and the *n*-type legs have the same length $L = 2$ cm. The cross-sectional area of the *n*-type leg is $A_n = 2.8 \times 10^{-5}$ m^2. Calculate the cross-sectional area for the p-type leg A_p if the figure of merit is to be maximized. Use the electrical and thermal properties of the *p*-type and *n*-type bismuth telluride given in Table C.9(b).
(c) Determine the magnitude and the location of the maximum temperature for the *p*-type leg, when $T_h = 40$℃, $T_c = -2$℃, and $J_e = 6$ A.
(d) Determine the rate of heat removal from the hot and cold ends per unit cross-sectional area $(A_n + A_p)$.

SOLUTION:

(a) The temperature distribution along a conductor with internal Joule heating is given by (3.104) as

$$T = T_c + \frac{x}{L}(T_h - T_c) + \frac{\rho_e j_e^2 L^2}{2k}\left(\frac{x}{L} - \frac{x^2}{L^2}\right).$$

To find the point in which the temperature is maximum, we differentiate T with respect to x and set the result equal to zero,

$$\frac{dT}{dx} = \frac{1}{L}(T_h - T_c) + \frac{\rho_e j_e^2 L^2}{2k}\left(\frac{1}{L} - \frac{2x}{L^2}\right) = 0.$$

Solving for x gives

$$x = \frac{L}{2} + \frac{k}{\rho_e j_e^2}\frac{(T_h - T_c)}{L}.$$

(b) To maximize the figure of merit Z_e (3.120), for a given temperature difference $T_h - T_c$, the ratio $R_{e,h\text{-}c}/R_{k,h\text{-}c}$ must be minimized. Minimizing this ratio with respect to the geometric parameters of the Peltier cooler, L_nA_p/L_pA_n, gives (3.121)

$$\frac{L_nA_p}{L_pA_n} = \left(\frac{\rho_{e,p}k_n}{\rho_{e,n}k_p}\right)^{1/2}.$$

The properties for the *p*- and *n*-type materials are given in Table C.9(a). For *p*- and *n*-type bismuth telluride alloys, $k_p = 1.70$ W/m-K, $\rho_{e,p} = 1 \times 10^{-5}$ ohm-m, $\alpha_{S,p} = 230 \times 10^{-6}$ V/℃, $k_n = 1.45$ W/m-K, $\rho_{e,n} = 1 \times 10^{-5}$ ohm-m, $\alpha_{S,n} = -210 \times 10^{-6}$ V/℃. For $L_n = L_p = 0.02$ m and $A_n = 2.8 \times 10^{-5}$ m^2, the area of the *p*-type leg is

$$A_p = A_n\left(\frac{k_n}{k_p}\right)^{1/2} = 2.8 \times 10^{-5}(\text{m}^2) \times \left(\frac{1.45}{1.70}\right)^{1/2} = 2.59 \times 10^{-5} \ \ \text{m}^2.$$

(c) The location in the *p*-type leg in which the temperature is maximum is given above. The current density is

$$j_e = \frac{J_e}{A_p} = \frac{6(\text{A})}{2.59 \times 10^{-5}(\text{m}^2)} = 2.32 \times 10^5 \ \ \text{A/m}^2.$$

Then

$$\begin{aligned} x(T_{\max}) &= \frac{L}{2} + \frac{k}{\rho_e j_e^2}\frac{(T_h - T_c)}{L} \\ &= \frac{0.02}{2}(\text{m}) + \frac{1.70(\text{W/m-K})}{10^{-5}(\text{ohm-m})(2.32 \times 10^5)^2(\text{A}^2)} \times \frac{(40+2)(℃)}{0.02(\text{m})} \\ &= 0.0167 \ \ \text{m} \ = 1.67 \ \ \text{cm}. \end{aligned}$$

The maximum temperature is obtained from the temperature distribution (3.104)

$$
\begin{aligned}
T_{\max} &= T_c + \frac{x}{L}(T_h - T_c) + \frac{\rho_e j_e^2 L^2}{2k}\left(\frac{x}{L} - \frac{x^2}{L^2}\right) \\
&= -2(℃) + \frac{0.0167(\text{m})}{0.02(\text{m})} \times [40(℃) + 2(℃)] \\
&+ \frac{10^{-5}(\text{ohm-m}) \times (2.32 \times 10^5)^2(\text{A}^2) \times 0.02^2(\text{m}^2)}{2 \times 1.70(\text{W/m-K})} \times \left\{\frac{0.0167(\text{m})}{0.02(\text{m})} - \left[\frac{0.0167(\text{m})}{0.02(\text{m})}\right]^2\right\} \\
&= 41.8℃.
\end{aligned}
$$

(d) The cooling power is given by (3.115)

$$
Q_c = -\alpha_\text{S} J_e T_c + \frac{T_h - T_c}{R_{k,h\text{-}c}} + \frac{1}{2} R_{e,h\text{-}c} J_e^2,
$$

where

$$
\begin{aligned}
R_{k,h\text{-}c} &= \left(\frac{k_p A_p}{L_p} + \frac{k_n A_n}{L_n}\right)^{-1} = 236.3℃/\text{W} \\
R_{e,h\text{-}c} &= \frac{\rho_{e,p} L_p}{A_p} + \frac{\rho_{e,n} L_n}{A_n} = 0.0149 \text{ ohm}.
\end{aligned}
$$

Then

$$
\begin{aligned}
Q_c &= -(230 \times 10^{-6} + 210 \times 10^{-6})(\text{V}/℃) \times 6(\text{A}) \times 271(℃) + \frac{42(℃)}{236.3(℃/\text{W})} \\
&+ \frac{1}{2} \times 0.0149(\text{ohm}) \times 6^2(\text{A}^2) = -0.27 \text{ W}
\end{aligned}
$$

or

$$
q_c = \frac{-0.27}{A_n + A_p} = -5{,}009 \quad \text{W/m}^2.
$$

The heat generated at the hot end is given by (3.125), i.e.,

$$
\begin{aligned}
Q_h &= -Q_c + R_{e,h\text{-}c} J_e^2 + \alpha_\text{S} J_e (T_h - T_c) \\
Q_h &= 0.27(\text{W}) + 0.01497(\text{ohm}) \times 6^2(\text{A}^2) + 440 \times 10^{-6}(\text{V}/℃) \times 6(\text{A}) \times 42(℃) = 0.92 \text{ W}
\end{aligned}
$$

or

$$
q_h = \frac{0.92}{A_n + A_p} = 17{,}069 \quad \text{W/m}^2.
$$

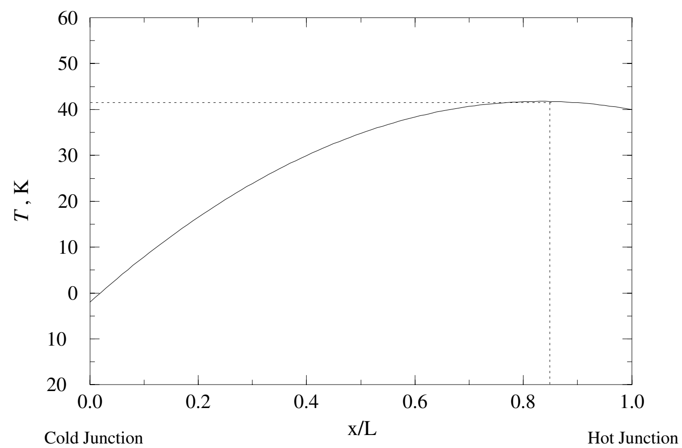


Figure Pr.3.29 Temperature distribution along the conductor, showing a maximum near the hot junction.

COMMENT:

The temperature distribution along the p-type leg of the Peltier cooler is parabolic, as given by (3.104). Figure Pr.3.29 shows the temperature distribution for the data given in item (c). Notice the maximum occurring near the hot end of the junction.

PROBLEM 3.30.FUN

GIVEN:

A thermoelectric cooler has bismuth telluride elements (i.e., p- and n-type pairs) that have a circular cross section of diameter $d = d_n = d_p = 3$ mm and a length $L = L_n = L_p = 2$ cm, as shown in Figure Pr.3.30(a). The temperatures of the hot and cold ends are $T_h = 40$°C and $T_c = -2$°C.

SKETCH:

Figure Pr.3.30(a) shows the heat flowing into the cold junction of thermoelectric cooler unit.

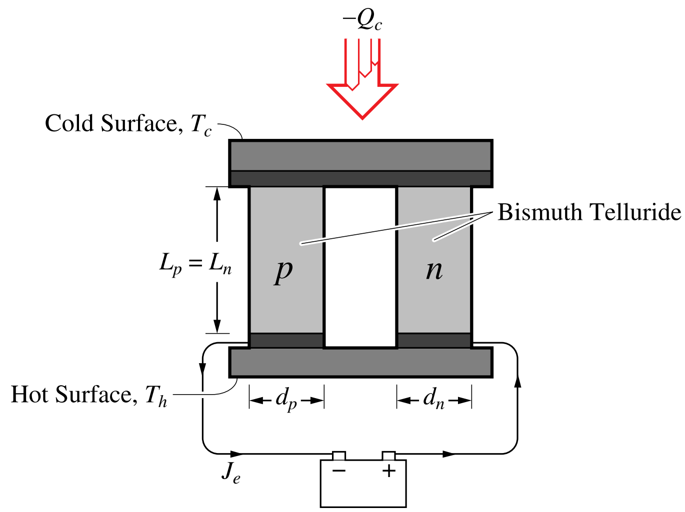


Figure Pr.3.30(a) A thermoelectric cooler unit.

OBJECTIVE:

Determine the cooling power for each junction, if the current corresponds to (a) the current that maximizes the cooling power Q_c,
(b) the current that is half of this optimum current, and
(c) the current that is twice the optimum current.

SOLUTION:

(a) The current that maximizes the cooling power is given by (3.117) as

$$J_e(Q_{c,\max}) = \frac{\alpha_S T_c}{R_{e,h\text{-}c}}.$$

The electrical resistance is given by (3.116) and using $\rho_{e,p}$ and $\rho_{e,n}$ from Table C.9(a) for bismuth telluride, we have

$$R_{e,h\text{-}c} = \frac{\rho_{e,p}L_p}{A_p} + \frac{\rho_{e,n}L_n}{A_n} = \frac{L}{A}(\rho_{e,p} + \rho_{e,n}) = \frac{0.02(\text{m})}{\pi \times (0.0015)^2(\text{m}^2)} \times 2 \times 10^{-5}(\text{ohm-m}) = 0.0566 \text{ ohm}.$$

For the n- and p-type bismuth telluride, from Table C.9(b), $\alpha_S = \alpha_{S,p} - \alpha_{S,n} = 440$ μV/K. The maximum current is

$$J_e(Q_{c,\max}) = \frac{440 \times 10^{-6}(\text{V/K}) \times 271.15(\text{K})}{0.0566(\text{ohm})} = 2.11 \text{ A}.$$

The cooling power is given by (3.115),

$$Q_c = -\alpha_S J_e T_c + \frac{T_h - T_c}{R_{k,h\text{-}c}} + \frac{1}{2} R_{e,h\text{-}c} J_e^2.$$

The thermal resistance is given by (3.116) and using k from Table C.9(a), we have

$$R_{k,h\text{-}c} = \left(\frac{k_p A_p}{L_p} + \frac{k_n A_n}{L_n}\right)^{-1} = \frac{L}{A}\frac{1}{(k_p + k_n)} = \frac{0.02(\text{m})}{\pi \times (0.0015)^2(\text{m}^2)} \times \frac{1}{(1.70 + 1.45)(\text{W/m-°C})} = 898.2\text{°C/W}.$$

Finally, the maximum cooling power is

$$Q_c = -(440 \times 10^{-6})(\text{K/V}) \times 2.11(\text{A}) \times 271.15(\text{K}) + \frac{42(\text{°C})}{898.2(\text{°C/W})} + \frac{0.0566(\text{ohm}) \times (2.11)^2(\text{A}^2)}{2} = -0.079 \ \ \text{W}.$$

(b) The cooling power for half the current (1.06 A) is

$$Q_c = -(440 \times 10^{-6})(\text{K/V}) \times 1.06(\text{A}) \times 271.15(\text{K}) + \frac{42(\text{°C})}{898.2(\text{°C/W})} + \frac{\times 0.0566(\text{ohm}) \times (1.06)^2(\text{A}^2)}{2} = -0.047 \ \ \text{W}.$$

(c) The cooling power for twice the current (4.22 A) is

$$Q_c = -(440 \times 10^{-6})(\text{K/V}) \times 4.22(\text{A}) \times 271.15(\text{K}) + \frac{42(\text{°C})}{898.2(\text{°C/W})} + \frac{\times 0.0566(\text{ohm}) \times (4.22)^2(\text{A}^2)}{2} = 0.048 \ \ \text{W}.$$

COMMENT:

For a given pair and geometry, the cooling power varies with the applied current. Figure Pr.3.30(b) shows the negative of the cooling power as a function of the current for this Peltier cooler. As the current increases, both the Peltier cooling and the Joule heating increase. However, the Joule heating increases with the square of the current and, for large values of current, its contribution overcomes that of the Peltier cooling.

Another possible optimization is the geometric optimization. Figure Pr.3.30(c) shows the negative of the cooling power as a function of the ratio of cross-sectional area over length A_k/L for the three different currents used in the problem. Note that for each current there is a maximum in the cooling power. For small values of A_k/L, the thermal resistance is large, thus reducing the heat conduction. However, the electrical resistance is also large and that increases the Joule heating. For large values of A_k/L the opposite occurs. The optimum point is given by the balance between the thermal and the electrical resistances.

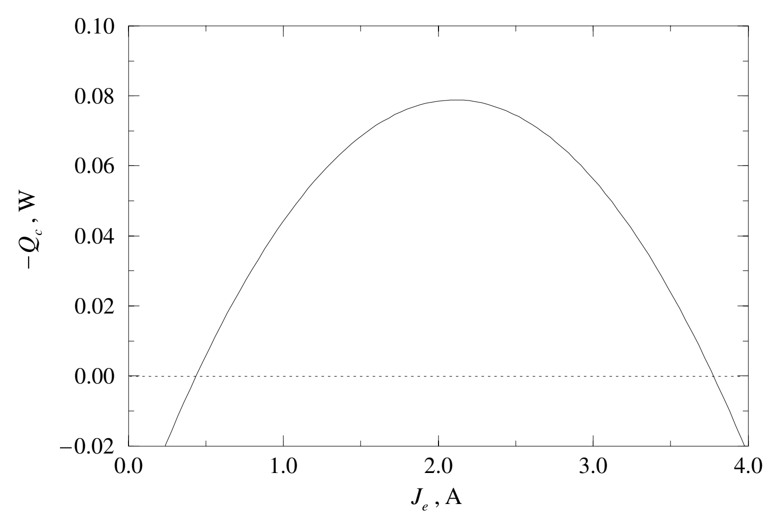


Figure Pr.3.30(b) Variation of cooling power ($-Q_c$) with respect to the current.

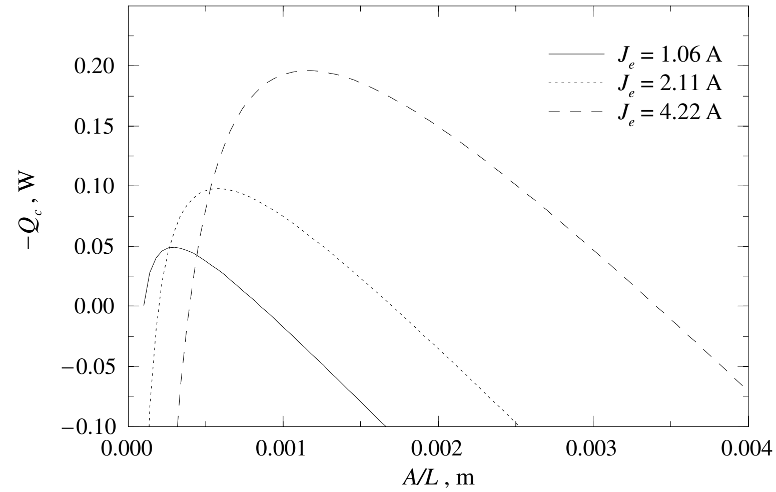


Figure Pr.3.30(c) Variation of cooling power ($-Q_c$) with respect to the ratio of conductor cross sectional area to length.

PROBLEM 3.31.FUN

GIVEN:

A thermoelectric device is used for cooling a surface to a temperature T_c.

For each bismuth telluride thermoelectric, circular cylinder conductor, use $d_n = d_p = 1.5$ mm, and $L_n = L_p = 4$ mm. The hot junction is at $T_h = 40^\circ$C.

SKETCH:

Figure Pr.3.31 shows the thermoelectric cooler unit.

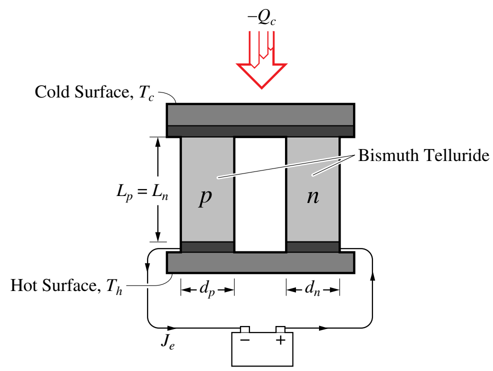


Figure Pr.3.31 A thermoelectric cooler unit.

OBJECTIVE:

For the conditions given below, determine (a) the minimum T_c, (b) the current for this condition, and (c) the minimum T_c for a current $J_e = 1$ A.

SOLUTION:

(a) The minimum T_c for a given T_h is given by

$$(T_h - T_c)_{\max} = \frac{\alpha_{\rm S}^2 T_c^2}{2R_{e,h\text{-}c}/R_{k,h\text{-}c}} = \frac{\alpha_{\rm S}^2 T_c^2}{2R_{e,h\text{-}c}R_{k,h\text{-}c}^{-1}}.$$

The electrical and thermal resistances are given by

$$R_{e,h\text{-}c} = \left(\frac{\rho_e L}{A}\right)_n + \left(\frac{\rho_e L}{A}\right)_p$$

$$R_{k,h\text{-}c}^{-1} = \left(\frac{kA}{L}\right)_n + \left(\frac{kA}{L}\right)_p.$$

From the data given,

$$A_n = A_p = \frac{\pi d_n^2}{4} = \frac{\pi(0.0015)^2(\text{m}^2)}{4} = 1.767 \times 10^{-6}\ \text{m}^2$$

$$L_n = L_p = 0.004\ \text{m}.$$

For the bismuth-telluride elements, from Table C.9(a), $\rho_{e,n} = \rho_{e,p} = 10^{-5}$ ohm-m, $k_n = 1.70$ W/m-K, $k_p = 1.45$ W/m-K, and $\alpha_{\rm S} = \alpha_{{\rm S},p} - \alpha_{{\rm S},n} = 230 \times 10^{-6}$(V/K)+ 210×10^{-6}(V/K) $= 440 \times 10^{-6}$ V/K. The resistances then become

$$R_{e,h\text{-}c} = 2\frac{10^{-5}(\text{ohm-m}) \times 0.004(\text{m})}{1.767 \times 10^{-6}(\text{m}^2)} = 0.0453 \quad \text{ohm}$$

$$R_{k,h\text{-}c}^{-1} = \frac{1.767 \times 10^{-6}(\text{m}^2)}{0.004(\text{m})}[1.70(\text{W/m-K}) + 1.45(\text{W/m-K})] = 0.00139 \quad \text{W/K}.$$

The minimum T_c is then given by

$$313.15(\text{K}) - T_c(\text{K}) = \frac{\left[440 \times 10^{-6}(\text{V/K})\right]^2 T_c^2}{(2)0.0453(\text{ohm}) \times 0.00139(\text{W/K})}$$

$$T_c^2 + 6.512 \times 10^2 T_c - 2.038 \times 10^5 = 0$$

or

$$T_c = 231 \text{ K}.$$

(b) The current for this temperature is given by

$$J_e = \frac{\alpha_\text{S} T_c}{R_{e,h\text{-}c}} = \frac{440 \times 10^{-6}(\text{V/K}) \times 231(\text{K})}{0.0453(\text{ohm})} = 2.245 \text{ A}.$$

(c) The minimum temperature for a current of $J_e = 1$ A is found by setting the cooling power to zero. From (3.115),

$$Q_c = -\alpha_\text{S} J_e T_c + R_{k,h\text{-}c}^{-1}(T_h - T_c) + \frac{1}{2} R_{e,h\text{-}c} J_e^2 = 0.$$

Solving for T_c gives

$$T_c = \frac{R_{k,h\text{-}c}^{-1} T_h + \frac{1}{2} R_{e,h\text{-}c} J_e^2}{\alpha_s J_e + R_{k,h\text{-}c}^{-1}} = \frac{0.00139(\text{W/K}) \times 313.15(\text{K}) + \frac{1}{2} \times 0.0453(\text{ohm}) 1^2(\text{A}^2)}{440 \times 10^{-6}(\text{V/K}) \times 1(\text{A}) + 0.00139(\text{W/K})} = 250.2 \text{ K}.$$

COMMENT:

Note that for a square cross section, $A = d_n^2 = 2.25 \times 10^{-6}$ m^2, we have

$$\begin{aligned} R_{e,h\text{-}c} &= 0.03556 \text{ ohm} \\ R_{k,h\text{-}c}^{-1} &= 0.00177 \text{ W/K} \\ J_e &= 2.859 \text{ A} \\ T_c &= 258.8 \text{ K}. \end{aligned}$$

PROBLEM 3.32.FUN

GIVEN:

A thin-film thermoelectric cooler is integrated into a device as shown in Figure Pr.3.32(a). In addition to heat conduction through the p- and n-type conductors, heat flows by conduction through the substrate. Assume a one-dimensional parallel conduction through the p- and n-type conductors and the substrate.

Model the conduction through the substrate as two conduction paths (one underneath each of the p- and n-type legs). Begin with (3.115) and use the optimum current.

SKETCH:

Figure Pr.3.32(a) shows the thermoelectric cooler unit, the heat source and sink, and the substrate.

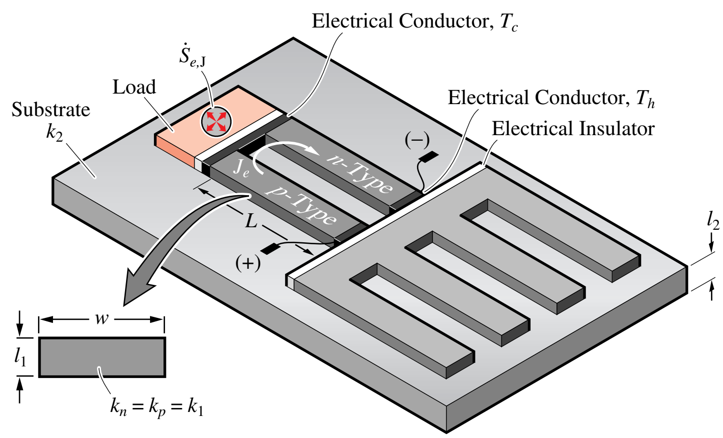


Figure Pr.3.32(a) A thin-film thermoelectric cooler placed over a substrate.

OBJECTIVE:

(a) Draw the thermal circuit diagram for heat flow between the T_h and T_c nodes.
(b) Show that the maximum temperature difference is

$$(T_h - T_c)_{max}(Q_c = 0) = \frac{Z_e T_c^2}{2\left(1 + \dfrac{l_2 k_2}{l_1 k_1}\right)}.$$

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.3.32(b). The conduction heat flow is through two parallel paths. The energy equation for T_c node is given by (3.115), which we rewrite as

$$Q_c = -\alpha_S J_e T_c + (T_h - T_c)\left[\left(\frac{1}{R_{k,h\text{-}c}}\right)_1 + \left(\frac{1}{R_{k,h\text{-}c}}\right)_2\right] + \frac{1}{2} R_{e,h\text{-}c} J_e^2.$$

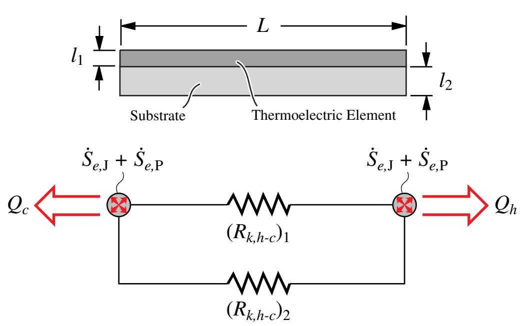


Figure Pr.3.32(b) Thermal circuit diagram.

For $(T_h - T_c)_{max}$ with $Q_c = 0$ and J_e corresponding to the optimum performance as given by (3.117), we have

$$J_e = \frac{\alpha_{\rm S} T_c}{R_{e,h\text{-}c}}$$

$$Q_c = 0 = -\alpha_{\rm S}\left(\frac{\alpha_{\rm S} T_c}{R_{e,h\text{-}c}}\right) T_c + (T_h - T_c)\left[\frac{1}{(R_{k,h\text{-}c})_1} + \frac{1}{(R_{k,h\text{-}c})_2}\right] + \frac{1}{2} R_{e,h\text{-}c}\frac{\alpha_{\rm S}^2 T_c^2}{R_{e,h\text{-}c}^2}$$

or

$$0 = -\frac{1}{2}\frac{\alpha_{\rm S}^2 T_c^2}{R_{e,h\text{-}c}} + (T_h - T_c)\left[\frac{1}{(R_{k,h\text{-}c})_1} + \frac{1}{(R_{k,h\text{-}c})_2}\right].$$

(b) Solving for $(T_h - T_c)_{max}$, we have

$$(T_h - T_c)_{max}(Q_c = 0) = \frac{\alpha_{\rm S}^2 T_C^2}{2\dfrac{R_{e,h\text{-}c}}{(R_{k,h\text{-}c})_1}\left[1 + \dfrac{(R_{k,h\text{-}c})_1}{(R_{k,h\text{-}c})_2}\right]}.$$

Using the thermoelectric figure of merit given by (3.120), we have

$$(T_h - T_c)_{max}(Q_c = 0) = \frac{Z_e T_c}{2\left[1 + \dfrac{(R_{k,h\text{-}c})_1}{(R_{k,h\text{-}c})_2}\right]}.$$

From (3.116), and for $k_p = k_n = k_1$, we have

$$\begin{aligned}
(R_{k,h\text{-}c})_1^{-1} &= 2\frac{A_{k,1}k_1}{L}, \quad A_{k,1} = l_1 w \\
(R_{k,h\text{-}c})_2^{-2} &= 2\frac{A_{k,2}k_2}{L}, \quad A_{k,2} = l_2 w.
\end{aligned}$$

Using these, we have

$$(T_h - T_c)_{max}(Q_c = 0) = \frac{Z_e T_c}{2\left(1 + \dfrac{l_2 k_2}{l_1 k_1}\right)}.$$

COMMENT:

The heat conduction through the substrate is not one dimensional. Also, parasitic heat is conducted to the cold junction from locations other than the hot junction.

PROBLEM 3.33.DES.S

GIVEN:

A miniature vapor sensor is cooled, for enhanced performance, by thermoelectric coolers. The sensor and its thermoelectric coolers are shown in Figure Pr.3.33(a). There are four bismuth-telluride thermoelectric modules and each module is made of four p-n layers (forming four p-n junctions) each p- and n-layer having a thickness l, length L, and width w. The sensor and its substrate are assumed to have the ρc_p of silicon and a cold junction temperature $T_c(t)$.

The hot junction temperature $T_h(t)$ is expected to be above the far-field solid temperature T_∞. The conduction resistance between the hot junctions and T_∞ is approximated using the results of Table 3.3(b), for steady-state resistance between an ambient placed on the bounding surface of a semi-infinite slab (T_h) and the rest of the slab (T_∞) [shown in Table 3.3(b), first entry]. This is

$$R_{k,h\text{-}\infty} = \frac{\ln\dfrac{4w}{2a}}{\pi k w} = \frac{\ln\dfrac{2w}{a}}{\pi k w}, \qquad Q_{k,h\text{-}\infty} = \frac{T_h(t) - T_\infty}{R_{k,h\text{-}\infty}}.$$

Initially there is a uniform sensor temperature, $T_c(t=0) = T_h(t=0) = T_\infty$. For heat storage of the thermoelectric modules, divide each volume into two with each portion having temperature T_c or T_h.

$l = 3$ μm, $w = 100$ μm, $L = 300$ μm, $a = 24$ μm, $T_\infty = 20$°C, $J_e = 0.010$ A.

SKETCH:

Figure 3.33(a) shows the thermoelectrically cooled vapor sensor.

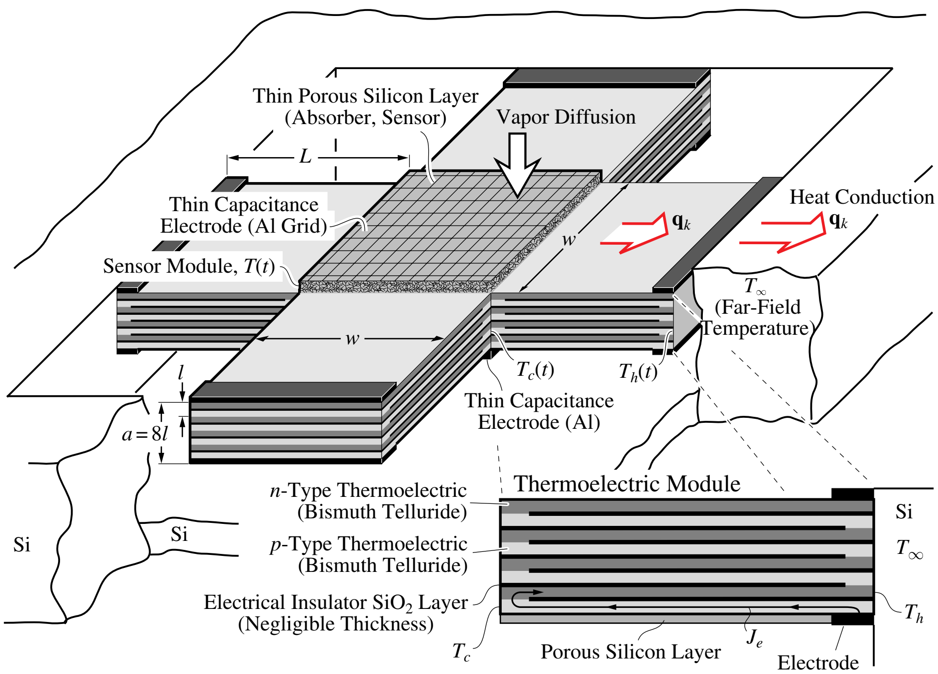


Figure Pr.3.33(a) A thermoelectrically cooled vapor sensor showing the four molecules.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine and plot the sensor temperature T_c.

(c) Determine the steady-state sensor temperature $T_c(t \to \infty)$.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.3.33(b). The sensible heat storage/release in the thermoelectric module is modeled by dividing its volume into two with one portion having the assumed uniform

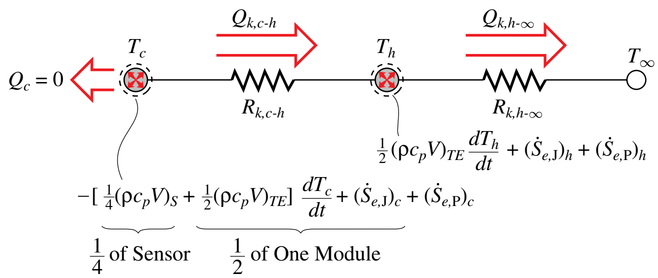


Figure Pr.3.33(b) Thermal circuit diagram.

temperature $T_c(t)$ and the other one $T_h(t)$. The heat conduction from the hot junction to the surrounding silicon is given by the conduction resistance $R_{k,h\text{-}\infty}$.
(b) The integral-volume energy equation (2.9) for the cold junction (including the heat storage/release for the sensor and half of the thermoelectric modules) is

$$\begin{aligned} Q_c + Q_{k,c\text{-}h} &= -(\rho c_p V)_s \frac{dT_c}{dt} - \left(\frac{1}{2}\rho c_p V\right)_{TE} \frac{dT_c}{dt} + (\dot{S}_{e,\mathrm{J}})_c + (\dot{S}_{e,\mathrm{P}})_c \quad \text{cold junctions} \\ Q_{k,h\text{-}c} + Q_{k,h\text{-}\infty} &= -\left(\frac{1}{2}\rho c_p V\right)_{TE} \frac{dT_h}{dt} + (\dot{S}_{e,\mathrm{J}})_h + (\dot{S}_{e,\mathrm{P}})_h \quad \text{hot junctions,} \end{aligned}$$

where

$$\begin{aligned} Q_c &= 0 \\ Q_{k,c\text{-}h} &= \frac{T_c - T_h}{R_{k,c\text{-}h}}, \qquad R^{-1}_{k,c\text{-}h} = \frac{4}{(R_{k,c\text{-}h})_p} + \frac{4}{(R_{k,c\text{-}h})_n} \\ V_s &= aw^2, \qquad V_{TE} = awL \\ (\dot{S}_{e,\mathrm{J}})_c = (\dot{S}_{e,\mathrm{J}})_h &= \frac{1}{2}J_e^2 R_e, \qquad R_{e,h\text{-}c} = 4(R_{e,h\text{-}c})_p + 4(R_{e,h\text{-}c})_n \\ (\dot{S}_{e,\mathrm{P}})_c &= -4\alpha_{\mathrm{S}} J_e T_c, \qquad \alpha_{\mathrm{S}} = \alpha_{\mathrm{S},p} - \alpha_{\mathrm{S},n} \\ (\dot{S}_{e,\mathrm{P}})_h &= 4\alpha_{\mathrm{S}} J_e T_h \\ (R_{k,h\text{-}c})_p &= \frac{L}{k_p l w}, \qquad (R_{k,h\text{-}c})_n = \frac{L}{k_n l w} \\ (R_{e,h\text{-}c})_p &= \frac{\rho_{e,p} L}{lw}, \qquad (R_{e,h\text{-}c})_n = \frac{\rho_{e,n} L}{lw} \\ R_{k,h\text{-}\infty} &= \frac{\ln(2w/a)}{\pi k w}. \end{aligned}$$

Note that we have used one thermoelectric module and 1/4 of the sensor volume in the energy equations.
From Table C.9(a), for bismuth telluride, we have

$$\begin{aligned} \alpha_{\mathrm{S},n} &= -210 \times 10^{-6}\ \mathrm{V/^\circ C} && \text{Table C.9(a)} \\ \alpha_{\mathrm{S},p} &= 230 \times 10^{-6}\ \mathrm{V/^\circ C} && \text{Table C.9(a)} \\ \rho_{e,n} &= 1.00 \times 10^{-5}\ \text{ohm-m} && \text{Table C.9(a)} \\ \rho_{e,p} &= 1.00 \times 10^{-5}\ \text{ohm-m} && \text{Table C.9(a)} \\ k_n &= 1.45\ \text{W/m-K} && \text{Table C.9(a)} \\ k_p &= 1.70\ \text{W/m-K} && \text{Table C.9(a).} \end{aligned}$$

From Table C.2, for bismuth at $T = 300$ K, we have

$$\begin{aligned} \rho_e &= 9{,}790\ \mathrm{kg/m^3} && \text{Table C.2} \\ c_{p,TE} &= 122\ \text{J/kg-K} && \text{Table C.2.} \end{aligned}$$

From Table C.2, for silicon at $T = 300$ K, we have

$$\rho_s = 2{,}330 \text{ kg/m}^3 \qquad \text{Table C.2}$$
$$c_{p,s} = 678 \text{ J/kg-K} \qquad \text{Table C.2}$$
$$k_s = 149 \text{ W/m-K} \qquad \text{Table C.2.}$$

The computed (using a solver, such as SOPHT) cold junction temperature $T_c(t)$ is plotted in Figure Pr.3.33(c), as a function of time. The steady state is reached at an elapsed time at nearly $t = 0.06 = 60$ ms.
Also plotted is $T_h = T_h(t)$ and the results show that T_h remains constant and nearly equal to T_∞. This is due to the rather small resistance between these two modes (i.e., $R_{k,h\text{-}\infty} \ll R_{k,h\text{-}c}$).

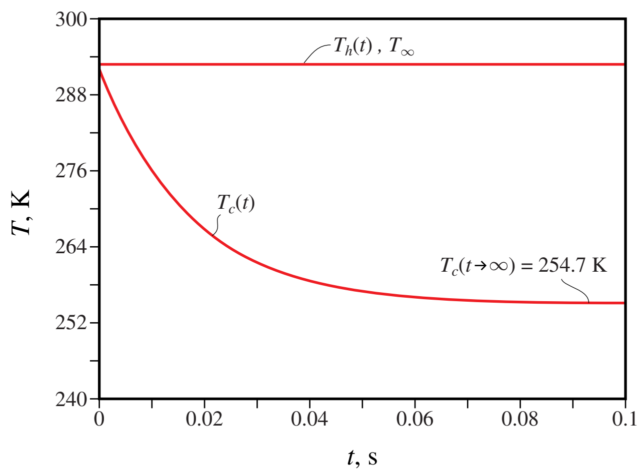


Figure Pr.3.33(c) Variation of the hot and cold junction temperatures, with respect to time.

(c) The steady-state, cold junction temperature is found from Pr.3.33(c), or by solving the steady state energy equation (3.174). The result is

$$T_c(t \to \infty) \quad = \quad 254.7 \text{ K} = -18.40℃.$$

COMMENT:

With a smaller volume for the sensor, the response time can be reduced. Although neglected here, the parasitic heat leaks (i.e., $Q_c < 0$) into the sensor prevent achieving low temperatures and also increases the response time. The electric power is $4J_e^2 R_{e,h\text{-}c} = 0.016$ W $= 16$ mW and is considered reasonable for microelectronics. Also, since $T_h \simeq T_\infty$, the closed-form solution to (3.172) can also be used. Note that since $(\dot{S}_{e,\text{P}})_c$ depends on T_c, we should use

$$\begin{aligned}
Q_{k,c\text{-}h} - (\dot{S}_{e,\text{P}})_c &= \frac{T_c - T_h}{R_{k,c\text{-}h}} + \alpha_\text{S} J_e T_c \\
&= T_c(\frac{1}{R_{k,c\text{-}h}} + \alpha_\text{S} J_e) - T_h(\frac{1}{R_{k,c\text{-}h}}) \\
&= (T_c - T_h)(\frac{1}{R_{k,c\text{-}h}} + \alpha_\text{S} J_e) + \alpha_\text{S} J_e T_h \\
&\equiv \frac{T_c - T_h}{R'_{k,c\text{-}h}} + Q_c.
\end{aligned}$$

Then we use this newly defined $R_{k,c\text{-}h}$ and $a_1 = -Q_c$ in (3.172).

PROBLEM 3.34.FUN

GIVEN:

A highly localized Joule heating applied to myocardium via a transvenous catheter can destroy (ablate) the endocardial tissue region that mediates life-threatening arrhythmias. Alternating current, with radio-frequency range of wavelength, is used. This is shown in Figure Pr.3.34(a)(i). The current flowing out of the spherical tip of the catheter flows into the surrounding tissue, as shown in Figures Pr.3.34(a)(ii) and (iii). Due to the rapid decay of the current flux, the Joule heating region is confined to a small region $R_e \le r \le R_1$ adjacent to the electrode tip. The total current J_e leaving the spherical tip results in a current density

$$j_e = \frac{J_e}{4\pi r^2}.$$

The tissue having a resistivity ρ_e will have a local energy conversion rate $\dot{s}_{e,\mathrm{J}} = \rho_e j_e^2$.

$T_2 = 37°\mathrm{C}, R_e = 1.0$ mm, $R_1 = 1.3$ mm, $\rho_e = 2.24$ ohm-m, $J_e = 0.07476$ A.

Use Table C.17 for k of muscle. Assume steady-state heat transfer.

SKETCH:

Figure Pr.3.34(a)(i) The Joule heating of myocardium region by a spherical electrode tip. (ii) The small heated region. (iii) Heat flow out of the heated region by conduction.

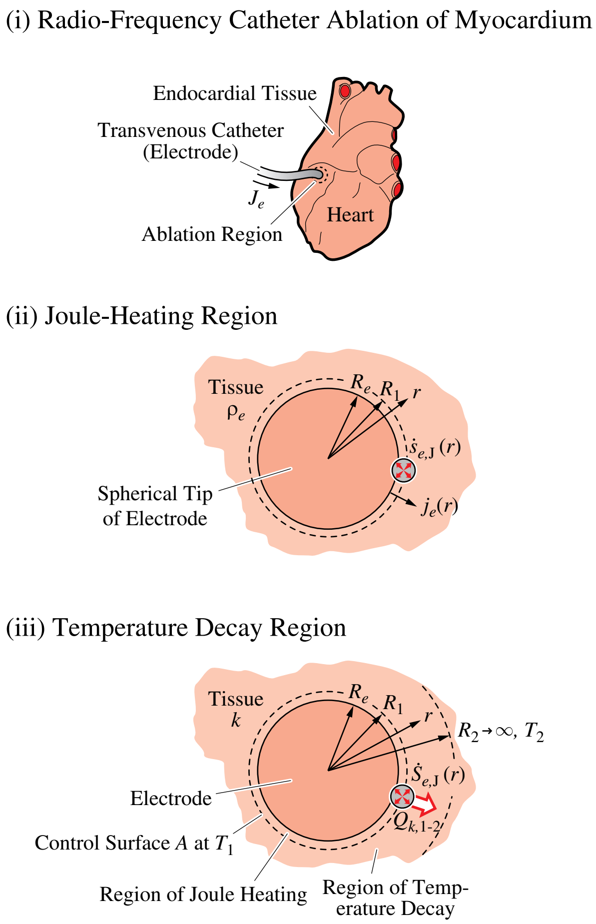


Figure Pr.3.34(a) Radio-frequency catheter ablation of myocardium showing the small a heated region and the conduction heat transfer from this region.

OBJECTIVE:

Assuming that all the energy conversion occurs in the region $R_e \le r \le R_1$, then the heat is conducted from this region toward the remaining tissue. The far-field temperature is T_2, i.e., as $r \rightarrow \infty$, $T \rightarrow T_2$.

(a) Derive the expression for the local $\dot{S}_{e,\mathrm{J}}(r)$ and comment on its distribution.

(b) Draw the thermal circuit diagram and write the surface energy equation for its surface located at $r = R_1$. Use the conduction resistance for a spherical shell (Table 3.2).
(c) Determine $T_1(R_1)$ for the following conditions.

SOLUTION:
(a) The Joule heating per unit volume is given by (2.33), i.e.,

$$\dot{s}_{e,\mathrm{J}} = \rho_e j_e^2.$$

The current flux j_e is related to the total current, for the spherical geometry considered, through

$$j_e = \frac{J_e}{4\pi r^2}.$$

Then

$$\dot{s}_{e,\mathrm{J}} = \rho_e \frac{J_e^2}{16\pi^2 r^4}.$$

This shows that the local Joule heating rate drops very quickly as r increases. This is the reason for the small Joule heating region.

(b) The thermal circuit diagram for the surface node T_1 is shown in Figure Pr.3.34(b).

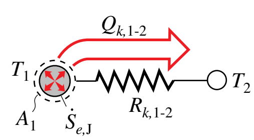


Figure Pr.3.34(b) Thermal circuit diagram.

The surface energy equation is

$$\dot{S}_{e,\mathrm{J}} = Q_{k,1\text{-}2} = \frac{T_1 - T_2}{R_{k,1\text{-}2}}.$$

From Table 3.2 for $R_2 \rightarrow \infty$, we have

$$R_{k,1\text{-}2} = \frac{1}{4\pi R_1 k}.$$

The integrated energy conversion rate $\dot{S}_{e,\mathrm{J}}$ is

$$\begin{aligned}
\dot{S}_{e,\mathrm{J}} &= \int_{R_e}^{R_1} \rho_e \frac{J_e^2}{16\pi^2 r^4} 4\pi r^2 dr \\
&= \frac{\rho_e J_e^2}{4\pi} \int_{R_e}^{R_1} r^{-2} dr \\
&= \frac{-\rho_e J_e^2}{4\pi} \left(\frac{1}{R_1} - \frac{1}{R_e} \right) \\
&= \frac{\rho_e J_e^2}{4\pi} \left(\frac{1}{R_e} - \frac{1}{R_1} \right).
\end{aligned}$$

(c) Solving the energy equation for T_1, we have

$$\begin{aligned}
T_1 &= T_2 + \dot{S}_{e,\mathrm{J}} R_{k,1\text{-}2} \\
&= T_2 + \frac{\rho_e J_e^2}{4\pi} \left(\frac{1}{R_e} - \frac{1}{R_1} \right) \frac{1}{4\pi R_1 k} \\
&= T_2 + \frac{\rho_e J_e^2}{16\pi^2 R_1 k} \left(\frac{1}{R_e} - \frac{1}{R_1} \right).
\end{aligned}$$

Using the numerical values, we have

$$\dot{S}_{e,\mathrm{J}} = \frac{2.24(\text{ohm-m}) \times 0.07476^2(\text{m}^2)}{4\pi} \times \left[\frac{1}{10^{-3}(\text{m})} - \frac{1}{1.3 \times 10^{-3}(\text{m})}\right] = 0.230 \text{ W}.$$

From Table C.17 (for muscle tissue), $k = 0.41$ W/m-K. Then

$$\begin{aligned} T_1 &= 37(^\circ\text{C}) + \frac{0.230(\text{W})}{4\pi \times 1.3 \times 10^{-3}(\text{m}) \times 0.41(\text{W/m-K})} \\ &= 37(^\circ\text{C}) + 34.36(^\circ\text{C}) = 71.36^\circ\text{C}. \end{aligned}$$

COMMENT:

Since in the Joule heating the local $\dot{s}_{e,\mathrm{J}}$ drops so rapidly with the increase in r, in practice a microwave heater, with a helical coil antenna design is used.

PROBLEM 3.35.FUN

GIVEN:

Refractive surgical lasers are used to correct the corneal refractive power of patients who are near-sighted, far-sighted, or astigmatic in their vision. These corneal reshaping procedures can be performed via several mechanisms, including ablation of the corneal surface to change the focal length, removal of a section of the cornea causing reformation, and reshaping of the corneal tissue by thermal shrinkage effects. In order to achieve minimal tissue thermal damage due to the laser ablation, we need to investigate the thermal behavior of the corneal tissue to understand the local thermal effects of laser heating, and to predict the potential for an unintentional injury during laser surgery.

Heat transfer in corneal tissue is modeled as a sphere of radius $R_1 = 5$ mm, with the energy source positioned at the center of the sphere [Figure Pr.3.35(a)(i)]. A small diameter laser beam with $\dot{S}_{e,\alpha} = A_r \alpha_r q_{r,i} = 220$ mW is used.

Assume a steady-state conduction heat transfer.

SKETCH:

Figure Pr.3.35(a) shows the eye laser surgery and the heat transfer model.

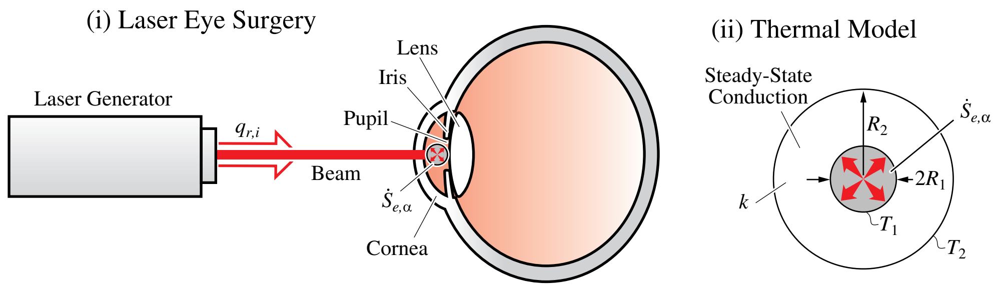


Figure Pr.3.35(a)(i) Laser eye surgery showing the absorbed irradiation. (ii) The thermal model.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine $R_{k,1\text{-}2}$.
(c) Determine T_1 of the laser beam at $R_1 = 1$ mm, using the energy equation.
(d) Plot the variation of T with respect to r.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.3.35(b).

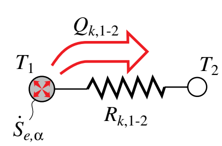


Figure Pr.3.35(b) Thermal circuit diagram.

(b) Using (3.64), we have

$$R_{k,1\text{-}2} = \frac{\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)}{4\pi k} = \frac{T_1 - T_2}{Q_{k,1\text{-}2}}.$$

Given that $R_1 = 1$ mm, $R_2 = 5$ mm, and $k = 0.6$ W/m-K, we have

$$R_{k,1\text{-}2} = \frac{\dfrac{1}{0.001(\text{mm})} - \dfrac{1}{0.005(\text{mm})}}{4\pi \times (0.6\text{W/m-℃})} = 106.1\text{℃/W}.$$

(c) From above, we have

$$Q_{k,1\text{-}2} = \frac{T_1 - T_2}{R_{k,1\text{-}2}},$$

with $Q_{k,1\text{-}2} = \dot{S}_{e,\alpha} = 220$ mW.
Solving for T_1, we have

$$T_1 = 60.34℃.$$

(d) We determine $T = T(r)$ using the differential-volume energy equation (3.35). For spherical shells with a constant conductivity k, a dominant radial temperature gradient, and no volumetric conversion, we have

$$-\frac{1}{r^2}\frac{d}{dr}kr^2\frac{dT}{dr} = \dot{s}$$

or

$$-\frac{1}{r^2}\frac{d}{dr}r^2\frac{dT}{dr} = 0.$$

Integrating this once, we have

$$\frac{dT}{dr} = \frac{a_1}{r^2}.$$

Integrating this once, we have

$$T(r) = -a_1\frac{1}{r} + a_2.$$

The boundary conditions are

$$T_1 = -a_1\frac{1}{R_1} + a_2 \quad \text{and} \quad T_2 = -a_1\frac{1}{R_2} + a_2.$$

Solving for a_1 and a_2, we have

$$a_1 = \frac{T_2 - T_1}{\dfrac{1}{R_1} - \dfrac{1}{R_2}}, \quad a_2 = \left(\frac{T_2 - T_1}{\dfrac{1}{R_1} - \dfrac{1}{R_2}}\right)\frac{1}{R_1} + T_1.$$

Then

$$T(r) = T_1 + \left(\frac{T_2 - T_1}{\dfrac{1}{R_1} - \dfrac{1}{R_2}}\right)\left(\frac{1}{R_1} - \frac{1}{r}\right).$$

Figure Pr.3.35(c) shows the plot of $T(r)$ versus $R_1 \le r \le R_2$.

COMMENT:

Note that $T(r)$ drops rapidly as r increases. Also note that $T(r = R_1) = 60.84℃$ is far above the reversible temperature limit of 42℃. The region with $T > 42℃$ is shown in Figure Pr.3.35(c).

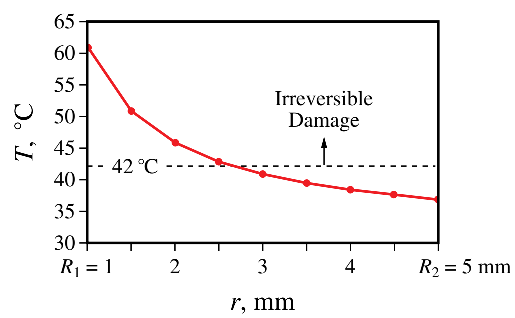


Figure Pr.3.35(c) Variation of the tissue temperature with respect to the radial position r.

PROBLEM 3.36.FUN

GIVEN:

Cardiac ablation refers to the technique of destroying heart tissue that is responsible for causing alternations of the normal heart rhythm. A well-localized region of the endocardial tissue is destroyed by microwave heating. The electric field is provided through a catheter that is inserted into the heart through a vein. One example of this cardiac ablation and the microwave catheter design is shown in Figure Pr.3.36(a). The region $R_e < r < R_o$ is heated with $R_e = 1$ mm and frequency $f = 3.00 \times 10^{11}$ Hz. A long catheter is assumed, so the heat transfer is dominant in the cylindrical cross-sectional plane [shown in Figure Pr.3.36(a)].

The dielectric loss factor for heart tissue is $\epsilon_{ec} = 17.6$ and its thermal conductivity is $k = 0.45$ W/m-K.

SKETCH:

Figure Pr.3.36(a) shows the microwave antenna and the heated tissue region.

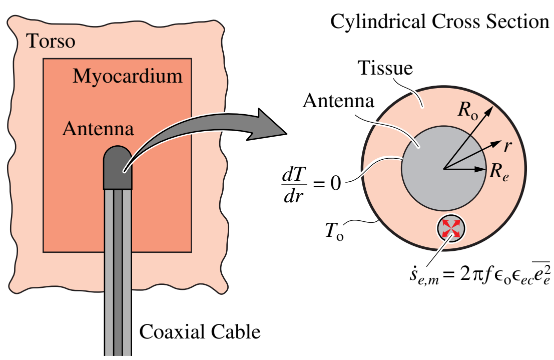


Figure Pr.3.36(a) A microwave catheter used for ablation of cardiac tissue.

OBJECTIVE:

(a) Assuming a zero temperature gradient $r = R_e$, start with the one-dimensional (radial direction) differential energy equation with $\dot{s}_{e,m} = \dot{s}_{e,m}(r)$, then integrate this differential equation to obtain the radial distribution of the temperature $T = T(r)$. It is known that $\overline{e}_e^2 = e_{e,o}^2(R_e/r)^2$. In addition to thermal conditions $dT/dr = 0$ at $r = R_e$ use $T = T_o$ at $r = R_o$.

(b) In order to perform a successful ablation, the microwave antenna needs to produce a temperature $T_e = 353$ K (80℃) at $r = R_e$. Given that temperature decreases to the normal tissue temperature $T_o = 310.65$ K (37.5℃) at $r = 1$ cm, determine the required electric field intensity $e_{e,o}$ to produce this temperature.

(c) Plot the variation of $T(r)$ with respect to r for several values of $e_{e,o}$.

SOLUTION:

(a) The energy equation for steady state conduction with an energy conservation in cylindrical coordinate is given by (3.33), i.e.,

$$\nabla \cdot q_k = -\frac{1}{r}\frac{d}{dr}rk\frac{dT}{dr} = \dot{s}_{e,m}(r),$$

$$\dot{s}_{e,m}(r) = 2\pi f \epsilon_o \epsilon_{ec} \overline{e}_e^2, \quad \overline{e}_e^2 = e_{e,o}^2\left(\frac{R_e}{r}\right)^2.$$

Then

$$-\frac{1}{r}\frac{d}{dr}rk\frac{dT}{dr} = 2\pi f \epsilon_o \epsilon_{ec} e_{e,o}^2\left(\frac{R_e}{r}\right)^2.$$

Using $a = 2\pi f \epsilon_o \epsilon_{ec} e_{e,o}^2 R_e^2$, we have

$$\begin{aligned}
-\frac{1}{r}\frac{d}{dr} rk \frac{dT}{dr} &= a\frac{1}{r^2} \\
-\frac{dT}{dr} &= \frac{a}{k}\frac{\ln r}{r} + \frac{a_1}{k}\frac{1}{r} \\
-\int_{T_e}^{T_o} dT &= \int_{R_e}^{R_o} \frac{a}{k}\frac{\ln r}{r} dr + \int_{R_e}^{R_o} \frac{a_1}{k}\frac{1}{r} dr \\
dT &= \frac{a(\ln r)^2}{2k} + \frac{a_1}{k}\ln r + a_2.
\end{aligned}$$

To solve for the constants a_1 and a_2, we use the following bounding surface thermal conditions:

$$\begin{aligned}
\text{at } r &= R_e, \quad \frac{\partial T}{\partial r} = 0 \\
\text{at } r &= R_o, \quad T = T_o.
\end{aligned}$$

Then

$$0 = \frac{a}{k}\frac{\ln r}{r} + \frac{a_1}{kr}, \qquad a_1 = -a \ln R_e$$

and

$$\begin{aligned}
T_e - T_o &= \frac{a}{k}\frac{(\ln R_o)^2}{2} - \frac{a \ln R_e \ln R_o}{k} + a_2 \\
a_2 &= \frac{a}{k}\ln R_e \ln R_o - \frac{a}{k}\frac{(\ln R_o)^2}{2} + (T_e - T_o).
\end{aligned}$$

Substituting for a_1 and a_2, the radial temperature distribution is

$$T(r) = T_o - \frac{a}{k}\left[\frac{(\ln r)^2}{2} - \ln R_e \ln r + \ln R_e \ln R_o - \frac{(\ln R_o)^2}{2}\right].$$

(b) Using $T_e = 355$ K and $T_o = 310.65$ K (at $r = R_e = 1$ mm and $R_o = 1$ cm, respectively), we have $a_1 = 0.047$ W/m-K, $a_1/k = 1.04$, and the required $e_{e,o}$ is 160 V/m.

(c) Figure Pr.3.36(b) shows the variation of temperature $T(r)$ with respect to r, for several values of $e_{e,o}$.

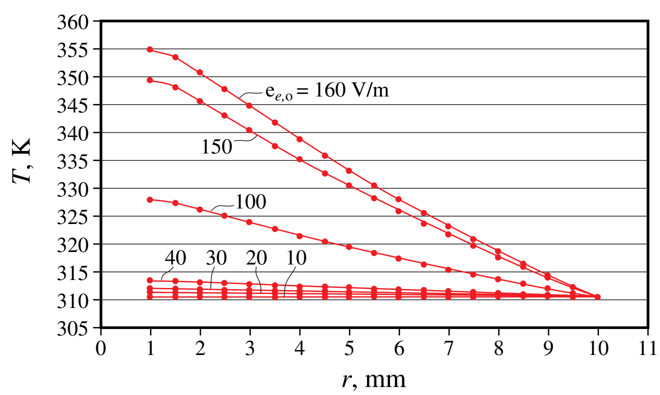


Figure Pr.3.36(b) Variation of $T(r)$ with respect to r for several values of $e_{e,o}$.

COMMENT:

Note that the zero derivative of the temperature at $r = R_e$ shows no heat loss across this surface.

PROBLEM 3.37.FUN

GIVEN:
The thermoelectric figure of merit $Z_e = \alpha_{\rm S}^2/(R_e/R_k)$ is maximized by minimizing R_e/R_k.

OBJECTIVE:
Begin with the relations for R_e and R_k, and the electrical and thermal properties, i.e., $\rho_{e,n}$, $\rho_{e,p}$. Then calculate k_n, k_p, and the geometrical parameters L_n, $A_{k,n}$, L_p, and $A_{k,p}$, as given by (3.116).
(a) Then assume that $L_n = L_p$ and minimize R_e/R_k with respect to $A_{k,p}/A_{k,n}$. Then show that

$$\frac{L_n A_{k,p}}{L_p A_{k,n}} = \frac{A_{k,p}}{A_{k,n}} = \left(\frac{\rho_{e,p} k_n}{\rho_{e,n} k_p}\right)^{1/2} \quad \text{for optimum } Z_e,$$

which is (3.121).
(b) Use this in (3.120), and show that

$$Z_e = \frac{\alpha_{\rm S}^2}{[(k\rho_e)_p^{1/2} + (k\rho_e)_n^{1/2}]}, \quad \text{optimized figure of merit.}$$

SOLUTION:
(a) From (3.116), we have

$$\begin{aligned}
\frac{R_e}{R_k} &= \left[\left(\frac{\rho_e L}{A_k}\right)_p + \left(\frac{\rho_e L}{A_k}\right)_n\right]\left[\left(\frac{A_k k}{L}\right)_p + \left(\frac{A_k k}{L}\right)_n\right] \\
&= (\rho_e k)_p + (\rho_e k)_n + \rho_{e,p} k_n \frac{A_{k,n}}{A_{k,p}}\frac{L_p}{L_n} + \rho_{e,n} k_p \frac{A_{k,p}}{A_{k,n}}\frac{L_n}{L_p}.
\end{aligned}$$

Using $L_n = L_p$, we have

$$\frac{R_e}{R_k} = (\rho_e k)_p + (\rho_e k)_n + \rho_{e,p} k_n \frac{A_{k,n}}{A_{k,p}} + \rho_{e,n} k_p \frac{A_{k,p}}{A_{k,n}}.$$

Taking the derivative with respect to $A_{k,p}/A_{k,n}$, and setting the resultant to zero, we have

$$\frac{d(R_e/R_k)}{d(A_{k,p}/A_{k,n})} = -\rho_{e,p} k_n \left(\frac{A_{k,n}}{A_{k,p}}\right)^2 + \rho_{e,n} k_p = 0,$$

or, while remembering that $L_n = L_p$,

$$\frac{L_n A_{k,p}}{L_p A_{k,n}} = \left(\frac{\rho_{e,p} k_n}{\rho_{e,n} k_p}\right)^{1/2}.$$

(b) Making this substitution, we have

$$\begin{aligned}
\frac{R_e}{R_k} &= (\rho_e k)_p + (\rho_e k)_n + \rho_{e,p} k_n \left(\frac{\rho_{e,n} k_p}{\rho_{e,p} k_n}\right)^{1/2} + \rho_{e,n} k_p \left(\frac{\rho_{e,p} k_n}{\rho_{e,n} k_p}\right)^{1/2} \\
&= (\rho_e k)_p + (\rho_e k)_n + 2(\rho_{e,p} k_n)^{1/2}(\rho_{e,n} k_p)^{1/2} \\
&= [(\rho_e k)_p^{1/2} + (\rho_e k)_n^{1/2}]^2.
\end{aligned}$$

Then (3.120) becomes

$$Z_e = \frac{\alpha_{\rm S}^2}{[(k\rho_e)_p^{1/2} + (k\rho_e)_n^{1/2}]^2}.$$

COMMENT:
The length requirement $L_p = L_n$ is indeed a practical necessity.

PROBLEM 3.38.FUN

GIVEN:

Begin with the differential-length energy equation (3.102), and use the prescribed thermal boundary conditions (3.103) for a finite length slab of thickness L.

OBJECTIVE:

(a) Derive the temperature distribution given by (3.104).

(b) Show that the location of the maximum temperature is

$$x(T_{max}) = \frac{L}{2} + \frac{k(T_h - T_c)}{\rho_e j_e^2 L}.$$

(c) Comment on this location for the case of (i) $j_e \rightarrow 0$, and (ii) $j_e \rightarrow \infty$.

SOLUTION:

(a) Starting with (3.102), i.e.,

$$-k\frac{d^2T}{dx^2} = \rho_e j_e^2, \quad \frac{d^2T}{dx^2} = -\frac{\rho_e j_e^2}{k},$$

and performing the integration once, we have

$$\frac{dT}{dt} = -\frac{\rho_e j_e^2}{k}x + a_1.$$

Integrating once more, we have

$$T(x) = -\frac{\rho_e j_e^2}{2k}x^2 + a_1 x + a_2.$$

Now, the conditions at $x = 0$ and $x = L$ are given by (3.103), i.e.,

$$T(x = 0) = T_c, \quad T(x = L) = T_h.$$

Using the first of these conditions, we have

$$T(x = 0) = T_c = -\frac{\rho_e j_e^2}{2k}(0)^2 + a_1(0) + a_2$$

or

$$a_2 = T_c.$$

Using the second of these conditions, we have

$$T(x = L) = T_h = -\frac{\rho_e j_e^2}{2k}L^2 + a_1 L + T_c$$

or

$$a_1 = \frac{T_h - T_c}{L} + \frac{\rho_e j_e^2 L}{2k}.$$

Then for a_1 and a_2, we have

$$T(x) = T_c + \frac{x}{L}(T_h - T_c) + \frac{\rho_e j_e^2}{2k}x(L - x).$$

(b) By differentiating the temperature distribution with respect to x, and setting the resultant to zero, we have

$$\begin{aligned}\frac{dT(x)}{dx} &= \frac{T_h - T_c}{L} + \frac{\rho_e j_e^2}{2k}(L - x) - \frac{\rho_e j_e^2 x}{2k} \\ &= \frac{T_h - T_c}{L} + \frac{\rho_e j_e^2 L}{2k} - \frac{\rho_e j_e^2 x}{k} = 0.\end{aligned}$$

Solving for x, we have

$$x(T_{max}) = \frac{k(T_h - T_c)}{\rho_e j_e^2 L} + \frac{L}{2}.$$

(c) For (i), where $j_e = 0$, we have $x(T_{max})_{j_e=0}$, i.e., there is not a location for the maximum temperature within $0 \leq x \leq L$. This indicates that no maximum will occurs (i.e., linear temperature distribution).
For (ii), where $j_e \to \infty$, we have $x(T_{max}) = L/2$, i.e., for a finite difference between T_h and T_c, the location of the maximum temperature is at the center (for a very large volumetric Joule heating rate).

COMMENT:

As shown in Figure 3.27, as j_e is increased, $x(T_{max})$ moves toward the center. Note that there is current for which $x(T_{max}) = L$, i.e., the hot surface will be the location of T_{max} (with no gradient in temperature, and therefore, no conduction at $x = L$). This current density is found by

$$x(T_{max}) = L = \frac{L}{2} + \frac{k(T_h - T_c)}{\rho_e j_e^2 L}$$

or

$$j_e = \left[\frac{2k(T_h - T_c)}{\rho_e L^2}\right]^{1/2}.$$

PROBLEM 3.39.FUN

GIVEN:

The optimum coefficient of performance for the thermoelectric cooler η_{cop}, given by (3.126), is based on a current that optimizes it. This current is also given in (3.126).

OBJECTIVE:

Derive the expression for the optimum current, i.e.,

$$J_e[\partial \eta_{cop}/\partial J_e = 0] = \frac{\alpha_{\rm S}(T_h - T_c)}{R_{e,h\text{-}c}[(1+Z_eT_o)^{1/2} - 1]},$$

where Z_e is given by (3.120) and

$$T_o = \frac{T_h + T_c}{2}.$$

SOLUTION:

Starting from (3.126), we differentiate η_{cop} with respect to J_e and we have

$$\frac{\partial \eta_{cop}}{\partial J_e} = \frac{\alpha_{\rm S} J_e T_c - R_{e,h\text{-}c}J_e}{R_{e,h\text{-}c}J_e^2 + \alpha_{\rm S}J_e(T_h - T_c)} - \frac{[\alpha_{\rm S}T_c - R_{e,h\text{-}c}^{-1}(T_h - T_c) - \frac{1}{2}R_{e,h\text{-}c}J_e^2][2R_{e,h\text{-}c}J_e + \alpha_{\rm S}(T_h - T_c)]}{[R_{e,h\text{-}c}J_e^2 + \alpha_{\rm S}J_e(T_h - T_c)]^2} = 0$$

or

$$(\alpha_{\rm S}T_c - R_{e,h\text{-}c}J_e)[R_{e,h\text{-}c}J_e^2 + \alpha_{\rm S}J_e(T_h - T_c)] - [\alpha_{\rm S}J_eT_c + R_{k,h\text{-}c}^{-1}(T_h - T_c) + \frac{1}{2}R_{e,h\text{-}c}J_e^2][2R_{e,h\text{-}c}J_e + \alpha_{\rm S}(T_h - T_c)] = 0$$

or

$$\begin{aligned}
&\alpha_{\rm S}T_cR_{e,h\text{-}c}J_e^2 - R_{e,h\text{-}c}^2J_e^3 + \alpha^2T_cJ_e(T_h - T_c) - R_{e,h\text{-}c}J_e^2\alpha_{\rm S}(T_h - T_c) - 2\alpha_{\rm S}J_e^2T_cR_{e,h\text{-}c} - \\
&\alpha_{\rm S}^2J_eT_c(T_h - T_c) + 2R_{k,h\text{-}c}^{-1}(T_h - T_c)J_eR_{e,h\text{-}c} + R_{k,h\text{-}c}^{-1}(T_h - T_c)^2\alpha_{\rm S} + R_{e,h\text{-}c}^2J_e^3 + \frac{1}{2}R_{e,h\text{-}c}J_e^2\alpha_{\rm S}(T_h - T_c) = 0.
\end{aligned}$$

Then

$$-\alpha_{\rm S}J_e^2T_cR_{e,h\text{-}c} - \frac{1}{2}R_{e,h\text{-}c}J_e^2\alpha_{\rm S}^2(T_h - T_c) + 2R_{k,h\text{-}c}^{-1}R_{e,h\text{-}c}J_e(T_h - T_c) + R_{k,h\text{-}c}^{-1}(T_h - T_c)^2\alpha_{\rm S} = 0.$$

By combining the terms containing J_e^2, we have

$$R_{e,h\text{-}c}\alpha_{\rm S}\left(\frac{T_h - T_c}{2} + T_c\right)J_e^2 - 2R_{k,h\text{-}c}^{-1}R_{e,h\text{-}c}(T_h - T_c)J_e - R_{k,h\text{-}c}^{-1}\alpha_{\rm S}(T_h - T_c) = 0.$$

The acceptable solution to this quadratic equation is

$$J_e = \frac{2R_{k,h\text{-}c}^{-1}R_{e,h\text{-}c}(T_h - T_c) + \left\{[2R_{k,h\text{-}c}^{-1}R_{e,h\text{-}c}(T_h - T_c)]^2 + 4R_{k,h\text{-}c}^{-1}R_{e,h\text{-}c}\alpha_{\rm S}^2(T_h - T_c)^2\left(\frac{T_h - T_c}{2} + T_c\right)\right\}^{1/2}}{2R_{e,h\text{-}c}\alpha_{\rm S}\left(\frac{T_h - T_c}{2} + T_c\right)}.$$

Then

$$\begin{aligned}
J_e &= R_{k,h\text{-}c}^{-1}R_{e,h\text{-}c}(T_h - T_c)\frac{1 + \left[1 + Z_e\left(\frac{T_h - T_c}{2} + T_c\right)\right]^{1/2}}{R_{e,h\text{-}c}\alpha_{\rm S}\left(\frac{T_h - T_c}{2} + T_c\right)} \\
&= R_{k,h\text{-}c}^{-1}R_{e,h\text{-}c}(T_h - T_c)\frac{1 + (1 + Z_eT_o)^{1/2}}{R_{e,h\text{-}c}\alpha_{\rm S}T_o} \\
&= \frac{R_{k,h\text{-}c}^{-1}(T_h - T_c)}{\alpha_{\rm S}}\frac{1 + (1 + Z_eT_o)^{1/2}}{T_o}, \quad T_o = \frac{T_h + T_c}{2}, \quad Z_e = \frac{\alpha_{\rm S}^2}{R_{k,h\text{-}c}^{-1}R_{e,h\text{-}c}}.
\end{aligned}$$

This can be rearranged by multiplying and dividing to achieve

$$\begin{aligned} J_e &= \frac{R_{k,h\text{-}c}^{-1}(T_h - T_c)}{\alpha_\mathrm{S}} \left[\frac{R_{e,h\text{-}c}}{R_{k,h\text{-}c}}\right] \frac{1 + (1 + Z_e T_o)^{1/2}}{T_o} \left[\frac{(1 + Z_e T_o)^{1/2} - 1}{(1 + Z_e T_o)^{1/2} - 1}\right] \\ &= \frac{\alpha_\mathrm{S}(T_h - T_c)}{R_{e,h\text{-}c}[(1 + Z_e T_o)^{1/2} - 1]}. \end{aligned}$$

COMMENT:

To arrive at the optimum η_{cop}, we substitute this current in the definition for η_{cop}, i.e., (3.126), and expand the expressions and then re-combine them.

PROBLEM 3.40.FUN

GIVEN:

In thermoelectric cooling, the lowest temperature for the cold junction T_c corresponds to $Q_c = 0$ and is given by (3.119). Further lowering of T_c is possible by using thermoelectric units in averaged stages. A two-stage unit is shown in Figure Pr.3.40(a). Assume that the temperature drops across the electrical conductor and insulator are negligible such that the top junction is at T_c, the intermediate junction is at T_1, and the lower junction is at T_h. Also assume no heat loss at the T_1 junction, $Q_1 = 0$, Use the bismuth telluride p-n pair and assume $Q_c = 0$.

$a = 1$ mm, $L = 1.5$ mm, $T_h = 40$℃, $J_e = 4$ A.

SKETCH:

Figure Pr.3.40(a) shows the two-stage thermoelectric cooler. The temperature drops across the electrical insulators and conductors are assumed to be negligible.

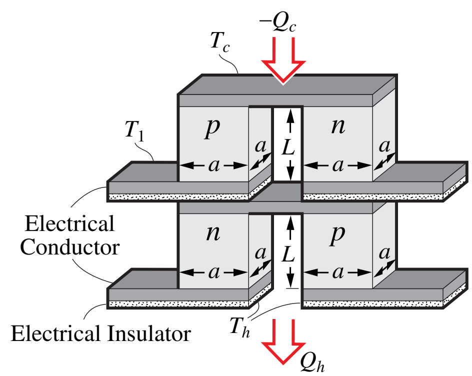


Figure Pr.3.40(a) A two-stage thermoelectric cooler unit.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Show that $T_c(Q_c = 0)$ is given by

$$T_c = \frac{\dfrac{R_c J_e^2 + \dfrac{T_h}{R_k}}{R_k \alpha_S J_e + 2} + \dfrac{1}{2} R_e J_e^2}{\alpha_S J_e + \left(1 - \dfrac{1}{R_k \alpha_S J_e + 2}\right) \dfrac{1}{R_k}}.$$

(c) Start by writing the junction energy equation (3.115) for the T_c and T_1 junctions. Determine $T_c(Q_c = 0)$ for the above conditions.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.3.40(b). Heat losses from junctions T_c and T_1 are assumed to be zero.

(b) The energy equations for T_c and T_1 nodes are found using (3.115), i.e.,

$$\begin{aligned} Q_c + \frac{T_c - T_1}{R_k} &= -\alpha_S J_e T_c + \frac{1}{2} R_e J_e^2 \\ Q_1 + \frac{T_1 - T_c}{R_k} + \frac{T_1 - T_h}{R_k} &= -\alpha_S J_e T_1 + R_e J_e^2, \end{aligned}$$

where we have used the same R_k and R_e for both stages.

Noting that $Q_c = Q_1 = 0$, we need to eliminate T_1 between these two equations and then solve for T_c. From the second equation, we have

$$\alpha_S J_e T_1 + \frac{2T_1}{R_k} = \frac{T_h + T_c}{R_k} + R_e J_e^2$$

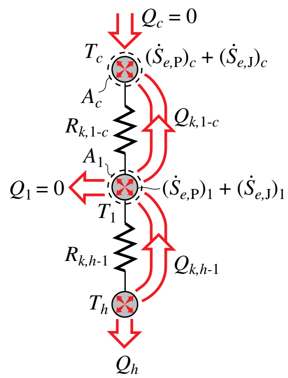


Figure Pr.3.40(b) Thermal circuit diagram.

or

$$T_1 = \frac{\dfrac{T_h + T_c}{R_k} + R_e J_e^2}{\dfrac{2}{R_k} + \alpha_{\mathrm{S}} J_e}.$$

Now substituting this in the first equation, we have

$$\begin{aligned}
0 = -\alpha_{\mathrm{S}} J_e T_c + \frac{T_1}{R_k} - \frac{T_c}{R_k} + \frac{1}{2} R_e J_e^2 = \\
-\alpha_{\mathrm{S}} J_e T_c + \frac{1}{R_k} \frac{R_e J_e^2 + \dfrac{T_h + T_c}{R_k}}{\alpha_{\mathrm{S}} J_e + \dfrac{2}{R_k}} - \frac{T_c}{R_k} + \frac{1}{2} R_e J_e^2 = \\
-\alpha_{\mathrm{S}} J_e T_c + \frac{R_e J_e^2 + \dfrac{T_h}{R_k}}{R_k \alpha_{\mathrm{S}} J_e + 2} + \left(\frac{1}{R_k \alpha_{\mathrm{S}} J_e + 2} - 1 \right) \frac{T_c}{R_k} + \frac{1}{2} R_e J_e^2.
\end{aligned}$$

Solving for T_c we have

$$T_c = \frac{\dfrac{R_e J_e^2 + \dfrac{T_h}{R_k}}{R_k \alpha_{\mathrm{S}} J_e + 2} + \dfrac{1}{2} R_e J_e^2}{\alpha_{\mathrm{S}} J_e - \left(\dfrac{1}{R_k \alpha_{\mathrm{S}} J_e + 2} - 1 \right) \dfrac{1}{R_k}}.$$

(c) From Example 3.14, for a bismuth telluride pair and for the given geometry, we have

$$\begin{aligned}
\alpha_{\mathrm{S}} &= 4.4 \times 10^{-4} \text{ V/K} \\
R_e &= 0.030 \text{ ohm} \\
R_k &= 4.762 \times 10^2 \text{ K/W}.
\end{aligned}$$

Then

$$T_c = \frac{\dfrac{0.030(\text{ohm}) \times 4^2(\text{A}^2) + \dfrac{(273.15 + 40)(\text{K})}{476.2(\text{K/W})}}{476.2(\text{K/W}) \times 4.4 \times 10^{-4}(\text{V/K}) \times 4(\text{A}) + 2} + \dfrac{1}{2} \times 0.030(\text{ohm}) \times 4^2(\text{A}^2)}{4.4 \times 10^{-4}(\text{V/K}) \times 4(\text{A}) - \left[\dfrac{1}{476.2(\text{K/W}) \times 4.4 \times 10^{-2}(\text{V/K}) \times 4(\text{A}) + 2} - 1 \right] \dfrac{1}{476.2(\text{K/W})}}.$$

$$
\begin{aligned}
T_c &= \frac{\dfrac{0.48 + 0.6576}{0.8381 + 2} + 0.24}{1.760 \times 10^{-3} - \left(\dfrac{1}{0.8381 + 2} - 1\right)\dfrac{1}{476.2}} \frac{\text{ohm-A}^2}{\text{V-A/K}} \\
&= \frac{0.4008 + 0.24}{1.760 \times 10^{-3} + 1.360 \times 10^{-3}} \frac{\text{V/A A}^2}{\text{V-A/K}} \\
&= 205.4 \text{ K}.
\end{aligned}
$$

COMMENT:

Note that for a single-stage unit, from (3.115) we have

$$
\begin{aligned}
T_c(Q_c = 0) &= \frac{\dfrac{1}{2} R_e J_e^2 + \dfrac{T_h}{R_k}}{\alpha_{\text{S}} J_e + \dfrac{1}{R_k}} \\
&= \frac{\dfrac{1}{2} \times 0.030 \times 4^2 + \dfrac{313.15}{476.2}}{4.4 \times 10^{-4} \times 4 + \dfrac{1}{476.2}} \\
&= \frac{0.24 + 0.6576}{1.760 \times 10^{-3} + 2.10 \times 10^{-3}} = 232.5 \text{ K}.
\end{aligned}
$$

As expected, this is higher than T_c found in part (c) for the two-stage thermoelectric cooler unit.

PROBLEM 3.41.DES

GIVEN:

An in-plane thermoelectric device is used to cool a microchip. It has bismuth telluride elements with dimensions given below. A contact resistance $R_{k,c} = l_c/A_k k_c$, given by (3.94), is present between the elements and the connector. These are shown in Figure Pr.3.41(a). The contact conductivities k_c is empirically determined for two different connector materials and are

$$k_c = 10k_{TE} \qquad \text{copper connector}$$
$$k_c = k_{TE} \qquad \text{solder connector,}$$

where k_{TE} is the average of p- and n-type materials.

$L = 150\ \mu$m, $w = 75\ \mu$m, $a = 4\ \mu$m, $T_h = 300$ K.

SKETCH:

Figure Pr.3.41(a) shows the miniaturized thermoelectric cooler unit.

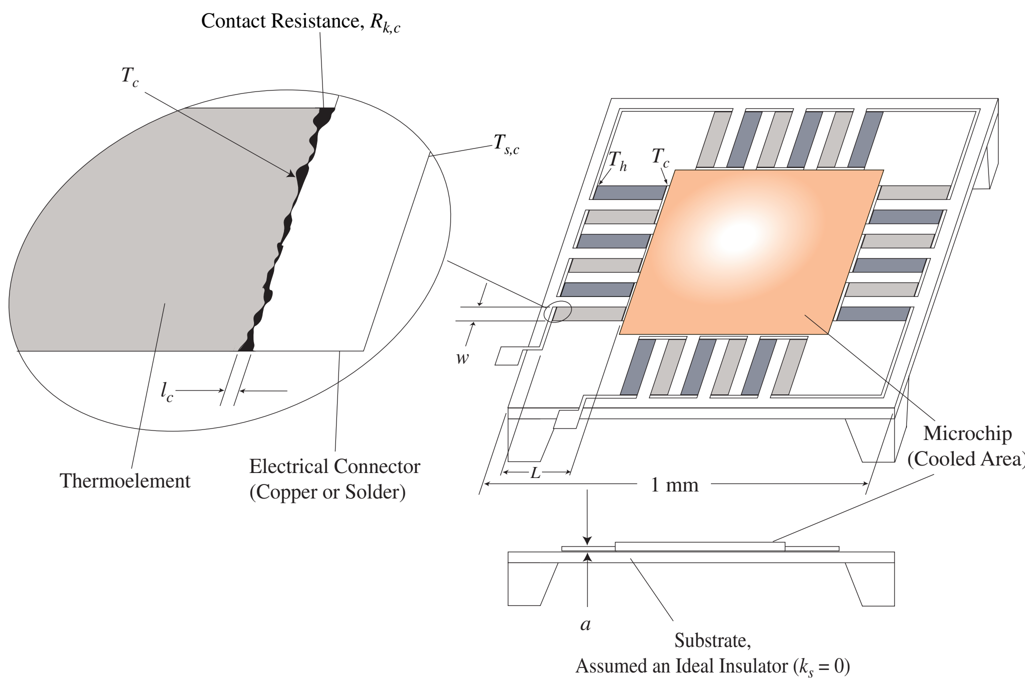


Figure Pr.3.41(a) A miniaturized thermoelectric device with a thermal contact resistance between the thermoelectric elements and the connectors.

OBJECTIVE:

(a) Draw the thermal circuit diagram showing the contact resistance at each end of the elements.
(b) Determine the optimum current for cold junction temperature $T_c = 275$ K.
(c) Determine the minimum T_c [i.e., (3.119)] for these conditions.
(d) For $T_c = 275$ K, determine $Q_{c,max}$ from (3.118).
(e) Using this $Q_{c,max}$ and (3.96), determine $\Delta T_c = T_{s,c} - T_c$ and plot $T_{s,c}$ versus l_c for $0 < l_c < 10\ \mu$m, for both the copper and solder connectors.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.3.41(b).

(b) The current for this temperature is given by (3.117), i.e.,

$$J_e = \frac{\alpha_S T_c}{R_{e,h\text{-}c}}.$$

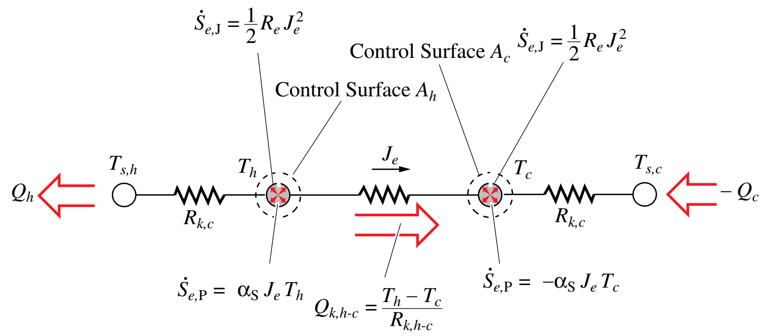


Figure Pr.3.41(b) Thermal circuit diagram.

The electrical resistance is given by (3.116) and, using the ρ_e values from Table C.9(a), we have

$$\begin{aligned}
\alpha_{S,p} &= 230 \times 10^{-6} \text{ V/K} && \text{Table C.9(a)}\\
\alpha_{S,n} &= -210 \times 10^{-6} \text{ V/K} && \text{Table C.9(a)}\\
\rho_e &= 10^{-5} \text{ ohm-m} && \text{Table C.9(a)}\\
k_p &= 1.70 \text{ W/m-K} && \text{Table C.9(a)}\\
k_n &= 1.45 \text{ W/m-K} && \text{Table C.9(a)},
\end{aligned}$$

$$\begin{aligned}
R_{e,h\text{-}c} &= \frac{\rho_{e,p} L_p}{A_p} + \frac{\rho_{e,n} L_n}{A_n} = \frac{L}{wa}(\rho_{e,p} + \rho_{e,n})\\
&= \frac{1.5 \times 10^{-4}(\text{m})}{7.5 \times 10^{-5}(\text{m}) \times 4 \times 10^{-6}(\text{m})} \times 2 \times (10^{-5})(\text{ohm-m}) = 10 \text{ ohm}.
\end{aligned}$$

Using $\alpha_S = \alpha_{S,p} - \alpha_{S,n} = 440\ \mu\text{V/K}$, the current is then

$$J_e = \frac{(440 \times 10^{-6})(\text{V/K}) \times 275(\text{K})}{10(\text{ohm})} = 1.210 \times 10^{-2} \text{ A}.$$

(c) The minimum T_c for a given T_h is given by (3.119) as

$$\begin{aligned}
(T_h - T_c)_{max} &= \frac{\alpha_S^2 T_c^2}{2R_{e,h\text{-}c}/R_{k,h\text{-}c}}\\
R_{k,h\text{-}c}^{-1} &= \frac{A_{k,p} k_p}{L_p} + \frac{A_{k,n} k_n}{L_n} = \frac{wa}{L}(k_p + k_n) = \frac{7.5 \times 10^{-5}(\text{m}) \times 4 \times 10^{-6}(\text{m})}{1.5 \times 10^{-4}(\text{m})}(1.70 + 1.45)(\text{W/m-K})\\
&= 6.360 \times 10^{-6} \text{ W/K}.
\end{aligned}$$

This minimum T_c is found from

$$\begin{aligned}
300(\text{K}) - T_c(\text{K}) &= \frac{[440 \times 10^{-6}(\text{V/K})]^2 T_c^2(\text{K}^2)}{2 \times 10(\text{ohm}) \times 6.360 \times 10^{-6}(\text{W/K})}\\
&= 1.522 \times 10^{-3} T_c^2(\text{K}^2)
\end{aligned}$$

or

$$T_{c,min} = 223.8 \text{ K}.$$

(d) For $T_c = 275$ K, $Q_{c,max}$ can be found by (3.118) as

$$\begin{aligned}
Q_{c,max} &= -\frac{\alpha_S^2 T_c^2}{2R_{e,h\text{-}c}} + R_{k,h\text{-}c}^{-1}(T_h - T_c)\\
&= -\frac{[440 \times 10^{-6}(\text{V/K})]^2 \times (275)^2(\text{K}^2)}{2 \times 10(\text{ohm})} + 6.360 \times 10^{-6}(\text{W/K}) \times (25)(\text{K}) = (-7.321 \times 10^{-4} + 1.590 \times 10^{-4})(\text{W})\\
&= -5.731 \times 10^{-4} \text{ W}.
\end{aligned}$$

(e) The value of the average thermoelectric conductivity is

$$k_{TE} = \frac{k_p + k_n}{2} = \frac{1.70 + 1.45}{2} = 1.575 \text{ W/m-K}.$$

The value of the gap conductivity can be found using the given relationships to be

$$\begin{aligned} k_c &= 10k_{TE} = 15.75 \text{ W/m-K} \quad \text{copper connector} \\ k_c &= k_{TE} = 1.575 \text{ W/m-K} \quad \text{solder connector.} \end{aligned}$$

By varying values of l_c, the thermal contact resistance can then be found using (3.94),

$$R_{k,c} = \frac{l_c}{k_c A_k} = \frac{l_c}{k_c wa}.$$

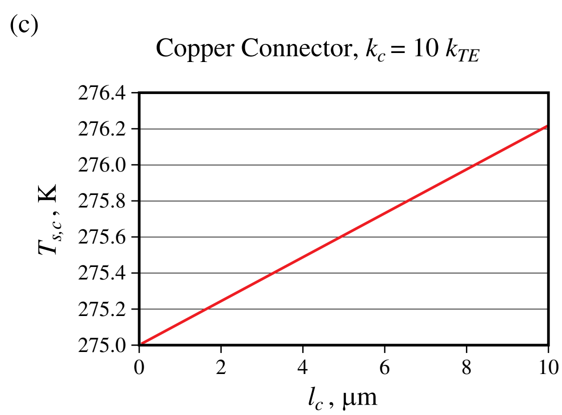
Finally, $T_{s,c}$ is found using (3.127) as

$$Q_c = \frac{T_c - T_{s,c}}{R_{k,c}}$$

or

$$T_{s,c} = T_c - Q_c R_{k,c} = 275(\text{K}) + 5.731 \times 10^{-4}(\text{W}) \times \frac{l_c}{3 \times 10^{-10}(\text{m}^2) k_c}.$$

The plots of $T_{s,c}$ versus l_c for both the copper and solder connectors are shown in Figures Pr.3.41(c) and (d). Note that while the contact resistance of the copper connector is negligible, that of the solder is not.



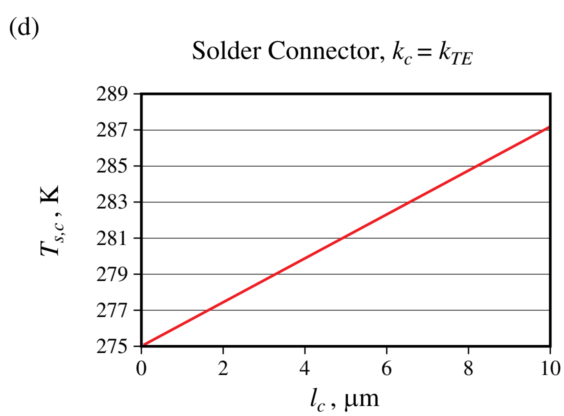


Figure Pr.3.41 Variation of the rise in cold junction temperature across the contact resistance, with respect to the contact gap length, for (c) copper and (d) solder connectors.

COMMENT:

As l_c increases, the value of $T_{s,c}$ increases linearly. For the copper connectors, and for $l_c = 10$ μm, the temperature rise across the contact is 1.213℃ (4.852 percent the temperature difference of 25℃). The cold side temperature is still far enough below $T_h = 300$ K to cool the microchip. If the solder connector is used, then the value of $T_{s,c}$ will rise about 12.13℃ (48.52 percent the temperature difference). For this reason, copper or other materials with high thermal and electrical conductivity are used for the connections between the thermoelectric pairs. Note that the substrate thermal conductivity is assumed zero. In practice this has a finite value and reduces the performance.

PROBLEM 3.42.FAM

GIVEN:

To melt the ice forming on a road pavement (or similarly to prevent surface freezing), pipes are buried under the pavement surface, as shown in Figure Pr.3.42(a). The pipe surface is at temperature T_1, while the surface is at temperature T_2. The magnitude of the geometrical parameters for the buried pipes are given below.

Use the thermophysical properties of soil (Table C.17).

$L = 5$ cm, $D = 1$ cm, $l = 2$ m, $w = 20$ cm, $T_1 = 10$°C, $T_2 = 0$°C.

Assume that the conduction resistances given in Table 3.3(a) are applicable.

SKETCH:

Figure Pr.3.42(a) shows the hot-water carrying buried pipes.

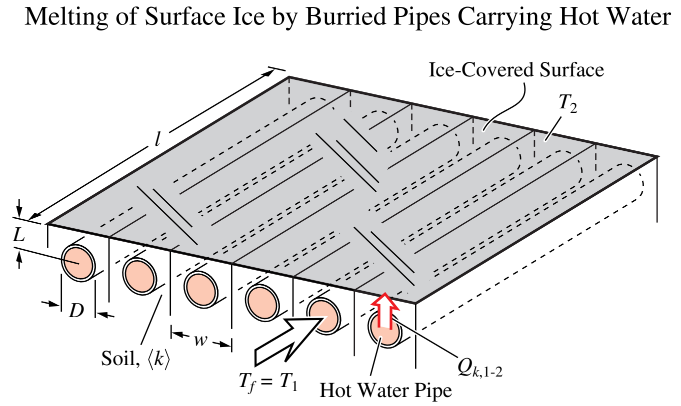


Figure Pr.3.42(a) Hot-water carrying buried pipes used for melting of an ice layer on a pavement surface.

OBJECTIVE:

(a) Draw the thermal circuit diagram for each pipe.

(b) From Table 3.3(a), determine the conduction resistance for (i) a single pipe (i.e., cylinder) independent of the adjacent pipes, and (ii) a pipe in a row of cylinders with equal depth and an axial center-to-center spacing w.

(c) Determine $Q_{k,1\text{-}2}$ per pipe for both cases (i) and (ii), and then compare.

SOLUTION:

(a) Figure Pr.3.42(b) shows the thermal circuit diagram for each pipe.

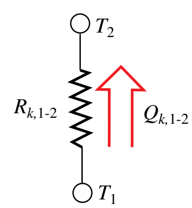


Figure Pr.3.42(b) Thermal circuit diagram.

(b) The conduction resistance for each pipe is given by Table 3.3(a), i.e.,

$$R_{k,1\text{-}2} = \frac{\ln\left(\dfrac{4L}{D}\right)}{2\pi k l} \qquad \text{for} \quad D < L < l.$$

Using (3.128), we have

$$Q_{k,1\text{-}2} = \frac{T_1 - T_2}{R_{k,1\text{-}2}}.$$

From Table C.17, we have

$$\text{soil:} \qquad \langle k \rangle = 0.52 \text{ W/m-K} = 0.52 \text{ W/m-°C} \quad \text{Table C.17}$$

Then

$$\begin{aligned} R_{k,1\text{-}2} &= \frac{\ln\left[\dfrac{4 \times 0.05(\text{m})}{0.01(\text{m})}\right]}{2 \times \pi \times 0.52(\text{W/m-°C}) \times 2(\text{m})} \\ &= \frac{\ln(20)}{6.535}(\text{°C/W}) = 0.4584\text{°C/W for each pipe.} \end{aligned}$$

(c) For the heat transfer per pipe, we have

$$Q_{k,1\text{-}2}|_{\text{for each pipe}} = \frac{(10-0)(\text{°C})}{0.4584(\text{°C/W})} = 21.81 \text{ W}.$$

COMMENT:

The center-to-center spacing of adjacent pipes w was assumed to be sufficiently large such that the heat transfer from each pipe could be assumed independent of the adjacent pipes. As w decreases, the effect of the adjacent pipes on the heat transfer of a single pipe must be considered. The correlation for the thermal resistance from a single cylinder to a row of cylinders at equal depth in a semi-infinite solid, as a function of center-to-center spacing w, is given in Table 3.3(a),i.e.,

$$R_{k,1\text{-}2} = \frac{\ln\left[\left(\dfrac{w}{\pi R}\right)\sinh\left(\dfrac{2\pi L}{w}\right)\right]}{2\pi k l} \quad \text{for each cylinder in a row of cylinders.}$$

The variation of $R_{k,1\text{-}2}$ for each pipe of a row of cylinders as a function of the spacing w is given in Figure Pr.3.42(c). Also shown is the $R_{k,1\text{-}2}$ for a single cylinder. Note that the thermal resistance for each pipe in the row increases as w decreases. This is because the adjacent pipes raise the temperature of the solid medium, resulting in a decrease in the heat transfer from each pipe.

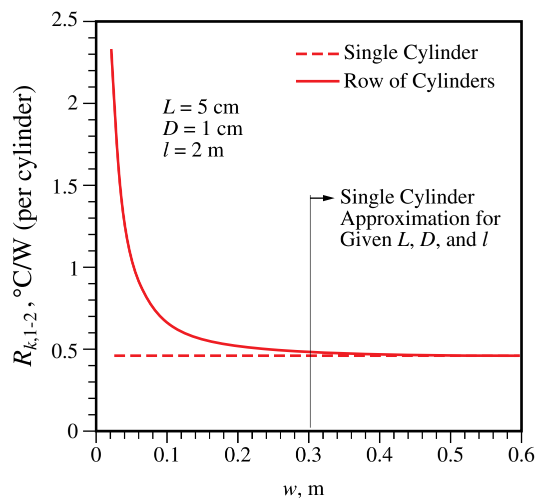


Figure Pr.3.42(c) Variation of conduction resistance with respect to pipe spacing.

PROBLEM 3.43.FUN

GIVEN:

The steady-state conduction in a two-dimensional, rectangular medium, as shown in Figure Pr.3.43, is given by the differential volume energy equation (B.55), i.e.,

$$\nabla q_k = -k\frac{\partial^2 T}{\partial x^2} - k\frac{\partial^2 T}{\partial y^2} = 0.$$

This is called a homogeneous, linear, partial differential equation and a general solution that separates the variable is possible, if the boundary (bounding-surface) conditions can also be homogeneous. This would require that the temperatures on all surfaces be prescribed, or the bounding surface energy equation be a linear resistive type. When the four surface temperatures are prescribed, such that three surfaces have a temperature and different from the fourth, the final solution would have a simple form.

SKETCH:

Figure Pr.3.43 shows the geometry and the prescribed temperatures on the four surfaces.

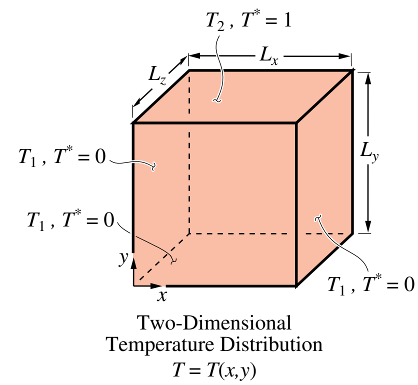


Figure Pr.3.43 A rectangular, two-dimensional geometry with prescribed surface temperatures.

OBJECTIVE:

(a) Using the dimensionless temperature distribution

$$T^* = \frac{T - T_1}{T_2 - T_1},$$

show that the energy equation and boundary conditions become

$$\frac{\partial^2 T^*}{\partial x^2} + \frac{\partial^2 T^*}{\partial y^2} = 0,\ T^* = T^*(x, y)$$

$$T^*(x = 0, y) = 0,\ T^*(x = L_x, y) = 0,\ T^*(x, y = 0) = 0,\ T^*(x, y = L_y) = 1.$$

(b) Use the method of the separation of variables (which is applicable to this homogeneous differential equation with all but one boundary conditions being also homogeneous), i.e.,

$$T^*(x, y) = X(x)Y(y)$$

to show that the energy equation becomes

$$-\frac{1}{X}\frac{d^2X}{dX^2} = \frac{1}{Y}\frac{d^2Y}{dy}.$$

(c) Since the left-hand side is only a function of x and the right-hand side is a function of y, then both sides should be equal to a constant. This is called the separation constant. Showing this constant as b^2, show also that

$$\frac{d^2X}{dx^2} + b^2X = 0$$

$$\frac{d^2Y}{dy^2} + b^2Y = 0.$$

Then show that the solutions for X and Y are

$$\begin{aligned} X &= a_1 \cos(bx) + a_2 \sin(bx) \\ Y &= a_3 e^{-by} + a_4 e^{by} \end{aligned}$$

or

$$T^* = [a_1 \cos(bx) + a_2 \sin(bx)](a_3 e^{-by} + a_4 e^{by}).$$

(d) Apply the homogeneous boundary conditions to show that

$$\begin{aligned} a_1 &= 0 \\ a_3 &= a_4 \\ a_2 a_4 \sin(bL_x)(e^{by} - e^{-by}) &= 0. \end{aligned}$$

Note that the last one would require that

$$\sin(bL_x) = 0$$

or

$$bL_x = n\pi, \quad n = 0, 1, 2, 3, \cdots .$$

(e) Using these, show that

$$\begin{aligned} T^*(x, y) &= a_2 a_4 \sin\left(\frac{n\pi x}{L_x}\right)(e^{n\pi y/L_x} - e^{-n\pi y/L_x}) \\ &\equiv a_n \sin\left(\frac{n\pi x}{L_x}\right) \sinh\left(\frac{n\pi y}{L_x}\right). \end{aligned}$$

Since $n = 1, 2, 3, \cdots$ and differential equation is linear, show that

$$T^* = \sum_{n=0}^{\infty} a_n \sin\left(\frac{n\pi x}{L_x}\right) \sinh\left(\frac{n\pi y}{L_x}\right).$$

(f) Using the last (i.e., nonhomogeneous) boundary condition and the orthogonality condition of the special function $\sin(z)$, it can be shown that

$$a_n = \frac{2[1 + (-1)^{n+1}]}{n\pi \sinh(n\pi L_y/L_x)}, \ n = 0, 1, 2, 3, \cdots .$$

Then express the final solution in terms of this and verify that all boundary conditions are satisfied.

SOLUTION:
(a) Using the prescribed temperatures T_1 and T_2, we have

$$T^*(x, y) = \frac{T(x, y) - T_1}{T_2 - T_1}$$

and the energy equation and boundary conditions become

$$\frac{\partial^2 T^*(x, y)}{\partial x^2} + \frac{\partial^2 T^*(x, y)}{\partial y^2} = 0$$

$$\begin{aligned} T^*(x = 0, y) = T^*(x = L_x, y) = T^*(x, y = 0) &= 0 \\ T^*(x, y = L_y) &= 1. \end{aligned}$$

(b) Using

$$T^*(x, y) = X(x)Y(y)$$

in the above energy equation, we have

$$Y(y)\frac{d^2 X(x)}{dx^2} + X(x)\frac{d^2 Y(y)}{dy^2} = 0$$

or

$$-\frac{1}{X(x)}\frac{d^2X(x)}{dx^2}=\frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2}.$$

This results in two ordinary differential equations being equal, while each side can only be a function of one independent variable. This will only be possible if these two equations are equal to a constant.

(c) Using intuition for the form of the solution, we set this constant equal to b^2. Then we have

$$\begin{aligned}\frac{d^2X(x)}{dx^2}+b^2X(x) &= 0\\ \frac{d^2Y(y)}{dy^2}-b^2Y(y) &= 0.\end{aligned}$$

The solutions to these two differential equations take the form of special sinusoidal and exponential (or hyperbolic) functions, respectively. These are

$$\begin{aligned}X(x) &= a_1\cos(bx)+a_2\sin(bx)\\ Y(y) &= a_3e^{-by}+a_4e^{by}\end{aligned}$$

or

$$T^*(x,y)=[a_1\cos(bx)+a_2\sin(bx)](a_3e^{-by}+a_4e^{by}).$$

(d) Using $T^*(x=0,y)=0$, we have

$$0=(a_1\cos 0+a_2\sin 0)(a_3e^{-by}+a_4e^{by})$$

or

$$a_1=0.$$

Using $T_{x,y=0}=0$, we have

$$0=a_2\sin(bx)(a_3+a_4)$$

or

$$a_3+a_4=0,\quad \text{or}\quad a_3=-a_4.$$

Using $T^*(x=L_x,y)=0$, we have

$$0=a_2\sin(bL_x)(a_3e^{-by}+a_4e^{by})$$

or

$$\sin(bL_x)=0.$$

This would require that

$$b=\frac{n\pi}{L_x},n=0,1,2,3,\cdots.$$

(e) Now combining these, we have

$$\begin{aligned}T^*(x,y) &= a_2a_4\sin\left[\frac{n\pi x}{L_x}\left(e^{\frac{n\pi y}{L_x}}-e^{\frac{-n\pi y}{L_x}}\right)\right]\\ &\equiv a_n\sin\left(\frac{n\pi x}{L_x}\right)\sinh\left(\frac{n\pi y}{L_x}\right),\end{aligned}$$

where we have used

$$\begin{aligned}a_n &= 2a_2a_4\\ \sinh(z) &\equiv \frac{e^z+e^{-z}}{2}.\end{aligned}$$

We expect a_n to depend on n and this will be shown below.

Since $n = 0, 1, 2, 3, \cdots$, and since the energy equation used is a linear differential equation, the sum of the solutions corresponding to $n = 0, n = 1, n = 2$, etc., is also a solution. Then

$$T^*(x, y) = \sum_{n=0}^{\infty} a_n \sin\left(\frac{n\pi x}{L_x}\right) \sinh\left(\frac{n\pi y}{L_x}\right).$$

(f) Using $T^*(x, y = L_y) = 1$, we have

$$1 = \sum_{n=0}^{\infty} a_n \sin\left(\frac{n\pi x}{L_x}\right) \sin\left(\frac{n\pi L_y}{L_x}\right)$$

We now multiply both sides by $\sin(n\pi x/L_x)$ and then integrate the resultants over $0 \le x \le L_x$. Then it can be shown that

$$a_n = \frac{2[1 + (-1)^{n+1}]}{n\pi \sinh(n\pi L_y/L_x)}.$$

Now combining these, we have

$$T^*(x, y) = \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{1 + (-1)^{n+1}}{n} \sin\left(\frac{n\pi x}{L_x}\right) \frac{\sinh\left(\frac{n\pi y}{L_x}\right)}{\sinh\left(\frac{n\pi L_y}{L_x}\right)}.$$

COMMENT:

The series solution would require a large number of terms in order to obtain a smooth temperature distribution. In a later exercise, we will compare this result with that found from a finite-small volume numerical solution.

PROBLEM 3.44.FAM

GIVEN:

In order to maintain a permanent frozen state (permafrost) and a firm ground, heat pipes are used to cool and freeze wet soil in the arctic regions. Figure Pr.3.44(a) shows a heat pipe, which is assumed to have a uniform temperature T_1, placed between the warmer soil temperature T_2, and the colder ambient air temperature $T_{f,\infty}$. The heat transfer between the heat pipe surface and the ambient air is by surface convection and this resistance is given by $A_{ku}R_{ku}$. The heat transfer between the pipe and soil is by conduction. Assume a steady-state heat transfer.

$D = 1$ m, $L_{ku} = 5$ m, $L_k = 2$ m, $T_{f,\infty} = -20$°C, $T_2 = 0$°C, $A_{ku}R_{ku} = 10^{-1}$ K/(W/m^2).

Use Table 3.3(b) to determine the resistance $R_{k,1\text{-}2}$.

SKETCH:

Figure Pr.3.44(a) shows the heat pipe, the far-field ambient air temperature $T_{f,\infty}$, and the soil temperature T_2.

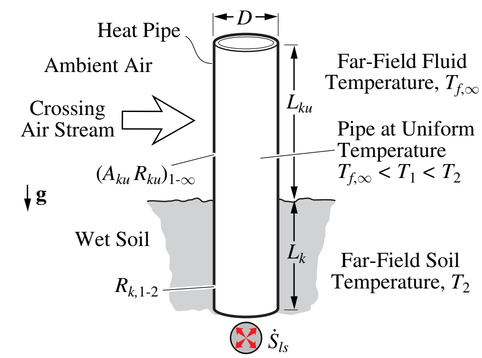


Figure Pr.3.44(a) A rendering of a heat pipe used for the maintenance of a permafrost layer in the arctic regions.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the heat pipe temperature T_1 and the amount of heat flow rate $Q_{k,2\text{-}1}$.

(c) If this heat is used entirely in phase change (solidification of liquid water), determine the rate of ice formation around the buried pipe.

SOLUTION:

(a) The thermal circuit diagram is shown Figure Pr.3.44(b). From node T_1, heat flows by surface convection and by conduction.

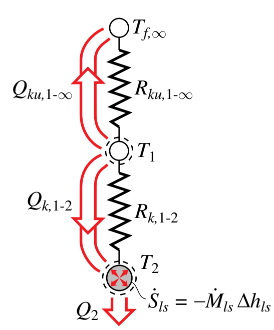


Figure Pr.3.44(b) Thermal circuit diagram.

(b) From Figure Pr.3.44(b), we have

$$Q_{k,1\text{-}2} + Q_{ku,1\text{-}\infty} = 0 = \frac{T_1 - T_2}{R_{k,1\text{-}2}} + \frac{T_1 - T_{f,\infty}}{R_{ku,1\text{-}\infty}}.$$

From Table 3.3(b), for an indented object in a semi-infinite medium (soil), we have

$$R_{k,1\text{-}2} = \frac{\ln(4L_k/D)}{2\pi k L_k},$$

where k is the soil conductivity. From Table C.17, we have

$$k = 0.52 \text{ W/m-K} \quad \text{Table C.17.}$$

Then

$$R_{k,1\text{-}2} = \frac{\ln \dfrac{4 \times 2(\text{m})}{1(\text{m})}}{2\pi \times 0.52(\text{W/m-K}) \times 2(\text{m})} = 0.3182 \text{ K/W} = 0.3182^\circ\text{C/W}.$$

For R_{ku}, we have

$$\begin{aligned} R_{ku} &= \frac{A_{ku}R_{ku}}{A_{ku}} = \frac{A_{ku}R_{ku}}{\pi D L_{ku} + \pi D^2/4} \\ &= \frac{10^{-1}[\text{K/(W/m}^2)]}{\pi \times 1(\text{m}) \times 5(\text{m}) + \pi \times 1^2(\text{m}^2)/4} = 6.063 \times 10^{-3} \text{ K/W} = 6.063^\circ\text{C/W}. \end{aligned}$$

Solving the energy equation for T_1, we have

$$\begin{aligned} T_1 &= \frac{\dfrac{T_2}{R_{k,1\text{-}2}} + \dfrac{T_{f,\infty}}{R_{ku,1\text{-}\infty}}}{\dfrac{1}{R_{k,1\text{-}2}} + \dfrac{1}{R_{ku,1\text{-}\infty}}} \\ &= \frac{\dfrac{0^\circ\text{C}}{0.3182(^\circ\text{C/W})} + \dfrac{-20^\circ\text{C}}{6.063 \times 10^{-3}(^\circ\text{C/W})}}{(3.143 + 1.649 \times 10^2)(\text{W/}^\circ\text{C})} = -19.63^\circ\text{C}. \\ Q_{k,2\text{-}1} &= \frac{T_2 - T_1}{R_{k,1\text{-}2}} = \frac{[0 - (-19.63)](^\circ\text{C})}{0.3182(^\circ\text{C/W})} = 61.69 \text{ W}. \end{aligned}$$

(c) From Figure Pr.3.44(b), we have

$$Q_2 + Q_{k,2\text{-}1} = \dot{S}_{ls}.$$

From Table 2.1, we have

$$\dot{S}_{ls} = -\dot{M}_{ls}\Delta h_{ls}.$$

From Table C.4, for water, we have

$$\Delta h_{sl} = 3.336 \times 10^5 \text{ J/kg} = -\Delta h_{ls},$$

then

$$\begin{aligned} \dot{M}_{ls} &= \frac{Q_{k,2\text{-}1}}{\Delta h_{sl}} = \frac{61.69(\text{W})}{3.336 \times 10^5(\text{J/kg})} \\ &= 1.849 \times 10^{-4} \text{ kg/s} = 0.1849 \text{ g/s} = 665.7 \text{ g/hr}. \end{aligned}$$

COMMENT:

Note that since $R_{ku,1\text{-}\infty} \ll R_{k,1\text{-}2}$, the heat-pipe temperature is nearly that of the ambient air, i.e., $T_1 \simeq T_{f,\infty}$.

PROBLEM 3.45.FAM.S

GIVEN:

In scribing of disks by pulsed laser irradiation (also called laser zone texturing) a small region, diameter D_1, is melted and upon solidification a protuberance (bump) is formed in this location. The surface of the liquid pool formed through heating is not uniform and depends on the laser energy and its duration, which in turn also influences the depth of the pool $L_1(t)$. These are shown in Figure Pr.3.45(a). Assume that the irradiated region is already at the melting temperature $T_1 = T_{sl}$ and the absorbed irradiation energy $(\dot{S}_{e,\alpha})_1$ is used to either melt the substrate $\dot{S}_{sl}$, or is lost through conduction $Q_{k,1\text{-}2}$ to the substrate. This simple, steady-state thermal model is also shown in Figure Pr.3.45(b). The irradiation is for an elapsed time of Δt.

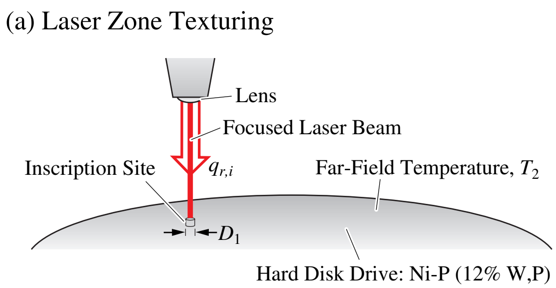
$D_1 = 10\ \mu$m, $(\dot{S}_{e,\alpha})_1 = 48$ W, $\Delta t = 1.3 \times 10^{-7}$ s, $T_2 = 50$℃.

Use the temperature, density, and heat of melting of nickel in Table C.2 and, the thermal conductivity of nickel at $T = 1,400$ K in Table C.14.

The energy conversion rate $\dot{S}_{sl}$ is given in Table 2.1 and note that $\dot{M}_{sl} = M_l/\Delta t$, where $M_l = \rho V_l(t)$. Use Table 3.3(b) for the conduction resistance and use $L(t = \Delta t)$ for the depth.

SKETCH:

Figure Pr.3.45(a) and (b) show the irradiation melting and the simple, steady-state heat transfer model.



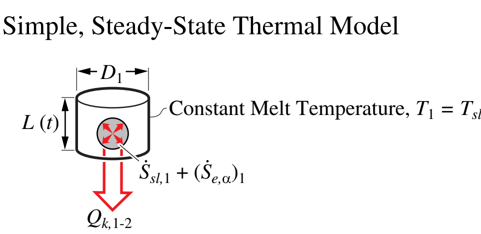


Figure Pr.3.45(a) Laser zone texturing of a disk. (b) The associated simple, steady-state heat transfer model..

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the depth of the melt L_1, after an elapsed time Δt.

SOLUTION:

(a) Figure Pr.3.45(c) shows the steady-state thermal circuit diagram.

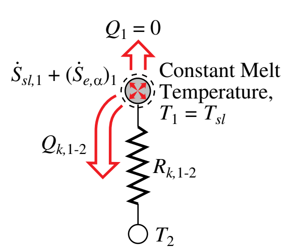


Figure Pr.3.45(c) Thermal circuit diagram.

(b) From Figure Pr.3.45(c), we have the energy equation

$$Q_{k,1\text{-}2} = (\dot{S}_{e,\alpha})_1 + \dot{S}_{sl,1}.$$

From Table 3.2, we have

$$Q_{k,1\text{-}2} = \frac{T_{sl} - T_2}{R_{k,1\text{-}2}}$$

From Table 2.1, we have

$$\begin{aligned}
\dot{S}_{sl,1} &= -\dot{M}_{sl}\Delta h_{sl} \\
&= -\dot{M}_l \Delta h_{sl} \\
&= -\rho \frac{V_l(t)}{\Delta t}\Delta h_{sl} \\
&= -\rho\pi D_1^2 \frac{L_1(t)}{\Delta t}\Delta h_{sl}
\end{aligned}$$

Then solving for $L_1(t)$, we have

$$\frac{T_{sl} - T_2}{R_{k,1\text{-}2}} = (\dot{S}_{e,\alpha})_1 - \rho\pi D_1^2 \frac{L(t)}{\Delta t}\Delta h_{sl}$$

or

$$L(t = \Delta t) = \frac{\left[(\dot{S}_{e,\alpha})_1 - \dfrac{T_{sl} - T_2}{R_{k,1\text{-}2}}\right]\Delta t}{\rho\pi D_1^2 \Delta h_{sl}}.$$

From Table 3.3(b), for a cylindrical indentation, we have

$$R_{k,1\text{-}2} = \frac{\ln(4L/D_1)}{4\pi k L}.$$

Here we assume that

$$R_{k,1\text{-}2} = \frac{\ln[4L(t = \Delta t)/D_1]}{4\pi k L(t = \Delta t)}.$$

From Table C.14, at T = 1,400 K for nickel, we have

$$k = 80 \text{ W/m-K} \quad \text{Table C.14.}$$

From Table C.2, for nickel, we have

$$\begin{aligned}
\rho &= 8{,}900 \text{ kg/m}^3 && \text{Table C.2} \\
T_{sl} &= 1{,}728 \text{ K} && \text{Table C.2} \\
\Delta h_{sl} &= 2.91 \times 10^5 \text{ J/kg} && \text{Table C.2.}
\end{aligned}$$

Then using the numerical values, we have

$$\begin{aligned}
L(t = \Delta t) &= \frac{\left[48(\text{W}) - \dfrac{(1{,}728 - 323.15)(\text{K})}{R_{k,1\text{-}2}(\text{K/W})}\right] \times 1.3 \times 10^{-7}(\text{s})}{8{,}900(\text{kg/m}^3) \times \pi \times (10^{-5})^2(\text{W}^2) \times 2.91 \times 10^5(\text{J/kg})} \\
R_{k,1\text{-}2} &= \frac{\ln[4L(t = \Delta t)/10^{-5}(\text{m})]}{4\pi \times 80(\text{W/m-K}) \times L(t = \Delta t)}.
\end{aligned}$$

Solving these using a solver (such as SOPHT), we have

$$\begin{aligned}
L(t = \Delta t) &= \frac{6.240 \times 10^{-6}(\text{J}) - \dfrac{1.826 \times 10^{-4}(\text{K-s})}{R_{k,1\text{-}2}(\text{K/W})}}{8.137 \times 10^{-1}(\text{J/m})} \\
R_{k,1\text{-}2} &= \frac{\ln[4 \times 10^5 \times L(t = \Delta t)]}{1.005 \times 10^3(\text{W/m-K}) \times L(t = \Delta t)}
\end{aligned}$$

or

$$
\begin{aligned}
L(t = \Delta t) &= 6.127 \times 10^{-6}\ \text{m} = 6.127\ \mu\text{m} \\
R_{k,1\text{-}2} &= 145.6\ \text{K/W}.
\end{aligned}
$$

COMMENT:

The steady-state conduction resistance is not expected to be accurate. The transient resistance is expected to be smaller and thus have a more significant rule. The initial heating, to the melting temperature T_{sl}, can be modeled similarly and its inclusion will reduce the final pool depth.

PROBLEM 3.46.FUN

GIVEN:

To estimate the elapsed time for the penetration of a change in the surface temperature of the brake rotor, the results of Table 3.4 and Figure 3.33(a) can be used. Consider the brake rotor shown in Figure Pr.3.46.

Use carbon steel AISI 1010 for the rotor at $T = 20$℃, and $2L = 3$ cm.

SKETCH:

Figure Pr.3.46 shows the friction heating of the rotor.

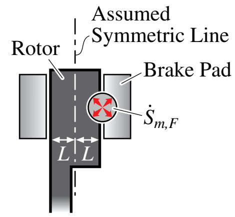


Figure Pr.3.46 Surface friction heating and its penetration into the brake rotor, during the braking period.

OBJECTIVE:

(a) For the conditions given above, determine the penetration time.

(b) If the brake is on for 4 s, is the assumption of a uniform rotor temperature valid during the braking period ?

(c) If the surface-convection cooling occurs after braking and over a time period of 400 s, is the assumption of a uniform rotor temperature valid during the cooling period?

SOLUTION:

(a) From Table C.16, for carbon steel AISI 1010, we have

$$\rho = 7{,}830 \text{ kg/m}^3 \qquad \text{Table C.16}$$

$$c_p = 434 \text{ J/kg-K} \qquad \text{Table C.16}$$

$$k = 64 \text{ W/m-K} \qquad \text{Table C.16}$$

$$\alpha = \frac{k}{\rho c_p} = 18.8 \times 10^{-6} \text{ m}^2\text{/s.} \qquad \text{Table C.16.}$$

Assuming that results of Figure 3.33(a)(ii) for a finite slab of thickness $2L$ apply here, we have for (3.150)

$$\text{Fo}_{L,o} = \frac{t\alpha}{L^2} = 0.07.$$

Then the elapsed time is

$$\begin{aligned} t &= 0.07\frac{L^2}{\alpha} \\ &= 0.07\frac{(1.5 \times 10^{-2})^2(\text{m}^2)}{1.88 \times 10^{-5}(\text{m}^2/\text{s})} = 0.8378 \text{ s.} \end{aligned}$$

(b) Figure 3.33(a)(ii) shows that for a nearly uniform temperature, a larger Fourier number is needed. As an approximation from Figure 3.33(a)(ii), we choose $\text{Fo}_L = 1.0$ to indicate nearly uniform temperature. Then the elapsed time is

$$t = 1.0\frac{L^2}{\alpha} = 11.97 \text{ s.}$$

This is large compared to the braking time of $t = 4$ s. Therefore, we cannot justifiably assume a uniform rotor temperature during the braking period.

(c) During the surface-convection cooling of $t = 400$ s, the changes occurring over the rotor surface will have a sufficient time to penetrate through the rotor (only $t = 11.97$ s is needed for a nearly complete penetration). Then we can justify the use of a uniform temperature assumption during the cooling period.

COMMENT:

Note that we have used the constant surface temperature results of Table 3.4 for the finite slab, while in practice, the surface temperature continues to rise during the brake period and drop during the cooling period.

PROBLEM 3.47.FUN

GIVEN:

The time-periodic variation of the surface temperature of a semi-infinite slab (such as that shown in Figure Ex.3.17) can be represented by an oscillating variation

$$T_s = T(t=0) + \Delta T_{max}\cos(\omega t),$$

where $\omega = 2\pi f$ is the angular frequency, f (1/s) is the linear frequency, and ΔT_{max} is the amplitude of the surface temperature change. The solution to the energy equation (3.134), with the above used for the first of the thermal conditions in (3.135), is

$$\frac{T(x,t) - T(x,t=0)}{\Delta T_{max}} = \exp\left[-x\left(\frac{\omega}{2\alpha}\right)^{1/2}\right]\cos\left[\omega t + x\left(\frac{\omega}{2\alpha}\right)^{1/2}\right].$$

OBJECTIVE:

Show that the penetration depth δ_α, defined by

$$\frac{T(x,t) - T(x,t=0)}{\Delta T_{max}} = 0.01,$$

is given by

$$\frac{\delta_\alpha}{(2\alpha t)^{1/2}} = 1.725,$$

which is similar to the penetration depth given by (3.148).

Evaluate the penetration depth after an elapsed time equal to a period, i.e., $t = \tau = 1/f$, where τ(s) is the period of oscillation.

SOLUTION:

Upon examining the solution

$$\frac{T(x,t) - T(x,t=0)}{\Delta T_{max}} = \exp\left[-x\left(\frac{\omega}{2\alpha}\right)^{1/2}\right]\cos\left[\omega t + x\left(\frac{\omega}{2\alpha}\right)^{1/2}\right],$$

we note that the exponential term is a spatial attenuation factor, while the cosine term is a combined temporal-spatial phase lag function. For the determination of the penetration depth, the spatial factor can be written as

$$\exp\left[-\delta_\alpha\left(\frac{\omega}{2\alpha}\right)^{1/2}\right] = \exp\left[-\delta_\alpha\left(\frac{2\pi f}{2\alpha}\right)^{1/2}\right] = \exp\left[-\delta_\alpha\left(\frac{2\pi}{2\alpha\tau}\right)^{1/2}\right] = \exp\left[-\delta_\alpha\left(\frac{\pi}{\alpha\tau}\right)^{1/2}\right].$$

Similarly,

$$\cos\left[\omega t + x\left(\frac{\omega}{2\alpha}\right)^{1/2}\right] = \cos\left[2\pi + \delta_\alpha\left(\frac{\pi}{\alpha\tau}\right)^{1/2}\right].$$

From the definition of penetration depth, we have:

$$\exp\left[-\delta_\alpha\left(\frac{\pi}{\alpha\tau}\right)^{1/2}\right]\cos\left[2\pi + \delta_\alpha\left(\frac{\pi}{\alpha\tau}\right)^{1/2}\right] = 0.01.$$

Using a solver (such as SOPHT), this gives

$$\delta_\alpha\left(\frac{\pi}{\alpha\tau}\right)^{1/2} = 1.528.$$

To compare with (3.148), we rearrange to obtain

$$\frac{\delta_\alpha}{2(\alpha\tau)^{1/2}} = 1.528\frac{1}{2\pi^{1/2}} = 0.4310.$$

This is similar in form to (3.148), except that the constant is 0.4310 (instead of 1.8).

COMMENT:

This shows that the assumption made in Example 3.17 is valid, regarding similarity of penetration depths for the periodic and for the sudden (pulsed) change in the surface temperature.

PROBLEM 3.48.FAM

GIVEN:

Pulsed lasers provide a large power $q_{r,i}$ for a short time Δt. In surface treatment of materials (e.g., laser-shock hardening), the surface is heated by laser irradiation using very small pulse durations. During this heating, the transient conduction through the irradiated material can be determined as that of a semi-infinite solid subject to constant surface heating $-q_s = q_{r,i}$; this is shown in Figure Pr.3.48, with the material being a metallic alloy (stainless steel AISI 316, Table C.16). The heated semi-infinite slab is initially at $T(t = 0)$. In a particular application, two laser powers (assume all the laser irradiation power is absorbed by the surface), with different pulse lengths Δt, are used. These are (i) $-q_s = 10^{12}$ W/m^2, $\Delta t = 10^{-6}$ s, and (ii) $-q_s = 10^{10}$ W/m^2, $\Delta t = 10^{-4}$ s.

SKETCH:

Figure Pr.3.48 shows the surface irradiated by a laser and the penetration of the heat into the substrate.

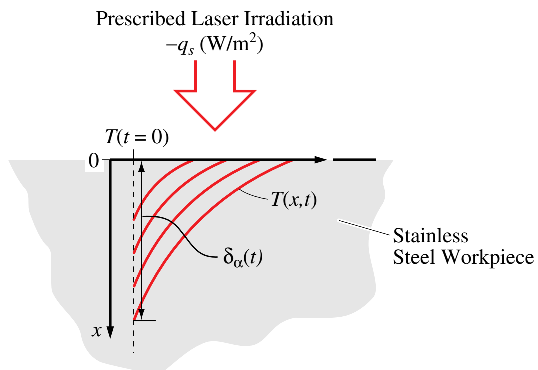


Figure Pr.3.48 Pulsed laser irradiation of a stainless steel workpiece and the anticipated transient temperature distribution within the workpiece.

OBJECTIVE:

(a) Determine the surface temperature $T(x = 0,\ t = \Delta t)$ after elapsed time $t = \Delta t$, for cases (i) and (ii).
(b) As an approximation, use the same expression for penetration depth $\delta_\alpha(t)$ as that for the semi-infinite slabs with a prescribed surface temperature, and determine the penetration depth after the elapsed time $t = \Delta t$, for cases (i) and (ii).
(c) Comment on these surface temperatures and penetration depths.

SOLUTION:

(a) From Table 3.4, for the case of a prescribed surface heat flux q_s, we have the expression for $T(x, t)$ as

$$T(x,t) = T(t=0) - \frac{q_s(4\alpha t)^{1/2}}{\pi^{1/2}k} e^{-\frac{x^2}{4\alpha t}} + \frac{q_s x}{k}\left\{1 - \mathrm{erf}\left[\frac{x}{(4\alpha t)^{1/2}}\right]\right\}.$$

Evaluating this at $x = 0$ and $t = \Delta t$, we have

$$T(x=0, t=\Delta t) = T(t=0) - \frac{q_s(4\alpha\Delta t)^{1/2}}{\pi^{1/2}k}.$$

From Table C.16, we have, for stainless steel 316

$$\alpha = 3.37 \times 10^{-6}\ \mathrm{m^2/s} \qquad \text{Table C.16}$$
$$k = 13\ \mathrm{W/m\text{-}K} \qquad \text{Table C.16.}$$

Then using the numerical values, we have

$$\begin{aligned}
\text{(i) } T(x=0, t=10^{-6}\text{s}) &= 20(℃) - \frac{(-10^{12})(\text{W/m}^2) \times [4 \times (3.37 \times 10^{-6})(\text{m}^2/\text{s}) \times 10^{-6}(\text{s})]^{1/2}}{\pi^{1/2} \times 13(\text{W/m-K})} \\
&= 20(℃) + 1.594 \times 10^5(℃) = 1.594 \times 10^5 ℃. \\
\text{(ii) } T(x=0, t=10^{-6}\text{s}) &= 20(℃) - \frac{(-10^{10})(\text{W/m}^2) \times [4 \times (3.37 \times 10^{-6})(\text{m}^2/\text{s}) \times 10^{-4}(\text{s})]^{1/2}}{\pi^{1/2} \times 13(\text{W/m-K})} \\
&= 20(℃) + 1.594 \times 10^4(℃) = 1.594 \times 10^4 ℃.
\end{aligned}$$

These are tremendously large surface temperatures sustained for a very short time (thus the name laser-shock harding).

(b) From (3.148), for a semi-infinite slab with a sudden change in the surface temperature, we have the penetration depth given as

$$\delta_\delta = 3.6\alpha^{1/2}t^{1/2}.$$

Then using the numerical values, we have

$$\begin{aligned}
\text{(i) } \delta_\alpha(t=\Delta t) &= 3.6 \times (3.37 \times 10^{-6})^{1/2}(\text{m}^2/\text{s})^{1/2} \times (10^{-6})^{1/2}(s)^{1/2} \\
&= 6.609 \times 10^{-6} \text{ m} = 6.609\ \mu\text{m} \\
\text{(ii) } \delta_\alpha(t=\Delta t) &= 3.6 \times (3.37 \times 10^{-6})^{1/2}(\text{m}^2/\text{s})^{1/2} \times (10^{-4})^{1/2}(s)^{1/2} \\
&= 6.609 \times 10^{-5} \text{ m} = 66.09\ \mu\text{m}.
\end{aligned}$$

(c) The thin region near the surface is shocked by this large temperature change and this allows for the rearrangement of the molecules. Then upon cooling, any crystalline defects (and also any surface impurities) will be removed.

COMMENT:

During very fast thermal shocks, the lattice nuclei may not be in thermal equilibrium with their electron clouds. The electrons having a much smaller mass heat up much faster than the nuclei. Also, any phase change occurring during the shock treatment will not follow the equilibrium phase diagrams, which are generally obtained through controlled and much slower heating/cooling processes.

PROBLEM 3.49.FUN

GIVEN:

In a solidification process, a molten acrylic at temperature $T(t=0)$ is poured into a cold mold, as shown in Figure 3.49, to form a clear sheet. Assume that the heat of solidification can be neglected.

$L = 2.5$ mm, $T_{ls} = 90$℃, $T(t=0) = 200$℃, $T_s = 40$℃.

SKETCH:

The planar mold and the acrylic melt are shown in Figure Pr.3.49.

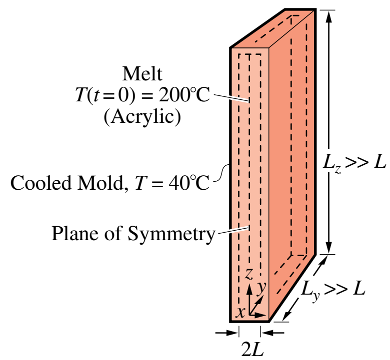


Figure Pr.3.49 Solidification of an acrylic melt in a mold having a constant temperature T_s.

OBJECTIVE:

(a) Determine the elapsed time for the cooling front to reach the central plane of the melt.

(b) Determine the elapsed time for the temperature of the central plane of the melt to reach the glass transition temperature T_{ls}.

SOLUTION:

From Table C.17, we have for acrylic (at $T = 293$ K)

$$\alpha = 1.130 \times 10^{-7} \text{ m}^2/\text{s} \qquad \text{Table C.17.}$$

(a) From Figure 3.33(a)(ii), or from (3.151), the time (Fourier number) for the penetration to reach the central plane is

$$\text{Fo}_L = 0.07 = \frac{\alpha t}{L^2}$$

or

$$t = \frac{L^2 \text{Fo}_L}{\alpha} = \frac{(2.5 \times 10^{-3})^2(\text{m}^2) \times 0.07}{1.130 \times 10^{-7}(\text{m}^2/\text{s})} = 3.872 \text{ s.}$$

(b) From Figure 3.33(a)(ii) for a finite slab, we have

$$\frac{T_{ls} - T(t=0)}{T_s - T(t=0)} = \frac{(90-200)(℃)}{(40-200)(℃)} = 0.6875,$$

and by interpolating the value of Fo_L for $x/L = 0$, we have

$$\text{Fo}_L = 0.57.$$

Then

$$\text{Fo}_L = \frac{\alpha t}{L^2}$$

or

$$\begin{aligned} t &= \frac{L^2 \mathrm{Fo}_L}{\alpha} = \frac{(2.5 \times 10^{-3})^2 (\mathrm{m}^2) \times 0.57}{1.130 \times 10^{-7} (\mathrm{m}^2/\mathrm{s})} \\ &= 31.53 \ \mathrm{s} \end{aligned}$$

COMMENT:

Note that for a significant change in the central-plane temperature, an elapsed time is needed which is many times that for just penetrating to the location of the central plane. We can approximately account for the heat of solidification Δh_{ls} by adjusting the specific heat capacity.

PROBLEM 3.50.FAM

GIVEN:

During solidification, as in casting, the melt may locally drop to temperatures below the solidification temperature T_{ls}, before the phase change occurs. Then the melt is in a metastable state (called supercooled liquid) and the nucleation (short of) of the solidification resulting in formation of crystals (and their growth) begins after a threshold liquid supercool is reached. Consider solidification of liquid paraffin (Table C.5) in three different molds. These molds are in the form of (i) a finite slab, (ii) a long cylinder, and (iii) a sphere, and are shown in Figure Pr.3.50. The melt in the molds in initially at its melting temperature $T(t=0)=T_{sl}$. Then at $t=0$ the mold surface is lowered and maintained at temperature T_s.

Assume that solidification will not occur prior to this elapsed time.

$T_s = 15$℃, $L = R = 2$ cm, $T = (x=0,t) = T(r=0,t) = T_o = 302$.

Use the properties of polystyrene (Table C.17).

SKETCH:

Figure Pr.3.50 shows the three geometries of the paraffin mold.

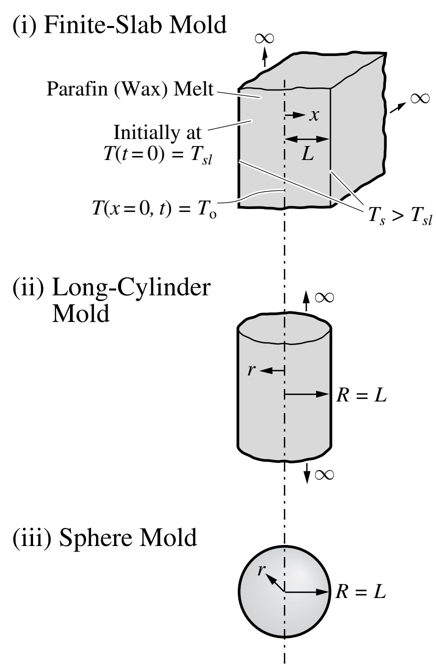


Figure Pr.3.50 Paraffin (wax) melt is cooled in a mold (three different geometries) and is gradually solidified.

OBJECTIVE:

Determine the elapsed time needed for the temperature at the center of the mold, i.e., $T(x=0,t)=T(r=0,t)$ to reach a threshold value T_o, for molds (i), (ii) and (iii). Assume that solidification will not occur prior to this elapsed time.

SOLUTION:

We will use the graphical results given in Figures 3.33(a) and (b) to determine t. The center temperature is desired, and noting that $T(t=0)=T_{sl}$,

$$\frac{T - T(t=0)}{T_s - T(t=0)} = \frac{T_o - T_{sl}}{T_s - T_{sl}}.$$

From Table C.5, we have

$$T_{sl} = 310.0 \text{ K} \qquad \text{Table C.5}$$

$$\Delta h_{sl} = 2.17 \times 10^5 \quad \text{J/kg} \quad \text{Table C.5.}$$

Then

$$\frac{T_o - T_{sl}}{T_s - T_{sl}} = \frac{(302 - 310)(\text{K})}{(298.15 - 310)(\text{K})} = 0.6751.$$

Now from Figures 3.33(a) and (b), we have

$$\begin{aligned}
&\text{(i) slab:} && x = 0, && \text{Fo}_L \simeq 0.55 && \text{Figure 3.33(a)(ii)}\\
&\text{(ii) cylinder:} && r = 0, && \text{Fo}_R \simeq 0.27 && \text{Figure 3.33(a)(i)}\\
&\text{(iii) sphere:} && r = 0, && \text{Fo}_R \simeq 0.18 && \text{Figure 3.33(a)(ii)},
\end{aligned}$$

where

$$\text{Fo}_L = \frac{\alpha t}{L^2}, \quad \text{Fo}_R = \frac{\alpha t}{R^2}, \quad \alpha = \frac{k}{\rho c_p}.$$

From Table C.17, we have (assume the same properties as polystyrene)

$$\alpha = 7.407 \times 10^{-8} \text{ m}^2/\text{s} \quad \text{Table C.17}$$

Then

$$\begin{aligned}
\text{(i) slab:} \quad t &= \frac{\text{Fo}_L L^2}{\alpha} = \frac{0.55 \times (2 \times 10^{-2})^2 (\text{m}^2)}{7.407 \times 10^{-8} (\text{m}^2/\text{s})} = 2{,}970 \text{ s}\\
\text{(ii) cylinder:} \quad t &= \frac{\text{Fo}_R R^2}{\alpha} = \frac{0.27 \times (2 \times 10^{-2})^2 (\text{m}^2)}{7.407 \times 10^{-8} (\text{m}^2/\text{s})} = 1{,}458 \text{ s}\\
\text{(iii) sphere:} \quad t &= \frac{\text{Fo}_R R^2}{\alpha} = \frac{0.18 \times (2 \times 10^{-2})^2 (\text{m}^2)}{7.407 \times 10^{-8} (\text{m}^2/\text{s})} = 972.1 \text{ s}.
\end{aligned}$$

COMMENT:

Note that the slab mold requires three times more elapsed time compared to the sphere. This is due to the monotonically decreasing volume of the sphere as the center is approached, thus requiring a smaller heat (or time) to change the local temperature. Here we used a large supercooling. In practice, solidification begins at a smaller supercooling, thus requiring less time. Inclusion of the solid-liquid phase change is discussed in Section 3.8.

PROBLEM 3.51.FAM

GIVEN:

An apple (modeled as a sphere of radius $R = 4$ cm), initially at $T(t = 0) = 23$℃ is placed in a refrigerator at time $t = 0$, and thereafter, it is assumed that its surface temperature is maintained at $T_s = 4$℃.

Use the thermophysical properties of water at $T = 293$ K from Table C.23. Use the graphical results given in Figure 3.33(b).

OBJECTIVE:

(a) Determine the elapsed time it takes for the thermal penetration depth to reach the center of the apple.
(b) Determine the elapsed time for the center temperature to reach $T = 10$℃.

SOLUTION:

(a) For the penetration depth to reach the center of the apple, assuming that the temperature changes by 1 %, we have

$$T^* = \frac{T - T(t = 0)}{T_s - T(t = 0)} = 0.01 \quad \text{at} \quad \frac{r}{R} = 0.$$

From Figure 3.33(b), the Fourier number for these conditions is nearly 0.03. From the definition of the Fourier number (3.131), we have

$$Fo_R = \frac{\alpha t}{R^2} = 0.03.$$

From Table C.23, for water at $T = 293$ K, we have $\alpha = 143 \times 10^{-9}$ m^2/s. Solving for t, we have

$$t = \frac{0.03 R^2}{\alpha} = \frac{0.03 \times [0.04(\text{m})]^2}{143 \times 10^{-9}(\text{m}^2/\text{s})} = 336 \text{ s} = 5.6 \text{ min}.$$

(b) For the center temperature condition $T = 10$℃, we have

$$T^* = \frac{T - T(t = 0)}{T_s - T(t = 0)} = \frac{10 - 23}{4 - 23} = 0.68 \quad \text{at} \quad \frac{r}{R} = 0.$$

Again, from Figure 3.33(b) the Fourier number for these conditions is approximately 0.19. From the definition of the Fourier number and solving for t, we have

$$t = \frac{0.19 R^2}{\alpha} = \frac{0.19 \times [0.04(\text{m})]^2}{143 \times 10^{-9}(\text{m}^2/\text{s})} = 2126 \text{ s} = 35 \text{ min}.$$

COMMENT:

It is not an easy task to keep the surface temperature constant. Inside a refrigerator, most likely there is a surface-convection boundary condition at the apple surface. This will be studied in Chapter 6.

PROBLEM 3.52.FAM

GIVEN:

In a summer day, the solar irradiation on the surface of a parking lot results in an absorbed irradiation flux $q_s = -500$ W/m^2, as shown in Figure Pr.3.52. The parking lot surface is covered with an asphalt coating that has a softening temperature of 55°C.

The initial temperature is $T(t = 0) = 20$°C.

Assume that all the absorbed heat flows into the very thick asphalt layer

Use the properties of asphalt in Table C.17.

SKETCH:

Figure Pr.3.52 shows a thick asphalt layer, treated as a semi-infinite slab, suddenly heated by solar irradiation. The temperature distribution beneath the surface is rendered for several elapsed times.

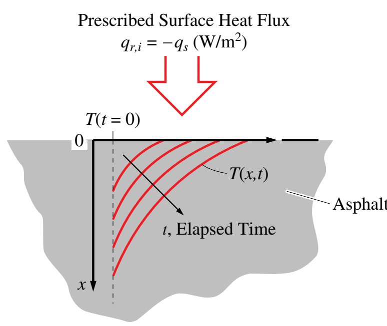


Figure Pr.3.52 Temperature variation within a thick asphalt layer, suddenly heated by solar irradiation.

OBJECTIVE:

Determine the elapsed time it takes for the surface temperature of the parking lot to rise to the softening temperature.

SOLUTION:

For a semi-infinite slab with a constant heat flux boundary condition the temperature as a function of x and t is given by Table 3.4 as

$$T(x,t) - T(t=0) = -\frac{2q_s}{k}\left(\frac{\alpha t}{\pi}\right)^{1/2} e^{\frac{-x^2}{4\alpha t}} + \frac{q_s x}{k}\left[1 - \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right].$$

For the surface of the asphalt layer, $x = 0$. The solution becomes

$$T_s(t) - T(t=0) = -\frac{2q_s}{k}\left(\frac{\alpha t}{\pi}\right)^{1/2}.$$

For asphalt, from Table C.17, $k = 0.06$ W/m-K and $\alpha = 0.03 \times 10^{-6}$ m^2/s. Solving for t and using the data given, we have

$$t = \frac{\pi}{\alpha}\left[\frac{(T_s - T_i)k}{-2q_s}\right]^2 = \frac{\pi}{0.03 \times 10^{-6}(\text{m}^2/\text{s})}\left\{\frac{[55(°\text{C}) - 20(°\text{C})] \times 0.06(\text{W/m-K})}{-2 \times (-500)(\text{W/m}^2)}\right\}^2 = 462 \text{ s} = 7.7 \text{ min}.$$

COMMENT:

The solution presented in Table 3.4 applies for a constant (with respect to time), prescribed heat flux q_s (W/m^2).

PROBLEM 3.53.FAM

GIVEN:

In a shaping process, a sheet of Teflon (Table C.17) of thickness $2L$, where $L = 0.3$ cm, is placed between two constant-temperature flat plates and is heated. The initial temperature of the sheet is $T(t = 0) = 20$℃ and the plates are at $T_s = 180$℃. This is shown in Figure Pr.3.53.

SKETCH:

Figure Pr.3.53 shows a Teflon sheet heated on its two surfaces.

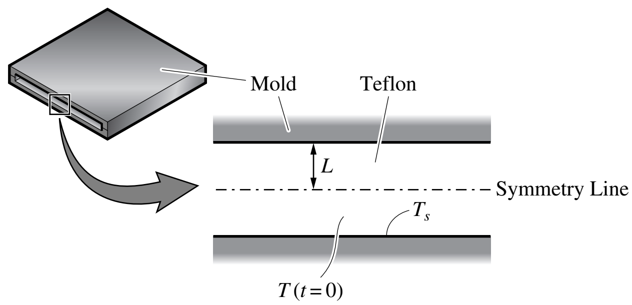


Figure Pr.3.53 Thermal forming of a Teflon sheet.

OBJECTIVE:

Determine the time it takes for the center of the sheet to reach 20℃ below T_s.

SOLUTION:

Treating the Teflon as a distributed system consisting of a slab bounded on two sides with a prescribed surface temperature T_s, for T^*, we have

$$T^* = \frac{T(t) - T(t=0)}{T_s - T(t=0)} = \frac{(160 - 20)(℃)}{(180 - 20)(℃)} = 0.875.$$

From Figure 3.33(a), for $x/L = 0$ (centerline), the Fourier number is

$$Fo = \frac{\alpha t}{L^2} \approx 0.9.$$

The thermal diffusivity of Teflon, from Table C.17, is $\alpha = 0.34 \times 10^{-6}$ m^2/s, and therefore

$$t = \frac{(0.9)(0.3 \times 10^{-2})^2(\text{m}^2)}{0.34 \times 10^{-6}(\text{m}^2/\text{s})} = 24 \text{ s}.$$

COMMENT:

The assumption of constant mold temperature is good for a preheated metal mold. This is due to the high effusivity $(\rho c_p k)^{1/2}$ of metals when compared to those of polymers.

PROBLEM 3.54.FUN.S

GIVEN:

When human skin is brought in contact with a hot surface, it burns. The degree of burn is characterized by the temperature of the contact material T_s and the contact time t. A first-degree burn displays no blisters and produces reversible damage. A second-degree burn is moist, red, blistered, and produces partial skin loss. A third-degree burn is dry, white, leathery, blisterless, and produces whole skin loss. A pure copper pipe with constant temperature $T_s = 80$℃ is brought in contact with human skin having $\rho c_p = 3.7 \times 10^6$ J/m^3-℃, $k = 0.293$ W/m-℃, and $T(t = 0) = 37$℃ for a total elapsed time of $t = 300$ s.

Use the solution for transient conduction through a semi-infinite slab with prescribed surface temperature to answer the following.

SKETCH:

Figure Pr.3.54(a) shows how the first-, second- and third-degree burns of human tissues are defined based on categories.

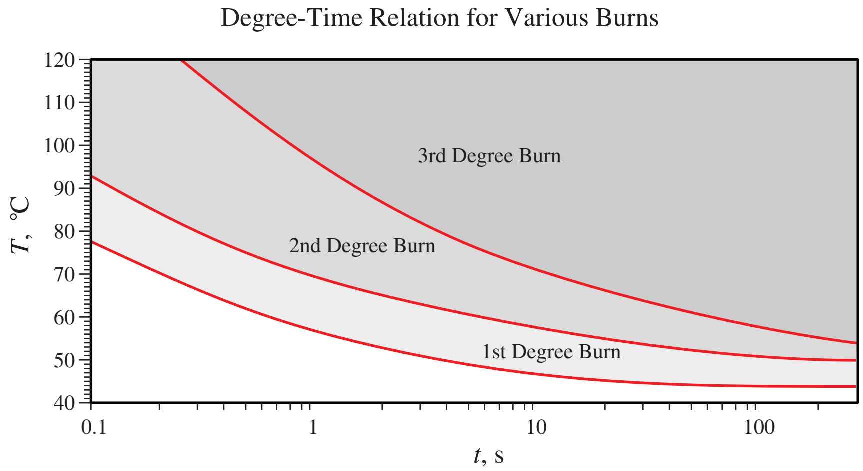


Figure Pr.3.54(a) Thermal damage (burn) to a human skin and the regions of various degrees of burn.

OBJECTIVE:

(a) Plot the temperature distribution $T(x, t)$(℃) as a function of position x(mm) at elapsed times t = 1, 10, 20, 40, 50, 100, 150, 200, 220, 240, 280, and 300 s.

(b) Use this plot, along with the plot shown in Figure Pr.3.54(a), to estimate the maximum depths for the first-, second-, and third-degree burns, after an elapsed time $t = 300$ s.

SOLUTION:

(a) The solution for the transient conduction through a semi-infinite slab, (3.142) is used, i.e.,

$$T(x, t) = T(t = 0) + [T_s + T(t = 0)] \left\{ 1 - \text{erf} \left[\frac{x}{2(\alpha t)^{1/2}} \right] \right\},$$

where

$$\alpha = \frac{k}{\rho c_p} = \frac{0.293(\text{W/m-℃})}{3.7 \times 10^6(\text{J/m}^3\text{-℃})} = 7.919 \times 10^{-8} \text{ m}^2/\text{s}, \quad T(t = 0) = 37\text{℃}, \quad \text{and} \quad T_s = 80\text{℃}.$$

The temperature distribution is shown in Figure Pr.3.54(b) for several elapsed times.

(b) We are interested in finding the maximum depth of the first-, second-, and third-degree burns. This is most easily done using the following steps.

(i) Pick the lowest possible temperature that causes a first-degree burn using Figure Pr.3.54(a).

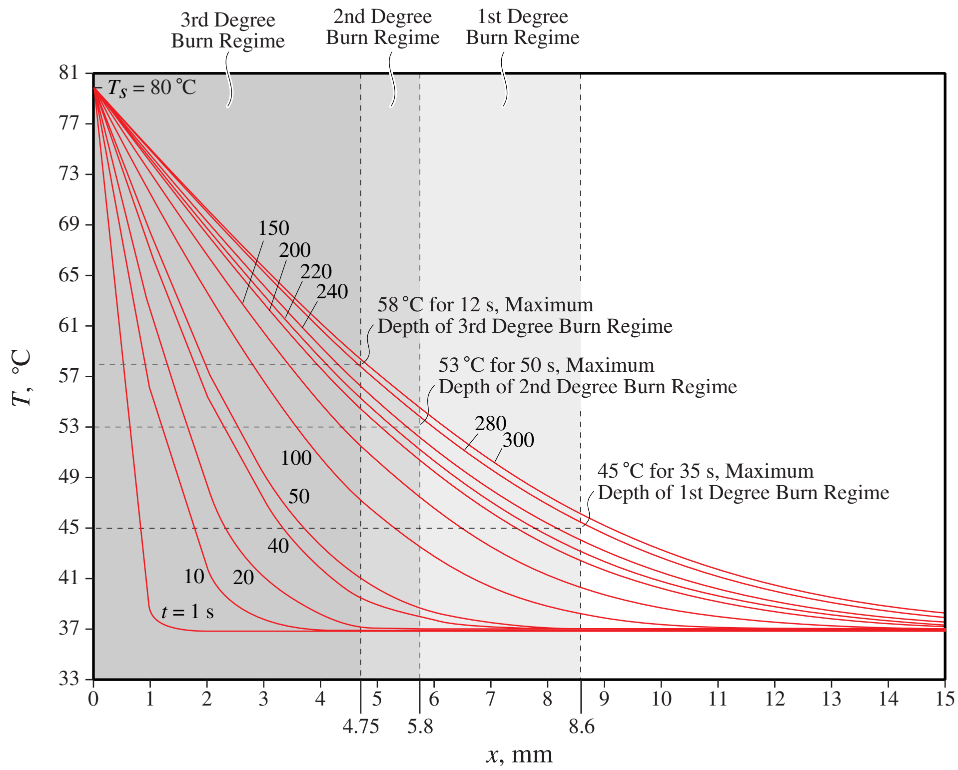


Figure Pr.3.54(b) Distribution of the tissue temperature for several elapsed times.

(ii) Using Figure Pr.3.54(a), find the time needed at this temperature to produce first-degree burns.
(iii) Using this temperature and time, find a depth from the graph in Figure Pr.3.54(b).
(iv) Add one degree to the temperature found in step (i).
(v) Using the same graph, find the time needed at this new temperature to produce a first-degree burn.
(vi) Using this temperature and time, find a depth from the graph in Figure Pr.3.54(b).
(vii) If this depth is greater than the one found in step (iii), go to step (iv).
(viii) If this depth is less than the one found in step (iii), stop.
(ix) Repeat this process for the second- and third-degree burns.

From Figure Pr.3.54(b), we find that the maximum depth for a first-degree burn is $x = 8.6 \pm 0.4$ mm (marked in Figure Pr.3.54(b) at $T = 45$℃ at $t = 35$ s).

For the second-degree burn, we have $x = 5.8 \pm 0.4$ mm (at $T = 53$℃ at $t = 50$ s).

For the third-degree burn, $x = 4.75 \pm 0.4$ mm (at $T = 58$℃ at $t = 12$ s).

COMMENT:

The above results are conservative estimates. We have assumed a constant temperature between the time of thermal front arrival and the burn initiation, while this location experiences an increase in temperature with time. Therefore, the actual extent of the burn regions could be deeper than indicated. The transient conduction solution also assumes constant thermal properties and that heat is not carried away by the blood perfusion. The thermal properties change with depth, as layers of tissues such as muscle and fat are encountered.

PROBLEM 3.55.FAM

GIVEN:

A hole is to be drilled through a rubber bottle stopper. Starting the hole in a soft room-temperature rubber often results in tears or cracks on the surface around the hole. It has been empirically determined that the rubber material at the surface can be hardened sufficiently for crack- and tear-free drilling by reducing the surface temperature at $x = 1$ mm beneath the surface to below $T = 220$ K . This reduction in temperature can be achieved by submerging the rubber surface into a liquid nitrogen bath for a period of time. This is shown in Figure Pr.3.55.

Assume that by submerging, the surface temperature drops from the initial uniform temperature $T(x, t = 0) = 20$℃ to the boiling temperature of the nitrogen T_{lg}, and then remains constant.

Use the saturation temperature of nitrogen T_{lg} at one atm pressure (Table C.26) and the properties of soft rubber (Table C.17).

SKETCH:

Figure Pr.3.55 shows the surface of a bottle stopper suddenly placed in contact with liquid nitrogen.

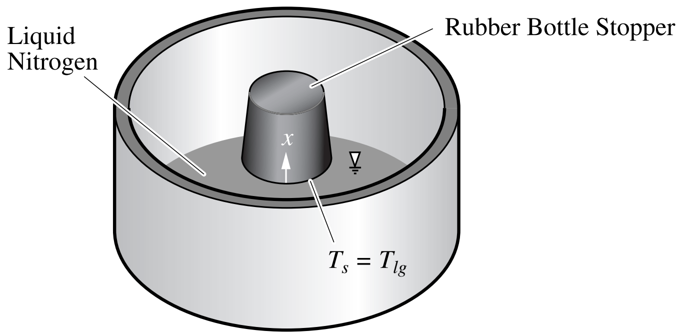


Figure Pr.3.55 A rubber bottle stopper is temporally submerged in a liquid nitrogen bath.

OBJECTIVE:

Determine after an elapsed time of $t = 10$ s, (a) the temperature 1 mm from the surface $T(x = 1 \text{ mm}, t = 10 \text{ s})$, (b) the temperature 3 mm from the surface $T(x = 3 \text{ mm}, t = 10 \text{ s})$, and (c) the rate of heat flowing per unit area out of the rubber surface $q_s(x = 0, t = 10 \text{ s})(\text{W/m}^2)$.

(d) Is $t = 10$ s enough cooling time to enable crack- and tear-free drilling of the hole?

SOLUTION:

Since we are asked to evaluate conditions after a elapsed time t, this is a transient problem. Since we are given no information about the size of the rubber stopper, we assume we can treat this as a semi-infinite medium with a constant surface temperature $T_s = T_{lg}$. The solution for the temperature distribution in a semi-infinite medium subject to a constant imposed surface T_s temperature is given by (3.140), i.e.,

$$T = T(t = 0) + [T_s - T(t = 0)][1 - \text{erf}(\eta)],$$

where

$$\eta(x) = \frac{x}{(4\alpha t)^{1/2}}.$$

From Table C.26, we have $T_{lg} = 77.35$ K for nitrogen. Initially, the stopper is at a uniform temperature $T(x, t = 0) = 20℃ = 293.15$ K. From Table C.17, we have $\alpha = 0.588 \times 10^{-7}$ m^2/s and $k = 0.13$ W/m-K for the rubber stopper.

(a) $x = 1$ mm and $t = 10$ s

$$\begin{aligned}
\eta(x = 0.001 \text{ m}) &= \frac{0.001(\text{m})}{[4 \times 0.588 \times 10^{-7}(\text{m}^2/\text{s}) \times 10(\text{s})]^{1/2}} = \frac{0.001(\text{m})}{1.5336 \times 10^{-3}(\text{m})} = 0.652 \\
\text{erf}(\eta) &= \text{erf}(0.652) = 0.642 \quad \text{Table 3.5} \\
T &= 293.15(\text{K}) + (77.35 - 293.15)(\text{K}) \times (1 - 0.642) = 215.8 \text{ K}.
\end{aligned}$$

(b) $x = 3$ mm and $t = 10$ s

$$\begin{aligned}
\eta(x = 0.003 \text{ m}) &= \frac{0.003(\text{m})}{[4 \times 0.588 \times 10^{-7}(\text{m}^2/\text{s}) \times 10(\text{s})]^{1/2}} = \frac{0.001(\text{m})}{1.5336 \times 10^{-3}(\text{m})} = 1.956 \\
\text{erf}(\eta) &= \text{erf}(1.956) = 0.994 \quad \text{Table 3.5} \\
T &= 293.15(\text{K}) + (77.35 - 293.15)(\text{K}) \times (1 - 0.994) = 291.9 \text{ K}.
\end{aligned}$$

(c) The solution for the heat flowing out of a semi-infinite medium subject to a constant temperature boundary condition is given by (3.145) as

$$\begin{aligned}
q_s = q_{\rho ck} &= -\frac{k[T_s - T(t=0)]}{(\pi \alpha t)^{1/2}} \\
&= -\frac{0.13(\text{W/m-K}) \times [77.35(\text{K}) - 293(\text{K})]}{[\pi \times 0.588 \times 10^{-7}(\text{m}^2/\text{s}) \times 10(\text{s})]^{1/2}} \\
&= 20{,}627 \text{ W out of the rubber.}
\end{aligned}$$

(d) After 10 s, the temperature of the rubber within the first millimeter of depth will be 215.84 K or lower. This is below the maximum temperature limit for crack-tear free drilling, i.e., $T_0 = 220$ K. Therefore, 10 s is enough cooling time to enable crack-tear free drilling of the hole.

COMMENT:

Note that the temperature at a distance of 1 mm from the surface has been lowered by 76.2 K, while the temperature 3 mm from the surface has been lowered by only 1.3 K. Therefore, as long as the rubber stopper has a thickness greater than 3 mm, the thermal penetration has not traveled across the rubber stopper and our assumption of a semi-infinite solid is shown to be valid.

PROBLEM 3.56.FAM

GIVEN:

The friction heat generation $\dot{S}_{m,F}$ (energy conversion) occurring in grinding flows *into* a workpiece (stainless steel AISI 316 at $T = 300$ K) and the grinder (use properties of brick in Table C.17 at $T = 293$ K). This is shown in Figure Pr.3.56(a). For a thick (i.e., assumed semi-infinite) grinder and a thick workpiece, the fraction of the heat flowing into the workpiece a_1 can be shown to be

$$a_1 = \frac{(\rho c_p k)_w^{1/2}}{(\rho c_p k)_w^{1/2} + (\rho c_p k)_g^{1/2}},$$

where the properties of the workpiece are designated by w and that of the grinder by g.

$t = 15$ s, $L = 1.5$ mm, $\dot{S}_{m,F}/A = 10^5$ W/m^2.

SKETCH:

Figure Pr.3.56(a) shows the grinding wheel, the workpiece, and the friction heating at the cylindrical surface.

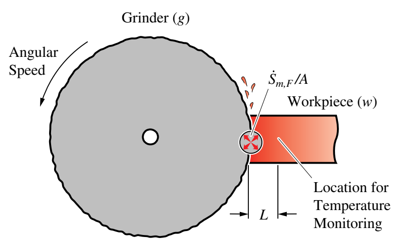


Figure Pr.3.56(a) Friction heating of a stainless steel workpiece and the division of the generated heat.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Use this relation for a_1 and the transient temperature distribution resulting from the sudden heating of a semi-infinite slab at a constant heat flux, given in Table 3.4, to determine the temperature of the workpiece at location L from the interface and after an elapsed time of t.

SOLUTION:

(a) The thermal circuit is shown in Figure Pr.3.56(b). The friction heating rate splits into two parts flowing into the grinder and the workpiece.

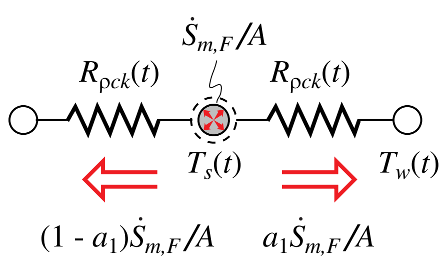


Figure Pr.3.56(b) Thermal circuit diagram.

(b) From Table C.16, for stainless steel AISI 316 at $T = 300$ K, we have $\rho_w = 8{,}238$ kg/m^3, $c_{p,w} = 468$ J/kg-K, $k_w = 13$ W/m-K, and $\alpha_w = 3.37 \times 10^{-6}$ m^2/s.

From Table C.17 for brick at $T = 293$ K, we have $\rho_g = 1{,}925$ kg/m^3, $c_{p,g} = 835$ J/kg-K, $k_g = 0.72$ W/m-K, and $\alpha_g = 0.45 \times 10^{-6}$ m^2/s.

From these, we have

$$\begin{aligned}(\rho c_p k)_w &= 50.12 \times 10^6 \text{ W}^2\text{-s/m}^4\text{-K}^2 \\ (\rho c_p k)_g &= 1.157 \times 10^6 \text{ W}^2\text{-s/m}^4\text{-K}^2.\end{aligned}$$

Then the fraction of heat flowing into the workpiece is

$$\begin{aligned}a_1 &= \frac{(\rho c_p k)_w^{1/2}}{(\rho c_p k)_w^{1/2} + (\rho c_p k)_g^{1/2}} \\ &= \frac{(50.12 \times 10^6)^{1/2}}{(1.157 \times 10^6)^{1/2} + (50.12 \times 10^6)^{1/2}} = 0.8681.\end{aligned}$$

Applying the surface energy equation (2.62) to node T_s, we have

$$\dot{S}_{m,F}/A = q_w + q_g = q_{tot} \quad \text{to node } T_s.$$

Therefore, for the heat flux flowing out of the surface is

$$\begin{aligned}q_w &= a_1 q_{tot} = a_1 \dot{S}_{m,F}/A \\ &= (0.8681) \times (10^5)(\text{W/m}^2) = 86{,}809 \text{ W/m}^2.\end{aligned}$$

This corresponds to a prescribed heat flux into the workpiece of

$$q_s = -q_w.$$

From Table 3.4, the transient temperature distribution in the workpiece is

$$T_s(x,t) = T_s(t=0) - \frac{q_s(4\alpha_w t)^{1/2}}{\pi^{1/2} k_w} e^{-\frac{x^2}{4\alpha_w t}} + \frac{q_s x}{k_w}\left\{1 - \text{erf}\left[\frac{x}{(4\alpha_w t)^{1/2}}\right]\right\}.$$

Then at $t = 15$ s and $x = 0.0015$ m, we have

$$\begin{aligned}4\alpha_w t &= 4 \times 3.376 \times 10^{-6}(\text{m/s}^2) \times 15(\text{s}) = 2.022 \times 10^{-4} \text{ m}^2 \\ \text{erf}\left[\frac{x}{(4\alpha_w t)^{1/2}}\right] &= \text{erf}\left[\frac{0.0015(\text{m})}{(2.022 \times 10^{-4})^{1/2}(\text{m})}\right] \\ &= \text{erf}(0.1055) = 0.1185 \qquad \text{Table 3.5.}\end{aligned}$$

Solving for the temperature, we have

$$\begin{aligned}T_s(x,t) &= T_s(t=0) - \frac{q_s(4\alpha_w t)^{1/2}}{\pi^{1/2} k_w} e^{-\frac{x^2}{4\alpha_w t}} + \frac{q_s x}{k_w}\left\{1 - \text{erf}\left[\frac{x}{(4\alpha_w t)^{1/2}}\right]\right\} \\ T_s(x = L, t = 1.5 \text{ s}) &= 300(\text{K}) - \frac{(-86{,}809)(\text{W/m}^2) \times (2.022 \times 10^{-4})^{1/2}(\text{m})}{\pi^{1/2}(13)(\text{W/m-K})} \exp\left[-\frac{(0.0015)^2(\text{m}^2)}{2.022 \times 10^{-4}(\text{m}^2)}\right] \\ &\quad + \frac{(-86{,}809)(\text{W/m}^2) \times (0.0015)(\text{m})}{(13)(\text{W/m-K})}[1 - (0.1185)] \\ &= 300(\text{K}) - [-52.98(\text{K})] + [-8.83(\text{K})] = 344.14 \text{ K} = 71^\circ\text{C}.\end{aligned}$$

COMMENT:

Note that a large fraction of the heat flows into the workpiece because of its larger effusivity. Also note that if a larger elapsed time is allowed, the temperature will be higher and can reach the damage threshold.

PROBLEM 3.57.FUN

GIVEN:

Two semi-infinite slabs having properties $(\rho, c_p, k)_1$ and $(\rho, c_p, k)_2$ and uniform, initial temperatures $T_1(t = 0)$ and $T_2(t = 0)$, are brought in contact at time $t = 0$.

OBJECTIVE:

(a) Show that their contact (interfacial) temperature is constant and equal to

$$T_{12} = \frac{(\rho c_p k)_1^{1/2} T_1(t=0) + (\rho c_p k)_2^{1/2} T_2(t=0)}{(\rho c_p k)_1^{1/2} + (\rho c_p k)_2^{1/2}}.$$

(b) Under what conditions does $T_{12} = T_2(t = 0)$? Give an example of material pairs that would result in the limit.

SOLUTION:

(a)Assuming a constant contact surface temperature, we use (3.140) for the temperature distribution and use $T_s = T_{12}$, i.e., for the two semi infinite slabs we have

$$\frac{T_1(t) - T_1(t=0)}{T_{12} - T_1(t=0)} = 1 - \text{erf}(\eta) = 1 - \frac{2}{\pi^{1/2}} \int_0^\eta e^{-\eta^2} d\eta$$

$$\frac{T_2(t) - T_2(t=0)}{T_{12} - T_2(t=0)} = 1 - \text{erf}(\eta) = 1 - \frac{2}{\pi^{1/2}} \int_0^\eta e^{-\eta^2} d\eta$$

$$\eta = \frac{x}{2(\alpha t)^{1/2}}.$$

Since the heat flowing out of one of the slabs flows into the other, we have

$$q_s = q_{12} = q_1(x=0) = q_2(x=0),$$

where $x = 0$ is the location of the interface.

From (3.143), we relate q_{12} and the derivative of the temperatures, i.e.,

$$\begin{aligned} q_{12} &= -k_1 \left.\frac{\partial T_1}{\partial x}\right|_{x=0} \\ &= -\left(-k_2 \left.\frac{\partial T_2}{\partial x}\right|_{x=0}\right). \end{aligned}$$

We note that from the chain rule for differentiation, we have

$$\begin{aligned} \frac{\partial}{\partial x} &= \frac{\partial \eta}{\partial x} \frac{\partial}{\partial \eta} \\ &= \frac{1}{2(\alpha t)^{1/2}} \frac{\partial}{\partial \eta}. \end{aligned}$$

Then

$$\begin{aligned} \left.\frac{\partial T_1(t)}{\partial x}\right|_{x=0} &= \frac{1}{2(\alpha_1 t)^{1/2}} \left(-\frac{2}{\pi^{1/2}} e^{-\eta^2}\right)_{\eta=0} [T_{12} - T_1(t=0)] \\ &= \frac{1}{2(\alpha_1 t)^{1/2}} \left(-\frac{2}{\pi^{1/2}}\right) [T_{12} - T_1(t=0)] \\ \left.\frac{\partial T_2(t)}{\partial x}\right|_{x=0} &= \frac{1}{2(\alpha_2 t)^{1/2}} \left(-\frac{2}{\pi^{1/2}}\right) [T_{12} - T_2(t=0)]. \end{aligned}$$

Then using the equality of the surface heat fluxes, we have

$$-k_1 \left[\frac{-1}{(\pi \alpha_1 t)^{1/2}}\right] \times [T_{12} - T_1(t=0)] = k_2 \left[\frac{-1}{(\pi \alpha_2 t)^{1/2}}\right] \times [T_{12} - T_2(t=0)]$$

or

$$-\frac{k_1}{(k/\rho c_p)_1^{1/2}}[T_{12} - T_1(t=0)] = \frac{k_2}{(k/\rho c_p)_2^{1/2}}[T_{12} - T_2(t=0)]$$

or

$$T_{12} = \frac{(\rho c_p k)_1^{1/2} T_1(t=0) + (\rho c_p k)_2^{1/2} T_2(t=0)}{(\rho c_p k)_1^{1/2} + (\rho c_p k)_2^{1/2}}.$$

(b) From Tables C.16 and C.17, we can choose material pairs for which one material has a much larger $(\rho c_p k)^{1/2}$, which is called the thermal effusivity. For example, metals have high ρ, low c_p, and high k. We choose copper from Table C.16 and wood for Table C.17. Then

copper (pure):	$\rho = 8{,}933$ kg/m^3	Table C.16
	$c_p = 385$ J/kg-K	Table C.16
	$k = 401$ W/m-K	Table C.16
wood (pine):	$\rho = 525$ kg/m^3	Table C.17
	$c_p = 2{,}750$ J/kg-K	Table C.17
	$k = 0.12$ W/m-K	Table C.17.

Then

$$\text{copper (pure):} \quad (\rho c_p k)_1^{1/2} = 3.714 \times 10^4 \text{ W-s}^{1/2}/\text{m}^2\text{-K}$$
$$\text{wood (pine):} \quad (\rho c_p k)_2^{1/2} = 4.162 \times 10^2 \text{ W-s}^{1/2}/\text{m}^2\text{-K}.$$

Using these, we have

$$\begin{aligned} T_{12} &= \frac{3.714 \times 10^4 \times T_1(t=0) + 4.162 \times 10^2 \times T_2(t=0)}{3.714 \times 10^4 + 4.162 \times 10^2} \\ &\simeq T_1(t=0). \end{aligned}$$

This shows that the contact temperature will be the initial copper temperature $T_1(t=0)$, i.e., the wood will instantly take on the surface temperature of the copper.

COMMENT:

Note that we initially assumed that the contact temperature T_{12} is constant and this allowed us to use the transient solutions for the constant surface temperature.

PROBLEM 3.58.FAM.S

GIVEN:

For thermal treatment, the surface of a thin-film coated substrate [initially at $T(t=0)$] is heated by a prescribed heat flux $q_s = -10^9$ W/m^2 (this heat flux can be provided for example by irradiation) for a short period. This heating period t_o (i.e., elapsed time) is chosen such that only the temperature of the titanium alloy (Ti-2 Al-2 Mn, mass fraction composition) thin film is elevated significantly (i.e., the penetration distance is only slightly larger than the thin-film thickness). The thin film is depicted in Figure Pr.3.58.

SKETCH:

Figure Pr.3.58 shows a thin film over a semi-infinite substrate, is heated by irradiation.

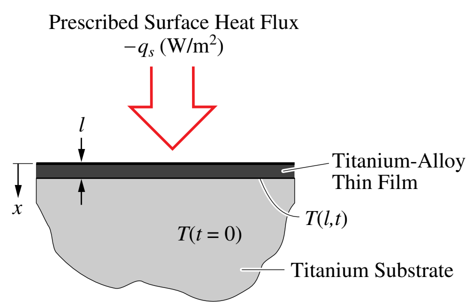


Figure Pr.3.58(a) A thin-film coated, semi-infinite substrate heated by irradiation.

OBJECTIVE:

Determine the required elapsed time t_o for the temperature of the interface between the thin film and the substrate, located at distance $l = 5\ \mu$m from the surface, to raise by $\Delta T(l,t_o) = T(l,t_o) - T(t=0) = 300$ K. Note that this would require determination of t from an implicit relation and would require iteration or use of a software.

SOLUTION:

Table 3.4 gives the temperature distribution for a semi-infinite slab with a prescribed heat flux at the surface. For q_s constant on the surface,

$$T - T(t=0) = -\frac{2q_s}{k}\left(\frac{\alpha t}{\pi}\right)^{1/2} e^{-\frac{x^2}{4\alpha t}} + \frac{q_s x}{k}\left\{1 - \text{erf}\left[\frac{x}{(4\alpha t)^{1/2}}\right]\right\}.$$

This equation requires k and α. From Table C.16, for a Ti-2 Al-2 Mn alloy at 300 K, $k = 8.4$ W/m-K and $\alpha = 4 \times 10^{-6}$ m^2/s.

From the problem statement, $T(l,t_o) - T(t=0) = \Delta T = 300$ K, $q_s = -10^9$ W/m^2, and $x = l = 5 \times 10^{-6}$ m. Substituting these values into the equation above, we have

$$300 = 2.687 \times 10^5 \times t^{1/2}\, e^{\frac{-1.563 \times 10^{-6}}{t}} - 5.952 \times 10^2 \times \left[1 - \text{erf}\left(\frac{1.250 \times 10^{-3}}{t^{1/2}}\right)\right].$$

The equation is an implicit relation for time. The solution can be obtained from an equation solver software or by hand calculation, iteratively. The method of successive substitutions can be used for the iterative solution. For this method, the above equation is written explicitly for one of the occurrences of the variable time. Choosing the time t appearing in the first term in the right-hand side, the equation is rewritten as

$$t_{new} = \left[\frac{3.332 \times 10^{-3} - 2.216 \times 10^{-3}\text{erf}\left(\frac{1.250 \times 10^{-3}}{t^{1/2}}\right)}{e^{\frac{-1.563 \times 10^{-6}}{t}}}\right]^2.$$

The solution requires guessing a value of t and solving for t_{new}. The process ends when t and t_{new} are sufficiently close. When the initial guess is close to the final answer, the convergence is stable and requires only a few iterations. A suitable initial guess can be obtained by calculating the time it takes for the surface temperature to be raised by ΔT. For the surface, $x = 0$, the equation for T becomes

$$T - T(t=0) = \frac{-2q_s}{k}\left(\frac{\alpha t_o}{\pi}\right)^{1/2}.$$

Solving for t_o gives

$$t_o = \frac{\pi}{\alpha}\left(\frac{\Delta T k}{-2q_s}\right)^2.$$

Using the known values,

$$t_o = \frac{\pi}{4\times10^{-6}(\text{m}^2/\text{s})}\left[\frac{300(\text{K})\times 8.4(\text{W/m-K})}{2\times10^9(\text{W/m}^2)}\right]^2 = 1.25\times10^{-6}\ \text{s} = 1.25\ \mu\text{s}.$$

The temperature at a depth of 5 μm will take longer to be raised by ΔT. The value of t_o gives a starting point for the iterations. Table Pr.3.58 shows the iterations. The error function is interpolated from Table 3.5.
The solution after 5 iterations is $t = 7.8$ μs.

Table Pr.3.58 Results for successive iterations.

t, s	$\eta = 1.250\times10^{-3}/t^{1/2}$	erf(η)	t_{new}, s
2×10^{-6}	0.884	0.7880	1.201×10^{-5}
4×10^{-6}	1.625	0.6223	8.334×10^{-6}
6×10^{-6}	0.5103	0.5289	7.856×10^{-6}
7.6×10^{-6}	0.4534	0.4775	7.801×10^{-6}
7.8×10^{-6}	0.4476	0.4721	7.800×10^{-6}

COMMENT:

The solution found using software [Figure Pr.3.58(b)] is $t_o = 7.785$ μs. The result obtained above shows that the use of Table 3.5 for the error function is acceptable. Note also that the same solution could be obtained if the time variable inside the exponential function were chosen as t_{new}.

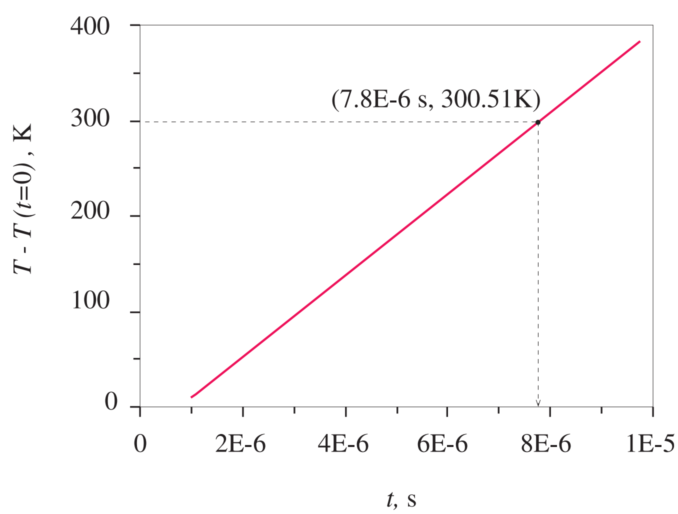


Figure Pr.3.58(b) Variation of temperature, at a location $x = 5$ μm from the surface, with respect to time.

Figure Pr.3.58(c) shows the temperature distributions as a function of the depth x for different values of time. Notice that the derivative dT/dx at the surface is constant, because q_s is constant at the surface. Also, the penetration depth increases with time. For the elapsed time of 8 μs, the ΔT at the surface is above 600 K.

From Figure Pr.3.58(c) we observe that for $t = 8$ μs there was substantial heat penetration into the substrate. In order to account for the change of properties between the thin film and the substrate, other solution techniques need to be used. One example of such a technique is the use of finite (i.e., small) volumes, presented in Section 3.7.

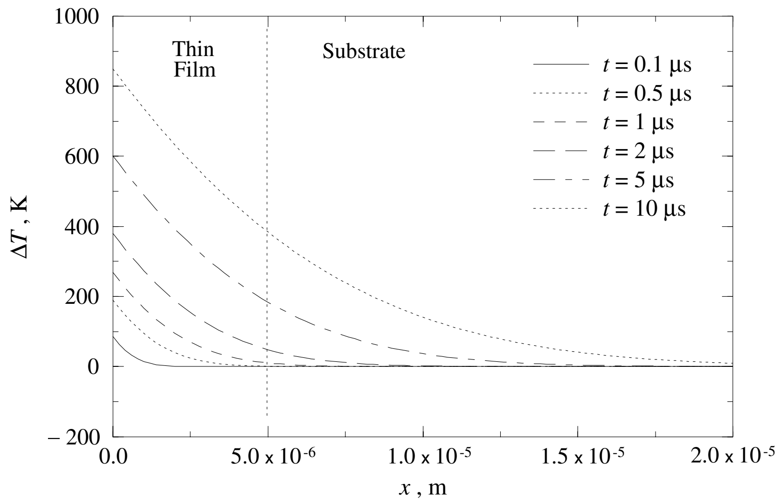


Figure Pr.3.58(c) Distribution of temperature near the surface, at several elapsed times.

PROBLEM 3.59.FAM

GIVEN:

An automobile tire rolling over a paved road is heated by surface friction, as shown in Figure Pr.3.59. The energy conversion rate divided by the tire surface area is $\dot{S}_{m,F}/A_t$, and this is related to the vehicle mass M and speed u_o through

$$\frac{\dot{S}_{m,F}}{A_t} = \frac{\mathrm{Mg}\mu_F u_o}{A_t}.$$

A fraction of this, $a_1\dot{S}_{m,F}/A_t$, is conducted through the tire. The tire has a cover-tread layer with a layer thickness L, which is assumed to be much smaller than the tire thickness, but is made of the same hard rubber material as the rest of the tire. The deep unperturbed temperature is $T(t=0)$.

The properties for hard rubber are listed in Table C.17.

$T(t=0) = 20$℃, g $= 9.807$ m/s^2, $u_o = 60$ km/hr, $L = 4$ mm, $A_t = 0.4$ m^2, $t_o = 10$ min, $\mu_F = 0.015$, $a_1 = 0.1$.

SKETCH:

Figure Pr.3.59 shows friction heating and heat transfer into a tire and the location from the surface.

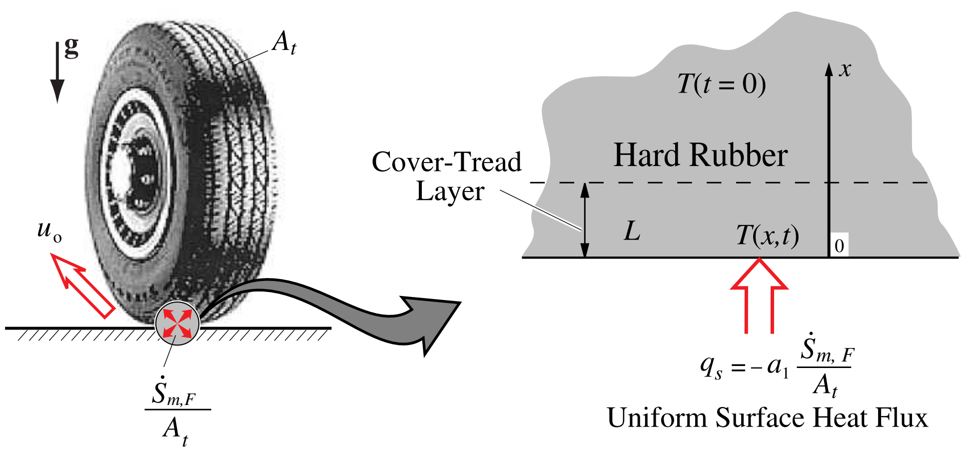


Figure Pr.3.59 Surface friction heating of a tire laminate.

OBJECTIVE:

Determine the temperature at this location L, after an elapsed time t_o, using (a) M = 1,500 kg, and (b) M = 3,000 kg.

SOLUTION:

(a) Applying the energy equation to the contact area, and assuming all the heat flows into the tire,

$$\begin{aligned} q_s &= -a_1\frac{\dot{S}_{m,F}}{A_t} \\ &= -a_1\frac{\mathrm{Mg}\mu_F u_o}{A_t} \\ &= -0.1\,\frac{1{,}500(\mathrm{kg}) \times 9.807(\mathrm{m/s^2}) \times 0.015 \times \dfrac{60{,}000(\mathrm{m/hr})}{3{,}600(\mathrm{s/hr})}}{0.4(\mathrm{m^2})} \\ &= -919.4\ \mathrm{W/m^2}. \end{aligned}$$

From Table 3.4, the transient temperature distribution (for prescribed q_s) in the workpiece is given as

$$T(x,t) = T(t=0) - \frac{q_s(4\alpha t)^{1/2}}{\pi^{1/2}k}e^{-\frac{x^2}{4\alpha t}} + \frac{q_s x}{k}\left\{1 - \mathrm{erf}\left[\frac{x}{(4\alpha t)^{1/2}}\right]\right\}.$$

For hard rubber, using Table C.17, we have, $k = 0.15$ W/m-K, $\alpha = 0.6219 \text{ x } 10^{-7}$ m^2/s.
Then at $t = 10 \times 60$ s and at $x = 0.004$ m, we have

$$
\begin{aligned}
4\alpha t &= 1.493 \times 10^{-4} \text{ m}^2 \\
\text{erf}\left[\frac{x}{(4\alpha t)^{1/2}}\right] &= \text{erf}\left[\frac{0.004(\text{m})}{(1.493 \times 10^{-4})^{1/2}(\text{m})}\right] \\
&= \text{erf}\,(0.3274) = 0.3560 \qquad \text{Table 3.5.}
\end{aligned}
$$

Solving for the temperature, we have

$$
\begin{aligned}
T(x,t) &= T(t=0) - \frac{q_s(4\alpha t)^{1/2}}{\pi^{1/2}k} e^{-\frac{x^2}{4\alpha t}} + \frac{q_s x}{k}\left\{1 - \text{erf}\left[\frac{x}{(4\alpha t)^{1/2}}\right]\right\} \\
T(x = 4 \text{ mm}, t = 600 \text{ s}) &= 293.15(\text{K}) - \frac{-919.4(\text{W/m}^2) \times (1.493 \times 10^{-4})^{1/2}(\text{m})}{\pi^{1/2} \times 0.15(\text{W/m-K})} \exp\left[-\frac{(0.004)^2(\text{m}^2)}{1.493 \times 10^{-4}(\text{m}^2)}\right] \\
&\quad + \frac{-919.4(\text{W/m}^2) \times 0.004(\text{m})}{0.13(\text{W/m-K})}\,(1 - 0.3560) \\
&= 293.15(\text{K}) - [-37.94(\text{K})] + [-15.79(\text{K})] = 315.3 \text{ K} = 42.15^\circ\text{C}.
\end{aligned}
$$

(b) Using M = 3,000 kg, $q_s = -1{,}838.8$ W/m^2, and solving for temperature, we have

$$
\begin{aligned}
T(x = 4 \text{ mm}, t = 600 \text{ s}) &= 293.15(\text{K}) - \frac{-1{,}838.8(\text{W/m}^2) \times (1.493 \times 10^{-4})^{1/2}(\text{m})}{\pi^{1/2} \times 0.15(\text{W/m-K})} \exp\left[-\frac{(0.004)^2(\text{m}^2)}{1.493 \times 10^{-4}(\text{m}^2)}\right] \\
&\quad + \frac{-1{,}838.8(\text{W/m}^2) \times 0.004(\text{m})}{0.15(\text{W/m-K})}\,(1 - 0.3560) \\
&= 293.15(\text{K}) - [-75.88(\text{K})] + [-31.58(\text{K})] = 337.5 \text{ K} = 64.30^\circ\text{C}.
\end{aligned}
$$

COMMENT:

As the tire heats up, the surface convection heat transfer rate increases. Therefore, the amount of heat conducting through the tire surface decreases with an increase in surface temperature.

PROBLEM 3.60.FAM

GIVEN:

In ultrasonic welding (also called ultrasonic joining), two thick slabs of polymeric solids to be joined are placed in an ultrasonic field that causes a relative motion at their joining surfaces. This relative motion combined with a joint pressure causes a surface friction heating at a rate of $\dot{S}_{m,F}/A$. This heat flows and penetrates equally into these two similar polymeric solids. The two pieces are assumed to be very thick and initially at a uniform temperature $T(t=0)$.

$\dot{S}_{m,F}/A = 10^4$ W/m^2, $T_{sl} = 300$℃, $T(t=0) = 25$℃, and use the properties of Teflon (Table C.17).

SKETCH:

Figure Pr.3.60(a) shows the two solid surfaces in sliding contact and the friction heat flow into each of the pieces.

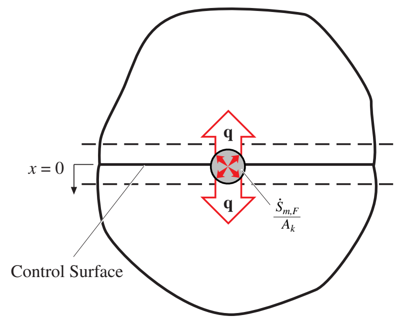


Figure Pr.3.60(a) The solids in sliding contact and friction heat flow into both of them.

OBJECTIVE:

How long would it take for the contacting surfaces of the two polymers in contact to reach their melting temperature T_{sl}?

SOLUTION:

The two pieces are very thick and therefore are assumed to behave as semi-infinite slabs. The surface friction heating occurs uniformly over the entire contact surface and the resulting generated heat flows equally into each piece.

The two pieces in contact are drawn schematically in Figure Pr.3.60(a). Performing a surface conservation of energy analysis on the contacting surface and noting that the heat flows equally into both the upper and lower pieces gives

$$q_{upper} + q_{lower} = 2q = \dot{S}_{m,F}/A$$
$$q = \frac{\dot{S}_{m,F}}{2A}.$$

By symmetry, we only need to analyze one of the pieces to determine when the surface temperature reaches the melting temperature. The control volume for the lower piece is rendered in Figure Pr.3.60(b).

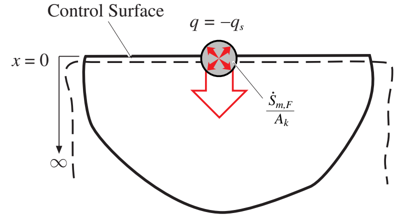


Figure Pr.3.60(b) Control volume for the solids in sliding contact.

As shown, the heat flux entering across the control surface into the volume is equal to the negative of that leaving one side of the control surface, i.e. $q_s = -q$. The resulting conservation of energy equation is solved, and the solution for $T(x,t)$ is given in Table 3.4 as

$$T(x,t) = T(t=0) - \frac{2q_s(\alpha t)^{1/2}}{\pi^{1/2}k} e^{-\frac{x^2}{4\alpha t}} + \frac{q_s x}{k}\left\{1 - \mathrm{erf}\left[\frac{x}{2(\alpha t)^{1/2}}\right]\right\}.$$

At location $x = 0$, this equation becomes

$$\begin{aligned} T(x=0,t) - T(t=0) &= -\frac{2q_s(\alpha t)^{1/2}}{\pi^{1/2}k} \\ \Delta T(0,t) &= T_{sl} - T(t=0) = -\frac{2(-\frac{\dot{S}_{m,F}}{2A})(\alpha t)^{1/2}}{\pi^{1/2}k}. \end{aligned}$$

For Teflon at $T = 293$ K, $k = 0.26$ W/m-K and $\alpha = 0.34 \times 10^{-6}$ m^2/s. Solving for t we have

$$\begin{aligned} t &= \left[\frac{\Delta T \pi^{1/2} k}{(\dot{S}_{m,F}/A)\alpha^{1/2}}\right]^2 = \left(\frac{\Delta T k}{\dot{S}_{m,F}/A}\right)^2 \frac{\pi}{\alpha} \\ &= \frac{(300℃ - 25℃)^2 \times (0.26 \text{ W/m-K})^2 \times \pi}{(10^4 \text{ W/m}^2)^2 \times (0.34 \times 10^{-6} \text{ m}^2/\text{s})} \\ &= \frac{(300-25)^2(℃)^2 \times (0.26)^2(\text{W/m-K})^2 \times \pi}{(10^4)^2(\text{W/m}^2)^2 \times (0.34 \times 10^{-6})(\text{m}^2/\text{s})} = 472.37 \text{ s} = 7.87 \text{ min}. \end{aligned}$$

COMMENT:

The process time is inversely proportional to $\dot{S}_{m,F}$ to the second power. The process time can be decreased by increasing $\dot{S}_{m,F}$.

PROBLEM 3.61.FAM

GIVEN:

A thin film is heated with irradiation from a laser source as shown in Figure Pr.3.61(a). Assume that all the radiation is absorbed (i.e., $\alpha_{r,1} = 1$). The heat losses from the film are by substrate conduction only. The film can be treated as having a uniform temperature $T_1(t)$ i.e., $N_{k,1} < 0.1$, and the conduction resistance $R_{k,1\text{-}2}$ through the substrate can be treated as constant.

SKETCH:

Figure Pr.3.61(a) shows the thin film, over a substrate, heated by laser irradiation.

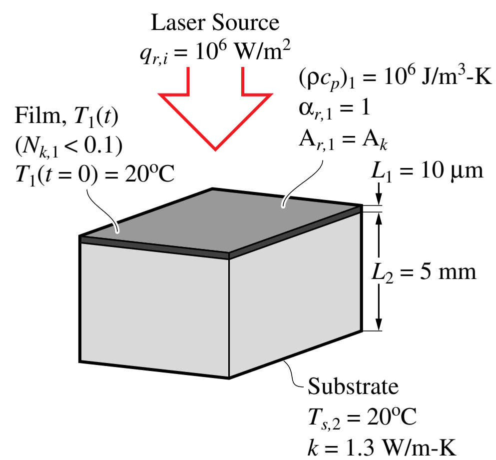


Figure Pr.3.61(a) Laser radiation heating of a thin film over a substrate.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the time needed to raise the temperature of the film $T_1(t)$ to 500℃.

SOLUTION:

(a) Figure Pr.3.61(b) shows the thermal circuit for the problem. Note that the thin film is lumped into a single node and the thick film is modeled as a conduction resistance constant with time.

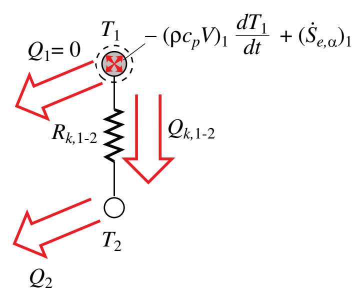


Figure Pr.3.61(b) Thermal circuit diagram.

(b) To determine the time needed to raise the film temperature to 500℃, the energy equation is applied to the thin film. The integral form of the energy equation is

$$Q|_A = -(\rho c_p V)_1 \frac{dT_1}{dt} + (\dot{S}_{e,\alpha})_1.$$

From Figure Pr.3.61(b), we notice that $Q|_A$ has only a conduction component. The energy source is due to radiation adsorption with $\alpha_r = 1$ and $\epsilon_r = 0$. Therefore, the energy equation becomes

$$\frac{T_1 - T_{s,2}}{R_{k,1\text{-}2}} = -(\rho c_p V)_1 \frac{dT_1}{dt} + \alpha_{r,1} q_{r,i} A_{r,1}.$$

The conduction resistance is given by

$$R_{k,1\text{-}2} = \frac{L_2}{k_s A_k} = \frac{5 \times 10^{-3}(\text{m})}{1.3(\text{W/m-K}) A_k} = \frac{3.85 \times 10^{-3}[\text{K/(W/m}^2)]}{A_k}.$$

The thermal capacitance is

$$(\rho c_p V)_1 = 10^6 \left(\text{J/m}^3\text{-K}\right) 10 \times 10^{-6}\,(\text{m})\, A_k = 10 \left(\text{J/m}^2\text{-°C}\right) A_k.$$

The source term is

$$\dot{S}_1 = \alpha_{r,1} q_{r,i} A_{r,1} = (1) \times 10^6 \left(\text{W/m}^2\right) A_{r,1}.$$

The solution for T_1 is given by (3.172). Solving for t, we have

$$t = -\tau_1 \ln \left[\frac{T_1 - T_{s,2} - a_1 \tau_1}{T_1(t=0) - T_{s,2} - a_1 \tau_1} \right],$$

where

$$\tau_1 = (\rho c_p V)_1 R_{k,1\text{-}2} = \frac{3.85 \times 10^{-3}\left(\text{K/(W/m}^2)\right)}{A_k} 10 \left(\text{J/m}^2\text{-K}\right) A_k = 3.85 \times 10^{-2} \text{ s}$$

$$a_1 = \frac{\dot{S}_1}{(\rho c_p V)_1} = \frac{10^6 \left(\text{W/m}^2\right) A_{r,1}}{10 \left(\text{J/m}^2\text{-K}\right) A_k} = 10^5 \text{ K/s}$$

and $A_{r,1} = A_k$ has been used. Then

$$\begin{aligned} t &= -3.85 \times 10^{-2}(\text{s}) \ln \left[\frac{500\,(°\text{C}) - 20\,(°\text{C}) - 10^5(1/\text{s}) \times 3.85 \times 10^{-2}(\text{s})}{20\,(°\text{C}) - 20\,(°\text{C}) - 10^5(1/\text{s}) \times 3.85 \times 10^{-2}(\text{s})} \right] \\ &= 0.0051 \text{ s} = 5.1 \text{ ms}. \end{aligned}$$

COMMENT:

Note that the response is rather fast. The radiation reflection from the surface and radiation emission from the surface will be addressed in Chapter 4.

PROBLEM 3.62.FUN

GIVEN:

Carbon steel AISI 4130 spheres with radius R_1 = 4 mm are to be annealed. The initial temperature is $T_1(t=0) = 25$°C and the annealing temperature is $T_a = 950$°C. The heating is done by an acetylene torch. The spheres are placed on a conveyor belt and passed under the flame of the acetylene torch. The surface-convection heat flux delivered by the torch (and moving into the spheres) is given as a function of the position within the flame by (see Figure Pr.3.62)

$$q_s(t) = q_{ku} = q_o \sin\left(\pi \frac{x}{L}\right),$$

where $L = 2$ cm is the lateral length of the flame and $q_o = -3 \times 10^6$ W/m^2 is the heat flux at the center of the flame.

Assume that the surface of the spheres is uniformly heated and neglect the heat losses. Use the properties at 300 K, as given in Table C.16.

SKETCH:

Figure Pr.3.62 shows a sphere placed on a conveyer and passed under a torch. The surface heat flux is also given, as a function of location.

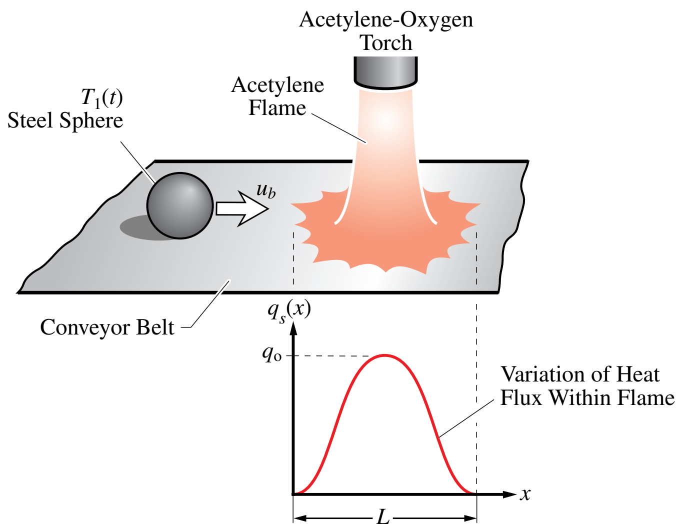


Figure Pr.3.62 Heating of carbon steel spheres placed on a conveyor.

OBJECTIVE:

Using a lumped-capacitance analysis ($N_{ku,1} < 0.1$) and starting from (3.160), find the speed of the conveyor belt u_b needed for heating the spheres from $T_1(t=0)$ to T_a.

SOLUTION:

Treating each sphere as a lumped-capacitance system, the integral-volume energy equation (3.161) becomes

$$Q|_A = \int_{A_1} (\mathbf{q} \cdot \mathbf{s}_n) dA = -(\rho c_p V)_1 \frac{dT}{dt} + \dot{S}_i.$$

As the convection heat flux vector points to the surface and there is no energy conversion inside the spheres, we have

$$\begin{aligned} \int_A (\mathbf{q} \cdot \mathbf{s}_n) dA &= -q_s(t) A \\ \dot{S}_i &= 0. \end{aligned}$$

The energy equation becomes

$$(\rho c_p V)_1 \frac{dT_1}{dt} = -q_s(t) A_1.$$

Using the equation for $q_s(t)$, we have

$$(\rho c_p V)_1 \frac{dT_1}{dt} = -q_o \sin\left(\frac{\pi x}{L}\right) A_1.$$

Note that the distance traveled along the flame x can be related to the elapsed time t by

$$x = u_b t.$$

Now, using this relation we have

$$(\rho c_p V)_1 \frac{dT_1}{dt} = q_o \sin\left(\frac{\pi u_b}{L} t\right) A_1.$$

Separating the variables and integrating gives

$$\begin{aligned}
dT_1 &= \frac{-q_o A_1}{(\rho c_p V)_1} \sin\left(\frac{\pi u_b}{L} t\right) dt \\
\int_{T_1(t=0)}^{T_a} dT_1 &= \frac{-q_o A_1}{(\rho c_p V)_1} \int_0^t \sin\left(\frac{\pi u_b}{L} t\right) dt \\
T_a - T_1(t=0) &= \frac{-q_o A_1}{(\rho c_p V)_1} \frac{L}{\pi u_b} \left[1 - \cos\left(\frac{\pi u_b}{L} t\right)\right].
\end{aligned}$$

The final time t is the time it takes to travel through the flame and is given by $t = L/u_b$. Then

$$T_a - T(t=0) = \frac{-2 q_o A_1}{(\rho c_p V)_1} \frac{L}{\pi u_b}.$$

For spheres, we have

$$\frac{V_1}{A_1} = \frac{\frac{4}{3}\pi R_1^3}{4\pi R_1^2} = \frac{R_1}{3}.$$

Then

$$T_a - T(t=0) = \frac{-6 q_o}{(\rho c_p)_1 R_1} \frac{L}{\pi u_b}.$$

The properties for carbon steel AISI 4130 at 300 K, from Table C.16, are ρ_1 = 7,840 kg/m^3 and $c_{p,1}$ = 460 J/kg-K. Solving for u_b gives

$$\begin{aligned}
u_b &= \frac{-6 q_o L}{\pi (\rho c_p)_1 R_1 [T_a - T(t=0)]} = \frac{6 \times 3 \times 10^6 (\mathrm{W/m^2}) \times 0.02 (\mathrm{m})}{\pi \times 7{,}840 (\mathrm{kg/m^3}) \times 460 (\mathrm{J/kg\text{-}K}) \times 0.004 (\mathrm{m}) \times [950(^\circ\mathrm{C}) - 25(^\circ\mathrm{C})]} \\
&= 0.008 \text{ m/s} = 51.5 \text{ cm/min}.
\end{aligned}$$

COMMENT:

We would need a smaller speed u_b if we considered the presence of heat losses. The lumped-capacitance analysis is valid when $N_{ku,1} < 0.1$ and this will be discussed in Chapter 6, in the context of surface-convection heat transfer q_{ku}.

PROBLEM 3.63.FAM.S

GIVEN:

An electrical resistance regulator is encapsulated in a rectangular casing (and assumed to have a uniform temperature T_1, i.e., $N_{k,1} < 0.1$) and attached to an aluminum slab with thickness $L = 3$ mm. The slab is in turn cooled by maintaining its opposite surface at the constant ambient temperature $T_2 = 30°C$. This slab is called a heat sink and is shown in Figure Pr.3.63(a). Most of the time, the regulator provides no resistance to the current flow, and therefore, its temperature is equal to T_2. Intermittently, the regulator control is activated to provide for an ohmic resistance and then energy conversion from electromagnetic to thermal energy occurs. A joint pressure is exerted to reduce the contact thermal resistance at the interface between the regulator and the heat sink. However, as the regulator temperature reaches a threshold value $T_{1,o} = 45°C$, the thermal stresses warp the regulator surface and the contact resistance changes from $A_k R_{k,c} = 10^{-3}$ K/(W/m^2) to a larger value of $A_k R_{k,c} = 10^{-2}$ K/(W/m^2). The regulator has $(\rho c_p V)_1/A_k = 1.3 \times 10^5$ J/K-m^2 and the amount of heat generated by Joule heating is $\dot{S}_{e,\mathrm{J}}/A_k = 2 \times 10^4$ W/m^2. Neglect the heat losses from the regulator to the ambient. Assume that the conduction resistance in the aluminum slab is steady state (i.e., constant resistance) and the energy storage in the slab is also negligible.

Use the thermal conductivity of aluminum at $T = 300$ K.

SKETCH:

Figure Pr.3.63(a) shows a regulator subject to the Joule heating and attached to a substrate with a thermal contact resistance.

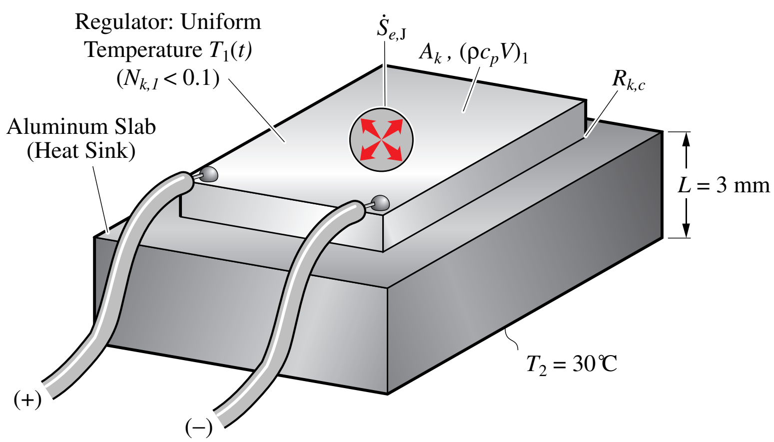


Figure Pr.3.63(a) An electrical resistance regulator attached to a substrate with a thermal contact resistance.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Use the lumped-capacitance analysis to determine the required time to reach the threshold temperature $T_c = 45°C$.

(c) Starting with $T_{1,o}$ as the initial temperature and using the new contact resistance $A_k R_{k,c}$, determine the time required to reach $T_1 = T_2 + 2\,T_c$.

(d) Determine the steady-state temperature.

(e) Make a qualitative plot of the regulator temperature versus time, showing (i) the transition in the contact resistance, and (ii) the steady-state temperature.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.3.63(b).

(b) To find the switch temperature, the integral-volume energy equation (3.161) is applied to the switch node T_1. For transient conditions, we have

$$Q_1 + \sum_j \frac{T_1 - T_j}{R_{k,1\text{-}j}} = -\rho c_p V \frac{dT_1}{dt} + \dot{S}_{e,\mathrm{J}}.$$

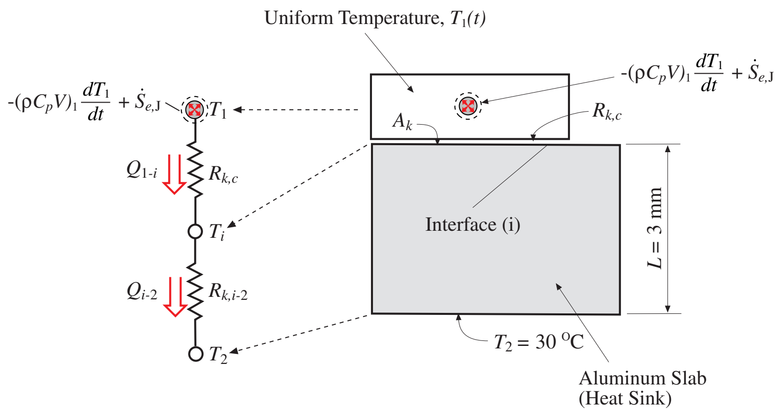


Figure Pr.3.63(b) Thermal circuit diagram.

The temperature T_2 is known. Therefore, the heat flux through the thermal resistances is written as a function of T_2. The energy equation for node 1 is

$$\frac{T_1 - T_2}{(R_{k,\Sigma})_{1\text{-}2}} = -\rho c_p V \frac{dT_1}{dt} + \dot{S}_{e,\mathrm{J}}.$$

For the resistances arranged in series, the overall thermal resistance $(R_{k,\Sigma})_{1\text{-}2}$ is

$$(R_{k,\Sigma})_{1\text{-}2} = R_{k,c} + R_{k,i\text{-}2}.$$

The conduction resistance $R_{k,i\text{-}2}$ for the slab is (Table 3.2)

$$R_{k,i\text{-}2} = \frac{L}{k_2 A_k},$$

and the contact resistance $R_{k,c}$ is given in the problem statement.

The thermal conductivity k_2 is needed to calculate the thermal resistance. For aluminum (Table C.14, $T = 300$ K) we obtain $k_2 = 237$ W/m-K. The solution for $T_1(t)$, given by (3.172), is

$$T_1(t) = T_2 + [T_i(t=0) - T_2]e^{-t/\tau_1} + a_1\tau_1(1 - e^{-t/\tau_1}),$$

where

$$\tau_1 = (\rho c_p V)_1 (R_{k,\Sigma})_{1\text{-}2}$$

$$a_1 = \frac{\dot{S}_{e,\mathrm{J}} - Q_1}{(\rho c_p V)_1}.$$

For the initial heating period, the conditions are: $T_1(t=0) = 30$℃, $T_1(t) = T_c = 45$℃, $A_k R_{k,c} = 10^{-3}$ K/(W/m^2). From the data given,

$$A_k R_{k,i\text{-}2} = \frac{L}{k_2} = \frac{0.003}{237} = 1.266 \times 10^{-5} \text{ K/(W/m}^2\text{)}$$

$$A_k (R_{k,\Sigma})_{1\text{-}2} = A_k R_{k,c} + A_k R_{k,i\text{-}2} = 10^{-3} + 1.266 \times 10^{-5} = 1.013 \times 10^{-3} \text{ K/(W/m}^2\text{)}$$

$$\tau_1 = (\rho c_p V)_1 (R_{k,\Sigma})_{1\text{-}2} = \frac{\rho c_p V}{A_k} A_k (R_{k,\Sigma})_{1\text{-}2} = (1.3 \times 10^5) \times (1.013 \times 10^{-3}) = 1.316 \times 10^2 \text{ s}$$

$$a_1 = \frac{\dot{S}_{e,\mathrm{J}} - Q_1}{(\rho c_p V)_1} = \left(\frac{\dot{S}_{e,\mathrm{J}}}{A_k} - \frac{Q_1}{A_k}\right)\frac{A_k}{(\rho c_p V)_1} = \frac{2\times 10^4 - 0}{1.3\times 10^5} = 1.538\times 10^{-1}\ \mathrm{K/s}.$$

Solving for time gives

$$\begin{aligned} t &= -\tau_1 \ln \frac{T_1(t) - T_2 - a_1\tau_1}{T_1(t=0) - T_2 - a_1\tau_1} \\ &= -1.316\times 10^2(\mathrm{s}) \ln \frac{45(^\circ\mathrm{C}) - 30(^\circ\mathrm{C}) - (1.538\times 10^{-1})(^\circ\mathrm{C/s}) \times (1.316\times 10^2)(\mathrm{s})}{0 - (1.538\times 10^{-1})(^\circ\mathrm{C/s}) \times (1.316\times 10^2)(\mathrm{s})} = 177.8\ \mathrm{s} \simeq 3.0\ \mathrm{min} \end{aligned}$$

(c) At $T_1 = 45^\circ\mathrm{C}$, the contact resistance changes to $A_k R_{k,c} = 10^{-2}\ \mathrm{K/(W/m^2)}$. The overall resistance and the time constant τ_1 become (note that a_1 remains the same)

$$A_k(R_{k,\Sigma})_{1\text{-}2} = A_k R_{k,c} + A_k R_{k,i\text{-}2} = 10^{-2} + 1.266\times 10^{-5} = 1.001\times 10^{-2}\ \mathrm{K/(W/m^2)}$$
$$\tau_1 = (\rho c_p V)_1 (R_{k,\Sigma})_{1\text{-}2} = \frac{\rho c_p V}{A_k} A_k (R_{k,\Sigma})_{1\text{-}2} = 1.3\times 10^5 \times 1.001\times 10^{-2} = 1.302\times 10^3\ \mathrm{s}.$$

For this second heating period, $T_1(t=0) = 45^\circ\mathrm{C}$ and $T_1(t) = T_2 + 2T_c = 120^\circ\mathrm{C}$. Then

$$\begin{aligned} t &= -\tau_1 \ln \frac{T_1(t) - T_2 - a_1\tau_1}{T_1(t=0) - T_2 - a_1\tau_1} \\ &= -1.302\times 10^3(\mathrm{s}) \ln \frac{120(^\circ\mathrm{C}) - 30(^\circ\mathrm{C}) - (1.538\times 10^{-1})(^\circ\mathrm{C/s}) \times (1.302\times 10^3)(\mathrm{s})}{45(^\circ\mathrm{C}) - 30(^\circ\mathrm{C}) - (1.538\times 10^{-1})(^\circ\mathrm{C/s}) \times (1.302\times 10^3)(\mathrm{s})} = 675.7\ \mathrm{s} \simeq 11.3\ \mathrm{min}. \end{aligned}$$

(d) The steady-state temperature is the condition for $t \to \infty$. Setting $t \to \infty$ in (3.172), we obtain

$$T_1(t\to\infty) = T_2 + a_1\tau_1 = 30 + (1.538\times 10^{-1}) \times (1.302\times 10^3) = 230.2^\circ\mathrm{C}.$$

(e) Figure Pr.3.63(c) shows T_1 as a function of time. At $T_1 = 45^\circ\mathrm{C}$, the change in the contact resistance causes a change in heat loss, from a smaller heat loss to a larger heat loss. This appears in the graph as an abrupt change in slope, from a smaller to a steeper slope.

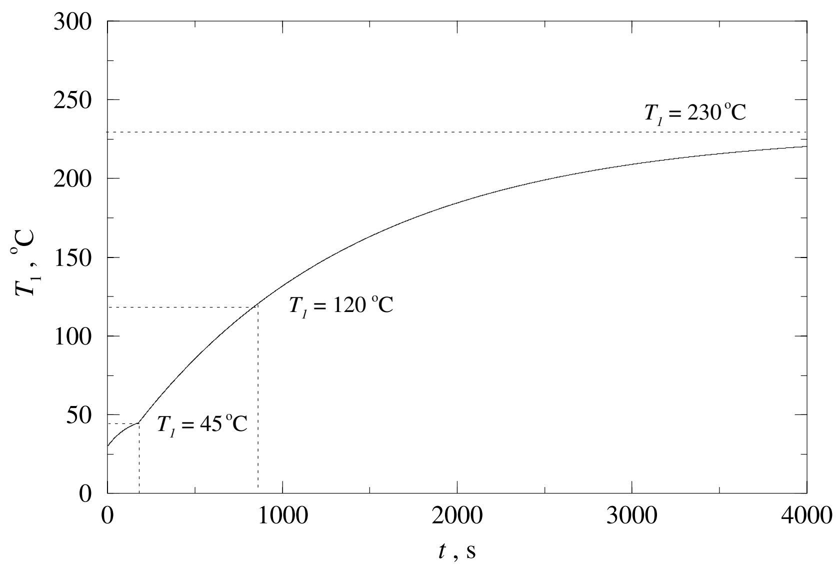


Figure Pr.3.63(c) Time variation of the regulator temperature for the periods with different constant resistance.

COMMENT:

Note that the increase in contact resistance results in a much larger regulator temperature. Applying joint pressure or thermal grease will assist in reducing the contact resistance.

PROBLEM 3.64.FAM

GIVEN:

In laser back-scribing, a substrate is heated and melted by radiation absorption. Upon solidification, a volume change marks the region and this is used for recording. An example is given in Figure Pr.3.64(a), where irradiation is provided through a thick glass layer and arrives from the backside to a thin layer of alumina. The alumina layer absorbs the radiation with an extinction coefficient $\sigma_{ex,1}$ that is much larger than that of glass. Assume that the alumina layer is at a uniform, but time-varying temperature (because of the high thermal conductivity of alumina compared to the glass, i.e., $N_k < 0.1$). Also assume that the conduction resistance in the glass is constant.

$a = 100$ μm, $l_1 = 0.6$ μm, $l_2 = 3$ μm, $q_{r,i} = 3 \times 10^9$ W/m^2, $\sigma_{ex,1} = 10^7$ 1/m, $\rho_r = 0.1$, $T_1(t = 0) = 20$°C, $T_2 = 20$°C, $T_{sl,1} = 1{,}900$°C.

SKETCH:

Figure Pr.3.64(a) shows laser back-scattering by volumetric absorption of irradiation.

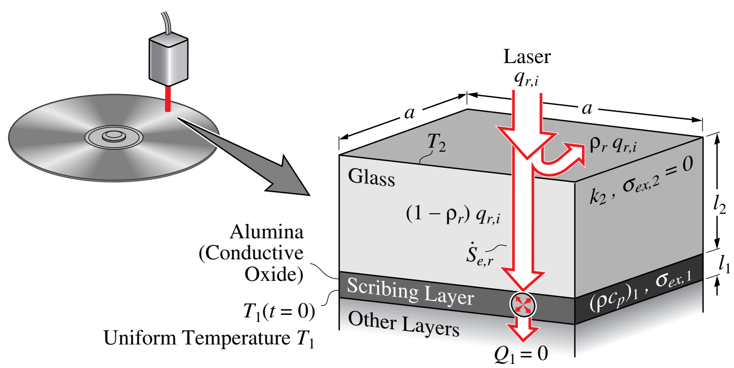


Figure Pr.3.64(a) Laser back-scribing on a compact disk storage device.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the time it takes to reach the melting temperature of the alumina $T_{sl,1}$.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.3.64(b). As stated in the problem, a constant conduction resistance is assumed for the glass layer. The temperature of the scribing layer is assumed uniform and time dependent. The absorbed irradiation is shown with $(\dot{S}_{e,r})_1$. This is the absorption integrated over the layer.

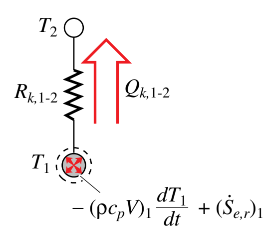


Figure Pr.3.64(b) Thermal circuit diagram.

(b) The energy absorbed in layer 1 is found from integrating (2.43), i.e.,

$$
\begin{aligned}
(\dot{S}_{e,r})_1 &= \int_{V_1} (\dot{S}_{e,r}/V) dV_1 = a^2 \int_0^{l_1} q_{r,i}(1-\rho_r)\sigma_{ex,1}\exp(-\sigma_{ex,1}x)dx \\
&= a^2 q_{r,i}(1-\rho_r)[1-\exp(-\sigma_{ex,1}l_1)] \\
&= (10^{-4})^2(\mathrm{m}^2) \times (3\times 10^9)(\mathrm{W/m}^2) \times (1-0.1) \times \\
&\quad \{1-\exp[-10^7(1/\mathrm{m}) \times 6\times 10^{-7}(\mathrm{m})]\} \\
&= 26.933 \text{ W}.
\end{aligned}
$$

The transient temperature of layer 1 is given by (3.172), i.e.,

$$
\begin{aligned}
T_1(t) &= T_2 + [T_1(t=0) - T_2]\exp(-t/\tau_1) + a_1\tau_1[1-\exp(-t/\tau_1)] \\
&= T_2 + a_1\tau_1(1-\exp(-t/\tau_1) \\
a_1 &= \frac{(\dot{S}_{e,r})_1}{(\rho c_p V)_1} \quad , \quad \tau_1 = (\rho c_p V)_1 R_{k,1\text{-}2}.
\end{aligned}
$$

From Table C.17, we have for alumina (at T=293 K)

$$
\begin{aligned}
\rho_1 &= 3{,}975 \text{ kg/m}^3 \qquad \text{Table C.17} \\
c_{p,1} &= 765 \text{ J/kg-K} \qquad \text{Table C.17.}
\end{aligned}
$$

From Table C.17, we have for a glass plate (at T = 293 K)

$$
k_2 = 0.76 \text{ W/m-K} \quad \text{Table C.17.}
$$

The volume is

$$
\begin{aligned}
V_1 = a^2 l_1 &= (10^{-4})^2(\mathrm{m}^2) \times (6\times 10^{-7})(\mathrm{m}) \\
&= 6\times 10^{-15} \text{ m}^3.
\end{aligned}
$$

The time constant is

$$
\begin{aligned}
\tau_1 &= (\rho c_p V)_1 R_{k,1\text{-}2} \\
R_{k,1\text{-}2} &= \frac{l_2}{\mathrm{A}_k k_2} = \frac{l_2}{a^2 k_2} \\
&= \frac{3\times 10^{-6}(\mathrm{m})}{(10^{-4})^2(\mathrm{m}^2)\times 0.76(\mathrm{W/m\text{-}K})} \\
&= 394.74 \text{ K/W}.
\end{aligned}
$$

Note that this large resistance is due to the small conduction area A_k. Then

$$
\begin{aligned}
\tau_1 &= 3{,}975(\mathrm{kg/m}^3) \times 765(\mathrm{J/kg\text{-}K}) \times (6\times 10^{-15})(\mathrm{m}^3) \times 394.74(\mathrm{K/W}) \\
&= 7.202 \times 10^{-6} \text{ s} \\
&= 7.202\ \mu\text{s}.
\end{aligned}
$$

Also

$$
a_1 = \frac{26.933(\mathrm{W})}{3{,}975(\mathrm{kg/m}^3)\times 765(\mathrm{J/kg\text{-}K})\times(6\times 10^{-15})(\mathrm{m}^3)} = 1.4762\times 10^9 {}^\circ\mathrm{C/s}.
$$

Solving for t, we have

$$
\frac{T_1(t)-T_2}{\mathrm{a}_1\tau_1} = 1-\exp(-t/\tau_1) \quad \text{or} \quad \exp(-t/\tau_1) = 1 - \frac{T_1(t)-T_2}{a_1\tau_1}
$$

or

$$
\begin{aligned}
t &= -\tau_1 \ln\left[1-\frac{T_1(t)-T_2}{a_1\tau_1}\right] \\
&= -7.202\times 10^{-6}(\mathrm{s})\ln\left[1-\frac{(1{,}900-20)({}^\circ\mathrm{C})}{1.4762\times 10^9({}^\circ\mathrm{C/s})\times 7.202\times 10^{-6}(\mathrm{s})}\right] \\
&= 1.402\times 10^{-6} \text{ s} = 1.402\ \mu\text{s}.
\end{aligned}
$$

COMMENT:

The assumption of constant conduction resistance in the glass layer can be relaxed by dividing the glass layer into many smaller layers (Section 3.7). In practice, the layer temperature is pulsed and its time variation should by taken into account. This results in a nonuniform distribution of the absorbed energy, once the reflections at the various internal boundaries are included.

PROBLEM 3.65.FUN

GIVEN:

In applications such as surface friction heat generation during automobile braking, the energy conversion rate $\dot{S}_{m,F}$ decreases with time. For the automobile brake, this is modeled as

$$\dot{S}_{m,F}(t) = (\dot{S}_{m,F})_o \left(1 - \frac{t}{t_o}\right) \qquad t \le t_o.$$

For a semi-infinite solid initially at $T(t=0)$, when its surface at $x=0$ experiences such time-dependent surface energy conversion, the surface temperature is given by the solution to (3.134). The solution, for these initial and bounding-surface conditions, is

$$T(x=0,t) = T(t=0) + \left(\frac{5}{4}\right)^{1/2} \frac{(\dot{S}_{m,F})_o}{A_k k} (\alpha t)^{1/2} \left(1 - \frac{2t}{3t_o}\right),$$

where $\dot{S}_{m,F}/A_k$ is the peak surface heat flux q_s.

$(\dot{S}_{m,f})_o/A_k = 10^5$ W/m^2, $t_o = 4$ s, $T(t=0) = 20$℃.

OBJECTIVE:

Consider a disc-brake rotor made of carbon steel AISI 1010.

(a) Plot the surface temperature $T(x=0,t)$ for the conditions given below and $0 \le t \le t_o$.

(b) By differentiating the above expression for $T(x=0,t)$ with respect to t, determine the time at which $T(x=0,t)$ is a maximum.

SOLUTION:

(a) From Table C.16 for carbon steel AISI 1010, we have

$$k = 64 \text{ W/m-K} \qquad \text{Table C.16}$$
$$\alpha = 1.88 \times 10^{-5} \text{ m}^2/\text{s} \qquad \text{Table C.16.}$$

Figure Pr.3.65 shows the variation of surface temperature with respect to time.

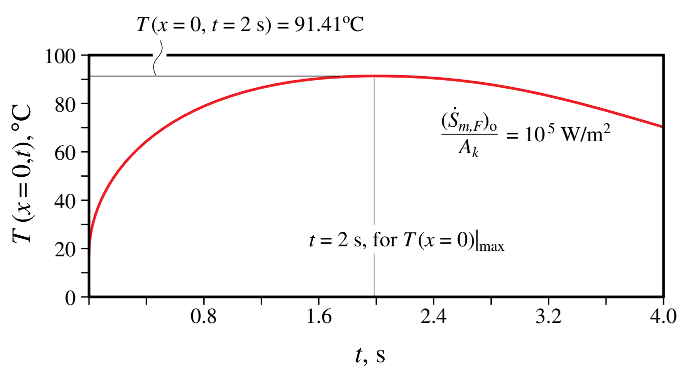


Figure Pr.3.65 Variation of surface temperature with respect to time.

(b) The differentiation of $T(x=0,t)$ with respect to t gives

$$\begin{aligned}
\frac{dT(x=0,t)}{dt} &= \left(\frac{5}{4}\right)^{1/2} \frac{(\dot{S}_{m,F})_o}{A_k k}\left[\alpha^{1/2}\left(\frac{1}{2}t^{-1/2}\right)\left(1 - \frac{2t}{2t_o}\right) - \frac{2\alpha^{1/2}t^{1/2}}{3t_o}\right] \\
&= \left(\frac{5}{4}\right)^{1/2} \frac{(\dot{S}_{m,F})_o}{A_k k}\alpha^{1/2}\left(\frac{1}{2}t^{-1/2} - \frac{t^{1/2}}{3t_o} - \frac{2t^{1/2}}{3t_o}\right) \\
&= \left(\frac{5}{4}\right)^{1/2} \frac{(\dot{S}_{m,F})_o}{A_k k}\alpha^{1/2}\left(\frac{1}{2}t^{-1/2} - \frac{t^{1/2}}{t_o}\right) = 0.
\end{aligned}$$

This gives

$$\frac{1}{2} - \frac{t}{t_o} = 0 \quad \text{or} \quad \frac{t}{t_o} = \frac{1}{2}.$$

COMMENT:

The maximum surface temperature, occurring at $t = t_o/2$, is

$$T(x = 0, t = t_o/2) \quad = \quad \left(\frac{5}{9}\right)^{1/2} \frac{(\dot{S}_{m,F})_o}{A_k k} (\alpha t)^{1/2} = 91.41℃.$$

PROBLEM 3.66.FUN

GIVEN:

The energy equation (3.171) for a lumped capacitance (integral-volume) system with a resistive-type surface heat transfer.

The initial condition is $T_1 = T_1(t=0)$ at $t = 0$.

OBJECTIVE:

Derive the solution (3.172) for $T = T_1(t)$ starting from the energy equation (3.171), which applies to a lumped-capacitance system with a resistive-type surface heat transfer.

SOLUTION:

We start from the energy equation (3.171), i.e.,

$$Q|_{A,1} = Q_1 + \frac{T_1(t) - T_2}{R_{t,1\text{-}2}} = -(\rho c_p V)_1 \frac{dT_1(t)}{dt} + \dot{S}_1$$

integral-volume(lumped capacitance)

energy equation with a resistive-type surface heat transfer,

with T_2 being constant and the initial condition of $T_1 = T_1(t=0)$.

We rewrite this ordinary differential equation (initial-value problem) as

$$\frac{dT_1}{dt} + \frac{1}{(\rho c_p V)_1 R_{t,1\text{-}2}}(T_1 - T_2) = \frac{\dot{S}_1 - Q_1}{(\rho c_p V)_1}$$

$$\frac{dT_1}{dt} + \frac{T_1 - T_2}{\tau_1} = a_1, \quad \tau_1 = (\rho c_p V)_1 R_{t,1\text{-}2}, \quad a_1 = \frac{\dot{S}_1 - Q_1}{(\rho c_p V)_1}.$$

Using substitution, we have

$$\frac{d\theta}{dt} + \frac{\theta}{\tau_1} = a_1, \quad \theta = T_1 - T_2.$$

First we find the homogeneous solution by setting $a_1 = 0$, i.e.,

$$\frac{d\theta}{dt} + \frac{\theta}{\tau_1} = 0, \quad \theta_h = Ae^{-t/\tau_1}.$$

Then we find the particular solution by setting $d\theta/dt = 0$, i.e.,

$$\frac{\theta}{\tau_1} = a_1, \quad \theta = a_1\tau_1.$$

Now combining the solutions, we have

$$\theta = Ae^{-t/\tau_1} + a_1\tau_1.$$

Applying the initial condition, we have

$$T_1(t=0) - T_2 = Ae^{-0/\tau_1} + a_1\tau_1$$

or

$$A = T_1(t=0) - T_2 - a_1\tau_1.$$

Then

$$T_1(t) - T_2 = [T_1(t=0) - T_2]e^{-t/\tau_1} + a_1\tau_1(1 - e^{-t/\tau_1}),$$

which is (3.172).

COMMENT:

Note that for the steady-state solution, i.e., $t \to \infty$, we have

$$\begin{aligned} T_1(t \to \infty) &= T_2 + a_1\tau_1 \\ &= T_2 + (\dot{S}_1 - Q_1)R_{k,1\text{-}2}. \end{aligned}$$

The steady-state solution is reached when $t \leq 4\tau_1$.

PROBLEM 3.67.FAM

GIVEN:

Water is heated (and assumed to have a uniform temperature, due to thermobuoyant motion mixing) from $T_1(t=0)$ to $T_1(t=t_f)=T_f$ by Joule heating in a cylindrical, portable water heater with inside radius R_1 and height l, as shown in Figure Pr.3.67(a). The ambient air temperature T_2 is rather low. Here we assume that the outside surface temperature (located at outer radius R_2) is the same as the ambient temperature (i.e., we neglect the resistance to heat transfer between the outside surface and the ambient). Two different heater wall designs, with different R_2 and k_s, are considered.

$R_1 = 7$ cm, $l = 15$ cm, $T_1(t=0) = T_2 = 2$℃, $t_f = 2{,}700$ s, $\dot{S}_{e,\mathrm{J}} = 600$ W.

Evaluate the water properties at $T = 310$ K (Table C.23). Neglect the heat transfer through the top and bottom surfaces of the water heater, and treat the wall resistance as constant.

SKETCH:

Figure Pr.3.67(a) shows the portable water heater and its Joule heater and side walls.

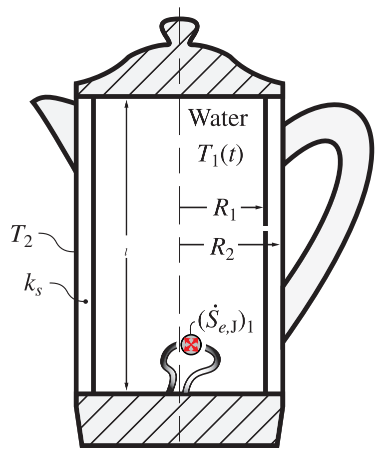


Figure Pr.3.67(a) A portable water heater.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine the water temperature T_f after an elapsed time t_f using, (i) a thin AISI 302 stainless-steel wall (Table C.16) with outer wall radius $R_2 = 7.1$ cm, and (ii) a thicker nylon wall (Table C.17) with $R_2 = 7.2$ cm.
(c) Compare the results of the two designs.

SOLUTION:

(a) The water has lumped thermal capacitance (uniform temperature) and is losing heat by conduction through the cylindrical side walls of the container. The thermal circuit is shown in Figure Pr.3.67(b).

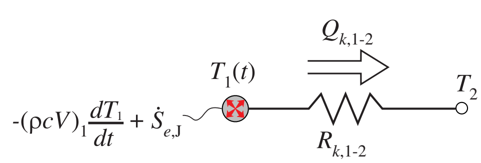


Figure Pr.3.67(b) Thermal circuit diagram.

(b) The water is a lumped system with a single resistive conduction heat transfer. Applying conservation of energy to node T_1, we have

$$\begin{aligned} Q\,|_A &= Q_{k,1\text{-}2} = -(\rho c V)_1 \frac{dT_1}{dt} + \dot{S}_1 \\ &= \frac{T_1 - T_2}{R_{k,1\text{-}2}} = -(\rho c V)_1 \frac{dT_1}{dt} + (\dot{S}_{e,\mathrm{J}})_1. \end{aligned}$$

The solution to this differential equation for $T_1(t)$ (with constant $\dot{S}$) is given as

$$T_1(t) = T_2 + [T_1(t=0) - T_2]e^{-t/\tau_1} + a_1\tau_1(1 - e^{-t/\tau_1}),$$

where

$$\tau_1 = (\rho c V)_1 R_{k,1\text{-}2} \qquad \text{and} \qquad a_1 = \frac{(\dot{S}_{e,\mathrm{J}})_1}{(\rho c V)_1}.$$

Since $T_1(t=0) = T_2$, this reduces to

$$T_1(t) = T_2 + a_1\tau_1(1 - e^{-t/\tau_1}).$$

The volume of the container is

$$\begin{aligned} V_1 &= \pi R_1^2 l = \pi \times 0.07^2(\mathrm{m}^2) \times 0.15(\mathrm{m}) \\ &= 2.309 \times 10^{-3}\ \mathrm{m}^3. \end{aligned}$$

From Table C.23 ($T_1 = 310$ K), $\rho_1 = 995.3$ kg/m^3 and $c = 4{,}178$ J/kg-K. The thermal capacitance of the water is then

$$\begin{aligned} (\rho c V)_1 &= 995.3(\mathrm{kg/m}^3) \times 4{,}178(\mathrm{J/kg\text{-}K}) \times 2.309 \times 10^{-3}(\mathrm{m}^3) \\ &= 9{,}602\ \mathrm{J/K}. \end{aligned}$$

For a cylindrical shell, the conduction resistance is

$$R_{k,1\text{-}2} = \frac{\ln(R_2/R_1)}{2\pi l k_s}.$$

(i) AISI 302 stainless steel, $R_1 = 0.071$ m.
From Table C.16, $k_s = 15$ W/m-K. Then we have

$$\begin{aligned} R_{k,1\text{-}2} &= \frac{\ln(R_2/R_1)}{2\pi l k_s} = \frac{\ln(0.071/0.07)}{2 \times \pi \times 0.15(\mathrm{m}) \times 15(\mathrm{W/m\text{-}K})} = 1.003 \times 10^{-3}\,^\circ\mathrm{C/W} \\ a_1 &= \frac{(\dot{S}_{e,\mathrm{J}})_1}{(\rho c V)_1} = \frac{600(\mathrm{W})}{9{,}601.95(\mathrm{J/K})} = 0.0625\ ^\circ\mathrm{C/s}, \\ \tau_1 &= (\rho c V)_1 R_{k,1\text{-}2} = 9{,}601.95(\mathrm{J/K}) \times 1.003 \times 10^{-3}(^\circ\mathrm{C/W}) = 9.631\ \mathrm{s}. \end{aligned}$$

Upon substitution at $t = 2{,}700$ s, we have

$$\begin{aligned} T_1(t) &= T_2 + a_1\tau_1(1 - e^{-t/\tau}) = 2^\circ\mathrm{C} + 0.0625(^\circ\mathrm{C/s}) \times 9.631(\mathrm{s}) \times (1 - e^{-2{,}700(\mathrm{s})/9.631(\mathrm{s})}) \\ &= 2^\circ\mathrm{C} + 0.602(^\circ\mathrm{C}) \times (1 - e^{-280.3}) = 2.6^\circ\mathrm{C}. \end{aligned}$$

(ii) Nylon, $R_2 = 0.072$ m.
From Table C.17, $k_s = 0.25$ W/m-K. Then we have

$$\begin{aligned} R_{k,1\text{-}2} &= \frac{\ln(R_2/R_1)}{2\pi l k_s} = \frac{\ln(0.072/0.07)}{2 \times \pi \times 0.15(\mathrm{m}) \times 0.25(\mathrm{W/m-K})} = 0.1196^\circ\mathrm{C/W}, \\ a_1 &= \frac{(\dot{S}_{e,\mathrm{J}})_1}{(\rho c V)_1} = \frac{600(\mathrm{W})}{9{,}601.95(\mathrm{J/K})} = 0.0625^\circ\mathrm{C/s}, \\ \tau_1 &= (\rho c V)_1 R_{k,1\text{-}2} = 9{,}601.95(\mathrm{J/K}) \times 0.1196(^\circ\mathrm{C/W}) = 1{,}148\ \mathrm{s}. \end{aligned}$$

Upon substitution at $t = 2{,}700$ s, we have

$$\begin{aligned} T_1(t) &= T_2 + a_1\tau_1(1 - e^{-t/\tau}) = 2℃ + 0.0625(℃/\text{s}) \times 1{,}148(\text{s}) \times (1 - e^{-2{,}700(\text{s})/1{,}148(\text{s})}) \\ &= 2℃ + 71.75(℃) \times (1 - e^{-2.35}) = 66.9℃. \end{aligned}$$

(c) For the given external surface temperature, the high thermal conductivity and the wall thickness of the stainless steel produces a wall thermal resistance for design (i) that is too low to allow for the water to be heated higher than 2.6℃. The low thermal conductivity and larger wall thickness of the Nylon produces a wall thermal resistance nearly 1000× higher than that of design(i), thus allowing the water to be heated to and above the desired temperature. Of the two designs, design (ii) is superior. Increasing the wall thickness and/or decreasing the thermal conductivity of the wall (by selecting an alternate material) would further improve the design.

COMMENT:

Note that we have neglected heat losses from the bottom and top surface and these can be significant.

PROBLEM 3.68.FAM

GIVEN:

A hot lead sphere is cooled by rolling over a cold surface, with a contact resistance $(R_{k,c})_{1\text{-}2}$, which is approximated by that between a pair of soft aluminum surfaces with $\langle\delta^2\rangle = 0.25\ \mu$m. The surface-convection and radiation heat transfer are represented by a constant heat transfer rate Q_1.

$D_1 = 2$ mm, $T_1(t=0) = 300$°C, $T_2 = 30$°C, $u_p = 0.5$ m/s, $A_{k,c} = 0.1$ mm^2, $Q_1 = 0.1$ W.

SKETCH:

Figure Pr.3.68(a) shows the rolling sphere and the contact resistance.

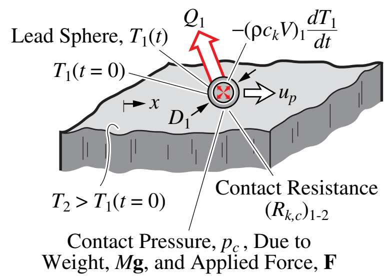


Figure Pr.3.68(a) A hot lead sphere is cooled by rolling over a cold surface.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Assume a uniform sphere temperature and determine the elapsed time required for the sphere temperature to reach 50°C above T_2.

(c) Use this elapsed time and evaluate the Fourier number Fo_R for the sphere. From this magnitude and by using Figure 3.33(b)(ii) for estimation, is the assumption of uniform temperature valid?

(d) By approximating the internal, steady-state resistance as $R_{k,1} = (D_1/2)/\pi D_1^2 k_1 = 1/(2\pi D_1 k_1)$, evaluate $N_{k,1}$ and comment on the validation of uniform sphere temperature assumption from the relative temperature variations inside and outside the object.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.3.68(b).

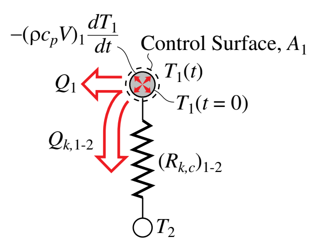


Figure Pr.3.68(b) Thermal circuit diagram.

(b) From (3.172), we have

$$T_1(t) = T_2 + [T_1(t=0) - T_2]e^{-t/\tau_1} + a_1\tau_1(1 - e^{-t/\tau_1}),$$

$$\tau_1 = (\rho c_p V)_1 (R_{k,c})_{1\text{-}2}, \quad a_1 = \frac{\dot{S}_1 - Q_1}{(\rho c_p V)_1} = -\frac{Q_1}{(\rho c_p V)_1},$$

since $\dot{S}_1 = 0$.

From Table C.16, we have for lead

$$
\begin{aligned}
\rho_1 &= 11{,}340 \text{ kg/m}^3 && \text{Table C.16}\\
c_{p,1} &= 129 \text{ J/kg-K} && \text{Table C.16}\\
k_1 &= 35.3 \text{ W/m-K} && \text{Table C.16}\\
\alpha_1 &= 2.41 \times 10^{-5} \text{ m}^2/\text{s} && \text{Table C.16.}
\end{aligned}
\tag{3.1}
$$

Then using $V_1 = \pi D_1^3/6$, we have

$$(\rho c_p V)_1 = 11{,}340(\text{kg/m}^3) \times 129(\text{J/kg-K}) \times \pi \times (2 \times 10^{-3})^3(\text{m}^3)/6 = 6.128 \text{ J/K}.$$

From Figure 3.25, we have for $p_c = 10^5$ Pa and soft Al-Al surfaces with $\langle\delta^2\rangle^{1/2} = 0.25\ \mu$m

$$(A_{k,c}R_{k,c})^{-1}_{1\text{-}2} = 8 \times 10^3 \ (\text{W/m}^2)/\text{K}.$$

Then

$$
\begin{aligned}
(R_{k,c})_{1\text{-}2} &= \frac{(A_{k,c}R_{k,c})_{1\text{-}2}}{A_{k,c}} = \frac{1}{8 \times 10^3[(\text{W/m}^2)/\text{K}] \times 10^{-7}(\text{m}^2)}\\
&= 1.25 \times 10^3 \text{ K/W}\\
\tau_1 &= 6.128(\text{J/K}) \times 1.25 \times 10^3(\text{K/W}) = 7{,}659.5 \text{ s}\\
a_1 &= -\frac{0.1(\text{W})}{6.128(\text{J/K})} = -0.0163 \text{ K/s}
\end{aligned}
$$

$$T_1(t) = (30+50)(^\circ\text{C}) = 30(^\circ\text{C}) + (300-30)(^\circ\text{C})e^{-t/7{,}659.5(\text{s})}(-0.0163)(\text{K/s}) \times 7{,}659.5(\text{s}) \times \left[1 - e^{-t/7{,}659.5(\text{s})}\right].$$

Solving for t, we have

$$
\begin{aligned}
t &= 6{,}239 \text{ s}\\
L &= u_p t = 0.5(\text{m/s}) \times 6.236(\text{s}) = 3{,}120 \text{ m}.
\end{aligned}
$$

(c) From Figure 3.33(b)(ii), the Fourier number is defined as

$$\text{Fo}_R = \frac{\alpha_1 t}{R_1^2} = \frac{4\alpha_1 t}{D_1^2} = \frac{4 \times 2.41 \times 10^{-5}(\text{m}^2/\text{s}) \times 6.236(\text{s})}{(2 \times 10^{-3})^2(\text{m}^2)} = 150.3.$$

From Figure 3.33(b)(ii), if the surface temperature was suddenly changed, then this elapsed time t, or its dimensionless value Fo_R, would be sufficient to establish a uniform temperature. Here we do not have a constant surface temperature, but we can state that there is a sufficient elapsed time to allow for a nearly complete penetration of surface temperature changes, i.e., it is safe to assume a uniform temperature.

(d) From (3.161) we have

$$
\begin{aligned}
N_{k,1} &= \frac{R_{k,1}}{R_{k,1\text{-}2}} = \frac{R_{k,1}}{(R_{k,c})_{1\text{-}2}}\\
&= \ = \frac{1/(2\pi D_1 k_1)}{1.25 \times 10^3(\text{K/W})}\\
&= \frac{1}{2\pi \times 2 \times 10^{-3}(\text{m}) \times 35.3(\text{W/m-K}) \times 1.25 \times 10^3(\text{K/W})} = 1.803 \times 10^{-3} \ll 0.1.
\end{aligned}
$$

This shows that the internal resistance is negligible and a lumped capacitance is also valid from the inside-outside temperature variation point of view.

COMMENT:

We have verified that, here, there is a negligible penetration temperature nonuniformity (i.e., large Fo_R) and a negligible internal temperature variation compared to the external temperature variation (i.e., small $N_{k,1}$). As the volume becomes smaller, this assumption becomes more readily satisfied. These are the assumptions used in the finite-small volume treatment of the heat transfer media (by division of the medium into small, but yet finite volumes). Also, note that we have not determined p_c and that a large p_c would require an applied force (other than the particle weight).

PROBLEM 3.69.FAM.S

GIVEN:

A thin, flexible thermofoil (etched foil) heater with a mica (a cleavable mineral) casing is used to heat a copper block. This is shown in Figure Pr.3.69(a). At the surface between the heater and the copper block, there is a contact resistance $(R_{k,c})_{1\text{-}2}$. Assume that the heater and the copper block both have small internal thermal resistances ($N_{k,1} < 0.1$), so they can be treated as having uniform temperatures $T_1(t)$ and $T_2(t)$, with the initial thermal equilibrium conditions $T_1(t = 0) = T_2(t = 0)$. The other heat transfer rates, from the heater and the copper block are prescribed (and constant) and are given by Q_1 and Q_2. If the heater temperature $T_1(t)$ exceeds a threshold value of $T_c = 600$℃, the heater is permanently damaged. To avoid this, the thermal contact resistance is decreased by the application of an external pressure (i.e., large contact pressure p_c).

For mica, use the density and specific heat capacity for glass plate in Table C.17.

$R = 3$ cm, $L_1 = 0.1$ cm, $L_2 = 3$ cm, $\dot{S}_{e,\mathrm{J}} = 300$ W, $Q_1 = 5$ W, $Q_2 = 50$ W, $T_1(t = 0) = T_2(t = 0) = 20$℃.

SKETCH:

Figure Pr.3.69(a) A thermofoil heater is used to heat a copper block. There is a contact resistance between the two.

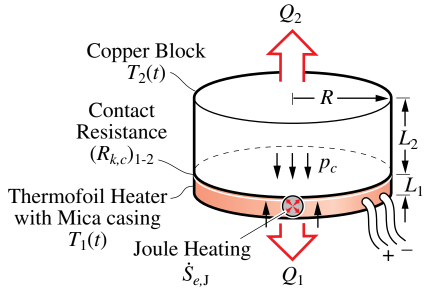


Figure Pr.3.69(a) A copper block is heated with a thin heater through a contact resistance.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Plot $T_1(t)$ and $T_2(t)$ with respect to time, for $0 \leq t \leq 100$ s, for (i) $A_k(R_{k,c})_{1\text{-}2} = 10^{-3}$ K/(W/m^2), and (ii) $A_k(R_{k,c})_{1\text{-}2} = 10^{-2}$ K/(W/m^2).

SOLUTION:

(a) The thermal circuit diagram is given in Figure Pr.3.69(b).

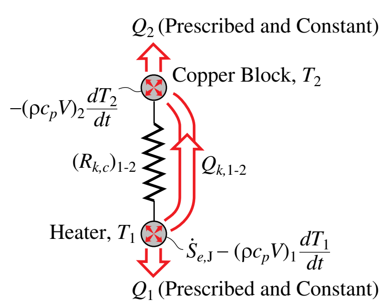


Figure Pr.3.69(b) Thermal circuit diagram.

(b) This is a two-node thermal system and for each node we use (3.161), i.e.,

$$Q_1 + Q_{k,1\text{-}2} = -(\rho c_p V)_1 \frac{dT_1}{dt} + \dot{S}_{e,\mathrm{J}}, \qquad N_{k,1} < 0.1 \tag{3.2}$$

$$Q_2 + Q_{k,2\text{-}1} = -(\rho c_p V)_2 \frac{dT_2}{dt}, \qquad N_{k,2} < 0.1, \tag{3.3}$$

where

$$
\begin{aligned}
V_1 &= \pi R^2 L_1, \quad V_2 = \pi R^2 L_2, \\
Q_{k,1\text{-}2} &= \frac{T_1(t) - T_2(t)}{(R_{k,1\text{-}2})} \\
T_1(t=0) &= T_2(t=0) \quad = \quad T(t=0).
\end{aligned}
$$

From Tables C.16 and C.17, we have

mica:	$\rho_1 = 2{,}710$ kg/m^3	Table C.17
	$c_{p,1} = 837$ J/kg-K	Table C.17
copper:	$\rho_2 = 8{,}933$ kg/m^3	Table C.16
	$c_{p,2} = 385$ J/kg-K	Table C.16.

Then

$$
\begin{aligned}
(\rho c_p V)_1 &= 2{,}710(\text{kg/m}^3) \times 837(\text{J/kg-K}) \times \pi \times (0.03)^2(\text{m}^2) \times 10^{-3}(\text{m}) = 6.413 \text{ J/K} \\
(\rho c_v V)_2 &= 8{,}933(\text{kg/m}^3) \times 385(\text{J/kg-K}) \times \pi \times (0.03)^2(\text{m}^2) \times 3 \times 10^{-2}(\text{m}) = 2.918 \times 10^2 \text{ J/K}.
\end{aligned}
$$

$$
\begin{aligned}
(R_{k,c})_{1\text{-}2} &= \frac{A_k(R_{k,c})_{1\text{-}2}}{A_k} = \frac{10^{-2}[\text{K/(W/m}^2)]}{\pi R^2} = \frac{10^{-2}[\text{K/(W/m}^2)]}{\pi \times (0.03)^2(\text{m}^2)} \\
&= 3.536 \text{ K/W}, \quad \text{for} \quad A_k(R_{k,c})_{1\text{-}2} = 10^{-2} \text{ K/(W/m}^2) \\
&= 0.3536 \text{ K/W}, \quad \text{for} \quad A_k(R_{k,c})_{1\text{-}2} = 10^{-3} \text{ K/(W/m}^2).
\end{aligned}
$$

The two energy equations become, for $A_k(R_{k,c})_{1\text{-}2} = 10^{-2}$ K/(W/m^2),

$$
\begin{aligned}
5 + \frac{T_1(t) - T_2(t)}{3.536} &= -6.413 \times \frac{dT_1(t)}{dt} + 300 \\
50 + \frac{T_2(t) - T_1(t)}{3.536} &= -2.918 \times 10^2 \times \frac{dT_2(t)}{dt}, \\
T_1(t=0) &= T_2(t=0) = 20^\circ\text{C}.
\end{aligned}
$$

The variations of $T_1(t)$ and $T_2(t)$ with respect to time are plotted in Figure Pr.3.69(c). Due to its smaller mass, the heater heats up quickly and when $(R_{k,c})_{1\text{-}2}$ is large, this results in temperatures in excess of the damaging threshold temperature $T_c = 600^\circ$C.

COMMENT:

Note that the assumption of a uniform temperature may be valid for the copper block, but not for the heater [especially for the smaller $(R_{k,c})_{1\text{-}2}$]. In this case, the heater should be divided into segments (along its thickness) and a separate energy equation should be written for each segment (i.e., finite-small volume energy equations).

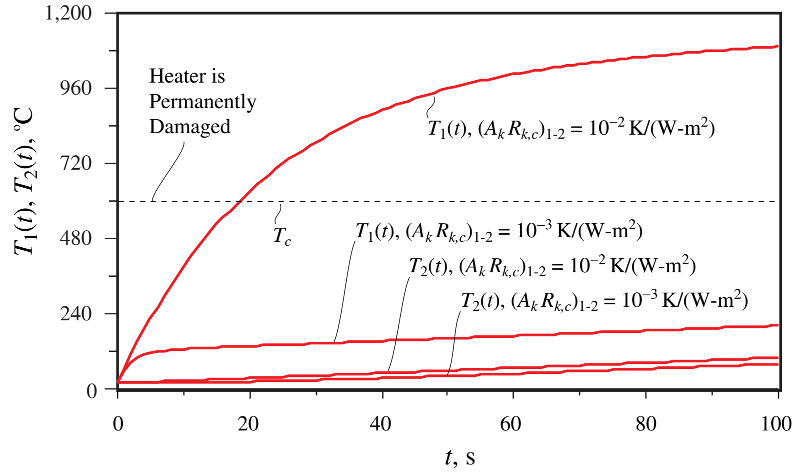


Figure Pr.3.69(c) Variation of the two node temperatures with respect to time.

PROBLEM 3.70.FAM

GIVEN:

An initially cold $T_1(t=0)$, pure aluminum spherical particle is rolling over a hot surface of temperature T_s at a constant speed u_o and is heated through a contact conduction resistance $R_{k,c}$. This is shown in Figure Pr.3.70(a).

$T_1(t=0) = 20°C$, $T_s = 300°C$, $T_f = 200°C$, $D_1 = 4$ mm, $R_{k,c} = 1{,}000$ K/W, $u_o = 0.1$ m/s.

Determine the pure aluminum properties at $T = 300$ K.

SKETCH:

Figure Pr.3.70(a) shows the falling spherical particle and the heat transfer by contact conduction.

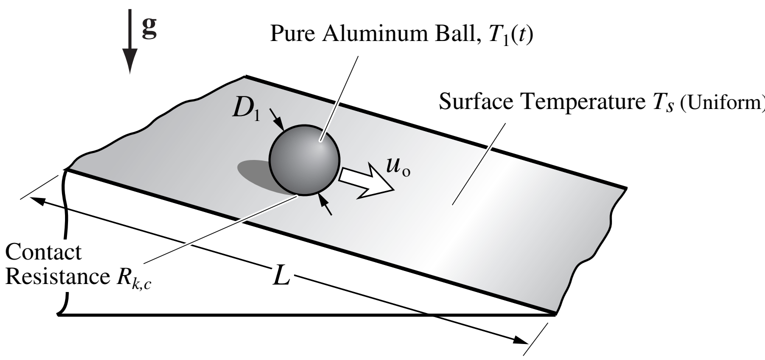


Figure Pr.3.70(a) A pure aluminum ball rolls over a hot surface and is heated by contact conduction.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Is the assumption of a uniform temperature valid? Use $R_{k,1} = 1/(4\pi D_1 k_1)$ for the internal conduction resistance.
(c) Determine the length L the ball has to travel before its temperature reaches T_f. Assume that the contact conduction is the only surface heat transfer (surface-convection and radiation heat transfer are assumed negligible).

SOLUTION:

(a) Figure Pr.3.70(b) shows the thermal circuit diagram. Heat transfer to the ball is by contact conduction only.

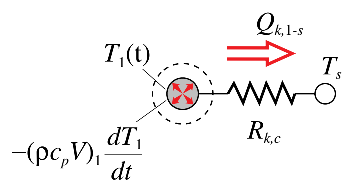


Figure Pr.3.70(b) Thermal circuit diagram.

(b) Lumped capacitance treatment is justified when $R_{k,i}/R_{k,i\text{-}j} \equiv N_{k,i} < 0.1$, i.e., (3.161). Using $R_{k,1} = 1/(4\pi D_1 k_1)$, and $k_1 = 237$ W/m-K (Table C.16), we have

$$N_{k,1} = \frac{\dfrac{1}{4\pi D_1 k_1}}{R_{k,c}} = \frac{\dfrac{1}{4 \times \pi \times 0.004(\text{m}) \times 237(\text{W/m-K})}}{1{,}000(\text{K/W})}$$

$$N_{k,1} = 8.385 \times 10^{-5} \ll 0.1, \text{ then the uniform temperature assumption is valid.}$$

(c) The transient temperature of the ball is given by (3.172), i.e.,

$$T_1(t) \quad = \quad T_s + [T_1(t=0) - T_s]\exp(-t/\tau_1) + a_1\tau_1[1 - \exp(-t/\tau_1)].$$

Since there is no heat loss and heat generation, i.e., $\dot{S}_1 - Q_1 = a_1 = 0$, this reduces to

$$T_1(t) \quad = \quad T_s + [T_1(t=0) - T_s]\exp(-t/\tau_1),$$

where

$$\tau_1 = (\rho c_p V)_1 R_{k,c}.$$

The volume is

$$\begin{aligned} V_1 = \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 \quad &= \quad \frac{4\pi}{3}\left[\frac{0.004}{2}\right]^3 \\ &= \quad 3.355 \times 10^{-8}\ \text{m}^3. \end{aligned}$$

From Table C.16, for pure aluminum, we have

$$\begin{aligned} \rho_1 \quad &= \quad 2{,}702\ \text{kg/m}^3 \\ c_{p,1} \quad &= \quad 903\ \text{J/kg-K} \\ \tau_1 \quad &= \quad 2{,}702(\text{kg/m}^3) \times 903(\text{J/kg-K}) \times 3.355 \times 10^{-8}(\text{m}^3) \times 1{,}000(\text{W/K}) \\ \tau_1 \quad &= \quad 81.85\ \text{s}. \end{aligned}$$

Then, using the values given, solving for time,

$$\begin{aligned} 473.15(\text{K}) \quad &= \quad 573.15(\text{K}) + [(293.15 - 573.15)(\text{K})]\ e^{\frac{-t}{81.85}} \\ t \quad &= \quad 84.27\ \text{s}. \end{aligned}$$

Using $L = t\ u_o$, with $u_o = 0.1$ m/s, we have

$$L = 8.427\ \text{m}.$$

COMMENT:

The heat transfer to the ambient by surface convection is neglected but becomes increasingly more important as T_1 increases. Surface radiation also becomes important. These would increase the required length (time).

PROBLEM 3.71.FUN

GIVEN:

In printed-circuit field-effect transistors, shown in Figure Pr.3.71(a), the electrons are periodically accelerated in the active layer and these electrons are scattered by collision with the lattice molecules (which is represented as collision with the lattice phonons), as well as collision with the other electrons and with impurities. These collisions result in the loss of the kinetic energy (momentum) of the electrons (represented by the Joule heating) and this energy is transferred to the lattice molecules due to local thermal nonequilibrium between the electrons having temperature T_e and the lattice having temperature T_l.

An estimate of the electron temperature T_e can be made using the concept of relaxation time. As discussed in the footnote on page 288, the energy equation for the electron in a lattice unit cell can be written as

$$\begin{aligned}\frac{T_e(t)-T_l}{\tau_e} - \frac{dT_e(t)}{dt} &= a_e = 0.1\frac{m_{e,o}u_e^2}{3k_{\rm B}}\left(\frac{2}{\tau_m}-\frac{1}{\tau_e}\right)\\ u_e &= \mu_o e,\end{aligned}$$

where the term on the left is the heat transfer from electron to the lattice, the first term on the right is storage, and the second term is energy conversion. Here the coefficient 0.1 in a_1 represents that 0.9 of the heat generated is conducted to surroundings. The electron drift velocity is related to the electric field and the electron mobility and the mass used is the effective electron mass $0.066me$. The two relaxation times are the electron momentum relaxation time τ_m, and the electron-lattice relation time τ_e. Assume that the lattice temperature is constant (due to the much larger volume of the lattice molecules, compared to the electrons).

$m_{e,o} = 0.066m_e$, $m_e = 9.109\times10^{-3}$ kg, $k_{\rm B} = 1.3807\times10^{-23}$ J/K, $\mu_o = 0.85$ m^2/V-s, $e = 5\times10^5$ V/m, $\tau_m = 0.3$ ps, $\tau_e = 8$ ps, $T_l = 300$ K, $T_e(t=0) = T_l$.

SKETCH:

Figure Pr.3.71(a) shows the transient heat generation (Joule heating) in the electron transport layer.

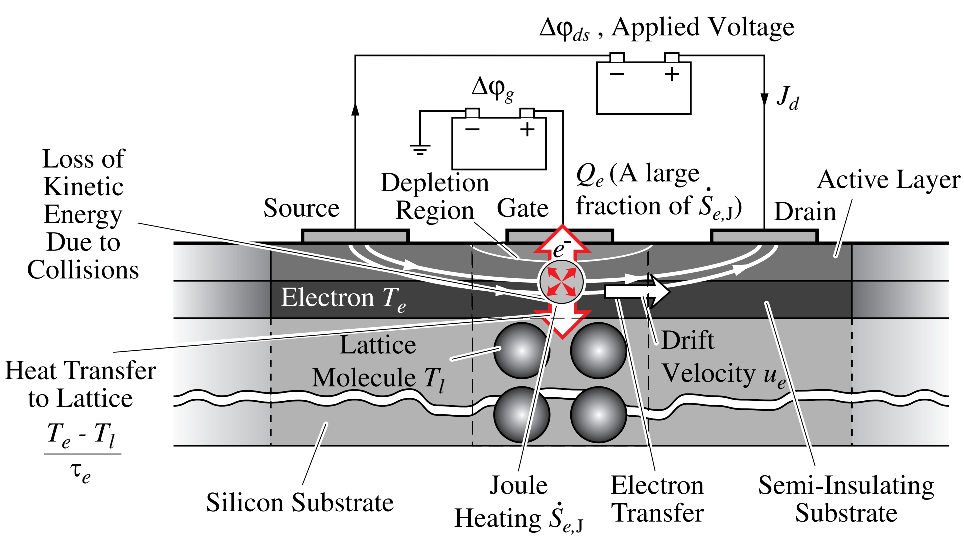


Figure Pr.3.71(a) The printed-circuit field-effect transistor and heat generation by electron kinetic energy loss due to collisions. The electron heat transfer as a unit cell is also shown.

OBJECTIVE:

For the conditions given below, plot the electron temperature, using the solution (3.172), with respect to time, up to an elapsed time of $t = 100$ ps.

SOLUTION:

We begin with the solution (3.172) for the electron temperature, i.e.,

$$\begin{aligned} T_e(t) &= T_l + [T_e(t=0) - T_l]e^{-t/\tau_l} + a_1\tau_l(1 - e^{-t/\tau_l}) \\ &= T_l + a_1\tau_l(1 - e^{-t/\tau_l}), \end{aligned}$$

since $T_e(t=0) = T_l$.

From the problem statement, we have for a_e

$$\begin{aligned} a_e &= 0.1\frac{m_{e,o}u_e^2}{3k_{\rm B}}\left(\frac{2}{\tau_m} - \frac{1}{\tau_e}\right), \\ u_e &= \mu_o e. \end{aligned}$$

and using the numerical values, we have

$$T_e(t) = 300({\rm K}) + 1.715 \times 10^{14}({\rm K/s}) \times 8 \times 10^{-12}({\rm s}) \times [1 - e^{-t/8\times10^{-12}({\rm S})}].$$

Figure Pr.3.71(b) shows the variation of the electron temperature with respect to time. As expected, within 4 time constants $4\tau_e$, the electron reaches its steady-state temperature.

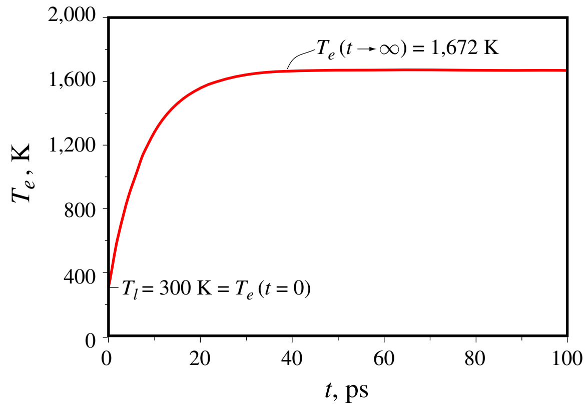


Figure Pr.3.71(b) Variation of the electron temperature with respect to time.

COMMENT:

Note that the steady-state temperature is also found by setting the time derivation equal to zero, i.e.,

$$\frac{T_e(t \to \infty) - T_l}{\tau_e} = a_e$$

or

$$T_e(t \to \infty) = T_e + a_1\tau_e = 1{,}672{\rm K}$$

Depending on the switching time, the electric field (applied voltage) is turned off and therefore, the Joule heating is on for a period. This period is generally longer than τ_e.

Also note that we have allowed for $0.9a_1$ to leave as Q_c (heat loss by conduction to surroundings). By including the conduction through the active and semi-insulating layers, this heat loss can be determined as part of the solution.

Since from (3.173), we have

$$a_e = \frac{\dot{S}_e - Q_e}{(\rho c_p V)_e},$$

here, $\dot{S}_e/V$ is rather large.

PROBLEM 3.72.FAM

GIVEN:

Due to defects in the brake pad or the rotor geometry, the friction heat generation $\dot{S}_{m,F}$ may not have a uniform distribution over the brake pad-rotor contact surface. This results in a hot spot at the locations of high contact, and due to the thermal expansion, these hot spots continue to have further increase in contact pressure. Eventually very high temperatures and a failure occurs. Consider the friction energy conversion occurring over a rotor surface. The rotor is idealized as a ring of inner radius R_i, outer radius R_o, and thickness l, as shown in Figure Pr.3.72(a). Under normal contact, the energy conversion will be equally distributed over the entire contact surface and a uniform temperature $T_1(t)$ can be assumed. Under hot-spot contact, assume that the energy is dissipated over a ring with the inner and outer radii $R_{i,1}$ and $R_{o,1}$ (with the same thickness l), resulting in a uniform temperature $T_1(t)$ (i.e., $N_{k,1} < 0.1$) and that the rest of the rotor is at a constant temperature T_2 with heat flowing by conduction from $T_1(t)$ to T_2 with a constant resistance $R_{k,1\text{-}2}$. This is only a very rough approximation.

$\dot{S}_{m,F}$ = 30 kW, R_o = 18 cm, R_i = 13 cm, $R_{o,1}$ = 16 cm, $R_{i,1}$ = 15 cm, $(\rho c_p)_1 = 3.5 \times 10^6$ J/m^3-K, $R_{k,1\text{-}2} = 1$℃/W, $T_1(t = 0) = 20$℃, $T_2 = 20$℃.

SKETCH:

Figure Pr.3.72(a) shows the areas and for the normal and the hot-spot braking.

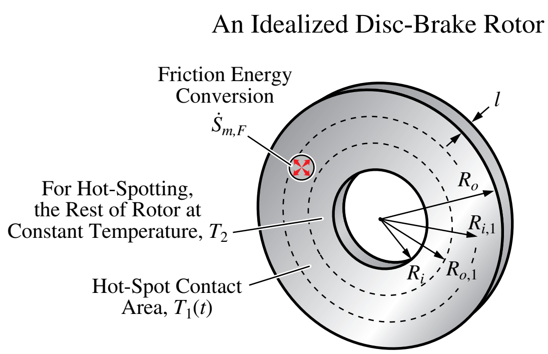


Figure Pr.3.72(a) A disc-brake rotor with (i) normal pad-rotor contact, and (ii) with hot-spot partial surface contact.

OBJECTIVE:

(a) Draw the thermal circuit diagram for (i) normal contact with no heat transfer, and (ii) hot-spot contact with $Q_{k,1\text{-}2}$ as the heat transfer.

(b) Determine the temperature $T_1(t)$ for cases (i) and (ii) after an elapsed time of $t = 4$ s.

(c) Comment on the difference in $T_1(t = 4$ s) for cases (i) and (ii).

SOLUTION:

(a) The thermal circuit diagrams are shown in Figure Pr.3.72(b). The prescribed heat transfer rate $Q_1 = 0$. For case (ii), a resistance-type heat transfer $Q_{k,1\text{-}2}$ exists and T_2 is prescribed and constant.

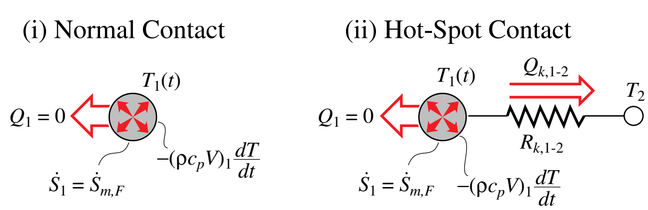


Figure Pr.3.72(b) Thermal circuit diagram.

(b) The time-dependent, uniform temperature $T_1(t)$ is given by (3.169) for the case of no resistive-type heat transfer and (3.172) for the case with a resistance-type heat transfer.

(i) From (3.169), we have

$$\begin{aligned} T_1(t) &= T_1(t=0) + \frac{\dot{S}_{m,F}}{(\rho c_p V)_1} t \\ V_1 &= \pi(R_o^2 - R_i^2)l. \end{aligned}$$

Using the numerical values, we have

$$\begin{aligned} V_1 &= \pi(R_o^2 - R_i^2)l \\ &= \pi[0.18^2(\mathrm{m}^2) - 0.13^2(\mathrm{m}^2)] \times 0.015(\mathrm{m}) \\ &= 7.300 \times 10^{-4}\ \mathrm{m}^3 \\ T_1(t = 4\ \mathrm{s}) &= 20(^\circ\mathrm{C}) + \frac{3 \times 10^4(\mathrm{W}) \times 4(\mathrm{s})}{3.5 \times 10^6(\mathrm{J/m^3\text{-}K}) \times 7.300 \times 10^{-4}(\mathrm{m}^3)} \\ &= 20(^\circ\mathrm{C}) + 46.97(^\circ\mathrm{C}) \\ &= 66.97^\circ\mathrm{C}. \end{aligned}$$

(ii) From (3.172), we have

$$\begin{aligned} T_1(t) &= T_2 + [T_1(t=0) - T_2]e^{-t/\tau_1} + a_1\tau_1(1 - e^{-t/\tau_1}) \\ \tau_1 &= (\rho c_p V)_1 R_{k,1\text{-}2}, \quad a_1 = \frac{\dot{S}_{m,F}}{(\rho c_p V)_1}. \end{aligned}$$

Since $T_1(t=0) = T_2$, we have

$$\begin{aligned} T_1(t=0) &= T_2 + a_1\tau_1(1 - e^{-t/\tau_1}) \\ &= T_2 + \dot{S}_{m,F} R_{k,1\text{-}2}(1 - e^{-t/\tau_1}). \end{aligned}$$

Now noting the smaller volume, and using the numerical values, we have

$$\begin{aligned} V_1 &= \pi(R_{o,1}^2 - R_{i,1}^2)l \\ &= \pi[0.16^2(\mathrm{m}^2) - 0.15^2(\mathrm{m}^2)] \times 0.015(\mathrm{m}) \\ &= 1.460 \times 10^{-4}\ \mathrm{m}^3 \\ \tau_1 &= 3.5 \times 10^6(\mathrm{J/m^3\text{-}K}) \times 1.460 \times 10^{-4}(\mathrm{m}^3) \times 1(\mathrm{K/W}) \\ &= 5.110 \times 10^2\ \mathrm{s} \\ T_1(t = 4\mathrm{s}) &= 20(^\circ\mathrm{C}) + 3 \times 10^4(\mathrm{W}) \times 1(\mathrm{K/W}) \times [1 - e^{-4(\mathrm{S})/511.0(\mathrm{S})}] \\ &= 20(^\circ\mathrm{C}) + 233.9(^\circ\mathrm{C}) = 253.9^\circ\mathrm{C}. \end{aligned}$$

(c) By comparing the results of (i) and (ii) in (b), we note that for (ii) there is an additional 180°C rise in the temperature. When multiple braking is made (as in stop-and-go or down-hill breaking), this increase is compounded each time the brake is applied. Then temperatures above the damage threshold of the brake pad material are reached.

COMMENT:

Note that since $\tau_1 = 511.0$ s is much larger than the elapsed time of $t = 4$ s, we could have neglected the heat transfer during the brake period. Then we could have used (3.169) also for case (ii). This would give, for case (ii),

$$\begin{aligned} T_1(t) &= 20(^\circ\mathrm{C}) + \frac{3 \times 10^4(\mathrm{W}) \times 4(\mathrm{s})}{3.5 \times 10^6(\mathrm{J/m^3\text{-}K}) \times 1.460 \times 10^{-4}(\mathrm{m}^3)} \\ &= 20(^\circ\mathrm{C}) \times 234.8(^\circ\mathrm{C}) \\ &= 254.8^\circ\mathrm{C}. \end{aligned}$$

For longer elapsed times, (3.172) should be used.
Note that the uniform rotor temperature assumption may not be valid for such a short elapsed time. Then a distributed, penetration treatment may be made.

PROBLEM 3.73.FUN.S

GIVEN:

A thermal barrier coating in the form of spray deposited, zirconia particles, is used as a thin layer to protect a substrate. Figures Pr.3.73(i) and (ii) show a typical barrier coating and a representative two-dimensional conduction network model. The heat conduction through the gas filling the voids is neglected. The thermal conductivity for the zirconia particles is $k_s = 1.675$ W/m-K, and the porosity of the coating is approximately $\epsilon = 0.25$. The geometrical properties of the representative network model are given in Table Pr.3.73.

SKETCH:

Figure Pr.3.73(i) shows the micrograph of the thermal barrier coating, and (ii) shows the two-dimensional, representative conduction network model.

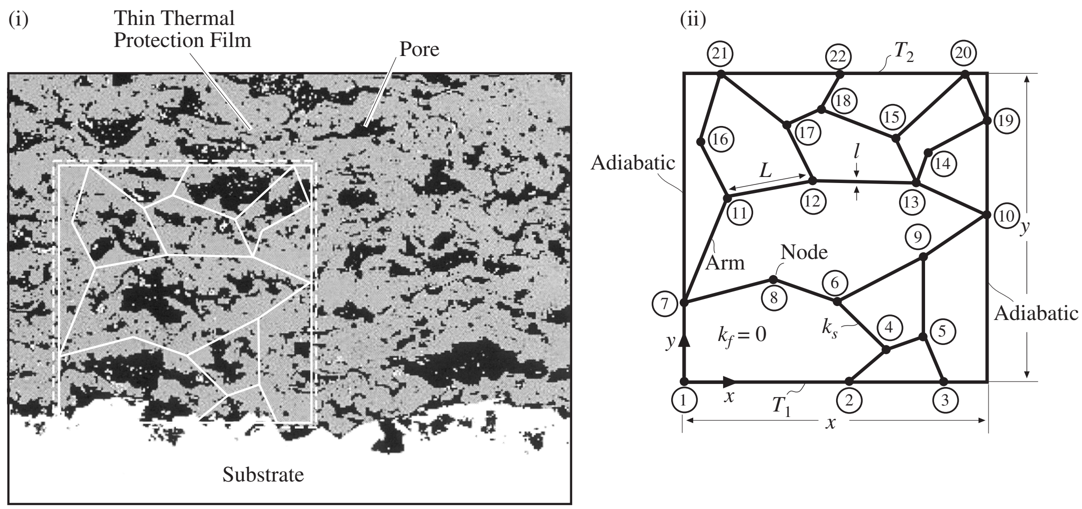


Figure Pr.3.73(i) Micrograph of a thermal barrier coating, and (ii) a two-dimensional, representative conduction network model.

OBJECTIVE:

(a) Determine the estimated effective thermal conductivity $\langle k \rangle_{yy}$ (along the y axis) for the film layer by using the two-dimensional thermal circuit diagram given in Figure Pr.3.73(ii).

Table Pr.3.73 The geometrical properties of the representative network model.

Arm	l, μm	L, μm	Arm	l, μm	L, μm
1-7	1.5	12	11-16	3.2	10
2-4	3	7	12-13	8	11
3-5	5	9	12-17	4	15
4-5	3	8	13-14	2	6
4-6	0.8	10	13-15	2	9
5-9	5	13	14-19	3	12
6-8	5	10	15-18	3.25	13
6-9	1	12	15-20	1.2	19
7-8	2.5	16	16-21	7.5	10
7-11	5	19	17-18	3	6
9-10	3.5	11	17-21	2.5	18
10-13	2	13	18-22	3	4
11-12	2.25	20	19-20	1	6

(The network model also represents the thermal circuit diagram for the layer.) Neglect the thermal conductivity of the gas filing the pores ($k_f = 0$). Take the length along the z axis (perpendicular to page) $w = 1$ m. Use as the temperature at the lower boundary L_z, $T_1 = 225$℃, and the temperature at the upper boundary, $T_2 = 400$℃ (for effective conductivity is independent of these values). Write one-dimensional, steady-state conduction heat flow for each arm and an energy equation for each node. Solve the set of linear algebraic equations for the temperature of each node. Calculate the total heat flux, i.e., Q_k, leaving the upper surface and determine the effective thermal conductivity from the expression

$$Q_k = \langle k \rangle_{yy} L_x L_z \frac{(T_2 - T_1)}{L_y}.$$

Take $L_x = L_y = 100$ μm. Note also that left and right boundaries of the network model are adiabatic, i.e., no heat flows across these boundaries.
(b) Compare the result of (a) with the analytical result for an isotropic, periodic unit-cell model given by

$$\frac{\langle k \rangle}{k} = 1 - \epsilon^{1/2}, \ \langle k \rangle_{xx} = \langle k \rangle_{yy}.$$

SOLUTION:
(a) For each arm, a one-dimensional steady-state conduction heat flow rate (a total of 26 relations), and for each node an energy equation (a total of 16 equations) are written. For example, for arm 13-14, we have

$$Q_{13\text{-}14} = A_{k,13\text{-}14} k_s \frac{T_{14} - T_{13}}{L_{13\text{-}14}}, \quad A_{k,13\text{-}14} = l_{13\text{-}14} L_z$$
$$Q_{13\text{-}14} + Q_{13\text{-}15} - Q_{12\text{-}13} - Q_{10\text{-}13} = 0.$$

Since the left and right boundaries of the network model are adiabatic, then for example for node 10, the energy equation becomes

$$Q_{9\text{-}10} = Q_{10\text{-}13}.$$

The set of linear algebraic equations is solved by a solver such as SOPHT.
We have $k_s = 1.675$ W/m-K, and the result is

$$\langle k \rangle_{yy} = 0.413 \text{ W/m-K}.$$

(b) Using the analytical result for an isotropic, periodic unit-cell model, we have

$$\langle k \rangle_{yy} = k_s(1 - \epsilon^{1/2}) = 1.675(\text{W/m-K}) \times (1 - 0.25^{1/2}) = 0.8375 \text{ W/m-K}.$$

COMMENT:
The analytical result gives us a higher value, since it assumes a geometry for the thermal barrier coating as composed of an isotropic, periodic unit cell. As is evident from the two-dimensional network model, the geometry cannot be assumed as isotropic, periodic unit cells. The predictions can be improved by using a larger number of nodes and by including the third dimension.

PROBLEM 3.74.FUN

GIVEN:

Steady-state conduction in a rectangular, two-dimensional medium with prescribed temperatures on the bounding surfaces.

$$L_x = L_y, \Delta x = \Delta y = L_x/N.$$

SKETCH:

Figure Pr.3.74(a) shows the two-dimensional medium divided into finite-small volumes.

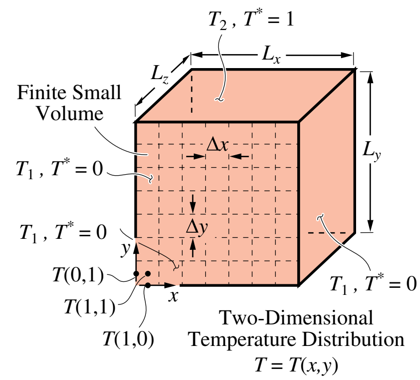


Figure Pr.3.74(a) Two-dimensional, steady-state conduction in a rectangular medium. The discretized finite-small volumes are also shown.

OBJECTIVE:

(a) Determine the temperature distribution for the two-dimensional, steady-state conduction in the rectangular geometry shown in Figure Pr.3.74. Use the dimensionless temperature and lengths

$$T^*(x,y) = \frac{T(x,y) - T_1}{T_2 - T_1},\ x^* = \frac{x}{L_x},\ y^* = \frac{y}{L_y}.$$

(b) Plot the results for $N = 3$, 15, and 21.

(c) Compare the results with the exact series solution

$$T^*(x,y) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1+(-1)^{n+1}}{n}\sin\left(\frac{n\pi x}{L_x}\right)\frac{\sinh(n\pi y/L_x)}{\sinh(n\pi L_y/L_x)}$$

by showing the results on the same plot.

$L_x = L_y = 20$ cm, $\Delta x = \Delta y = L_x/N$.

SOLUTION:

(a) The finite-volume energy equation for two-dimensional heat transfer with the Cartesian coordinates, is given by (3.184), for the interior nodes. This written for $\Delta x = \Delta y$, is

$$\frac{T^*_{i,j} - T^*_{i,j}}{\frac{\Delta x}{k\Delta y L_z}} + \frac{T^*_{i,j} - T^*_{i+1,j}}{\frac{\Delta x}{k\Delta y L_z}} + \frac{T^*_{i,j} - T^*_{i,j-1}}{\frac{\Delta y}{k\Delta x L_z}} + \frac{T^*_{i,j} - T^*_{i,j+1}}{\frac{\Delta y}{k\Delta x L_z}} = 0$$

Since we have chosen $\Delta x = \Delta y$, we have

$$4T^*_{i,j} - T^*_{i-1,j} - T^*_{i+1,j} - T^*_{i,j-1} - T^*_{i,j+1} = 0.$$

Here $i = 3, \cdots N-1$, and $j = 2, 3, \cdots N-1$, with i designating the x-direction index and j designating the y-direction index.

For nodes $i = 2$ and N and $j = 2$ and N, we have a different energy equation. For example, for $i = 2$, and $2 < j < N$, we have

$$3.5T^*_{2,j} - 0.5T^*_{1,j} - T^*_{3,j} - T^*_{2,j-1} - T^*_{2,j+1} = 0.$$

These are N^2 interior nodes and the above energy equations are written for each node.
The surface nodes $T^*_{0,j}, T^*_{N+1,j}, T^*_{i,0}$ and $T^*_{i,N+1}$ are all prescribed. All are set to zero except $T^*_{i,N+1} = 1$.
A solver, such as SOPHT, is used to determine $T_{i,j}$ for the interior nodes.

(b) Figure Pr.3.74(b) shows the plot of numerical results obtained using $N = 3, 15$, and 21. Since the plotter uses a curve fit to the discrete data, all results appear the same, although for $N = 3$, only 3 interior nodes are used. The results for the series solution are also shown and are nearly identical to those obtained numerically (they cannot be distinguished on the figure). At least 50 terms are needed in the series solution for a converged solution.

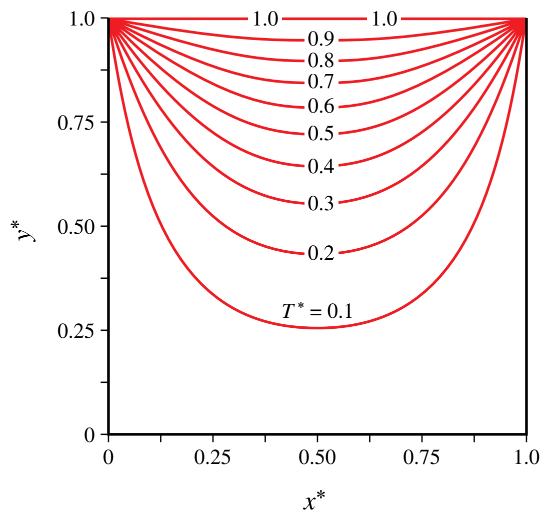


Figure Pr.3.74(b) Distribution of $T^*(x^*, y^*)$ obtained by finite-small volume method using $N = 3, 15$, and 21, and also by the series solution.

The temperature found at ($x^* = 0.125,\ y^* = 0.125$) for $N = 3$ is $T^* = 0.03571$. For $N = 15$ at the same location, we find $T^* = 0.03051$, and for $N = 21$, $T^* = 0.03044$. The exact solution for this location is $T^* = 0.03036$. This shows that for $N > 15$ a relatively accurate result is obtained.

COMMENT:

In practice, the number of increments N is increased until the results no longer change within a small, acceptable criterion.

PROBLEM 3.75.FUN

GIVEN:

A porcelain workpiece (in form of a circular disk) is ablated by laser irradiation. The piece is held inside a cooling ring, as shown in Figure Pr.3.75(a), to maintain its outer surface at a temperature T_s. For the ablation, the temperature of the ceramic much reach a threshold temperature T_{sg} (i.e., a sublimation temperature), over the area of interest. The radiation is absorbed only over the surface $A_{r,\alpha}$, and there is surface convection over the rest of the area A_{ku}. Assume a steady-state heat transfer and a uniform temperature along the z and ϕ axes [i.e., $T = T(r)$ only].

Assume all irradiation is absorbed on the surface. For the central node, node 1, use $R_r/2$ as the inner surface location for the determination of the conduction resistance.

$R = 3$ cm, $R_r = R/5$, $l = 3$ mm, $q_{r,i} = 10^6$ W/m^2, $T_s = 90$℃, $A_{ku}R_{ku} = 10^{-3}$ K/(W/m^2), $T_{f,\infty} = 120$℃.

SKETCH:

Figure Pr.3.75(a) shows the workpiece and the cooling ring around it.

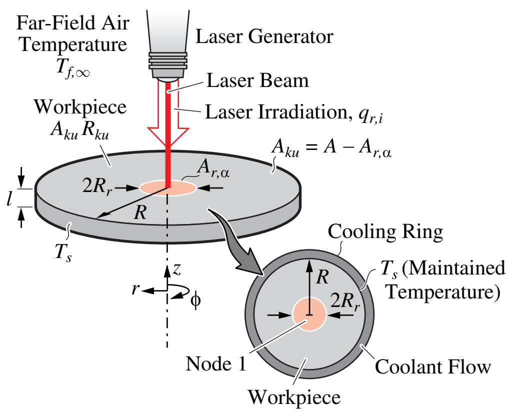


Figure Pr.3.75(a) A porcelain workpiece is irradiated for ablation. The workpiece is cooled at its periphery by a coolant carrying ring.

OBJECTIVE:

Divide the porcelain piece into N segments, i.e., $\Delta r = R/N$, and apply the finite-small volume energy equation (3.176) to each segment.

(a) Draw the thermal circuit diagram for the entire disk.

(b) Determine the segment temperature T_i for $N = 5$.

SOLUTION:

(a) Figure Pr.3.75(b) shows the thermal circuit diagram for $N = 5$ or $\Delta r = R/5$. Note that $Q_1 = 0$, because the only heat transfer from A_1 is by conduction $Q_{k,1\text{-}2}$.

(b) The energy equation (3.176) for V_1, under steady-state conditions, is

$$Q|_{A,1} = Q_{k,1\text{-}2} = (\dot{S}_{r,\alpha})_1 = A_{r,\alpha} q_{r,i}.$$

For $Q_{k,1\text{-}2}$, from Table 3.2, we have

$$Q_{k,1\text{-}2} = \frac{T_1 - T_2}{R_{k,1\text{-}2}}, \quad R_{k,1\text{-}2} = \frac{\ln \dfrac{3\Delta r/2}{R_r/2}}{2\pi k l}.$$

Note that the nodes are located at the center of each segment, except for node 1, where to avoid singularity, we have used $R_r/2$. The conductivity of porcelain is given in Table C.15, i.e.,

$$k = 1.5 \text{ W/m-K} \quad \text{Table C.15}.$$

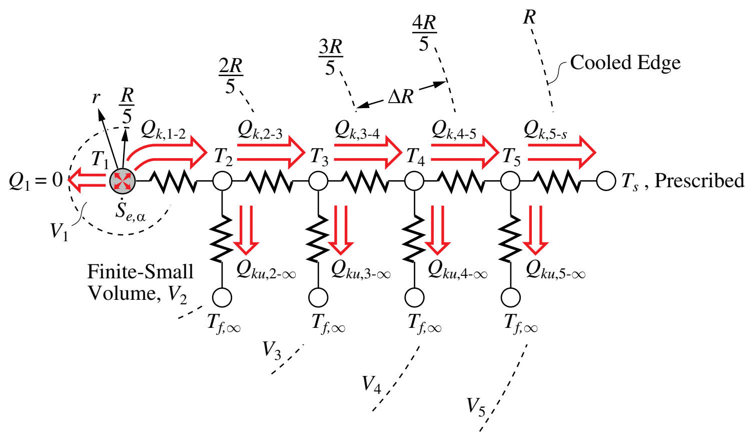


Figure Pr.3.75(b) Thermal circuit diagram.

Then

$$
\begin{aligned}
R_{k,1\text{-}2} &= \frac{\ln \dfrac{3 \times 3 \times 10^{-2}(\text{m})/5 \times 2}{3 \times 10^{-2}(\text{m})/5 \times 2}}{2\pi \times 1.5(\text{W/-K}) \times 3 \times 10^{-3}(\text{m})} \\
&= 38.85 \text{ K/W} \\
(\dot{S}_{r,\alpha})_1 &= \pi R_r^2 q_{r,i} = \pi \frac{(3 \times 10^{-2})^2(\text{m}^2)}{25} \times 10^6(\text{W/m}^2) \\
&= 1.131 \times 10^2 \text{ W.}
\end{aligned}
$$

The energy equation (3.176), for V_2, is

$$
Q|_{A,2} = Q_{k,2\text{-}1} + Q_{k,2\text{-}3} + Q_{ku,2\text{-}\infty} = 0,
$$

where

$$
\begin{aligned}
Q_{k,2\text{-}1} &= -Q_{k,1\text{-}2}, \quad R_{k,2\text{-}1} = R_{k,1\text{-}2} \\
Q_{k,2\text{-}3} &= \frac{T_2 - T_3}{R_{k,2\text{-}3}}, \quad R_{k,2\text{-}3} = \frac{\ln \dfrac{5\Delta r/2}{3\Delta r/2}}{2\pi k l} \\
Q_{ku,2\text{-}\infty} &= \frac{T_2 - T_{f,\infty}}{R_{ku,2\text{-}\infty}}, \quad R_{ku,2\text{-}\infty} = \frac{A_{ku} R_{ku}}{\pi \left[\left(\dfrac{2R}{5} \right)^2 - \left(\dfrac{R}{5} \right)^2 \right]}.
\end{aligned}
$$

Then

$$
\begin{aligned}
R_{k,2\text{-}3} &= \frac{\ln \dfrac{5}{3}}{2\pi \times 1.5(\text{W/m-K}) \times 3 \times 10^{-3}(\text{m})} = 18.06 \text{ K/W} \\
R_{ku,2\text{-}\infty} &= \frac{10^{-3}[\text{K/(W/m}^2)]}{\pi \times \dfrac{3}{25} \times (3 \times 10^{-2})^2(\text{m}^2)} = 2.947 \text{ K/W.}
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
R_{k,3\text{-}4} &= \frac{\ln \dfrac{7}{5}}{2\pi \times 1.5 \times 3 \times 10^{-3}} = 11.90 \text{ K/W} \\
R_{k,4\text{-}5} &= \frac{\ln \dfrac{9}{7}}{2\pi \times 1.5 \times 3 \times 10^{-3}} = 8.887 \text{ K/W} \\
R_{k,5\text{-}\infty} &= \frac{\ln \dfrac{1}{9/10}}{2\pi \times 1.5 \times 3 \times 10^{-3}} = 3.726 \text{ K/W} \\
R_{ku,3\text{-}\infty} &= \frac{10^{-3}}{\pi \times \dfrac{5}{25} \times (3 \times 10^{-3})^2} = 1.768 \text{ K/W} \\
R_{ku,4\text{-}\infty} &= \frac{10^{-3}}{\pi \times \dfrac{7}{25} \times (3 \times 10^{-3})^2} = 1.263 \text{ K/W} \\
R_{ku,5\text{-}\infty} &= \frac{10^{-3}}{\pi \times \dfrac{9}{25} \times (3 \times 10^{-3})^2} = 0.9822 \text{ K/W.}
\end{aligned}
$$

Using a solver (such as SOPHT), we solve for T_1 to T_5, from the five energy equations. The results are

$$
\begin{aligned}
T_1 &= 4{,}804^\circ\text{C} \\
T_2 &= 409.8^\circ\text{C} \\
T_3 &= 142.9^\circ\text{C} \\
T_4 &= 121.3^\circ\text{C} \\
T_5 &= 114.4^\circ\text{C.}
\end{aligned}
$$

COMMENT:

By increasing N, a more accurate prediction of T_1 is obtained. However, the lack of surface convection and the localized irradiation does result in the desired high temperature T_1 (for ablation).

PROBLEM 3.76.FUN.S

GIVEN:

The effective thermal conductivity of some porous media can be determined using the random network model. In one of these models, a regular lattice is used, but the locations of the nodes, within the regular lattice, are generated randomly and then connected, forming a network. The thicknesses (for a two- or three-dimensional geometry) of these connectors (i.e., arms) are then assigned based on the porosity and any other available information. The network can represent the solid or the fluid part of the medium. A 3×3 square unit-cell, two-dimensional random network model is shown in Figure Pr.3.76(a). This is determined by randomly selecting the location of each node within its unit cell space. The coordinates of each node, and the length and the thickness for each arm are given in Table Pr.3.76.

Assume that the heat transfer between adjacent nodes is one dimensional and steady. For the arms, use the thermal conductivity of aluminum (Table C.16). Assume that the thermal conductivity of the fluid is much smaller than that of the solid. The left and right boundaries of the medium are ideally insulated. The temperature for the lower boundary of the medium, $[(x, y) = (0.0)]$, is maintained at $T_c = 100$℃, (i.e., $T_c = T_1 = T_6 = T_{11} = 100$℃), while the temperature for the upper boundary, $[(x, y) = (3, 0)]$, is maintained at $T_h = 200$℃, (i.e., $T_h = T_5 = T_{10} = T_{15} = 200$℃).

Table Pr.3.76 Coordinates of each node, and the length and the thickness of each arm, for a two-dimensional, random network model.

Node	(x, y)	$L_{i,j}$(mm)	$l_{i,j}$(mm)
1	(0.76,0.00)	$L_{1,2} = 0.715$	$l_{1,2} = 0.414$
2	(0.13,0.34)	$L_{2,3} = 1.581$ $L_{2,7} = 1.059$	$l_{2,3} = 0.213$ $l_{2,7} = 0.055$
3	(0.92,1.71)	$L_{3,4} = 0.622$ $L_{3,8} = 1.105$	$l_{3,4} = 0.152$ $l_{3,8} = 0.197$
4	(0.47,2.14)	$L_{4,5} = 0.932$ $L_{4,9} = 0.970$	$l_{4,5} = 0.344$ $l_{4,9} = 0.376$
5	(0.83,3.00)		
6	(1.73,0.00)	$L_{6,7} = 0.788$	$l_{6,7} = 0.151$
7	(1.17,0.54)	$L_{7,8} = 1.047$ $L_{7,12} = 1.596$	$l_{7,8} = 0.232$ $l_{7,12} = 0.215$
8	(1.93,1.26)	$L_{8,9} = 1.059$ $L_{8,13} = 0.314$	$l_{8,9} = 0.469$ $l_{8,13} = 0.125$
9	(1.44,2.15)	$L_{9,10} = 0.935$ $L_{9,14} = 1.592$	$l_{9,10} = 0.071$ $l_{9,14} = 0.316$
10	(1.83,3.00)		
11	(2.38,0.00)	$L_{11,12} = 0.854$	$l_{11,12} = 0.417$
12	(2.75,0.77)	$L_{12,13} = 0.887$	$l_{12,13} = 0.222$
13	(2.18,1.45)	$L_{13,14} = 1.602$	$l_{13,14} = 0.118$
14	(2.84,2.91)	$L_{14,15} = 0.577$	$l_{14,15} = 0.322$
15	(2.27,3.00)		

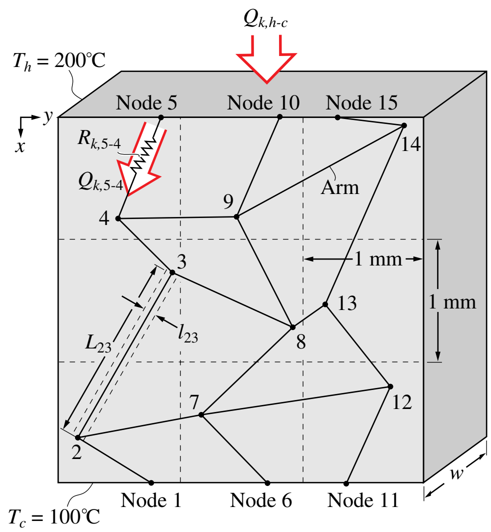
SKETCH:

Figure Pr.3.76(a) shows a two-dimensional regular (periodic) lattice with random location of nodes within the lattice, making for a random network model.

OBJECTIVE:

(a) Draw the thermal circuit diagram using the geometrical data. Write the energy equation for each node, along with the conduction heat transfer relation for each arm.
(b) Determine the total conduction heat transfer rate, $Q_{k,h\text{-}c}$.
(c) Using

$$Q_{k,h\text{-}c} \equiv \frac{A_k(T_h - T_c)\langle k \rangle}{L},$$



A 3 x 3 Square Unit-Cell, Two-Dimensional Random Network Model

Figure Pr.3.76(a) The random network model in an ordered lattice.

for $L = 10^{-3}$ m and $A_k = 3 \times 10^{-6}$ m^2, determine the effective thermal conductivity $\langle k \rangle$.

SOLUTION:

(a) Figure Pr.3.76(a) shows the 3×3 square unit cell and the random network.

The porosity ϵ is determined by first determining the solid fraction by adding the area of all solid arms. Then this area is divided by the total area and then subtracted from unity.

(b) For each arm, a, one-dimensional steady-state conduction relation (total 18 relations), and for each node an energy equation (total 9 equations) are written. For example, for arm 1 - 2, we have

$$Q_{1\text{-}2} = A_{k,1\text{-}2} k_s \frac{T_2 - T_1}{L_{1,2}}$$

$$Q_{1\text{-}2} - Q_{2\text{-}3} - Q_{2\text{-}7} = 0.$$

The set of linear algebraic equations is solved by a solver such as SOPHT.

From Table C.16, we have $k_s = 237$ W/m-K, at $T = 300$ K. The result is

$$Q_{k,h\text{-}c} = 4.283 \text{ W}.$$

(c) Using the above definition, we have

$$\langle k \rangle = 42.83 \text{ W/m-K}.$$

COMMENT:

The effective thermal conductivity for the series arrangement is given by

$$\frac{1}{\langle k \rangle} = \frac{\epsilon}{k_f} + \frac{(1-\epsilon)}{k_s}.$$

The effective conductivity for the parallel arrangement is given by

$$\langle k \rangle = k_f + k_s(1-\epsilon).$$

For the isotropic, unit-cell geometry, we have

$$\langle k \rangle = k_s(1-\epsilon^{1/2}).$$

Using these relations, the variation of the dimensionless effective conductivity with respect to porosity is plotted in Figure Pr.3.76(b). Figure Pr.3.76(b) shows that the dimensionless effective conductivity for the random network is lower than that for the unit-cell model, for a given porosity.Also note that there are overlaps at the interceptions of the arms. This should be considered in calculating the total solid area. Neglecting this, we use Figure Pr.3.76(b). The curve fitting of Figure 3.76(b) is used to calculate the actual porosity. In Figure 3.76(c), the results for the actual porosity is also shown.

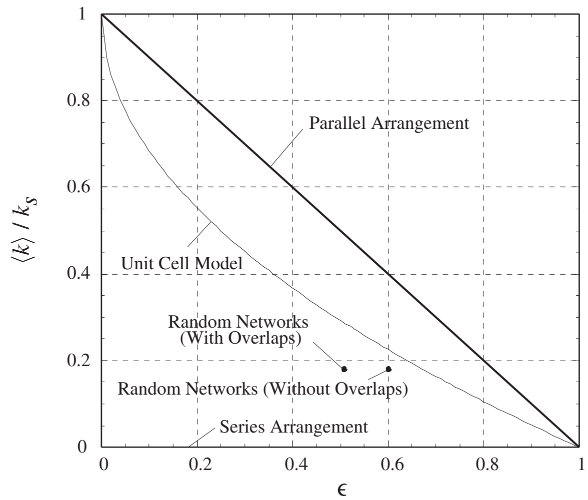


Figure Pr.3.76(b) Variation of the dimensionless effective conductivity with respect to porosity.

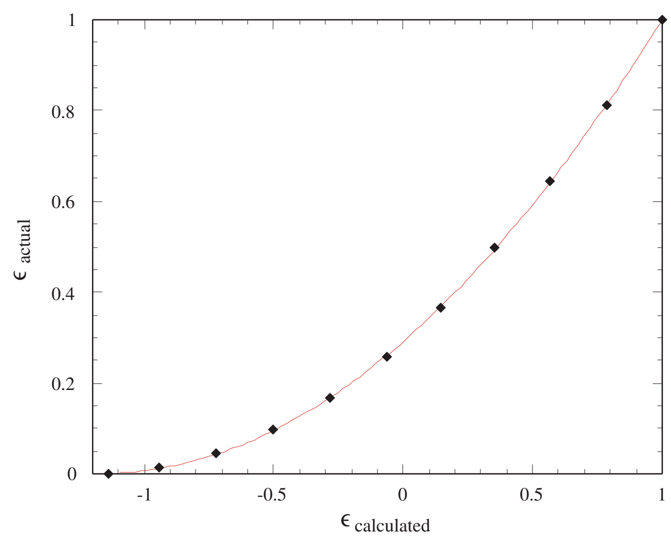


Figure Pr.3.76(c) Actual porosity versus the calculated porosity.

PROBLEM 3.77.FAM

GIVEN:

The friction heating during skating over ice layers causes melting, and the thickness of this melt at the end of the blade δ_α may be estimated when the blade surface temperature $T_s = T_{l,o}$ is known. Figure Pr.3.77 shows the blade length L in contact with the ice, with the skating speed designated as u_s.

$T_{l,o} = 10$℃, $T_{sl} = 0$℃, $L = 0.20$ m, $u_p = 2$ m/s.

Use properties of water given in Tables C.4 and C.27 (at $T = 275$ K).

SKETCH:

Figure Pr.3.77 shows the length L for the contact and the blade edge temperature T_s.

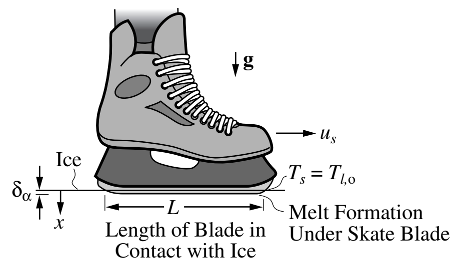


Figure Pr.3.77 Melt formation during ice skating.

OBJECTIVE:

In order to estimate the elapsed time used in determining δ_α, we can use $t = L/u_s$. For the conditions given, determine the liquid film thickness δ_α assuming that the one-dimensional melting analysis of Section 3.8 is applicable.

SOLUTION:

From (3.198), we have

$$\delta_\alpha(t) = 2\eta_o(\alpha_l t)^{1/2},$$

where η_o is found from Table 3.7 and depends on the Stefan number given by (3.197), i.e.,

$$\text{Ste}_l = \frac{c_{p,l}(T_{l,0} - T_{sl})}{\Delta h_{sl}}.$$

From Tables C.4 and C.27 ($T = 275$K), we have for water

$$\Delta h_{sl} = 3.336 \times 10^5 \text{ J/kg} \qquad \text{Table C.4}$$

$$\rho_l = 1{,}000 \text{ kg/m}^3 \qquad \text{Table C.27}$$

$$c_{p,l} = 4{,}211 \text{ J/kg} \qquad \text{Table C.27}$$

$$k_l = 0.547 \text{ W/m-K} \qquad \text{Table C.27}$$

$$\alpha_l = \left(\frac{k}{\rho c_p}\right)_l = \frac{(0.547)(\text{W/m-K})}{1{,}000(\text{kg/m}^3) \times 4{,}211(\text{J/kg})} = 1.299 \times 10^{-7} \text{ m}^2/\text{s}$$

$$\text{Ste}_l = \frac{4{,}211(\text{J/kg}) \times (10-0)(\text{K})}{3.336 \times 10^5(\text{J/kg})} = 0.1262$$

$$\frac{\text{Ste}_l}{\pi^{1/2}} = 0.07121.$$

Then from Table 3.7, we have

$$\eta_o = 0.2062.$$

The elapsed time is

$$\begin{aligned}
t &= \frac{L}{u_s} = \frac{0.20(\text{m})}{2(\text{m/s})} = 0.10 \text{ s} \\
\delta_\alpha(t = 0.10 \text{ s}) &= 2 \times 0.2062 \times [1.299 \times 10^{-7}(\text{m}^2/\text{s}) \times 0.10(\text{s})]^{1/2} \\
&= 4.700 \times 10^{-5} \text{ m} \\
&= 47 \ \mu\text{m}.
\end{aligned}$$

COMMENT:

The one-dimensional analysis overestimates the heat flow rate and the liquid thickness. Also, due to the skater weight, the liquid will be forced out. This is called close-contact melting and this tends to increase the heat transfer rate.

PROBLEM 3.78.FUN

GIVEN:

For the conduction-melting of a semi-infinite solid initially at the melting temperature T_{sl} and suddenly exposed to $T_{l,o} > T_s$ at its surface ($x = 0$), the temperature distribution in the melt is given by (3.194).

OBJECTIVE:

(a) Derive this temperature distribution using the energy equation (3.189) and the thermal conditions at $x = 0$ and $x = \delta_\alpha(t)$, i.e., as given by (3.190) to (3.191), i.e.,

$$T_l(x,t) = a_1 + a_2 \mathrm{erf}(\eta)$$

$$\frac{T_l(x,t) - T_{l,0}}{T_{sl} - T_{l,o}} = \frac{\mathrm{erf}(\eta)}{\mathrm{erf}(\eta_o)}.$$

Use the similarity variable (3.195) and an error function solution, i.e., similar to (3.140), with erf(η) defined by (3.141)

(b) Using (3.192), show that η_o is determined from (3.196).

SOLUTION:

(a) The differential energy equation for the semi-infinite medium is given by (3.189), i.e.,

$$\frac{\partial^2 T_l}{\partial x^2} - \frac{1}{\alpha_l}\frac{\partial T_l}{\partial t} = 0.$$

Based on the similarity solution for transient conduction in a semi-infinite slab, given in Section 3.5.1, we choose the similarity variable (3.136), i.e.,

$$\eta = \frac{x}{2(\alpha_l t)^{1/2}},$$

and a solution of the type

$$T_l(x,t) = a_1 + a_2 \mathrm{erf}(\eta).$$

Now using (3.190), we have

$$T_l(x=0,t) = T_{l,0} = a_1 + a_2 \mathrm{erf}(0).$$

Using Table 3.5, we note that erf(0) = 0, then

$$a_1 = T_{l,0}.$$

Next we use (3.191), i.e.,

$$T_l[x = \delta_\alpha(t)] = T_{sl} = T_{l,0} + a_2 \mathrm{erf}(\eta_o),$$

where

$$\eta_o = \frac{\delta_\alpha(t)}{2(\alpha_l t)^{1/2}}$$

and η_o is a constant.

Solving for a_2, we have the melt temperature distribution

$$a_2 = \frac{T_{sl} - T_{l,0}}{\mathrm{erf}(\eta_o)} = \frac{T_l(x,t) - T_{l,0}}{\mathrm{erf}(\eta)}$$

$$\frac{T_l(x,t) - T_{l,0}}{T_{sl} - T_{l,0}} = \frac{\mathrm{erf}(\eta)}{\mathrm{erf}(\eta_o)}.$$

(b) Now using (3.192) or (3.200), we have the condition for the determination of η_o, i.e.,

$$k_l \left.\frac{\partial T_l}{\partial x}\right|_{x=\delta_\alpha(t)} = -\rho_l \Delta h_{sl} u_F = -\rho_l \Delta h_{sl} \frac{\alpha_l^{1/2}}{t^{1/2}} \eta_o.$$

From the definition of erf(η) given by (3.141), we have

$$\frac{\partial}{\partial x}\mathrm{erf}(\eta) = \frac{\partial}{\partial x}\frac{2}{\pi^{1/2}}\int_0^\eta e^{-z^2} dz.$$

We need to use the chain rule, i.e.,

$$\frac{\partial}{\partial x} = \frac{\partial \eta}{\partial x}\frac{\partial}{\partial \eta} = \frac{1}{2(\alpha_l t)^{1/2}}\frac{\partial}{\partial \eta}$$

or

$$\frac{\partial}{\partial x}\mathrm{erf}(\eta) = \frac{2}{\pi^{1/2}}\frac{1}{2(\alpha_l t)^{1/2}} e^{\eta^2}.$$

Then from the interface energy equation, we have

$$-k_l(T_{sl} - T_{l,o})\frac{e^{-\eta_o^2}}{\pi^{1/2}(\alpha_l t)^{1/2}\mathrm{erf}(\eta_o)} = \rho_l \Delta h_{sl} \frac{\alpha_l^{1/2}}{t^{1/2}}\eta_o, \quad \alpha_l = \frac{k_l}{(\rho c_p)_l}$$

or

$$c_{p,l}\frac{(T_{l,0} - T_{sl})}{\pi^{1/2}\Delta h_{sl}} = \eta_o e^{\eta_o^2}\mathrm{erf}(\eta_o).$$

This is (3.196).

COMMENT:
Note that

$$\delta_\alpha(t) = 2\eta_o(\alpha_l t)^{1/2},$$

where η_o is the root to (3.196). This relation is in a form similar to (3.148), if we use $\eta_o = 1.8$ or $\mathrm{Ste}_l/\pi^{1/2} = 45.46$. In arriving at (3.148), we assume that $T^* = 0.01$. Then the case of $\mathrm{Ste}_l/\pi^{1/2} = 45.46$ corresponds to the small conduction heat transfer rate through the surface $x = \delta_\alpha(t)$ which is a result of a very small temperature gradient.

PROBLEM 3.79.FAM

GIVEN:

To remove an ice layer from an inclined automobile windshield, shown in Figure Pr.3.79, heat is supplied by a thin-film Joule heater. The heater maintains the surface temperature of the window at $T_{l,0}$. It is determined empirically that when the melt thickness reaches $\delta_\alpha = 1$ mm, the ice sheet begins to fall from the window.

Assume that the heat transfer through the liquid water is one dimensional and occurs by conduction only, that the ice is at the melting temperature T_{ls}, and neglect the sensible heat of the liquid water. Use the properties of water at $T = 273$ K from Table C.23 and Table C.6.

SKETCH:

Figure Pr.3.79 shows an ice layer melting over a glass sheet with the heat provided by a thin-film Joule heater.

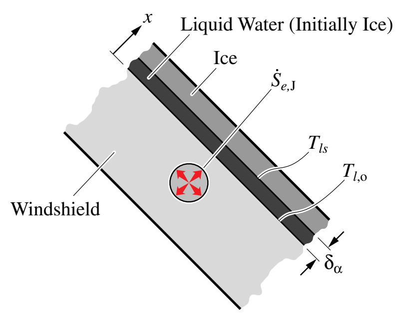


Figure Pr.3.79 Melting of ice on an automobile windshield.

OBJECTIVE:

Determine how long it will take for the ice to be removed t and estimate the amount of thermal energy $\int_0^t q dt$(J/m^2) required when (a) $T_{l,0} = 4$℃, and (b) $T_{l,0} = 15$℃.

SOLUTION:

The position of the melting front is given by (3.198)

$$\delta_\alpha(t) = 2\eta_o(\alpha_l t)^{1/2}.$$

The constant η_o is the solution to (3.194), i.e.,

$$a_o e^{\eta_o^2}\mathrm{erf}(\eta_o) = \frac{\mathrm{Ste}_l}{\pi^{1/2}},$$

where the liquid Stefan number is defined by (3.197)

$$\mathrm{Ste}_l = \frac{c_{p,l}(T_{l,0} - T_{sl})}{\Delta h_{sl}}.$$

The total energy per unit area spent to melt the 1 mm ice layer is given by integrating (3.200), i.e.,

$$\int_o^t -q_k dt = \int_o^t \frac{\rho_l \Delta h_{sl} a_o {\alpha_l}^{1/2}}{t^{1/2}} dt = 2\rho_l \Delta h_{sl}\eta_o \alpha_l t^{1/2} = \rho_l \Delta h_{sl}\delta_\alpha(t).$$

The properties for water are found from Table C.23, $T = 273$ K, $\rho_l = 1{,}002$ kg/m^3, $c_{p,l} = 4{,}217$ J/kg-K, $\alpha_l = 131 \times 10^{-9}$ m^2/s, from Table C.6, $\Delta h_{sl} = 333.60 \times 10^3$ J/kg, $T_{sl} = 273.2$ K.

For each of the surface temperatures we have

(a) $T_{l,0} = 4$℃$= 277.15$ K

The Stefan number is

$$\mathrm{Ste}_l = \frac{c_{p,l}(T_{l,0} - T_{sl})}{\Delta h_{sl}} = \frac{4{,}217(\mathrm{J/kg\text{-}K}) \times (277.15 - 273.2)(K)}{333.60 \times 10^3(\mathrm{J/kg})} = 0.0499.$$

For $\text{Ste}_l/\pi^{1/2} = 0.0282$, from interpolation in Table 3.5 we obtain $\eta_o = 0.16063$.

Solving for time gives

$$t = \frac{1}{\alpha_l}\left(\frac{\delta_\alpha}{2\eta_o}\right)^2 = \frac{1}{131 \times 10^{-9}}\left(\frac{0.001}{2 \times 0.16063}\right)^2 = 74 \text{ s} \simeq 1.2 \text{ min}.$$

The total energy per unit area is then

$$\int_0^t -q_k dt = \rho_l \Delta h_{sl} \delta_\alpha(t) = 1{,}002(\text{kg/m}^3) \times 333.60 \times 10^3(\text{J/kg}) \times 0.001(\text{m}) = 3.341 \times 10^5 \text{ J/m}^2.$$

(b) $T_{l,0} = 15$℃:
The Stefan number is

$$\text{Ste}_l = \frac{c_{p,l}(T_{l,0} - T_{sl})}{\Delta h_{sl}} = \frac{4{,}217(\text{J/kg-K}) \times (288.15 - 273.2)(K)}{333.60 \times 10^3(\text{J/kg})} = 0.1890.$$

For $\text{Ste}_l/\pi^{1/2} = 0.1066$, from interpolation in Table 3.5 we obtain $\eta_o = 0.30022$.

Solving for time gives

$$t = \frac{1}{\alpha_l}\left(\frac{\delta_\alpha}{2\eta_o}\right)^2 = \frac{1}{131 \times 10^{-9}}\left(\frac{0.001}{2 \times 0.30022}\right)^2 = 21 \text{ s}.$$

As the ice thickness is still the same, the total energy per unit area required remains the same, $\int_0^t q_k dt = 3.341 \times 10^5 \text{ J/m}^2$.

COMMENT:

When the liquid water begins to flow due to the action of gravity, the assumption of pure conduction heat transfer is no longer valid. However, the water layer thickness is small enough that the treatment made here will suffice.

PROBLEM 3.80.DES

GIVEN:

In a thermostat used to control the passage of coolant through secondary piping leading to the heater core of an automobile, solid-liquid phase change is used for displacement of a piston, which in turn opens the passage of the coolant. The thermostat is shown in Figure Pr.3.80(i). The phase-change material is a wax that undergoes approximately 15% volume change upon solidification/melting. The response time of the thermostat is mostly determined by the time required for complete melting of the wax. This in turn is determined by the speed of penetration of the melting front into the wax and the time for its complete penetration. Two different designs are considered, and are shown in Figures Pr.3.80(ii) and (iii). In the design shown in Figure Pr.3.80(iii), the wax reservoirs have a smaller diameter D_2 compared to that of the first design, shown in Figure Pr.3.80(ii). Therefore, it is expected that in the three-reservoir design the wax will melt faster.

To solve the melting problem using the analysis of Section 3.8, we need to assume that the front is planar. Although this will result in an overestimation of the time required for melting, it will suffice for comparison of the two designs. Due to unavailability of complete properties for the wax, use the phase-change properties of Table C.5 for paraffin and the properties of engine oil in Table C.23 for the wax liquid phase. The melting temperature is a function of pressure and is represented by the Clausius-Clapeyron relation (A.14). Here, neglect the pressure variation and use a pressure of one atm and a constant melting temperature.

Use the properties of engine oil at $T = 310$ K. $T_{l,0} = 80$°C, $D_1 = 8$ mm, $D_2 = 3$ mm.

SKETCH:

Figure Pr.3.80 shows the coolant thermostat and the (i) single and (ii) three reservoir designs.

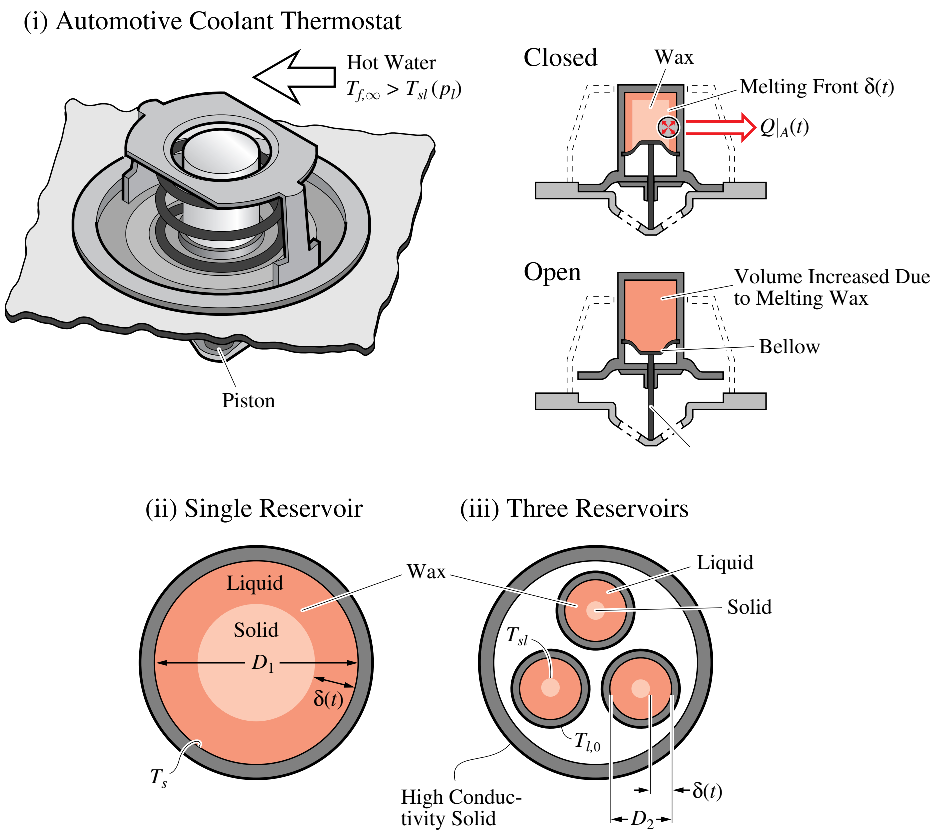


Figure Pr.3.80 (i) An automotive coolant thermostat. The solid-liquid phase change actuates the piston. (ii) The single-reservoir design. (iii) The three-reservoir designs.

OBJECTIVE:

Determine the time it takes for the complete melting of the wax in the one- and three-reservoir designs.

SOLUTION:

The penetration distance is given by (3.198), i.e.,

$$\delta(t) = 2\eta_o(\alpha t)^{1/2}.$$

For complete penetration, we have $\delta = D/2$, or

$$\frac{D}{2} = 2\eta_o(\alpha_l t)^{1/2} \quad \text{or} \quad t = \frac{D^2}{16\eta_o^2\alpha_l},$$

where t is the elapsed time needed. Then

$$t_1 = \frac{D_1^2}{16\eta_o^2\alpha_l} \quad , \quad t_2 = \frac{D_2^2}{16\eta_o^2\alpha_l}.$$

From these, the smaller D results in a smaller elapsed time t.
The thermophysical properties from Tables C.5. and C.23, are
paraffin (at 1 atm pressure):

$$T_{sl} = 310.0 \text{ K} \qquad \text{Table C.5}$$
$$\Delta h_{sl} = 2.17 \times 10^5 \text{ J/kg} \qquad \text{Table C.5}$$

engine oil (at $T = 310$ K):

$$\alpha_l = 8.70 \times 10^{-8}\text{m}^2 \qquad \text{Table C.23}$$
$$c_{p,l} = 1{,}950 \text{ J/kg-K} \qquad \text{Table C.23.}$$

Here a_o is found from Table 3.7 and depends on the Stefan number, which is given by (3.197), i.e.,

$$\begin{aligned}\text{Ste}_l &= \frac{c_{p,l}(T_{l,0} - T_{sl})}{\Delta h_{sl}} \\ &= \frac{1{,}950(\text{J/kg-K}) \times (273.15 + 80 - 310)(\text{K})}{2.17 \times 10^5(\text{J/kg})} = 0.3878.\end{aligned}$$

Then

$$\frac{\text{Ste}_l}{\pi^{1/2}} = 0.2188.$$

From Table 3.7, we have

$$\eta_o = 0.4005 \qquad \text{Table 3.7.}$$

Now, for the elapsed times, we have,

$$\begin{aligned}t_1 &= \frac{(0.008)^2(\text{m}^2)}{16 \times (0.4005)^2 \times 8.70 \times 10^{-8}(\text{m}^2/\text{s})} = 286.6 \text{ s} \\ t_2 &= \frac{(0.003)^2(\text{m}^2)}{16 \times (0.4005)^2 \times 8.70 \times 10^{-8}(\text{m}^2/\text{s})} = 40.31 \text{ s.}\end{aligned}$$

COMMENT:

Note the much lower response time for the smaller diameter wax reservoir. The planar approximation of the front results in an underestimation of the response time. This is because, in the radial system, as the center is reached, a smaller conduction area becomes available and this increases the penetration speed.

PROBLEM 3.81.FAM

GIVEN:

In a grinding operation, material is removed from the top surface of a small piece of pure copper with dimensions shown in Figure Pr.3.81(a). The grinding wheel is pressed against the copper workpiece with a force $F_c = 50$ N and there is an interfacial velocity $\Delta u_i = 20$ m/s. The coefficient of friction between the two surfaces is $\mu_F = 0.4$. The copper is initially at temperature $T_1(t = 0) = 20$℃.

SKETCH:

Figure Pr.3.81(a) shows copper being ground and expanding due to friction heating.

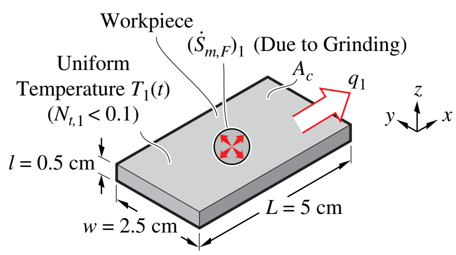


Figure Pr.3.81(a) Friction heating of a copper workpiece by grinding resulting in thermal expansion.

OBJECTIVE:

(a) Assuming a uniform copper temperature $T_1(t)$, i.e., $N_{t,1} < 0.1$, and a constant surface heat loss rate per unit area $q_1 = 675$ W/m^2, draw the thermal circuit diagram.
(b) Assuming an unconstrained expansion, determine the elapsed time needed to cause the copper length L to thermally expand by $\Delta L = 0.5$ mm. Neglect all nonthermally induced stresses and strains.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.3.81(b).

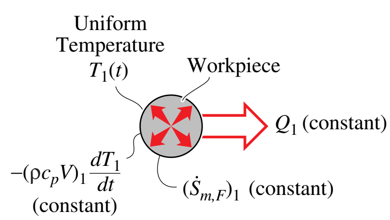


Figure Pr.3.81(b) Thermal circuit diagram.

(b) Before using the lumped capacitance, i.e., the single-node energy equation with a constant surface heat transfer rate (3.169), we determine the strain in the x-direction and the temperature needed to induce the desired strain. From (3.204) we have

$$\Delta L^* = \beta_s (T_1 - T_o)$$

where

$$\begin{aligned} \beta_s &= 1.7 \times 10^{-5}(1/\text{K}) \quad \text{and} \quad T_o = 20℃ \\ \Delta L^* &= \Delta L/L = 5 \times 10^{-4}(\text{m})/0.05(\text{m}) = 0.01. \end{aligned}$$

Then

$$\begin{aligned} 0.01 &= 1.7 \times 10^{-5} \times (T_1 - 20)(℃) \\ T_1 &= 608.2℃. \end{aligned}$$

From (3.169), we have

$$T_1 = T_1(t=0) + \frac{-q_1 A_1 + S_1}{(\rho c_p V)_1} t,$$

where

$$\begin{aligned}
A_1 &= 2(wL) + 2(lL) + 2(wl) \\
&= 2(0.025 \times 0.05)(\text{m}^2) + 2(0.005 \times 0.05)(\text{m}^2) + 2(0.005 \times 0.025)(\text{m}^2) \\
&= 0.00325 \text{ m}^2 \\
V_1 &= wlL \\
&= 0.025(\text{m}) \times 0.005(\text{m}) \times 0.05(\text{m}) \\
&= 6.25 \times 10^{-6} \text{ m}^3.
\end{aligned}$$

The energy conversion by mechanical friction is given in Table C.1(d), i.e.,

$$\begin{aligned}
(\dot{S}_{m,F})_1/A &= \mu_F \times p_c \times \Delta u_i \\
p_c &= F_c/A_c = 50(\text{N})/0.00125(\text{m}^2) = 40{,}000 \text{ N/m}^2 \\
A_c &= Lw \\
&= 0.05 \times 0.025(\text{m}^2) \\
&= 0.00125 \text{ m}^2.
\end{aligned}$$

Then

$$\begin{aligned}
(\dot{S}_{m,F})_1/A &= 0.4 \times 40{,}000(\text{N/m}^2) \times 20(\text{m/s}) = 320{,}000 \text{ W/m}^2 \\
(\dot{S}_{m,F})_1 &= 320{,}000(\text{W/m}^2) \times 0.00125(\text{m}^2) \\
&= 400 \text{ W}.
\end{aligned}$$

From (3.169), using the properties for pure copper given in Table C.16, $\rho = 8933$ kg/m^3, $c_p = 385$ J/kg-K, $k = 401$ W/m-K, we have

$$\begin{aligned}
608.2(^\circ\text{C}) &= 20(^\circ\text{C}) + \frac{[-675(\text{W/m}^2) \times (0.00325)(\text{m}^2) + 400(\text{W})]}{8{,}933(\text{kg/m}^3) \times 385(\text{J/kg-K}) \times 6.25 \times 10^{-6}(\text{m}^2)} \times t \\
588.2(^\circ\text{C}) &= \frac{397.8}{21.5}(^\circ\text{C/s})t \\
t &= 31.8 \text{ s}.
\end{aligned}$$

COMMENT:

In order to verify the validity of a uniform temperature (in the presence of energy conversion) within the workpiece, we determine the internal conduction resistance, from Table 3.2, as

$$\begin{aligned}
R_{k,1} &= L/A_k k = l/A_c k \\
&= 0.005(\text{m})/[0.00125(\text{m}) \times 401(\text{W/m-K})] = 0.01 \text{ K/W}.
\end{aligned}$$

For the temperature difference between the top and bottom surface of the workpiece designated by ΔT, and allowing the entire energy conversion $\dot{S}_{m,F} = 400$ W to flow through the workpiece, we have

$$(\dot{S}_{m,F})_1 = \frac{\Delta T}{R_k}$$

or

$$\Delta T = \dot{S}_{m,F} R_k = 400(\text{W}) \times 0.01(\text{K/W}) = 4 \text{ K}.$$

Since we are dealing with a relatively high temporal temperature rise $T_1 - T_1(t=0) = 588.2$ K, this temperature variation across the workpiece is insignificant and the two surfaces can be assumed the same temperature.

PROBLEM 3.82.FUN

GIVEN:

The thermal stress in an idealized disc-brake rotor can be determined using some simplifying assumptions. The cast-iron rotor is shown in Figure Pr.3.82(a), along with a prescribed temperature distribution $T = T(r)$.

The temperature distribution is steady and one dimensional and the stress tensor would have planar, axisymmetric stresses given by principal components $\tau_{rr}(r), \tau_{\theta\theta}(r)$. It can be shown that these stresses are expressed as

$$\begin{aligned}
\tau_{rr}(r) &= -\frac{E_s\beta_s}{r^2}\int r(\tau-\tau_o)dr + a_1 - \frac{a_2}{r^2} \\
\tau_{\theta\theta}(r) &= \frac{E_s\beta_s}{r^2}\int r(T-T_o)dr - E_s\beta_s(T-T_o) + a_1 - \frac{a_2}{r^2} \\
\tau_{zz}(r) &= 0.
\end{aligned}$$

For cast iron, $E_s = 2\times10^{11}$ Pa, and β_s is listed in Table C.16. Also $R = 17$ cm, $T_o = 100$℃, $T_R = 400$℃.

SKETCH:

Figure Pr.3.82(a) shows the rotor and its temperature distribution.

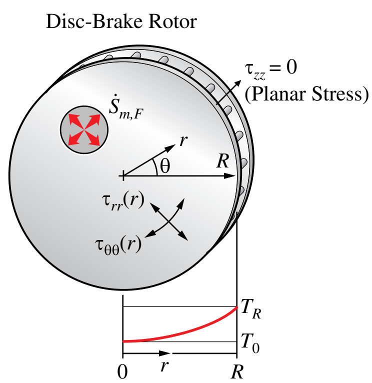


Figure Pr.3.82(a) The temperature distribution within a disc-brake rotor and the induced, planar axisymmetric thermal stresses.

OBJECTIVE:

(a) Determine the integration constants a_1 and a_2 using the mechanical conditions. Note that at $r = R$, there is no radial stress (for surface), and that at $r = 0$ the stresses should have a finite magnitude.

(b) Plot the distribution of the radial and tangential rotor thermal stresses with respect to the radial location.

SOLUTION:

(a) Using the temperature distribution

$$T = T_R + (T_o - T_R)\left(1 - \frac{r^2}{R^2}\right),$$

We have

$$T - T_o = (T_R - T_o)\frac{r^2}{R^2}.$$

Since at $r = 0$, the stresses have to be finite, we have $a_2 = 0$.

Performing the integrations, we have

$$\begin{aligned}
\tau_{rr}(r) &= -\frac{E_s\beta_s}{r^2}\int r(T_R - T_o)\frac{r^2}{R^2}dr + a_1 \\
&= -\frac{E_s\beta_s(T_R - T_o)r^2}{4R^2} + a_1 \\
\tau_{\theta\theta}(r) &= \frac{E_s\beta_s(T_R - T_o)r^2}{4R^2} - \frac{E_s\beta_s(T_R - T_o)r^2}{R^2} + a_1 \\
&= -\frac{3E_s\beta_s(T_R - T_o)r^2}{4R^2} + a_1
\end{aligned}$$

Using $\tau_{rr} = 0$ at $r = R$, we have

$$0 = -\frac{E_s\beta_s(T_R - T_o)R^2}{4R^2} + a_1$$

or

$$a_1 = \frac{E_s\beta_s(T_R - T_o)}{4}.$$

Then

$$\begin{aligned}
\tau_{rr}(r) &= \frac{E_s\beta_s(T_R - T_o)}{4}\left(1 - \frac{r^2}{R^2}\right) \\
\tau_{\theta\theta}(r) &= \frac{E_s\beta_s(T_R - T_o)}{4}\left(1 - 3\frac{r^2}{R^2}\right).
\end{aligned}$$

(b) From Table C.16, we have

$$\text{carbon steel:} \quad \beta_s = 1.15 \times 10^{-5} \text{ 1/K}.$$

Using the numerical results, we have

$$\begin{aligned}
\frac{E_s\beta_s(T_R - T_o)}{4} &= \frac{2 \times 10^{11}(\text{Pa}) \times 1.15 \times 10^{-5}(1/\text{K})}{4}(400 - 100)(\text{K}) \\
&= 1.725 \times 10^8 \text{ Pa}.
\end{aligned}$$

Figure 3.82(b) shows the variation of $\tau_{rr}(r)$ and $\tau_{\theta\theta}(r)$ with respect to r. Note that at $r = 0$, the two stresses are equal and at $r = R$, we have $\tau_{rr} = 0$, as expected.

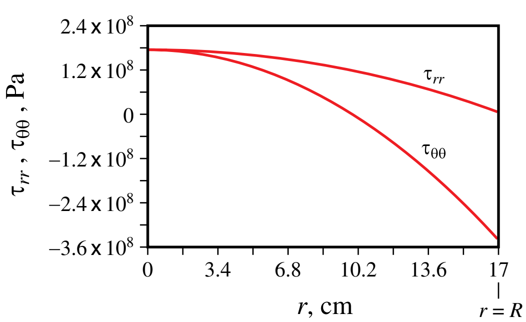


Figure Pr.3.82(b) Variation of $\tau_{rr}(r)$ and $\tau_{\theta\theta}(r)$ with respect to r.

COMMENT:

Note that the largest stress is for $\tau_{\theta\theta}$ and occurs at $r = R$.

PROBLEM 3.83.FUN

GIVEN:

Consider the thermal stress due to a nonuniform temperature $T = T(r)$ in an aluminum rod encapsulated in a glass shell. This is shown in Figure Pr.3.83. Since the thermal expansion coefficient β_s is much smaller for the fused silica glass (Figure 3.43), we assume that the periphery of the aluminum rod is ideally constrained. For the axisymmetric geometry and temperature-stress conditions we have here the stress and strain distributions as given by [5].

$$\begin{aligned}
\tau_{rr}(r) &= -\frac{E_s\beta_s}{r^2}\int r(T-T_o)dr + a_1 + \frac{a_2}{r^2}\\
\tau_{\theta\theta}(r) &= -\frac{E_s\beta_s}{r^2}\int r(T-T_o)dr - E_s\beta_s(T-T_o) + a_1 - \frac{a_2}{r^2}\\
\Delta R(r) &= \frac{\beta_s(1+\nu_{\rm P})}{r}\int r(T-T_o)dr + \frac{a_1(1-\nu_{\rm P})r}{E_s} - \frac{(1+\nu_{\rm P})a_2}{E_s r}.
\end{aligned}$$

The temperature distribution within the aluminum rod is estimated as

$$T(r) = T_R + (T_o - T_R)\left(1 - \frac{r^2}{R^2}\right).$$

The two constants of integration, a_1 and a_2, are determined using the mechanical conditions of a finite stress at $r = 0$ and an ideal constraint at $r = R$.

$\tau_{max,g} = -300$ MPa, $E_s = 68$ GPa, $\nu_{\rm P} = 0.25$, $T_o = 80$°C, $R = 4$ cm.

SKETCH:

Figure Pr.3.83 shows the aluminum rod encapsulated in a low thermal expansion coefficient glass.

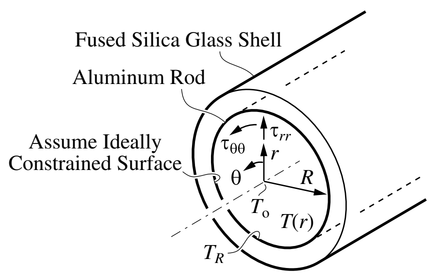


Figure Pr.3.83 Thermal stress induced in an aluminum rod encapsulated in a fused-silica glass shell.

OBJECTIVE:

(a) Determine the integration constants a_1 and a_2 and write the expression for $\tau_{rr}(r = R)$.

(b) Using the conditions given, determine the temperature T_R at which the ultimate compression stress of the glass $\tau_{max,g}$ is reached, i.e., $\tau_{rr}(r = R) = \tau_{max,g}$.

SOLUTION:

(a) We rewrite the temperature distribution as

$$T(r) = T_R + (T_o - T_R)\left(1 - \frac{r^2}{R^2}\right).$$

Since at $r = 0$, the stresses have to be finite, $a_2 = 0$.

Next we use the second mechanical condition, i.e., $\Delta R_r(r = R) = 0$, i.e.,

$$0 = \frac{\beta_s(1+\nu_{\rm P})}{R}\int_0^R r(T_R - T_o)\frac{r^2}{R^2}dr + \frac{a_1(1-\nu_{\rm P})R}{E_s}$$

or

$$0 = \frac{\beta_s(1+\nu_{\rm P})}{R^3}(T_R - T_o)\frac{1}{4}R^4 + \frac{a_1(1-\nu_{\rm P})R}{E_s}$$

or

$$a_1 = -\frac{E_s\beta_s(1+\nu_{\rm P})(T_R - T_o)}{4(1-\nu_{\rm P})}.$$

Using this in τ_{rr}, we have

$$\begin{aligned}
\tau_{rr}(r=R) &= -\frac{E_s\beta_s}{R^2}\int_0^R r(T_R - T_o)\frac{r^2}{R^2}dr - \frac{E_s\beta_s(1+\nu_{\rm P})(T_R - T_o)}{4(1-\nu_{\rm P})} \\
&= -\frac{E_s\beta_s(T_R - T_o)}{4}\left(1 + \frac{1+\nu_{\rm P}}{1-\nu_{\rm P}}\right).
\end{aligned}$$

(b) From Table C.16, we have

$$\beta_s = 2.25 \times 10^{-5}\ 1/\text{K} \qquad \text{Table C.16.}$$

Using the numerical values, we have

$$\begin{aligned}
-3 \times 10^8(\text{Pa}) &= -\frac{6.8 \times 10^{10}(\text{Pa}) \times 2.25 \times 10^{-5}(1/\text{K}) \times (T_R - 80)(\text{K})}{4} \times \left[1 + \frac{(1+0.25)}{(1-0.25)}\right] \\
&= -6.375 \times 10^5(T_R - 80) \\
T_R &= 550.6^\circ\text{C}.
\end{aligned}$$

COMMENT:

We did not determine $\tau_{\theta\theta}(r)$, but it can simply be done since a_1 and a_2 are known. Note that T_R is independent of R, but depends on the temperature distribution.

Chapter 4

Radiation

PROBLEM 4.1.FUN

GIVEN:

A piece of polished iron, with a surface area $A_r = 1$ m^2, is heated to a temperature $T_s = 1{,}100$℃.

OBJECTIVE:

(a) Determine the maximum amount of thermal radiation this surface can emit.
(b) Determine the actual amount of thermal radiation this surface emits. Interpolate the total emissivity from the values listed in Table C.18.
(c) What fraction of the radiation energy emitted is in the visible (λ between 0.39 and 0.77 μm), near infrared (λ between 0.77 and 25 μm), and far infrared (λ between 25 and 1,000 μm) ranges of the electromagnetic spectrum?

SOLUTION:

(a) The maximum amount of thermal radiation that a surface can emit is the blackbody emissive power $E_b(T_s)$, which is given by (4.6), i.e.,

$$E_b(T_s) = \sigma_{\rm SB} T_s^4.$$

For $T_s = (1{,}100 + 273.15)(\mathrm{K}) = 1{,}373.15$ K, we have

$$E_b = 5.67 \times 10^{-8}\ (\mathrm{W/m^2\text{-}K^4}) \times (1{,}373.15)^4(\mathrm{K^4}) = 201{,}584\ \mathrm{W/m^2}.$$

(b) For the polished iron, from Table C.18, we have $\epsilon_r \simeq 0.41$. Then the radiation emitted by the surface is given by (4.13), i.e.,

$$Q_{r,\epsilon} = \epsilon_r A_r E_b = 0.41 \times 1(\mathrm{m^2}) \times 210{,}584(\mathrm{W/m^2}) = 82{,}649\ \mathrm{W}.$$

(c) The fraction of radiation energy emitted in the visible range of the electromagnetic spectrum is given by (4.8), i.e.,

$$F_{\lambda_1 T\text{-}\lambda_2 T} = F_{0\text{-}\lambda_2 T} - F_{0\text{-}\lambda_1 T}.$$

For $T_s = 1{,}373.15$ K, with interpolation from Table 4.1, we have

$$\begin{aligned}\lambda_1 T &= 0.39(\mu\mathrm{m}) \times 1{,}373.15(\mathrm{K}) = 535.53\ \mu\mathrm{m\text{-}K}, \quad \text{then} \quad F_{0\text{-}\lambda_1 T} = 0\\ \lambda_2 T &= 0.77(\mu\mathrm{m}) \times 1{,}373.15(\mathrm{K}) = 1{,}057.33\ \mu\mathrm{m\text{-}K}, \quad \text{then} \quad F_{0\text{-}\lambda_2 T} = 0.0013.\end{aligned}$$

Then,

$$F_{0.39T\text{-}0.77T} = 0.0013 - 0 = 0.0013 = 0.13\%.$$

For the fraction of radiation energy emitted in the near infrared range of the electromagnetic spectrum, we have

$$\begin{aligned}\lambda_1 T &= 0.77(\mu\mathrm{m}) \times 1{,}373.15(\mathrm{K}) = 1{,}057.33\ \mu\mathrm{m\text{-}K}, \quad \text{then} \quad F_{0\text{-}\lambda_1 T} = 0.0013\\ \lambda_2 T &= 25(\mu\mathrm{m}) \times 1{,}373.15(\mathrm{K}) = 34{,}329\ \mu\mathrm{m\text{-}K}, \quad \text{then} \quad F_{0\text{-}\lambda_2 T} = 0.9968.\end{aligned}$$

Then,

$$F_{0.77T\text{-}25T} = 0.9968 - 0.0013 = 0.9955 = 99.55\%.$$

For the fraction of radiation emitted in the far infrared range of the spectrum we have

$$\begin{aligned}\lambda_1 T &= 25(\mu\mathrm{m}) \times 1{,}373.15(\mathrm{K}) = 34{,}329\ \mu\mathrm{m\text{-}K}, \quad \text{then} \quad F_{0\text{-}\lambda_1 T} = 0.9968\\ \lambda_2 T &= 1{,}000(\mu\mathrm{m}) \times 1{,}373.15(\mathrm{K}) = 1{,}373{,}000\ \mu\mathrm{m\text{-}K}, \quad \text{then} \quad F_{0\text{-}\lambda_2 T} = 1.\end{aligned}$$

Then,

$$F_{25T\text{-}1000T} = 1 - 0.9968 = 0.0032 = 0.32\%.$$

COMMENT:

The results show that practically 100% of the thermal radiation emitted by a surface at $T_s = 1{,}110$℃ is emitted in the near infrared range of the spectrum (0.77 μm $\leq \lambda \leq$ 25 μm).

PROBLEM 4.2.FUN

GIVEN:

A blackbody radiation source at $T_i = 500$℃ is used to irradiate three different surfaces, namely, (i) aluminum (commercial sheet), (ii) nickel oxide, and (iii) paper. The irradiating surfaces have an area $A_r = 1$ m^2 and are assumed gray, diffuse, and opaque. Use the emissivities in Table C.18 [for (ii) extrapolate; for others use the available data].

OBJECTIVE:

(a) Sketch the radiation heat transfer arriving and leaving the surface, showing the black-body emitter and the irradiated surface [see Figure 4.9(b)]. Show heat transfer as irradiation $Q_{r,i}$, absorption $Q_{r,\alpha}$, reflection $Q_{r,\rho}$, emission $Q_{r,\epsilon}$, and radiosity $Q_{r,o}$.
(b) If the three surfaces are kept at $T_s = 100$℃, determine the amounts of reflected and absorbed energies.
(c) Determine the rates of energy emitted and the radiosity for each surface.
(d) Determine the net radiation heat transfer rate for each surface. Which surface experiences the highest amount of radiation heating?

SOLUTION:

(a) Figure Pr.4.2 shows the radiation energy arriving and leaving the surface. From (4.14), the reflected radiation is given by

$$Q_{r,\rho} = A_r \rho_r q_{r,i},$$

where $q_{r,i}$ is the irradiation flux impinging on the surface. All this irradiation arrives from emission by a blackbody surface at $T_i = 773$ K. Then using (4.13), we have

$$Q_{r,\rho} = A_r \rho_r \sigma_{\rm SB} T_i^4.$$

The absorbed energy is given by (4.14), i.e.,

$$Q_{r,\alpha} = A_r \alpha_r \sigma_{\rm SB} T_i^4.$$

Since the surfaces are opaque ($\tau_r = 0$), we have from (4.19)

$$\alpha_r + \rho_r = 1.$$

Assuming that the surfaces are gray, we have from (4.20)

$$\alpha_r = \epsilon_r.$$

Then the reflected and absorbed energy can be rewritten as

$$Q_{r,\rho} = A_r(1 - \epsilon_r)\sigma_{\rm SB} T_i^4$$

$$Q_{r,\alpha} = A_r \epsilon_r \sigma_{\rm SB} T_i^4.$$

(b) Surface 1: Aluminum, Commercial Sheet
From Table C.18, for $T = 373$ K (it is the only data available), $\epsilon_r = 0.09$. Thus

$$\begin{aligned} Q_{r,\rho} &= 1({\rm m}^2) \times (1 - 0.09) \times 5.67 \times 10^{-8}({\rm W/m^2\text{-}K^4}) \times (773.15)^4({\rm K}^4) = 18{,}437 \text{ W} \\ Q_{r,\alpha} &= 1({\rm m}^2) \times 0.09 \times 5.67 \times 10^{-8}({\rm W/m^2\text{-}K^4}) \times (773.15)^4({\rm K}^4) = 1{,}823 \text{ W}. \end{aligned}$$

Surface 2: Nickel Oxide
From Table C.18, for $T = 373.15$ K (an extrapolation of the data available is possible), $\epsilon_r = 0.345$. Then

$$\begin{aligned} Q_{r,\rho} &= 1 \times (1 - 0.345) \times 5.67 \times 10^{-8} \times (773.15)^4 = 13{,}220 \text{ W} \\ Q_{r,\alpha} &= 1 \times 0.345 \times 5.67 \times 10^{-8} \times (773.15)^4 = 6{,}990 \text{ W}. \end{aligned}$$

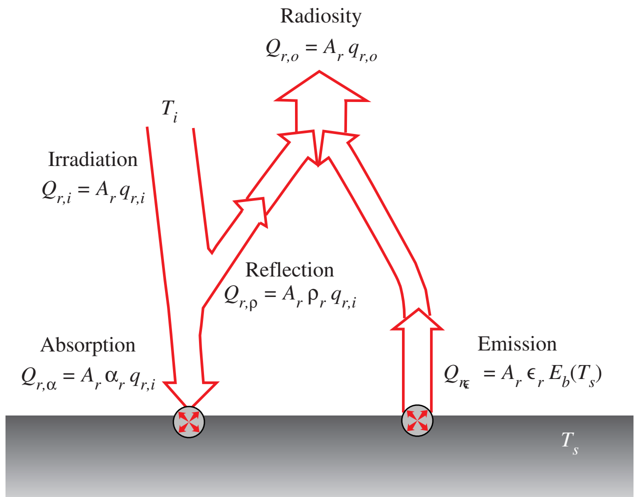


Figure Pr.4.2 An opaque, diffuse surface at temperature T_s, irradiated by a blackbody surface at temperature T_i.

Surface 3: Paper
From Table C.18, for $T = 308$ K (the data available is limited), $\epsilon_r = 0.95$. Then

$$Q_{r,\rho} = 1 \times (1 - 0.95) \times 5.67 \times 10^{-8} \times (773.15)^4 = 1{,}012 \text{ W}$$
$$Q_{r,\alpha} = 1 \times 0.95 \times 5.67 \times 10^{-8} \times (773.15)^4 = 19{,}247 \text{ W}.$$

(c) Surface 1:
From (4.13), we have the surface emission as

$$\begin{aligned} Q_{r,\epsilon} &= A_r \epsilon_r \sigma_{\rm SB} T_s^4 \\ &= 1(\text{m}^2) \times 0.09 \times 5.67 \times 10^{-8} (\text{W/m}^2\text{-K}^4) \times (373.15)^4 (\text{K}^4) = 98.94 \text{ W}. \end{aligned}$$

From (4.22), we have the radiosity as

$$\begin{aligned} Q_{r,o} &= Q_{r,\rho} + Q_{e,\epsilon} \\ &= 18{,}437(\text{W}) + 98.94(\text{W}) = 18{,}536 \text{ W}. \end{aligned}$$

Surface 2:
$$\begin{aligned} Q_{r,\epsilon} &= 1 \times 0.345 \times 5.67 \times 10^{-8} \times (373.15)^4 = 379.3 \text{ W} \\ Q_{r,o} &= 13{,}270(\text{W}) + 379(\text{W}) = 13{,}649 \text{ W}. \end{aligned}$$

Surface 3:
$$\begin{aligned} Q_{r,\epsilon} &= 1 \times 0.95 \times 5.67 \times 10^{-8} \times (373.15)^4 = 1{,}044 \text{ W} \\ Q_{r,o} &= 1{,}013(\text{W}) + 1{,}044(\text{W}) = 2{,}057 \text{ W}. \end{aligned}$$

(d) The net radiation heat transfer is given by (4.24), i.e.,

$$\begin{aligned} Q_r &= Q_{r,o} - Q_{r,i} \\ Q_{r,i} &= A_r q_{r,i} = A_r \sigma_{\rm SB} T_i^4 \end{aligned}$$

Surface 1: $Q_r = 18{,}536(\text{W}) - 1(\text{m}^2) \times 5.67 \times 10^{-8} (\text{W/m}^2\text{-K}^4) \times (773.15)^4 (\text{K}^4) = -1{,}724 \text{ W}.$

Surface 2: $Q_r = 13{,}649(\text{W}) - 20{,}260(\text{W}) = -6{,}611 \text{ W}.$

Surface 3: $Q_r = 2{,}057(\text{W}) - 20{,}260(\text{W}) = -18{,}203 \text{ W}.$

COMMENT:

Examining the net radiation heat transfer to the three surfaces, we note that surface 3 (paper) is the most heated by radiation (net radiation heat transfer into this surface, which is negative, as the largest magnitude). The emissivity data, as a function of temperature is not available for most materials. Usually, it becomes necessary to estimate a value from limited data. This increases the uncertainty in the analysis of radiation heat transfer. For applications that need more accuracy, the measurement of the radiation properties, for the surfaces at the temperatures of interest, may be required.

PROBLEM 4.3.FUN

GIVEN:

Three different surfaces are heated to a temperature $T_s = 800°\text{C}$. The total radiation heat flux leaving these surfaces (i.e., the radiosity) is measured with a calorimeter and the values $(q_{r,o})_1 = 6{,}760\ \text{W/m}^2$, $(q_{r,o})_2 = 21{,}800\ \text{W/m}^2$, and $(q_{r,o})_3 = 48{,}850\ \text{W/m}^2$ are recorded.

Assume that the reflected radiation is negligible compared to surface emission and that the surfaces are opaque, diffuse, and gray.

OBJECTIVE:

(a) Determine the total emissivity for each of these surfaces.

(b) Comment on the importance of considering the surface reflection in the measurement of surface emissivity ϵ_r.

SOLUTION:

The radiant heat flux leaving a surface is the sum of the emitted and the reflected radiation, i.e.,

$$A_r q_{r,o} = A_r q_{r,\epsilon} + A_r \rho_r q_{r,i}.$$

(a) Assuming that the reflection part is much smaller than the emitted part, and using the Stefan-Boltzmann law (4.6), we have

$$q_{r,o} = q_{r,\epsilon} = \epsilon_r \sigma_{\rm SB} T_s^4 \quad \text{for } q_{r,\epsilon} \gg q_{r,\rho} = \rho_r q_{r,i}.$$

This equation is then used to find ϵ_r. For each of the surfaces we have

Surface 1: $(q_{r,o})_1 = 6{,}760\ \text{W/m}^2$

$$\epsilon_{r,1} = \frac{(q_{r,o})_1}{\sigma_{\rm SB} T_s^4} = \frac{6{,}760}{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (1{,}073.15)^4(\text{K}^4)} = 0.09,$$

Surface 2: $(q_{r,o})_2 = 21{,}800\ \text{W/m}^2$

$$\epsilon_{r,2} = \frac{(q_{r,o})_2}{\sigma_{\rm SB} T_s^4} = \frac{21{,}800}{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (1{,}073.15)^4(\text{K}^4)} = 0.29,$$

Surface 3: $(q_{r,o})_2 = 48{,}850\ \text{W/m}^2$

$$\epsilon_{r,3} = \frac{(q_{r,o})_3}{\sigma_{\rm SB} T_s^4} = \frac{48{,}850}{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (1{,}073.15)^4(\text{K}^4)} = 0.65.$$

(b) When $q_{r,\rho}$ cannot be neglected, then the irradiation flux $q_{r,i}$ as well as the reflectivity ($\rho_r = 1 - \epsilon_r$, for opaque, gray surfaces) must be included. If the irradiation heat flux $q_{r,i}$ or ρ_r are large, the above procedure for the determination of ϵ_r does not lead to accurate results.

COMMENT:

Emissivities of the order of 0.1 are characteristic of commercial aluminum. The other two emissivities are found, for example, for refractory brick and opaque quartz, both of which are ceramics.

PROBLEM 4.4.FAM

GIVEN:

In an incandescent lamp, the electrical energy is converted to the Joule heating in the thin-wire filament and this is in turn converted to thermal radiation emission. The filament is at $T = 2{,}900$ K, and behaves as an opaque, diffuse, and gray surface with a total emissivity $\epsilon_r = 0.8$.

OBJECTIVE:

Determine the fractions of the total radiant energy and the amount of emitted energy in the (a) visible, (b) near infrared, and (c) the remaining ranges of the electromagnetic spectrum.

SOLUTION:

The radiation energy emitted by the filament is given by (4.13), i.e.,

$$q_{r,\epsilon} = \epsilon_r \sigma_{\rm SB} T^4.$$

The fraction of radiant energy emitted over a wavelength range between λ_1 and λ_2, for a gray surface, is given by (4.8), i.e.,

$$(q_{r,\epsilon})_{\lambda_1\text{-}\lambda_2} = F_{\lambda_1 T\text{-}\lambda_2 T}\epsilon_r \sigma_{\rm SB} T^4.$$

(a) For the visible range of the spectrum, $\lambda_1 = 0.39$ μm and $\lambda_2 = 0.77$ μm, then,

$$F_{\lambda_1 T\text{-}\lambda_2 T} = F_{0\text{-}\lambda_2 T} - F_{0\text{-}\lambda_1 T}.$$

From Table 4.1,

$$\begin{aligned} F_{0\text{-}\lambda_2 T} &= F_{0\text{-}2233} = 0.10738 \\ F_{0\text{-}\lambda_1 T} &= F_{0\text{-}1131} = 0.002766. \end{aligned}$$

Then

$$\begin{aligned} (q_{r,\epsilon})_{\rm visible} &= (0.10738 - 0.002766) \times 0.8 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times 2{,}900^4(\text{K}^4) \\ &= 3.356 \times 10^5 \ \text{W/m}^2. \end{aligned}$$

(b) For the near infrared range of the spectrum, $\lambda_1 = 0.77$ μm, and $\lambda_2 = 25$ μm. Then from Table 4.1

$$F_{0\text{-}\lambda_2 T} = F_{0\text{-}72{,}500} \simeq 1.0.$$

Therefore,

$$\begin{aligned} (q_{r,\epsilon})_{\text{near infrared}} &= (1 - 0.10738) \times 0.8 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times 2{,}900^4(\text{K}^4) \\ &= 2.864 \times 10^6 \ \text{W/m}^2. \end{aligned}$$

(c) For the remaining range of the spectrum, we have

$$\begin{aligned} (q_{r,\epsilon})_{\rm remaining} &= 0.002766 \times 0.8 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times 2{,}900^4(\text{K}^4) \\ &= 8{,}874 \ \text{W/m}^2. \end{aligned}$$

COMMENT:

Note that most of the radiant energy is emitted in the near infrared range of the spectrum. A relatively small fraction (10.46 percent) is in the visible portion of the spectrum, and nearly none in the ultraviolet and far infrared ranges.

PROBLEM 4.5.FUN

GIVEN:

The equation of radiative transfer describes the change in the radiation intensity I_r as it experiences local scattering, absorption, and emission by molecules or larger particles. In the absence of a significant local emission, and by combining the effects of scattering and absorption into a single volumetric radiation property, i.e., the extinction coefficient σ_{ex} (1/m), we can describe the ability of the medium to attenuate radiation transport across it. Under this condition, we can write the equation of radiative transport for a one-dimensional volumetric radiation heat transfer as

$$\frac{dI_\lambda}{dx} = -\sigma_{ex} I_\lambda \quad \text{radiative transport for nonemitting media}$$

or by integrating this over all the wavelength and solid angles (as indicated in the second footnote of Section 4.1.2), we have arrive at the radiation heat flux q_r

$$\frac{dq_r}{dx} = -\sigma_{ex} q_r.$$

SKETCH:

Figure Pr.4.5 shows the attenuation in a medium with a portion of the incoming radiation $q_{r,i}$ reflected on the surface ($x = 0$) and the remaining entering the medium.

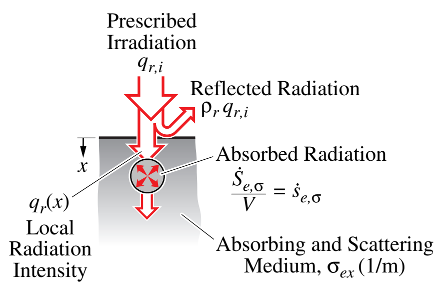


Figure Pr.4.5 Radiation attenuating (absorbing and scattering) medium with a surface ($x = 0$) reflection.

OBJECTIVE:

(a) Integrate this equation of radiative transfer, using $q_r(x = 0) = q_{r,i}(1 - \rho_r)$, where ρ_r is the surface reflectivity, as shown in Figure Pr.4.5, and show that

$$q_r(x = 0) = q_{r,i}(1 - \rho_r)e^{-\sigma_{ex}x}.$$

(b) Starting from (2.1), and assuming one-dimensional, volumetric radiation heat transfer only, show that

$$q_{r,i}(1 - \rho_r)\sigma_{ex} e^{-\sigma_{ex}x} = -\dot{s}_{e,\sigma} = -\frac{\dot{S}_{e,\sigma}}{V},$$

which is also given by (2.43), when we note that the attenuation of radiation is represented by $\dot{s}_{e,\sigma}$ as a source of energy.

SOLUTION:

(a) We begin with the equation of radiative transfer

$$\frac{dq_r}{dx} = -\sigma_{ex} q_r$$

and integrate this once to obtain

$$q_r = a_1 e^{-\sigma_{ex}x}.$$

Then we use the condition at the surface, $x = 0$, and we have

$$q_r(x = 0) = q_{r,i}(1 - \rho_r).$$

or

$$q_r(x) = q_{r,i}(1-\rho_r)e^{-\sigma_{ex}x}.$$

(b) Starting from (2.1), for volumetric radiation only and under steady-state heat transfer, we have

$$\nabla \cdot q_r = \frac{d}{dx}q_r(x) = \dot{s}_{e,\sigma}.$$

using the solution for $q_r(x)$ we have

$$\frac{d}{dx}q_{r,i}(1-\rho_r)e^{-\sigma_{ex}x} = \dot{s}_{e,\sigma}$$

or

$$q_{r,i}(1-\rho_i)\frac{d}{dx}e^{-\sigma_{ex}x} = \dot{s}_{e,\sigma}$$

or

$$-q_{r,i}(1-\rho_r)\sigma_{ex}e^{-\sigma_{ex}x} = \dot{s}_{e,\sigma}.$$

We note that the attenuation of radiation, which shows a net heat transfer into the differential volume, is represented on the right-hand side of the energy equation as positive $\dot{s}_{e,\sigma}$ (i.e., as a source). Therefore, we have used a negative sign in the definition for $\dot{s}_{e,\sigma}$ given in Table 2.1, and $\dot{s}_{e,\sigma}$ is positive.

COMMENT:

Attenuation of radiation heat flux could have been included in the divergence of $\mathbf{q}_r$ if we had given a more general description of $\mathbf{q}_r$. This requires the introduction of a general equation of radiative transfer which is left to the advance studies.

Also note that $\sigma_{ex} = 1/\lambda_{ph}$, where λ_{ph} is the phonon mean-free path. The large magnitude for σ_{ex} (large attenuation and small λ_{ph}) gives rise to a significant absorption of irradiation. Figure 2.13 gives examples of σ_{ex}(1/m) for various absorbing-scattering media.

PROBLEM 4.6.FUN

GIVEN:

When the optical thickness defined by (2.44), $\sigma_{ex}^* = \sigma_{ex} L$, for a heat transfer medium of thickness L and extinction coefficient σ_{ex} is larger than 10, the emission and transfer of radiation can be given by the radiation heat flux as (for a one-dimensional heat flow)

$$q_{r,x} = -\frac{16}{3}\frac{\sigma_{\rm SB} T^3}{\sigma_{ex}}\frac{dT}{dx} \quad \text{diffusion approximation for optically thick } (\sigma_{ex}^* > 10) \text{ heat transfer media.}$$

This is called the diffusion approximation.

The equation of radiation transfer, for an emitting medium with a strong absorption, becomes

$$\frac{dI_{r,b}}{d(x/\cos\theta)} = -\sigma_{ex} I_r + \sigma_{ex} I_{r,b}, \quad \pi I_{r,b} = E_{r,b} = \sigma_{\rm SB} T^4,$$

where $x/\cos\theta$ is the photon path as it travels between surfaces located at x and $x + \Delta x$, as shown in Figure Pr.4.6.

The radiation heat flux is found by the integration of I_r over a unit sphere, i.e.,

$$\mathbf{q}_r = \int_0^{2\pi}\int_0^{\pi} \mathbf{s} I_r \cos\theta \sin\theta d\theta d\phi.$$

Note that this integral is over a complete sphere. Also note that $I_{r,b}$ is independent of θ and ϕ. For the x direction, using Figure Pr.4.6, we have $q_{r,x} = \int_0^{2\pi}\int_0^{\pi} I_r \cos\theta \sin\theta d\theta d\phi$.

SKETCH:

Figure Pr.4.6 shows the geometry considered and the angles used.

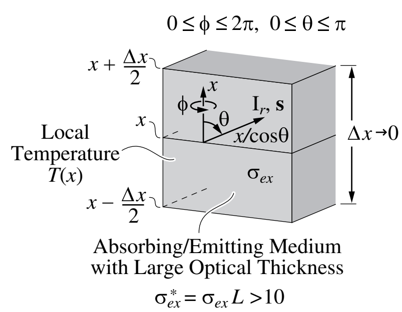


Figure Pr.4.6 Radiation intensity I_r traveling in an optically thick, emitting medium.

OBJECTIVE:

Using the equation of radiative transfer and the definition of $\mathbf{q}_r$, both in the given, derive the given expression for $q_{r,x}$ for the diffusion approximation.

SOLUTION:

The equation of radiative transfer can be rearranged as

$$I_r = \frac{dI_{r,b}\cos\theta}{\sigma_{ex} dx} + I_{r,b}.$$

Upon integration, we have for $q_{r,x}$

$$q_{r,x} = \int_0^{2\pi}\int_0^{\pi}\left(-\frac{dI_r \cos\theta}{\sigma_{ex} dx} + I_{r,b}\right)\cos\theta d\sin\theta d\theta d\phi.$$

Now, noting that $I_{r,b}$ is independent of θ and ϕ (as discussed in Section 4.1.2) we have

$$\begin{aligned} q_{r,x} &= -\frac{dI_{r,b}}{\sigma_{ex}dx}\int_0^{2\pi}\int_0^{\pi}\cos^2\theta\sin\theta d\theta d\phi \\ &+ I_{r,b}\int_0^{2\pi}\int_0^{\pi}\cos\theta\sin\theta d\theta d\phi \\ &= -\frac{dI_{r,b}}{\sigma_{ex}dx}\times\frac{4}{3}\pi + I_{r,b}\times 0 \\ &= -\frac{4\pi}{3\sigma_{ex}}\frac{dI_{r,b}}{dx}. \end{aligned}$$

Now, using $\pi I_{r,b} = E_b = \sigma_{\rm SB}T_1^4$, we have

$$q_{r,x} = -\frac{4}{3\sigma_{ex}}\frac{dE_b}{dx} = -\frac{4\sigma_{\rm SB}}{3\sigma_{ex}}\frac{dT^4}{dx} = -\frac{16}{3}\frac{\sigma_{\rm SB}T^3}{\sigma_{ex}}\frac{dT}{dx}.$$

COMMENT:

In replacing dI_r/dx with $dI_{r,b}/dx$ in the equation of radiative transfer, we are eliminating all the distant radiation heat transfer effects by approximating the local intensity as

$$I_r = I_{r,b} - \frac{\cos\theta}{\sigma_{ex}}\frac{dI_{r,b}}{dx}.$$

Note that E_r is the integral of I_r over a unit hemisphere as discussed in Section 4.1.2.

PROBLEM 4.7.FAM

GIVEN:

The human eye is sensitive to the visible range of the photon wavelength and has a threshold for detection of about $E_{b,\lambda} = 0.0936$ W/m-μm at wavelength of $\lambda = 0.77$ μm (largest wavelength in the red band, Figure 4.1). This corresponds to the Draper point in Figure 4.2(a).

OBJECTIVE:

(a) The turtle eye is sensitive to the infrared range and if the threshold for detection is the same, but at $\lambda = 1.5$ μm, determine the corresponding temperature at which the turtle can detect blackbody emission.
(b) Using this temperature, at what wavelength would $E_{b,\lambda}$ peak?
(c) Would this turtle eye be able to detect radiation emission from a tank containing liquid water at one atm pressure (Table C.3)?

SOLUTION:

(a) From (4.2), we have

$$\begin{aligned} E_{b,\lambda}(T,\lambda) &= \frac{a_1}{\lambda^5(e^{a_2/\lambda T}-1)} \\ &= 0.0936 \text{ W/m}^2\text{-}\mu\text{m}, \end{aligned}$$

$$a_1 = 3.742 \times 10^8 \text{ W-}\mu\text{m}^4/\text{m}^2, \quad a_2 = 1.439 \times 10^4 \ \mu\text{m-K}, \quad \lambda = 1.5 \ \mu\text{m}.$$

$$\begin{aligned} e^{a_2/\lambda T} &= 1 + \frac{a_1}{E_{b,\lambda}\lambda^5} \\ \frac{a_2}{\lambda T} &= \ln\left(1 + \frac{a_1}{E_{b,\lambda}\lambda^5}\right) \\ T &= \frac{a_2}{\lambda \ln\left(1 + \dfrac{a_1}{E_{b,\lambda}\lambda^5}\right)}. \end{aligned}$$

Using the numerical values, we have

$$\begin{aligned} T &= \frac{1.439 \times 10^4(\mu\text{m-K})}{1.5(\mu\text{m}) \ln\left[1 + \dfrac{3.742 \times 10^8(\text{W-}\mu\text{m}^4/\text{m}^2)}{0.0936(\text{W/m}^2\text{-}\mu\text{m}) \times (1.5)^5(\mu\text{m}^5)}\right]} \\ &= \frac{9.593 \times 10^3(\text{K})}{\ln(1 + 5.265 \times 10^8)} = \frac{9.593 \times 10^3(\text{K})}{2.008 \times 10^1} \\ &= 477.7 \text{ K}. \end{aligned}$$

(b) From Figure 4.2(a), we have $E_{b,\lambda}$ having its maximum value at $\lambda_{max}T = 2{,}898$ μm-K.
Then

$$\begin{aligned} \lambda_{max} &= \frac{2{,}898(\mu\text{m-K})}{477.7(\text{K})} \\ &= 6.067 \ \mu\text{m}. \end{aligned}$$

(c) The boiling point is the highest temperature that the liquid will have. From Table C.3, we have for water at one atm pressure,

$$T_{lg} = 373.2 \text{ K} < 477.7 \text{ K}$$

Therefore, the emission from this tank is not detectable by the turtle eye.

COMMENT:

From Figure 4.2(a), note that for a given $E_{b,\lambda}$ and T, there are two wavelength ranges, one is the short and one is the long wavelength range. Here we have selected the short wavelength. Also, it should be kept in mind that λ is multivalued when solving for λ as the unknown.

PROBLEM 4.8.FUN

GIVEN:

Dielectrics, e.g., ceramics such as SiC, have very small extinction index κ and also small refraction index n (optical properties). On the other hand metals have large κ and n. Figure Pr.4.8 shows the interface between two media, 1 and 2, which have different optical properties.

The normal (i.e., $\theta = 0$) emissivity, for the dielectrics and metals, is predicted using these optical properties and a simple relation given by

$$\epsilon_r = \frac{4\dfrac{n_2}{n_1}}{\left(\dfrac{n_2}{n_1}+1\right)^2 + \kappa^2},$$

where, as shown in Figure 4.6, 1 and 2 (or i and j) refer to the two media and here we use air as media 1 (with $n_1 = 1$).

The measured values of n and κ at $\lambda = 5$ μm are given as

$$\begin{aligned}
\text{SiC}: &\quad n_2 = 2.4, \quad \kappa = 0.07, \quad \lambda = 5\ \mu\text{m}\\
\text{Al}: &\quad n_2 = 9, \quad \kappa = 65, \quad \lambda = 5\ \mu\text{m}\\
\text{air}: &\quad n_1 = 1.
\end{aligned}$$

SKETCH:

Figure Pr.4.8 shows the interface between two media with different optical properties.

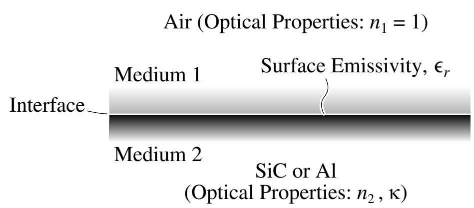


Figure Pr.4.8 Interface between two media with different optical properties.

OBJECTIVE:

(a) Determine the total hemispherical emissivity ϵ_r for SiC and Al in contact with air.

(b) Compare these values with the measured values of ϵ_r given in Table C.18.

SOLUTION:

(a) Using the above relation, we have

$$\text{SiC:} \quad \epsilon_r = \frac{4 \times \dfrac{2.4}{1}}{\left(\dfrac{2.4}{1}+1\right)^2 + (0.07)^2} = 0.8301$$

$$\text{Al:} \quad \epsilon_r = \frac{4 \times \dfrac{9}{1}}{\left(\dfrac{9}{1}+1\right)^2 + (65)^2} = 0.008324.$$

(b) From Table C.18, we have

$$\begin{aligned}
\text{SiC:} \quad \epsilon_r &= 0.83 \quad \text{to} \quad 0.96\\
\text{Al:} \quad \epsilon_r &= 0.008324 \quad \text{to} \quad 0.18 \quad \text{(fine to rough polish)}.
\end{aligned}$$

While the results of SiC are close to what is given in Table C.18, the predicted value for Al is for an ideally polished surface.

COMMENT:

Although the general trend of high ϵ_r for dielectrics and smaller ϵ_r for the metals are predicted, for more accuracy the wavelength dependence of n and κ should be included and then integrated as indicated by (4.11). Also note that for $n_2 = 1$ and $\kappa_2 = 0$, we obtain $\epsilon_r = \alpha_r = 1$, from the relation given above. This indicates that the photons would continue to travel through the interface when this surface does not mark any change in the optical properties. As stated in Figure 4.6, n and κ are related to the three fundamental electromagnetic properties of matter.

PROBLEM 4.9.FAM

GIVEN:

Some surface materials and coatings are selected for their radiation properties, i.e., their emissivity ϵ_r and absorptivity α_r. Consider the following selections based on surface radiation properties. All surfaces are at $T_s = 300$ K.

(i) Space suit (α_r for low heat absorption).

(ii) Solar collector surface (α_r for high heat absorption and ϵ_r for low heat emission).

(iii) Surface of thermos (α_r for low heat absorption).

OBJECTIVE:

(a) Choose the materials for the applications (i) to (iii) from Table C.19.

(b) Determine the emissive power for the selected surfaces (i) to (iii).

(c) Determine the surface reflectivity for the selected surfaces (i) to (iii), assuming no transmission (opaque surface).

SOLUTION:

(a) In Table C.19 we search for the closest match.

(i) For low solar absorption, from Table C.19, we choose coatings such as white potassium zirconium silicate, $\alpha_r = 0.13$, $\epsilon_r = 0.89$.

(ii) For the solar collector surface coating, from Table C.19, we choose black oxidized copper, $\epsilon_r = 0.16$, $\alpha_r = 0.91$, a highly nongray (selective) surface.

(iii) For the surface of the thermos, we can choose from Table C.19, aluminum foil, $\epsilon_r = 0.025$, $\alpha_r = 0.10$.

(b) The emissive power is given by (4.13) as

$$E_{r,s} = \epsilon_{r,s}\sigma_{\rm SB}T_s^4.$$

Then

(i) white potassium zirconium silicate:

$$\begin{aligned} E_{r,s} &= 0.89 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (300)^4(\text{K}^4) \\ &= 408.8 \text{ W/m}^2 \end{aligned}$$

(ii) black-oxidized copper:

$$\begin{aligned} E_{r,s} &= 0.16 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (300)^4(\text{K}^4) \\ &= 73.48 \text{ W/m}^2 \end{aligned}$$

(iii) aluminum foil:

$$\begin{aligned} E_{r,s} &= 0.025 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (300)^4(\text{K}^4) \\ &= 11.48 \text{ W/m}^2. \end{aligned}$$

(c) From (4.15), we have for opaque surfaces (written for the total quantities)

$$\begin{aligned} \alpha_r + \rho_r + \tau_r &= 1 \\ \alpha_r + \rho_r &= 1, \quad \text{for} \quad \tau_r = 0 \quad \text{(opaque surface)}. \\ \rho_r = 1 - \alpha_r. & \end{aligned}$$

Then

(i) Write potassium zirconium silicate:

$$\rho_r = 1 - 0.13 = 0.87$$

(ii) black-oxidized copper:

$$\rho_r = 1 - 0.91 = 0.09$$

(iii) aluminum foil:

$$\rho_r = 1 - 0.1 = 0.90.$$

COMMENT:

Note that some of these materials are highly nongray (i.e., α_r and ϵ_r are vastly different).

PROBLEM 4.10.FAM

GIVEN:

The view factors between two surfaces making up part of an enclosure are given for some geometries in Table 4.2 and Figures 4.11(a) to (e).

SKETCH:

Figures Pr.4.10(a) to (e) show five surface pairs for which the view factors are sought.

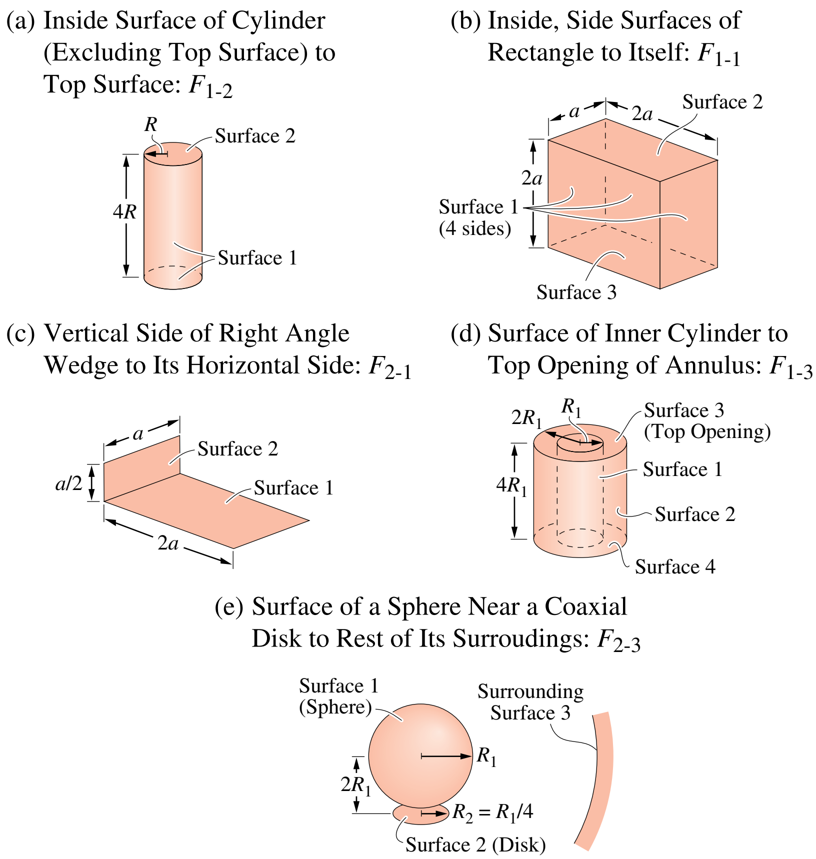


Figure Pr.4.10(a) to (e) View factors between surface pairs for five different surface pairs in different geometries.

OBJECTIVE:

Determine the view factors ($F_{i\text{-}j}$, with i and j specified for each case on top of the figures) for the five surface pairs shown in Figures Pr.4.10(a) to (e).

Note that, for the geometry shown in Figure Pr.4.10(a), the view factor can be found using only the summation and the reciprocity rules (4.33) and (4.34), and by using simple inspection (i.e., no tables or figures are needed) of the limiting view factor (i.e., surfaces that are completely enclosed by another surface).

SOLUTION:

(a) To determine $F_{1\text{-}1}$, we begin by noting that $F_{2\text{-}1}$ (from the disk to the rest of the cylinder) is equal to unity i.e., $F_{2\text{-}1} = 1$.

Now we return to surface 1 and use the summation rule (4.33), i.e.,

$$F_{1\text{-}1} + F_{1\text{-}2} = 1 \quad \text{summation rule}$$

or

$$F_{1\text{-}1} = 1 - F_{1\text{-}2}.$$

To determine $F_{1\text{-}2}$, we use the reciprocity rule (4.34), i.e.,

$$A_{r,1}F_{1\text{-}2} = A_{r,2}F_{2\text{-}1} \quad \text{reciprocity rule}$$

or

$$\begin{aligned} F_{1\text{-}2} &= \frac{A_{r,2}}{A_{r,1}}F_{2\text{-}1} \\ &= \frac{\pi R^2}{2\pi R(4R) + \pi R^2}F_{2\text{-}1} \\ &= \frac{1}{8+1}F_{2\text{-}1} \\ &= \frac{1}{9} \times 1 = \frac{1}{9}. \end{aligned}$$

Note that

$$F_{1\text{-}1} = 1 - \frac{1}{9} = \frac{8}{9}.$$

Note that the lower disk area is part of surface 1.

(b) To determine $F_{1\text{-}1}$, we begin by writing the summation rule for surface 1, i.e.,

$$F_{1\text{-}1} + F_{1\text{-}2} + F_{1\text{-}3} = 1 \quad \text{summation rule}$$

or

$$F_{1\text{-}1} = 1 - 2F_{1\text{-}2},$$

where we have used the symmetry to write $F_{1\text{-}2} = F_{1\text{-}3}$.
To determine $F_{1\text{-}2}$, we use the reciprocity rule

$$F_{1\text{-}2} = \frac{A_{r,2}}{A_{r,1}}F_{2\text{-}1}, \quad \text{reciprocity rule.}$$

To determine $F_{2\text{-}1}$, we use the summation rule for surface 2, i.e.,

$$F_{2\text{-}1} + F_{2\text{-}2} + F_{2\text{-}3} = 1 \quad \text{summation rule}$$

or

$$F_{2\text{-}1} = 1 - F_{2\text{-}3},$$

where $F_{2\text{-}2} = 0$, because it is planar.
We determine $F_{2\text{-}3}$ from Figure 4.11(b) for

$$\begin{aligned} \frac{w}{l} &= \frac{2a}{2a} = 1, \quad \frac{a}{l} = \frac{a}{2a} = \frac{1}{2} \\ F_{2\text{-}3} &\simeq 0.12 \quad \text{Figure 4.11(b).} \end{aligned}$$

Then

$$\begin{aligned} F_{1\text{-}1} &= 1 - 2F_{1\text{-}2} = 1 - 2\frac{A_{r,2}}{A_{r,1}}F_{2\text{-}1} = 1 - 2\frac{A_{r,2}}{A_{r,1}}(1 - F_{2\text{-}3}) \\ &= 1 - 2\frac{2a^2}{2(4a^2) + 2(2a^2)}(1 - 0.12) = 1 - \frac{1}{3}(1 - 0.12) = 0.7067. \end{aligned}$$

(c) To determine $F_{1\text{-}2}$, we use Figure 4.11(c) and the reciprocity rule, i.e.,

$$\begin{aligned} F_{2\text{-}1} &= \frac{A_{r,1}}{A_{r,2}}F_{1\text{-}2} \\ &= \frac{2a \times a}{\frac{1}{2}a \times a}F_{1\text{-}2} = 4F_{1\text{-}2} \end{aligned}$$

$$\begin{aligned} \frac{w}{a} &= \frac{2a}{a} = 2, \quad \frac{l}{a} = \frac{\frac{1}{2}a}{a} = \frac{1}{2} \\ F_{1\text{-}2} &\simeq 0.08 \quad \text{Figure 4.11(c).} \end{aligned}$$

Then

$$F_{2\text{-}1} = 4 \times 0.08 = 0.32.$$

(d) To determine $F_{2\text{-}1}$, we begin with the summation rule for surface 1, i.e.,

$$\begin{aligned} F_{1\text{-}1} + F_{1\text{-}2} + F_{1\text{-}3} + F_{1\text{-}4} &= 1 \\ 2F_{1\text{-}3} &= 1 - F_{1\text{-}2} \\ F_{1\text{-}3} &= \frac{1}{2}(1 - F_{1\text{-}2}), \end{aligned}$$

where we have used the symmetry condition for surfaces 1 and 3, and observed that $F_{1\text{-}1} = 0$ (because it is a convex surface) to determine $F_{1\text{-}2}$.
We now use the reciprocity rule and Figure 4.11(e) i.e.,

$$\begin{aligned} F_{1\text{-}2} &= \frac{A_{r,2}}{A_{r,1}} F_{2\text{-}1} \\ \frac{R_1}{R_2} &= \frac{1}{2}, \quad \frac{l}{R_2} = \frac{4R_1}{2R_1} = 2 \\ F_{2\text{-}1} &\simeq 0.415 \quad \text{Figure 4.11(e)}. \end{aligned}$$

Then

$$\begin{aligned} F_{1\text{-}3} &= \frac{1}{2}(1 - F_{1\text{-}2}) = \frac{1}{2}\left(1 - \frac{A_{r,2}}{A_{r,1}} F_{2\text{-}1}\right) \\ &= \frac{1}{2}\left(1 - \frac{2\pi \times 2R_1 \times 4R_1}{2\pi \times R_1 \times 4R_1} \times 0.415\right) \\ &= \frac{1}{2}(1 - 0.830) = 0.085. \end{aligned}$$

(e) To determine $F_{1\text{-}3}$, we begin with the summation rule for surface 2, i.e.,

$$F_{2\text{-}1} + F_{2\text{-}2} + F_{2\text{-}3} = 1$$

or

$$F_{2\text{-}3} = 1 - F_{2\text{-}1},$$

where $F_{2\text{-}2} = 0$ because it is planar.
To determine $F_{2\text{-}1}$, we use the reciprocity rule and the results of Table 4.2, i.e.,

$$\begin{aligned} F_{2\text{-}1} &= \frac{A_{r,1}}{A_{r,2}} F_{1\text{-}2} \\ \frac{R_2}{l} &= \frac{\frac{1}{4}R_1}{2R_1} = \frac{1}{8} \\ F_{1\text{-}2} &= \frac{1}{2}\left\{1 - \frac{1}{\left[1 + \left(\frac{R_2}{l}\right)^2\right]^{1/2}}\right\} \\ &= \frac{1}{2}\left\{1 - \frac{1}{[1 + 0.125^2]^{1/2}}\right\} = \frac{1}{2}(1 - 0.9923) = 0.003861. \end{aligned}$$

Then

$$\begin{aligned} F_{2\text{-}3} &= 1 - F_{2\text{-}1} = 1 - \frac{A_{r,1}}{A_{r,2}} F_{1\text{-}2} \\ &= 1 - \frac{4\pi R_1^2}{\pi\left(\frac{1}{4}R_1\right)^2} F_{1\text{-}2} = 1 - 64F_{1\text{-}2} = 0.7529. \end{aligned}$$

COMMENT:

In general, to determine a view factor, first inspect and find an available view factor and then work toward the unknown using the summation and reciprocity rules.

PROBLEM 4.11.FUN

GIVEN:

Two planar surfaces having the same area $A = A_1 = A_2$ are to have three different geometries/arrangements, while having nearly the same view factor $F_{1\text{-}2}$. These are coaxial circular disks, coaxial square plates, and perpendicular square plates, and are shown in Figure Pr.4.11.

SKETCH:

Figure Pr.4.11 shows the three geometries/arrangements.

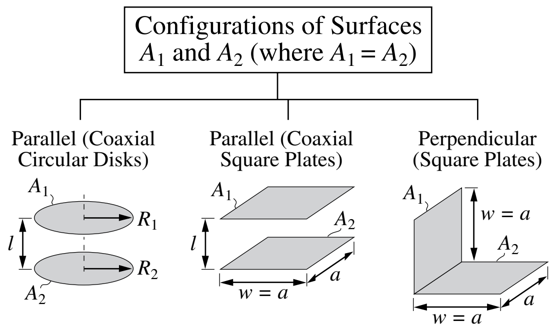


Figure Pr.4.11 Two planar surfaces having the same area and three different geometries/arrangements.

OBJECTIVE:

(a) Determine this nearly equal view factor $F_{1\text{-}2}$ (shared among the three geometries).

(b) Under this requirement, are the disks or the plates placed closer together?

SOLUTION:

(a) The view factors for parallel disks, parallel plates, and perpendicular plates, are given in Figures 4.11(a), (b), and (c). We note that for the perpendicular arrangement, $F_{1\text{-}2}$ is determined once the plate geometry is known. So, we begin in Figure 4.11(c) and note that for $l = a = w$, we have

$$l^* = 1, \quad w^* = 1.$$

Then from Figure 4.11(c), we have

$$F_{1\text{-}2} \simeq 0.2 \quad \text{Figure 4.11(c).}$$

Now, this view factor is found in Figure 4.11(b), i.e.,

$$F_{1\text{-}2} = 0.2, \quad \text{for} \quad w^* = \frac{w}{l} = 1, \;\; a^* = \frac{a}{l} = 1 \quad \text{Figure 4.11(b).}$$

Nearly the same view factor is found in Figure 4.11(a), i.e.,

$$F_{1\text{-}2} \simeq 0.2 \quad \text{for} \quad \frac{1}{R_1^*} = \frac{l}{R_1} = 1.70 = \frac{l}{R_2}, \quad \text{or} \quad R_2^* = \frac{R_2}{l} = 0.588 \quad \text{Figure 4.11(a).}$$

(b) Since the areas are all the same, we have

$$\begin{aligned} A &= \pi R^2 \quad \text{disk} \\ &= a^2 \quad \text{square plate} \end{aligned}$$

or

$$R = \frac{a}{\pi^{1/2}}.$$

Then we have

$$\begin{aligned} l &= 1.70R = 1.70\frac{a}{\pi^{1/2}} = 0.9591a \quad \text{disks} \\ l &= a \quad \text{plates.} \end{aligned}$$

Then the disks are placed slightly closer together, but under the approximations made reading from the graphs, this is negligible.

COMMENT:

Note that while the parallel arrangements allow for achieving higher $F_{1\text{-}2}$ by reducing l, the perpendicular arrangement results in a constant $F_{1\text{-}2}$, once the plate geometries are fixed.

PROBLEM 4.12 FUN

GIVEN:

The blackbody surface can be simulated using a large cavity (i.e., an enclosure with a small opening). The internal surfaces of the cavity have a total emissivity $\epsilon_{r,1}$ which is smaller than unity; however, due to the large cavity surface area, compared to its opening (i.e., mouth), the opening appears as a blackbody surface. To show this, consider the cylindrical enclosure shown in Figure Pr.4.12(a). The surrounding is assumed to be a blackbody at T_∞.

SKETCH:

Figure Pr.4.12(a) shows the use of apparent emissivity for construction of a blackbody emitter using a deep graybody cavity.

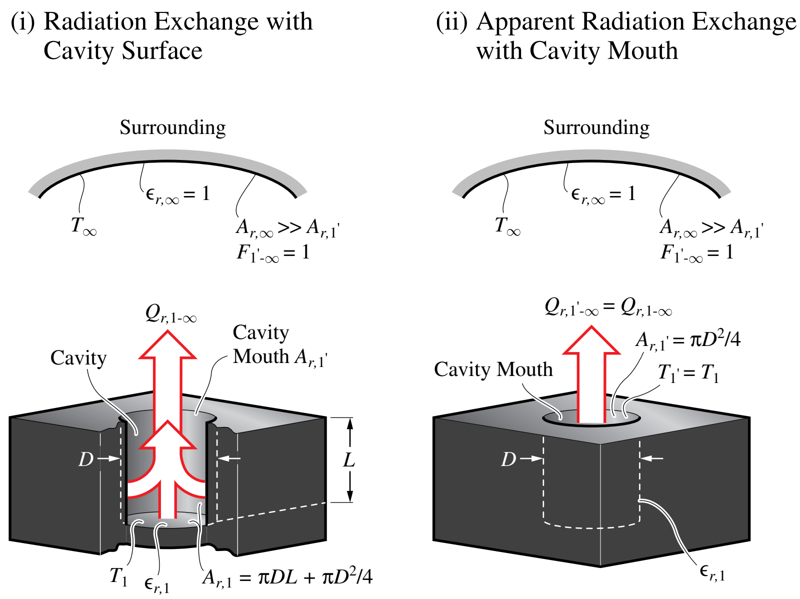


Figure Pr.4.12(a)(i) Surface radiation from a cavity. (ii) The concept of apparent emissivity for the cavity opening.

OBJECTIVE:

(a) Equate the net radiation heat transfer $Q_{r,1\text{-}\infty}$ from the cavity surface in Figure Pr.4.12(a)(i) to that in Figure Pr.4.12(a)(ii). In Figure Pr.4.12(a)(ii), we use the cavity opening area and an apparent emissivity $\epsilon_{r,1}{}'$. Then derive an expression for this apparent emissivity.

(b) Show that this apparent emissivity tends to unity for $L \gg D$.

SOLUTION:

(a) The thermal circuit diagrams for both cases are shown in Figure Pr.4.12(b). The cavity surface and the surrounding of the opening are treated as blackbody surfaces. The surface radiation heat flow for the two-surface enclosure of Figure Pr.4.12(a)(i) is given by (4.48), i.e.,

$$Q_{r,1\text{-}\infty} = Q_{r,1\text{-}1'} \quad = \quad \frac{E_{b,1}(T_1) - E_{b,\infty}(T_\infty)}{\left(\frac{1-\epsilon_{r,1}}{A_r\epsilon_{r,1}}\right)_1 + \frac{1}{A_{r,1}F_{1\text{-}1'}} + 0}.$$

Using the reciprocity rule, (4.34),

$$Q_{r,1\text{-}\infty} \quad = \quad \frac{E_{b,1} - E_{b,\infty}(T_\infty)}{\left(\frac{1-\epsilon_{r,1}}{A_r\epsilon_{r,1}}\right)_1 + \frac{1}{A_{r,1'}F_{1'\text{-}1}}}.$$

Thermal Circuit Model

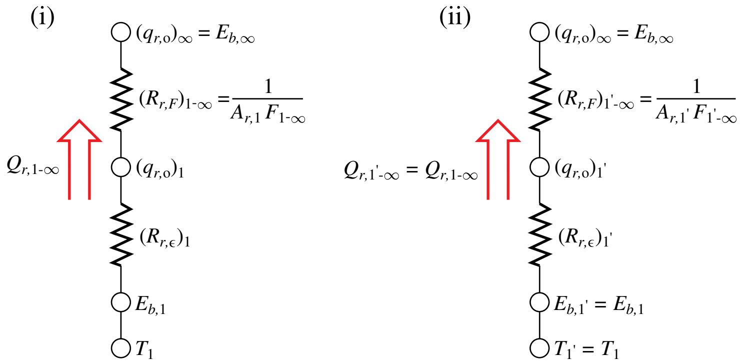


Figure Pr.4.12(b) Thermal circuit diagrams for the two-cases.

For surface $1'$, from (4.49) and Figure 4.12(b)(ii), we also have

$$\begin{aligned} Q_{r,1'\text{-}\infty} &= \frac{E_{b,1}(T_1) - E_{b,\infty}(T_\infty)}{\left(\dfrac{1-\epsilon_r}{A_r\epsilon_r}\right)_{1'} + \dfrac{1}{A_{r,1'}F_{1',\infty}}} = \frac{E_{b,1}(T_1) - E_{b,\infty}(T_\infty)}{\dfrac{1}{A_{r,1'}\epsilon_{r,1'}}}, \quad \text{for } F_{1'\text{-}\infty} = 1 \quad \text{with } A_{r,\infty} \gg A_{r,1} \\ &\equiv Q_{r,1\text{-}\infty}. \end{aligned}$$

Solving for $\epsilon_{r,1'}$, from these two equations, we have

$$\begin{aligned} \frac{1}{A_{r,1'}\epsilon_{r,1'}} &= \frac{1}{\dfrac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} + \dfrac{1}{A_{r,1'}F_{1'\text{-}1}}} \\ \epsilon_{r,1'} &= \frac{1}{\dfrac{A_{r,1'}}{A_{r,1}}\dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1}} + \dfrac{A_{r,1'}}{A_{r,1'}}\dfrac{1}{F_{1'\text{-}1}}} \\ &= \frac{1}{\dfrac{A_{r,1'}}{A_{r,1}}\dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1}} + 1} \quad \text{since } F_{1'\text{-}1} = 1. \end{aligned}$$

Now, we use the diameter and length of the cylindrical cavity to have

$$\begin{aligned} A_{r,1'} &= \pi D^2/4 \\ A_{r,1} &= \pi D L + \pi D^2/4. \end{aligned}$$

Then, the apparent emissivity of the cavity is

$$\epsilon_{r,1'} = \frac{1}{\dfrac{D}{4L+D}\dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1}} + 1}.$$

(b) For large L/D, i.e., a deep cavity, we have,

$$\lim_{L/D\to\infty} \epsilon_{r,1'} = \lim_{L/D\to\infty} \frac{1}{\dfrac{1}{4L/D+1}\dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1}} + 1} = \frac{1}{0+1} = 1,$$

i.e., to the surrounding, the cavity appears as a blackbody surface.

COMMENT:

For a given $\epsilon_{r,1}$, the ratio L/D needed to make $\epsilon_{r,1'}$ near unity, is determined from the above equation. The smaller $\epsilon_{r,1}$, the larger L/D needs to be for creating an apparent blackbody surface.

PROBLEM 4.13.FAM

GIVEN:

Liquid oxygen and hydrogen are used as fuel in space travel. The liquid is stored in a cryogenic tank, which behaves thermally like a thermos. Radiation shields (highly reflecting aluminum or gold foils) are placed over the tank to reduce irradiation to the tank surface [Figure Pr.4.13(a)]. The surface of the tank has a total emissivity of $\epsilon_{r,1} = 0.7$ and the shields have an emissivity of $\epsilon_{r,s} = 0.05$. Consider placing one [Figure Pr.4.13(a)(i)] and two [Figure Pr.4.13(a)(ii)] radiation shields on the tank.

Assume that the surface of the tank is $T_1 = 80$℃ above the saturation temperature of the liquid at one atm pressure, the tank is facing away from the sun, and the deep sky temperature is $T_2 = 3$ K.

SKETCH:

Figure Pr.4.13(a) shows the idealized tank surface and the radiation shields.

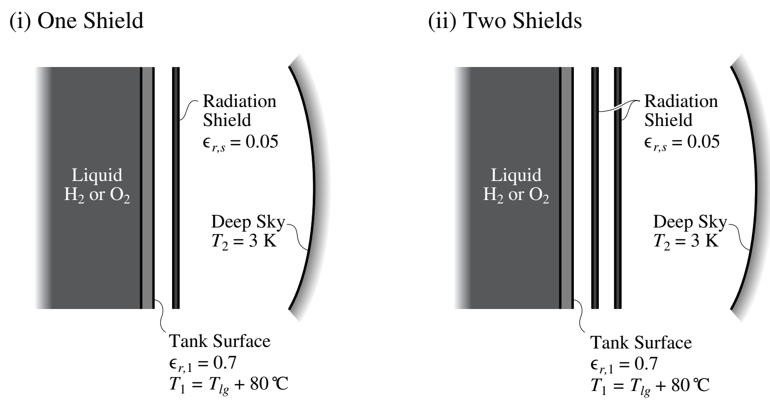


Figure Pr.4.13(a) Surface-radiation heat transfer from a cryogenic liquid tank. (i) With one shield. (ii) With two shields.

OBJECTIVE:

(a) Draw the thermal circuit diagrams.

(b) Determine the rate of heat flowing out of the tank per unit area for liquid oxygen and liquid hydrogen.

SOLUTION:

(a) The thermal circuit for one radiation shield is shown in Figure Pr.4.13(b) and for two radiation shields, in Figure Pr.4.13(c).

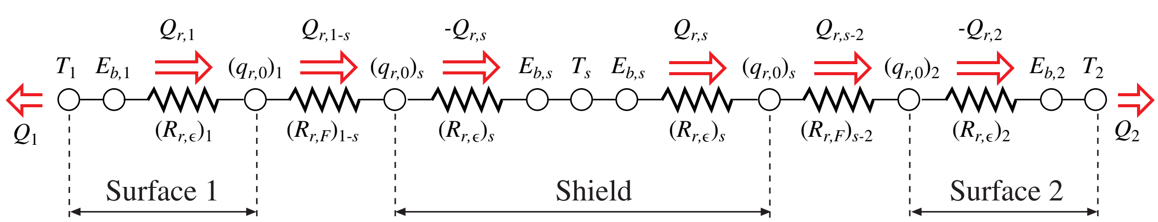


Figure Pr.4.13(b) Thermal circuit diagram for one shield.

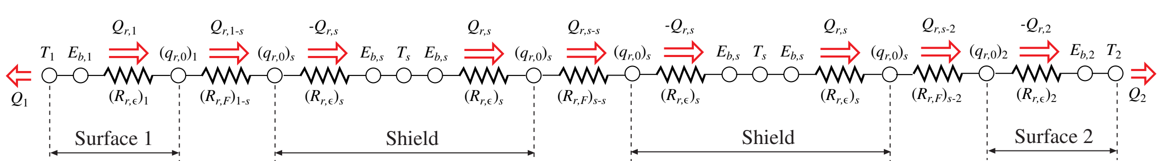


Figure Pr.4.13(c) Thermal circuit diagram for two shields.

(b) (i) For one radiation shield, the temperatures T_1 and T_2 are given (i.e., $E_{b,1}$ and $E_{b,2}$ are known). From (4.50), the heat transfer rate between surfaces 1 and 2 can then be expressed as a function of the potential difference $E_{b,1} - E_{b,2}$ and the overall resistance $(R_{r,\Sigma})_{1\text{-}2}$, i.e.,

$$Q_{r,1\text{-}2} = \frac{E_{b,1} - E_{b,2}}{(R_{r,\Sigma})_{1\text{-}2}}.$$

The overall resistance, for the radiation resistances arranged in series, is

$$\begin{aligned}(R_{r,\Sigma})_{1\text{-}2} &= \sum_j R_{r,j} = \frac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} + \frac{1}{A_{r,1}F_{1\text{-}s}} + \frac{1-\epsilon_{r,s}}{A_{r,s}\epsilon_{r,s}} + \frac{1-\epsilon_{r,s}}{A_{r,s}\epsilon_{r,s}} + \frac{1}{A_{r,s}F_{s\text{-}2}} + \frac{1-\epsilon_{r,2}}{A_{r,2}\epsilon_{r,2}} \\ &= \frac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} + \frac{1}{A_{r,1}F_{1\text{-}s}} + \left[2\frac{1-\epsilon_{r,s}}{A_{r,s}\epsilon_{r,s}} + \frac{1}{A_{r,s}F_{s\text{-}2}}\right] + \frac{1-\epsilon_{r,2}}{A_{r,2}\epsilon_{r,2}}.\end{aligned}$$

The view factors for infinite, parallel plates are unity, $F_{1\text{-}s} = F_{s\text{-}2} = 1$, and all the surface areas are equal, $A_{r,1} = A_{r,s} = A_r$. The total emissivities for the surfaces are $\epsilon_{r,1} = 0.7$, $\epsilon_{r,s} = 0.05$, and $\epsilon_{r,2} = 1$ (note that the deep sky behaves as a black body). Thus, the equivalent resistance becomes

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1-0.7}{0.7A_r} + \frac{1}{A_r} + 2\frac{1-0.05}{0.05A_r} + \frac{1}{A_r} + 0 = \frac{1.43+39}{A_r} = \frac{40.43}{A_r}.$$

Oxygen: From Table C.4, we have $T_{lg} = 90$ K. Then, the tank surface temperature becomes $T_1 = 90(\text{K}) + 80(\text{K}) = 170$ K. The deep sky temperature is $T_2 = 3$ K. Therefore, heat transfer rate is,

$$\begin{aligned}Q_{r,1\text{-}2} &= \frac{\sigma_{\text{SB}}(T_1^4 - T_2^4)}{(R_{r,\Sigma})_{1\text{-}2}} \\ &= \frac{5.67\times10^{-8}(\text{W/m}^2\text{-K}^4)\times\left[170^4(\text{K}^4) - 3^4(\text{K}^4)\right]}{\dfrac{40.43}{A_r}} \\ &= 1.17(\text{W/m}^2)\times A_r(\text{m}^2)\end{aligned}$$

or

$$q_{r,1\text{-}2} = \frac{Q_{r,1\text{-}2}}{A_r} = 1.17 \text{ W/m}^2.$$

Hydrogen: From Table C.4, we have $T_{lg} = 20.4$ K. Then, the tank surface temperature becomes $T_1 = 20.4(\text{K}) + 80(\text{K}) = 100.4$ K. Therefore, the heat transfer rate is

$$\begin{aligned}Q_{r,1\text{-}2} &= \frac{\sigma_{\text{SB}}(T_1^4 - T_2^4)}{(R_{r,\Sigma})_{1\text{-}2}} \\ &= \frac{5.67\times10^{-8}(\text{W/m}^2\text{-K}^4)\times\left[100.4^4(\text{K}^4) - 3^4(\text{K}^4)\right]}{\dfrac{40.43}{A_r}} \\ &= 0.14(\text{W/m}^2)\times A_r(\text{m}^2)\end{aligned}$$

or

$$q_{r,1\text{-}2} = \frac{Q_{r,1\text{-}2}}{A_r} = 0.14 \text{ W/m}^2.$$

(ii) For two radiation shields, the thermal circuit is shown in Figure Pr.4.13(c). For the overall thermal resistance, an equation similar to the one above is obtained. For two radiation shields, with the same surface radiation properties (same $\epsilon_{r,s}$), the term within brackets is multiplied by two. Therefore, we have

$$(R_{r,\Sigma})_{1\text{-}2} = \sum_j R_{r,j} = \frac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} + \frac{1}{A_{r,1}F_{1\text{-}s}} + 2\left[2\frac{1-\epsilon_{r,s}}{A_{r,s}\epsilon_{r,s}} + \frac{1}{A_{r,s}F_{s\text{-}2}}\right] + \frac{1-\epsilon_{r,2}}{A_{r,2}\epsilon_{r,2}}.$$

From the data available

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1-0.7}{0.7A_r} + \frac{1}{A_r} + 2\left[2\frac{1-0.05}{0.05A_r} + \frac{1}{A_r}\right] + 0 = \frac{1.43+78}{A_r} = \frac{79.43}{A_r}.$$

Oxygen:

$$\begin{aligned} Q_{r,1\text{-}2} &= \frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{(R_{r,\Sigma})_{1\text{-}2}} \\ &= \frac{5.67\times10^{-8}(\mathrm{W/m^2\text{-}K^4})\times\left[170^4(\mathrm{K}^4) - 3^4(\mathrm{K}^4)\right]}{\dfrac{79.43}{A_r}} \\ &= 0.5960(\mathrm{W/m^2})\times A_r(\mathrm{m}^2) \end{aligned}$$

or

$$q_{r,1\text{-}2} = \frac{Q_{r,1\text{-}2}}{A_r} = 0.5960\ \mathrm{W/m^2}.$$

Hydrogen:

$$\begin{aligned} Q_{r,1\text{-}2} &= \frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{(R_{r,\Sigma})_{1\text{-}2}} \\ &= \frac{5.67\times10^{-8}(\mathrm{W/m^2\text{-}K^4})\times\left[100.4^4(\mathrm{K}^4) - 3^4(\mathrm{K}^4)\right]}{\dfrac{79.43}{A_r}} \\ &= 0.073(\mathrm{W/m^2})\times A_r(\mathrm{m}^2) \end{aligned}$$

or

$$q_{r,1\text{-}2} = \frac{Q_{r,1\text{-}2}}{A_r} = 0.073\ \mathrm{W/m^2}.$$

COMMENT:

Note that the thermal resistance due to the radiation shield is $39/1.43 = 27$ times larger than the resistance due to the surface grayness of the tank alone. If the tank faces the sun, there would be absorption of solar irradiation at the tank surface and this heat would flow into the tank.

PROBLEM 4.14.DES

GIVEN:

A single-junction thermocouple psychrometer is used to measure the relative humidity in air streams flowing through ducts, as shown in Figure Pr.4.14. In the simplest design, the thermocouple psychrometer consists of a thermocouple bead, which is exposed to the humid air stream, connected simultaneously to a DC power source and a voltmeter. Initially, a voltage is applied to the thermocouple, causing a decrease in the bead temperature, due to the energy conversion from electromagnetic to thermal energy by the Peltier effect. This cooling causes the condensation of the water vapor and the formation of a liquid droplet on the thermocouple bead. When the temperature drops to temperature $T_{t,o}$, the power source is turned off and the voltage generated by the thermocouple is recorded. Since the droplet temperature is lower than the ambient temperature, the droplet receives heat from the ambient by surface convection and surface radiation. This causes the evaporation of the droplet. The voltage measured between the thermocouple leads is related to the temperature of the thermocouple bead/water droplet. An equilibrium condition is reached when the net heat flow at the droplet surface balances with the energy conversion due to phase change. This equilibrium temperature is called the wet-bulb temperature for the air stream T_{wb}.

Figure Pr.4.14 shows the thermocouple placed in the air stream. The duct diameter is much larger than the thermocouple bead. The water droplet has a diameter $D_d = 0.5$ mm and its surface is assumed to be a blackbody. The tube surface is opaque, diffuse, and gray and has a surface emissivity $\epsilon_{r,2} = 0.5$ and a temperature $T_2 = 300$ K. The evaporation rate of the water is estimated as $\dot{m}_{lg} = 0.00017$ kg/m^2-s.

SKETCH:

Figure Pr.4.14 shows the thermocouple psychrometer with a screen radiation shield.

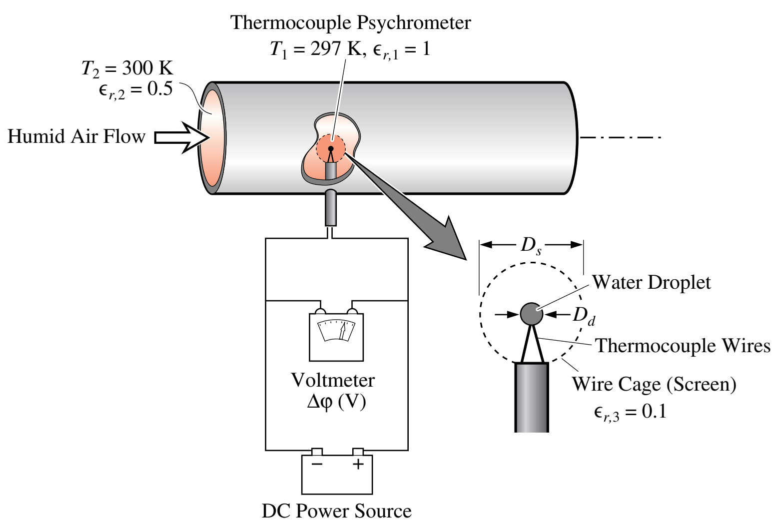


Figure Pr.4.14 Thermocouple psychrometer with a screen radiation shield.

OBJECTIVE:

(a) If the droplet temperature is $T_1 = 290$ K, determine the net heat transfer by surface radiation between the bead and the tube surface and express it as a percentage of the energy conversion due to liquid-vapor phase change.

(b) If the bead is protected by a porous spherical wire cage with diameter $D_s = 3$ mm and the ratio between the open area and total area $a_1 = A_{void}/A_{total} = 0.7$, calculate the reduction in the net heat transfer by surface radiation $\Delta Q_{r,1}$. The surface of the wires is opaque, diffuse, and gray, and has an emissivity $\epsilon_{r,3} = 0.1$. Using the available results for radiation between two surfaces separated by a screen, and for $A_{r,2} \gg A_{r,3} > A_{r,1}$, the overall radiation resistance is given by

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1-\epsilon_{r,1}}{\epsilon_{r,1}A_{r,1}} + \frac{1}{a_1 A_{r,1} + \cfrac{1}{\cfrac{1}{A_{r,1}(1-a_1)} + 2\left(\cfrac{1-\epsilon_r}{A_r\epsilon_r}\right)_3}}.$$

(c) In order to reduce the amount of heat transfer between the droplet and the tube surface by surface radiation, should we increase or decrease $a_1 = A_{void}/A_{total}$ and $\epsilon_{r,3}$?

SOLUTION:
(a) If the presence of the thermocouple wire is neglected, the droplet and the tube form a two-surface enclosure. The net radiation heat transfer is then given by (4.47) as

$$Q_{r,1} = \frac{\sigma_{\rm SB}(T_2^4 - T_1^4)}{\dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1}A_{r,1}} + \dfrac{1}{A_{r,1}F_{1\text{-}2}} + \dfrac{1-\epsilon_{r,2}}{A_{r,2}\epsilon_{r,2}}}.$$

The view factor between the droplet and the tube, for a very long tube, is $F_{1\text{-}2} = 1$. Also, assuming that $A_{r,2} \gg A_{r,1}$, the net radiation heat transfer becomes

$$Q_{r,1} = \frac{\sigma_{\rm SB}(T_2^4 - T_1^4)}{\dfrac{1}{\epsilon_{r,1}A_{r,1}}}.$$

Using the numerical values

$$\begin{aligned} Q_{r,1} &= \frac{5.67\times10^{-8}(\text{W/m}^2\text{-K}^4)\times(300^4 - 297^4)(\text{K})^4}{\dfrac{1}{1\times\pi\times(0.5\times10^{-3})^2(\text{m})^2}} \\ &= 1.42\times10^{-5}\ \text{W}. \end{aligned}$$

The energy conversion due to phase change is given by

$$\dot{S}_{lg} = -\dot{M}_{lg}\Delta h_{lg} = -\dot{m}_{lg}\Delta h_{lg}A_1 = -\dot{m}_{lg}\Delta h_{lg}\pi D_d^2.$$

From Table C.4, $\Delta h_{lg} = 2{,}256$ kJ/kg, and then,

$$\begin{aligned} \dot{S}_{lg} &= -0.00017(\text{kg/m}^2\text{-s})\times 2{,}256\times10^3(\text{J/kg})\times\pi(0.5\times10^{-3})^2(\text{m})^2 \\ &= -3.01\times10^{-4}\ \text{W}. \end{aligned}$$

The ratio of the radiation heat transfer and the energy conversion due to liquid-gas phase change is

$$\frac{-Q_{r,1}}{\dot{S}_{lg}} = \frac{-1.42\times10^{-5}(\text{W})}{-3.01\times10^{-4}(\text{W})} = 0.047.$$

Therefore, for these conditions, the radiation heat transfer is equal to 4.7% of the heat used for evaporation of the liquid.

(b) Using the available result for screens, the net radiation heat transfer between two surfaces separated by a screen with void fraction $a_1 = A_{void}/(A_{void} + A_{solid})$ is,

$$Q_{r,1} = \frac{\sigma_{\rm SB}(T_2^4 - T_1^4)}{(R_{r,\Sigma})_{1\text{-}2}},$$

where

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1-\epsilon_{r,1}}{\epsilon_{r,1}A_{r,1}} + \frac{1}{\dfrac{1}{\dfrac{1}{A_{r,1}F_{1\text{-}2}}} + \dfrac{1}{\dfrac{1}{A_{r,1}F_{1\text{-}3}} + 2\left(\dfrac{1-\epsilon_{r,3}}{A_{r,3}\epsilon_{r,3}}\right) + \dfrac{1}{A_{r,2}F_{2\text{-}3}}}} + \frac{1-\epsilon_{r,2}}{A_{r,2}\epsilon_{r,2}}.$$

The view factors are $F_{1\text{-}2} = a_1$ and $F_{2\text{-}3} = F_{1\text{-}3} = (1 - a_1)$, and assuming that $A_{r,2} \gg A_{r,3} > A_{r1}$, we have

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1-\epsilon_{r,1}}{\epsilon_{r,1}A_{r,1}} + \frac{1}{a_1A_{r,1} + \dfrac{1}{\dfrac{1}{A_{r,1}(1-a_1)} + \dfrac{2(1-\epsilon_{r,3})}{A_{r,3}\epsilon_{r,3}}}}.$$

Using the numerical values, we have

$$
\begin{aligned}
(R_{r,\Sigma})_{1\text{-}2} &= \frac{1-1}{\pi \times (0.5 \times 10^{-3})^2 (\mathrm{m})^2 \times 1} \\
&+ \frac{1}{0.7 \times \pi \times (0.5 \times 10^{-3})^2 (\mathrm{m})^2 + \dfrac{1}{\dfrac{1}{\pi \times (0.5 \times 10^{-3})^2 \times 0.3} + \dfrac{2(1-0.1)}{\pi \times (3 \times 10^{-3})(\mathrm{m})^2 \times 0.1}}} \\
&= 1.325 \times 10^6 \ 1/\mathrm{m}^2.
\end{aligned}
$$

Then, the net heat transfer by surface radiation becomes

$$
Q_{r,1} = \frac{5.67 \times 10^{-8} (\mathrm{W/m^2\text{-}K^4}) \times (300^4 - 297^4)(\mathrm{K})^4}{1.325 \times 10^6 (1/\mathrm{m}^2)} = 1.37 \times 10^{-5} \ \mathrm{W}.
$$

The ratio to the energy conversion is

$$
\frac{-Q_{r,1}}{\dot{S}_{lg}} = \frac{-1.37 \times 10^{-5} (\mathrm{W})}{-3.01 \times 10^{-4} (\mathrm{W})} = 0.045.
$$

The reduction of thermal radiation due to shielding by the screen is only $\Delta Q_{r,1} = 4.5\%$.

(c) Increasing the radiation thermal resistance $(R_{r,\Sigma})_{1\text{-}2}$ causes a decrease in the net heat transfer by radiation. From the expression for $(R_{r,\Sigma})_{1\text{-}2}$, to decrease the thermal radiation we should decrease a_1 and decrease $\epsilon_{r,3}$ (i.e., we should use a polished, metal shield).

COMMENT:

To allow for the surface-convection evaporation of the droplet, it is necessary to use a screen with a large a_1. The reduction of the effect of the surface radiation is desirable.

PROBLEM 4.15.DES

GIVEN:

The polymer coating of an electrical wire is cured using infrared irradiation. The wire is drawn through a circular ceramic oven as shown in Figure Pr.4.15(a). The polymer coating is thin and the drawing speed u_w is sufficiently fast. Under these conditions, the wire remains at a constant and uniform temperature of $T_1 = 400$ K, while moving through the oven. The diameter of the wire is $d = 5$ mm and its surface is assumed opaque, diffuse, and gray with an emissivity $\epsilon_{r,1} = 0.9$. The oven wall is made of aluminum oxide (Table C.18), has a diameter $D = 20$ cm, and length $L = 1$ m, and its surface temperature is $T_2 = 600$ K. One of the ends of the furnace is closed by a ceramic plate with a surface temperature $T_3 = 600$ K and a surface emissivity $\epsilon_{r,3} = 0.5$. The other end is open to the ambient, which behaves as a blackbody surface with $T_4 = 300$ K. Ignore the heat transfer by surface convection.

SKETCH:

Figure Pr.4.15(a) shows the wire and its surface radiation surroundings.

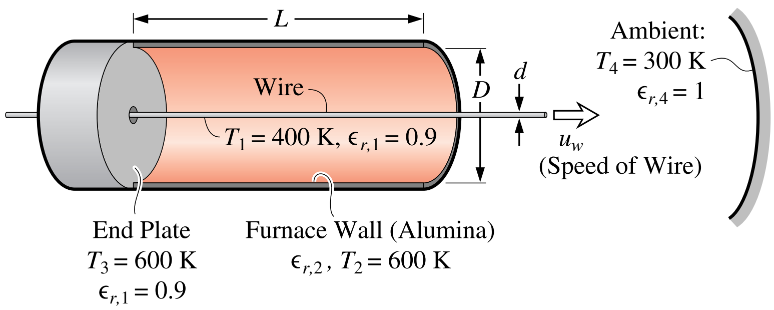


Figure Pr.4.15(a) A wire drawn through an oven.

OBJECTIVE:

(a) Draw the thermal circuit diagram for the four-surface radiation enclosure and write all of the relations for determination of the net heat transfer by radiation to the wire surface $Q_{r,1}$.

(b) Assuming that the wire exchanges heat by radiation with the tube furnace surface only (i.e., a two-surface enclosure), calculate the net heat transfer by surface radiation to the wire surface $Q_{r,1}$.

(c) Explain under what conditions the assumption made on item (b) can be used. Does the net heat transfer by surface radiation at the wire surface increase or decrease with an increase in the furnace diameter D (all the other conditions remaining the same)? Explain your answer.

SOLUTION:

(a) The thermal circuit diagram for the four-surface enclosure is shown in figure Pr.4.15(b). The energy equations are given below.

Surface 1:

$$-Q_1 = Q_{r,1} = Q_{r,1\text{-}2} + Q_{r,1\text{-}3} + Q_{r,1\text{-}4}, \quad Q_{r,1} = \frac{E_{b,1} - (q_{r,o})_1}{\dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1} A_{r1}}}.$$

Surface 2:

$$-Q_2 = Q_{r,2} = Q_{r,2\text{-}1} + Q_{r,2\text{-}3} + Q_{r,2\text{-}4}, \quad Q_{r,2} = \frac{E_{b,2} - (q_{r,o})_2}{\dfrac{1-\epsilon_{r,2}}{\epsilon_{r,2} A_{r2}}}.$$

Surface 3:

$$-Q_3 = Q_{r,3} = Q_{r,3\text{-}1} + Q_{r,3\text{-}2} + Q_{r,3\text{-}4}, \quad Q_{r,3} = \frac{E_{b,3} - (q_{r,o})_3}{\dfrac{1-\epsilon_{r,3}}{\epsilon_{r,3} A_{r3}}}.$$

Surface 4:

$$-Q_4 = Q_{r,4} = Q_{r,4\text{-}1} + Q_{r,4\text{-}2} + Q_{r,4\text{-}3}, \quad Q_{r,4} = \frac{E_{b,4} - (q_{r,0})_4}{\dfrac{1-\epsilon_{r,4}}{\epsilon_{r,4} A_{r4}}}.$$

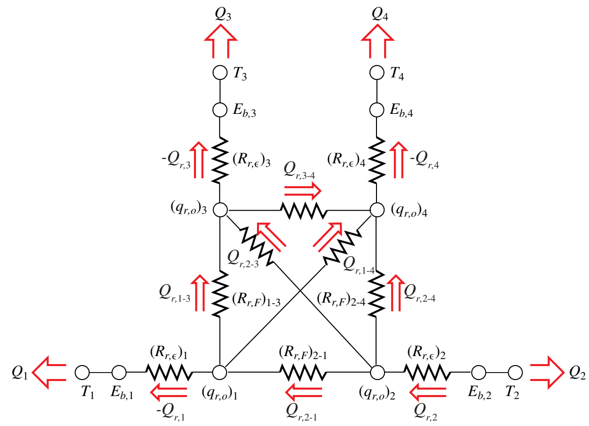


Figure Pr.4.15(b) Four-surface thermal circuit diagram.

The surface-radiation heat transfer rates are

$$Q_{r,1\text{-}2} = \frac{(q_{r,0})_1 - (q_{r,o})_2}{\dfrac{1}{F_{1\text{-}2}A_{r,1}}}, \quad Q_{r,1\text{-}2} = -Q_{r,2\text{-}1}$$

$$Q_{r,1\text{-}3} = \frac{(q_{r,0})_1 - (q_{r,o})_3}{\dfrac{1}{F_{1\text{-}3}A_{r,1}}}, \quad Q_{r,1\text{-}3} = -Q_{r,3\text{-}1}$$

$$Q_{r,1\text{-}4} = \frac{(q_{r,0})_1 - (q_{r,o})_4}{\dfrac{1}{F_{1\text{-}4}A_{r,1}}}, \quad Q_{r,1\text{-}4} = -Q_{r,4\text{-}1}$$

$$Q_{r,2\text{-}3} = \frac{(q_{r,0})_2 - (q_{r,o})_3}{\dfrac{1}{F_{2\text{-}3}A_{r,2}}}, \quad Q_{r,2\text{-}3} = -Q_{r,3\text{-}2}$$

$$Q_{r,2\text{-}4} = \frac{(q_{r,0})_2 - (q_{r,o})_4}{\dfrac{1}{F_{2\text{-}4}A_{r,2}}}, \quad Q_{r,2\text{-}4} = -Q_{r,4\text{-}2}$$

$$Q_{r,3\text{-}4} = \frac{(q_{r,0})_3 - (q_{r,o})_4}{\dfrac{1}{F_{3\text{-}4}A_{r,3}}}, \quad Q_{r,3\text{-}4} = -Q_{r,4\text{-}3}$$

The view factors are

$$\begin{aligned}
F_{1\text{-}1} + F_{1\text{-}2} + F_{1\text{-}3} + F_{1\text{-}4} &= 1 \\
F_{2\text{-}1} + F_{2\text{-}2} + F_{2\text{-}3} + F_{2\text{-}4} &= 1 \\
F_{3\text{-}1} + F_{3\text{-}2} + F_{3\text{-}3} + F_{3\text{-}4} &= 1 \\
F_{4\text{-}1} + F_{4\text{-}2} + F_{4\text{-}3} + F_{4\text{-}4} &= 1
\end{aligned}$$

$$\begin{aligned}
&A_{r,1}F_{1\text{-}2} = A_{r,2}F_{2\text{-}1}, \quad A_{r,1}F_{1\text{-}3} = A_{r,3}F_{3\text{-}1} \\
&A_{r,1}F_{1\text{-}4} = A_{r,4}F_{4\text{-}1}, \quad A_{r,2}F_{2\text{-}3} = A_{r,3}F_{3\text{-}2} \\
&A_{r,2}F_{2\text{-}4} = A_{r,4}F_{4\text{-}2}, \quad A_{r,3}F_{3\text{-}4} = A_{r,4}F_{4\text{-}3}.
\end{aligned}$$

There are 16 view factors and 10 view factor equations. Therefore, 6 view factors need to be determined independently. Considering that $F_{11} = F_{33} = F_{44}$=0, we need to determine 3 view factors from graphs or equations. The emissive powers are

$$E_{b,1} = \sigma_{\rm SB}T_1^4, \qquad E_{b,2} = \sigma_{\rm SB}T_2^4, \qquad E_{b,3} = \sigma_{\rm SB}T_3^4, \qquad E_{b,4} = \sigma_{\rm SB}T_4^4.$$

The variables are

$$T_1, T_2, T_3, T_4, Q_1, Q_2, Q_3, Q_4, Q_{r,1}, Q_{r,2}, Q_{r,3}, Q_{r,4}, Q_{r,1\text{-}2}, Q_{r,1\text{-}3}, Q_{r,1\text{-}4}, Q_{r,2\text{-}1}, Q_{r,2\text{-}3},$$
$$Q_{r,2\text{-}4}, Q_{r,3\text{-}1}, Q_{r,3\text{-}2}, Q_{r,3\text{-}3}, Q_{r,4\text{-}1}, Q_{r,4\text{-}2}, Q_{r,4\text{-}3}, E_{b,1}, E_{b,2}, E_{b,3}, E_{b,4},$$
$$(q_{r,o})_1, (q_{r,o})_2, (q_{r,o})_3, (q_{r,o})_4.$$

Therefore, there are 32 unknowns and 28 equations. Four unknowns must then be specified. For this problem, the 4 temperatures are known.

(b) The simplified formulation assumes a two-surface enclosure formed by the oven and the wire. For this situation, the net heat transfer to the wire surface is

$$-Q_{r,1} = Q_{r,2\text{-}1} = \frac{\sigma_{\rm SB}(T_2^4 - T_1^4)}{\dfrac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} + \dfrac{1}{A_{r,1}F_{1,2}} + \dfrac{1-\epsilon_{r,2}}{A_{r,2}\epsilon_{r,2}}}.$$

For this two surface enclosure, $F_{1\text{-}2} = 1$. From Table C.18 for alumina at $T_2 = 600$ K, we have $\epsilon_{r,2} = 0.58$. Then, using the values given,

$$\begin{aligned}
-Q_{r,1} &= \frac{5.67\times10^{-8}({\rm W/m^2\text{-}K^4})\times(600^4-400^4)({\rm K^4})}{\dfrac{1-0.9}{\pi\times(5\times10^{-3})({\rm m})\times1({\rm m})\times0.9} + \dfrac{1}{\pi\times(5\times10^{-3})({\rm m})\times1({\rm m})} + \dfrac{1-0.58}{\pi\times(0.2)({\rm m})\times1({\rm m})\times0.58}} \\
&= \frac{5.67\times10^{-8}({\rm W/m^2\text{-}K^4})(600^4-400^4)({\rm K^4})}{7.07+63.7+1.15} = 82\ {\rm W}.
\end{aligned}$$

(c) The resistance $(R_{r,\epsilon})_2$ is already small, so a further increase in D_2 will increase $Q_{r,1}$ only by a small amount.

COMMENT:

The assumption made in item (b) is acceptable when the view factor from the wire to the furnace, $F_{1\text{-}2}$, is approximately one. In this case, there is a negligible radiation heat transfer between the wire surface and surfaces 3 and 4. The view factor $F_{1\text{-}2}$ can be obtained from the relation in Figure 4.11(e) ($F_{1\text{-}2} = F_{2\text{-}1}A_{r,2}/A_{r,1}$). Figure Pr.4.15(c) shows the variation of the view factor $F_{1\text{-}2}$, with respect to the ratio of radius of the wire R_1 to the oven radius R_2, keeping the other dimensions constant. We observe that the view factor is always larger than 0.9 and approaches 1 as R_1 approaches R_2. Therefore, with an increase in the furnace diameter, the heat transfer to the wire decreases.

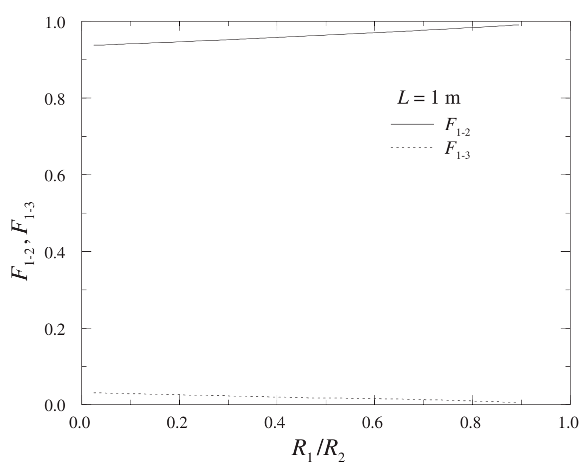


Figure Pr.4.15(c) Variation of view factor $F_{1\text{-}2}$ with respect to R_1/R_2.

PROBLEM 4.16.FUN

GIVEN:

As in the application of radiation shields discussed in Section 4.4.5, there are applications where the surface radiation through multiple (thin, opaque solid) layers (or solid slabs) is of interest. This is rendered in Figure Pr.4.16. Since for large N (number of layers) the local radiation heat transfer becomes independent of the presence of the far away layers, we can then use the local (or diffusion approximation) approximation of radiation heat transfer and use the temperature difference (or local temperature gradient) between adjacent layers and write

$$q_{r,x} \equiv -\langle k_r \rangle \frac{dT}{dx} = -\langle k_r \rangle \frac{T_2 - T_1}{l} = -\langle k_r \rangle \frac{T_c - T_h}{L},$$

where l is the spacing between adjacent layers. This radiant conductivity is

$$\langle k_r \rangle = \frac{4\epsilon_e \sigma_{\rm SB} T^3 l}{2 - \epsilon_r}, \quad T = \left[\frac{(T_1^2 + T_2^2)(T_1 + T)}{4} \right]^{1/3}.$$

SKETCH:

Figure Pr.4.16 shows the multilayer system, where all surfaces have the same emissivity and surface area. The radiant conductivity $\langle k_r \rangle$ and the radiant-conductivity based resistance R_{kr} are also shown.

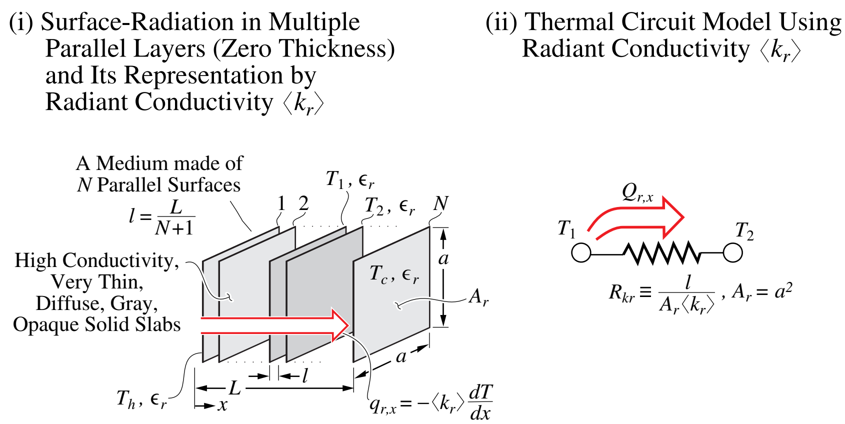


Figure Pr.4.16(i) Surface radiation in a multilayer (each layer opaque) system. (ii) Its thermal circuit representation by radiant conductivity.

OBJECTIVE:

(a) Start from (4.47) for radiation between surfaces 1 and 2 and assume that $l \ll a$, such that $F_{1\text{-}2} = 1$. Then show that

$$q_{r,1\text{-}2} = \frac{\epsilon_r \sigma_{\rm SB}(T_1^4 - T_2^4)}{2 - \epsilon_r}.$$

(b) Use that linearization of (4.72) to show that

$$q_{r,1\text{-}2} = \frac{4\epsilon_r \sigma_{\rm SB} T^3 (T_1 - T_2)}{2 - \epsilon_r} \quad \text{for} T_1 \rightarrow T_2,$$

i.e., for small diminishing difference between T_1 and T_2.

(c) Then using the definition of $q_{r,x}$ given above, derive the given expression for radiant conductivity $\langle k_r \rangle$.

SOLUTION:

(a) Starting from (4.47), we have for surface radiation between surfaces 1 and 2

$$Q_{r,1\text{-}2} = \frac{E_{b,1} - E_{b,2}}{\dfrac{1 - \epsilon_r}{A_r \epsilon_r} + \dfrac{1}{A_r F_{1\text{-}2}} + \dfrac{1 - \epsilon_r}{A_r \epsilon_r}},$$

where we have the same area A_r and emissivity ϵ_r for both surfaces.
Since $l \ll a$, then from Figure 4.11(b), $F_{1\text{-}2} = 1$ and we have

$$q_{r,1\text{-}2} = \frac{Q_{r,1\text{-}2}}{A_r} = \frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{\dfrac{2}{\epsilon_r} - 1} = \frac{\epsilon_r \sigma_{\rm SB}(T_1^4 - T_2^4)}{2 - \epsilon_r}.$$

(b) Using $T_1/T_2 \simeq 1$, we use the results of (4.72), i.e.,

$$\begin{aligned}(T_1^4 - T_2^4) &= (T_1^2 + T_2^2)(T_1^2 - T_2^2) \\ &= (T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2) \\ &\equiv 4T^3(T_1 - T_2)\end{aligned}$$

where we have defined

$$(T_2^2 + T_1^2)(T_1 + T_2) = 4T^3.$$

Note that for $T_1 \rightarrow T_2$, i.e., a diminishing temperature differences between two adjacent layers, we will have $T = T_1 = T_2$. Using the results of (a), we then have

$$q_{r,1\text{-}2} = \frac{4\epsilon_r \sigma_{\rm SB} T^3 (T_1 - T_2)}{2 - \epsilon_r}.$$

(c) Next, we use the definition and the results of (b), i.e.,

$$\begin{aligned}q_{r,x} &= q_{r,1\text{-}2} \equiv -\langle k_r \rangle \frac{T_2 - T_1}{l} \\ &= \frac{4\epsilon_r \sigma_{\rm SB} T^3 (T_1 - T_2)}{2 - \epsilon_r} = -\langle k_r \rangle \frac{T_2 - T_1}{l}.\end{aligned}$$

Then solving for $\langle k_r \rangle$, we have

$$\langle k_r \rangle = \frac{4\epsilon_r \sigma_{\rm SB} T^3 l}{2 - \epsilon_r}.$$

COMMENT:

Note that we did not allow for any conduction resistance through each layer. This can be significant for high emissivity, but low conductivity solids (e.g., polymeric materials such as paper and fabrics).

PROBLEM 4.17.FAM

GIVEN:

A short, one-side closed cylindrical tube is used as a surface radiation source, as shown in Figure Pr.4.17(a). The surface (including the cylindrical tube and the circular closed end) is ideally insulated on its outside surface and is uniformly heated by Joule energy conversion, resulting in a uniform inner surface temperature $T_1 = 800°\text{C}$. The heat transfer from the internal surface to the surroundings is by surface radiation only.

$T_\infty = 100°\text{C}$, $\epsilon_{r,1} = 0.9$, $D = 15$ cm, $L = 15$ cm.

SKETCH:

Figure Pr.4.17(a) shows the one-side closed cavity with its wall heated by Joule energy conversion exchanging radiation with surroundings.

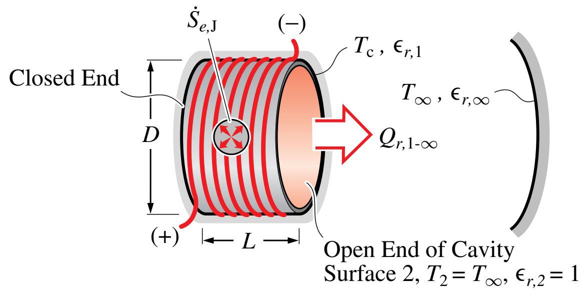


Figure Pr.4.17(a) Surface radiation from a one-end closed cavity, to its surroundings.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the required Joule heating rate $\dot{S}_{e,\text{J}}$.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.4.17(b).

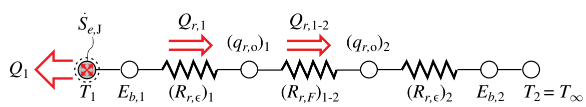


Figure Pr.4.17(b) Thermal circuit diagram.

(b) The radiation source here is the inner surface of a one-side closed cylindrical tube. It is at a uniform temperature T_1. Therefore, the inner surface of the tube and the closed end can be modeled as a single node at T_1. This surface exchanges heat by radiation with the surrounding through the open end (surface 2). Surface 2 is a blackbody surface ($\epsilon_{r,2} = 1$) and we have a two-surface enclosure. The corresponding thermal circuit diagram is shown in Figure Pr.4.17(b). The conservation of energy equation applied to node T_1 (for steady-state condition) gives

$$Q_{r,1\text{-}\infty} + Q_1 = Q_{r,1\text{-}2} + Q_1 = \dot{S}_{e,\text{J}}.$$

Since the outer surface is insulated, $Q_1 \to 0$, and then

$$\frac{E_{b,1} - E_{b,\infty}}{(R_{r,\epsilon})_1 + (R_{r,F})_{1\text{-}2} + (R_{r,\epsilon})_2} = \dot{S}_{e,\text{J}},$$

where

$$
\begin{aligned}
(R_{r,\epsilon})_1 &= \frac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} \\
(R_{r,F})_{1\text{-}2} &= \frac{1}{A_{r,1}F_{1\text{-}2}} \\
(R_{r,\epsilon})_2 &= \frac{1-\epsilon_{r,2}}{A_{r,2}\epsilon_{r,2}}.
\end{aligned}
$$

Also

$$
A_{r,1} = \frac{\pi D^2}{4} + \pi DL = \frac{\pi \times (0.15)^2(\mathrm{m})^2}{4} + \pi \times (0.15)(0.15)(\mathrm{m})^2 = 0.0884 \ \mathrm{m}^2.
$$

As noted, all net radiation exchange between surface 1 and the surrounding must pass through the remaining open end. For simplicity, this end can be thought of as an imaginary surface 2 of area $A_{r,2} = \pi D^2/4 = 0.01767 \ \mathrm{m}^2$ and at $T_2 = T_\infty = 373.15$ K, that would provide the same effect as the surroundings for radiation heat exchange. This is drawn schematically in the lower part of Figure Pr.4.17(b), where $Q_{r,1\text{-}2}$ is equal to $Q_{r,1\text{-}\infty}$. Note that $F_{2\text{-}1} = 1$ by inspection, since the imaginary end surface is a flat end of the tube. Then

$$
\begin{aligned}
(R_{r,\epsilon})_1 &= \frac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} = \frac{1-0.9}{0.0884(\mathrm{m}^2) \times 0.9} = 1.258 \ \mathrm{m}^{-2} \\
(R_{r,F})_{1\text{-}2} &= \frac{1}{A_{r,1}F_{1\text{-}2}} = \frac{1}{A_{r,2}F_{2\text{-}1}} = \frac{1}{0.01767(\mathrm{m}^2) \times 1} = 56.588 \ \mathrm{m}^{-2} \\
(R_{r,\epsilon})_2 &= \frac{1-1}{A_{r,\infty} \times 1} = 0.
\end{aligned}
$$

Then the energy equation becomes

$$
\begin{aligned}
Q_{r,1\text{-}\infty} = Q_{r,1\text{-}2} = \dot{S}_{e,\mathrm{J}} &= \frac{E_{b,1} - E_{b,2}}{(R_{r,\epsilon})_1 + (R_{r,F})_{1\text{-}2} + (R_{r,\epsilon})_2} \\
\dot{S}_{e,\mathrm{J}} &= \frac{\sigma_{\mathrm{SB}}(T_1^4 - T_2^4)}{(R_{r,\epsilon})_1 + (R_{r,F})_{1\text{-}2} + (R_{r,\epsilon})_2} \\
\dot{S}_{e,\mathrm{J}} &= \frac{5.67 \times 10^{-8}(\mathrm{W/m^2\text{-}K^4})[(1{,}073.15 \ \mathrm{K})^4 - (373.15 \ \mathrm{K})^4]}{1.258(1/\mathrm{m}^2) + 56.588(1/\mathrm{m}^2) + 0} \\
\dot{S}_{e,\mathrm{J}} &= 1{,}041 \ \mathrm{W}.
\end{aligned}
$$

COMMENT:

For cavities, the opening can be treated as a blackbody surface having the temperature of the surrounding. This allows for radiation leaving the cavity to the surrounding with no reflection from the opening and also allows for the surrounding to emit into the cavity.

PROBLEM 4.18.FAM

GIVEN:

A cylindrical piece of wood (length L and diameter D) is burning in an oven as shown in Figure Pr.4.18(a). The wood can be assumed to be in the central region of the cube furnace (with each oven side having length a). The internal oven surface temperature is T_2. The burning rate is $\dot{M}_{r,c}$, and the heat of combustion is $\Delta h_{r,c}$. Assume that the only surface heat transfer from the wood is by steady-state radiation.

$T_2 = 80$℃, $\dot{M}_{r,c} = 2.9 \times 10^{-4}$ kg/s, $\Delta h_{r,c} = -1.4 \times 10^7$ J/kg, $\epsilon_{r,1} = 0.9$, $\epsilon_{r,2} = 0.8$, $D = 5$ cm, $L = 35$ cm, $a = 1$ m.

Use geometrical relations (not the tables) to determine the view factors.

SKETCH:

Figure Pr.4.18(a) shows the cylindrical piece of wood.

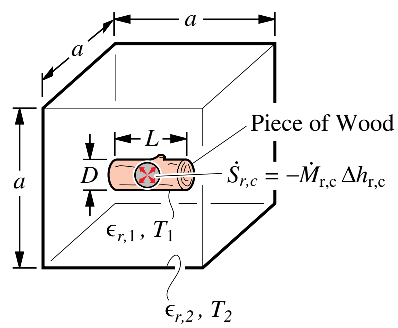


Figure Pr.4.18(a) A cylindrical piece of wood burning in an oven.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the wood surface temperature T_1.

(c) What would T_1 be if T_2 were lowered by 80℃?

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.4.18(b).

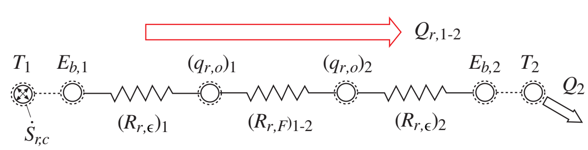


Figure Pr.4.18(b) Thermal circuit diagram.

(b) Applying the conservation of energy equation to node T_1, and noting steady-state, we have

$$
\begin{aligned}
Q|_A = Q_{r,1\text{-}2} &= \dot{S}_{r,c} \\
\frac{E_{b,1} - E_{b,2}}{R_{r,\Sigma}} &= -\dot{M}_{r,c}\Delta h_{r,c} \\
\frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{R_{r,\Sigma}} &= -\dot{M}_{r,c}\Delta h_{r,c}.
\end{aligned}
$$

The total radiation thermal resistance is then found with

$$\begin{aligned}
A_1 &= A_{\text{shaft}} + A_{\text{ends}} = \pi D L + 2 \times \left(\frac{\pi D^2}{4}\right) \\
&= \pi D (L + D/2) = \pi \times 0.05(\text{m}) \times [0.35(\text{m}) + 0.05(\text{m})/2] \\
&= 0.0589 \text{ m}^2 \\
A_2 &= A_{\text{box}} = 6(a \times a) = 6 \times 1(\text{m}) \times 1(\text{m}) \\
&= 6 \text{ m}^2 \\
F_{1\text{-}2} &= 1 \qquad \text{by inspection.}
\end{aligned}$$

Then

$$\begin{aligned}
(R_{r,\epsilon})_1 &= \frac{1-\epsilon_{r,1}}{A_1 \epsilon_{r,1}} = \frac{1-0.9}{0.0589(\text{m}^2) \times 0.9} = 1.886 \text{ 1/m}^2 \\
(R_{r,F})_{1\text{-}2} &= \frac{1}{A_1 F_{1\text{-}2}} = \frac{1}{0.0589(\text{m}^2) \times 1} = 16.977 \text{ 1/m}^2 \\
(R_{r,\epsilon})_2 &= \frac{1-\epsilon_{r,2}}{A_2 \epsilon_{r,2}} = \frac{1-0.8}{6(\text{m}^2) \times 0.8)} = 0.04167 \text{ 1/m}^2
\end{aligned}$$

or

$$\begin{aligned}
R_{r,\Sigma} &= (R_{r,\epsilon})_1 + (R_{r,F})_{1\text{-}2} + (R_{r,\epsilon})_2 \\
&= 1.886(1/\text{m}^2) + 16.977(1/\text{m}^2) + 0.04167(1/\text{m}^2) \\
&= 18.90 \text{ 1/m}^2.
\end{aligned}$$

Then solving for T_1, we have

$$\begin{aligned}
T_1 &= \left[T_2^4 - \frac{R_{r,\Sigma}}{\sigma_{\text{SB}}} \dot{M}_{r,c} \Delta h_{r,c}\right]^{1/4} \\
&= \left\{[80 + 273.15]^4(\text{K})^4 - \frac{18.90(1/\text{m}^2)}{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4)} \times [2.9 \times 10^{-4}(\text{kg/s})][-1.4 \times 10^7(\text{J/kg})]\right\}^{1/4} \\
&= 1{,}081.7 \text{ K} = 808.6^\circ\text{C}.
\end{aligned}$$

(c) If T_2 is lowered by 80℃ to 0℃, we have

$$\begin{aligned}
T_1 &= \left[T_2^4 - \frac{R_{r,\Sigma}}{\sigma_{\text{SB}}} \dot{M}_{r,c} \Delta h_{r,c}\right]^{1/4} \\
&= \left\{[0 + 273.15]^4(\text{K}) - \frac{18.904(1/\text{m}^2)}{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4)} \times [2.9 \times 10^{-4}(\text{kg/s})][-1.4 \times 10^7(\text{J/kg})]\right\}^{1/4} \\
&= 1{,}079.8 \text{ K} = 806.6^\circ\text{C}.
\end{aligned}$$

COMMENT:

The surface-convection heat transfer (due to the thermobuoyant fluid motion) can be significant and would tend to reduce the surface temperature.

PROBLEM 4.19.FUN

GIVEN:

Consider two square (each length a) parallel plates at temperatures T_1 and T_2 and having an equal emissivity ϵ_r. Assume that the distance between them l is much smaller than a ($l \ll a$).

OBJECTIVE:

(a) Show that radiative heat flux between surface 1 and 2 is

$$q_{r,1\text{-}2} = \frac{\epsilon_r \sigma_{\rm SB}(T_1^4 - T_2^4)}{2 - \epsilon_r}.$$

(b) Show that if a radiation shield having the same size and emissivity is placed between them, then

$$q_{r,1\text{-}2} = \frac{\epsilon_r \sigma_{\rm SB}(T_1^4 - T_2^4)}{2(2 - \epsilon_r)}.$$

SOLUTION:

(a) For two, parallel plates placed very close to each other ($l \ll a$), we have from Figure 4.11(b), for the view factor

$$F_{1\text{-}2} = 1.$$

Then (4.48) for a two-surface enclosure becomes

$$\begin{aligned} Q_{r,1\text{-}2} &= \frac{E_{b,1} - E_{b,2}}{\dfrac{1-\epsilon_r}{A_r\epsilon_r} + \dfrac{1}{A_r} + \dfrac{1-\epsilon_r}{A_r\epsilon_r}} \\ &= \frac{E_{b,1} - E_{b,2}}{\dfrac{2}{A_r\epsilon_r} - \dfrac{1}{A_r}} \\ &= \frac{A_r(E_{b,1} - E_{b,2})}{\dfrac{2}{\epsilon_r} - 1} \\ &= \frac{A_r\epsilon_r\sigma_{\rm SB}(T_1^4 - T_2^4)}{2 - \epsilon_r} \end{aligned}$$

or

$$q_{r,1\text{-}2} = \frac{Q_{r,1\text{-}2}}{A_r} = \frac{\epsilon_r \sigma_{\rm SB}(T_1^4 - T_2^4)}{2 - \epsilon_r}.$$

(b) With one shield added, starting from (4.50), we have

$$\begin{aligned} Q_{r,1\text{-}2} &= \frac{E_{b,1} - E_{b,2}}{2[(R_{r,\epsilon})_1 + (R_{r,F})_{1\text{-}2} + (R_{r,\epsilon})_2]} \\ &= \frac{E_{b,1} - E_{b,2}}{2\left(\dfrac{1-\epsilon_r}{A_r\epsilon_r} + \dfrac{1}{A_r} + \dfrac{1-\epsilon_r}{A_r\epsilon_r}\right)}, \end{aligned}$$

where again we have used $F_{1\text{-}s} = F_{s\text{-}2} = 1$.
Then following the steps in part (a), we have

$$q_{r,1\text{-}2} = \frac{\epsilon_r \sigma_{\rm SB}(T_1^4 - T_2^4)}{2(2 - \epsilon_r)}.$$

COMMENT:

Note that as highly reflective ($\epsilon_r \to 0$) surfaces are used, $q_{r,1\text{-}2} = \epsilon_r\sigma_{\rm SB}(T_1^4 - T_2^4)/4$, which shows a direct proportionality between $q_{r,1\text{-}2}$ and ϵ_r.

PROBLEM 4.20.FUN

GIVEN:

Two very large, parallel plates at maintained temperatures T_1 and T_2 are exchanging surface radiation heat. A third large and thin plate is placed in between and parallel to the other plates [Figure Pr.4.20(a)]. This plate has periodic voids (e.g., as in a screen) and the fraction of void area to total surface area is $\epsilon = A_{\rm voids}/A_{\rm total}$. The screen is sufficiently thin such that its temperature T_3 is uniform across the thickness. All plates have opaque, diffuse, and gray surfaces with the same total emissivity ϵ_r.

SKETCH:

Figure Pr.4.20(a) shows the surface-radiation heat transfer between two plates separated by a screen.

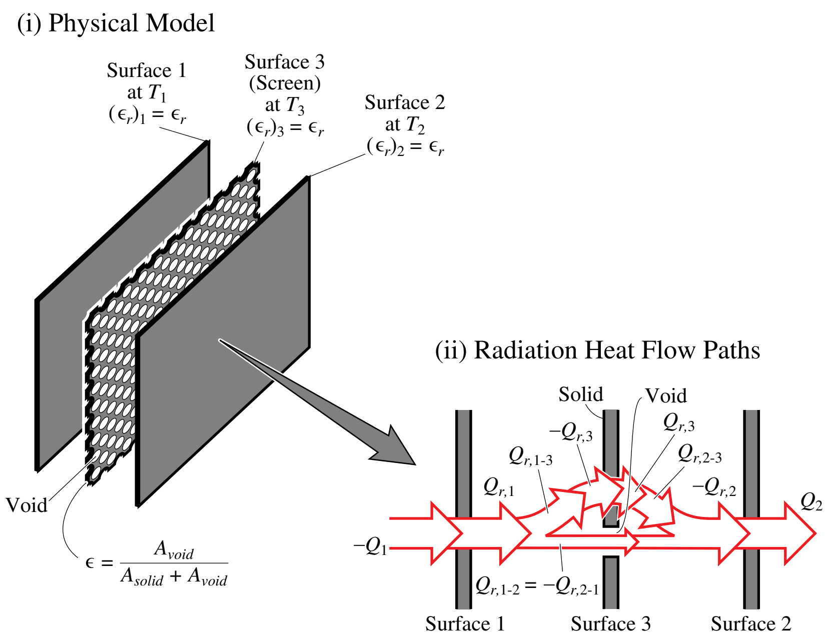


Figure Pr.4.20(a)(i) and (ii) Surface radiation heat transfer between two plates separated by a screen.

OBJECTIVE:

(a) Draw the thermal circuit.

(b) Derive the expression for the net heat transfer rate by surface radiation between surfaces 1 and 2, i.e., $Q_{r,1\text{-}2}$, given by

$$\frac{Q_{r,1\text{-}2}}{A_r} = \frac{E_{b,1} - E_{b,2}}{2\left[\dfrac{1-\epsilon_r}{\epsilon_r} + \dfrac{1}{2\epsilon + \epsilon_r(1-\epsilon)}\right]}.$$

(Suggestion: Use a three-surface enclosure and allow for heat transfer between surfaces 1 and 2 directly through the screen voids and indirectly through the solid portion of the screen. The screen has radiation exchange on both of its sides, with a zero net heat transfer).

(c) Comment on the limits as $\epsilon \to 0$ and $\epsilon \to 1$.

(d) Would $Q_{r,1\text{-}2}$ increase or decrease with an increase in the emissivity of the screen?

(Suggestion: Analyze $Q_{r,1\text{-}2}$ in the limits for $\epsilon_r \to 0$ and $\epsilon_r \to 1$.)

SOLUTION:

(a) The thermal circuit diagram for the problem is shown in Figure Pr.4.20(b).

(b) The temperatures T_1 and T_2 are known. Therefore, the radiation heat transfer rate from surface 1 to surface 2, $Q_{r,1\text{-}2}$, is found from Figure Pr.4.20(b) as

$$Q_{r,1\text{-}2} = \frac{E_{b,1} - E_{b,2}}{(R_{r,\Sigma})_{1\text{-}2}},$$

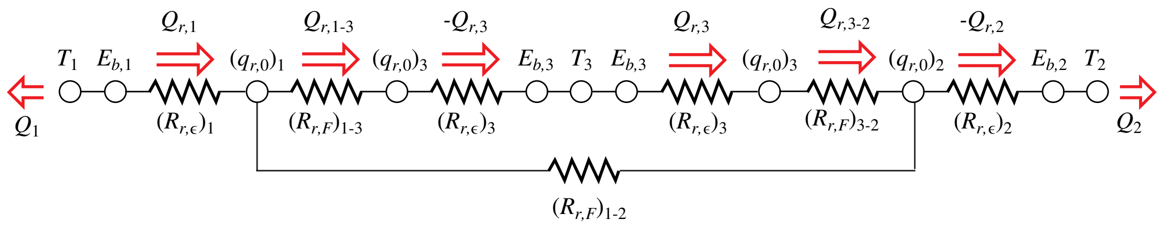


Figure Pr.4.20(b) Thermal circuit diagram.

where the overall resistance for the thermal circuit is

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} + \frac{1}{\dfrac{1}{\left(\dfrac{1}{A_{r,1}F_{1\text{-}2}}\right)} + \dfrac{1}{\dfrac{1}{A_{r,1}F_{1\text{-}3}} + \dfrac{1-\epsilon_{r,3}}{A_{r,3}\epsilon_{r,3}} + \dfrac{1-\epsilon_{r,3}}{A_{r,3}\epsilon_{r,3}} + \dfrac{1}{A_{r,3}F_{3\text{-}2}}}} + \frac{1-\epsilon_{r,2}}{A_{r,2}\epsilon_{r,2}}.$$

Here, surface 3 refers to the solid part of the screen only ($A_{r,3} = A_{\rm solid}$). Three view factors are needed, $F_{1\text{-}2}$, $F_{1\text{-}3}$, and $F_{3\text{-}2}$. The view factor from surface 3 to surface 2 is unity, because surface 2 is infinite and parallel to surface 3 ($F_{3\text{-}2} = 1$). By symmetry, the view factor from surface 3 to surface 1 is also unity. Applying the reciprocity rule (4.34) to $F_{3\text{-}1}$, we obtain

$$A_{r,3}F_{3\text{-}1} = A_{r,1}F_{1\text{-}3}.$$

Solving for $F_{1\text{-}3}$ gives

$$F_{1\text{-}3} = \frac{F_{3\text{-}1}A_{r,3}}{A_{r,1}} = \frac{A_{r,3}}{A_{r,1}}.$$

Using the relation $1-\epsilon = A_{\rm solid}/(A_{\rm solid} + A_{\rm void})$, we have

$$F_{1\text{-}3} = 1-\epsilon.$$

Applying the summation rule (4.33) to surface 1 gives

$$F_{1\text{-}1} + F_{1\text{-}2} + F_{1\text{-}3} = 1.$$

For surface 1, $F_{1\text{-}1} = 0$. Solving for $F_{1\text{-}2}$ gives

$$F_{1\text{-}2} = 1 - F_{1\text{-}3} = \epsilon_1.$$

All the surfaces have the same total emissivity ϵ_r. Then we have

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1-\epsilon_r}{A_{r,1}\epsilon_r} + \frac{1}{\dfrac{1}{\left(\dfrac{1}{A_{r,1}\epsilon}\right)} + \dfrac{1}{\dfrac{1}{A_{r,1}(1-\epsilon)} + \dfrac{1-\epsilon_r}{A_{r,3}\epsilon_r} + \dfrac{1}{A_{r,3}}}} + \frac{1-\epsilon_r}{A_{r,2}\epsilon_r}.$$

The surface areas are $A_{r,1} = A_{r,2} = A_r$ and $A_{r,3} = (1-\epsilon_1)A_r$. Then

$$(R_{r,\Sigma})_{1\text{-}2} = 2\left(\frac{1-\epsilon_r}{A_r\epsilon_r}\right) + \frac{1}{A_r\epsilon + \dfrac{1}{\dfrac{2}{A_r(1-\epsilon)} + \dfrac{2}{1-\epsilon}\left(\dfrac{1-\epsilon_r}{A_r\epsilon_r}\right)}}.$$

Finally,

$$\begin{aligned} Q_{r,1\text{-}2} &= \frac{E_{b,1}-E_{b,2}}{(R_{r,\Sigma})_{1\text{-}2}} \\ &= \frac{E_{b,1}-E_{b,2}}{2\left(\dfrac{1-\epsilon_r}{A_r\epsilon_r}\right) + \dfrac{1}{A_r\epsilon + \dfrac{1}{\dfrac{2}{A_r(1-\epsilon)} + \dfrac{2}{1-\epsilon}\left(\dfrac{1-\epsilon_r}{A_r\epsilon_r}\right)}}}. \end{aligned}$$

After dividing by A_r, we have

$$\begin{aligned}
q_{r,1\text{-}2} &= \frac{Q_{r,1\text{-}2}}{A_r} \\
&= \frac{E_{b,1} - E_{b,2}}{2\left(\dfrac{1-\epsilon_r}{\epsilon_r}\right) + \dfrac{1}{\epsilon + \dfrac{1}{\dfrac{2}{1-\epsilon} + \dfrac{2}{1-\epsilon}\left(\dfrac{1-\epsilon_r}{\epsilon_r}\right)}}} \\
&= \frac{E_{b,1} - E_{b,2}}{2\left[\dfrac{1-\epsilon_r}{\epsilon_r} + \dfrac{1}{2\epsilon + \epsilon_r(1-\epsilon)}\right]}.
\end{aligned}$$

(c) It is always a good idea to check the limits of your solution to see whether they agree with your physical understanding of the problem. In the limit when $\epsilon \to 1$, we have

$$\lim_{\epsilon_1 \to 1} q_{r,1\text{-}2} = \frac{E_{b,1} - E_{b,2}}{\dfrac{2}{\epsilon_r} - 1},$$

which is the surface radiation heat flux between two infinite, parallel, flat plates. In the limit when $\epsilon \to 0$, we have

$$\lim_{\epsilon_1 \to 0} q_{r,1\text{-}2} = \frac{E_{b,1} - E_{b,2}}{2\left(\dfrac{2}{\epsilon_r} - 1\right)},$$

which is the surface radiation heat flux between two infinite, parallel, flat plates when one radiation shield is placed between them.

(d) From the expression for $q_{r,1\text{-}2}$, we have for the case of $\epsilon_r = 0$

$$q_{r,1\text{-}2} = \frac{E_{b,1} - E_{b,2}}{\infty} = 0,$$

i.e, no surface-radiation heat transfer occurs.

For the case of $\epsilon_r = 1$, we have

$$q_{r,1\text{-}2} = \frac{E_{b,1} - E_{b,2}}{\dfrac{1}{\epsilon}} = \epsilon(E_{b,1} - E_{b,2}).$$

This shows that the radiation heat transfer decreases by a factor of ϵ when the surfaces (including the screen) are blackbodies.

COMMENT:

Note that even a screen with a large ϵ_1 can reduce the heat transfer rate between the surfaces.

PROBLEM 4.21.FUN

GIVEN:

In surface-radiation heat transfer between surfaces 1 and 2, the enclosure geometry dependence of the radiation heat flux $q_{r,2\text{-}1}$ is examined using four different geometries. These are shown in Figures Pr.4.21(a)(i) through (iv) and are: parallel plates, coaxial cylinders, coaxial spheres, and a disk facing an enclosing hemisphere. The plates are assumed to be placed sufficiently close to each other and the cylinders are assumed to be sufficiently long, such that for all the four enclosure geometries the radiation is only between surfaces 1 and 2 (i.e., two-surface enclosures).

$T_1 = 120°\text{C}$, $T_2 = 90°\text{C}$, $\epsilon_{r,1} = \epsilon_{r,2} = 0.8$.

SKETCH:

Figure Pr.4.21(a) shows the four geometries.

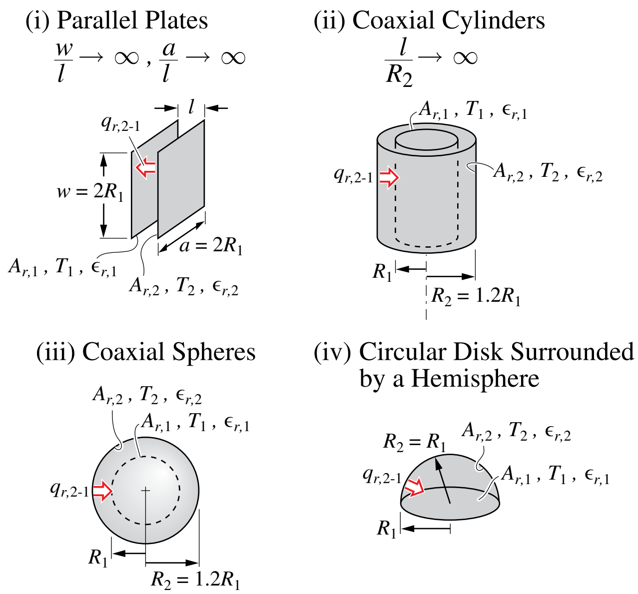


Figure Pr.4.21(a)(i) through (iv) Four enclosure geometries used in determining the dependence of $q_{r,2\text{-}1}$ on the enclosure geometry.

OBJECTIVE:

(a) Draw the thermal circuit diagram (one for all geometries).

(b) Determine $q_{r,2\text{-}1} = Q_{r,2\text{-}1}/A_{r,2}$ for the geometries of Figures Pr.4.21(a)(i)-(iv), for the given conditions.

SOLUTION:

(a) Figure Pr.4.21(b) shows the thermal circuit diagram for all the geometries.

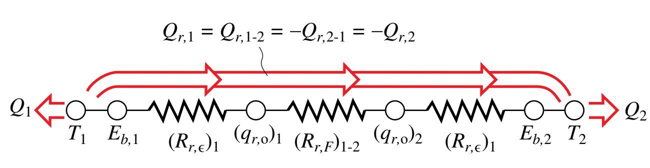


Figure Pr.4.21(b) Thermal circuit diagram.

(b) The radiation heat transfer $Q_{r,2\text{-}1}$ is given by (4.47), i.e.,

$$Q_{r2\text{-}1} = \frac{\sigma_{\rm SB}(T_2^4 - T_1^4)}{\dfrac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} + \dfrac{1}{A_{r,1}F_{1\text{-}2}} + \dfrac{1-\epsilon_{r,2}}{A_{r,2}\epsilon_{r,2}}}.$$

(i) For the parallel plate, we have

$$\begin{aligned} A_{r,1} &= A_{r,2} = wa = 4R_1^2 \\ F_{1\text{-}2} &= 1 \quad (\text{for } w^* \to \infty, \quad a^* \to \infty) \quad \text{Figure 4.11(b)} \end{aligned}$$

Then

$$\begin{aligned} \frac{Q_{r,2\text{-}1}}{A_{r,2}} = q_{r,2\text{-}1} &= \frac{\sigma_{\rm SB}(T_2^4 - T_1^4)}{\dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1}} + \dfrac{1}{1} + \dfrac{1-\epsilon_{r,2}}{\epsilon_{r,2}}} \\ &= \frac{5.67\times 10^{-8}(\text{W/m}^2\text{-K}^4)[(363.15)^4 - (393.15)^4](\text{K}^4)}{\dfrac{2\times(1-0.8)}{0.8} + 1} \\ &= -\frac{3.685\times 10^2(\text{W/m}^2)}{0.5+1} = -245.7 \text{ W/m}^2. \end{aligned}$$

(ii) For long, coaxial cylinders, we have

$$\begin{aligned} A_{r,1} &= 2\pi R_1 l, \quad A_{r,2} = 2\pi R_2 l \\ F_{1\text{-}2} &= 1 \quad \text{since all radiation leaving surface 1 is assumed to arrive at surface 2.} \end{aligned}$$

Then

$$\begin{aligned} \frac{Q_{r,2\text{-}1}}{A_{r,2}} &= q_{r,2\text{-}1} = \frac{\sigma_{\rm SB}(T_2^4 - T_1^4)}{\left(\dfrac{R_2}{R_1}\right)\dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1}} + \dfrac{R_2}{R_1} + \dfrac{1-\epsilon_{r,2}}{\epsilon_{r,2}}} \\ &= -\frac{3.685\times 10^2(\text{W/m}^2)}{(1.2)\dfrac{1-0.8}{0.8} + 1.2 + \dfrac{1-0.8}{0.8}} \\ &= \frac{-3.685\times 10^2(\text{W/m}^2)}{0.3+1.2+0.25} = -210.6 \text{ W/m}^2. \end{aligned}$$

(iii) For coaxial spheres,we have

$$\begin{aligned} A_{r,1} &= 4\pi R_1^2, \quad A_{r,2} = 4\pi R_2^2 \\ F_{1\text{-}2} &= 1 \quad \text{since all radiation leaving surface 1 arrives at surface 2.} \end{aligned}$$

Then

$$\begin{aligned} \frac{Q_{r,2\text{-}1}}{A_{r,2}} &= q_{r,2\text{-}1} = \frac{\sigma_{\rm SB}(T_2^4 - T_1^4)}{\left(\dfrac{R_2}{R_1}\right)^2\dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1}} + \left(\dfrac{R_2}{R_1}\right)^2 + \dfrac{1-\epsilon_{r,2}}{\epsilon_{r,2}}} \\ &= \frac{-3.685\times 10^2(\text{W/m}^2)}{(1.2)^2\dfrac{1-0.8}{0.8} + (1.2)^2 + \dfrac{1-0.8}{0.8}} \\ &= -\frac{3.685\times 10^2(\text{W/m}^2)}{0.36+1.44+0.25} = -179.8 \text{ W/m}^2. \end{aligned}$$

(iv) For a disk surrounded by a hemisphere, we have

$$\begin{aligned} A_{r,1} &= \pi R_1^2, \quad A_{r,2} = 2\pi R_2^2 \\ F_{1\text{-}2} &= 1 \quad \text{since all radiation leaving surface 1 arrives at surface 2.} \end{aligned}$$

Then

$$\begin{aligned}
\frac{Q_{r,2\text{-}1}}{A_{r,2}} &= q_{r,2\text{-}1} = \frac{\sigma_{\rm SB}(T_2^4 - T_1^4)}{2\left(\dfrac{R_2}{R_1}\right)^2 \dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1}} + 2\left(\dfrac{R_2}{R_1}\right)^2 + \dfrac{1-\epsilon_{r,2}}{\epsilon_{r,2}}} \\
&= \frac{-3.685 \times 10^2 (\mathrm{W/m^2})}{2 \times \dfrac{1-0.8}{0.8} + 2 + \dfrac{1-0.8}{0.8}} \\
&= -\frac{3.685 \times 10^2 (\mathrm{W/m^2})}{0.5 + 2 + 0.25} = -134.0\ \mathrm{W/m^2}.
\end{aligned}$$

COMMENT:

Note that, due to the change in the surface areas, the magnitude of the surface radiation heat flux decreases as we move from the planar surface to the curved surfaces (resulting in an increase in surface area $A_{r,2}$).

PROBLEM 4.22.FAM

GIVEN:

A hemispherical Joule heater (surface 1) is used for surface-radiation heating of a circular disk (surface 2). This is shown in Figure Pr.4.22(a). In order to make an efficient use of the Joule heating, a hemispherical cap (surface 3) is placed around the heater surface and is ideally insulated.

$R_1 = 5$ cm, $R_2 = 5R_1$, $T_1 = 1{,}100$ K, $T_2 = 500$ K $\epsilon_{r,1} = \epsilon_{r,2} = 1$.

Assume that $F_{1\text{-}2}$ corresponds to that from a sphere to a disk (i.e., assume that the upper hemisphere does not see the disk).

SKETCH:

Figure Pr.4.22(a) shows the heater, the disk, and the reradiating surface.

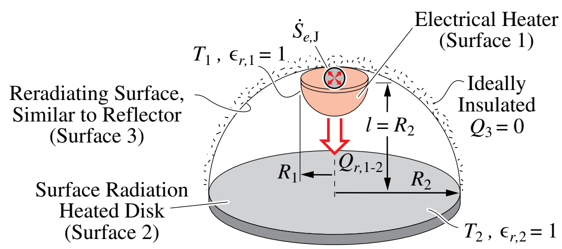


Figure Pr.4.22(a) A Joule heater is used for surface-radiation heating of a disk. A reradiating hemisphere is used to improve the heating rate.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine $Q_{r,1\text{-}2}$ for the given conditions.

(c) Determine $Q_{r,1\text{-}2}$ without the reradiator and compare the results with the results in (b).

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.4.22(b).

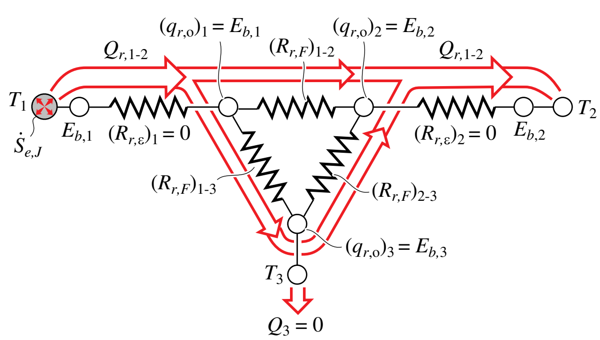


Figure Pr.4.22(b) Thermal circuit diagram.

(b) Noting that $(R_{r,\epsilon})_1 = (R_{r,\epsilon})_2 = 0$, because $\epsilon_{r,1} = \epsilon_{r,2} = 1$, (4.60) applies to this three-surface enclosure with one surface reradiating, i.e.,

$$Q_{r,1\text{-}2} = \frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{\dfrac{1}{A_{r,1}F_{1\text{-}2} + \dfrac{1}{\dfrac{1}{A_{r,1}F_{1\text{-}3}} + \dfrac{1}{A_{r,2}F_{2\text{-}3}}}}}.$$

Here we obtain $F_{1\text{-}2}$ from Table 4.2, noting that $R_2^* = R_2/l = 1$, i.e.,

$$\begin{aligned} F_{1\text{-}2} &= \frac{1}{2}\left[1 - \frac{1}{(1+R_2/l)^{1/2}}\right] = \frac{1}{2}\left(1 - \frac{1}{2^{1/2}}\right) \\ &= 0.1464. \end{aligned}$$

Then using the summation rule (4.33), we have

$$F_{1\text{-}1} + F_{1\text{-}2} + F_{1\text{-}3} = 1, \quad F_{1\text{-}1} = 0 \quad \text{(planar surface)}$$

or

$$F_{1\text{-}3} = 1 - F_{1\text{-}2} = 1 - 0.1464 = 0.8536.$$

To find $F_{2\text{-}3}$, we use the summation rule again, i.e.,

$$F_{2\text{-}1} + F_{2\text{-}2} + F_{2\text{-}3} = 1, \quad F_{2\text{-}2} = 0 \quad \text{(planar surface)}$$

or

$$\begin{aligned} F_{2\text{-}3} &= 1 - F_{2\text{-}1} = 1 - \frac{A_{r,1}}{A_{r,2}} F_{1\text{-}2} \\ &= 1 - \frac{2\pi R_1^2}{\pi R_2^2} F_{1\text{-}2} = 1 - \frac{2}{25} \times 0.1464 = 0.9883, \end{aligned}$$

where we have used the reciprocity rule (4.34).
Now we use these numerical values to evaluate $Q_{r,1\text{-}2}$, i.e.,

$$\begin{aligned} Q_{r,1\text{-}2} &= \frac{\sigma_{\text{SB}}(T_1^4 - T_2^4)}{\dfrac{1}{2\pi(0.05)^2(\text{m}^2) \times 0.1464 + \dfrac{1}{\dfrac{1}{2\pi(0.05)^2(\text{m}^2) \times 0.8536} + \dfrac{1}{\pi(0.25)^2(\text{m}^2) \times 0.9883}}}} \\ &= \frac{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4)[(1{,}100)^4 - (500)^4](\text{K}^4)}{\dfrac{1}{0.0023(\text{m}^2) + \dfrac{1}{74.58(1/\text{m}^2) + 5.153(1/\text{m}^2)}}} \\ &= \frac{7.946 \times 10^4(\text{W/m}^2)}{67.379(1/\text{m}^2)} = 1{,}180 \text{ W}. \end{aligned}$$

(c) When the reradiating surface is not present, then the heat transfer between surface 1 and 2 is found from Figure Pr.4.22(b), where the only resistance between the two surfaces is $(R_{r,F})_{1\text{-}2}$, i.e.,

$$\begin{aligned} Q_{k,1\text{-}2} &= \frac{E_{b,1} - E_{b,2}}{\dfrac{1}{A_{r,1}F_{1\text{-}2}}} \\ &= \frac{7.947 \times 10^4(\text{W/m}^2)}{\dfrac{1}{0.0023}(1/\text{m}^2)} = 182.78 \text{ W}. \end{aligned}$$

COMMENT:

Note that the radiation heat transfer rate has increased by 6.6 folds, when the reradiating surface is used.

PROBLEM 4.23.FAM

GIVEN:

A flat radiation heater is placed along a vertical wall to heat the passing pedestrians who may stop temporarily and face the heater. The heater is shown in Figure Pr.4.23(i) and is geometrically similar to a full-size mirror. The heater surface is at $T_1 = 600°C$ and the pedestrians have a surface temperature of $T_2 = 5°C$. Assume that the surfaces are opaque, diffuse, and blackbody surfaces (total emissivities are equal to one). Also assume that both the heater and the pedestrian have a rectangular cross section with dimensions $a = 50$ cm and $w = 170$ cm and that the distance between them is $l = 40$ cm, as shown in Figure Pr.4.6(ii).

SKETCH:

Figure Pr.4.23(a) shows the heater and the pedestrian and the idealized surface radiation geometry.

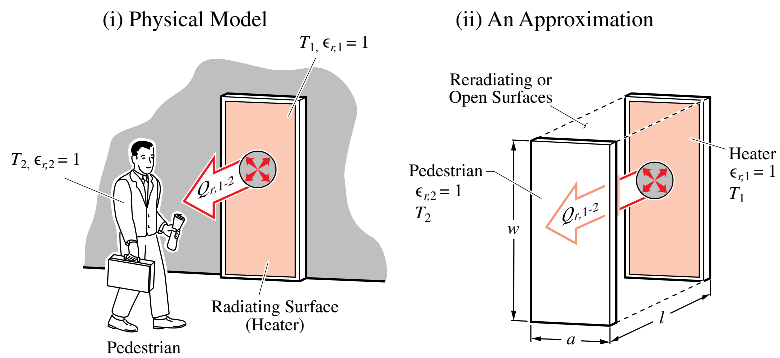


Figure Pr.4.23(a)(i) Physical model of a radiant wall, pedestrian heater. (ii) Idealized model.

OBJECTIVE:

(a) Draw the thermal circuit diagram for a three-surface enclosure (including the surroundings as a blackbody surface).

(b) Determine the net radiation heat transfer from the heater to the pedestrian $Q_{r,1\text{-}2}$.

(c) Determine the net radiation heat transfer to the pedestrian, when a reradiating (i.e., ideally insulated) surface is placed around the heater and pedestrian to increase the radiant heat flow $Q_{r,1\text{-}2}$.

SOLUTION:

(a) A fictitious surface can be wrapped around the open air space between the pedestrian and the heater. This fictitious surface is treated as an additional radiation surface (surface 3) and the problem becomes a three-surface enclosure with diffuse, gray surfaces. For these blackbody three surfaces, the thermal circuit is presented in Figure Pr.4.23(b).

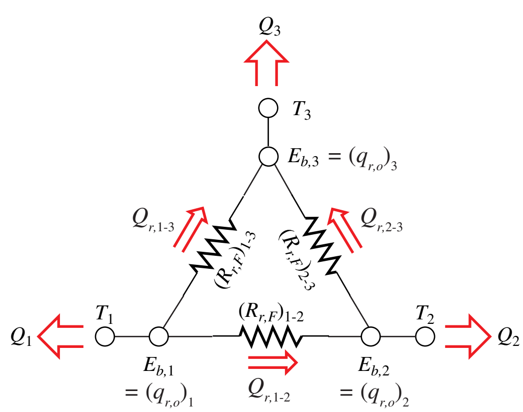


Figure Pr.4.23(b) A three-surface blackbody enclosure. Radiation heat transfer flow to surface 2 from surface 1 and when surface 3 is reradiating ($Q_3 = 0$), heat flows indirectly from surface 1 to 2.

(b) The surface-radiation heat transfer rate from surface 1 to surface 2 is found from Figure Pr.4.23(b) as

$$Q_{r,1\text{-}2} = \frac{E_{b,1} - E_{b,2}}{(R_{r,\Sigma})_{1\text{-}2}},$$

where, for the unity emissivities, we have

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1}{A_{r,1}F_{1\text{-}2}}.$$

The view factor $F_{1\text{-}2}$ is obtained from Figure 4.11(b). For surfaces 1 and 2, $w^* = w/l = 1.7(\text{m})/0.4(\text{m}) = 4.25$, $a^* = a/l = 0.5(\text{m})/0.4(\text{m}) = 1.25$. From Figure 4.11(b) we obtain $F_{1\text{-}2} = 0.39$. The area for surface 1 is $A_1 = aw = 0.5(\text{m}) \times 1.7(\text{m}) = 0.85 \text{ m}^2$. Therefore

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1}{0.85(\text{m}^2) \times 0.39} = 3.03 \text{ 1/m}^2$$

The heat transfer rate is

$$\begin{aligned} Q_{r,1\text{-}2} &= \frac{\sigma_{\text{SB}}(T_1^4 - T_2^4)}{(R_{r,\Sigma})_{1\text{-}2}} \\ &= \frac{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times \left[873.15^4(\text{K}^4) - 278.15^4(\text{K}^4)\right]}{3.03(1/\text{m}^2)} = 10{,}765 \text{ W}. \end{aligned}$$

(c) If surface 3 is perfectly insulated, $Q_3 = 0$ (surface 3 is called a reradiating surface). In this case, the overall thermal resistance is found from Figure Pr.4.23(b) as

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1}{\dfrac{1}{\dfrac{1}{A_{r,1}F_{1\text{-}2}}} + \dfrac{1}{\dfrac{1}{A_{r,1}F_{1\text{-}3}} + \dfrac{1}{A_{r,2}F_{2\text{-}3}}}}.$$

The view-factor $F_{1\text{-}2}$ was obtained above. The view factors $F_{1\text{-}3}$ and $F_{2\text{-}3}$ need to be determined. From the summation rule (4.33), we have

$$F_{1\text{-}1} + F_{1\text{-}2} + F_{1\text{-}3} = 1.$$

Since $F_{1\text{-}1} = 0$, we have

$$F_{1\text{-}3} = 1 - F_{1\text{-}2} = 1 - 0.39 = 0.61.$$

From the reciprocity rule and noting that $A_{r,2} = A_{r,1}$, $F_{2\text{-}3} = F_{1\text{-}3} = 0.61$. Then, $A_{r,2} = A_{r,1} = 0.85 \text{ m}^2$ and we have

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1}{\dfrac{1}{\dfrac{1}{0.85(\text{m}^2) \times 0.39}} + \dfrac{1}{\dfrac{2}{0.85(\text{m}^2) \times 0.61}}} = 1.69 \text{ 1/m}^2.$$

Finally, the heat transfer rate is

$$\begin{aligned} Q_{r,1\text{-}2} &= \frac{\sigma_{\text{SB}}(T_1^4 - T_2^4)}{(R_{r,\Sigma})_{1\text{-}2}} \\ &= \frac{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \left[873.15^4(\text{K}^4) - 278.15^4(\text{K}^4)\right]}{1.69(1/\text{m}^2)} = 19{,}300 \text{ W}. \end{aligned}$$

COMMENT:

Note that by placing the reradiating surface, the heat transfer from the heater to the pedestrian has nearly doubled.

PROBLEM 4.24.FAM

GIVEN:

A source for thermal irradiation is found by a Joule heater placed inside a solid cylinder of radius R_1 and length l. Then a hollow cylinder of radius R_2 and length l is placed coaxially around it with this outer cylinder and the top part of the opening ideally insulated. This is shown in Figure Pr.4.24(a) with the radiation leaving through the opening at the bottom spacing between the cylinders (surface 2). This results in surface 1 being the high temperature surface with direct and reradiation exchange with surface 2.

$T_2 = 400$ K, $\epsilon_{r,1} = 0.8$, $\dot{S}_{e,\mathrm{J}} = 1{,}000$ W, $R_1 = 1$ cm, $R_2 = 5$ cm, $l = 10$ cm.

SKETCH:

Figure Pr.4.24(a) shows the heated inner cylinder and the reradiating and the opening surfaces.

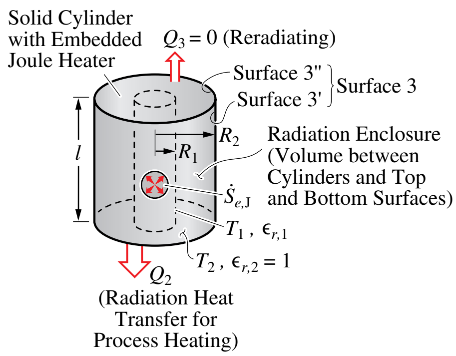


Figure Pr.4.24(a) Surface 1 is heated by Joule heating and through direct and reradiation allows for radiation to leave for surface 2.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the view factors $F_{1'\text{-}2}$, $F_{1\text{-}3}$, and $F_{2\text{-}3}$, using Figures 4.11(d) and (e), and the designations of Figure Pr.4.24(a).

(c) Determine the heater surface temperature T_1.

SOLUTION:

(a) Figure Pr.4.24(b) shows the thermal circuit diagram. Both direct and reradiating radiation are shown.

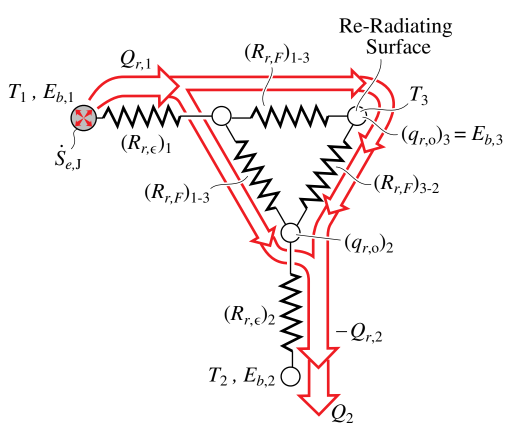


Figure Pr.4.24(b) Thermal circuit diagram.

(b) Using Figure Pr.4.24(b), and (4.60), we have

$$\begin{aligned}
Q_{r,1} &= \dot{S}_{e,\mathrm{J}} \\
&= \frac{\sigma_{\mathrm{SB}}(T_1^4 - T_2^4)}{\left(\dfrac{1-\epsilon_r}{A_r\epsilon_r}\right)_1 + \dfrac{1}{A_{r,1}F_{1\text{-}2} + \dfrac{1}{\dfrac{1}{A_{r,1}F_{1\text{-}3}} + \dfrac{1}{A_{r,2}F_{2\text{-}3}}}} + \left(\dfrac{1-\epsilon_r}{A_r\epsilon_r}\right)_2}.
\end{aligned}$$

Here we have $\epsilon_{r,2} = 1$, and also

$$A_{r,1} = 2\pi R_1 l, \quad A_{r,2} = \pi(R_2^2 - R_1^2).$$

For the view factors, we begin by using Figure 4.11(e). We define surface $3^{'}$ [see Figure Pr.4.24(a)] and the inner surface of the outer cylinder and obtain $F_{1\text{-}3'}$ by using the reciprocity rule (4.34), i.e.,

$$F_{1\text{-}3'} = \frac{A_{r,3'}}{A_{r,1}} F_{3'\text{-}1}.$$

To obtain $F_{3'\text{-}1}$, we use Figure 4.11(e) with

$$\begin{aligned}
\frac{1}{R*} &= \frac{R_1}{R_2} = \frac{0.01(\mathrm{m})}{0.05(\mathrm{m})} = 0.2 \\
\frac{l}{R_2} &= \frac{0.10(\mathrm{m})}{0.05(\mathrm{m})} = 2
\end{aligned}$$

and then we obtain $F_{3'\text{-}1} \simeq 0.15$.

To obtain $F_{1\text{-}3}$ we use the summation rule (4.33) for surface 1, i.e.,

$$F_{1\text{-}1} + F_{1\text{-}3'} + 2F_{1\text{-}3''} = 1,$$

where we have noted that the top $(3^{''})$ and bottom (2) surfaces between the two cylinders are identical. Here $F_{1\text{-}1} = 0$, and we obtain

$$\begin{aligned}
F_{1\text{-}2} &= F_{1\text{-}3''} = \frac{1 - F_{1\text{-}3'}}{2} = \frac{1 - \dfrac{A_{r,3'}}{A_{r,1}} F_{3'\text{-}1}}{2} \\
&= \frac{1 - \dfrac{R_2}{R_1} F_{3'\text{-}1}}{2} \\
&= \frac{1 - \dfrac{0.05(\mathrm{m})}{0.01(\mathrm{m})} \times 0.15}{2} \\
&= 0.125.
\end{aligned}$$

Then

$$F_{1\text{-}3} = F_{1\text{-}3'} + F_{1\text{-}3''} = 0.75 + 0.125 = 0.875 \quad \text{view factor between surface 1 and the reradiating surfaces 3.}$$

To determine, $F_{2\text{-}3}$, we use the summation rule for surface 2, i.e.,

$$F_{2\text{-}2} + F_{2\text{-}1} + F_{2\text{-}3} = 1$$

or

$$
\begin{aligned}
F_{\text{2-3}} &= 1 - F_{\text{2-2}} - F_{\text{2-1}} \\
&= 1 - F_{\text{2-1}} \\
&= 1 - \frac{A_{r,1}}{A_{r,2}} F_{\text{1-2}} \\
&= 1 - \frac{2\pi R_1 l}{\pi (R_2^2 - R_1^2)} 0.125 \\
&= 1 - \frac{2\pi \times 0.01 \times 0.1}{\pi (0.05^2 - 0.01^2)} \times 0.125 \\
&= 0.8958.
\end{aligned}
$$

(c) We now solve the energy equation for T_1, i.e.,

$$
T_1^4 = T_2^4 + \frac{\dot{S}_{e,\text{J}}}{\sigma_{\text{SB}}} \left[\left(\frac{1-\epsilon_r}{A_r \epsilon_r} \right)_1 + \frac{1}{A_{r,1} F_{\text{1-2}} + \dfrac{1}{\dfrac{1}{A_{r,1} F_{\text{1-3}}} + \dfrac{1}{A_{r,2} F_{\text{2-3}}}}} \right]
$$

$$
\begin{aligned}
T_1^4 &= (400)^4 (\text{K}^4) + \frac{1{,}000(\text{W})}{5.67 \times 10^{-8} (\text{W/m}^2\text{-K}^4)} \times \left[\frac{1 - 0.8}{2\pi \times 0.01(\text{m}) \times 0.1(\text{m}) \times 0.8} + \right. \\
&\left. \frac{1}{2\pi \times 0.01 \times 0.1(\text{m}^2) \times 0.125 + \dfrac{1}{\left(\dfrac{1}{2\pi \times 0.01 \times 0.1(\text{m}^2) \times 0.875} \right)} + \dfrac{1}{\left(\dfrac{1}{\pi (0.05^2 - 0.01^2)(\text{m}^2) \times 0.895} \right)}} \right] \\
&= 2.560 \times 10^{10} (\text{K}^4) + 1.76 \times 10^{10} (\text{K}^4\text{-m}^2) \times \\
&\left[39.79 (1/\text{m}^2) + \frac{1}{7.854 \times 10^{-4} (\text{m}^2) + \dfrac{1}{1.819 \times 10^2 (\text{m}^2)} + \dfrac{1}{1.482 \times 10^2 (\text{m}^2)}} \right] \\
T_1^4 &= 2.56 \times 10^{10} (\text{K}^4) + 1.76 \times 10^{10} (\text{K}^4) \times \left(39.79 + \frac{1}{7.854 \times 10^{-4} + 1.225 \times 10^{-2}} \right) \\
&= [2.560 \times 10^{10} + 1.76 \times 10^{10} \times (39.79 + 76.71)] (\text{K}^4)
\end{aligned}
$$

or

$$
T_1 = 1{,}200 \text{ K}.
$$

COMMENT:

Note that this is a rather large temperature for the inner cylinder. This is below the melting temperature of oxide ceramics. Note that reradiation by surfaces $3'$ and $3''$ reduces T, significantly, by reducing the view-factor resistance from $1/A_{r,1}F_{\text{1-2}}$ to what was used above.

PROBLEM 4.25.FUN.S

GIVEN:

Consider three opaque, diffuse, and gray surfaces with temperatures $T_1 = 400$ K, $T_2 = 400$ K, and $T_3 = 300$ K, with surface emissivities $\epsilon_{r,1} = 0.2$ and $\epsilon_{r,2} = \epsilon_{r,3} = 0.5$, and areas $A_{r,1} = A_{r,2} = A_{r,3} = 1$ m^2.

OBJECTIVE:

(a) For (i) surfaces 1 and 2 forming a two-surface enclosure (i.e., $F_{1\text{-}2} = 1$), and (ii) surfaces 1, 2, and 3 forming a three-surface enclosure (assume a two-dimensional equilateral triangular enclosure), is there a net radiation heat transfer rate $Q_{r,1\text{-}2}$ between surfaces 1 and 2?

(b) If there is a nonzero net heat transfer rate, what is the direction of this heat transfer?

(c) Would this heat transfer rate change if $T_3 = 500$ K?

(d) What is the temperature T_3 for which $Q_{r,1\text{-}2} = 0$?

SOLUTION:

(a)(i) Figure Pr.4.25(a) shows the thermal circuit diagram for surfaces 1 and 2 forming a two-surface enclosure. In this case, the net radiation heat transfer rate is

$$Q_{r,1\text{-}2} = \frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{(R_{r,\Sigma})_{1\text{-}2}}.$$

Since $T_1 = T_2$, there is a zero net heat transfer between surfaces 1 and 2, regardless of the value of the surface emissivities.

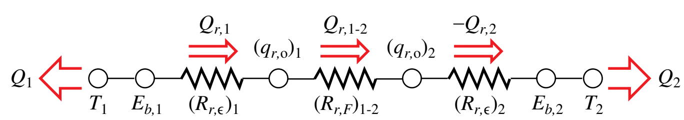


Figure Pr.4.25(a) Thermal circuit diagram for the two-surface enclosure.

(ii) Figure Pr.4.25(b) shows the thermal circuit diagram for surfaces 1, 2 and 3 forming a three-surface enclosure. Then, the net radiation heat transfer rates on surfaces 1, 2 and 3 are

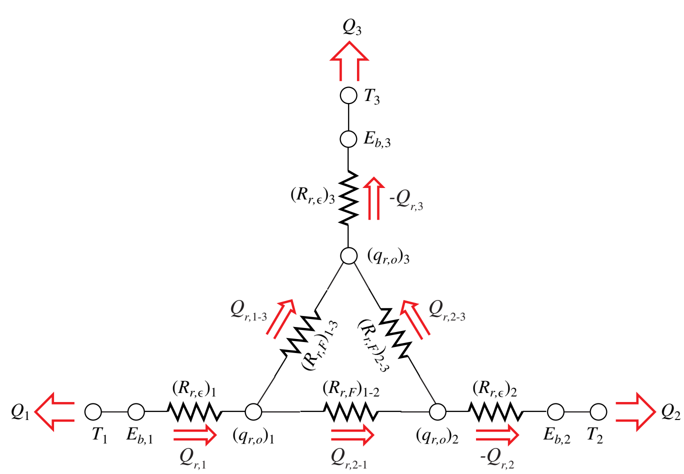


Figure Pr.4.25(b) Thermal circuit diagram for the three-surface enclosure.

$$
\begin{aligned}
Q_{r,1} &= \frac{E_{b,1}-(q_{r,o})_1}{\dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1}A_{r,1}}} = \frac{(q_{r,o})_1-(q_{r,o})_2}{\dfrac{1}{F_{1\text{-}2}A_{r,1}}} + \frac{(q_{r,o})_1-(q_{r,o})_3}{\dfrac{1}{F_{1\text{-}3}A_{r,1}}} \\
Q_{r,2} &= \frac{E_{b,2}-(q_{r,o})_2}{\dfrac{1-\epsilon_{r,2}}{\epsilon_{r,2}A_{r,2}}} = \frac{(q_{r,o})_2-(q_{r,o})_1}{\dfrac{1}{F_{2\text{-}1}A_{r,2}}} + \frac{(q_{r,o})_2-(q_{r,o})_3}{\dfrac{1}{F_{2\text{-}3}A_{r,2}}} \\
Q_{r,3} &= \frac{E_{b,3}-(q_{r,o})_3}{\dfrac{1-\epsilon_{r,3}}{\epsilon_{r,3}A_{r,3}}} = \frac{(q_{r,o})_3-(q_{r,o})_2}{\dfrac{1}{F_{3\text{-}1}A_{r,3}}} + \frac{(q_{r,o})_3-(q_{r,o})_2}{\dfrac{1}{F_{3\text{-}2}A_{r,3}}}.
\end{aligned}
$$

Note that, even with $E_{b,1} = E_{b,2}$, since $\epsilon_{r,1} \neq \epsilon_{r,2}$, there may be a non-zero net heat transfer rate $Q_{r,1\text{-}2}$ between surfaces 1 and 2. The view factors and areas are all the same, i.e., $F_{1\text{-}2} = F_{1\text{-}3} = F_{2\text{-}3} = 0.5$ and $A_{r,1} = A_{r,2} = A_{r,3} = 1\ \text{m}^2$. The surface resistances then become

$$
\begin{aligned}
(R_{r,\epsilon})_1 &= \frac{1-\epsilon_{r,1}}{\epsilon_{r,1}A_{r,1}} = 4 \\
(R_{r,\epsilon})_2 &= \frac{1-\epsilon_{r,2}}{\epsilon_{r,2}A_{r,2}} = 1 \\
(R_{r,\epsilon})_3 &= \frac{1-\epsilon_{r,3}}{\epsilon_{r,2}A_{r,3}} = 1.
\end{aligned}
$$

Therefore, the equations for the net heat transfer rates become

$$
\begin{aligned}
\frac{E_{b,1}-(q_{r,o})_1}{4} &= \frac{(q_{r,o})_1-(q_{r,o})_2}{2} + \frac{(q_{r,o})_1-(q_{r,o})_3}{2} \\
\frac{E_{b,2}-(q_{r,o})_2}{1} &= \frac{(q_{r,o})_2-(q_{r,o})_1}{2} + \frac{(q_{r,o})_2-(q_{r,o})_3}{2} \\
\frac{E_{b,3}-(q_{r,o})_3}{1} &= \frac{(q_{r,o})_3-(q_{r,o})_1}{2} + \frac{(q_{r,o})_3-(q_{r,o})_2}{2}.
\end{aligned}
$$

Upon re-arranging, we have

$$
\begin{aligned}
1.25(q_{r,o})_1 - 0.5(q_{r,o})_2 - 0.5(q_{r,o})_3 &= 0.25E_{b,1} \\
-0.5(q_{r,o})_2 + 2(q_{r,o})_2 - 0.5(q_{r,o})_3 &= E_{b,2} \\
-0.5(q_{r,o})_1 - 0.5(q_{r,o})_2 + 2(q_{r,o})_3 &= E_{b,3},
\end{aligned}
$$

where $E_{b,1} = E_{b,2} = 1{,}451.52\ \text{W/m}^2$ and $E_{b,3} = 459.27\ \text{W/m}^2$.
Solving the linear system of equations above (e.g., using SOPHT) we obtain, $(q_{r,o})_1 = 1{,}090.7\ \text{W/m}^2, (q_{r,o})_2 = 1{,}198.95\ \text{W/m}^2$, and $(q_{r,o})_3 = 802.047\ \text{W/m}^2$.
Therefore, the net heat transfer rate between surfaces 1 and 2 is

$$
\begin{aligned}
Q_{r,1\text{-}2} &= \frac{(q_{r,o})_1-(q_{r,o})_2}{\dfrac{1}{F_{1\text{-}2}A_{r,1}}} = \frac{(1{,}090.7-1{,}198.95)(\text{W/m}^2)}{2(1/\text{m}^2)} \\
&= -54.13\ \text{W}.
\end{aligned}
$$

(b) The negative sign indicates that heat is transferred from surface 2 to surface 1, i.e., from the larger to the smaller one.

(c) For $T_3 = 500$ K, we have $E_{b,3} = 3{,}543.75\ \text{W/m}^2$, and solving the new system of linear equations, we obtain $(q_{r,o})_1 = 2{,}212.33\ \text{W/m}^2$, $(q_{r,o})_2 = 1{,}984.09\ \text{W/m}^2$ and $(q_{r,o})_3 = 2{,}820.98\ \text{W/m}^2$, and the net heat transfer rate between surfaces 1 and 2 becomes

$$
Q_{r,1\text{-}2} = \frac{(2{,}212-1{,}984)(\text{W/m}^2)}{2(1/\text{m}^2)} = 114.1\ \text{W}.
$$

Note that the heat transfer now occurs from surface 1 to surface 2, i.e., from the smaller to the larger emissivity.

(d) For $Q_{r,1\text{-}2} = 0$, we need $T_3 = 400$ K. In this case $(q_{r,o})_1 = (q_{r,o})_2 = (q_{r,o})_3 = E_{b,1} = E_{b,2} = E_{b,3} = 1{,}452$ W/m^2.

COMMENT:

When the temperatures are equal, but emissivities or areas are not, the presence of a third surface results in a net radiation heat transfer between these surfaces.

PROBLEM 4.26.FAM

GIVEN:

Surface-radiation absorption is used to melt solid silicon oxide powders used for glass making. The heat is provided by combustion occurring over an impermeable surface 1 with dimensions $a = w = 1$ m, as shown in Figure Pr.4.26. The desired surface temperature T_1 is 1,600 K. The silicon oxide powders may be treated as a surface 2, with the same area as the radiant heater, at a distance $l = 0.25$ m away from the heater, and at a temperature $T_2 = 873$ K. The surroundings are at $T_3 = 293$ K. Assume that all surfaces are ideal blackbody surfaces.

SKETCH:

Figure Pr.4.26(a) shows the radiating surface 1 heating surface 2 in a three-surface enclosure.

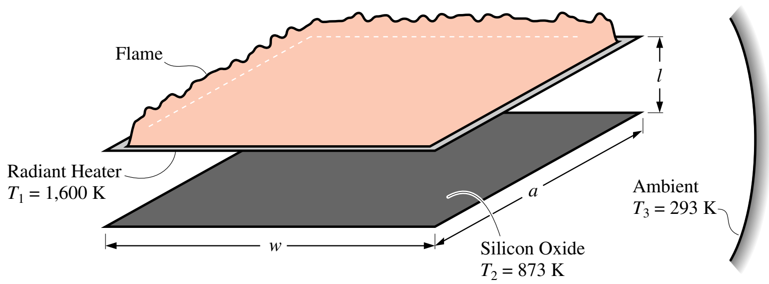


Figure Pr.4.26(a) Surface radiant heater heated by a flame over it and forming a three-surface radiation enclosure.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the net radiation heat transfer to the silicon oxide surface.

SOLUTION:

(a) The thermal circuit for a three-surface enclosure is shown in Figure Pr.4.26(b).

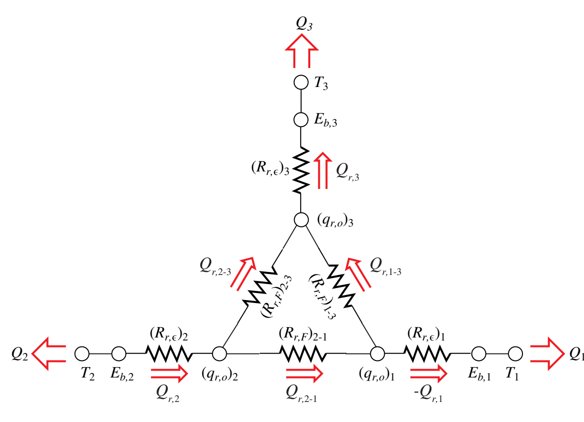


Figure Pr.4.26(b) Graybody thermal circuit diagram.

(b) Since all the surfaces are blackbodies ($\epsilon_{r,i} = 1$) the surface-grayness resistances are zero and the radiosities become equal to the blackbody emissive powers. The thermal circuit then reduces to the one shown in Figure Pr.4.26(c). The net radiation heat transfer rate leaving surface 2 is

$$\begin{aligned} Q_{r,2} &= Q_{r,2\text{-}1} + Q_{r,2\text{-}3} \\ &= \frac{E_{b,2} - E_{b,1}}{(R_{r,F})_{2\text{-}1}} + \frac{E_{b,2} - E_{b,3}}{(R_{r,F})_{2\text{-}3}}. \end{aligned}$$

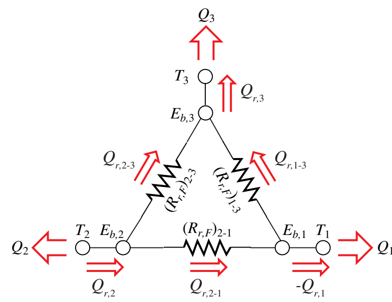


Figure Pr.4.26(c) Blackbody thermal circuit diagram.

The view-factor resistances are

$$(R_{r,F})_{2\text{-}1} = \frac{1}{A_{r,2}F_{2\text{-}1}}, \qquad (R_{r,F})_{2\text{-}3} = \frac{1}{A_{r,2}F_{2\text{-}3}}.$$

The view factor between surfaces 2 and 1 can be evaluated from Figure 4.11(b). From the dimensions of the two plates and the distance separating them, $w^* = w/l = 1(\text{m})/0.25(\text{m}) = 4$, $a^* = a/l = 1(\text{m})/0.25(\text{m}) = 4$. The view factor is then approximately $F_{1\text{-}2} = 0.63$. Using the reciprocity rule (4.34), $F_{2\text{-}1}A_{r,2} = F_{1\text{-}2}A_{r,1}$ and as $A_{r,1} = A_{r,2}$, $F_{2\text{-}1} = F_{1\text{-}2} = 0.63$. Using the summation rule for surface 2 we have $F_{2\text{-}1} + F_{2\text{-}2} + F_{2\text{-}3} = 1$. Since $F_{2\text{-}2} = 0$ (because surface 2 is flat), we have $F_{2\text{-}3} = 1 - F_{2\text{-}1} = 0.37$.
Then, the view-factor resistances become

$$\begin{aligned}
(R_{r,F})_{2\text{-}1} &= \frac{1}{A_{r,2}F_{2\text{-}1}} = \frac{1}{1\,(\text{m}) \times 1\,(\text{m}) \times 0.63} = 1.59 \quad 1/\text{m}^2 \\
(R_{r,F})_{2\text{-}3} &= \frac{1}{A_{r,2}F_{2\text{-}3}} = \frac{1}{1\,(\text{m}) \times 1\,(\text{m}) \times 0.37} = 2.70 \quad 1/\text{m}^2.
\end{aligned}$$

Finally, the net heat transfer rate leaving surface 2 is

$$\begin{aligned}
Q_{r,2} &= \frac{\sigma_{\text{SB}}\left(T_2^4 - T_1^4\right)}{(R_{r,F})_{2\text{-}1}} + \frac{\sigma_{\text{SB}}\left(T_2^4 - T_3^4\right)}{(R_{r,F})_{2\text{-}3}} \\
&= \frac{5.67 \times 10^{-8}\left(\text{W/m}^2\text{-K}^4\right)\left[873^4\left(\text{K}^4\right) - 1{,}600^4\left(\text{K}^4\right)\right]}{1.59\left(1/\text{m}^2\right)} + \\
&\quad \frac{5.67 \times 10^{-8}\left(\text{W/m}^2\text{-K}^4\right)\left[873^4\left(\text{K}^4\right) - 293^4\left(\text{K}^4\right)\right]}{2.70\left(1/\text{m}^2\right)} \\
&= -213{,}353\,(\text{W}) + 12{,}031\,(\text{W}) = -201{,}322\ \text{W}.
\end{aligned}$$

COMMENT:
One of the difficulties in operating an oven like this is stabilizing the flame over of the ceramic plate. One alternative is to use a porous radiant burner. However, an impermeable ceramic plate prevents the combustion products from contaminating the glass.

PROBLEM 4.27.FUN

GIVEN:

Surface-radiation emission can be redirected to a receiving surface using reradiating surfaces. Figure Pr.4.27(a) renders such a redirection design using a reradiating surface 3. Surface 3 is ideally insulated and is treated as a single surface having a uniform temperature T_3. Surface 1 has a temperature T_1 higher than that of surface 2, T_2.

$R_1 = 25$ cm, $R_2 = 25$ cm, $F_{1\text{-}2} = 0.1$, $\epsilon_{r,1} = 1.0$, $\epsilon_{r,2} = 1.0$, $T_1 = 900$ K, $T_2 = 400$ K.

Note that since surfaces 1 and 2 are blackbody surfaces, $(q_{r,o})_1 = E_{b,1}$ and $(q_{r,o})_2 = E_{b,2}$.

SKETCH:

Figure Pr.4.27(a) shows the two blackbody, surface-radiation heat transfer surfaces, and the reradiating third surface.

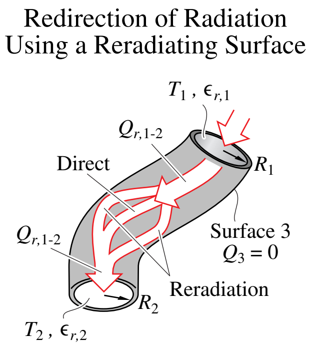


Figure Pr.4.27(a) Two blackbody surfaces that are exchanging surface-radiation heat and are completely enclosed by a reradiation surface.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine $Q_{r,1\text{-}2}$ for the given conditions.
(c) Compare this with $Q_{r,1\text{-}2}$ without reradiation.
(d) Show the expression for $Q_{r,1\text{-}2}$ for the case of $F_{1\text{-}2} = 0$ and comment on this expression.

SOLUTION:

(a) The thermal circuit diagram for surface radiation from surface 1 to surface 2, with the presence of the reradiating surface 3, is shown in Figure Pr.4.27(b).

(b) The net surface radiation between surface 1 and 2 is determined from Figure 4.27(b) or from (4.60). Then for the case of $\epsilon_{r,1} = \epsilon_{r,2} = 1$, we have

$$Q_{r,1\text{-}2} = Q_{r,1} = \frac{E_{b,1} - E_{b,2}}{\dfrac{1}{A_{r,1}F_{1\text{-}2} + \dfrac{1}{\dfrac{1}{A_{r,1}F_{1\text{-}3}} + \dfrac{1}{A_{r,2}F_{2\text{-}3}}}}},$$

where

$$\begin{aligned} A_{r,1} = A_{r,2} &= \pi R^2 = \pi(0.25)^2(\text{m}^2) \\ &= 0.1963 \text{ m}^2. \end{aligned}$$

We use the summation rule (4.33) to find

$$\begin{aligned} F_{1\text{-}3} &= 1 - F_{1\text{-}1} - F_{1\text{-}2} = 1 - 0 - 0.1 = 0.9 \\ F_{2\text{-}3} &= 1 - F_{2\text{-}2} - F_{2\text{-}1} = 1 - F_{2\text{-}2} - \frac{A_{r,1}F_{1\text{-}2}}{A_{r,2}} = 0.9. \end{aligned}$$

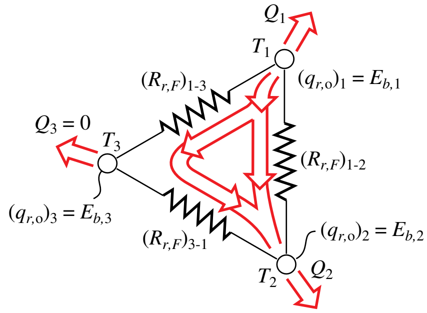


Figure Pr.4.27(b) Thermal circuit diagram.

$$
\begin{aligned}
Q_{r,1\text{-}2}|_{\text{with reradiation}} &= \frac{5.67\times10^{-8}(\text{W/m}^2\text{-K}^4)\times(900^4-400^4)(\text{K}^4)}{\left[0.1963(\text{m}^3)\times0.1+\dfrac{1}{\dfrac{1}{0.1963(\text{m}^3)\times0.9}+\dfrac{1}{0.1963(\text{m}^3)\times0.9}}\right]^{-1}}\\
&= \frac{3.575\times10^4(\text{W/m}^2)}{[0.01963(\text{m}^3)+0.08834(\text{m}^2)]^{-1}}\\
&= 3.860\ \text{kW}.
\end{aligned}
$$

(c) For $Q_{r,1\text{-}2}|_{\text{no reradiation}}$, we have from Figure Pr.4.27(b)

$$
\begin{aligned}
Q_{r,1\text{-}2}|_{\text{no reradiation}} &= \frac{E_{b,1}-E_{b,2}}{\dfrac{1}{A_{r,1}F_{1\text{-}2}}}\\
&= \frac{3.575\times10^4(\text{W/m}^2)}{[0.01963(\text{m}^2)]^{-1}}\\
&= 0.7018\ \text{kW}.
\end{aligned}
$$

This is substantially less than the one with reradiation, i.e., thus it is only 18.18% of (b).

(d) For the case of $F_{1\text{-}2}\rightarrow0$, we have $F_{1\text{-}3}\rightarrow1$, $F_{2\text{-}3}\rightarrow1$. Note that we still assume that we have a three-surface enclosure. Then (4.60) becomes

$$
\begin{aligned}
Q_{r,1\text{-}2} &= (E_{b,1}-E_{b,2})\left[\frac{1}{A_{r,1}F_{1\text{-}3}}+\frac{1}{A_{r,1}F_{2\text{-}3}}\right]^{-1}\\
&= (E_{b,1}-E_{b,2})\frac{A_{r,1}F_{1\text{-}3}}{2}\\
&= \frac{A_{r,1}}{2}(E_{b,1}-E_{b,2}) \quad \text{for} \quad F_{1\text{-}2}\rightarrow0.
\end{aligned}
$$

Examining Figure Pr.4.27(b) shows that for $(R_{r,F})_{1\text{-}2}\rightarrow\infty$, the radiation is completely transferred by reradiation, subject to two view-factor resistances.

COMMENT:

The reradiating surface 3 does facilitate surface-radiation heat transfer between surfaces 1 and 2, but there is a finite geometrical resistance associated with this participation. Note that we have assumed a three-surface enclosure as we allowed $F_{1\text{-}2}\rightarrow0$, which also allowed for surface radiation heat transfer. In practice, as $F_{1\text{-}2}\rightarrow0$, these will only by a two-surface enclosure.

PROBLEM 4.28.FAM

GIVEN:

Fire barriers are used to temporarily protect spaces adjacent to fires. Figure Pr.4.28 shows a suspended fire barrier of thickness L and effective conductivity $\langle k \rangle$ (and $\langle \rho \rangle$ and $\langle c_p \rangle$) subjected to a flame irradiation $(q_{r,i})_f$. The barrier is a flexible, wire-reinforced mat made of a ceramic (high melting temperature, such as ZrO_2) fibers. The barrier can withstand the high temperatures resulting from the flow of $(q_{r,i})_f$ into the mat until, due to thermal degradation of the fibers and wires, it fails. In some cases the barrier is actively water sprayed to delay this degradation.

The transient conduction through the mat, subject to a constant $(q_{r,i})_f$, can be treated analytically up to the time that thermal penetration distance δ_α reaches the back of the mat $x = L$. This is done by using the solution given in Table 3.4 for a semi-infinite slab, and by neglecting any surface radiation emission and any surface convection. Assume that these simplifications are justifiable and the transient temperature $T(x,t)$ can then be obtained, subject to a constant surface flux $q_s = (q_{r,i})_f$ and a uniform initial temperature $T(t=0)$.

$L = 3$ cm, $\langle k \rangle = 0.2$ W/m-K, $\langle \rho \rangle = 600$ kg/m^3, $c_p = 1{,}000$ J/kg-K, $(q_{r,i})_f = -10^5$ W/m^2, $T(t=0) = 40$°C.

SKETCH:

Figure Pr.4.28 shows the suspended fire-barrier mat.

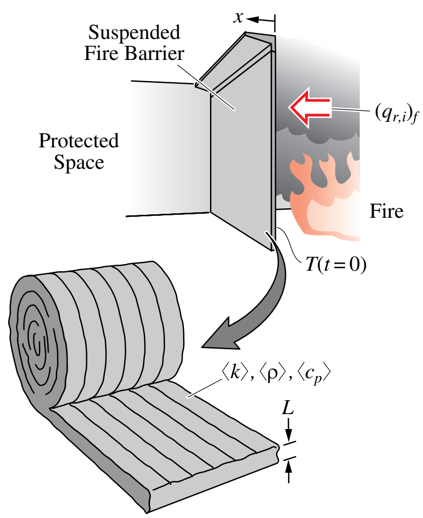


Figure Pr.4.28 A fire barrier is used to protect a space adjacent to a fire.

OBJECTIVE:

(a) Determine the elapsed time t for the thermal penetration using (3.148).

(b) Determine the surface temperature $T(x=0,t)$ at this elapsed time.

(c) Using the melting temperature of ZrO_2 in Figure 3.8, would the mat disintegrate at this surface?

SOLUTION:

(a) Using the thermal penetration depth given by (3.148), we have

$$\delta_\alpha = L = 3.6(\alpha t)^{1/2}, \quad \alpha = \frac{\langle k \rangle}{\langle \rho \rangle \langle c_p \rangle}$$

$$\alpha = \frac{0.2(\text{W/m-K})}{600(\text{kg/m}^3) \times 1{,}000(\text{J/kg-K})} = 3.333 \times 10^{-7} \text{ m}^2/\text{s}.$$

Solving for t, we have

$$t = \frac{L^2}{(3.6)^2 \alpha} = \frac{(0.03)^2(\text{m}^2)}{(3.6)^2 \times 3.333 \times 10^{-7}(\text{m}^2/\text{s})} = 208.3 \text{ s}.$$

(b) Using Table 3.4, for $q_s = (q_{r,i})_f$, and for $x = 0$ we have the transient surface temperature given by

$$\begin{aligned}
T(x=0,t) &= T(t=0) - \frac{(q_{r,i})_f(4\alpha t)^{1/2}}{\pi^{1/2}\langle k\rangle} \\
&= 40(℃) - \frac{(-10^5)(\mathrm{W/m^2}) \times [4 \times 3.333 \times 10^{-7}(\mathrm{m^2/s}) \times 208.3(\mathrm{s})]^{1/2}}{\pi^{1/2} \times 0.2(\mathrm{W/m\text{-}K})} \\
&= 40(℃) + 4{,}701(℃) \\
&= 4{,}741℃.
\end{aligned}$$

(c) From Table 3.9, for ZrO_2, we have a melting temperature of $T_{lg} = 2{,}715$℃. We expect the mat to melt (and sublimated) at this surface.

COMMENT:

The flame irradiation is readily estimated using (4.62), with the known relevant thermal-chemical-physical properties of the flame. Here we did not include the heat of melting (and sublimation) of the mat materials. Inclusion of these reduces the surface temperature. Also by soaking the mat with water, the temperature is further reduced (due to evaporation energy conversion).

PROBLEM 4.29.FAM

GIVEN:

Using reflectors (mirrors) to concentrate solar irradiation allows for obtaining very large (concentrated) irradiation flux. Figure Pr.4.29 shows a parabolic concentrator that results in concentration irradiation flux $(q_{r,i})_c$, which is related to the geometric parameters through the energy equation applied to solar energy, i.e.,

$$(q_{r,i})_s wL = (q_{r,i})_c DL,$$

where D is the diameter, DL is the projected cross-sectional area of the receiving tube, and wL is the projected concentrator cross-sectional area receiving solar irradiation.

The concentrated irradiation is used to produce steam from saturated (at $T = T_{lg}$) water, where the water mass flow rate is $\dot{M}_l$. The absorptivity of the collector is $\alpha_{r,c}$ and its emissivity $\epsilon_{r,c}$ is lower (nongray surface). In addition to surface emission, the collector loses heat to the ambient through surface convection and is given as a prescribed Q_{ku}. Assume that collector surface temperature is $T_c = T_{lg}$.

$(q_{r,i})_s = 200$ W/m^2, $Q_{ku} = 400$ W, $T_c = T_{lg} = 127$°C, $\alpha_{r,c} = 0.95$, $\epsilon_{r,c} = 0.4$, $D = 5$ cm, $w = 3$ m, $L = 5$ m.

Use Table C.27 for properties of saturated water.

SKETCH:

Figure Pr.4.29(a) shows the concentration and the steam producing collector.

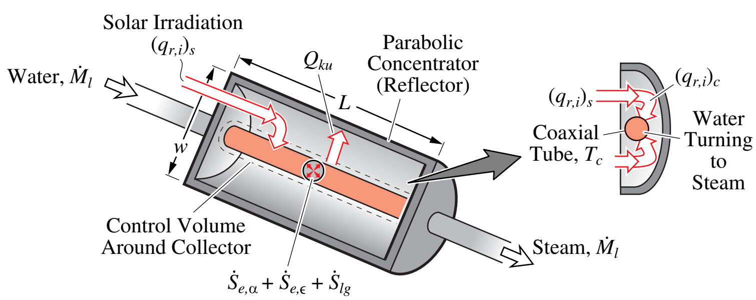


Figure Pr.4.29(a) A concentrator-solar collector system used for steam production.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the stream production rate $\dot{M}_l$.

SOLUTION:

(a) Figure Pr.4.29(b) shows the thermal circuit diagram. The only surface heat transfer is the surface convection and there are three energy conversion mechanisms (because the surface is a nongray surface, radiation absorption and emission are treated as energy conversions).

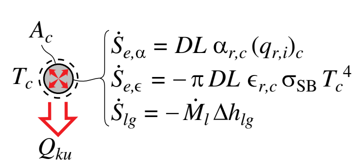


Figure Pr.4.29(b) Thermal circuit diagram.

(b) The energy equation from Figure Pr.4.29(b), and similarly from (4.60), becomes

$$\begin{aligned} Q|_{A,c} &= \dot{S}_{r,\alpha} + \dot{S}_{r,\epsilon} + \dot{S}_{lg} \\ Q_{ku} &= DL\alpha_{r,c}(q_{r,i})_c - \pi DL\epsilon_{r,c}\sigma_{\rm SB}T_c^4 - \dot{M}_l\Delta h_{lg}. \end{aligned}$$

Note that for the irradiation we have used the projected area DL and for the emission we have used the surface area πDL. Solving for $\dot{M}_l$, and using $DL(q_{r,i})_c = wL(q_{r,i})_s$, we have

$$\dot{M}_l = \frac{\alpha_{r,c} wL(q_{r,i})_s - \pi DL\epsilon_{r,c}\sigma_{\rm SB}T_c^4 - Q_{ku}}{\Delta h_{lg}}.$$

From Table C.27, at $T = (273.15 + 127)(\mathrm{K}) = 400.15$ K, we have

$$\Delta h_{lg} = 2.183 \times 10^6 \ \mathrm{J/kg}.$$

Using the numerical values, we have

$$\begin{aligned} \dot{M}_l &= \frac{0.95 \times 3(\mathrm{m}) \times 5(\mathrm{m}) \times 200(\mathrm{W/m^2}) - \pi \times 0.05(\mathrm{m}) \times 0.4 \times 5(\mathrm{m}) \times 5.67 \times 10^{-8}(\mathrm{W/m^2\text{-}K^4}) \times (400.15)^4(\mathrm{K})^4 - 400(\mathrm{W}}{2.183 \times 10^6(\mathrm{J/kg})} \\ &= \frac{(2{,}850 - 456.8 - 400)(\mathrm{W})}{2.183 \times 10^6(\mathrm{J/kg})} = 9.131 \times 10^{-4} \ \mathrm{kg/s} = 0.9131 \ \mathrm{g/s}. \end{aligned}$$

COMMENT:

Note that from Figure 4.18, the value we used for $(q_{r,i})_s$ is close to the annual average over the earth surface, i.e., $\overline{\langle (q_{r,i})_s \rangle}_A = 172.4$ W/m^2. The seasonal and daily peaks in $(q_{r,i})_s$ lead to much larger instantaneous stream production rates.

PROBLEM 4.30.FAM

GIVEN:

Pulsed lasers may be used for the ablation of living-cell membrane in order to introduce competent genes, in gene therapy. This is rendered in Figure Pr.4.30(a). The ablation (or scissors) laser beam is focused on the cell membrane using a neodymium yttrium aluminum garnet (Nd:YAG) laser with $\lambda = 532$ nm $= 0.532$ μm, and a focus spot with diameter $D = 500$ μm. There is a Gaussian distribution of the irradiation across D, but here we assume a uniform distribution.

Assume a steady-state heat transfer. Although the intent is to sublimate $\dot{S}_{sg}$ the targeted membrane region for a controlled depth (to limit material removal to the thin, cell membrane), the irradiation energy is also used in some exothermic chemical reaction $\dot{S}_{r,c}$ and in some heat losses presented by Q (this includes surface emission).

$D = 500$ nm, $L = 10$ nm, $\rho = 2 \times 10^3$ kg/m^3, $(q_{r,i})_l = 10^{10}$ W/m^2, $\Delta h_{sg} = 3 \times 10^6$ J/kg, $\dot{S}_{r,c} = -7 \times 10^{-4}$ W, $Q = 3 \times 10^{-4}$ W, $\alpha_r = 0.9$.

SKETCH:

Figure Pr.4.30(a) shows the ablating membrane.

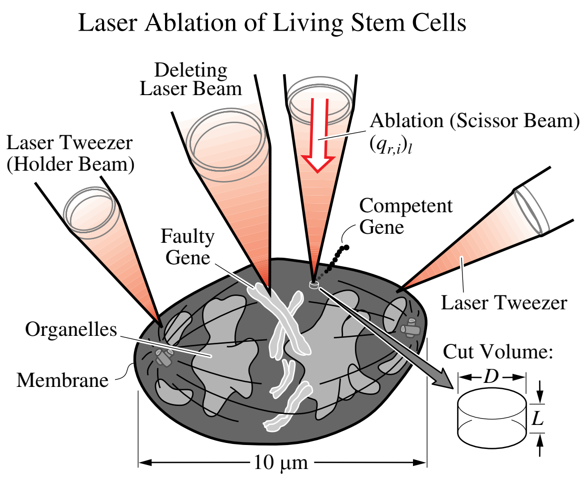


Figure Pr.4.30(a) A living cell is ablated at a region on its membrane, for introduction of competent genes.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the required duration of the laser pulse Δt, for the given conditions.

SOLUTION:

(a) The thermal circuit diagram is given in Figure Pr.4.30(b). Under steady state, the irradiation energy is used for sublimation, endothermic chemical reaction, and heat losses.

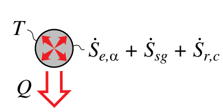


Figure Pr. 4.30(b) Thermal circuit diagram.

(b) From Figure Pr.4.30(b), and similarly from (4.60), the energy equation for the targeted volume is

$$Q|_A = Q = \dot{S}_{e,\alpha} + \dot{S}_{sg} + \dot{S}_{r,c}$$

or

$$\begin{aligned} Q &= \alpha_r \frac{\pi D^2}{4}(q_{r,i})_l - \dot{M}\Delta h_{sg} + \dot{S}_{r,c} \\ &= \alpha_r \frac{\pi D^2}{4}(q_{r,i})_l - \rho \frac{\pi D^2 L}{4\Delta t}\Delta h_{sg} + \dot{S}_{r,c} \end{aligned}$$

Solving for Δt, we have

$$\begin{aligned} \frac{1}{\Delta t} &= \frac{\alpha_r \frac{\pi D^2}{4}(q_{r,i})_l + \dot{S}_{r,c} - Q}{\rho \frac{\pi D^2 L}{4}\Delta h_{sg}} \\ &= \frac{0.9 \times \frac{\pi(5\times10^{-7})^2(\mathrm{m}^2)}{4} \times 10^{10}(\mathrm{W/m^2}) - 7\times10^{-4}(\mathrm{W}) - 3\times10^{-4}(\mathrm{W})}{\frac{2\times10^3(\mathrm{kg/m^3})\times\pi\times(5\times10^{-7})^2(\mathrm{m}^2)\times10^{-8}(\mathrm{m})\times3\times10^6(\mathrm{J/kg})}{4}} \\ &= \frac{1.767\times10^{-3}(\mathrm{W}) - 7\times10^{-4}(\mathrm{W}) - 3\times10^{-4}(\mathrm{W})}{1.178\times10^{-11}} \\ \Delta t &= 1.536\times10^{-8}\ \mathrm{s} = 15.36\ \mathrm{ns}. \end{aligned}$$

COMMENT:

Due to lack of specific data, we have estimated the heat of sublimation based on a physical bond similar to water.

PROBLEM 4.31.FUN

GIVEN:

When thermal radiation penetrates a semitransparent medium, e.g., a glass plate, reflection, absorption, and transmission occur at the interface between two adjacent media along the radiation path, e.g., each of the glass/air interfaces for a glass plate surrounded by air. These multiple absorptions and reflections result in an overall attenuation, absorption, and reflection of the radiation incident in the glass plate. These effects are modeled as overall transmittance, absorptance, and emittance. The glass plate is then assumed diffuse and gray.

Consider a glass plate bounded by its two infinite, parallel surfaces, as shown in Figure Pr.4.31. The glass plate has a transmittance $\tau_{r,2} = 0.1$ and an absorptance $\alpha_{r,2} = 0.7$. Its temperature T_2 is assumed uniform across the thickness. The surfaces are opaque, diffuse, and gray with total emissivity $\epsilon_{r,1} = \epsilon_{r,3} = 0.5$.

SKETCH:

The semi-transparent layer is shown in Figure Pr.4.31, along with the various radiation flux terms.

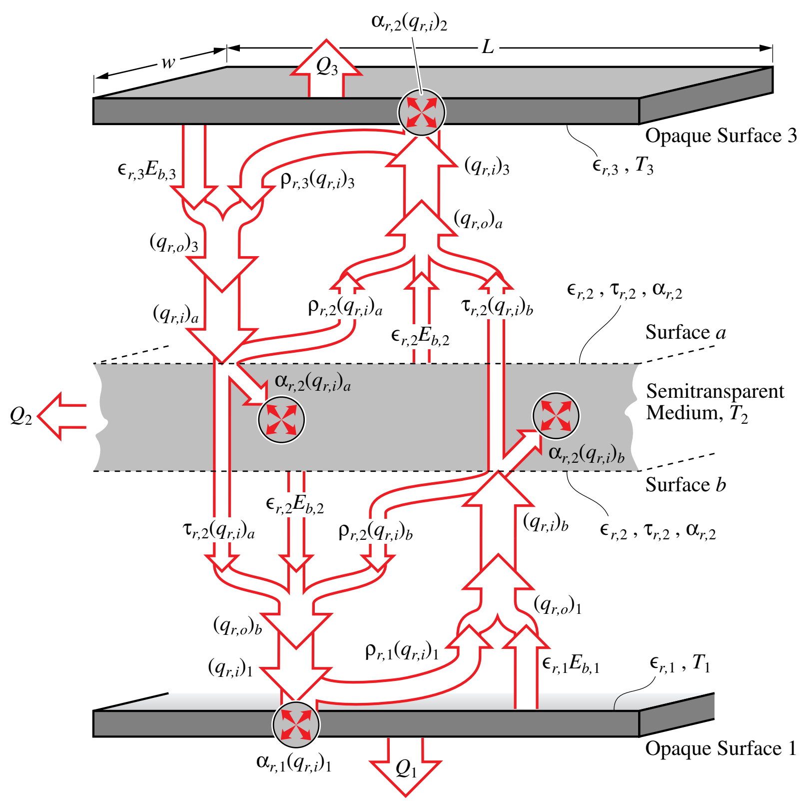


Figure Pr.4.31 Radiative heat transfer across a glass plate. The glass is placed between two parallel solid surfaces.

OBJECTIVE:

(a) Show that the net heat transfer by surface radiation from both sides of the glass plate is given by

$$\frac{Q_{r,a}}{A_{r,2}} = \frac{1}{\rho_{r,2}\,(1-\zeta^2)}\left\{[\rho_{r,2}(1-\zeta^2)-1](q_{r,o})_a + \zeta(q_{r,o})_b + (1-\zeta)\epsilon_{r,2}\right\}$$

$$\frac{Q_{r,b}}{A_{r,2}} = \frac{1}{\rho_{r,2}\,(1-\zeta^2)}\left\{[\rho_{r,2}(1-\zeta^2)-1](q_{r,o})_b + \zeta(q_{r,o})_a + (1-\zeta)\epsilon_{r,2}\right\},$$

where $\zeta = \rho_{r,2}/\tau_{r,2}$.

(b) Write a system of equations that would allow for the solution of the problem and identify the set of variables that are being solved for and the number of variables that need to be known.

(c) If there is no other heat transfer from the glass plate, write the energy equation to be solved the glass plate temperature T_2 and write the expression for T_2.
(d) Compare the results of this problem with the results of Problem Pr.4.31. Is there an analogy between the transmittance of the glass plate and the porosity of the screen?

SOLUTION:
(a) A radiation balance at any surface gives

$$\frac{Q_r}{A} = q_{r,o} - q_{r,i}.$$

For surface b of the glass plate, the radiosity $(q_{r,o})_b$ is given by

$$(q_{r,o})_b = \tau_{r,2}(q_{r,i})_a + \rho_{r,2}(q_{r,i})_b + \epsilon_{r,2}E_{b,2}.$$

Analogously, the radiosity for surface a is given by

$$(q_{r,o})_a = \tau_{r,2}(q_{r,i})_b + \rho_{r,2}(q_{r,i})_a + \epsilon_{r,2}E_{b,2}.$$

Solving for the irradiation $(q_{r,i})_a$, we have

$$(q_{r,i})_a = \frac{(q_{r,o})_a}{\rho_{r,2}} - \frac{\epsilon_{r,2}E_{b,2}}{\rho_{r,2}} - \frac{\tau_{r,2}}{\rho_{r,2}}(q_{r,o})_b.$$

Substituting this into the equation for $(q_{r,o})_b$ and rearranging, we have

$$(q_{r,o})_b = \frac{\tau_{r,2}}{\rho_{r,2}}(q_{r,o})_a + \left(\rho_{r,2} - \frac{\tau_{r,2}^2}{\rho_{r,2}}\right)(q_{r,0})_b + \left(1 - \frac{\tau_{r,2}}{\rho_{r,2}}\right)\epsilon_{r,2}E_{b,2}.$$

Solving for $(q_{r,o})_b$, we obtain

$$(q_{r,o})_b = \frac{1}{\rho_{r,2}\left(1 - \dfrac{\tau_{r,2}^2}{\rho_{r,2}^2}\right)}\left[(q_{r,o})_b - \frac{\tau_{r,2}}{\rho_{r,2}}(q_{r,o})_a - \left(1 - \frac{\tau_{r,2}}{\rho_{r,2}}\right)\epsilon_{r,2}E_{b,2}\right].$$

Then, substituting this expression into the expression for $Q_{r,b}/A$, we finally obtain

$$\frac{Q_{r,b}}{A} = \frac{1}{\rho_{r,2}(1-\zeta^2)}\{[\rho_{r,2}(1-\zeta^2) - 1](q_{r,o})_b + \zeta(q_{r,o})_a + (1-\zeta)\epsilon_{r,2}E_{b,2}\}$$

where, $\zeta = \tau_{r,2} \ / \ \rho_{r,2}$.
By symmetry, for surface a we obtain

$$\frac{Q_{r,a}}{A} = \frac{1}{\rho_{r,2}(1-\zeta^2)}\{[\rho_{r,2}(1-\zeta^2) - 1](q_{r,o})_a + \zeta(q_{r,o})_b + (1-\zeta)\epsilon_{r,2}E_{b,2}\}.$$

(b) The solution would require energy equations for all surfaces and relations for radiation exchange between the surfaces.
Surface 1:

$$\begin{aligned}
\frac{Q_{r,b}}{A_{r,1}} &= \frac{1}{\rho_{r,2}}[\epsilon_{r,1}E_{b,1} - (1-\rho_{r,1})(q_{r,o})_1] \\
\frac{Q_{r,1\text{-}b}}{A_{r,1}} &= (q_{r,o})_1 - (q_{r,o})_b \\
-Q_1 &= Q_{r,1} = Q_{r,1\text{-}b} \\
E_{b,1} &= \sigma_{\rm SB}T_1^4.
\end{aligned}$$

Surface 3:

$$\begin{aligned}\frac{Q_{r,3}}{A_{r,3}} &= \frac{1}{\rho_{r,3}}[\epsilon_{r,3}E_{b,3} - (1-\rho_{r,3})(q_{r,o})_3]\\ \frac{Q_{r,3\text{-}a}}{A_{r,3}} &= (q_{r,o})_3 - (q_{r,o})_a\\ -Q_3 &= Q_{r,3} = Q_{r,3\text{-}a}\\ E_{b,3} &= \sigma_{\rm SB}T_3^4.\end{aligned}$$

Surface 2:

$$\begin{aligned}\frac{Q_{r,b}}{A_{r,2}} &= \frac{1}{\rho_{r,2}(1-\zeta^2)}\{[\rho_{r,2}(1-\zeta^2)-1](q_{r,o})_b + \zeta(q_{r,o})_a + (1-\zeta)\epsilon_{r,2}E_{b,2}\}\\ \frac{Q_{r,a}}{A_{r,2}} &= \frac{1}{\rho_{r,2}(1-\zeta^2)}\{[\rho_{r,2}(1-\zeta^2)-1](q_{r,o})_a + \zeta(q_{r,o})_b + (1-\zeta)\epsilon_{r,2}E_{b,2}\}\end{aligned}$$

$$\begin{aligned}Q_{r,a\text{-}3} &= -Q_{r,3\text{-}a}\\ Q_{r,b\text{-}1} &= -Q_{r,1\text{-}b}\\ -Q_2 &= Q_{r,b} + Q_{r,a}\\ Q_{r,b} &= -Q_{r,b\text{-}1}\\ Q_{r,a} &= -Q_{r,a\text{-}3}\\ E_{b,2} &= \sigma_{\rm SB}T_2^4.\end{aligned}$$

The unknowns are,

$$T_1, T_2, T_3, Q_1, Q_2, Q_3, Q_{r,1}, Q_{r,2}, Q_{r,3}, Q_{r,1\text{-}b}, Q_{r,b\text{-}1}, Q_{r,3\text{-}a}, Q_{r,a\text{-}3},$$
$$E_{b,1}, E_{b,2}, E_{b,3}, (q_{r,o})_1, (q_{r,o})_a, (q_{r,o})_b, (q_{r,o})_3$$

There are 20 unknowns and 17 equations. Therefore, 3 unknowns need to be specified. These could be, for example, the temperatures T_1 and T_3 and the heat transfer rate Q_2 could be set to zero.

(c) Using the expressions for $(q_{r,o})_a$ and $(q_{r,o})_b$ in the energy equation for the glass plate, we obtain

$$-\frac{Q_2}{A_{r,2}} = \frac{1}{\rho_{r,2}(1-\zeta^2)}\{2(1-\zeta)\epsilon_{r,2}E_{b,2} - [1-\zeta-\rho_{r,2}(1-\zeta^2)][(q_{r,o})_a + (q_{r,o})_b]$$

For the case when $Q_2 = 0$, after solving the equation above for $E_{b,2}$, we have

$$\begin{aligned}E_{b,2} &= \frac{1-\rho_{r,2}(1+\zeta)}{2\epsilon_{r,2}}[(q_{r,o})_a + (q_{r,o})_b]\\ &= \frac{\alpha_{r,2}}{2\epsilon_{r,2}}[(q_{r,o})_a + (q_{r,o})_b],\end{aligned}$$

or

$$T_2 = \frac{1}{\sigma_{\rm SB}}\left\{\frac{\alpha_{r,2}}{2\epsilon_{r,2}}[(q_{r,o})_a + (q_{r,o})_b]\right\}^{1/4}.$$

(d) The porosity of the screen a_1 in Problem 4.5 behaves like the transmittance of the glass plate $\tau_{r,2}$. Note that, in contrast to Problem 4.5, this problem has been solved assuming that the transmitted fraction of the radiosity of surface 1 becomes part of the radiosity of surface a (the same is used for 3 and b).

COMMENT

Semi-transparent layers are treated as shown in Figure Pr.4.31, by allowing the transmitted radiation to interact with the surroundings. For semi-transparent thin films, similar relations are derived.

PROBLEM 4.32.FUN

GIVEN:

Semitransparent, fire-fighting foams (closed cell) have a very low effective conductivity and also absorb radiation. The absorbed heat results in the evaporation of water, which is the main component (97% by weight) of the foam. As long as the foam is present, the temperature of the foam is nearly that of the saturation temperature of water at the gas pressure. A foam covering (i.e., protecting) a substrate while being exposed to a flame of temperature T_f is shown in Figure Pr.4.32.

The foam density $\langle\rho\rangle$ and its thickness L, both decreases as a results of irradiation and evaporation. However, for the sake of simplicity here we assume constant $\langle\rho\rangle$ and L. The absorbed irradiation, characterized by the flame irradiation flux $(q_{r,i})_f$ and by the foam extinction coefficient σ_{ex}, results in the evaporation of foam. The flame is a propane-air flame with a composition given below.

$\langle\rho\rangle = 30$ kg/m^3, $\sigma_{ex} = a_1\langle\rho\rangle$, $a_1 = 3$ m^2/kg, $L = 10$ cm, $R = 1$ m, $T_f = 1{,}800$ K, $p_{CO_2} = 0.10$ atm, $p_{H_2O} = 0.13$ atm, $\epsilon_s = 10^{-7}$, $\rho_r = 0$.

Assume no heat losses.

SKETCH:

Figure Pr.4.32 shows the foam layer, the flame, and the protected substrate.

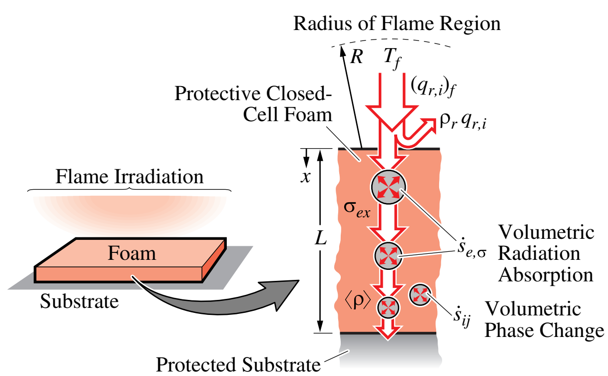


Figure Pr.4.32 A fire-fighting foam layer protecting a substrate from flame irradiation. A close-up of the closed-cell foam is also shown.

OBJECTIVE:

(a) Write the energy equation for the constant-volume foam layer.

(b) Determine the flame irradiation flux $(q_{r,i})_f$ impinging on the foam.

(c) Determine the rate of irradiation absorbed into the foam layer $\dot{S}_{e,\sigma}$. Use (2.43) and integrate it over the foam thickness L.

(d) Assuming that irradiation heat absorbed results in the foam evaporation, determine the elapsed time for the complete evaporation of the foam, Use (2.25) with Δh_{lg} being that of water at $T = 100$°C.

SOLUTION:

(a) The energy equation for the constant foam volume is the integral-volume energy equation (2.9). The result for a steady-state conduction, and with no heat loss, is

$$Q|_A = 0 = \dot{S}_{e,\sigma} + \dot{S}_{lg}.$$

From (2.43) and (2.25), we have

$$\begin{aligned}
\dot{S}_{e,\sigma} &= \int_V \dot{s}_{e,\sigma} dV = \pi R^2 \int_o^L \dot{s}_{e,\sigma} dx \\
&= \pi R^2 \int_o^L (q_{r,i})_f (1-\rho_r)\sigma_{ex} e^{-\sigma_{ex}x} dx \\
\dot{S}_{lg} &= -\dot{N}_{lg}\Delta h_{lg} = -\frac{M}{\Delta t}\Delta h_{lg} = -\frac{\langle\rho\rangle V}{\Delta t}\Delta h_{lg},
\end{aligned}$$

where we have assumed a constant evaporation rate $M/\Delta t$, and Δt is the elapsed time for complete evaporation.

(b) The flame irradiation flux is determined from (4.62), i.e.,

$$(q_{r,i})_f = \epsilon_{r,f}\sigma_{\rm SB}T_f^4.$$

The propane-air flame considered has the same condition as those of Example 4.11. From Example 4.11, we have

$$(q_{r,i}) = 9.52 \times 10^4 \text{ W/m}^2.$$

(c) The integral over L gives

$$\begin{aligned}
\dot{S}_{e,\sigma} &= \pi R^2 (q_{r,i})_f (1-\rho_r)\int_o^L \sigma_{ex} e^{-\sigma_{ex}} dx \\
&= -\pi R^2 (q_{r,i})_f (1-\rho_r) e^{-\sigma_{ex}}|_0^L \\
&= -\pi R^2 (q_{r,i})_f (1-\rho_r)(e^{-\sigma_{ex}L} - 1) \\
&= \pi R^2 (q_{r,i})_f (1-\rho_r)(1 - e^{-\sigma_{ex}L}).
\end{aligned}$$

Note that when $\sigma_{ex}L \to \infty$, all the radiation is absorbed in the foam layer. Using the numerical results, we have

$$\begin{aligned}
\dot{S}_{e,\sigma} &= \pi \times 1^2(\text{m}^2) \times 9.52 \times 10^4(\text{W/m}^2) \times (1-0) \times \left[1 - e^{-3(\text{m}^2/\text{kg})\times 30(\text{kg/m}^3)\times 0.1(\text{m})}\right] \\
&= 2.989 \times 10^5(\text{W})(1 - 0.0001234) \\
&= 2.989 \times 10^5 \text{ W}
\end{aligned}$$

This shows that all the irradiation has been absorbed by the foam layer, because $\sigma_{ex}^* = \sigma_{ex}L = 9$.

(d) From the energy equation

$$\frac{\langle\rho\rangle V}{\Delta t}\Delta h_{lg} = \dot{S}_{e,\sigma}$$

or

$$\Delta t = \frac{\langle\rho\rangle V \Delta h_{lg}}{\dot{S}_{e,\sigma}} = \frac{\langle\rho\rangle \pi R^2 L \Delta h_{lg}}{\dot{S}_{e,\sigma}}.$$

The heat of evaporation is obtained from Table C.4., i.e.,

$$\text{water:} \quad \Delta h_{lg} = 2.256 \times 10^6 \text{ J/kg} \quad \text{Table C.4.}$$

Using the numerical values, we have

$$\Delta t = \frac{30(\text{kg/m}^3) \times \pi \times 1^2(\text{m}^2) \times 0.1(\text{m}) \times 2.256 \times 10^6(\text{J/kg})}{2.989 \times 10^5(\text{W})} = 71.14 \text{ s},$$

COMMENT:

The decrease in the foam thickness L and density $\langle\rho\rangle$ will influence the absorption of irradiation. However, as long as $\sigma_{ex}L > 4$, nearly all the radiation is absorbed within the foam layer.

PROBLEM 4.33.FAM

GIVEN:

Flame radiation from a candle can be sensed by the temperature sensors existing under the thin, skin layer of the human hands. The closer the sensor (or say hand) is, the higher irradiation flux $q_{r,i}$ it senses. This is because the irradiation leaving the approximate flame surface $(q_{r,i})_f A_{r,f}$, $A_{r,f} = 2\pi R_1 L$, is conserved. This is rendered in Figure Pr.4.33. Then, assuming a spherical radiation envelope R_2, we have

$$2\pi R_1 L (q_{r,i})_f = 4\pi R_2^2 q_{r,i} \quad \text{for} \quad R_2^2 \gg R_1^2.$$

$L = 3.5$ cm, $R_1 = 1$ cm, $R_2 = 10$ cm, $T_f = 1{,}100$ K, $p_{CO_2} = 0.15$ atm, $p_{H_2O} = 0.18$ atm, $\epsilon_s = 2 \times 10^{-7}$.

SKETCH:

Figure Pr.4.33 shows the candle flame envelope and a hand sensing it a distance R_2 from the center.

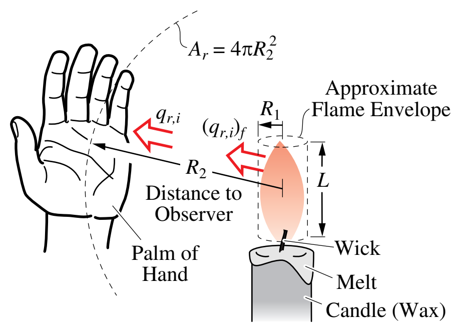


Figure Pr.4.33 A candle flame and the sensing of its irradiation at a distance R_2 from the flame center line.

OBJECTIVE:

(a) Determine the flame irradiation flux $(q_{r,i})_f$ at the flame envelope for the given heavy-soot condition.
(b) Determine $q_{r,i}$ at $r = R_2$, using the given relation.

SOLUTION:

(a) The flame irradiation flux is found from (4.62), i.e.,

$$(q_{r,i})_f = \langle \epsilon_{r,f} \rangle \sigma_{\rm SB} T_f^4,$$

where for (4.63), we have for $\Delta\epsilon_r = 0$,

$$\langle \epsilon_{r,f} \rangle = \langle \epsilon_{r,CO_2} \rangle + \langle \epsilon_{r,H_2O} \rangle + \langle \epsilon_{r,soot} \rangle.$$

The emissivity for the CO_2 and H_2O band emissions are determined using the partial pressures and the mean beam length $\langle \lambda_{ph} \rangle$. The mean beam length is found from Table 4.4, where for a cylindrical flame we have

$$\langle \lambda_{ph} \rangle = 1.9 R_1 \quad \text{Table 4.4.}$$

Then for $T_f = 1{,}100$ K, we have

$$\begin{aligned}
p_{CO_2} \langle \lambda_{ph} \rangle &= 0.15(\text{atm}) \times 1.9 \times 0.01(\text{m}) \\
&= 0.002850 \text{ atm-m} \\
\langle \epsilon_{r,CO_2} \rangle &\simeq 0.028 \quad \text{Figure 4.20(a)} \\
p_{H_2O} \langle \lambda_{ph} \rangle &= 0.18 \times 1.9 \times 0.01 = 0.003420 \text{ atm-m} \\
\langle \epsilon_{r,H_2O} \rangle &\simeq 0.014 \quad \text{Figure 4.20(b)} \\
\langle \epsilon_{r,soot} \rangle &\simeq 0.220 \quad \text{Figure 4.20(c)} \\
\langle \epsilon_{r,f} \rangle &= 0.028 + 0.014 + 0.220 = 0.262 \\
(q_{r,i})_f &= 0.220 \times 5.67 \times 10^{-8} (\text{W/m}^2\text{-K}^4) \times (1{,}100)^4 (\text{K}^4) \\
&= 1.826 \times 10^4 \text{ W/m}^2.
\end{aligned}$$

(b) Using the radiation heat flow conservation equation given above, and solving for $q_{r,i}(r = R_2)$, we have

$$\begin{aligned} q_{r,i} &= \frac{2\pi R_1 L}{4\pi R_2^2}(q_{r,i})_f \\ &= \frac{0.01(\text{m}) \times 0.035(\text{m})}{2 \times (0.10)^2(\text{m}^2)} \times 1.826 \times 10^4(\text{W/m}^2) \\ &= 319.6 \text{ W/m}^2. \end{aligned}$$

COMMENT:

The irradiation heat flux $q_{r,i}$ drops as the distance to the hand R_2 increases. Therefore, to sense the heat, the hand needs to be brought close to the flame envelope.

PROBLEM 4.34.DES

GIVEN:

New coating technologies employ ultraviolet curable coatings and ultraviolet radiation ovens. The coatings contain monomers and oligomers that cross link to form a solid, cured film upon exposure to the ultraviolet radiation. The radiation is produced by a mercury vapor or a gallium UV (ultraviolet) lamp. The intensity of the radiation is selected to suit the type of coating applied, its pigmentation, and its thickness. One advantage of the UV-curable coatings is that a smaller amount of solvent is used and discharged to the atmosphere during curing.

In a wood coating-finishing process, the infrared fraction of the emitted radiation is undesirable. The infrared radiation can heat the wood panels to a threshold temperature where the resins leach into the coating before it cures, thus producing an inferior finish. To prevent this, inclined selective surfaces, which reflect the ultraviolet fraction of the radiation, are used. Figure Pr.4.34(i) shows a UV oven. A wood panel, with length $L_1 = 80$ cm, and width $w = 1$ m, occupies the central part of the oven and a bank of UV lamps, with length $L_2 = 50$ cm and width $w = 1$ m, are placed on both sides of the workpiece. The top surfaces act as selective reflecting surfaces, i.e., absorb the infrared radiation and reflect the ultraviolet radiation. They are cooled in their back by a low temperature air flow in order to minimize emission of infrared radiation.

The UV lamps emit $(\dot{S}_{e,\epsilon}/A_r)_2 = (q_{r,o})_2 = 7 \times 10^5$ W/m^2 which is 95% in the ultraviolet range of the spectrum and 5% in the visible and infrared range of the spectrum. The wood boards have a curing temperature $T_1 = 400$ K and behave as a blackbody surface. The selective surfaces have a temperature $T_3 = 500$ K and the emissivity and reflectivity shown in Figure Pr.4.34(ii).

SKETCH:

Figure Pr.4.34(i) and (ii) show the oven and the selective reflector.

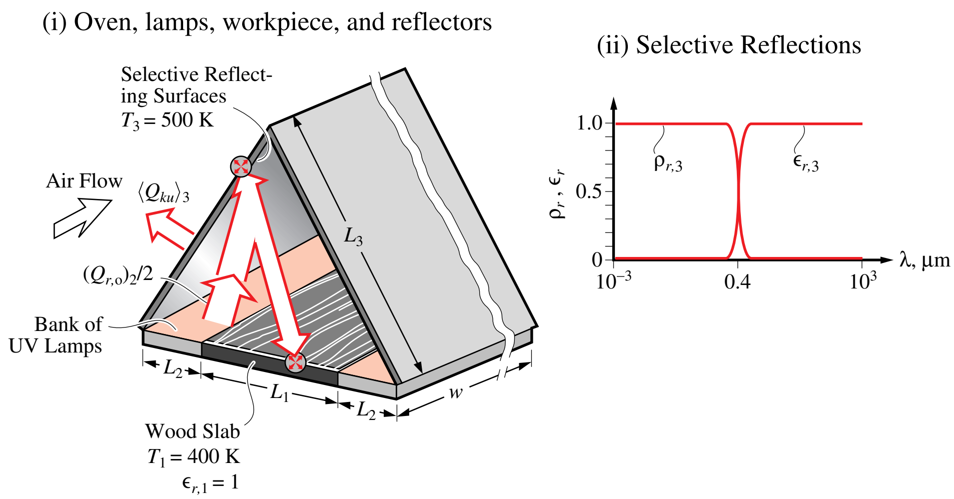


Figure Pr.4.34 Ultraviolet irradiation. (i) UV oven. (ii) Selective reflections.

OBJECTIVE:

(a) Determine the amount of heat transfer by surface convection Q_{ku}(W) needed to keep the selective surfaces at $T_3 = 500$ K.

(b) Determine the radiation heat transfer in the ultraviolet range $Q_{r,1}$(UV) and infrared and visible range $Q_{r,1}(\mathrm{IR}+\mathrm{V})$ reaching the workpiece (surface 1).

(c) Determine the maximum allowed temperature for the selective surfaces $T_{3,max}$, such that the amount of infrared and visible radiation reaching the workpiece is less than 3% of the ultraviolet radiation [i.e., $Q_{r,1}(\mathrm{IR}+\mathrm{V})/Q_{r,1}(\mathrm{UV}) < 0.03$].

SOLUTION:

(a) The integral-volume energy equation (4.66), applied to the reflection surface at T_3, gives

$$Q|_{A,3} = \dot{S}_3.$$

Assuming that the only heat transfer from the surface is by surface convection,

$$Q|_{A,3} = Q_{ku,3} = q_{ku,3} A_{ku,3}.$$

For this node, the energy conversions occur by absorption and emission of thermal radiation, i.e.,

$$\dot{S}_3 = (\dot{S}_{e,\alpha})_3 + (\dot{S}_{e,\epsilon})_3.$$

The energy conversion by radiation absorption is given by (4.64), i.e.,

$$(\dot{S}_{e,\alpha})_3 = A_{r,3} \int_0^\infty (\alpha_{r,\lambda})_3 (q_{r,\lambda,i})_3 d\lambda,$$

where $(q_{r,\lambda,i})_3$ is the irradiation on surface 3 and the integration is done over all the wavelengths λ. The irradiation on surface 3 is given by the spectral form of (4.35), i.e.,

$$(q_{r,i,\lambda})_3 A_{r,3} = F_{1\text{-}3} A_{r,1} (q_{r,0,\lambda})_1 + F_{2\text{-}3} A_{r,2} (q_{r,0,\lambda})_2.$$

Dividing both sides by $A_{r,3}$ and using the reciprocity rule (4.34), we obtain,

$$(q_{r,i,\lambda})_3 = F_{3\text{-}1} (q_{r,0,\lambda})_1 + F_{3\text{-}2} (q_{r,0,\lambda})_2.$$

The radiosity from surface 1 is

$$(q_{r,0,\lambda})_1 \quad = \quad (\rho_{r,\lambda})_1 (q_{r,i,\lambda})_1 + (\epsilon_{r,\lambda})_1 (E_{b,\lambda,1})_1.$$

Substituting into the equation for $(q_{r,i,\lambda})_3$ we obtain,

$$(q_{r,i,\lambda})_3 \quad = \quad F_{3\text{-}1}[(\rho_{r,\lambda})_1 q_{r,i,\lambda} + (\epsilon_{r,\lambda})_1 E_{b,1,\lambda}] + F_{3\text{-}2} (q_{r,o,\lambda})_2.$$

From Figure Pr.4.34(ii), the selective surface has a constant reflectivity and emissivity for the wavelength intervals 0 to 0.4 μm and 0.4 μm to very large wavelengths. Then the surface absorption is written as

$$(\dot{S}_{e,\alpha})_3 = A_{r,3,\lambda} \int_0^{0.4} \alpha_{r,3} (q_{r,i,\lambda})_3 d\lambda + \int_{0.4}^\infty (q_{r,i,\lambda})_3 d\lambda.$$

From Figure Pr.4.34(ii), $\alpha_{r,\lambda,3} = 0$ for $0 \ll \lambda < 0.4$ μm and $\alpha_{r,\lambda,3} = \epsilon_{r,\lambda,3} = 1$ for $0.4\ \mu\text{m} < \lambda < \infty$. Then, using the equation for $(q_{r,i,\lambda})_3$ and $(\rho_{r,\lambda})_1 = 1 - (\epsilon_{r,\lambda})_1 = 0$, we have

$$\begin{aligned}
(\dot{S}_{e,\alpha})_3 &= A_{r,3} \int_{0.4}^\infty (q_{r,\lambda,i})_3 d\lambda \\
&= A_{r,3} \left[F_{3\text{-}1} \int_{0.4}^\infty E_{b,1,\lambda} d\lambda + F_{3\text{-}2} \int_{0.4}^\infty (q_{r,o,\lambda})_2 d\lambda \right].
\end{aligned}$$

The fraction of the total blackbody radiation emitted in a wavelength interval $\lambda_1 T$-$\lambda_2 T$ is found from (4.7), i.e.,

$$F_{\lambda_1 T\text{-}\lambda_2 T} = \frac{\displaystyle\int_0^{\lambda_2} E_{b,1,\lambda} \lambda d\lambda - \int_0^{\lambda_1} E_{b,1,\lambda} d\lambda}{\sigma_{\rm SB} T^4}.$$

Then

$$\int_{0.4}^\infty E_{b,1,\lambda} d\lambda = (1 - F_{0\text{-}0.4T}) \sigma_{\rm SB} T_1^4.$$

For surface 2, only 5% of the emitted radiation is at wavelengths above 0.4 μm and then

$$\int_{0.4}^\infty (q_{r,o,\lambda})_2 d\lambda = 0.05 (q_{r,o,\lambda})_2.$$

Then

$$(\dot{S}_{e,\alpha})_3 = A_{r,3}[F_{3\text{-}1}(1 - F_{0\text{-}0.4T}) \sigma_{\rm SB} T_1^4 + F_{3\text{-}2}(0.05)(q_{r,o})_2].$$

From Table 4.1, we obtain, $F_{0\text{-}0.4T_1} = F_{0\text{-}160} = 0$, as expected. For the geometry of the enclosure, we obtain $F_{1\text{-}3} = F_{2\text{-}3} = 1$. Then, using the reciprocity rule (4.34), we have

$$\begin{aligned} F_{3\text{-}1} &= \frac{A_{r,1}F_{1\text{-}3}}{A_{r,3}} = 1 \times \frac{0.8(\text{m})}{3.6(\text{m})} = 0.22 \\ F_{3\text{-}2} &= \frac{A_{r,2}F_{2\text{-}3}}{A_{r,3}} = 1 \times \frac{1.0(\text{m})}{3.6(\text{m})} = 0.28. \end{aligned}$$

Using the numerical values, the radiation absorbed is

$$\begin{aligned} (\dot{S}_{e,\alpha})_3 &= 3.6(\text{m}^2)[0.22 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (400)^4(\text{K})^4 + 0.28 \times 0.05 \times 7 \times 10^5(\text{W/m}^2)] \\ &= 36{,}430 \text{ W}. \end{aligned}$$

The energy conversion due to radiation emission is found from (4.65), i.e.,

$$\dot{S}_{e,\alpha} = -A_{r,3}\int_0^\infty (\epsilon_{r,\lambda})_3(E_{b,\lambda})_3 d\lambda.$$

Using Figure Pr.4.34(ii), and using the definition of the fraction of the blackbody emissive power, we have

$$(\dot{S}_{e,\epsilon})_3 = -A_{r,3}\left[(1 - F_{0\text{-}0.4T_3})\sigma_{\text{SB}}T_3^4\right].$$

For $T_3 = 500$ K, from Table 4.3, we have $F_{0\text{-}0.4T_3} = 0$. Then,

$$\begin{aligned} (\dot{S}_{e,\epsilon})_3 &= -3.6(\text{m}^2) \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}) \times (500)^4(\text{K})^4 \\ &= -12{,}758 \text{ W}. \end{aligned}$$

Then, from the energy equation,

$$Q_{ku,3} = (q_{ku}A_{ku})_3 = (\dot{S}_{e,\alpha})_3 + (\dot{S}_{e,\epsilon})_3$$

Solving for $q_{ku,3}$, we have

$$q_{ku,3} = \frac{36{,}430(\text{W}) - 12{,}758(\text{W})}{3.6(\text{m}^2)} = 6{,}576 \text{ W/m}^2.$$

(b) The irradiation on surface 1 is given by

$$(q_{r,i,\lambda})_1 = F_{1\text{-}3}(q_{r,o,\lambda})_3 + F_{1\text{-}2}(q_{r,o,\lambda})_2.$$

Since $F_{1\text{-}2} = 0$, then from the equation for $(q_{r,o,\lambda})_3$, we have

$$(q_{r,i,\lambda})_1 = F_{1\text{-}3}[(\rho_{r,\lambda})_3(q_{r,i,\lambda})_3 + (\epsilon_{r,\lambda})_3(E_{b,\lambda})_3].$$

The radiation leaving the surface in the infrared and visible ranges of the spectrum is

$$Q_{r,1}(IR + V) = A_{r,1}\int_{0.4}^\infty (q_{r,i,\lambda})_1 d\lambda.$$

Using Figure Pr.4.34(ii), we then have

$$\begin{aligned} Q_{r,1}(IR + V) &= A_{r,1}F_{1\text{-}3}\sigma_{\text{SB}}T_3^4 \\ &= 0.8(\text{m}^2) \times 1 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (500)^4(\text{K}^4) \\ &= 2{,}835 \text{ W}. \end{aligned}$$

The radiation heat transfer reaching surface 1 in the ultraviolet range is

$$Q_{r,1}(UV) = A_{r,1}\int_{0.4}^\infty (q_{r,i,\lambda})_1 d\lambda.$$

Again, using the equation for $(q_{r,i})_1$ and Figure Pr.4.34(ii), we have

$$Q_{r,1}(UV) = A_{r,1}F_{1\text{-}3}\rho_{r,3}\int_{0.4}^{\infty}(q_{r,i,\lambda})_3 d\lambda.$$

The fraction reflected by surface 3 is

$$\int_{0.4}^{\infty}(q_{r,i,\lambda})_3 d\lambda = 0.95 \times (q_{r,o})_2 F_{3\text{-}2}.$$

Then

$$Q_{r,1}(UV) = A_{r,1}F_{1\text{-}3}\rho_{r,3} \times 0.95 \times (q_{r,o})_2 F_{3\text{-}2}.$$

From the numerical values, we have

$$\begin{aligned} Q_{r,1}(UV) &= 0.8(\text{m}^2) \times 1 \times 0.95 \times 7 \times 10^5(\text{W/m}^2) \times 0.28 \\ &= 148{,}960 \text{ W}. \end{aligned}$$

(c) For a fraction of IR and V radiation equal to 3 % of the radiation in the UV, we have

$$\frac{Q_{r,1}(IR+V)}{Q_{r,1}(UV)} = \frac{\sigma_{\text{SB}}T_{3,\max}^4}{Q_{r,1}(UV)} = 0.03$$

Solving for T_3,

$$T_{3,\max} = [\frac{0.03 \times 148{,}960(\text{W})}{5.67 \times 10^{-8}(\text{W/m}^2\text{-K})}]^{1/4} = 529.9 \text{ K}.$$

COMMENT:

Note that $Q_{ku,3} = 2.367 \times 10^4$ W, while $Q_{r,1}(UV) = 1.490 \times 10^5$ W. This shows that a large fraction of energy emitted by the lamp arrives on the workpiece.

PROBLEM 4.35.DES

GIVEN:

Consider the efficiency of a solar collector with and without a glass cover plate. In a simple model for a solar collector, all the surfaces around the collector, participating in the radiation exchange, are included by defining a single average environment temperature T_a. A solar collector and the various heat flows considered are shown in Figure Pr.4.35.

The irradiation from the sun has a magnitude $(q_{r,i})_s = 800$ W/m^2 and the irradiation from the atmosphere (diffuse irradiation) is given by $(q_{r,i})_a = \sigma_{\rm SB} T_a^4$, where $T_a = 290$ K is the effective atmospheric temperature. The cover plate (made of low iron glass) has a total transmittance for the solar irradiation of $\tau_{r,2} = 0.79$. For the infrared emission, the glass surface can be considered diffuse and gray with an emissivity $\epsilon_{r,2} = 0.9$. The absorber plate (coated with the matte black) is opaque, diffuse, and gray and has absorptivity $\alpha_{r,1} = 0.95$. The glass plate is at a temperature $T_2 = 310$ K and heat losses by surface-convection heat transfer occur at the rate of 400 W/m^2. The interior of the solar collector is evacuated and the bottom heat losses by conduction are at a rate of $q_{k,1\text{-}a} = 40$ W/m^2. The collector surface is rectangular with dimensions $w = 1$ m and $L = 2$ m. For the cases (i) with a glass cover, and (ii) with no glass cover, determine the following. (Use $T_1 = 340$ K.)

SKETCH:

Figure Pr.4.35 shows the collector with the cover plate.

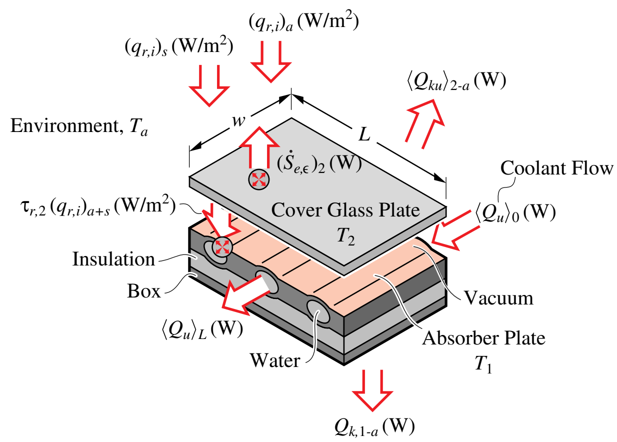


Figure Pr.4.35 A solar collector with a cover glass plate and the associated heat transfer terms.

OBJECTIVE:

For the cases (i) with a glass cover, and (ii) with no glass cover, determine the following.

(a) Determine the amount of heat transferred to the coolant $\langle Q_u\rangle_{L\text{-}0}$(W).

(b) Determine the thermal efficiency η, defined as the heat transferred to the coolant divided by the total irradiation.

SOLUTION:

(i) With a Glass Cover:

(a) From the assumptions above, the integral-volume energy equation (2.9) applied to the collector gives

$$Q|_{A,1} = \dot{S}_1,$$

where

$$Q|_{A,1} = \langle Q_u\rangle_L - \langle Q_u\rangle_0 + \langle Q_{ku}\rangle_{2\text{-}a} + Q_{k,1\text{-}a}$$

and

$$\dot{S}_1 = (\dot{S}_{e,\alpha})_1 + (\dot{S}_{e,\epsilon})_2.$$

The energy conversion by radiation absorption is due to absorption of solar radiation at the absorber plate (mostly UV and V) and absorption of radiation from the environment at the cover plate (IR and V). Then, noting that only a fraction $\tau_{r,2}$ of the solar radiation is transmitted through the cover plate, we have

$$(\dot{S}_{e,\alpha})_1 = \tau_{r,2}\alpha_{r,1}(q_{r,i})_s A_{r,1} + \alpha_{r,2}(q_{r,i})_a A_{r,2}.$$

Using the numerical values and using (4.18), $\alpha_{r,2} = \epsilon_{r,2}$, $A_{r,1} = A_{r,2} = w \times L$ and $(q_{r,i})_a = \sigma_{\rm SB} T_a^4$. Then

$$\begin{aligned}(\dot{S}_{e,\alpha})_1 &= 0.79 \times 0.95 \times 800(\mathrm{W/m^2}) \times 1(\mathrm{m}) \times 2(\mathrm{m}) + \\ &\quad 0.9 \times 5.67 \times 10^{-8}(\mathrm{W/m^2\text{-}K^4}) \times 290^4(\mathrm{K^4}) \times 1(\mathrm{m}) \times 2(\mathrm{m}) \\ &= (1{,}200.9 + 721.8)(\mathrm{W}) = 1{,}922.7\ \mathrm{W}.\end{aligned}$$

The radiation emitted by the absorber plate is reabsorbed internally or at the cover plate. The cover plate emits radiation to the surroundings which is given by

$$(\dot{S}_{e,\epsilon})_2 = -A_{r,2}\epsilon_{r,2}\sigma_{\rm SB}T_2^4.$$

Using the values given,

$$\begin{aligned}(\dot{S}_{e,\epsilon})_2 &= -1(\mathrm{m}) \times 2(\mathrm{m}) \times 0.9 \times 5.67 \times 10^{-8}(\mathrm{W/m^2\text{-}K^4}) \times 310^4(\mathrm{K^4}) \\ &= -942.5\ \mathrm{W}.\end{aligned}$$

Then, solving for the net convective heat transfer to the fluid, we have

$$\begin{aligned}\langle Q_u\rangle_{L\text{-}0} &\equiv \langle Q_u\rangle_L - \langle Q_u\rangle_0 = (\dot{S}_{e,\alpha})_1 + (\dot{S}_{e,\epsilon})_2 - \langle Q_{ku}\rangle_{2\text{-}a} - Q_{k,2\text{-}a} \\ &= 1{,}922.7(\mathrm{W}) - 942.5(\mathrm{W}) - [200(\mathrm{W/m^2}) + 40(\mathrm{W/m^2})] \times 1(\mathrm{m}) \times 2(\mathrm{m}) \\ &= 500.2\mathrm{W}\end{aligned}$$

(b) The thermal efficiency of the collector is

$$\begin{aligned}\eta &= \frac{\langle Q_u\rangle_L - \langle Q_u\rangle_0}{(q_{r,i})_s + (q_{r,i})_a} = \frac{500.2(\mathrm{W})}{800(\mathrm{W/m^2}) + 5.67 \times 10^{-8}(\mathrm{W/m^2\text{-}K^4}) \times 290^4(\mathrm{K^4}) \times 1(\mathrm{m}) \times 2(\mathrm{m})} \\ &= 0.3122 = 31.22\%.\end{aligned}$$

(ii) Without a Glass Cover:
(a) The radiation absorbed is

$$(\dot{S}_{e,\alpha})_1 = A_{r,1}\alpha_{r,1}[(q_{r,i})_s + (q_{r,i})_a].$$

From the numerical values, we have

$$\begin{aligned}(\dot{S}_{e,\alpha})_1 &= 1(\mathrm{m}) \times 2(\mathrm{m}) \times 0.95[800(\mathrm{W/m^2}) + 5.67 \times 10^{-8}(\mathrm{W/m^2\text{-}K^4}) \times 290^4(\mathrm{K^4})] \\ &= 2{,}282(\mathrm{W}).\end{aligned}$$

The radiation emission from the absorber plate at T_1 is

$$\begin{aligned}(\dot{S}_{e,\epsilon})_1 &= -A_{r,1}\epsilon_{r,1}\sigma_{\rm SB}T_1^4 \\ &= -1(\mathrm{m}) \times 2(\mathrm{m}) \times 0.95 \times 5.67 \times 10^{-8}(\mathrm{W/m^2\text{-}K^4}) \times 340^4(\mathrm{K})^4 \\ &= -1{,}439.6\ \mathrm{W}.\end{aligned}$$

Then the integral-volume energy equation (2.9) gives, where we now have $\langle Q_{ku}\rangle_{1\text{-}a}$ as surface 2 has been removed,

$$\begin{aligned}\langle Q_u\rangle_{L\text{-}0} &= \langle Q_u\rangle_L - \langle Q_u\rangle_0 = (\dot{S}_{e,\alpha})_1 + (\dot{S}_{e,\epsilon})_1 - \langle Q_{ku}\rangle_{1\text{-}a} - Q_{k,1\text{-}a} \\ &= 2{,}282(\mathrm{W}) - 1{,}439.6(\mathrm{W}) - 1(\mathrm{m}) \times 2(\mathrm{m}) \times [200(\mathrm{W/m^2}) + 40(\mathrm{W/m^2})] \\ &= 362\ \mathrm{W}.\end{aligned}$$

(b) The thermal efficiency is then

$$\eta = \frac{\langle Q_{u,L}\rangle - \langle Q_{u,0}\rangle}{(q_{r,i})_s + (q_{r,i})_a} = \frac{362(\mathrm{W})}{1{,}602(\mathrm{W})} = 22.60\%.$$

COMMENT:

Both the surface convection and conduction heat losses are different when a cover plate is used. The surface-convection heat loss from the higher-temperature absorber plate is larger than that from the cover plate. This will be discussed in Chapter 6. With the cover plate, the temperature of the absorber increases, and the conduction heat losses also increase. The use of an additional cover plate (as in a double-glazed flat plate solar collector), increases the thermal efficiency even more.

PROBLEM 4.36.FUN

GIVEN:

Cirrus clouds are the thin, high clouds (usually above 6 km) in the form of trails or streaks composed of "delicate white filaments, or tenuous white patches and narrow bands." Due to the atmospheric conditions at these high altitudes (low temperature and high relative humidity), these clouds contain large amounts of ice crystals. An important effect of these clouds in the atmosphere is the absorption and emission of thermal radiation, which plays an important role in the upper troposphere water and the heat budget. This may significantly affect the earth's climate and the atmospheric circulation.

A cirrus uncinus cloud (a hook-like cloud appearing at an altitude between 5 to 15 km, and indicating a slowly approaching storm) has an average thickness L. The extinction coefficient for the cloud is $\sigma_{ex} = 2.2 \times 10^{-3}$ 1/m. The intensity of the irradiation at the top of the cloud is $(q_{r,i})_s = 830$ W/m^2, as shown in Figure Pr.4.36. Assume that the surface reflectivity of the cloud is zero.

Assume that the volume-averaged density and the specific heat of the cloud are $\rho = 0.8$ kg/m^3 and $c_p = 2{,}000$ J/kg-K.

SKETCH:

Figure Pr.4.36 shows the irradiation heating cloud layer.

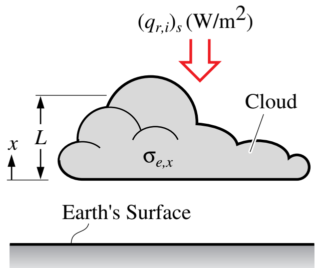


Figure Pr.4.36 Irradiation heating of a cloud layer.

OBJECTIVE:

(a) Determine the amount of radiation absorbed by the cloud $\dot{S}_{e,\alpha}/A$, for $L = 1{,}000$ m.

(b) The length-averaged cloud temperature $\langle T \rangle_L$ is defined as

$$\langle T \rangle_L = \frac{1}{L}\int_0^L T dx.$$

Determine the time variation of the length-averaged cloud temperature $d\langle T \rangle_L/dt$ for the cloud thickness $L = 1{,}000$ m. Neglect the heat transfer by surface convection and the energy conversion due to phase change (i.e., the air is in equilibrium with the cloud).

SOLUTION:

(a) From Table C.1(d), the local volumetric absorption is

$$\frac{\dot{S}_{e,\sigma}}{V} = (1-\rho_r)(q_{r,i})_s\sigma_{ex}e^{-\sigma_{ex}x}.$$

Integrating this over length L along the x direction, we have

$$\frac{\dot{S}_{e,\sigma}}{A} = \int_0^L \frac{\dot{S}_{e,\sigma}}{v}dx = (1-\rho_r)(q_{r,i})_s\sigma_{ex}\int_0^L e^{-\sigma_{ex}x} = (1-\rho_r)(q_{r,i})_s(1-e^{-\sigma_{ex}L}).$$

For zero surface reflectivity, $\rho_r = 0$, we have

$$\frac{\dot{S}_{r,\alpha}}{A} = (q_{r,i})_s(1-e^{-\sigma_{ex}L}).$$

Using the numerical values, we have

$$\frac{\dot{S}_{e,\sigma}}{A} = 830(\text{W/m}^2)\left[1 - e^{-2.2\times10^{-3}(1/\text{m})1{,}000(\text{m})}\right] = 738\ \text{W/m}^2.$$

Therefore, 89% of the incident radiation is absorbed by the cloud.

(b) For a differential-volume at a distance x along the cloud is

$$\nabla\cdot\mathbf{q} = -\rho c_p\frac{dT}{dt} + \dot{s}_{e,\sigma}.$$

From Table C.1(d), the volumetric absorption of irradiation at the depth x is

$$\dot{s}_{e,\sigma} = \frac{\dot{S}_{e,\sigma}}{V} = (1-\rho_r)q_{r,i}\sigma_{\epsilon x}e^{-\sigma_{\epsilon x}x} = (q_{r,i})_s\sigma_{ex}e^{-\sigma_{\epsilon x}x}.$$

Neglecting any other heat transfer from this cloud, we have

$$0 = -\rho c_p\frac{dT}{dt} + (q_{r,i})_s\sigma_{\epsilon x}e^{-\sigma_{\epsilon x}x}.$$

Integrating this energy equation over the cloud thickness L and dividing the result by L, we have

$$\begin{aligned}0 &= -\rho c_p\frac{d}{dt}\left(\frac{1}{L}\int_0^L T dx\right) + (q_{r,i})_s\sigma_{\epsilon x}\left(\frac{1}{L}\int_0^L e^{-\sigma_{\epsilon x}x}dx\right)\\ &= -\rho c_p\frac{d}{dt}\langle T\rangle_L + \frac{(q_{r,i})_s}{L}\left(1 - e^{-\sigma_{\epsilon x}L}\right).\end{aligned}$$

Thus, the time variation of $\langle T\rangle_L$ is

$$\frac{d\langle T\rangle_L}{dt} = \frac{(q_{r,i})_s}{\rho c_p L}\left(1 - e^{-\sigma_{\epsilon x}L}\right).$$

From the numerical values given, we have

$$\begin{aligned}\frac{d\langle T\rangle_L}{dt} &= \frac{830(\text{W/m}^2)}{0.8(\text{kg/m}^3)\times2{,}000(\text{J/kg-K})\times1{,}000(\text{m})}\times\left[1 - e^{-2.2\times10^{-3}(1/\text{m})\times1{,}000(\text{m})}\right]\\ &= 4.6\times10^{-4}\ \text{K/s} = 40\ \text{K/day}.\end{aligned}$$

COMMENT:

Note the relation between the energy equations used in (a) and (b). The length-averaged temperature is the temperature used in the integral form of the energy equation, for which the energy conversion in (a) applies. The energy absorption by the cloud is by the water droplets, mostly by the ice particles. This energy affects the growth of the ice particles which in turn affects the air temperature and air circulation within and underneath the cloud. This may have important implications on the upper atmosphere circulation and the earth surface energy budget. The effect of radiation absorption on the growth rate of ice particles is explored in Problem 4.37.

PROBLEM 4.37.FUN

GIVEN:

Consider a cirrus cloud with an average thickness $L = 1$ km and at an altitude of $r = 8$ km from the earth's surface. The top of the cloud is exposed to the deep space and receives solar radiation at the rate $(q_{r,i})_s = 1{,}353$ W/m^2. The bottom of the cloud is exposed to the earth's surface. The extinction coefficient for the cloud is $\sigma_{ex} = 2.2 \times 10^{-3}$ 1/m. This is shown in Figure Pr.4.37(a).

(i) A spherical ice particle at the top of the cloud has a temperature of $T_1 = -35$°C, and a diameter $d = 100$ μm. The ice surface is opaque, diffuse, and gray with an emissivity $\epsilon_{r,1} = 1$. The particle is moving under the effect of the draft air currents and has no preferred orientation relative to the solar irradiation. The deep sky behaves as a blackbody with a temperature of $T_3 = 3$ K.

(ii) Another ice particle at the bottom of the cloud has the same temperature, dimensions, and surface radiation properties, but it is exposed to the earth's surface. The earth's surface is opaque, diffuse, and gray, has a surface-averaged temperature $T_2 = 297$ K, and a total emissivity $\epsilon_{r,2} = 0.9$.

SKETCH:

Figure Pr.4.37(a) shows a cloud layer with ice particles located (i) at top and (ii) at the bottom of the cloud layer undergoing radiation heat transfer.

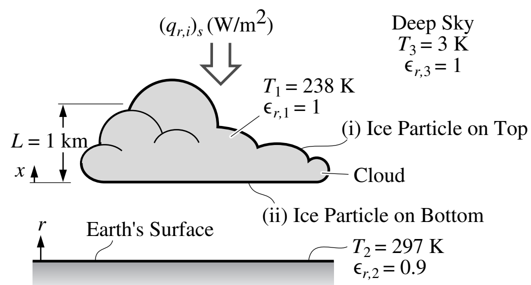


Figure Pr.4.37(a) Ice-particle heat transfer for a particle (i) at the top, and (ii) at the bottom of a cloud layer.

OBJECTIVE:

For each of these two particles perform the following analyses.

(a) Track the heat transfer vector and show the energy conversions. Note that the particles lose heat by surface convection, and that phase change (frosting or sublimation) also occurs.

(b) Draw the thermal circuit diagram for the particles.

Assuming that the cloud is optically thick (i.e., there is a significant attenuation of the radiation across the cloud) the radiation heat transfer between two positions 1 and 2 in the cloud with emissive powers $E_{b,1}$ and $E_{b,2}$ is given by

$$Q_{r,1\text{-}2} = \frac{(E_{b,1} - E_{b,2})}{3\sigma_{ex}/4A_r}.$$

This resistance can be added in series with the surface-grayness and view-factor resistances between two surfaces enclosing the cloud.

(c) Determine the net heat transfer for each particle.

(d) Neglecting the surface-convection heat transfer and assuming a steady-state condition, determine the rate of growth by frosting (or by sublimation) of the ice particles. For the heat of sublimation use $\Delta h_{sg} = 2.843 \times 10^6$ J/kg and for the density of ice use $\rho_s = 913$ kg/m^3.

(e) From the above results, are these particles expected to grow or decay in size? Would the radiation cooling or heating rate differ for particles of a different size?

SOLUTION:

(a) The heat transfer vector tracking and the energy conversion terms are shown in Figure Pr.4.37(b).

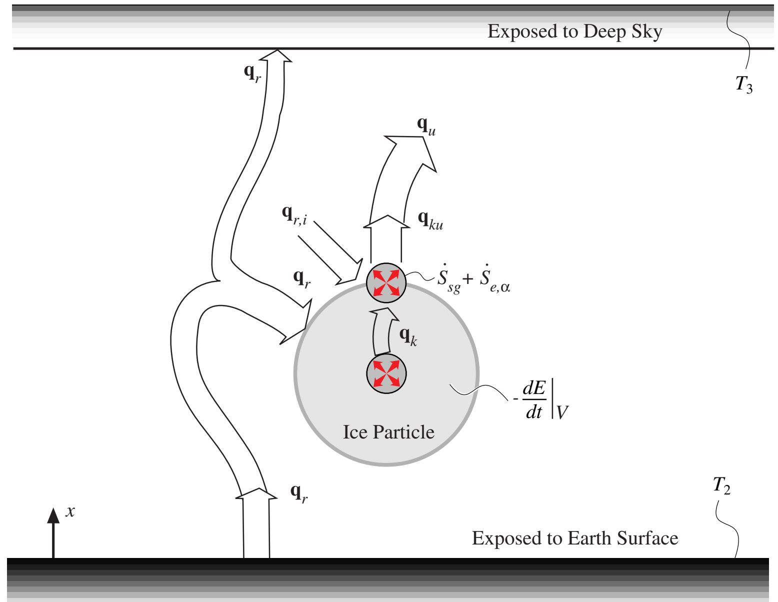


Figure Pr.4.37(b) Track of the heat flux vector around an ice particle in a cloud layer.

(b) The thermal circuit diagram for heat transfer from a particle at the bottom of the cloud layer is shown in Figure Pr.4.37(c).

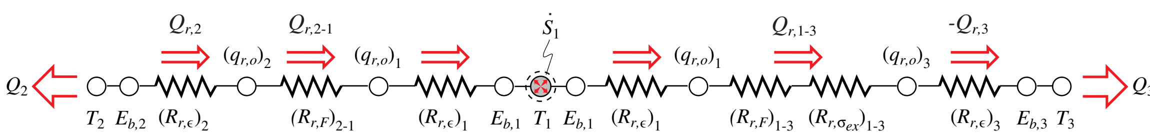


Figure Pr.4.37(c) Thermal circuit diagram for an ice particle at the bottom of the cloud layer.

(c)(i) For an ice particle at the top of the cloud, the integral-volume energy equation (2.9) energy equation is

$$Q|_{A,1} = -(\rho c_p V)_1 \frac{dT_1}{dt} + \dot{S}_1.$$

The energy conversion is due to radiation absorption and solid-gas phase change (sublimation), i.e.,

$$\dot{S}_1 = \dot{S}_{e,\alpha} + \dot{S}_{sg}.$$

The energy conversion due to radiation absorption is

$$\dot{S}_{e,\alpha} = \alpha_{r,1}(q_{r,i})_s A_1.$$

The net heat transfer at the particle surface is

$$Q|_{A,1} = Q_{ku} + Q_{r,1\text{-}3} + Q_{r,1\text{-}2}$$

where $Q_{r,1\text{-}3}$ and $Q_{r,1\text{-}2}$ are the surface-radiation heat transfer between particle and sky and particle and earth. The surface radiation heat transfer between the particle and sky is given by

$$Q_{r,1\text{-}3} = \frac{\sigma_{\rm SB}(T_1^4 - T_3^4)}{(R_{r,\Sigma})_{1\text{-}3}}$$

where,

$$(R_{r,\Sigma})_{1\text{-}3} = \frac{1-\epsilon_{r,1}}{\epsilon_{r,1}A_1} + \frac{1}{F_{1\text{-}3}A_1} + \frac{1-\epsilon_{r,3}}{\epsilon_{r,3}A_3}.$$

Considering that $A_3 \gg A_1$ and $F_{1\text{-}3}$=1 we have,

$$(R_{r,\Sigma})_{1\text{-}3} = \frac{1}{A_1\epsilon_{r,1}}$$

The surface-radiation heat transfer between the particle and the earth is affected by the presence of the cloud, which absorbs and scatters radiation. Assuming that the diffusion approximation for the radiation flux within the cloud applies (5.64), this surface radiation heat transfer is given by

$$Q_{r,1\text{-}2} = \frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{(R_{r,\Sigma})_{1\text{-}2}},$$

where the overall radiation resistance, and using the resistance for the volumetric absorption of radiation, is given by

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1-\epsilon_{r,1}}{\epsilon_{r,1}A_1} + \frac{1}{F_{1\text{-}2}A_1} + \frac{1-\epsilon_{r,2}}{\epsilon_{r,2}A_2} + \frac{3\sigma_{ex}L}{4A_1}.$$

Considering that $A_2 \gg A_1$ and $F_{1\text{-}2} = 0.5$ we have

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1}{A_1}\left(\frac{1}{\epsilon_{r,1}} + 1 + \frac{3\sigma_{ex}L}{4}\right).$$

Therefore, the net radiation heat transfer and absorption for the ice particle at the top of the cloud is

$$Q_{r,1,t} = -\alpha_{r,1}(q_{r,i})_s A_1 + \frac{\sigma_{\rm SB}(T_1^4 - T_3^4)}{\left(\dfrac{1}{A_1\epsilon_{r,1}}\right)} + \frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{\dfrac{1}{A_1}\left(\dfrac{1}{\epsilon_{r,1}} + 1 + \dfrac{3\sigma_{ex}L}{4}\right)}$$

or

$$\frac{Q_{r,1,t}}{A_1} = -\alpha_{r,1}(q_{r,i})_s + \frac{\sigma_{\rm SB}(T_1^4 - T_3^4)}{\left(\dfrac{1}{\epsilon_{r,1}}\right)} + \frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{\dfrac{1}{\epsilon_{r,1}} + \dfrac{3\sigma_{ex}L}{4} + 1}.$$

Using the numerical values, we have

$$\begin{aligned}\frac{Q_{r,1,t}}{A_1} &= -1 \times 1{,}353(\text{W/m}^2) + \frac{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (238.15^4 - 3^4)(\text{K}^4)}{1} + \\ &\quad \frac{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (238.15^4 - 297^4)(\text{K}^4)}{\dfrac{1}{1} + \dfrac{3 \times 2.2 \times 10^{-3}(1/\text{m}) \times 1{,}000(\text{m})}{4} + 1} \\ &= -1{,}353(\text{W/m}^2) + 182(\text{W/m}^2) - 71(\text{W/m}^2) \\ &= -1{,}242\ \text{W/m}^2.\end{aligned}$$

The negative sign means that the particle is being heated by radiation.
(ii) For the particle at the bottom of the cloud, the radiation absorption is

$$\dot{S}_{e,\alpha} = \alpha_{r,1}q_{r,i}A_1.$$

The irradiation is attenuated by volumetric absorption, as discussed in Section 2.3.2(E). The irradiation flux at the bottom of the cloud is given by (2.42), i.e.,

$$q_{r,i} = (q_{r,i})_s(1-\rho_{r,1})e^{-\sigma_{ex}L}.$$

With $\epsilon_{r,1} = 1$ (i.e., $\rho_{r,1} = 0$), we have

$$q_{r,i} = (q_{r,i})_s e^{-\sigma_{ex} L}.$$

Then, as given in Table C.1(d)

$$\dot{S}_{e,\alpha} = \alpha_{r,1}(q_{r,i})_s \sigma_{ex} e^{-\sigma_{ex} L} A_1.$$

The net surface-radiation heat transfer, in analogy to the particle at the top, is

$$\begin{aligned} Q_{r,1\text{-}2} &= \frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{\left(\dfrac{1}{A_1 \epsilon_{r,1}}\right)} \\ Q_{r,1\text{-}3} &= \frac{\sigma_{\rm SB}(T_1^4 - T_3^4)}{\dfrac{1}{A_1}\left(\dfrac{1}{\epsilon_{r,1}} + 1 + \dfrac{3\sigma_{ex} L}{4}\right)}. \end{aligned}$$

Therefore, the net radiation heat transfer is,

$$\frac{Q_{r,1,b}}{A_1} = -\alpha_{r,1}(q_{r,i})_s \sigma_{ex} e^{-\sigma_{ex} L} + \frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{\left(\dfrac{1}{\epsilon_{r,1}}\right)} + \frac{\sigma_{\rm SB}(T_1^4 - T_3^4)}{\dfrac{1}{\epsilon_{r,1}} + 1 + \dfrac{3\sigma_{ex} L}{4}}$$

Using the numerical value, we have

$$\begin{aligned} \frac{Q_{r,1,b}}{A_1} &= -1 \times 1{,}353(\text{W/m}^2) \times 2.2 \times 10^{-3}(1/\text{m}) \times e^{-2.2 \times 10^{-3}(1/\text{m}) \times 1{,}000(\text{m})} \\ &\quad + \frac{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (238.15^4 - 297^4)(\text{K}^4)}{1} \\ &\quad + \frac{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (238.15^4 - 3^4)(\text{K}^4)}{2 + \dfrac{3 \times 2.2 \times 10^{-3}(1/\text{m}) \times 1{,}000(\text{m})}{4}} \\ &= (-0.33 - 258.8 + 49.8)(\text{ W/m}^2) \\ &= -209 \text{ W/m}^2. \end{aligned}$$

Note that the particle at the bottom of the cloud is also being heated, but at a much smaller rate.

(d) For a steady-state condition, the integral-volume energy equation (2.9) becomes

$$Q|_{A_1} = \dot{S}_1.$$

Neglecting surface convection and using the results calculated in item (b), we have

$$Q_{r,1} = \dot{S}_{sg}$$

The energy conversion by sublimation (Table 2.1) is given by

$$\frac{\dot{S}_{sg}}{A_1} = -\dot{M}_{sg} \Delta h_{sg}$$

The rate of sublimation is related to the rate of growth by

$$\dot{M}_{sg} = -\rho_s \frac{dV_1}{dt}$$

Then

$$\frac{1}{A_1}\frac{dV_1}{dt} = \frac{Q_{r,1}}{A_1 \rho_s \Delta h_{sg}} = \frac{q_{r,1}}{\rho_s \Delta h_{sg}}$$

For the particle at the top

$$\frac{1}{A_1}\frac{dV_1}{dt} = \frac{-1{,}242(\text{W/m}^2)}{913(\text{kg/m}^3) \times 2.843 \times 10^6(\text{J/kg})}$$
$$= -4.78 \times 10^{-7}\ \text{m/s} = -0.478\ \mu\text{m/s}.$$

For the particle at the bottom

$$\frac{1}{A_1}\frac{dV_1}{dt} = \frac{-209(\text{W/m}^2)}{913(\text{kg/m}^3) \times 2.843 \times 10^6(\text{J/kg})}$$
$$= -8.05 \times 10^{-8}\ \text{m/s} = -0.0805\ \mu\text{m/s}.$$

(e) Both particles are expected to decrease in size. The particle at the top would disappear faster than the particle at the bottom. Larger particles would take longer to sublimate due to their larger volume.

COMMENT:

The surface-convection heat transfer from the particle surface influences the particle growth (or decay) rates.

PROBLEM 4.38.DES

GIVEN:

A flat-plate solar collector is modeled as a surface with equivalent total absorptance $\alpha_{r,1}$ and emittance $\epsilon_{r,1}$, which represent the surface and wavelength average of the absorptivity and emissivity of all the internal and external surfaces (it also accounts for the transmissivity of the cover plate). The solar collector and the model are shown in Figure Pr.4.38.

The solar collector receives solar irradiation $(q_{r,i})_s = 800$ W/m^2 and atmospheric irradiation $(q_{r,i})_a = \sigma_{\rm SB} T_a^4$, where $T_a = 290$ K is the effective atmospheric temperature (it accounts for the emission and scattering of the radiation by the atmosphere). This radiation is incident on the collector plate surface, which has area $L \times w = 1 \times 2(\mathrm{m}^2)$. The collector surface temperature is $T_1 = 310$ K and it loses heat by surface-convection heat transfer at a rate $\langle q_{ku} \rangle_{1\text{-}\infty} = 400$ W/m^2 and also by surface radiation emitted to the surroundings $(\dot{S}_{e,\epsilon})_1$. The bottom of the collector loses heat by conduction at a rate $q_{k,1\text{-}\infty} = 50$ W/m^2. The total absorptance of the solar collector is $\alpha_{r,1} = 0.9$ and the total emittance is $\epsilon_{r,1} = 0.3$.

SKETCH:

Figure Pr.4.38 shows the solar collector and the various heat transfer from the collector plate.

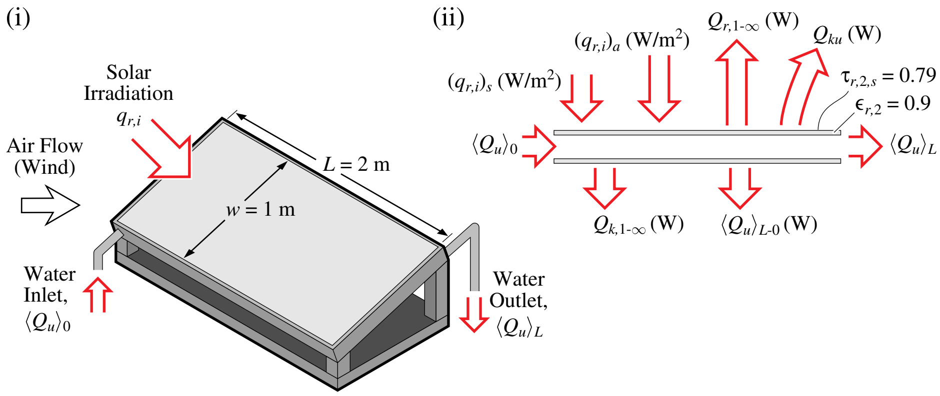


Figure Pr.4.38(i) and (ii) A solar collector and the collector plate heat transfer.

OBJECTIVE:

(a) Determine the amount of heat transferred to the fluid $\langle Q_u \rangle_{L\text{-}0} = \langle Q_u \rangle_L - \langle Q_u \rangle_0$ (W).

(b) Determine the thermal efficiency η, defined as the ratio of the heat transferred to the fluid to the total irradiation.

SOLUTION:

(a) The integral-volume energy equation (2.9) applied to the collector gives

$$Q|_{A,1} = \dot{S}_1,$$

where

$$Q|_{A,1} = q_{ku,1\text{-}\infty} A_{ku} + q_{k,1\text{-}\infty} A_k + \langle Q_u \rangle_{L,0}$$

and

$$\dot{S}_1 = (\dot{S}_{e,\alpha})_1 + (\dot{S}_{e,\epsilon})_1.$$

The radiation absorbed is (noting that $\epsilon_{r,1} = \alpha_{r,1}$)

$$\begin{aligned}(\dot{S}_{e,\alpha})_1 &= A_{r,1}[(\alpha_{r,1})_s (q_{r,i})_s + (\alpha_{r,1})_{V+IR} \sigma_{\rm SB} T_a^4] \\ &= 1(\mathrm{m}) \times 2(\mathrm{m}) \times [0.90 \times 800(\mathrm{W/m}^2) + 0.3 \times 5.67 \times 10^{-8}(\mathrm{W/m}^2\text{-K}^4) \times (290)^4(\mathrm{K})^4] \\ &= 1{,}680 \text{ W}.\end{aligned}$$

The radiation emitted is

$$
\begin{aligned}
(\dot{S}_{e,\epsilon})_1 &= -A_{r,1}(\alpha_{r,1})_{V+IR}\sigma_{\rm SB}T_1^4 \\
&= -1({\rm m}) \times 2({\rm m}) \times 0.3 \times 5.67 \times 10^{-8}({\rm W/m^2\text{-}K^4}) \times (310)^4({\rm K})^4 \\
&= -314.18 \text{ W}.
\end{aligned}
$$

Then the convection heat transfer is

$$
\begin{aligned}
\langle Q_u \rangle_{L\text{-}0} &= (\langle Q_{u,L} \rangle - \langle Q_{u,0} \rangle) = (\dot{S}_{e,\alpha})_1 + (\dot{S}_{e,\epsilon})_1 - q_{ku,1\text{-}\infty}A_{ku} - q_{k,1\text{-}\infty}A_k \\
&= 1{,}680.62({\rm W}) - 314.18({\rm W}) - 1({\rm m}) \times 2({\rm m}) \times [400({\rm W/m^2}) + 50({\rm W/m^2})] \\
&= 466.4 \text{ W}.
\end{aligned}
$$

(b) The thermal efficiency is defined as

$$
\begin{aligned}
\eta &= \frac{\langle Q_u \rangle_{L\text{-}0}}{A_{r,1}[(q_{r,i})_s + \sigma_{\rm SB}T_a^4]} \\
&= \frac{466.4({\rm W})}{1({\rm m}) \times 2({\rm m}) \times [800({\rm W/m^2}) + 5.67 \times 10^{-8}({\rm W/m^2\text{-}K^4}) \times 290^4({\rm K^4})]} = 0.1942.
\end{aligned}
$$

The efficiency in converting total solar irradiation to sensible heat of the water stream is 19.42%.

COMMENT:

Note that the effective temperature of the collector T_1 is smaller than the water temperature. This temperature is basically an average temperature for the cover plate, which is the emitting surface.

PROBLEM 4.39.DES

GIVEN:

Table Pr.4.39 gives a short list of materials used as selective surface coatings, i.e., coatings that have different absorption properties for shorter and longer wavelength ranges. Other materials are listed in Table C.19. For each of the two applications below, choose from Table Pr.4.39 the coating that results in the optimum performance.

Table Pr.4.39 Spectral absorptivity and emissivity properties for some selective coatings.

Coating	$\alpha_{r,\lambda}(0.3 \leq \lambda \leq 3\ \mu\text{m})$	$\epsilon_{r,\lambda}(3 \leq \lambda \leq 50\ \mu\text{m})$
black, chrome electro-deposited	0.95	0.15
copper oxide	0.87	0.15
aluminum hard-anodized	0.03	0.80
Teflon	0.12	0.85

Figure Pr.4.39(i) shows a solar collector. The absorber plate is exposed directly to the solar and atmospheric irradiation $(q_{r,i})_a$, as shown in Figure Pr.4.39(i). The intensity of the solar radiation is $(q_{r,i})_s = 800$ W/m^2. The net radiation emitted by the atmosphere is given by $(q_{r,i})_a = \sigma_{\text{SB}} T_a^4$, where $T_a = 290$ K is the effective atmospheric temperature (i.e., the "sky" temperature). The absorber plate has a surface $L \times w = 1 \times 2$ m^2 and it is at a temperature of $T_1 = 80$℃. The collector is insulated from below and the surface-convection heat transfer is neglected. Choose a material from Table Pr.4.39 that would result in the maximum convection heat removal $\langle Q_u \rangle_{L\text{-}0}$ (W) (i.e., maximum heating of the fluid) from the collector and determine the heat flow rate.

Figure Pr.4.39(ii) shows a radiative cooler. A satellite in orbit around the earth uses radiation cooling to reject heat [Figure Pr.4.39(ii)]. As the satellite rotates, it is temporarily exposed to the sun. The intensity of the solar irradiation is $(q_{r,i})_s = 1{,}353$ W/m^2. The plate has an area $L \times w = 50 \times 50$ cm^2 and its temperature is $T_1 = 250$ K. The deep sky temperature is $T_{sky} = 3$ K. Choose a material that would give the maximum heat transfer $Q_{r,1}$(W) and determine this heat flow rate.

SKETCH:

Figures Pr.4.39(i) and (ii) show the two applications of selective radiation coatings.

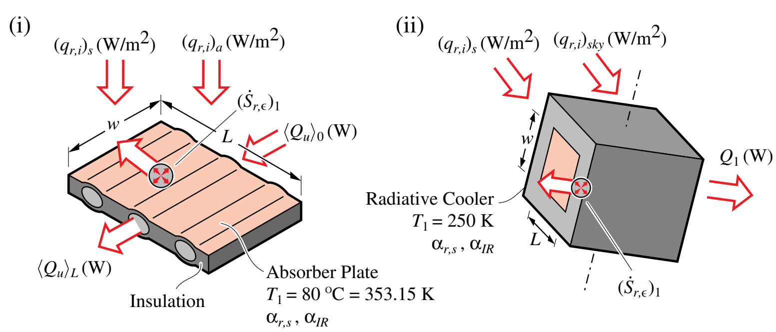


Figure Pr.4.39(i) A solar collector. (ii) A radiative cooler. Both use selective radiation absorption.

OBJECTIVE:

(a) For the solar collector in Figure Pr.4.39(i), choose a material from Table Pr.4.39 that would result in the maximum convection heat removal $\langle Q_u \rangle_{L\text{-}0}$ (W) (i.e., maximum heating of the fluid) from the collector and determine the heat flow rate.

(b) For the radiative cooler in Figure Pr.4.39(ii), choose a material that would give the maximum heat transfer $Q_{r,1}$(W) and determine this heat flow rate.

SOLUTION:
(a) For a flat plate solar collector we need a large absorptivity in the wavelength range characteristic of the solar irradiation and a small emissivity in the wavelength characteristic of radiation emitted at low temperatures. Both the black, chrome electro-deposited and the copper-oxide coatings would perform well under these conditions. To calculate the net convection heat transfer $\langle Q_u \rangle_{L\text{-}0}$, we use the integral-volume energy equation,

$$Q|_{t,1} = \dot{S}_1$$

where

$$\begin{aligned} Q|_{A,1} &= Q_{u,L} - Q_{u,0} \\ &\equiv \langle Q_u \rangle_{L\text{-}0} \end{aligned}$$

and

$$\dot{S}_1 = (\dot{S}_{e,\alpha})_1 + (\dot{S}_{e,\epsilon})_1$$

The energy conversion due to the absorption of irradiation is given by (4.64), i.e.,

$$(\dot{S}_{e,\alpha})_1 = A_{r,1} \int_0^\infty (\alpha_{r,\lambda})_1 (q_{r,i,\lambda})_1 d\lambda.$$

We assume that the surfaces are gray in the wavelength range shown in Table Pr.4.39 and that solar irradiation occurs at the short wavelength and atmospheric irradiation occurs at the long wavelength range. Then we have (using $\epsilon_{r,\lambda} = \alpha_{r,\lambda}$),

$$\dot{S}_{e,\alpha} = A_{r,1}[\alpha_{r,\lambda}(0.3 \leq \lambda < 3\ \mu\text{m})(q_{r,i})_s + \epsilon_{r,\lambda}(3 \leq \lambda < 50\ \mu\text{m})\sigma_{\text{SB}} T_a^4].$$

Using the values given for black, chrome electro-deposited coating, we have

$$\begin{aligned} (\dot{S}_{e,\alpha})_1 &= 1(\text{m}) \times 2(\text{m})[0.95 \times 800(\text{W/m}^2) + 0.15 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (290)^4(\text{K})^4] \\ &= 1{,}640 \text{ W}. \end{aligned}$$

The emitted radiation is emitted in the larger wavelength range, and we have

$$\begin{aligned} (\dot{S}_{e,\epsilon})_1 &= -A_{r,1}\epsilon_{r,\lambda}(3 \leq \lambda < 50\ \mu\text{m})\sigma_{\text{SB}} T_1^4 \\ &= -1(\text{m}) \times 2(\text{m}) \times 0.15 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (353.15)^4(\text{K})^4 = -264 \text{ W}. \end{aligned}$$

Then the convection heat transfer is

$$\begin{aligned} \langle Q_u \rangle_{L\text{-}0} &= (\dot{S}_{e,\alpha})_1 + (\dot{S}_{e,\epsilon})_1 \\ &= 1{,}640(\text{W}) - 264(\text{W}) = 1{,}376 \text{ W}. \end{aligned}$$

(b) In order to reject the maximum amount of heat possible, the radiative cooler should have a large emissivity in the large wavelength range and a small absorptivity at short wavelength ranges. Both the hard-anodized aluminum and the Teflon would perform well under these conditions. The net radiation heat transfer is due to absorption and emission of radiation. Then the integral-volume energy equation is

$$Q|_{A,1} = (\dot{S}_{e,\alpha})_1 + (\dot{S}_{e,\epsilon})_1.$$

The radiation absorption is

$$\begin{aligned} (\dot{S}_{e,\alpha})_1 &= A_{r,1}[\alpha_{r,\lambda}(0.3 \leq \lambda < 3\ \mu\text{m})(q_{r,i})_s + \epsilon_{r,\lambda}(3 \leq \lambda < 50\ \mu\text{m})\sigma_{\text{SB}} T_{sky}^4] \\ &= 0.5(\text{m}) \times 0.5(\text{m}) \times [0.03 \times 1{,}353(\text{W/m}^2) + 0.80 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (3)^4(\text{K})^4] \\ &= 10.15 \text{ W}. \end{aligned}$$

The radiation emitted is

$$\begin{aligned} (\dot{S}_{e,\epsilon})_1 &= -A_{r,1}\epsilon_{r,\lambda}(3 \leq \lambda < 50\ \mu\text{m})\sigma_{\text{SB}} T_1^4 \\ &= -0.5(\text{m}) \times 0.5(\text{m}) \times 0.80 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (250)^4(\text{K})^4 \\ &= -44.30 \text{ W}. \end{aligned}$$

Then

$$\begin{aligned} Q_1 = Q|_{A,1} &= (\dot{S}_{e,\alpha})_1 + (\dot{S}_{e,\epsilon})_1 \\ &= 10.15 - 44.30(\text{W}) = -34.15 \text{ W}. \end{aligned}$$

COMMENT:

Note that for (b), the negative $Q|_{A,1}$ indicates that heat must be provided to surface A_1. This heat can be provided by sensible heat, i.e., cooling down, of the satellite. Other selective surfaces are listed in Table C.19.

PROBLEM 4.40.DES.S

GIVEN:

The high temperature of the automobile exhaust can be used to promote catalytic reactions and conversions (this is called a close-coupled converter) of some gaseous pollutants (such as unburned hydrocarbons) over catalytic metal-oxide surfaces. When the converter is placed close to, but downstream of an internal combustion engine, the exhaust pipe leading to the converter is insulated. One scenario is the addition of a radiation shield around of the outside of the exhaust pipe. This is shown in Figure Pr.4.40(a). The gap between the pipe and the shield contains air, and heat transfer occurs by conduction and by surface radiation.

The external surface of the shield exchanges heat with its surroundings by surface radiation and surface convection. The surface-convection heat loss of the external surface of the shield is estimated as $q_{ku} = 4{,}000$ W/m^2. The surroundings behave as a blackbody at $T_3 = 300$ K. The exhaust pipe has an outside diameter $D_1 = 5$ cm, a surface temperature of $T_1 = 800$ K, and its surface is diffuse, opaque, and gray with a surface emissivity of $\epsilon_{r,1} = 0.7$. The shield has an inside diameter $D_{2,i} = 5.4$ cm and an outside diameter $D_{2,o} = 5.5$ cm and is made of chromium coated carbon steel AISI 1042. Its surface is opaque, diffuse, and gray and has a surface emissivity $\epsilon_{r,2} = 0.1$. For the thermal conductivity of air use $k_a = 0.04$ W/m-K.

SKETCH:

Figure Pr.4.40(a) shows the exhaust pipe, the radiation shield, and the surroundings of the shield.

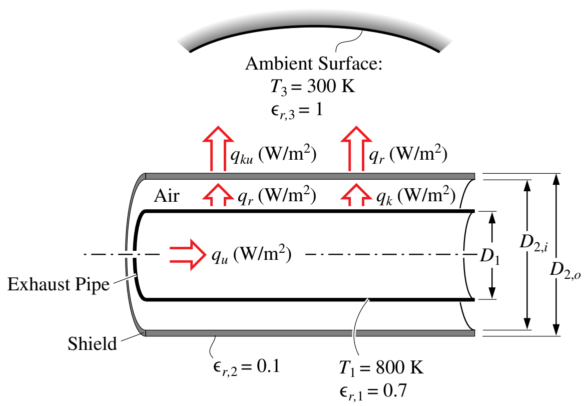


Figure Pr.4.40(a) Insulation of an automobile exhaust pipe.

OBJECTIVE:

(a) Determine the net heat transfer from the exhaust pipe to the ambient for a $L = 1$ m long pipe.

(b) Comment on the effect of the pipe wall conduction thermal resistance on the total heat transfer from the exhaust pipe.

(c) Keeping all the other conditions the same, would the heat transfer from the exhaust pipe increase or decrease with an increase of the inside diameter of the shield (while keeping the thickness constant)? (Suggestion: Plot the variation of Q_1 for 5.2 cm $\leq D_{2,i} \leq$ 7 cm.)

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.4.40(b) under a steady-state condition and with no energy generation present. Applying the integral-volume energy equation to nodes $T_1, T_{2,i}, T_{2,o}$ and T_3, we have

$$Q_1 = Q_{k,1\text{-}2} + Q_{r,1\text{-}2} = Q_{ku} + Q_{r,2\text{-}3} = Q_3.$$

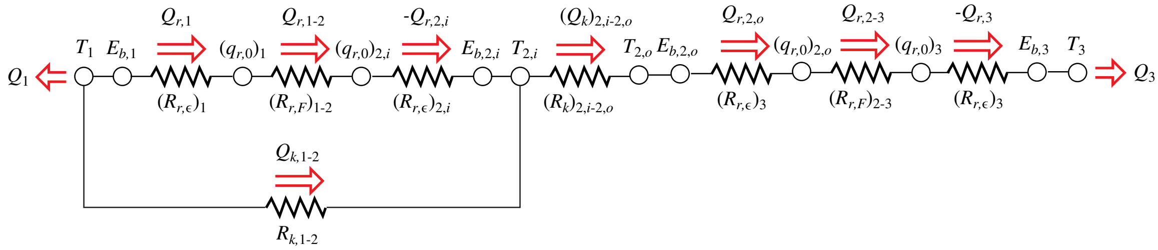


Figure Pr.4.40(b) Thermal circuit diagram.

The conduction, surface radiation and surface convection heat transfer rates are given by

$$\begin{aligned} Q_{k,1\text{-}2} &= \frac{T_1 - T_{2,i}}{R_{k,1\text{-}2}} \\ Q_{r,1\text{-}2} &= \frac{\sigma_{\rm SB}({T_1}^4 - {T_{2,i}}^4)}{(R_{r,\Sigma})_{1\text{-}2}} \\ Q_{ku} &= A_{ku,3} q_{ku} \end{aligned}$$

$$\begin{aligned} Q_{r,2\text{-}3} &= \frac{\sigma_{\rm SB}(T_{2,i}^4 - {T_3}^4)}{(R_{r,\Sigma})_{2\text{-}3}} \\ (Q_k)_{2,i\text{-}2,o} &= \frac{T_{2,i} - T_{2,o}}{(R_k)_{2,i\text{-}2,o}}. \end{aligned}$$

The conduction thermal resistance through the air gap is

$$R_{k,1\text{-}2} = \frac{\ln(R_{2,i}/R_1)}{2\pi k_a L} = \frac{\ln[2.7(\text{cm})/2.5(\text{cm})]}{2\pi \times 0.04(\text{W/m-K}) \times 1(\text{m})} = 0.3062^\circ\text{C/W}.$$

The overall radiation thermal resistance through the air gap is

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} + \frac{1}{A_{r,1}F_{1\text{-}2}} + \frac{1-\epsilon_{r,2}}{A_{r,2}\epsilon_{r,2}}.$$

Using $F_{1\text{-}2}$ =1 and the areas

$$\begin{aligned} A_{r,1} &= \pi D_1 L = \pi \times 5 \times 10^{-2}(\text{m}) \times 1(\text{m}) = 0.157 \text{ m}^2 \\ A_{r,2} &= \pi D_2 L = \pi \times 5.4 \times 10^{-2}(\text{m}) \times 1(\text{m}) = 0.170 \text{ m}^2, \end{aligned}$$

we have

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1-0.7}{0.157(\text{m}^2) \times 0.7} + \frac{1}{0.157(\text{m}^2) \times 1} + \frac{1-0.1}{0.170(\text{m}^2) \times 0.1} = 62.04 \text{ 1/m}^2.$$

The conduction thermal resistance through the pipe wall is (using k_s for steel from Table C.16),

$$(R_k)_{2,i\text{-}2,o} = \frac{\ln(R_{2,o}/R_{2,i})}{2\pi k_s L} = \frac{\ln[2.75(\text{cm})/2.7(\text{cm})]}{2\pi \times 50(\text{W/m-K}) \times 1(\text{m})} = 5.84 \times 10^{-5}\,^\circ\text{C/W}$$

The overall radiation resistance from the pipe surface to the surrounding is

$$(R_{r,\Sigma})_{2\text{-}3} = \frac{1-\epsilon_{r,2}}{A_{r,2,o}\epsilon_{r,2}} + \frac{1}{A_{r,2,o}F_{2\text{-}3}} + \frac{1-\epsilon_{r,3}}{A_{r,3}\epsilon_{r,3}}$$

Using $F_{2\text{-}3}$ =1, $A_{r,3} \gg A_{r,2,o}$ and

$$A_{r,2,o} = \pi D_{2,o} L = \pi \times 5.5 \times 10^{-2}(\text{m}) \times 1(\text{m}) = 0.173 \text{ m}^2,$$

we have

$$(R_{r,\Sigma})_{2\text{-}3} = \frac{1-0.1}{0.173(\text{m}^2)\times 0.1} + \frac{1}{0.173(\text{m}^2)} = 57.80\ 1/\text{m}^2.$$

Noting that $(R_k)_{2,i\text{-}2,o} \ll \text{R}_{k,1\text{-}2}$, $(R_{r,\Sigma})_{1\text{-}2}$, and $(R_{r,\Sigma})_{2\text{-}3}$, we can assume that $T_{2,i} = T_{2,o} \simeq T_2$. Then the energy equation for node T_2 becomes

$$Q|_{A,2} = Q_{k,1\text{-}2} + (Q_{r,\Sigma})_{1\text{-}2} - Q_{ku} + (Q_{r,\Sigma})_{2\text{-}3} = 0.$$

Using the equation for the heat transfer rates, we have

$$\frac{T_1 - T_2}{R_{k,1\text{-}2}} + \frac{\sigma_{\text{SB}}(T_1^4 - T_2^4)}{(R_{r,\Sigma})_{1\text{-}2}} - Q_{ku} - \frac{\sigma_{\text{SB}}(T_2^4 - T_3^4)}{(R_{r,\sigma})_{2\text{-}3}} = 0.$$

This is a fourth order polynomial equation on T_2. Solving for T_2 using a solver (such as SOPHT), we obtain T_2 = 619.7 K. The heat transfer rate Q_1 is then given by

$$Q_1 = \frac{T_1 - T_2}{R_{k,1\text{-}2}} + \frac{\sigma_{\text{SB}}(T_1^4 - T_2^4)}{(R_{r,\Sigma})_{1\text{-}2}} = 827.7\ \text{W}.$$

(b) The conduction resistance through the pipe wall, if taken into account, would reduce the heat transfer rate Q_1. However, here, due to the relatively high thermal conductivity of steel and the small thickness of the pipe wall, its effect on Q_1 is negligible.

(c) Increasing the diameter of the shield $D_{2,i}$ increases the conduction resistance through the air gap, but also decreases the radiation resistance, and increases the surface convection. The combined result of these effects is shown in Figure Pr.4.40(c). Note that there is a minimum in the heat transfer rate Q_1 for $D_{2,i}$ = 5.8 cm, for which Q_1 = 808 W.

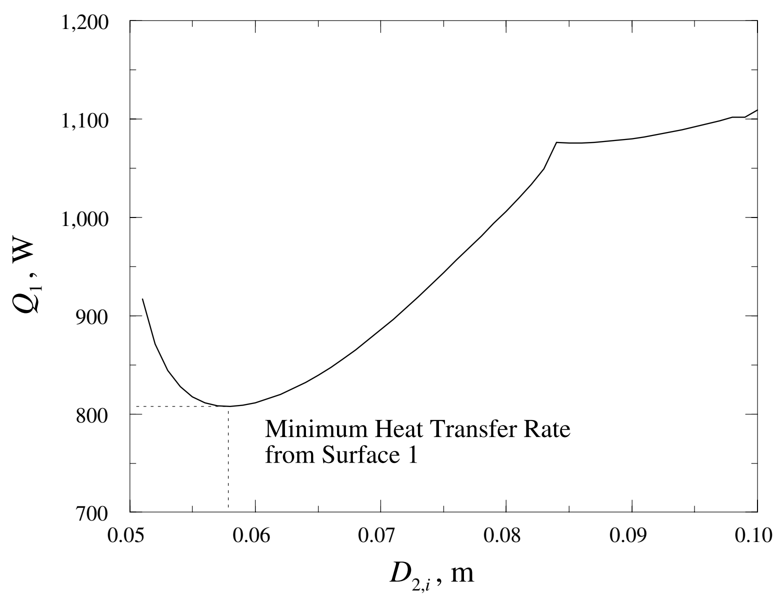


Figure Pr.4.40(c) Variation of the heat loss with respect to the shielded diameter $D_{2,i}$.

COMMENT:

Here, the dependency of the surface-convection heat transfer rate on the external diameter $D_{2,o}$ through q_{ku} is neglected. This will be discussed in Chapter 6. Also, with the increase in the gap size, between the shield and the pipe, there is an increase in the importance of the surface-convection heat transfer in the gap (when compared to the conduction heat transfer). This thermobuoyant flow and heat transfer is explained in Chapters 6. When the conduction resistance across the pipe wall is not neglected, we obtain $T_{2,i}$ = 619.758 K, $T_{2,o}$ = 619.709 K and $Q_{2,i\text{-}2,o}$ = 827.708 W. Note the negligible difference between these results and the results obtained in part (a).

PROBLEM 4.41.DES

GIVEN:

During continuous thermal processing of silicon wafers, to heat the wafers to a desired temperature, the wafers are stacked vertically and moved through an evacuated, cylindrical radiation oven at speed u_w. This is shown in Figure Pr.4.41(i). Consider a unit cell formed by two adjacent wafers. This is shown in Figure Pr.4.41(ii). The wafers have a diameter $D_1 = 100$ mm, the distance separating the wafers is $l = 30$ mm, and the wafers are placed coaxially to the oven wall, which is also cylindrical. The diameter of the ceramic oven is $D_2 = 300$ mm. The oven surface is opaque, diffuse, and gray with surface emissivity $\epsilon_{r,2} = 0.9$ and surface temperature $T_2 = 800$ K. The wafers surface is also diffuse, opaque, and gray with emissivity $\epsilon_{r,1} = 0.01$.

SKETCH:

Figure Pr.4.41(a) shows the wafer-furnace.

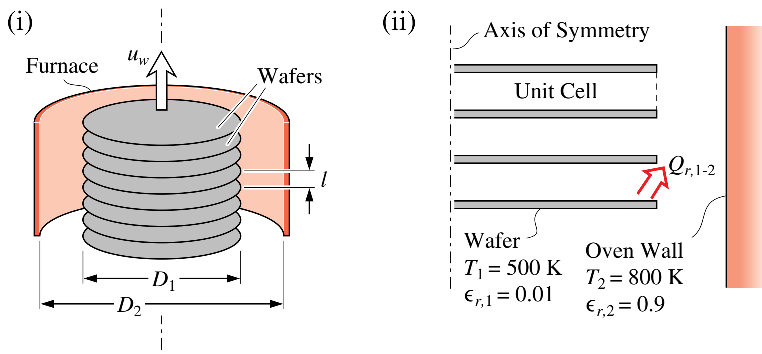


Figure Pr.4.41(a)(i) Silicon wafers moving and heating in a radiation oven. (ii) The unit cell formed by two adjacent wagers is also shown.

OBJECTIVE:

(a) Assuming that the wafers have a uniform temperature $T_1 = 500$ K, determine the net heat transfer by surface radiation between the oven and a wafer $Q_{r,1}$.

(b) During this process, the wafers enter the oven at an initial temperature lower than 500 K. As they move through the oven, they are heated by surface radiation from the oven walls until they reach a near steady-state temperature. Assuming that the axial variation of temperature (along the thickness) is negligible, quantitatively sketch the radial distribution of the wafer temperature at several elapsed times (as the wafer moves through the oven).

(c) Apply a combined integral-differential length energy equation to a wafer. Use the integral length along the thickness and a differential length along the radius. Express the surface-radiation heat transfer in terms of a differential radiation resistance, which depends on a differential view factor.

SOLUTION:

(a) The surface of a wafer exchanges radiation heat transfer with the other wafers facing it and with the oven surface. As the wafer has a uniform temperature T_1, the surface of two adjacent wafers, which are at the same temperature T_1, and the oven form a two-surface enclosure, as shown in Figure Pr.4.41(a)(ii). Then, the net surface radiation heat transfer is given by (4.47), i.e.,

$$Q_{r,1\text{-}2} = \frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{(R_{r,\Sigma})_{1\text{-}2}},$$

where the overall radiation resistance is

$$(R_{r,\Sigma})_{1\text{-}2} \quad = \quad \frac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} + \frac{1}{A_{r,1}F_{1\text{-}2}} + \frac{1-\epsilon_{r,2}}{A_{r,2}\epsilon_{r,2}}.$$

The view factor $F_{1\text{-}2}$ is obtained from Figure 4.11(a) and by using the summation rule.

From Figure 4.11(a), using $R_1 = 100(\text{mm})/30(\text{mm}) = 3.33$, and $R_2 = 3.33$, we obtain $F_{1\text{-}1} = 0.72$. Then using the summation rule, we have

$$F_{1\text{-}2} = 1 - F_{1\text{-}1} = 1 - 0.72 = 0.28.$$

The areas are

$$\begin{aligned} A_{r,1} = \frac{2\pi D_1^2}{4} &= \frac{2 \times \pi \times (100 \times 10^{-3})^2(\text{m}^2)}{4} = 0.0157 \text{ m}^2 \\ A_{r,2} = \pi D_2 l &= \pi \times 300 \times 10^{-3}(\text{m}) \times 30 \times 10^{-3}(\text{m}) = 0.0283 \text{ m}^2. \end{aligned}$$

Then, the overall radiation resistance becomes

$$\begin{aligned} (R_{r,\Sigma})_{1\text{-}2} &= \frac{1 - 0.01}{0.01 \times 0.0157(\text{m}^2)} + \frac{1}{0.28 \times 0.0157(\text{m}^2)} + \frac{1 - 0.9}{0.9 \times 0.0283(\text{m}^2)} \\ &= 6{,}302.54(1/\text{m}^2) + 227.36(1/\text{m}^2) + 3.93(1/\text{m}^2) \\ &= 6{,}537 \text{ } 1/\text{m}^2. \end{aligned}$$

The net heat transfer by surface radiation is then

$$\begin{aligned} Q_{r,1\text{-}2} &= \frac{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4)(500^4 - 800^4)(\text{K}^4)}{6{,}537(1/\text{m}^2)} \\ &= -3.008 \text{ W}. \end{aligned}$$

(b) Figure Pr.4.41(b) shows the qualitative, radial temperature distribution of wafer temperature at several elapsed times. Also shown is the expected steady-state temperature distribution.

(c) The integral-differential length analysis for a wafer with thickness w gives

$$\begin{aligned} \lim_{\Delta A \to 0} \frac{\int_{\Delta A} (\mathbf{q} \cdot \mathbf{s}_n) dA}{\Delta V} &= \lim_{\Delta A \to 0} \left\{ \frac{q_r 2\pi[(r + \Delta r)^2 - r^2]}{\pi[(r + \Delta r)^2 - r^2]w} + \frac{-q_{k,r} 2\pi r w + q_{k,r+\Delta r} 2\pi(r + \Delta r) w}{\pi[(r + \Delta r)^2 - r^2]w} \right\} \\ &= \lim_{\Delta A \to 0} \left[\frac{2q_r}{w} + \frac{-q_{k,r} 2r + q_{k,r+\Delta r} 2(r + \Delta r)}{2r\Delta r + \Delta r^2} \right]. \end{aligned}$$

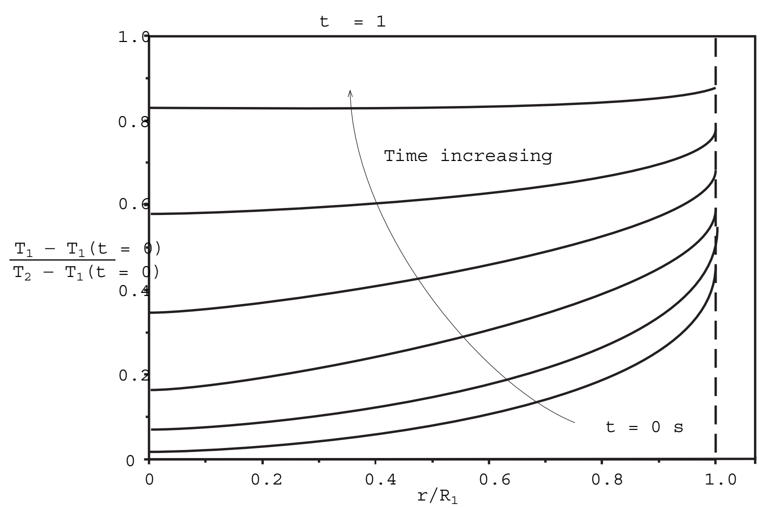


Figure Pr.4.41(b) Qualitative radial distribution of the wafer temperature, at several elapsed times.

Considering that for small $\Delta r \Delta r^2 \ll 2r\Delta r$, we have

$$\lim_{\Delta A \to 0} \frac{\int_{\Delta A} (\mathbf{q} \cdot \mathbf{s}_n) dA}{\Delta V} = \lim_{\Delta A \to 0} \left[\frac{2q_r}{w} + \left(\frac{-r q_{k,r} + r q_{k,r+\Delta r} + \Delta r q_{r,r+\Delta r}}{r\Delta r} \right) \right].$$

The second term can be rewritten as

$$\begin{aligned}\lim_{\Delta A\to 0}\left(\frac{-rq_{k,r}+rq_{k,r+\Delta r}+\Delta r q_{r,r+\Delta r}}{r\Delta r}\right) &= \lim_{\Delta A\to 0}\left(\frac{q_{k,r+\Delta r}-q_{k,r}}{\Delta r}+\frac{q_{k,r+\Delta r}}{r}\right)\\ &= \frac{\partial q_k}{\partial r}+\frac{q_k}{r}=\frac{1}{r}\frac{\partial}{\partial r}(rq_k).\end{aligned}$$

Thus, the integral-differential length energy equation becomes

$$\frac{2q_r}{w}+\frac{1}{r}\frac{\partial}{\partial r}(rq_k)=-\rho c_p\frac{\partial T}{\partial t}.$$

The heat flux by radiation is

$$q_r=\lim_{\Delta A\to 0}\frac{Q_{r,1\text{-}2}}{\Delta A_r},$$

where

$$Q_{r,1\text{-}2}=\frac{\sigma_{\rm SB}(T^4-T_2^4)}{(\Delta R_{r,\Sigma})_{1\text{-}2}}$$

and

$$(\Delta R_{r,\Sigma})_{1\text{-}2}=\frac{1-\epsilon_{r,1}}{\Delta A_r\epsilon_{r,1}}+\frac{1}{\Delta A_r F_{\Delta A_r\text{-}2}}+\frac{1-\epsilon_{r,2}}{A_{r,2}\epsilon_{r,2}},$$

where $F_{\Delta A_r\text{-}2}$ is the view factor between the differential area ΔA_r and surface 2. Then

$$\begin{aligned}q_r &= \lim_{\Delta A\to 0}\frac{\sigma_{\rm SB}(T^4-T_2^4)}{\dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1}}+\dfrac{1}{F_{\Delta A_r\text{-}2}}+\left(\dfrac{1-\epsilon_{r,2}}{\epsilon_{r,2}}\right)\dfrac{\Delta A_r}{A_{r,2}}}\\ &= \frac{\sigma_{\rm SB}(T^4-T_2^4)}{\dfrac{1-\epsilon_{r,1}}{\epsilon_{r,1}}+\dfrac{1}{F_{\Delta A_r,2}}}.\end{aligned}$$

The view factor $F_{\Delta A_r\text{-}2}$ is the view factor from a cylindrical shell (ring), with radial length Δr, to the oven surface. This is given in reference [9] as a function of geometric parameters.

COMMENT:

The radiation heat flow rate into the wafer $Q_{r,i}$ is rather small, but the mass of a silicon wafer is not very large. Therefore, speedy heat-up is possible. An increase in l will increase $Q_{r,1}$.

PROBLEM 4.42.FAM

GIVEN:

A gridded silicon electric heater is used in a microelectromechanical device, as shown in Figure Pr.4.42. The heater has an electrical resistance R_e and a voltage $\Delta\varphi$ is applied resulting in the Joule heating. For testing purposes, the heater is raised to a steady-state, high temperature (i.e., glowing red). The gridded heater is connected to a substrate through four posts (made of silicon oxide, for low conductivity k_p), resulting in conduction heat loss through four support posts. The substrate is at T_s and has an emissivity $\epsilon_{r,s}$. The upper heater surface is exposed to large surface area surroundings at T_{surr}. Treat the heater as having a continuous surface (i.e., solid, not gridded), with a uniform temperature T_1.

$T_s = 400$°C, $T_{surr} = 25$°C, $w = 0.5$ mm, $a = 0.01$ mm, $l = 0.01$ mm, $\epsilon_{r,1} = 0.8$, $\epsilon_{r,s} = 1$, $R_e = 1{,}000$ ohm, $\Delta\varphi = 5$ V, $k_p = 2$ W/m-K.

You do not need to use tables or figures for the view factors. Use (2.28) for $\dot{S}_{e,\mathrm{J}}$.

SKETCH:

Figure Pr.4.42(a) shows the heater and the supporting posts.

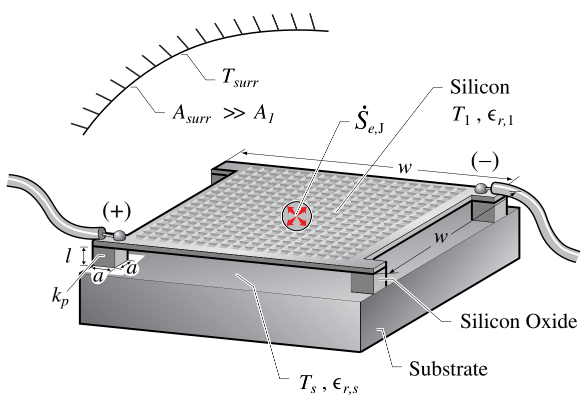


Figure Pr.4.42(a) A miniature gridded heater connected to a substrate by four support posts and raised to a glowing temperature.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the heater temperature T_1.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure 4.42(b). Heat transfer from the heater is by surface radiation to the surroundings and the substrate, and by conduction through the posts.

(b) The energy equation, from Figure Pr.4.41(b), is

$$Q_{r,1\text{-}surr} + Q_{r,1\text{-}s} + Q_{k,1\text{-}s} = \dot{S}_{e,\mathrm{J}}.$$

From (2.28), we have

$$\dot{S}_{e,\mathrm{J}} = \frac{\Delta\varphi^2}{R_e}.$$

The view factors between the upper surface and surroundings is unity $F_{1\text{-}surr} = 1$. The view factor between the lower surface and the substrate $F_{1\text{-}s}$ is also assumed unity, using $w \ll l$ in Figure 4.11(b).
The surface radiation for unity view factor and $A_{r,surr} \ll A_1$, and for $\epsilon_{r,s} = 1$, are given by (4.49), i.e.,

$$\begin{aligned} Q_{r,1\text{-}surr} &= A_{r,1}\epsilon_{r,1}\sigma_{\mathrm{SB}}(T_1^4 - T_{surr}^4) \\ Q_{r,1\text{-}s} &= A_{r,1}\epsilon_{r,1}\sigma_{\mathrm{SB}}(T_1^4 - T_s^4), \quad A_{r,1} = w^2. \end{aligned}$$

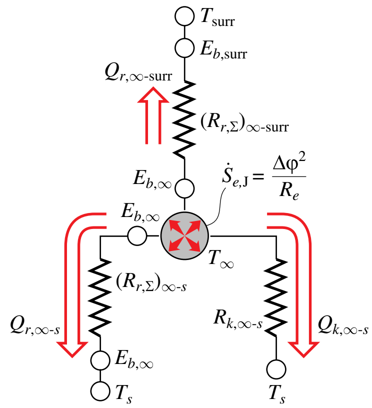


Figure Pr.4.42(b) Thermal circuit diagram.

The conduction resistance is found from Table 3.2, i.e.,

$$Q_{k,1\text{-}s} = \frac{T_1 - T_2}{R_{k,1\text{-}s}} = \frac{4A_p k_p}{l}(T_1 - T_s), \quad A_p = a^2.$$

The energy equation becomes

$$w^2 \epsilon_{r,1} \sigma_{\text{SB}} (2T_1^4 - T_{surr}^4 - T_s^4) + \frac{4a^2 k_p}{l}(T_1 - T_s) = \frac{\Delta\varphi^2}{R_e}$$

or

$$(5 \times 10^{-4})^2(\text{m}^2) \times 0.8 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times [2T_1^4 - (298.15)^4(\text{K}^4) - (673.15)^4(\text{K}^4)] + \frac{4 \times (10^{-5})^2(\text{m}^2) \times 2(\text{W/m-K})}{10^{-5}(\text{m})} \times (T_1 - 673.15)(\text{K}) = \frac{5^2(V^2)}{1{,}000(\text{ohm})}$$

or

$$1.134 \times 10^{-14} \times (2T_1^4 - 7.902 \times 10^9 - 2.053 \times 10^{11}) + 8.000 \times 10^{-5} \times (T_1 - 673.15) = 0.025.$$

Solving for T_1, we have

$$T_1 = 860.5 \text{ K}.$$

COMMENT:

Note that the dull red is identified as the Draper point in Figure 4.2(a) with $T = 798$ K. In practice, the heater is a gridded silicon with a polysilicon coating. Also note that since $w/l = 50$, the $F_{1\text{-}s}$, from Figure 4.11(b), is unity.

PROBLEM 4.43.FAM

GIVEN:

A spherical cryogenic (hydrogen) liquid tank has a thin (negligible thickness), double-wall structure with the gap space filled with air, as shown in Figure Pr.4.43. The air pressure is one atm.

$\epsilon_{r,1} = 0.05$, $\epsilon_{r,2} = 0.05$, $R_1 = 1$ m, $R_2 = 1.01$ m, $T_1 = -240$℃, $T_2 = -80$℃.

Use (3.19) for low and moderate gas pressure gases to determine any pressure dependence of the gas conductivity. Use Table C.22 for the atmospheric pressure properties of air and use $T = 150$ K.

SKETCH:

Figure Pr.4.43(a) shows the tank, walls, and the air gap.

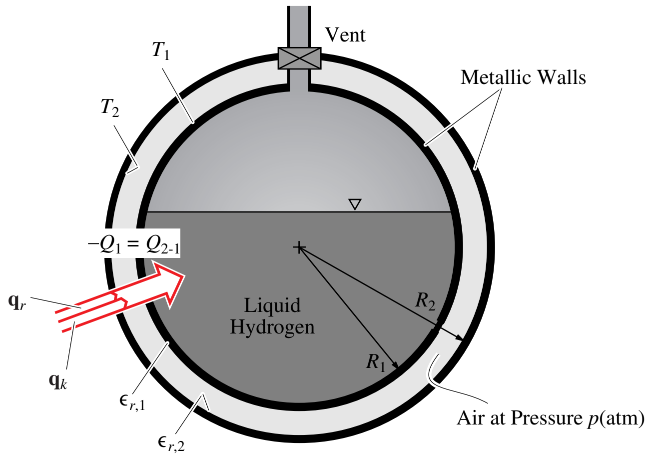


Figure Pr.4.43(a) A cryogenic (liquid hydrogen) tank has a double-wall structure with the space between the thin walls filled with atmospheric or subatmospheric pressure air.

OBJECTIVE:

(a) Draw the thermal circuit diagram for heat flow between the outer and inner walls.

(b) Determine the rate of heat transfer to the tank $Q_{2\text{-}1}$.

(c) Would $Q_{2\text{-}1}$ change if the air pressure is reduced to 1/10 atm? How about under ideal vacuum ($p = 0$, $k = 0$)?

SOLUTION:

(a) Figure Pr.4.43(b) shows the thermal circuit diagram. The heat flows by conduction and radiation from surface 2 to surface 1.

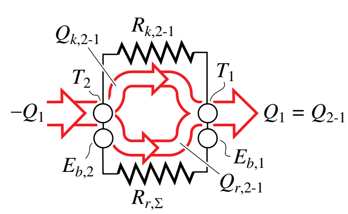


Figure Pr.4.43(b) Thermal circuit diagram.

(b) From Figure Pr.4.43(b), we have

$$Q_{2\text{-}1} = \frac{T_2 - T_1}{R_{k,2\text{-}1}} + \frac{E_{b,2} - E_{b,1}}{R_{r,\Sigma}},$$

where from Table 3.3, we have

$$R_{k,2\text{-}1} = R_{k,1\text{-}2} = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)/4\pi k,$$

and for a two-surface enclosure of the gap space we have from (4.47),

$$R_{r,\Sigma} = \left(\frac{1-\epsilon_r}{A_r\epsilon_r}\right)_1 + \frac{1}{A_{r,1}F_{1\text{-}2}} + \left(\frac{1-\epsilon_r}{A_r\epsilon_r}\right)_2 .$$

Here $F_{1\text{-}2} = 1$ and $A_{r,1} = 4\pi R_1^2$ and $A_2 = 4\pi R_2^2$.

From Table C.22, for air at $T = 150$ K, we have

$$k = 0.0158 \text{ W/m-K} \quad \text{Table C.22}$$

Then

$$\begin{aligned} R_{k,2\text{-}1} &= \left(\frac{1}{1} - \frac{1}{1.01}\right)/4\pi \times 0.0158(\text{W/m-K}) \\ &= 4.986 \times 10^{-2} \text{ K/W} \\ R_{r,\Sigma} &= \frac{1-0.05}{4\pi \times (1)^2(\text{m}^2) \times 0.05} + \frac{1}{4\pi \times (1)^2(\text{m}^2)} + \frac{1-0.05}{4\pi \times (1.01)^2(\text{m}^2) \times 0.05} \\ &= (1.512 + 0.07957 + 1.482)(1/\text{m}^2) = 3.074 \text{ 1/m}^2. \end{aligned}$$

Then

$$\begin{aligned} Q_{2\text{-}1} &= \frac{(193.15 - 33.15)(\text{K})}{4.986 \times 10^{-2}(\text{K/W})} + \frac{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4)[(193.15)^4 - (33.15)^4](\text{K}^4)}{3.074(1/\text{m}^2)} \\ &= 3{,}209(\text{W}) + 25.65(\text{W}) = 3{,}235 \text{ W}. \end{aligned}$$

(c) From (3.19), we note that there is no pressure dependence of k for the monatomic gases at low and moderate pressures. This is also true for the diatomic gas mixtures such as air. Then

$$Q_{2\text{-}1} = 3{,}235 \text{ W} \quad p = 0.1 \text{ atm}.$$

For $p = 0$ which gives $k = 0$, we have radiation heat transfer only, and

$$Q_{2\text{-}1} = 25.65 \text{ W} \quad p = 0.$$

COMMENT:

Air is made of oxygen and nitrogen and their condensation temperature (boiling point) at $p = 1$ atm is given in Table C.4 as $T_{lg} = 90.0$ K and $T_{lg} = 197.6$ K, respectively. Then it becomes necessary to evacuate the gap space in order to avoid condensate formation which collects in the gap, at the bottom of the tank under gravity. In practice, the outer surface is insulated to lower the surface temperature much below −80℃.

Note the small contribution due to radiation (due to the small emissivities).

Also as the gas pressure drops, the possibility of gas molecules colliding with the walls becomes greater than that for the intermolecular collisions. Then the size (gap between the walls) should be included. This is left as an end of Chapter 3 problem and includes the Knudsen number $\text{Kn}_L = \lambda_m/L$ as a parameter, where λ_m is the molecular mean-free path given by (1.19) and $L = R_2 - R_1$.

PROBLEM 4.44.FUN

GIVEN:

In surface radiation through multiple, opaque layer systems, such as the one shown in Figure Pr.4.44, the rate of conduction through the layers can be significant. To include the effect of the layer conductivity, and also the layer spacing indicated by porosity, use the approximation that the local radiation heat transfer is determined by the local temperature gradient, i.e.,

$$q_{r,x} = -\langle k_r \rangle \frac{dT}{dx} = -\langle k_r \rangle \frac{T_2 - T_1}{l_1 + l_2}, \quad \langle k_r \rangle = \langle k_r \rangle (k_s, \epsilon, \epsilon_r, T_2, l_2),$$

where $\langle k_r \rangle$ is the radiant conductivity.

SKETCH:

Figure Pr.4.44 shows a multiple, finite thickness l_1 and conductivity k_s parallel layer (opaque) system and its thermal circuit model.

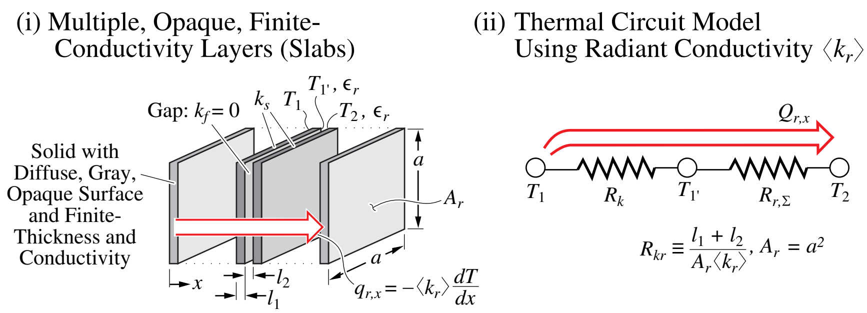


Figure Pr.4.44(i) Surface-radiation and solid conduction through a system of parallel slabs. (ii) Thermal circuit diagram.

OBJECTIVE:

(a) Using the thermal circuit diagram representing $Q_{r,x}$ in Figure Pr.4.44(ii), start from (4.47) and use $F_{1'\text{-}2} = 1$ for $l_2 \ll a$, and then use (4.72) to linearize $T_{1'}^4 - T_2^4$ and arrive at

$$q_{r,x} = \frac{4\epsilon_r \sigma_{\rm SB} T^3 (T_1^{'} - T_2)}{2 - \epsilon_r}.$$

(b) Then add R_k and use $T_1 - T_2$ to arrive at the radiant conductivity expression $\langle k_r \rangle$

$$\langle k_r \rangle = \frac{1}{\dfrac{1-\epsilon}{k_s} + \dfrac{(2-\epsilon_r)\epsilon}{4\epsilon_r \sigma_{\rm SB} T^3 l_2}}, \quad \epsilon = \frac{l_2}{l_1 + l_2}$$

or

$$\frac{\langle k_r \rangle}{4\sigma_{\rm SB} T^3 l_2} = \frac{1}{\dfrac{4\sigma_{\rm SB} T^3 l_2 (1-\epsilon)}{k_s} + \dfrac{(2-\epsilon_r)\epsilon}{\epsilon_r}}$$

or

$$\frac{k_r}{4\sigma_{\rm SB} T^3 l_1} = \frac{\epsilon_r N_r^{-1} (1-\epsilon)^{-1}}{\epsilon_r + N_r^{-1}(2-\epsilon_r)}, \quad N_r = \frac{4\sigma_{\rm SB} T^3 l_1}{k_s}.$$

SOLUTION:

(a) Starting from (4.47) and assuming $F_{1'\text{-}2} = 1$ [from Figure 4.11(b)], we have

$$\begin{aligned} q_{r,x} &= q_{r,1'\text{-}2} = \frac{1}{A_r} \frac{E_{b,1'} - E_{b,2}}{\dfrac{1-\epsilon_r}{A_r \epsilon_r} + \dfrac{1}{A_r} + \dfrac{1-\epsilon_r}{A_r \epsilon_r}} \\ &= \frac{\sigma_{\rm SB}(T_{1'}^4 - T_2^4)}{\dfrac{2}{\epsilon_r} - 1} = \frac{\epsilon_r \sigma_{\rm SB}(T_{1'}^4 - T_2^4)}{2 - \epsilon_r}. \end{aligned}$$

Now, from (4.72), using

$$\begin{aligned} T_{1'}^4 - T_2^4 &= (T_{1'}^2 + T_2^2)(T_{1'}^2 - T_2^2) \\ &= (T_{1'}^2 + T_2^2)(T_{1'} + T_2)(T_{1'} - T_2) \\ &= 4T^3(T_{1'} - T_2), \end{aligned}$$

where we have defined

$$4T^3 \equiv (T_{1'}^2 + T_2^2)(T_{1'} + T_2)$$

as the average temperature for the case of $T_{1'} \rightarrow T_2 \rightarrow T$. Then

$$q_{r,x} = \frac{4\epsilon_r \sigma_{\rm SB} T^3 (T_{1'} - T_2)}{2 - \epsilon_r}.$$

(b) From Table 3.2, we use the conduction resistance to write, using Figure Pr.4.44(i),

$$q_{r,x} = \frac{1}{A_r}\frac{T_1 - T_{1'}}{R_k} = \frac{T_1 - T_{1'}}{l_1/k_s}.$$

Now, noting that the conduction and radiation resistance are in series, we have

$$\begin{aligned} q_{r,x} &= \frac{T_1 - T_2}{A_r(R_k + R_{r,\Sigma})} = \frac{T_1 - T_2}{\dfrac{l_1}{k_s} + \dfrac{2-\epsilon_r}{4\epsilon_r \sigma_{\rm SB} T^3}} \\ &= -\frac{1}{\dfrac{l_1}{(l_1 + l_2)k_s} + \dfrac{(2-\epsilon_r) l_2}{4\epsilon_r \sigma_{\rm SB} T^3 l_2 (l_1 + l_2)}} \frac{T_2 - T_1}{l_1 + l_2} \\ &= -\frac{1}{\dfrac{1-\epsilon}{k_s} + \dfrac{(2-\epsilon_r)\epsilon}{4\epsilon_r \sigma_{\rm SB} T^3 l_2}} \frac{T_2 - T_1}{l_1 + l_2}, \quad \epsilon = \frac{l_2}{l_1 + l_2} \\ &\equiv -\langle k_r \rangle \frac{T_2 - T_1}{l_1 + l_2} \end{aligned}$$

or

$$\langle k_r \rangle = \frac{1}{\dfrac{1-\epsilon}{k_s} + \dfrac{(2-\epsilon_r)\epsilon}{4\epsilon_r \sigma_{\rm SB} T^3 l_2}}.$$

or

$$\frac{\langle k_r \rangle}{4\sigma_{\rm SB} T^3 l_2} = \frac{1}{\dfrac{4\sigma_{\rm SB} T^3 l_2 (1-\epsilon)}{k_s} + \dfrac{(2-\epsilon_r)\epsilon}{\epsilon_r}}$$

Now using the conduction-radiation number N_r defined by (4.75), we have

$$\frac{k_r}{4\sigma_{\rm SB} T^3 l_1} = \frac{\epsilon_r N_r^{-1} (1-\epsilon)^{-1}}{\epsilon_r + N_r^{-1}(2-\epsilon_r)}.$$

Note that using l instead of l_2 gives

$$\frac{k_r}{4\sigma_{\rm SB} T^3 l_1} = \frac{1}{\dfrac{4\sigma_{\rm SB} T^3 l_1 (1-\epsilon)}{k_s} + \dfrac{(2-\epsilon_r)(1-\epsilon)}{\epsilon_r}}.$$

COMMENT:

Note that for $k_s \rightarrow \infty$, i.e., an ideally conducting solid, we have

$$\langle k_r \rangle = \frac{4\epsilon_r \sigma_{\rm SB} T^3 l_2}{(2-\epsilon_r)\epsilon}, \quad k_s \rightarrow \infty.$$

This shows an increase in $\langle k_r \rangle$ as the fraction of high conductivity solid ($k_s \rightarrow \infty$) increases (i.e., ϵ decreases). Note that when $\epsilon_r \rightarrow 0$, then there is no heat transfer (infinite radiation resistance), because heat has to be transferred by radiation between surfaces in order to be conducted across the layer (here zero conductivity is assumed for the fluid occupying the space between the surfaces).

PROBLEM 4.45.FUN.S

GIVEN:

The measured effective thermal conductivity of porous solids, such as that for packed zirconium oxide fibers given in Figure 3.13(b), does include the radiation contribution. The theoretical prediction can treat the conduction and radiation heat transfer separately. In a prediction model (derivation for cubic particles is left as an end of the chapter problem), the effective, combined (total) conductivity for a periodic porous solid is given by

$$\begin{aligned}\langle k_{kr}\rangle &= \langle k_k\rangle + \langle k_r\rangle \\ \langle k_r\rangle &= 4D\sigma_{\rm SB}T^3\frac{\epsilon_r(1-\epsilon)^{1/3}N_r^{-1}}{\epsilon_r + N_r^{-1}(2-\epsilon_r)}, \quad N_r = \frac{4\sigma_{\rm SB}T^3 D}{k_s} \\ \langle k\rangle &= k_f\left(\frac{k_s}{k_f}\right)^{0.280-0.757\log\epsilon+0.057\log(k_s/k_f)}\end{aligned}$$

Using these, we can compare the predicted and measured results.

Here ϵ is the porosity, D is the fiber-diameter, ϵ_r is the fiber emissivity, k_f is the fluid, and k_s is the solid conductivity.

$D = 10$ μm, $\langle\rho\rangle = 1{,}120$ kg/m^3.

Use $\langle\rho\rangle = \epsilon\rho_f + (1-\epsilon)\rho_s = (1-\epsilon)\rho_s$, for $\rho_f \ll \rho_s$, to determine ϵ. For air use $k_f = 0.0267$(W/m-K) $+ 5.786\times 10^{-5}$(W/m-K^4) $\times (T-300)$(K), and use the only data available for zirconium oxide k_s and ρ_s in Table C.17. For emissivity use $\epsilon_r = 0.9 - 5.714\times 10^{-4}(T-300)$(K) based on Table C.18 for zirconium oxide.

OBJECTIVE:

(a) Plot the variation of $\langle k_{kr}\rangle$ with respect to T for 300 K $\le T \le$ 1,000 K, for the zirconium oxide and air system.
(b) Compare the results with the experimental results of Figure 3.13(b).
(c) Is radiation contribution significant in this material?

SOLUTION:

(a) From Table C.17, we have for zirconia

$$\rho_s = 5{,}680 \text{ kg/m}^3, \quad k_s = 1.675 \text{ W/m-K} \quad \text{Table C.17 for the porosity.}$$

Using $\langle\rho\rangle = \epsilon\rho_f + (1-\epsilon)\rho_f$ and noting that ρ_f (from Table C.22) is much smaller than ρ_s, we have

$$1-\epsilon = \frac{\langle\rho\rangle}{\rho_s} = \frac{1{,}120(\text{kg/m}^3)}{5{,}680(\text{kg/m}^3)} = 0.1972$$

or

$$\epsilon = 0.8028.$$

Using a solver-plotter, the results for $\langle k_{kr}\rangle$ versus T is plotted in Figure Pr.4.45. Note the rather linear increase with respect to T. This is due to the assumed linear increase in k_f with T.

(b) Using the experimental results plotted in Figure 3.13(b), we choose $T = 400$℃ $= 673.15$ K and this experimental result is also shown in Figure Pr.4.45. The agreement is rather good.

(c) The radiation conductivity $\langle k_r\rangle$ is not large, $\langle k_r\rangle = 0.0004399$ W/m-K at $T = 1{,}000$ K, due to the small particle diameter D. The lower emissivity at higher temperatures also makes $\langle k_r\rangle$ small.

COMMENT:

In Figure 3.13(b), the results for higher and lower $\langle\rho\rangle$ do not agree as well with the predictions, however, in all of these data, the role of radiation is not significant in the temperature range considered, because D is small.

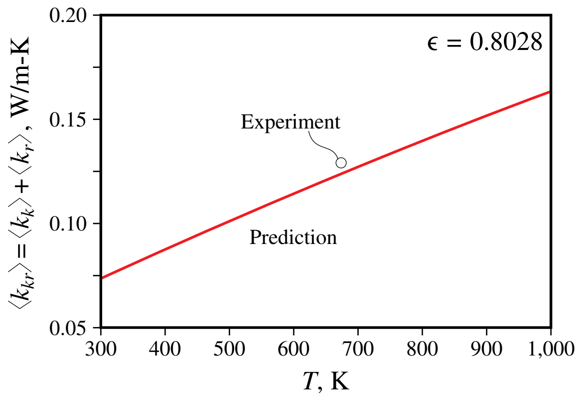


Figure Pr.4.45 Variation of the total thermal conductivity of zirconium oxide air packed bed of fibers, with respect to temperature.

PROBLEM 4.46.FAM.S

GIVEN:

A spherical carbon steel AISI 1010 piece of diameter $D = 1$ cm, initially at $T_1(t = 0) = 1{,}273$ K, is cooled by surface radiation to a completely enclosing cubic oven made of white refractory brick with each side having a length $L = 10$ cm and a surface temperature $T_2 = 300$ K. Assume that all the surfaces are opaque, diffuse, and gray. For the carbon steel sphere use the higher value for the emissivity of oxidized iron, listed in Table C.18.

OBJECTIVE:

(a) Using a software, plot the variation of the piece temperature with respect to time.
(b) Determine the time it takes for the piece to reach $T_1 = 600$ K and compare this result (i.e., the numerical solution) with the one predicted by (4.82) (i.e., the analytic solution).

SOLUTION:

(a) Figure Pr.4.46 shows the variation in the workpiece temperature with respect to time, along with the associated computer code from SOPHT.

(b) For the carbon steel sphere (node T_1), we have (from Tables C.16 and C.18, and the given geometry), $\rho_1 = 7{,}830$ kg/m^3, $c_{p,1} = 434$ J/kg-K, $\epsilon_{r,1} = 0.89$, $V_1 = \pi D^3/6 = 5.236 \times 10^{-7}$ m^3, $A_1 = \pi D^2 = 3.141 \times 10^{-4}$ m^2.

For the white refractory brick walls (node T_2), we have (from Table C.18, and the given geometry), $\epsilon_{r,1} = 0.29$ (for $T = 1{,}373$ K), $A_1 = 6(L \times L) = 6(0.1 \text{ m} \times 0.1 \text{ m}) = 0.06$ m^2,
and using (4.82) gives

$$\frac{\sigma_{\rm SB} T_2^3}{R_{r,\Sigma}(\rho c_p V)_1} t = \frac{1}{4}\left[\ln\left|\frac{T_2 + T_1}{T_2 - T_1}\right| - \ln\left|\frac{T_2 + T_1(t=0)}{T_2 - T_1(t=0)}\right| + 2\tan^{-1}\frac{T_1}{T_2} - 2\tan^{-1}\frac{T_1(t=0)}{T_2}\right],$$

where $T_1(=0) = 1{,}273$ K, $T_2 = 300$ K, and $T_1 = T_1(t) = 600$ K.
The radiation resistance between the sphere and the walls is given by (4.48) as

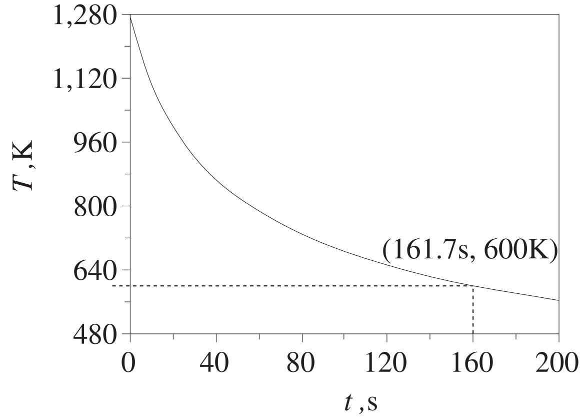
$$\begin{aligned} R_{r,\Sigma} &= (R_{r,\epsilon})_1 + (R_{r,F})_{1\text{-}2} + (R_{r,\epsilon})_2 \\ &= \left(\frac{1-\epsilon_r}{\epsilon_r A}\right)_1 + \left(\frac{1}{A_1 F_{1\text{-}2}}\right) + \left(\frac{1-\epsilon_r}{\epsilon_r A}\right)_2, \end{aligned}$$

where $F_{1\text{-}2} = 1$.
Substituting into (4.82) and solving for t, we give

$$t = 162 \text{ s}.$$

COMMENT:

The numerical solution for this relatively simple problem, is very accurate. Figure Pr.4.46 gives $t = 161.7$ s.




```
// Problem 4.46
// Note that the initial condition on T1 will be specified in the Solve window
// Volumetric Transient Node, 1
T1' = dT1dt // Must be first equation in the set
QA1 = -rho1*cp1*V1*dT1dt + S1dot // Conservation of energy, W
QA1 = Qr1 // Summation of heat transfer leaving node, W
rho1 = 7830 // Density of medium, kg/m^3
cp1 = 434 // Specific heat, J/kg-K
d1 = 0.01 // sphere diameter, m
V1 = pi*d1^3/6 // Volume, m^3
S1dot = 0 // Energy conversion, W

// Surface radiation for node 1
Qr1 = Qr12 // Surface radiation heat transfer, W
// Net radiation heat transfer between surface 1 & 2
Qr12 = (Eb1-Eb2)/Rrs12 // Radiation heat transfer between surfaces, W
Eb1 = sigmaSB*T1^4 // Emissive power of node 1, W/m^2
Eb2 = sigmaSB*T2^4 // Emissive power of node 2, W/m^2
T2 = 300 // Temperature of node 2, K

// Two-surface radiation resistance
Rrs12 = (1-epsilonr1)/(Ar1*epsilonr1)+1/(Ar1*F12)+(1-epsilonr2)/(Ar2*epsilonr2)
// equivalent radiation resistance, 1/m^2
epsilonr1 = 0.89 // Surface 1 emissivity
epsilonr2 = 0.29 // Surface 2 emissivity
Ar1 = pi*d1^2 // Surface 1 area, m^2
l2 = 0.1 // cube side length, m
Ar2 = 6*l2^2 // Surface 2 area, m^2
F12 = 1 // View factor

sigmaSB = 5.67e-8 // Stefan-Boltzmann constant
```

Figure Pr.4.46 Variation of workpiece temperature with respect to time and the computer code from SOPHT.

PROBLEM 4.47.FUN

GIVEN:

An idealized bed of solid particles is shown in Figure Pr.4.47. The heat is transferred through the bed by surface radiation and the cubic solid particles have a finite conductivity k_s, while conduction through the fluid is neglected ($k_f = 0$). We use the radiant conductivity $\langle k_r \rangle$ defined through

$$q_{r,x} = -\langle k_r \rangle \frac{dT}{dx} = -\langle k_r \rangle \frac{T_2 - T_1}{l_1 + l_2},$$

$$\langle k_r \rangle = \langle k_r \rangle (k_s, \epsilon, \epsilon_r, T, l_2).$$

SKETCH:

Figure Pr.4.47(a) shows the bed and a simplified thermal circuit model.

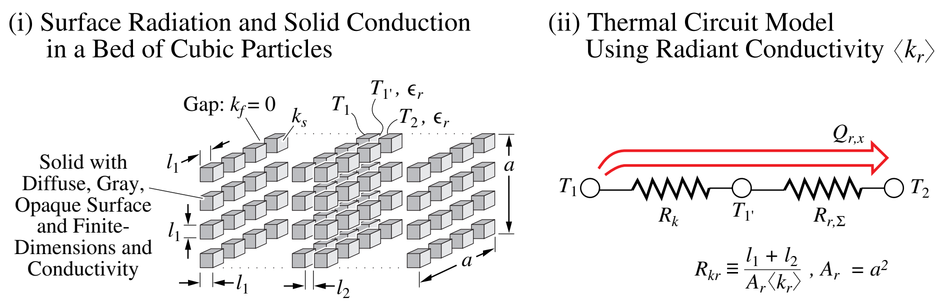


Figure Pr.4.47(a)(i) A bed of cubical particles with surface radiation and solid conduction. (ii) Simplified thermal circuit diagram.

OBJECTIVE:

(a) For the thermal circuit model shown in Figure Pr.4.47(ii), determine the radiation resistance $R_{r,\Sigma}$ between two adjacent surfaces $1'$ and 2. Assume that $F_{1'\text{-}2} = 1$ and use the linearization given in (4.72).

(b) Add the conduction resistance using series resistances to arrive at

$$\frac{\langle k_r \rangle}{4\sigma_{\rm SB} T^3 l_2 (1-\epsilon)^{2/3}} = \frac{1}{\dfrac{4\sigma_{\rm SB} T^3 l_2 (1-\epsilon)^{1/3}}{k_s} + \dfrac{(2-\epsilon_r)\epsilon^{1/3}}{\epsilon_r}}$$

$$\epsilon = 1 - \frac{l_1^3}{(l_1+l_2)^3} = \frac{l_2^3}{(l_1+l_2)^3}$$

$$\frac{k_r}{4\sigma_{\rm SB} T^3 l_1} = \frac{\epsilon_r N_r^{-1} (1-\epsilon)^{-1}}{\epsilon_r + N_r^{-1}(2-\epsilon_r)}, \quad N_r = \frac{4\sigma_{\rm SB} T^3 l_1}{k_s}.$$

SOLUTION:

(a) Starting from (4.47), with $F_{1'\text{-}2} = 1$ and all surfaces having the same emissivity ϵ_r, we have

$$Q_{r,1'\text{-}2} = \frac{E_{b,1'} - E_{b,2}}{\dfrac{1-\epsilon_r}{A_r(1-\epsilon)^{2/3}\epsilon_r} + \dfrac{1}{A_r(1-\epsilon)^{2/3}} + \dfrac{1-\epsilon_r}{A_r(1-\epsilon)^{2/3}\epsilon_r}}$$

$$= \frac{A_r(1-\epsilon)^{2/3}(E_{b,1'} - E_{b,2})}{\dfrac{2(1-\epsilon_r)}{\epsilon_r} + 1}$$

$$= \frac{A_r(1-\epsilon)^{2/3}(E_{b,1'} - E_{b,2})}{\dfrac{2}{\epsilon_r} - 2 + 1}$$

$$= \frac{A_r(1-\epsilon)^{2/3}\sigma_{\rm SB}(T_{1'}^4 - T_2^4)}{\dfrac{2-\epsilon_r}{\epsilon_r}}, \quad A_r = a \times a, \quad \epsilon = 1 - \frac{l_1^3}{(l_1+l_3)}, \quad (1-\epsilon)^{2/3} = \frac{l_1^2}{(l_1+l_2)^3},$$

where $(1-\epsilon)^{2/3}$ is the solid area fraction (as compared to $1-\epsilon$ which is solid volume fraction).

Now similar to (4.72), we linearize this for $T_{1'} \rightarrow T_2 \rightarrow T$, i.e.,

$$\begin{aligned}
(T_{1'}^4 - T_2^4) &= (T_{1'}^2 + T_2^2)(T_{1'}^2 - T_2^2) \\
&= (T_{1'}^2 + T_2^2)(T_{1'} + T_2)(T_{1'} - T_2) = 4T^3(T_{1'} - T_2) \\
4T^3 &\equiv (T_{1'}^2 + T_2^2)(T_1 + T_2)
\end{aligned}$$

where T is an average temperature. Then

$$\begin{aligned}
Q_{r,1'\text{-}2} &= \frac{A_r(1-\epsilon)^{2/3} 4\sigma_{\rm SB} T^3 (T_{1'} - T_2)}{\dfrac{2-\epsilon_r}{\epsilon_r}} \\
&= \frac{T_{1'} - T_2}{\dfrac{1}{4\sigma_{\rm SB} A_r T^3 (1-\epsilon)^{2/3}} \left(\dfrac{2-\epsilon_r}{\epsilon_r} \right)}.
\end{aligned}$$

(b) Now adding the conduction heat transfer (Table 3.2) as a series resistance, we have

$$\begin{aligned}
Q_{r,1\text{-}2} &= \frac{T_1 - T_2}{\dfrac{l_1}{A_r(1-\epsilon)^{2/3} k_s} + \dfrac{1}{A_r(1-\epsilon)^{2/3} 4\sigma_{\rm SB} T^3} \left(\dfrac{2-\epsilon_r}{\epsilon_r} \right)} \\
q_{r,1\text{-}2} &= \frac{Q_{r,1\text{-}2}}{A_r} = \frac{T_1 - T_2}{\dfrac{l_1}{(1-\epsilon)^{2/3} k_s} + \dfrac{1}{(1-\epsilon)^{2/3} 4\sigma_{\rm SB} T^3} \left(\dfrac{2-\epsilon_r}{\epsilon_r} \right)} \\
&\equiv -\langle k_r \rangle \frac{T_2 - T_1}{l_1 + l_2}.
\end{aligned}$$

Then using $l_1 = (l_1 + l_2)(1-\epsilon)^{1/3}$, and $l_2 = (l_1 + l_2)\epsilon^{1/3}$, we have

$$\frac{\langle k_r \rangle}{4\sigma_{\rm SB} T^3 l_2 (1-\epsilon)^{2/3}} = \frac{1}{\dfrac{4\sigma_{\rm SB} T^3 l_2 (1-\epsilon)^{1/3}}{k_s} + \dfrac{(2-\epsilon_r)\epsilon^{1/3}}{\epsilon_r}}$$

or

$$\frac{\langle k_r \rangle}{4\sigma_{\rm SB} T^3 l_1} = \frac{\epsilon_r N_r^{-1} (1-\epsilon)^{1/3}}{\epsilon_r + N_r^{-1}(2-\epsilon_r)}, \quad N_r = \frac{4\sigma_{\rm SB} T^3 l_1}{k_s}.$$

COMMENT:

Note that for $l_1 \to 0$ ($\epsilon \to 1$), we have

$$\langle k_r \rangle = \frac{4\sigma_{\rm SB}\epsilon_r T^3 l_2}{2-\epsilon_r}.$$

For $k_s \to \infty$, we have

$$\langle k_r \rangle = \frac{4\sigma_{\rm SB}\epsilon_r T^3 l_2 (1-\epsilon)^{1/3}}{2-\epsilon_r}.$$

Figure Pr.4.44(b) shows the variation of the dimensionless radiant conductivity $4\sigma_{\rm SB} l_1 T^3/\langle k_r \rangle$ with respect to the inverse of conduction-radiation $N_r^{-1} = k_s/(4\sigma_{\rm SB}T^3 l_1)$ for various surface emissivity ϵ_r. The results are for $\epsilon = 0.478$ (corresponding to a square-array arrangement of touching spherical particles).

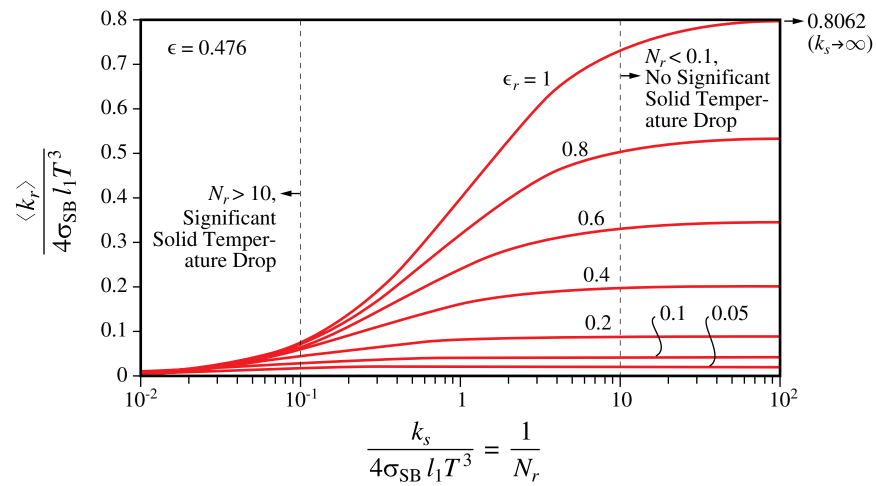


Figure Pr.4.47(b) Variation of dimensionless radiant conductivity with respect to the inverse of conduction-radiation number.

Note that from the above relation for $\langle k_r \rangle$, for $\epsilon_r = 1$, we have

$$\frac{\langle k_r \rangle}{4\sigma_{\rm SB}T^3 l_1} = (1-\epsilon)^{1/3},$$

where

$$l_2(1-\epsilon)^{1/3} = l_2 \frac{l_1}{l_1+l_2} = \frac{l_2}{l_1+l_2} l_1 = \epsilon^{1/3} l_1.$$

Now using $\epsilon = 0.476$, we have a value of $\langle k_r \rangle / 4\sigma_{\rm SB}T^3 l_1 = 0.8062$ which matches the value in Figure Pr.4.44(b). From the figure, note the role of N_r on the radiant conductivity.

PROBLEM 4.48.FUN

GIVEN:

In the limit of a optically thick medium, $\sigma_{ex}^* = \sigma_{ex}L > 10$, the extinction coefficient σ_{ex} and the radiant conductivity are used interchangeably and are related through

$$\begin{aligned} q_{r,x} &\equiv -\langle k_r \rangle \frac{dT}{dx} \\ &= -\frac{4}{3\sigma_{ex}}\frac{dE_b}{dx} = -\frac{16}{3}\frac{\sigma_{\rm SB}T^3}{\sigma_{ex}}\frac{dT}{dx}. \end{aligned}$$

Then

$$\langle k_r \rangle = \frac{16\sigma_{\rm SB}T^3}{3\sigma_{ex}}.$$

For a packed bed of cubical particles of finite conductivity k_s, surface emissivity ϵ_r, and linear dimension l_1, with interparticle spacing l_2, the radiant conductivity can be shown to be approximated by

$$\langle k_r \rangle = \frac{1}{\dfrac{1}{k_s(1-\epsilon)^{1/3}} + \dfrac{(2-\epsilon_r)\epsilon^{1/3}}{4\sigma_{\rm SB}\epsilon_r T^3 l_2 (1-\epsilon)^{2/3}}}$$

or

$$\sigma_{ex} = \frac{16\sigma_{\rm SB}T^3}{3k_s(1-\epsilon)^{1/3}} + \frac{4(2-\epsilon_r)\epsilon^{1/3}}{3\epsilon_r l_2 (1-\epsilon)^{2/3}}.$$

OBJECTIVE:

(a) Determine σ_{ex} for a bed of alumina cubical particles at $T = 500$ K, with $\epsilon_r = 0.7$ $\epsilon = 0.4$, $k_s = 36$ W/m-K, and (i) $l_2 = 3$ cm, and (ii) $l_2 = 3$ μm.
(b) Repeat (a) for amorphous silica particles, $\epsilon_r = 0.45$, $k_s = 1.38$ W/m-K, keeping other parameters the same.
(c) Compare these with the results of Figure 2.13 and comment.

SOLUTION:

(a) Alumina, (i) $l_2 = 0.03$ m

$$\begin{aligned} \sigma_{ex} &= \frac{16 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (500)^3(\text{K}^3)}{3 \times 36(\text{W/m-K})(1-0.4)^{1/3}} + \\ &\quad \frac{4(2-0.7) \times 0.4^{1/3}}{3 \times 0.7 \times 3 \times 10^{-2}(\text{m}) \times (1-0.4)^{2/3}} \\ &= 1.245(1/\text{m}) + 85.51(1/\text{m}) = 86.75 \text{ 1/m}. \end{aligned}$$

(ii) For $l_2 = 3 \times 10^{-6}$ m, we have

$$\sigma_{ex} = 1.245(1/\text{m}) + 8.551 \times 10^5(1/\text{m}) = 8.551 \times 10^5 \text{ 1/m}.$$

(b) Amorphous silica, (i) $l_2 = 0.03$ m

$$\begin{aligned} \sigma_{ex} &= \frac{16 \times 5.67 \times 10^{-8} \times (500)^3}{3 \times 1.38(1-0.4)^{1/3}} + \frac{4(2-0.45) \times 0.4^{1/3}}{3 \times 0.45 \times 3 \times 10^{-2}(\text{m}) \times (1-0.4)^{2/3}} \\ &= 32.47(1/\text{m}) + 158.6(1/\text{m}) = 191.0 \text{ 1/m}. \end{aligned}$$

(ii) For $l_2 = 3 \times 10^{-6}$ m, we have

$$\sigma_{ex} = 32.47(1/\text{m}) + 1.585 \times 10^6(1/\text{m}) = 1.586 \times 10^5 \text{ 1/m}.$$

(c) In Figure 2.13, for a packed bed of spherical particles (material is not identified), we have σ_{ex} of about 20 1/m for a particle diameter of 3 cm and about 2×10^5 1/m for a particle diameter of 3 μm. These are in general agreement with the above results.

COMMENT:

In the expression used for $\langle k_r \rangle$, a one-dimensional conduction in series with surface radiation is used. Therefore, the model should be considered an approximation. Note that the conduction contribution decreases as the particle spacing decreases.

Also note that from the expression for radiant conductivity, if we define the phonon mean-free path as

$$\lambda_{ph} = \frac{1}{\sigma_{ex}} = \frac{2\epsilon_r l_2 (1-\epsilon)^{2/3}}{4(2-\epsilon_r)\epsilon^{1/3}},$$

then λ_{ph} decreases with decreasing ϵ_r (smaller surface emission) and increasing ϵ (there is less surface to emit radiation).

PROBLEM 4.49.FAM

GIVEN:

Spherical, pure, rough-polish aluminum particles of diameter D_1 and emissivity $\epsilon_{r,1}$ are heated by surface radiation while traveling through an alumina ceramic tube kept at a high temperature T_2. The tube has an inner diameter D_2, a length l, and an emissivity $\epsilon_{r,2}$. This is shown in Figure Pr.4.49(a). A particle arrives at the entrance to the tube with an initial, uniform temperature $T_1(t=0)$, and exits the tube with a final, uniform temperature $T_1(t=t_f)$. Assume that, throughout the time of travel, the fraction of radiative heat transfer between the particle and the open ends of the tube is negligible (i.e., the view factor $F_{1\text{-ends}} = 0$).

$D_1 = 10\ \mu$m, $D_2 = 3$ mm, $l = 5$ cm, $T_1(t=0) = 20°$C, $T_1(t=t_f) = T_{sl}$ (aluminum, Table C.16), $T_2 = 1{,}283$ K.

Evaluate the emissivities from Table C.18 and the properties of aluminum at $T = 300$ K (Table C.16).

SKETCH:

Figure Pr.4.49(a) shows a particle flowing in a tube while being heated by surface radiation from the tube wall.

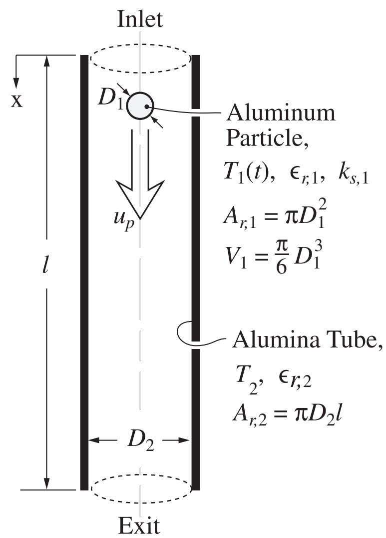


Figure Pr.4.49(a) Particles heated in a tube by surface radiation.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the speed $u_p = l/t_f$ at which the particle must move through the tube in order to exit at the melting temperature $T_1(t=t_f) = T_{sl}$.

(c) Approximate the internal conduction resistance to be $(D_1/2)/(A_{r,1}k_{s,1})$, where $k_{s,1}$ is the thermal conductivity of the solid aluminum, and determine if the assumption of uniform temperature within the particle is valid.

SOLUTION:

(a) To determine the particle speed $u_p = l/t_f$, we must determine the time t_f for the particle to be heated to the melting temperature of aluminum $T_1(t=t_f) = T_{sl} = 933$ K (Table C.16).

Since the sphere is very small and consists of aluminum which has a high thermal conductivity, we will initially assume it to behave as a lumped-capacitance thermal mass. The thermal circuit diagram is shown in Figure Pr.4.49(b).

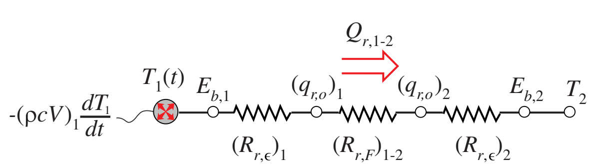


Figure Pr.4.49(b) Thermal circuit diagram.

The sphere is a lumped system with a single resistive radiation heat transfer. Applying conservation of energy to node T_1, we have

$$\begin{aligned}
Q\,|_A &= Q_{r,1\text{-}2} &= -(\rho c V)_1 \frac{\partial T_1}{\partial t}\\
&= \frac{E_{b,1} - E_{b,2}}{(R_{r,\Sigma})_{1\text{-}2}} &= -(\rho c V)_1 \frac{\partial T_1}{\partial t}\\
&= \frac{\sigma_{\rm SB}(T_1^4 - T_2^4)}{(R_{r,\epsilon})_1 + (R_{r,F})_{1\text{-}2} + (R_{r,\epsilon})_2} &= -(\rho c V)_1 \frac{\partial T_1}{\partial t}.
\end{aligned}$$

The solutions to this differential equation is given as

$$\frac{\sigma_{\rm SB} T_2^3}{(R_{r,\Sigma})_{1\text{-}2}(\rho c V)_1} t_f = \frac{1}{4}\left[\ln\left|\frac{T_2 + T_1(t=t_f)}{T_2 - T_1(t=t_f)}\right| - \ln\left|\frac{T_2 + T_1(t=0)}{T_2 - T_1(t=0)}\right| + 2\tan^{-1}\left(\frac{T_1(t=t_f)}{T_2}\right) - 2\tan^{-1}\left(\frac{T_1(t=0)}{T_2}\right)\right].$$

The volume and areas relevant to the problem are

$$\begin{aligned}
V_1 &= \pi D_1^3/6 = \pi \times (10 \times 10^{-6})^3(\mathrm{m}^3)/6\\
&= 5.236 \times 10^{-16}\ \mathrm{m}^3,\\
A_{r,1} = A_1 &= \pi D_1^2 = \pi \times (10 \times 10^{-6})^2(\mathrm{m}^2)\\
&= 3.142 \times 10^{-10}\ \mathrm{m}^2,\\
A_{r,2} = A_2 &= \pi D_2 l = \pi \times 0.003(\mathrm{m}) \times 0.05(\mathrm{m})\\
&= 4.712 \times 10^{-4}\ \mathrm{m}^2,
\end{aligned}$$

From Table C.16 ($T_1 = 300$ K), $\rho_1 = 2{,}702$ kg/s and $c = 903$ J/kg-K. The thermal capacitance of the particle is then

$$\begin{aligned}
(\rho c V)_1 &= 2{,}702(\mathrm{kg/m^3}) \times 903(\mathrm{J/kg\text{-}K}) \times 5.236 \times 10^{-16}(\mathrm{m}^3)\\
&= 1.278 \times 10^{-9}\ \mathrm{J/K}.
\end{aligned}$$

From Table C.18, the emissivity of the rough polish aluminum is $\epsilon_{r,1} = 0.18$ and of the nylon is $\epsilon_{r,2} = 0.78$. The grayness resistances are then

$$\begin{aligned}
(R_{r,\epsilon})_1 &= \frac{1 - \epsilon_{r,1}}{\epsilon_{r,1} A_{r,1}} = \frac{1 - 0.18}{0.18 \times 3.142 \times 10^{-10}(\mathrm{m}^2)}\\
&= 1.450 \times 10^{10}\ 1/\mathrm{m}^2,\\
(R_{r,\epsilon})_2 &= \frac{1 - \epsilon_{r,2}}{\epsilon_{r,2} A_{r,2}} = \frac{1 - 0.78}{0.78 \times 4.712 \times 10^{-4}(\mathrm{m}^2)}\\
&= 598.58\ 1/\mathrm{m}^2.
\end{aligned}$$

From the summation rule for the view factors from surface 1,

$$\sum_{j=1}^{n} F_{i\text{-}j} = F_{1\text{-}1} + F_{1\text{-}2} + F_{1\text{-ends}} = 1.$$

Since $F_{1\text{-}1} = 0$ and $F_{1\text{-ends}} \approx 0$, then $F_{1\text{-}2} \approx 1$. Then the view factor resistance between surfaces 1 and 2 is

$$\begin{aligned}
(R_{r,F})_{1\text{-}2} &= \frac{1}{A_{r,1} F_{1\text{-}2}} = \frac{1}{3.142 \times 10^{-10}(\mathrm{m}^2) \times 1}\\
&= 3.183 \times 10^9\ 1/\mathrm{m}^2,
\end{aligned}$$

and the total radiative resistance is

$$\begin{aligned}
(R_{r,\Sigma})_{1\text{-}2} &= (R_{r,\epsilon})_1 + (R_{r,F})_{1\text{-}2} + (R_{r,\epsilon})_2\\
&= 1.450 \times 10^{10}(1/\mathrm{m}^2) + 3.183 \times 10^9(1/\mathrm{m}^2) + 598.58\ (1/\mathrm{m}^2)\\
&= 1.768 \times 10^{10}\ 1/\mathrm{m}^2.
\end{aligned}$$

Upon substitution into our thermal conservation of energy equation

$$\begin{aligned}\frac{\sigma_{\rm SB}T_2^3}{(R_{r,\Sigma})_{1\text{-}2}(\rho cV)_1}t_f &= \frac{1}{4}\left\{\ln\left|\frac{T_2+T_1(t=t_f)}{T_2-T_1(t=t_f)}\right| - \ln\left|\frac{T_2+T_1(t=0)}{T_2-T_1(t=0)}\right|\right.\\ &\left. +2\tan^{-1}\left[\frac{T_1(t=t_f)}{T_2}\right] - 2\tan^{-1}\left[\frac{T_1(t=0)}{T_2}\right]\right\},\end{aligned}$$

we can solve for t_f [noting $T_1(t=0)=(20+273.15)(\text{K})=293.15$ K] as

$$\begin{aligned}\frac{5.67\times10^{-8}(\text{W/m}^2\text{-K}^4)\times(1{,}283)^3(\text{K}^3)}{1.768\times10^{10}(1/\text{m}^2)\times1.278\times10^{-9}(\text{J/K})}\times t_f &= \frac{1}{4}\left\{\ln\left|\frac{1{,}283(\text{K})+933(\text{K})}{1{,}283(\text{K})-933(\text{K})}\right| - \ln\left|\frac{1{,}283(\text{K})+293(\text{K})}{1{,}283(\text{K})-293(\text{K})}\right| + \right.\\ &\left. 2\times\tan^{-1}\left[\frac{933(\text{K})}{1{,}283(\text{K})}\right] - 2\times\tan^{-1}\left[\frac{293(\text{K})}{1{,}283(\text{K})}\right]\right\}\\ 5.30(1/\text{s})\times t_f &= 0.25\times(1.846-0.465+1.257-0.449)\\ t_f &= 0.103 \text{ s}.\end{aligned}$$

The velocity of the particle through the tube must then be such that it travels the entire length of the tube in t_f seconds, i.e.,

$$u_p = l/t_f = 0.05(\text{m})/0.103(\text{s}) = 0.48 \text{ m/s}.$$

(b) The assumption of uniform temperature distribution can be validated by considering the conduction-radiation number N_r, given by (4.74) where

$$N_r = \frac{R_{k,i}}{R_{r,\Sigma}/(4\sigma_{\rm SB}T_m^3}),$$

where $R_{k,i}$ is the internal conduction resistance and T_m is the radiation mean temperature. From Table C.16 for aluminum, $k_{s,1}=237$ W/m-K. Then

$$\begin{aligned}R_{k,i} &= \frac{D_1/2}{A_{r,1}k_{s,1}} = \frac{10\times10^{-6}(\text{m})/2}{3.142\times10^{-10}(\text{m}^2)\times237(\text{W/m-K})}\\ &= 67.15 \text{ K/W},\\ T_m^3 &= \frac{(T_2^2+T_1^2)(T_2+T_1)}{4} = 0.25\times[(1{,}283^2+933^2)(1{,}283+933)](\text{K}^3)\\ &= 1.394\times10^9 \text{ K}^3.\end{aligned}$$

Upon substitution into N_r, we have

$$\begin{aligned}N_r &= \frac{R_{k,i}}{R_{r,\Sigma}/4\sigma_{\rm SB}T_m^3},\\ &= \frac{67.15(\text{K/W})}{1.768\times10^{10}(1/\text{m}^2)/[4\times5.67\times10^{-8}(\text{W/m}^2\text{-K}^4)\times1.394\times10^9(\text{K}^3)]} = 1.2\times10^{-6}.\end{aligned}$$

Here, $N_r < 0.1$, therefore the assumption of uniform temperature within the particle is valid.

COMMENT:

Surface-convection heating will also occur during the flight and should be included.

PROBLEM 4.50.FUN

GIVEN:

A highly insulated thermos depicted in Figure Pr.4.50 has five layers of insulation shields on the outside. The wall has two glass layers separated by an evacuated space [Figure Pr.4.50(a)(i)], or a cork board [Figure Pr.4.50(a)(ii)].

$T_1 = 90$℃, $T_3 = 20$℃, $l = 1$ mm, $L = 7$ mm, $\epsilon_{r,s} = 0.04, \epsilon_{r,2} = 0.9, \epsilon_{r,3} = 1$.

SKETCH:

Figure Pr.4.50(a) shows the thermos with five radiation shields with and without a cork board between the glass layers.

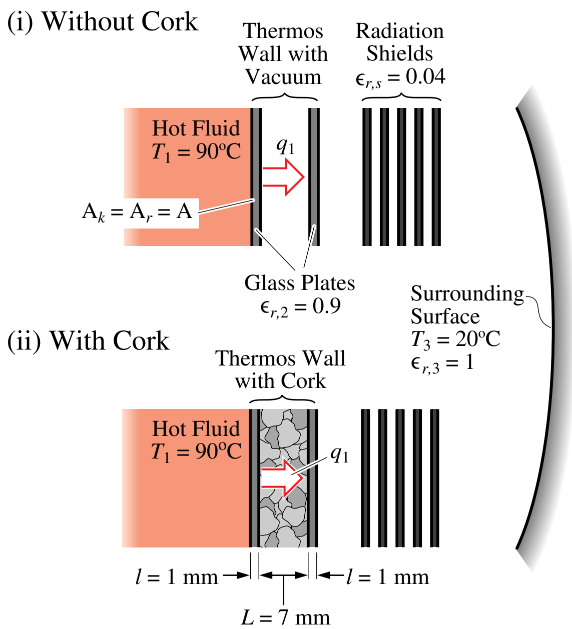


Figure Pr.4.50(a) Surface radiation from a thermos having radiation shields (i) With a cork board wall. (ii) Without a cork board wall.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the heat transfer per unit area associated with each of these designs. Comment on the preference of the cork board or the vacuum. Also, assume that all the surfaces are diffuse and gray. The glass is assumed opaque to radiation.

SOLUTION:

(i) Thermos wall with vacuum:

(a) The heat transfer is by conduction and radiation. Figure Pr.4.50(b) shows the thermal circuit diagram. Note that $\epsilon_{r,i} = \epsilon_{r,o} = \epsilon_{r,2}$.

(b) Using Figure Pr.4.50(b), we have

$$\begin{aligned} Q_{k,1\text{-}i} &= \frac{T_1 - T_i}{R_{k,1\text{-}i}} \\ Q_{r,i\text{-}o} &= \frac{E_{b,i} - E_{b,o}}{(R_{r,\Sigma})_{i\text{-}o}} \\ Q_{k,o\text{-}2} &= \frac{T_o - T_2}{R_{k,o\text{-}2}} \\ Q_{r,2\text{-}3} &= \frac{E_{b,2} - E_{b,3}}{(R_{r,\Sigma})_{2\text{-}3}}. \end{aligned}$$

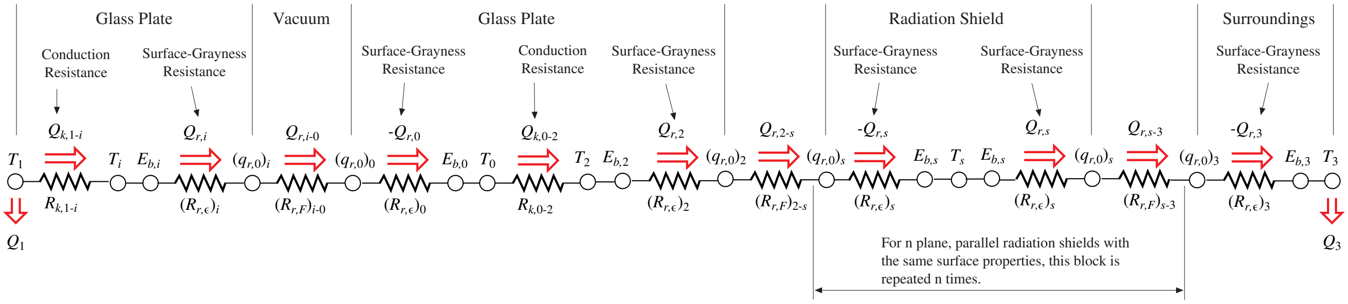


Figure Pr.4.50(b) Thermal circuit diagram without cork board.

From Figure Pr.4.50(b), we have

$$-Q_1 = Q_{k,1\text{-}i} = Q_{r,i\text{-}o} = Q_{k,o\text{-}2} = Q_{r,2\text{-}3} = Q_3.$$

The blackbody emissive power is

$$E_{b,i} = \sigma_{\rm SB} T_i^4.$$

All the thermal resistances can be calculated from the data given. Then, the equations above form a system of 4 equations and 4 unknowns. The unknowns are T_i, T_o, T_2, and Q_1. The equations are nonlinear because of the T^4 terms.

To solve for the resistances, the thermal conductivity of the glass plate is needed. From Table C.17, we have $k_g = 0.76$ W/m-K. The resistances are

$$\begin{aligned}
A_k R_{k,1\text{-}i} &= \frac{l}{k_g} = \frac{0.001(\text{m})}{0.76(\text{W/m-K})} = 0.0013\ \text{K/(W/m}^2\text{)},\ A_k = A_r = A \\
A_r \left(R_{r,\Sigma}\right)_{i\text{-}o} &= A_r \left(R_{r,\epsilon}\right)_i + A_r \left(R_{r,F}\right)_{i\text{-}o} + A_r \left(R_{r,\epsilon}\right)_o \\
&= \frac{1-\epsilon_{r,i}}{\epsilon_{r,i}} + \frac{1}{F_{i\text{-}o}} + \frac{1-\epsilon_{r,o}}{\epsilon_{r,o}} \\
&= \frac{1-0.9}{0.9} + 1 + \frac{1-0.9}{0.9} = 1.22 \\
A_k R_{k,o\text{-}2} &= \frac{l}{k_g} = \frac{0.001(\text{m})}{0.76(\text{W/m-K})} = 0.0013\ \text{K/(W/m}^2\text{)} \\
A_r \left(R_{r,\Sigma}\right)_{2\text{-}3} &= A_r \left(R_{r,\epsilon}\right)_2 + A_r \left(R_{r,F}\right)_{2\text{-}s1} + 5\left[2\left(R_{r,\epsilon}\right)_s + \left(R_{r,F}\right)_{s1\text{-}s2}\right] + A_r \left(R_{r,\epsilon}\right)_3 \\
&= \frac{1-\epsilon_{r,2}}{\epsilon_{r,2}} + \frac{1}{F_{2\text{-}3}} + 5\left[2\left(\frac{1-\epsilon_{r,s}}{\epsilon_{r,s}}\right) + \frac{1}{F_{s1\text{-}s2}}\right] + \frac{1-\epsilon_{r,3}}{\epsilon_{r,3}} \\
&= \frac{1-0.9}{0.9} + 1 + 5\left[2\left(\frac{1-0.04}{0.04}\right) + 1\right] + \frac{1-1}{1} = 246.11.
\end{aligned}$$

The energy equations are then written as

$$-\frac{Q_1}{A} = \frac{T_1 - T_i}{0.0013[\text{K/(W/m}^2)]} = \frac{E_{b,i} - E_{b,o}}{1.22} = \frac{T_o - T_2}{0.0013[\text{K/(W/m}^2)]} = \frac{E_{b,2} - E_{b,3}}{246.11}.$$

The thermal resistances between surfaces 1 and i, surfaces i and o, and surfaces o and 2, are small compared to the resistance between surfaces 2 and 3. This allows us to use the approximation

$$T_2 \simeq T_o \simeq T_i \simeq T_1.$$

Using this approximation (another reason to adopt this is discussed in COMMENT), we have

$$-\frac{Q_1}{A} \simeq \frac{E_{b,1} - E_{b,3}}{246.11}.$$

Using the numerical values, we have

$$-\frac{Q_1}{A} \simeq \frac{5.67\times10^{-8}(\mathrm{W/m^2\text{-}K^4})\left[363.15^4(\mathrm{K})^4-293.15^4(\mathrm{K})^4\right]}{246.11}$$
$$= 2.305\ \mathrm{W/m^2}.$$

(ii) Thermos wall with cork:
(a) The thermal circuit for the system with the cork is shown in Figure Pr.4.50(c).

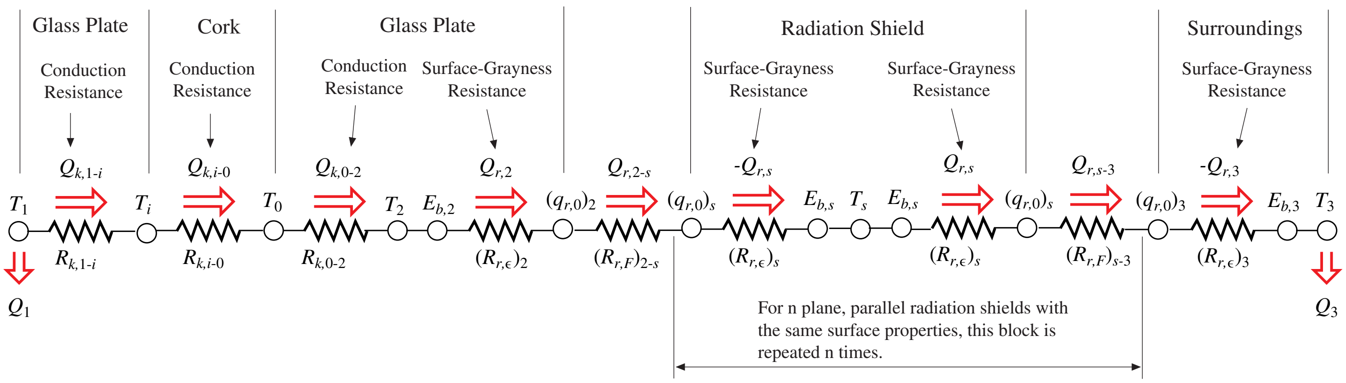


Figure Pr.4.50(c) Thermal circuit diagram with cork board.

(b) The heat transfer within the space between the glass plates, instead of being by radiation, is by conduction through the cork. Between surfaces 1 and 2, conduction is the only heat transfer mode. This allows us to write the heat flux between surfaces 1 and 2 as

$$Q_{k,1\text{-}2} = \frac{T_1 - T_2}{(R_{k,\Sigma})_{1\text{-}2}},$$

where

$$A_k\,(R_{k,\Sigma})_{1\text{-}2} = \frac{l}{k_g} + \frac{L}{k_c} + \frac{l}{k_g}.$$

From Table C.17, for cork board, we have $k_c = 0.043$ W/m-K. Thus

$$A_k\,(R_{k,\Sigma})_{1\text{-}2} = \frac{0.001(\mathrm{m})}{0.76(\mathrm{W/m\text{-}K})} + \frac{0.007(\mathrm{m})}{0.043(\mathrm{W/m\text{-}K})} + \frac{0.001(\mathrm{m})}{0.76(\mathrm{W/m\text{-}K})}$$
$$= 0.165\ \mathrm{K/(W/m^2)}.$$

The heat flux can then be written as, noting that $A_k = A_r = A$,

$$-\frac{Q_1}{A} = \frac{T_1 - T_2}{0.165[\mathrm{K/(W/m^2)}]} = \frac{E_{b,2} - E_{b,3}}{246.11}$$

The conduction resistance is again much smaller than the radiation resistance. This allows us to assume that $T_2 \simeq T_1$. Using this approximation, the heat flux can be calculated from

$$-\frac{Q_1}{A} \simeq \frac{E_{b,1} - E_{b,3}}{246.11} = 2.305\ \mathrm{W/m^2}.$$

COMMENT:
Using a solver (to solve the nonlinear system of equations) results in the following values:

$$\text{(i) } T_i = 363.147\ \mathrm{K},\quad T_o = 362.889\ \mathrm{K},\quad T_2 = 362.886\ \mathrm{K},\quad -Q_1/A = 2.294\ \mathrm{W},$$
$$\text{(ii) } T_i = 363.147\ \mathrm{K},\quad T_o = 362.774\ \mathrm{K},\quad T_2 = 362.771\ \mathrm{K},\quad -Q_1/A = 2.290\ \mathrm{W}.$$

Note that the temperatures T_i to T_2 are nearly equal to T_1. Also, the calculated heat fluxes are very close to those found using the approximations. Due to the existence of the five radiation shields, the use of vacuum or cork between the glass plates has little effect on the heat loss. The relative magnitude of the conduction and radiation resistances can be compared by using the conduction-radiation number N_r defined in (4.75). For the radiation between surfaces 2 and 3 with $T_1 = T_2$, the average temperature is

$$T_m = \left[\frac{\left(T_2^2 + T_3^2\right)(T_2 + T_3)}{4}\right]^{1/3} = \left[\frac{\left(363.15^2 + 293.15^2\right)(363.15 + 293.15)}{4}\right]^{1/3} = 329.4 \text{ K}.$$

Comparing the conduction resistance between surfaces o and 2 with the radiation resistance between 2 and 3, we have from (4.74)

$$N_r = \frac{4\sigma_{\rm SB} T_m^3 R_{k,o\text{-}2}}{(R_{r,\Sigma})_{2\text{-}3}} = \frac{4\sigma_{\rm SB} T_m^3 A_k R_{k,0\text{-}2}}{A_r (R_{r,\Sigma})_{2\text{-}3}}$$

$$= \frac{4 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times 329.4^3(\text{K})^3 \times 0.0013[\text{K/(W/m}^2)]}{246.11} = 4.28 \times 10^{-5}.$$

As $N_r \ll 1$, the conduction resistance can be neglected. Because of the series arrangement for the resistances, the system is radiation-resistance dominated.

PROBLEM 4.51.FAM.S

GIVEN:

A person with a surface temperature $T_1 = 31°C$ is standing in a very large room ($A_{r,2} \gg A_{r,1}$) and is losing heat by surface radiation to the surrounding room surfaces, which are at $T_2 = 20°C$ [Figure Pr.4.51(a)]. Model the person as a cylinder with diameter $D = 0.4$ m and length $L = 1.7$ m placed in the center of the room, as shown in Figure Pr.4.51(a). Neglect surface-convection heat transfer and the heat transfer from the ends of the cylinder.

Assume that all the surfaces are opaque, diffuse, and gray. Assume negligible contact resistance between the clothing and the body.

SKETCH:

Figure Pr.4.51(a) shows the person losing heat by surface radiation, to the surrounding walls, (i) with no clothing, and (ii) with a layer of clothing.

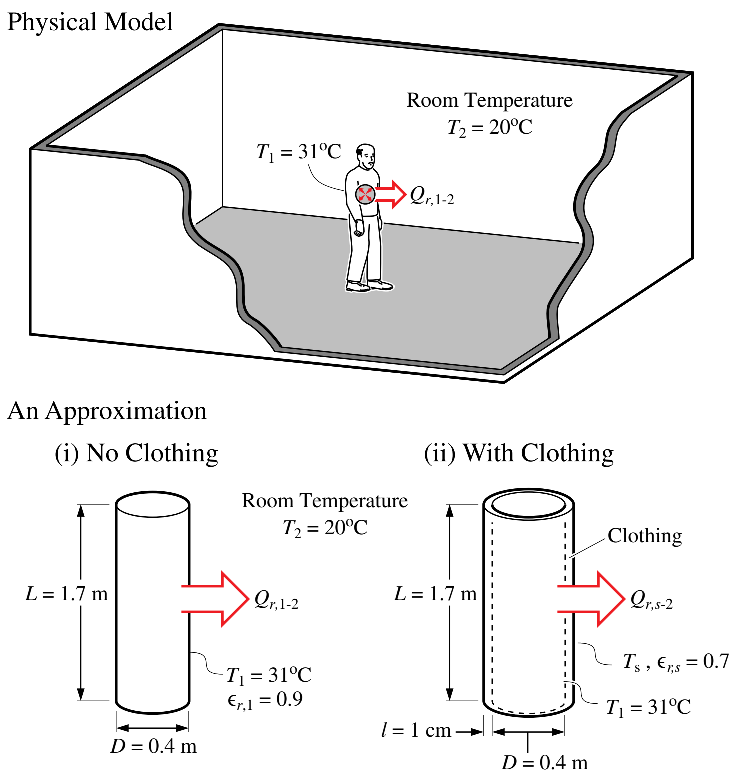


Figure Pr.4.51(a) A physical model and approximation for surface-radiation heat exchange between a person and his or her surrounding surfaces, with (i) no clothing and (ii) a layer of clothing.

OBJECTIVE:

For a steady-state condition, (a) draw the thermal circuit and (b) determine the rate of heat loss for the case of (i) no clothing covering a body with a surface emissivity $\epsilon_{r,1} = 0.9$, and (ii) for the case of added clothing of thickness $l = 1$ cm with a conductivity $k = 0.1$ W/m-K and a surface emissivity $\epsilon_{r,s} = 0.7$.

Comment on the effect of the clothing, for the given temperature difference.

SOLUTION:

(i) No Clothing:

(a) For no clothing, the thermal circuit is shown in Figure Pr.4.51(b).

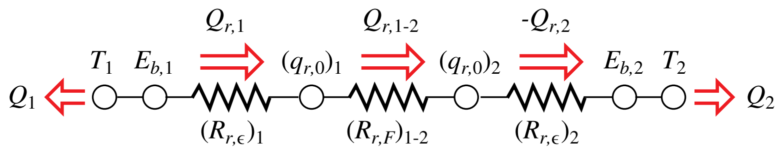


Figure Pr.4.51(b) Thermal circuit diagram for no clothing included.

(b) From Figure Pr.4.51(b), the heat transfer rate from the body to the walls is given by

$$Q_{r,1\text{-}2} = \frac{E_{b,1} - E_{b,2}}{(R_{r,\varepsilon})_1 + (R_{r,F})_{1\text{-}2} + (R_{r,\varepsilon})_2}.$$

The radiation thermal resistances are

$$(R_{r,\varepsilon})_1 = \frac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} = \frac{1-0.9}{[\pi \times 0.4(\text{m}) \times 1.7(\text{m})] \times 0.9} = 0.05201 \ 1/\text{m}^2$$

$$(R_{r,F})_{1\text{-}2} = \frac{1}{A_{r,1}F_{1\text{-}2}} = \frac{1}{[\pi \times 0.4(\text{m}) \times 1.7(\text{m})] \times 1} = 0.4681 \ 1/\text{m}^2$$

$$(R_{r,\varepsilon})_2 \ll (R_{r,\varepsilon})_1.$$

Here $F_{1\text{-}2} = 1$, because the cylinder is surrounded by the room walls. Also $A_{r,2} \gg A_{r,1}$(note that no assumption is made about $\epsilon_{r,2}$). This allows us to neglect the wall surface-grayness resistance. Solving for $Q_{r,1\text{-}2}$, we have

$$Q_{r,1\text{-}2} = \frac{5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times [304.15^4(\text{K}^4) - 293.15^4(\text{K}^4)]}{0.05201(1/\text{m}^2) + 0.4681(1/\text{m}^2)} = 127.8 \text{ W}.$$

(ii) With Clothing:
(a) By covering the body with clothing, a conduction thermal resistance is created in the path of the heat transfer. Figure Pr.4.51(c) shows the thermal circuit.

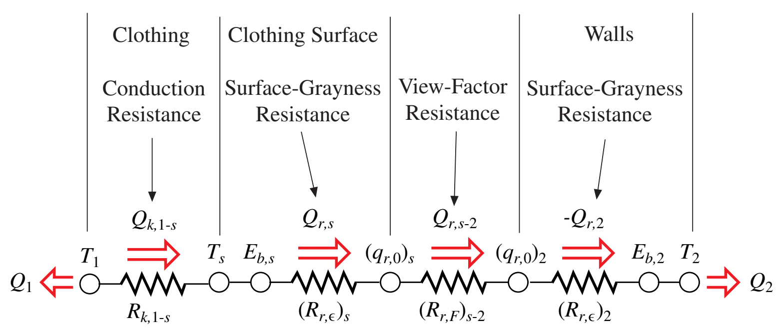


Figure Pr.4.51(c) Thermal circuit diagram for clothing included.

(b) Applying the energy equation to the T_s node, we have

$$Q_{k,1\text{-}s} = Q_{r,s\text{-}2}.$$

The conduction and radiation heat transfer rates are

$$Q_{k,1\text{-}s} = \frac{T_1 - T_s}{R_{k,1\text{-}s}}$$

$$Q_{r,s\text{-}2} = \frac{E_{b,s} - E_{b,2}}{(R_{r,\varepsilon})_s + (R_{r,F})_{s\text{-}2} + (R_{r,\varepsilon})_2}.$$

The thermal resistances are

$$R_{k,1\text{-}s} = \frac{\ln(R_2/R_1)}{2\pi k L} = \frac{\ln[(0.2+0.01)(\text{m})/0.2(\text{m})]}{2 \times \pi \times 0.1(\text{W/m-K}) \times 1.7(\text{m})} = 0.0457\ ℃/\text{W}$$

$$(R_{r,\varepsilon})_s = \frac{1-\epsilon_{r,s}}{A_{r,s}\epsilon_{r,s}} = \frac{1-0.7}{[\pi \times (0.4+0.02)(\text{m}) \times 1.7(\text{m})] \times 0.7} = 0.191\ 1/\text{m}^2$$

$$(R_{r,F})_{s\text{-}2} = \frac{1}{A_{r,s}F_{s\text{-}2}} = \frac{1}{[\pi \times (0.4+0.02)(\text{m}) \times 1.7(\text{m})]1} = 0.446\ 1/\text{m}^2$$

$$(R_{r,\varepsilon})_2 \ll (R_{r,\varepsilon})_s.$$

Then

$$Q_{r,1\text{-}2} = \frac{T_1 - T_s}{0.0456}(℃/\text{W}) = \frac{\sigma_{\text{SB}}\left(T_s^4 - T_2^4\right)}{0.191(1/\text{m}^2) + 0.446(1/\text{m}^2)} = \frac{\sigma_{\text{SB}}\left(T_s^4 - T_2^4\right)}{0.637(1/\text{m}^2)},$$

where $T_1 = 304.15$ K, $T_2 = 293.15$ K, and $\sigma_{\text{SB}} = 5.67 \times 10^{-8}$ W/m^2-K^4. This is an implicit equation for T_s. The solution can be obtained iteratively. First, we rewrite this as an algebraic equation in T_s,

$$T_s = T_1 - 4.059 \times 10^{-9}(\text{K}^{-3}) \times (T_s^4 - T_2^4)$$

and using T_1 and T_2, we have

$$T_s = 304.15(\text{K}) - 4.059 \times 10^{-9}(\text{K}^{-3}) \times (T_s^4 - 7.385 \times 10^9)(\text{K}^4).$$

Using the method of successive substitutions, the equation above is rearranged as

$$T_s^{\text{new}} = 304.15(\text{K}) - 4.059 \times 10^{-9}(\text{K}^{-3}) \times [(T_s^{\text{old}})^4 - 7.385 \times 10^9(\text{K}^4)]$$

Table Pr.4.51 presents the results for three iterations.

Table Pr.4.51 Results obtained for three iterations.

T_s^{old}, K	T_s^{new}, K
300	301.25
301.10	300.70
300.70	300.94

After about 10 iterations, the solution converges to $T_s = 300.87$ K.

The heat transfer rate can then be calculated from

$$= Q_{k,1\text{-}s} = \frac{(304.15(\text{K}) - 300.87(\text{K})}{0.0456(℃/\text{W})} = 71.93\ \text{W}.$$

Alternatively, we can use a solver (such a SOPHT).

COMMENT:

The clothing has the shape of a cylindrical shell and requires the appropriate equation for the conduction thermal resistance. If the equation for a slab is used instead, the conduction thermal resistance is

$$R_{k,1\text{-}2} = \frac{l}{A_{k,ave}k} = \frac{0.01(\text{m})}{[\pi \times (\frac{0.42+0.40}{2})(\text{m}) \times 1.7(\text{m})] \times 0.1(\text{W/m-K})} = 0.0457\ ℃/\text{W}$$

This is a good approximation for this problem, because $l/R = 0.01/0.2 = 0.05 \ll 1$.

To determine whether the conduction thermal resistance could be neglected when compared to the radiation thermal resistance, the conduction-radiation number N_r could be used. The linearized N_r is given by (4.75), i.e.,

$$N_r = \frac{4\sigma_{\rm SB} T_m^3 R_{k,1\text{-}s}}{R_{r,\Sigma}},$$

where the linearized average temperature T_m is

$$T_m = \left[\frac{\left(T_s^2 + T_2^2\right)\left(T_s + T_2\right)}{4}\right]^{1/3}.$$

The highest value for T_s is achieved when the conduction resistance is negligible and it is equal to T_1. Using $T_s = T_1 = 304.15$ K, we find that $T_m = 298.7$ K and N_r is

$$N_r = \frac{4 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times (298.7)^3(\text{K}^3) \times 0.0456(^\circ\text{C/W})}{0.637(1/\text{m}^2)} = 0.433.$$

This indicates that the conduction thermal resistance is 43.3% of an equivalent (linearized) radiation thermal resistance and therefore, cannot be neglected (the assumption that $R_{k,1\text{-}s} \ll R_{r,\Sigma}$ does not apply). Note that using the definition of N_r and the calculated temperature we obtain,

$$N_r = \frac{T_1 - T_s}{T_s - T_2} = \frac{304.15(\text{K}) - 300.87(\text{K})}{300.87(\text{K}) - 293.15(\text{K})} = 0.425.$$

The small difference is due to the linearization of the difference $(E_{b,s} - E_{b,2})$ used in the definition of the radiation thermal resistance used in the equation for N_r.

PROBLEM 4.52.FAM

GIVEN:

A thin film is heated with irradiation from a laser source with intensity $q_{r,i} = 10^6$ W/m^2, as shown in Figure Pr.4.52(a). The heat losses from the film are by surface emission and by conduction through the substrate. Assume that the film can be treated as having a uniform temperature $T_1(t)$ and that the conduction resistance through the substrate can be treated as constant.

SKETCH:

Figure Pr.4.52(a) shows the radiation heating of a thin film with heat loss by substrate conduction.

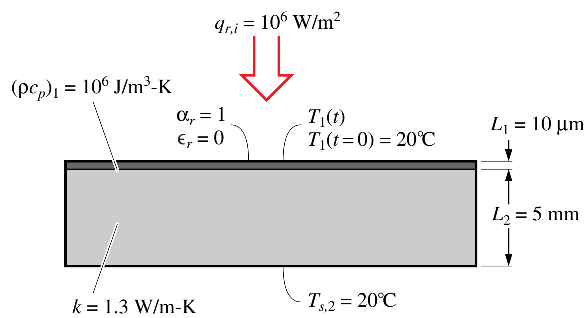


Figure Pr.4.52(a) Laser irradiation heating of a thin film on a substrate.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) For an initial temperature $T_1(t=0) = 20$℃, determine the time needed to raise the temperature of the film T_1 to 500℃.

SOLUTION:

(a) Figure Pr.4.52(b) shows the thermal circuit for the problem. Note that the thin film is lumped into a single node and the thick film is modeled as a conduction resistance constant with time.

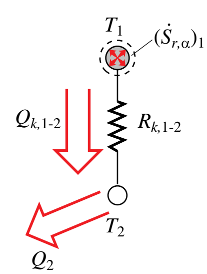


Figure Pr.4.52(b) Thermal circuit diagram.

(b) The energy equation is applied to the thin film to determine the time needed to raise the film temperature to 500℃. The integral-volume energy equation is (4.76)

$$Q|_A = -(\rho c_p V)_1 \frac{dT_1}{dt} + \dot{S}_1.$$

From Figure Pr.4.52(b), we notice that $Q|_A$ has only a conduction component. The energy convection terms are due to radiation absorption with $\alpha_r = 1$ and radiation emission with $\epsilon_r = 0$. Then from (4.66) we have

$$\frac{T_1 - T_2}{R_{k,1\text{-}2}} = -(\rho c_p V)_1 \frac{dT_1}{dt} + \alpha_r q_{r,i} A_r.$$

The conduction resistance is given by

$$R_{k,1\text{-}2} = \frac{L_2}{k_s A_k} = \frac{5 \times 10^{-3}\,(\text{m})}{1.3\,(\text{W/m-K})\,A_k} = \frac{3.85 \times 10^{-3}[°\text{C}/(\text{W/m}^2)]}{A_k}.$$

The thermal capacitance is

$$(\rho c_p V)_1 = 10^6(\text{J/m}^3\text{-}°\text{C}\ ) \times 10 \times 10^{-6}\,(\text{m})\,A_k = 10(\text{J/m}^2\text{-}°\text{C})A_k.$$

The energy conversion term is

$$\dot{S}_1 = \alpha_r q_{r,i} A_r = (1) \times 10^6(\text{W/m}^2)A_r.$$

The solution to this integral-volume energy equation is given in Section 3.5.2, i.e.,

$$t = -\tau_1 \ln\left[\frac{T_1 - T_{s,2} - a_1\tau_1}{T_1(t=0) - T_{s,2} - a_1\tau_1}\right],$$

where

$$\begin{aligned}
\tau_1 &= (\rho c_p V)_1 R_{k,1\text{-}2} = \frac{3.85 \times 10^{-3}[°\text{C}/(\text{W/m}^2)]}{A_k} 10(\text{J/m}^2\text{-}°\text{C})A_k = 3.85 \times 10^{-2}\ \text{s} \\
a_1 &= \frac{\dot{S}_1}{(\rho c_p V)_1} = \frac{10^6(\text{W/m}^2)A_r}{10(\text{J/m}^2\text{-}°\text{C})A_k} = 10^5\ 1/\text{s},
\end{aligned}$$

and $A_r = A_k$ has been used. Then

$$\begin{aligned}
t &= -3.85 \times 10^{-2}(\text{s}) \ln\left[\frac{500\,(°\text{C}) - 20\,(°\text{C}) - 10^5(1/\text{s}) \times 3.85 \times 10^{-2}(\text{s})}{20\,(°\text{C}) - 20\,(°\text{C}) - 10^5(1/\text{s}) \times 3.85 \times 10^{-2}(\text{s})}\right] \\
&= 0.0051\ \text{s} = 5\ \text{ms}.
\end{aligned}$$

COMMENT:

The assumption of constant substrate resistance is probably not a valid assumption for small elapsed times. In this case, there is a penetration of the transient conduction front and the equivalent thermal resistance changes with time.

PROBLEM 4.53.DES

GIVEN:

A pipeline carrying cryogenic liquid nitrogen is to be insulated. Two scenarios, shown in Figure Pr.4.53, are considered. The first one [Figure Pr.4.53(i)] consists of placing the pipe (tube) concentrically inside a larger diameter casing and filling the space with microspheres insulation material. The microspheres have an effective thermal conductivity of $\langle k \rangle = 0.03$ W/m-K. The tube has an outside diameter $D_1 = 2$ cm and the casing has an inside diameter $D_2 = 10$ cm. Another scenario [Figure Pr.4.53(ii)] consists of placing a thin polished metal foil between the tube and the casing, thus forming a cylindrical shell with diameter $D_3 = 6$ cm, and then evacuating the spacings. Both the tube and casing have an emissivity $\epsilon_{r,1} = \epsilon_{r,2} = 0.4$ and the thin foil has an emissivity $\epsilon_{r,3} = 0.05$. The tube is carrying liquid nitrogen and has a surface temperature $T_1 = 77.3$ K and the casing has a surface temperature $T_2 = 297$ K.

SKETCH:

Figure Pr.4.53 shows the tube insulation.

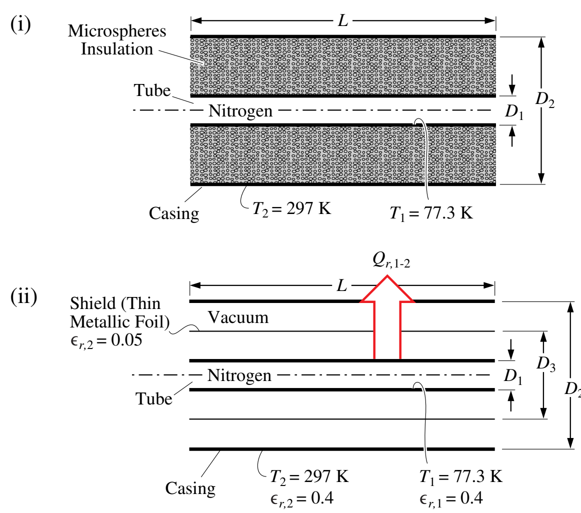


Figure Pr.4.53(i) and (ii) Two scenarios for insulation of a cryogenic fluid tube.

OBJECTIVE:

(a) Determine the net heat transfer to liquid nitrogen for the two scenarios using a tube length $L = 1$ m.
(b) How thick should the microsphere insulation be to allow the same heat transfer as that for the evacuated, radiation shield spacing?

SOLUTION:

(a) (i) For the microsphere insulation, the conduction thermal resistance, from Table 3.1, is

$$LR_{k,1\text{-}2} = \frac{\ln(R_2/R_1)}{2\pi\langle k \rangle} = \frac{\ln[5(\text{cm})/1(\text{cm})]}{2\pi(0.03)(\text{W/m-K})} = 8.54 \text{ K/(W/m)}.$$

Then, the conduction heat transfer is,

$$\frac{Q_{k,1\text{-}2}}{L} = \frac{T_1 - T_2}{R_{k,1\text{-}2}} = \frac{(77.3 - 297)(\text{K})}{8.54(\text{K/W})} = -25.7 \text{ W/m}.$$

(ii) With one radiation shield placed between surfaces 1 and 2, the overall radiation thermal resistance is

$$(R_{r,\Sigma})_{1\text{-}2} = \frac{1-\epsilon_{r,1}}{A_{r,1}\epsilon_{r,1}} + \frac{1}{A_{r,1}F_{1\text{-}3}} + 2\left(\frac{1-\epsilon_{r,3}}{A_{r,3}\epsilon_{r,3}}\right) + \frac{1}{A_{r,3}F_{3\text{-}2}} + \frac{1-\epsilon_{r,2}}{A_{r2}\epsilon_{r,2}}.$$

For $F_{1\text{-}3} = F_{3\text{-}2} = 1$ and using the values given,

$$L(R_{r,\Sigma})_{1\text{-}2} = 23.87 + 15.92 + 2 \times 100.80 + 5.31 + 4.77 = 251.47 \text{ 1/m}.$$

Then, the net radiation heat transfer is

$$\frac{Q_{r,1\text{-}2}}{L} = \frac{\sigma_{\rm SB}(\text{W/m}^2\text{-K}^4) \times (77.3^4 - 297^4)(\text{K})^4}{251.47(1/\text{m}^2)} = -1.743 \text{ W/m}.$$

(b) Equating the expression for the conduction heat transfer to the result for the net radiation heat transfer, we have

$$-1.743(\text{W/m}) = \frac{(77.3 - 297)(\text{K})}{LR_{k,1\text{-}2}}.$$

Solving for $LR_{k,1\text{-}2}$ we obtain

$$LR_{k,1\text{-}2} = 126.0 \text{ K/(W/m)}.$$

From the expression for $R_{k,1\text{-}2}$ and solving for R_2, we obtain finally

$$R_2 = 2.042 \times 10^8 \text{ m}.$$

COMMENT:

Note how effective the radiation shield and vacuum are in reducing the heat transfer from the pipe. One assumption used is that conduction and surface convection heat transfer through the evacuated gap are negligible.

PROBLEM 4.54.FAM

GIVEN:

Automatic fire sprinklers, shown in Figure Pr.4.54(a), are individually heat activated, and tied into a network of piping filled with pressurized water. When the heat flow from a fire raises the sprinkler temperature to its activation temperature $T_m = 165$°F, a lead alloy solder link will melt, and the pre-existing stress in the frame and spring washer will eject the link and retainer from the frame, allowing the water to flow. An AISI 410 stainless steel sprinkler having a mass $M_s = 0.12$ kg and an initial temperature $T_1(t=0) = 72$°F is used to extinguish a fire having a temperature $T_\infty = 1{,}200$°F and an area $A_{r,\infty}$ much greater than the area of the sprinkler $A_{r,1} = 0.003$ m^2. Assume that the dominant source of heat transfer is radiation and that the lumped capacitance analysis is valid.

SKETCH:

Figure Pr.4.54(a) shows the fire sprinkler actuated by heat transfer and raised to a threshold temperature.

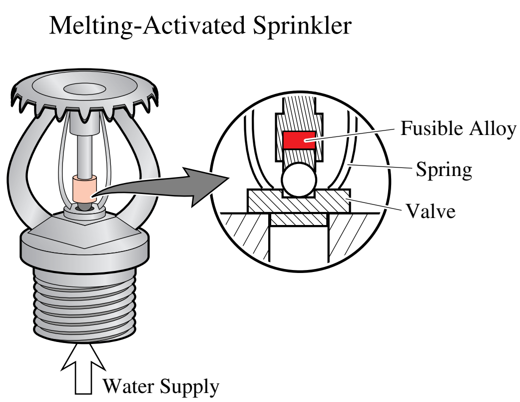


Figure Pr.4.54(a) A fire extinguisher actuated by rise in temperature caused by surface-radiation heating.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the elapsed time t needed to raise the sprinkler temperature to the actuation temperature.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure 4.54(b).

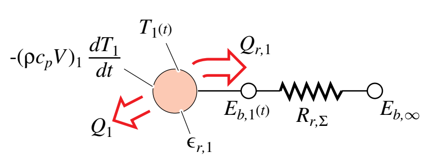


Figure Pr.4.54(b) Thermal circuit diagram.

(b) We use the transient lumped-capacitance analysis and from (4.80), we have

$$\frac{\sigma_{\rm SB} T_\infty^3}{R_{r,\epsilon}(\rho c_p V)_1} t = \frac{1}{4}\left[\ln\left|\frac{T_\infty + T_1}{T_\infty - T_1}\right| - \ln\left|\frac{T_\infty + T_1(t=0)}{T_\infty - T_1(t=0)}\right| + 2\tan^{-1}\frac{T_1}{T_\infty} - 2\tan^{-1}\frac{T_1(t=0)}{T_\infty}\right]$$

where

$$\begin{aligned}
R_{r,\epsilon} &= (R_{r,\epsilon})_1 + (R_{r,F})_{1\text{-}\infty} + (R_{r,\epsilon})_2 \\
(R_{r,\epsilon})_1 &= \left(\frac{1-\epsilon_r}{A_r\epsilon_r}\right)_1 = \frac{1-0.13}{0.003(\text{m}^2)\times 0.13} = 2{,}230.8\ 1/\text{m}^3 \\
(R_{r,F})_{1\text{-}\infty} &= \frac{1}{A_{r,1}F_{1\text{-}\infty}} = \frac{1}{0.003(\text{m}^2)} = 333.33\ 1/\text{m}^3, \quad F_{1\text{-}\infty} = 1 \\
(R_{r,\epsilon})_2 &= \left(\frac{1-\epsilon_r}{A_r\epsilon_r}\right)_2 \simeq 0, \quad A_{r,o} \gg A_{r,1} \\
R_{r,\epsilon} &= 2{,}564.13\ 1/\text{m}^3.
\end{aligned}$$

The properties are (AISI 410 stainless steel, Table C.16), $\rho = 7{,}770$ kg/m^3, $k = 25$ W/m-K, and $c_p = 460$ J/kg-K and (AISI 410 stainless steel, Table C.19), $\epsilon_{r,1} = 0.13$.
The volume V_1, is

$$\rho = \frac{M}{V}, \quad V = \frac{M}{\rho} = \frac{0.12(\text{kg})}{7{,}770(\text{kg/m}^3)} = 1.544\times 10^{-5}\ \text{m}^3.$$

Using (4.80) with $T_\infty = 1{,}200$°F $= 422$ K, $T_1(t=0) = 72$°F $= 295.4$ K, and $T_m = 165$°F $= 347$ K, we have

$$\begin{aligned}
\frac{5.67\times 10^{-8}(\text{W/m}^2\text{-K}^4)\times(922)^3(\text{K}^3)}{2{,}564.13(1/\text{m}^2)\times(7{,}770\times 460\times 1.544\times 10^{-5})(\text{J/K})}t &= \frac{1}{4}\left[\ln\left|\frac{922+347}{922-347}\right| - \ln\left|\frac{922+295.4}{922-295.4}\right| + 2\tan^{-1}\frac{347}{922}\right. \\
&\quad \left. -2\tan^{-1}\frac{295.4}{922}\right] \\
3.14\times 10^{-4}\,t &= \frac{1}{4}[0.7916 - 0.664 + 0.7199 - 0.62] \\
3.14\times 19^{-4}\,t &= 0.05687\ \text{s} \\
t &= 181.1\ \text{s} \simeq 3\ \text{min}.
\end{aligned}$$

COMMENT:

The time needed to start the sprinkler is rather high, $t = 23$ min. The heat transfer by surface convection reduces t. In order to obtain a more accurate prediction, the thermobuoyant flow surface convection should be included.

PROBLEM 4.55.FUN

GIVEN:

Heat transfer by conduction and surface radiation in a packed bed of particles with the void space occupied by a gas is approximated using a unit-cell model. Figure Pr.4.55(i) shows a rendering of the cross section of a packed bed of monosized spherical particles with diameter D and surface emissivity ϵ_r. A two-dimensional, periodic structure with a square unit-cell model is used. The cell has a linear dimension l, with the gas and solid phases distributed to allow for an interparticle contact and also for the presence of the pore space, as shown in Figure Pr.4.55(ii). The thermal circuit model for this unit cell is shown in Figure Pr.4.55(iii).

The surface radiation is approximated by an optically thick medium treatment. This allows for a volumetric presentation of radiation (this is discussed in Section 5.4.6). This uses the concept of radiant conductivity $\langle k_r \rangle$. One of the models for $\langle k_r \rangle$ is

$$\langle k_r \rangle = \frac{4\sigma_{\rm SB} T^3 D}{\dfrac{2}{\epsilon_r} - 1} = \frac{4\epsilon_r \sigma_{\rm SB} T^3 D}{2 - \epsilon_r}.$$

SKETCH:

Figure Pr.4.55 shows the cross section of the packed bed of spheres, the unit-cell model, the thermal circuit model for the unit cell. The radiant conductivity is combined with the gas conductivity in $R_{kr,f}$.

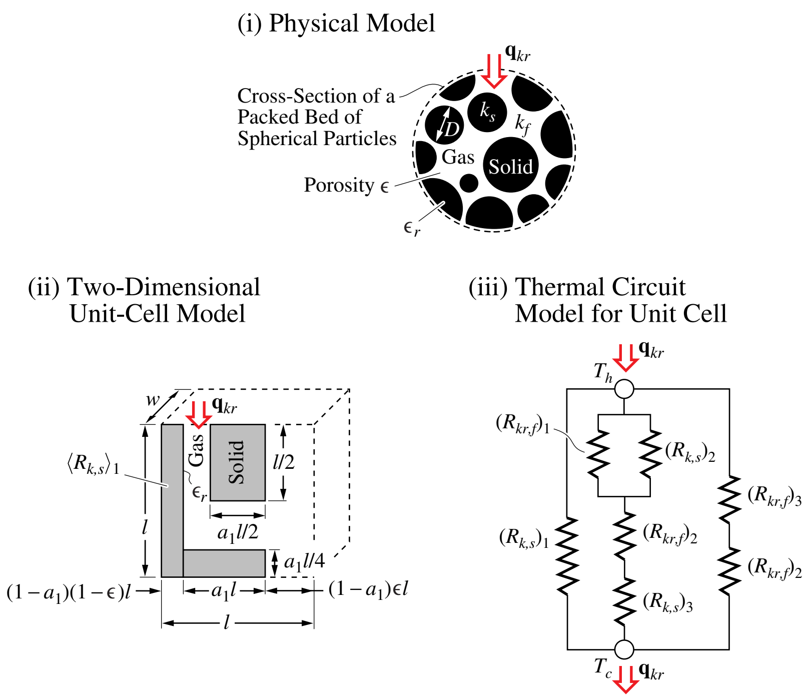


Figure Pr.4.55(i) Physical model of a packed bed of spherical particle with the pore space filled with a gas. (ii) A simplified, two-dimensional unit-cell model. (iii) Thermal circuit model for the unit cell.

OBJECTIVE:

Using the geometric parameters shown in Figure Pr.4.55(ii), show that the total thermal conductivity for the thermal circuit model of Figure Pr.4.55(iii) is

$$\langle k_{kr} \rangle = \frac{q_{kr}}{(T_h - T_c)l} = (1-a_1)(1-\epsilon_r)k_s + \frac{a_1}{\dfrac{1}{(k_f + \langle k_r \rangle) + k_s} + \dfrac{1}{4(k_f + \langle k_r \rangle)} + \dfrac{1}{4k_s}} + (1-a_1)\epsilon(k_f + \langle k_r \rangle).$$

Here we have combined the surface radiation with the gas conduction such that in Figure Pr.4.55(iii), $R_{kr,f}$ uses $k_f + \langle k_r \rangle$ as the conductivity.

SOLUTION:

The effective thermal conductivity is defined as

$$q_{kr} \equiv \frac{T_h - T_c}{A_{kr}\langle R_{kr}\rangle} = \frac{(T_h - T_c)l}{\langle k \rangle},\ A_{kr} = lw,$$

where w is the depth of the unit cell.
The heat flows through the various resistances shown in Figure Pr.4.55(iii). Combining these, we have

$$\frac{1}{\langle R_{kr}\rangle} = \frac{1}{(R_{k,s})_1} + \frac{1}{\dfrac{1}{\dfrac{1}{(R_{kr,f})_1} + \dfrac{1}{(R_{k,s})_2}} + (R_{kr,f})_2 + (R_{kr,f})_3} + (R_{k,s})_3.$$

The six resistances are determined using Table 3.1 for the slab resistance along with the geometrical parameters of Figure Pr.4.45(ii). Then, we have

$$\begin{aligned}
(R_{k,s})_1 &= \frac{l}{(1-a_1)(1-\epsilon)lwk_s} \\
(R_{kr,f})_1 &= \frac{l/2}{\dfrac{a_1 l}{2} w(k_f + \langle k_r \rangle)} \\
(R_{k,s})_2 &= \frac{l/2}{\dfrac{a_1 l}{2} wk_s} \\
(R_{k,f})_2 &= \frac{l/4}{a_1 lw(k_f + \langle k_r \rangle)} \\
(R_{k,s})_3 &= \frac{l/4}{a_1 lwk_s} \\
(R_{k,f})_2 &= \frac{l}{(1-a_1)\epsilon lw(k_f + \langle k_r \rangle)}.
\end{aligned}$$

Combining these, we have

$$\langle k_{kr}\rangle = \frac{1}{w\langle R_{kr}\rangle} = (1-a_r)(1-\epsilon_r)k_s + \frac{1}{\dfrac{1}{a_1(k_f + \langle k_r\rangle) + k_s} + \dfrac{1}{4a_1(k_f + \langle k_r \rangle)} + \dfrac{1}{4a_1 k_s}} + (1-a_1)\epsilon(k_f + \langle k_r \rangle).$$

which is the desired expression.

COMMENT:

To verify the results, take the case of $a_1 = 1$ and $\epsilon = 0$. Then by setting $k_f + \langle k_r \rangle = k_s$, we will have $\langle k_{kr}\rangle = k_s$, as expected. Also for the case of $\epsilon = 0$ and $a_1 = 0$, we have $\langle k_{kr} \rangle = k_s$, as expected.
Note that $\langle k_r \rangle$ is given in terms of the surface emissivity ϵ_r. In Section 5.4.6 we will give another expression for $\langle k_r \rangle$.

PROBLEM 4.56.FAM.S

GIVEN:

The range-top electrical heater has an electrical conductor that carries a current and produces Joule heating $\dot{S}_{e,\mathrm{J}}/L$(W/m). This conductor is covered by an electrical insulator. This is shown in Figure Pr.4.56(a). In electrical insulator should be a good thermal conductor, in order to a avoid large temperature drop across it. It should also have good wear properties, therefore various ceramics (especially, oxide ceramics) are used. Here we consider alumina (Al_2O_3).

Consider a heater with a circular cross section, as shown Figure Pr.4.56(a). Neglect the conduction resistance between the electrical conductor (central cylinder) and the electrical insulator (cylindrical shell). Assume a steady-state surface radiation heat transfer only (from the heater surface).

$R_i = 1$ mm, $\dot{S}_{e,\mathrm{J}}/L = 5 \times 10^3$ W/m, $T_2 = 30$℃, $\epsilon_{r,1} = 0.76$.

SKETCH:

Figure Pr.4.56(a) shows the heater and its insulation shell.

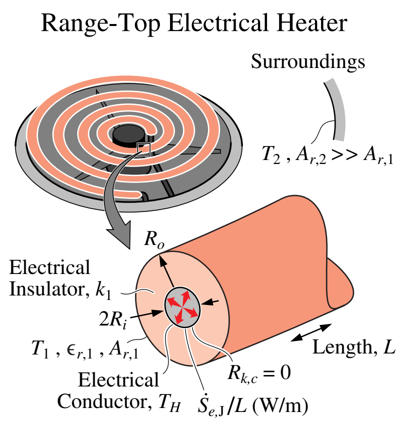


Figure Pr.4.56(a) A range-top electrical heater with a cylindrical heating element made of an inner electrical conductor and an outer electrical insulator.

OBJECTIVE:

(a) Draw the thermal circuit diagram for the heater.

(b) Plot the heater temperature T_H with respect to the outer radius R_o, for $2 \le R_o \le 20$ mm, and the conditions given below.

(c) Plot T_H with respect to the conduction-radiation number $N_r = R_{k,H-1}/[R_{r,\Sigma}/(4\sigma_{\text{SB}}T_m^3)]$, where T_m is defined by (4.73).

SOLUTION:

(a) Figure Pr.4.56(b) shows the thermal circuit diagram. The heat flow per unit length $\dot{S}_{e,\mathrm{J}}/L$ encounters conduction and a surface-radiation resistances.

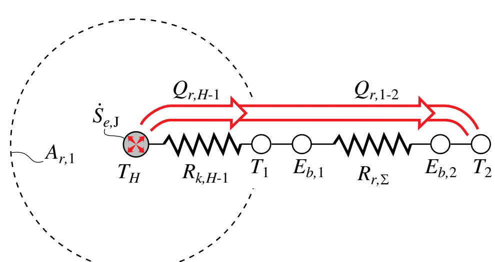


Figure Pr.4.56(b) Thermal circuit diagram.

(b) The energy equation for the heating surface $A_{r,1}$, is written using Figure 4.56(b), i.e.,

$$Q|_{A,1} = Q_{r,1\text{-}2} = \dot{S}_{e,\mathrm{J}},$$

or and since the same flows through the electrical insulator, we have

$$\begin{aligned}
\frac{T_H - T_1}{R_{k,H\text{-}1}} &= \frac{T_H - T_1}{\dfrac{\ln(R_o/R_i)}{2\pi k_1 L}} = \dot{S}_{e,\mathrm{J}} \\
&= \frac{E_{b,1} - E_{b,2}}{R_{r,\Sigma}} = 2\pi R_o L \epsilon_{r,1} \sigma_{\mathrm{SB}} (T_1^4 - T_2^4) = \dot{S}_{e,\mathrm{J}},
\end{aligned}$$

where we have used Table 3.2 for $R_{k,H\text{-}1}$ and (4.49) for $R_{r,\Sigma}$, with $(R_{r,\epsilon})_2 \to 0$ for $A_{r,2} \gg A_{r,1}$ and $F_{1\text{-}2} = 1$.

Then solving for T_1 and T_H, we have

$$\begin{aligned}
T_1 &= \left(T_2^4 + \frac{\dot{S}_{e,\mathrm{J}}/L}{2\pi R_o \epsilon_{r,1} \sigma_{\mathrm{SB}}} \right)^{1/4} \\
T_H &= T_1 + \frac{(\dot{S}_{e,\mathrm{J}}/L) \ln(R_o/R_i)}{2\pi k_1} \\
&= \left(T_2^4 + \frac{\dot{S}_{e,\mathrm{J}}/L}{2\pi R_o \epsilon_{r,1} \sigma_{\mathrm{SB}}} \right)^{1/4} + \frac{(\dot{S}_{e,\mathrm{J}}/L) \ln(R_o/R_i)}{2\pi k_1}.
\end{aligned}$$

The thermal conductivity is found at T = 1,300 K in Table C.14, as

$$k_1 = 6.0 \text{ W/m-K} \quad \text{Table C.14.}$$

We also have $\dot{S}_{e,\mathrm{J}}/L = 5 \times 10^3$ W/m, $T_2 = (273.15 + 30)$ K = 303.15 K, and $R_i = 1$ mm.

Using these numerical values, we have

$$\begin{aligned}
T_H &= \left[(303.15)^4 (\mathrm{K}^4) + \frac{5 \times 10^3 (\mathrm{W/K})}{2\pi R_o \times 0.76 \times 5.67 \times 10^{-8} (\mathrm{W/m^2\text{-}K^4})} \right]^{1/4} + \frac{5 \times 10^3 (\mathrm{W/m}) \times \ln(R_o/0.001)}{2\pi \times 6 (\mathrm{W/m\text{-}K})} \\
&= \left(8.446 \times 10^9 + \frac{1.846 \times 10^{10}}{R_o} \right)^{1/4} + 1.326 \times 10^2 (\mathrm{K}) \ln\left(\frac{R_o}{0.001} \right).
\end{aligned}$$

Figure Pr.4.56(c) shows the variation of T_H with respect to R_o. Note that as R_o increases, the surface area $A_{r,1}$ increases proportional to R_o, while the conduction resistance increases as $\ln(R_o)$. This results in a continuous decrease of T_H as R_o increases.

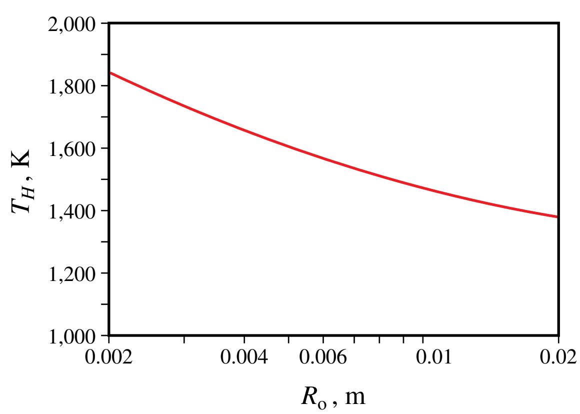


Figure Pr.4.56(c) Variation of the heater temperature with respect to insulator outer radius.

(c) From (4.74), we have

$$N_r = \frac{R_{k,H\text{-}1}}{R_{r,\Sigma}/(4\sigma_{\rm SB}T_m^3)} = \frac{\dfrac{\ln(R_o/R_i)}{2\pi k_1 L}}{\dfrac{1}{2\pi R_o L\epsilon_{r,1}(4\sigma_{\rm SB}T_m^3)}}$$

$$= 4\sigma_{\rm SB}T_m^3\frac{R_o\epsilon_{r,1}}{k_1}\ln(R_o/R_i).$$

From (4.73), we have

$$T_m = \left[\frac{(T_1^2+T_2^2)(T_1+T_2)}{4}\right]^{1/3}.$$

Using the numerical values, we have

$$N_r = 4\times 5.67\times 10^{-8}(\text{W/m}^2\text{-K}^4)T_m^3\times\frac{R_o\times 0.76\times\ln\dfrac{R_o}{0.001}}{6(\text{W/m-K})}$$

$$= 2.873\times 10^{-8}R_o\ln\left(\frac{R_o}{0.001}\right)T_m^3$$

$$T_1 = \left(T_2^4+\frac{\dot{S}_{e,\rm J}/L}{2\pi R_o\epsilon_{r,1}\sigma_{\rm SB}}\right)^{1/4}$$

$$= \left(8.446\times 10^9+\frac{1.846\times 10^{10}}{R_o}\right)^{1/4}.$$

Figure Pr.4.56(d) shows the variation of N_r with respect to R_o. As $R_o \to R_i$, the conduction resistance (and therefore, N_r) decreases and $T_H \to T_1$.

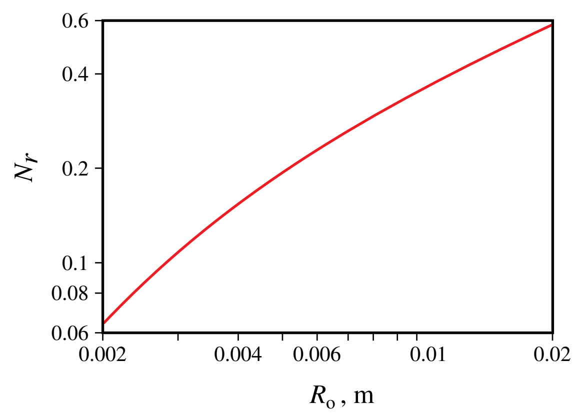


Figure Pr.4.56(d) Variation of conduction-radiation number with respect to insulator outer radius.

COMMENT:

Note that increasing the dielectric layer thickness $R_o - R_i$, decreases the heater temperature T_H by reducing the overall resistance. Also note that even for $N_r < 0.1$, there is still a conduction resistance and this influences T_H.

PROBLEM 4.57.FAM

GIVEN:

Ice is formed in a water layer as it flows over a cooled surface at temperature T_c. Assume that the surface of the water is at the saturation temperature T_{ls} and that the heat transfer across the water layer (thickness L) is by steady-state conduction only. The top of the water layer is exposed to the room-temperature surroundings at temperature T_∞, as shown in Figure Pr.4.57(a). Assume that water and the surrounding surfaces are opaque, diffuse, and gray (this is a reasonable assumption for water in the near infrared range which is applicable in this problem).

$T_{ls} = 0$℃, $T_c = -10$℃, $T_\infty = 300$ K, $L = 2$ mm, $\epsilon_{r,l} = 1$.

Assume that the ice is being formed at the top surface of the water layer. Evaluate the water properties at $T = 280$ K (Tables C.4, and Table C.13).

SKETCH:

Figure Pr.4.57(a) shows ice formation by conduction through the water layer. There is also surface radiation between the water surface and the surrounding surfaces.

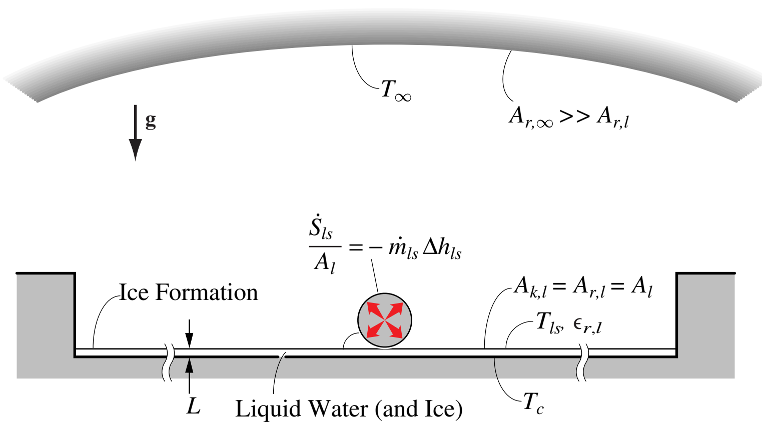


Figure Pr.4.57(a) Ice is formed in a thin water layer cooled from below.

OBJECTIVE:

(a) Draw the thermal circuit diagram for the water surface.

(b) Determine the rate of ice formation per unit area $\dot{m}_{ls} = \dot{M}_{ls}/A_l$.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.4.57(b). Heat transfer from the surroundings to the surface of the water is by surface radiation and heat transfer across the water layer is by conduction.

(b) The energy equation, from Figure Pr.4.57(b), is

$$Q_{r,l\text{-}\infty} + Q_{k,l\text{-}c} = \dot{S}_{ls}.$$

The view factor between the surface of the water and surroundings is unity, $F_{l\text{-}\infty} = 1$. The surface radiation for the unity view factor, for $A_{r,l} \ll A_{r,\infty}$, and $\epsilon_{r,1} = 1$, is given by (4.49), i.e.,

$$Q_{r,l\text{-}\infty} \quad = \quad A_{r,l}\ \sigma_{\rm SB}\ (T_{ls}^4 - T_\infty^4).$$

The conduction resistance is found from Table 3.2, and when used in $Q_{k,l\text{-}c}$, gives

$$Q_{k,l\text{-}c} = \frac{T_{ls} - T_c}{R_{k,l\text{-}c}} = \frac{A_{k,l}k_w}{L}(T_{ls} - T_c).$$

Then, the energy equation becomes

$$A_{r,l}\sigma_{\rm SB}\ (T_{ls}^4 - T_\infty^4) + \frac{A_{k,l}k_w}{L}(T_{ls} - T_c) = \dot{S}_{ls}.$$

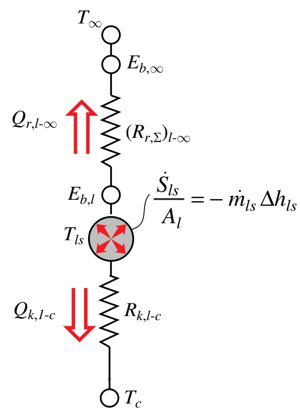


Figure Pr.4.57(b) Thermal circuit diagram.

Since $\dot{S}_{ls}/A_l = -\dot{m}_{ls}\Delta h_{ls}$, the energy equation becomes

$$A_{r,l}\sigma_{\rm SB}\ (T_{ls}^4 - T_\infty^4) + \frac{A_{k,l}k_w}{L}(T_{ls} - T_c) = -\dot{m}_{ls}A_l\Delta h_{ls},$$

From Tables C.4 and C.13, we have $k_w = 0.582$ W/m-K, and $\Delta h_{ls} = -\Delta h_{sl} = -333.6 \times 10^3$ J/kg. Using the numerical values given, we have

$$A_{r,l} \times 5.67 \times 10^{-8}({\rm W/m^2\text{-}K^4}) \times (273.15^4 - 300^4)({\rm K^4})\ + \frac{(273.15 - 263.15)({\rm K})}{\left(\dfrac{2 \times 10^{-3}({\rm m})}{A_{k,l} \times 0.582({\rm W/m\text{-}K})}\right)} =$$

$$-\dot{m}_{ls}A_l \times (-333.6 \times 10^3)({\rm J/kg}),$$

$$-143.6A_{r,l}\ + 2{,}910A_{k,l} = -\dot{m}_{ls}A_l \times (-333.6 \times 10^3)({\rm J/kg}),$$

or

$$\dot{m}_{ls} = 8.284 \times 10^{-3}\ {\rm kg/m^2\text{-}s} = 8.284\ {\rm g/m^2\text{-}s}.$$

where we have used A_l=$A_{r,l}$=$A_{k,l}$.

COMMENT:

Note that surface radiation is not negligible. In practice the ice is formed adjacent to the cooled surface and heat is conducted through the ice. Also note that the heat gained by radiation is approximately 5% of the heat removed by conduction.

Chapter 5

Convection: Unbounded Fluid Streams

PROBLEM 5.1.FAM

GIVEN:

In order to protect exhaust line walls from exposure to high-temperature exhaust gases, these walls are covered by a sacrificial layer. This is shown in Figure Pr.5.1(a). Upon exposure to high temperature exhaust gas and a rise in temperature, this sacrificial layer undergoes a pyrolytic thermal degradation, produces pyrolytic gases, and becomes porous. The pyrolytic gas flows toward the heated surface, thus providing for transpiration cooling and prevention of the large heat load Q_s from reaching the wall. Treat the pyrolytic gas as air at $T = 600$ K and assume a steady-state gas flow that is uniform through the layer. Assume that the area for conduction-convection is πDl and use the planar presentation of the resistance as given by (5.14).

$T_{f,1} = 300$ K, $T_{f,2} = 900$ K, $\langle k \rangle = 0.5$ W/m-K, $u_f = 50$ cm/s, $D = 80$ cm, $l = 1$ m, $L = 1.5$ cm.

SKETCH:

Figure Pr.5.1(a) shows the sacrificing layer lining and the heat transfer to the layer.

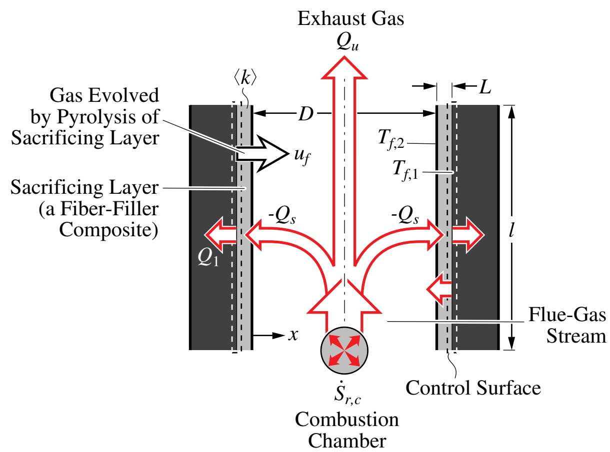


Figure Pr.5.1(a) Exhaust line walls covered by sacrificial layers.

OBJECTIVE:

(a) Draw the thermal circuit diagram and show the energy equation for surface node $T_{f,1}$.

(b) For the conditions given above, determine the rate of heat flowing into the wall.

SOLUTION:

(a) The thermal circuit diagram for node $T_{f,1}$ is shown in Figure Pr.5.1(b).

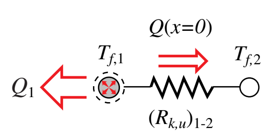


Figure Pr.5.1(b) Thermal circuit diagram.

(b) The heat flow rate Q_1 is determined from the energy equation for node 1, which is found by examining Figure Pr.5.1(b) as

$$Q|_A = Q_1 + Q(x=0) = 0.$$

Then from (5.23), we have

$$Q_1 = -\frac{A_{k,u}\langle k \rangle}{L}\frac{\text{Pe}_L}{e^{\text{Pe}_L}-1}(T_{f,1} - T_{f,2})$$

$$A_{k,u} = \pi Dl, \quad \text{Pe}_L = \frac{u_f L}{\langle \alpha \rangle}, \quad \langle \alpha \rangle = \frac{\langle k \rangle}{(\rho c_p)_f}.$$

From Table C.22, for air at $T = 600$ K, we have

$$\rho_f = 0.589 \text{ kg/m}^3 \qquad \text{Table C.22}$$
$$c_{p,f} = 1038 \text{ J/kg-K} \qquad \text{Table C.22.}$$

Then

$$\begin{aligned}
\alpha &= \frac{0.5(\text{W/m-K})}{0.589(\text{kg/m}^3) \times 1038(\text{J/kg-K})} \\
&= 8.178 \times 10^{-4} \text{ m}^2/\text{s}. \\
\text{Pe}_L &= \frac{0.5(\text{m/s}) \times 0.015(\text{m})}{8.178 \times 10^{-4}(\text{m}^2/\text{s})} = 9.171.
\end{aligned}$$

Using the numerical values, $Q(x = 0)$ is

$$Q_1 = -\frac{\pi \times 0.8(\text{m}) \times 1(\text{m}) \times 8(\text{W/m-K})}{0.015(\text{m})} \times \frac{9.171}{e^{9.171} - 1}(300 - 900)(\text{K}) = 767.3 \text{ W}.$$

COMMENT:

Note that as Pe_L increase, less heat flows into the substrate. Materials which produce significant pyrolytic gases, as a result of thermal degradation, are used.

PROBLEM 5.2.FAM

GIVEN:

The axial conduction-convection resistance may be large, when compared to other heat transfer resistances. In flow through a tube, as shown in Figure Pr.5.2, the axial conduction-convection resistance $R_{k,u}$ is compared to the average convection resistance $\langle R_u \rangle_L$ (this will be discussed in Chapter 7). The average velocity of the fluid flowing in the tube is u_f and the average fluid inlet and outlet temperatures are $T_{f,1}$ and $T_{f,2}$.

The average convection resistance is given (for the case of small NTU, to be discussed in Chapter 7) by

$$\langle R_u \rangle_L = \frac{1}{3.66\pi L k_f}.$$

$u_f = 0.2$ m/s, $D = 5$ cm, and $L = 30$ cm.

Evaluate the properties for air at $T = 350$ K from Table C.22, and for engine oil at $T = 350$ K from Table C.23.

SKETCH:

Figure Pr.5.2 shows the two resistances in a tube flow and heat transfer.

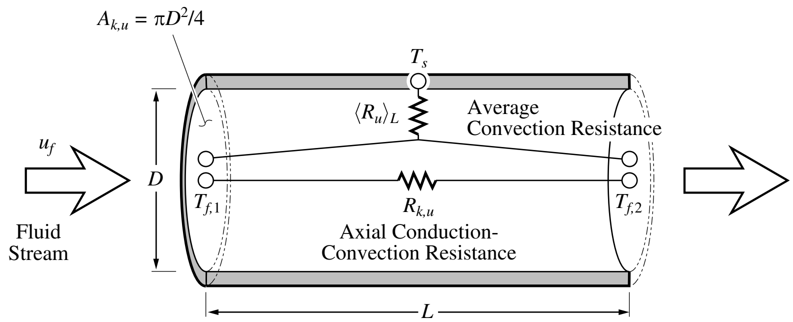


Figure Pr.5.2 Comparison of lateral (surface-convective) and axial (conduction-convective) resistances.

OBJECTIVE:

(a) For the conditions given above determine the ratio of $R_{k,u}/R_{ku}$ when the fluid is air.

(b) Determine the ratio of $R_{k,u}/R_{ku}$ when the fluid is engine oil.

SOLUTION:

Using the definition of $R_{k,u}$ given in Table 5.1, and using $A_{k,u} = \pi D^2/4$, the ratio of the two resistances is

$$\begin{aligned}\frac{R_{k,u}}{\langle R_u \rangle_L} &= \frac{\dfrac{L}{A_{k,u}k_f}\dfrac{e^{\mathrm{Pe}_L}-1}{\mathrm{Pe}_L e^{\mathrm{Pe}_L}}}{\dfrac{1}{3.66\pi L k_f}} \\ &= \frac{\dfrac{L}{(\pi D^2/4)k_f}}{\dfrac{1}{3.66\pi L k_f}}\frac{e^{\mathrm{Pe}_L}-1}{\mathrm{Pe}_L e^{\mathrm{Pe}_L}} \\ &= 3.66 \times 4\frac{L^2}{D^2}\frac{e^{\mathrm{Pe}_L}-1}{\mathrm{Pe}_L e^{\mathrm{Pe}_L}},\end{aligned}$$

where from (5.9),

$$\mathrm{Pe}_L = \frac{u_f L}{\alpha_f}.$$

From Tables C.22 and C.23 at $T = 350$ K

$$\text{air}: \alpha_f = 2.944 \times 10^{-5}\ \mathrm{m^2/s} \qquad \text{Table C.22}$$

$$\text{engine oil}: \alpha_f = 7.74 \times 10^{-8}\ \mathrm{m^2/s} \qquad \text{Table C.23.}$$

Then for (a),

$$\mathrm{Pe}_L = \frac{0.2(\mathrm{m/s}) \times 0.30(\mathrm{m})}{2.944 \times 10^{-5}(\mathrm{m^2/s})} = 2{,}038.$$

This will be a very large argument for the exponential function. By noting that for $\mathrm{Pe}_L \gg 1$,

$$\frac{e^{\mathrm{Pe}_L} - 1}{e^{\mathrm{Pe}_L}} \simeq 1,$$

the ratio of the resistances can be expressed as

$$\frac{R_{k,u}}{\langle R_u \rangle_L} = 3.66 \times 4 \frac{L^2}{D^2} \frac{1}{\mathrm{Pe}_L}.$$

And so, for (a),

$$\begin{aligned} \frac{R_{k,u}}{\langle R_u \rangle_L} &= 3.66 \times 4 \times \frac{(0.30)^2(\mathrm{m^2})}{(0.05)^2(\mathrm{m^2})} \frac{1}{2{,}038} \\ &= 0.2586 \text{ for air.} \end{aligned}$$

For (b),

$$\begin{aligned} \mathrm{Pe}_L &= \frac{0.2(\mathrm{m/s}) \times 0.30(\mathrm{m})}{7.44 \times 10^{-8}(\mathrm{m^2/s})} = 8.065 \times 10^5 \\ \frac{R_{k,u}}{\langle R_u \rangle_L} &= 3.66 \times 4 \times \frac{(0.30)^2(\mathrm{m^2})}{(0.05)^2(\mathrm{m^2})} \frac{1}{8.065 \times 10^5} \\ &= 6.535 \times 10^{-3} \text{ for engine oil.} \end{aligned}$$

COMMENT:

Since we are comparing two resistances, in addition to Pe_L, the ratio L/D is also important. This is because the resistance $\langle R_u \rangle_L$ decreases with an increase in L, while the resistance $R_{k,u}$ increases with an increase in L. Therefore, a large Pe_L is needed to make the axial conduction-convection resistance negligible, unless L/D is small. Here the surface-convection resistance dominates for the oil. But for air at this speed, the axial conduction-convection resistance is relatively significant.

PROBLEM 5.3.FAM

GIVEN:

Transpiration surface cooling refers to flowing a fluid through a permeable solid toward the surface to intercept and remove a large amount of heat flowing to the surface. This imposed heat input Q_s is called the heat load. The flowing fluid opposes the axial conduction heat transfer and results in a lower surface temperature, compared to that of conduction heat transfer only (i.e., no permeation). This is shown in Figure Pr.5.3(a). Air is made to flow through a porous ceramic slab to protect a medium (a substrate) beneath the ceramic.

$Q_s = -10^3$ W, $u_f = 10$ cm/s, $T_{f,1} = 20°$ C, $\langle k \rangle = 0.5$ W/m-K, $w = l = 20$ cm, $L = 5$ cm.

Evaluate the air properties at $T = 500$ K.

SKETCH:

Figure Pr.5.3(a) shows air flowing toward the surface and intercepting the imposed heat load.

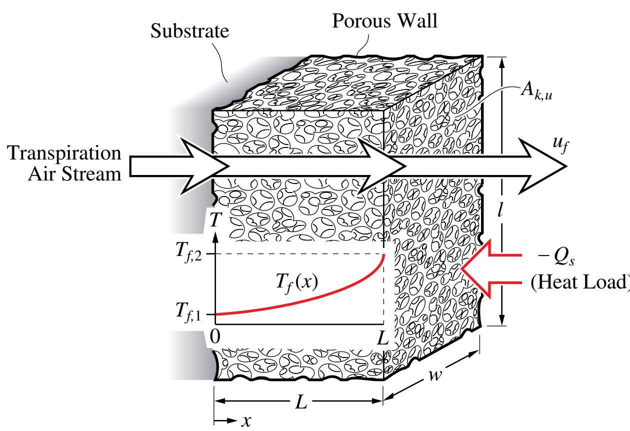


Figure Pr.5.3(a) Transpiration surface cooling.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) For the conditions given below, determine the surface temperature $T_{f,2}$.

(c) For comparison, determine $T_{f,2}$ using $u_f = 0$, i.e., (5.15).

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.5.3(b).

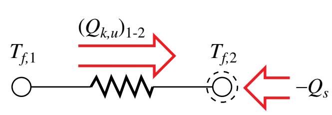


Figure Pr.5.3(b) Thermal circuit diagram.

(b) The energy equation for node $T_{f,2}$, shown in Figure Pr.5.3(b), is

$$Q|_A = -(Q_{k,u})_{1\text{-}2} + Q_s = 0,$$

where from (5.13)

$$(Q_{k,u})_{1\text{-}2} = \frac{T_{f,1} - T_{f,2}}{(R_{k,u})_{1\text{-}2}}$$

$$(R_{k,u})_{1\text{-}2} = \frac{L(e^{\mathrm{Pe}_L} - 1)}{A_{k,u}\langle k \rangle \mathrm{Pe}_L e^{\mathrm{Pe}_L}}$$

$$\mathrm{Pe}_L = \frac{u_f L}{\alpha_f}, \qquad \alpha_f = \frac{\langle k \rangle}{(\rho c_p)_f}, \qquad A_{k,u} = lw.$$

Solving the energy equation for $T_{f,2}$, we have

$$\begin{aligned} T_{f,2} &= T_{f,1} - Q_s (R_{k,u})_{1\text{-}2} \\ &= T_{f,1} - Q_s \frac{L}{lw\langle k\rangle}\frac{e^{\mathrm{Pe}_L}-1}{\mathrm{Pe}_L e^{\mathrm{Pe}_L}}. \end{aligned}$$

From Table C.22, for air at $T = 500$ K, we have $\rho_f = 0.706$ kg/m^3 and $c_{p,f} = 1017$ J/kg-K. Then

$$\begin{aligned} \langle\alpha\rangle &= \frac{\langle k\rangle}{(\rho c_p)_f} = \frac{0.5(\mathrm{W/m\text{-}K})}{0.706(\mathrm{kg/m^3}) \times 1017(\mathrm{J/kg\text{-}K})} = 6.964 \times 10^{-4}\ \mathrm{m^2/s} \\ \mathrm{Pe}_L &= \frac{0.1(\mathrm{m/s}) \times 0.05(\mathrm{m})}{6.964 \times 10^{-4}(\mathrm{m^2/s})} = 7.180. \end{aligned}$$

The temperature of surface 2 is then

$$\begin{aligned} T_{f,2} &= 293.15(\mathrm{K}) - (-10^3)(\mathrm{W}) \times \frac{0.05(\mathrm{m})}{0.2(\mathrm{m}) \times 0.2(\mathrm{m}) \times 0.5(\mathrm{W/m\text{-}K})} \times \frac{e^{7.180}-1}{7.180 \times e^{7.180}} \\ &= 293.15(\mathrm{K}) + 347.9(\mathrm{K}) = 641.1\ \mathrm{K} = 367.9^\circ\mathrm{C}. \end{aligned}$$

(c) For the case of $u_f = 0$, we have from (5.15),

$$(R_{k,u})_{1\text{-}2} = R_{k,1\text{-}2} = \frac{L}{A_{k,u}\langle k\rangle},$$

and

$$T_{f,2} = T_{f,1} - Q_s \frac{L}{lw\langle k\rangle},$$

or

$$\begin{aligned} T_{f,2} &= 293.15(\mathrm{K}) - (-10^3)(\mathrm{W}) \times \frac{0.05(\mathrm{m})}{0.2(\mathrm{m}) \times 0.2(\mathrm{m}) \times 0.5(\mathrm{W/m\text{-}K})} \\ &= 293.15(\mathrm{K}) + 2{,}500(\mathrm{K}) = 2{,}793\ \mathrm{K} = 2{,}520^\circ\mathrm{C}. \end{aligned}$$

COMMENT:

Note that by providing a cold air flow, the surface temperature is reduced significantly. The air flow removes the heat by convection and prevents a large fraction of the heat load from entering the substrate. The heat flow rate into the substrate is given by (5.23).

PROBLEM 5.4.FUN

GIVEN:

The one-dimensional, axial conduction-convection thermal resistance is given by (5.14), i.e.,

$$R_{k,u} = \frac{L}{A_{k,u}k_f}\frac{e^{\mathrm{Pe}_L} - 1}{\mathrm{Pe}_L e^{\mathrm{Pe}_L}}.$$

OBJECTIVE:

Show that in the limit as $\mathrm{Pe}_L \rightarrow 0$, this resistance becomes the conduction thermal resistance for a slab (Table 3.2).

SOLUTION:

We begin by taking the limit of (5.14) for $\mathrm{Pe}_L \rightarrow 0$, i.e.,

$$\lim_{\mathrm{Pe}_L \rightarrow 0} R_{k,u} = \lim_{\mathrm{Pe}_L \rightarrow 0} \frac{L}{A_{k,u}k_f}\frac{e^{\mathrm{Pe}_L} - 1}{\mathrm{Pe}_L e^{\mathrm{Pe}_L}} = \frac{L}{A_{k,u}k_f} \lim_{\mathrm{Pe}_L \rightarrow 0} \frac{e^{\mathrm{Pe}_L} - 1}{\mathrm{Pe}_L e^{\mathrm{Pe}_L}}.$$

Applying the L'Hopital rule, we have

$$\lim_{\mathrm{Pe}_L \rightarrow 0} \frac{e^{\mathrm{Pe}_L} - 1}{\mathrm{Pe}_L e^{\mathrm{Pe}_L}} = \lim_{\mathrm{Pe}_L \rightarrow 0} \frac{e^{\mathrm{Pe}_L}}{e^{\mathrm{Pe}_L} + \mathrm{Pe}_L e^{\mathrm{Pe}_L}} = \lim_{\mathrm{Pe}_L \rightarrow 0} \frac{1}{1 + \mathrm{Pe}_L} = 1.$$

Therefore,

$$\lim_{\mathrm{Pe}_L \rightarrow 0} R_{k,u} = \frac{L}{A_{k,u}k_f}.$$

This is the conduction resistance for a slab given in Table 3.2. The area for conduction is the same as the area for conduction and convection.

COMMENT:

The Péclet number is a ratio of the fluid axial conduction and convection resistances, as given by (5.9), i.e.,

$$\mathrm{Pe}_L = \frac{R_{k,f}}{R_{u,f}}.$$

When the convection resistance becomes very large, the heat transfer occurs primarily by conduction, being controlled by the conduction thermal resistance given above. Similarly, as the convection resistance become very small ($\mathrm{Pe}_L \rightarrow \infty$), the primary transport will be by convection.

PROBLEM 5.5.FAM

GIVEN:

In a space shuttle, a permeable O-ring is used as a thermal barrier and in order to optimize its function, the permeation of combustion flue gas allows for gradual pressure equalization around it. This O-ring is shown in Figure Pr.5.5. The braided carbon fiber O-ring has an average porosity ϵ. The mass flow rate through the O-ring is $\dot{M}_f$. Assume an ideal, square cross-sectional area $L \times L$. The length of the O-ring is $l = 1$ m.

$\epsilon = 0.5$, $L = 0.7$ cm, $\dot{M}_f = 3.25$ g/s, $T_{f,1} = 1{,}700$℃, $T_{f,2} = 100$℃.

For the gas use the properties of air at $T = 900$℃. Use thermal conductivity of carbon at $T = 900$℃. Use (3.28) to determine the effective thermal conductivity $\langle k \rangle$.

SKETCH:

Figure Pr.5.5 shows the permeable O-ring.

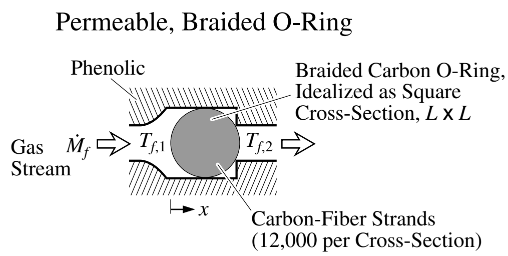


Figure Pr.5.5 A permeable O-ring, made of braided carbon fiber, is used as a thermal barrier and gradual pressure equalizer.

OBJECTIVE:

Determine the rate of heat transfer $(Q_{k,u})_{1\text{-}2} = Q_{x=L}$.

SOLUTION:

The heat transfer rate $(Q_{k,u})_{1\text{-}2}$ is given by (5.13), i.e.,

$$\begin{aligned}(Q_{k,u})_{1\text{-}2} &= \frac{T_{f,1} - T_{f,2}}{(R_{k,u})_{1\text{-}2}} \\ &= (T_{f,1} - T_{f,2}) A_{k,u} \langle k \rangle \frac{\mathrm{Pe}_L e^{\mathrm{Pe}_L}}{e^{\mathrm{Pe}_L} - 1},\end{aligned}$$

where we have used the effective conductivity $\langle k \rangle$ for the carbon-fiber and air composite.
Since we are given the porosity, we use (3.28) for $\langle k \rangle$, i.e.,

$$\frac{\langle k \rangle}{k_f} = \left(\frac{k_s}{k_f} \right)^{0.280 - 0.757 \log \epsilon - 0.057 \log(k_s/k_f)}.$$

The Péclet number is given by (5.9), i.e.,

$$\mathrm{Pe}_L = \frac{u_f L}{\langle \alpha \rangle}, \ \langle \alpha \rangle = \frac{\langle k \rangle}{(\rho c_p)_f},$$

where

$$\dot{M}_f = A_{k,u} \rho_f u_f \quad \text{or} \quad u_f = \frac{\dot{M}_f}{A_u \rho_f}, \quad A_{k,u} = A_u = lL.$$

Interpolating from Table C.14, we have, for carbon at $T = 900$ K,

$$k_s = 2.435 \text{W/m-K} \quad \text{Table C.14.}$$

Interpolating from Table C.22, we have, for air at $T = 900K$,

$$\begin{aligned} \rho_f &= 0.392 \text{ kg/m}^3 && \text{Table C.22} \\ c_{p,f} &= 1{,}111 \text{ J/kg-K} && \text{Table C.22} \\ k_f &= 0.0625 \text{ W/m-K} && \text{Table C.22.} \end{aligned}$$

Then,

$$\begin{aligned} \frac{k_s}{k_f} &= \frac{2.435(\text{W/m-K})}{0.0625(\text{W/m-K})} = 38.96 \\ \langle k \rangle &= 0.0625(\text{W/m-K}) \times (38.96)^{0.280-0.757\times\log(0.5)-0.057\log(38.96)} \\ &= 0.0625(\text{W/m-K}) \times 4.609 = 0.2881 \text{ W/m-K.} \end{aligned}$$

Also,

$$\begin{aligned} u_f &= \frac{3.25 \times 10^{-3} \text{ kg/s}}{1(\text{m}) \times 0.007(\text{m}) \times 0.392(\text{kg/m}^3)} \\ &= 1.184 \text{ m/s} \end{aligned}$$

Then

$$\begin{aligned} \text{Pe}_L &= \frac{u_f L}{\langle \alpha \rangle} = \frac{u_f L (\rho c_p)_f}{\langle k \rangle} \\ &= \frac{1.184(\text{m/s}) \times 0.007(\text{m}) \times 0.392 \times 1{,}111(\text{J/m}^3\text{-K})}{0.2884(\text{W/m-K})} = 12.53. \end{aligned}$$

For the heat flow rate we have, for $A_u = lL$,

$$(Q_{k,u})_{1\text{-}2} = (1{,}700 - 100)(^\circ\text{C}) \times 1(\text{m}) \times 0.007(\text{m}) \times \frac{12.53 \times e^{12.53}}{e^{12.53} - 1} = 140.3 \text{ W.}$$

COMMENT:

Although intended as a thermal barrier, here Pe_L is large enough to cause a large heat flow. The dimensionless temperature distribution along the flow direction is given in Figure 5.3 and shows the strong influence of convection.

PROBLEM 5.6.FUN

GIVEN:

The temperature distribution in a fluid stream with axial conduction and convection and subject to prescribed temperatures $T_{f,1}$ and $T_{f,2}$ at locations $x = 0$ and $x = L$, respectively, is given by (5.12).

OBJECTIVE:

Starting from the dimensionless, one-dimensional steady-state differential-volume energy equation (5.7), and by using (5.10) and (5.11), derive (5.12).

SOLUTION:

Equation (5.7) is a dimensionless energy equation and is a second-order, ordinary differential equation with the boundary conditions given by (5.10) and (5.11), i.e.,

$$\begin{aligned} \frac{d^2T_f^*}{dx^{*2}} - \mathrm{Pe}_L \frac{dT_f^*}{dx^*} &= 0 \\ T_f^*(x^* = 0) &= 1 \\ T_f^*(x^* = L) &= 0 \end{aligned}$$

The first integration gives an exponential solution

$$\frac{dT_f^*}{dx*} = a_1 e^{\mathrm{Pe}_L x^*}.$$

Integrating this again, gives

$$T_f^* = \frac{a_1}{\mathrm{Pe}_L} e^{\mathrm{Pe}_L x^*} + a_2.$$

Using the boundary conditions, we have

$$\begin{aligned} 1 &= \frac{a_1}{\mathrm{Pe}_L} + a_2 \quad \text{or} \quad a_1 = (1 - a_2)\mathrm{Pe}_L \\ 0 &= a_1 \frac{e^{\mathrm{Pe}_L}}{\mathrm{Pe}_L} + a_2 \quad \text{or} \quad a_2 = -a_1 \frac{e^{\mathrm{Pe}_L}}{\mathrm{Pe}_L}. \end{aligned}$$

Eliminating a_2,

$$\begin{aligned} a_1 &= \left(1 + a_1 \frac{e^{\mathrm{Pe}_L}}{\mathrm{Pe}_L}\right) \mathrm{Pe}_L \\ a_1 &= \mathrm{Pe}_L + a_1 e^{\mathrm{Pe}_L} \\ a_1 &= \frac{\mathrm{Pe}_L}{1 - e^{\mathrm{Pe}_L}}, \end{aligned}$$

so that,

$$a_2 = -\frac{e^{\mathrm{Pe}_L}}{1 - e^{\mathrm{Pe}_L}}.$$

Using these expressions for a_1 and a_2, we have

$$\begin{aligned} T_f* &= \frac{\mathrm{Pe}_L}{1 - e^{\mathrm{Pe}_L}} \frac{e^{\mathrm{Pe}_L x*}}{\mathrm{Pe}_L} - \frac{e^{\mathrm{Pe}_L}}{1 - e^{\mathrm{Pe}_L}} \\ &= \frac{e^{\mathrm{Pe}_L x*} - e^{\mathrm{Pe}_L}}{1 - e^{\mathrm{Pe}_L}} \\ &= \frac{e^{\mathrm{Pe}_L} - e^{\mathrm{Pe}_L x*}}{e^{\mathrm{Pe}_L} - 1} \\ &= \frac{e^{\mathrm{Pe}_L} - 1 - (e^{\mathrm{Pe}_L x*} - 1)}{e^{\mathrm{Pe}_L} - 1} \\ &= 1 - \frac{e^{\mathrm{Pe}_L x*} - 1}{e^{\mathrm{Pe}_L} - 1}. \end{aligned}$$

COMMENT:

Note that for large Pe_L, we have $e^{\mathrm{Pe}_L} \gg 1$ and $e^{\mathrm{Pe}_L x*} \gg 1$. Then

$$T_f* = 1 - e^{\mathrm{Pe}_L x^* - \mathrm{Pe}_L} = 1 - e^{\mathrm{Pe}_L (x^* - 1)},$$

which has an exponential behavior, as shown in Figure 5.3. For small values of Pe_L, we expect e^{Pe_L} and $e^{\mathrm{Pe}_L x^*}$ as

$$e^{\mathrm{Pe}_L} = 1 + \mathrm{Pe}_L + ... \, .$$

Then

$$T_f^* = 1 - \frac{\mathrm{Pe}_L x^*}{\mathrm{Pe}_L} = 1 - x^*,$$

which is the expected linear behavior, as shown in Figure 5.3.

PROBLEM 5.7.FUN

GIVEN:

Impermeable, extended surfaces (fins) are used to assist in surface-convection heat transfer by providing an extra surface area. By allowing flow through the fins (e.g., in boiling heat transfer, the surface tension is used to draw the liquid through the fins and this liquid evaporates on the surface), the heat flow rate at the base of the fin can increase substantially. To demonstrate this, consider the permeable fins shown in Figure Pr.5.7(a).

Assume that the fluid stream starts from the fin top and leaves very close to the base ($x = 0$). Then a unidirectional flow with velocity u_f can be assumed (this imply that the fluid stream continues to flow through the base). Here water is allowed to flow through fins made of sintered metallic particles.

$L = 2$ mm, $R = 0.5$ mm, $T_{f,1} = 70$C, $T_{f,2} = 80$°C, $\langle k \rangle = 20$ W/m-K.

Evaluate the water properties at $T = 350$ K.

SKETCH:

Figure Pr.5.7(a) shows a simplified model for the permeable fins attached to a surfaces.

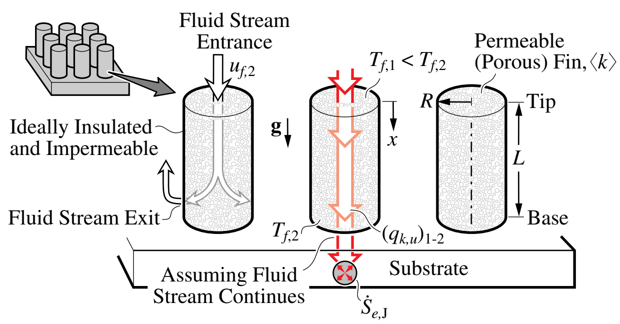


Figure Pr.5.7(a) A permeable fin is used to direct a fluid stream toward the base. A simple model is also used.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the heat flow rates through each fin $(Q_{k,u})_{1\text{-}2}$ and $(q_{k,u})_{1\text{-}2}$, for (ii) $u_f = 0.1$ m/s, and $u_f = 0$.

SOLUTION:

(a) Figure Pr.5.7(b) shows the thermal circuit diagram. The only heat transfer to be determined is $(Q_{k,u})_{1\text{-}2}$.

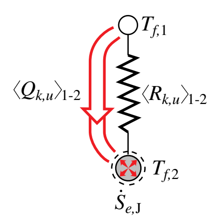


Figure Pr.5.7(b) Thermal circuit diagram.

(b) From Table 5.1, we have

$$
\begin{aligned}
(Q_{k,u})_{1\text{-}2} &= \frac{T_{f,1} - T_{f,2}}{(R_{k,u})_{1\text{-}2}} \\
&= \frac{T_{f,1} - T_{f,2}}{\dfrac{L}{A_{ku}\langle k \rangle} \dfrac{e^{\mathrm{Pe}_L - 1}}{\mathrm{Pe}_L e^{\mathrm{Pe}_L}}}, \quad \mathrm{Pe}_L = \frac{u_f L}{\langle \alpha \rangle}, \quad \langle \alpha \rangle = \frac{\langle k \rangle}{(\rho c_p)_f}.
\end{aligned}
$$

Here we have

$$A_{k,u} = \pi R^2.$$

From Table C.23, for water at $T = 350$ K, we have

$$\rho_f = 975.7 \text{ kg/m}^3 \qquad \text{Table C.23}$$
$$c_{p,f} = 4{,}194 \text{ J/kg-K} \qquad \text{Table C.23.}$$

(i) Using the numerical values, we have, for $u_f = 1$ m/s,

$$\begin{aligned}
\langle\alpha\rangle &= \frac{20(\text{W/m-K})}{(975.7)(\text{kg}/^3) \times 4{,}194(\text{J/kg-K})} \\
&= 4.887 \times 10^{-6} \text{ m}^2/\text{s} \\
\text{Pe}_L &= \frac{0.1(\text{m/s}) \times 2 \times 10^{-3}(\text{m})}{4.887 \times 10^{-6}(\text{m}^2/\text{s})} = 40.92 \\
(Q_{k,u})_{1\text{-}2} &= \frac{(70-80)(^\circ\text{C})}{\dfrac{2 \times 10^{-3}(\text{m})}{\pi(5 \times 10^{-4})^2(\text{m}^2) \times 20(\text{W/m-K})} \dfrac{e^{40.92} - 1}{40.92 \times e^{40.92}}} \\
&= \frac{-10(^\circ\text{C})}{3.111(^\circ\text{C/W})} = -3.214 \text{ W} \\
(q_{k,u})_{1\text{-}2} &= \frac{(Q_{k,u})_{1\text{-}2}}{A_{k,u}} = -\frac{3.214(\text{W})}{\pi(5 \times 10^{-4})^2(\text{m}^2)} \\
&= -4.092 \times 10^6 \text{ W/m}^2.
\end{aligned}$$

(ii) For $u_f = 0$, we have from (5.15)

$$\begin{aligned}
(R_{k,u})_{1\text{-}2} &= R_{k,1\text{-}2} = \frac{L}{A_{ku}\langle k\rangle} \\
&= \frac{2 \times 10^{-3}(\text{m})}{\pi \times (5 \times 10^{-4})^2(\text{m}^2) \times 20(\text{W/m-K})} \\
&= 127.3^\circ\text{C/W}.
\end{aligned}$$

Then

$$\begin{aligned}
(Q_{k,u})_{1\text{-}2} &= \frac{-10(^\circ\text{C})}{127.3(^\circ\text{C/W})} = -7.854 \times 10^{-2} \text{ W} \\
(q_{k,u})_{1\text{-}2} &= \frac{(Q_{k,u})_{1\text{-}2}}{A_{k,u}} = \frac{-7.855 \times 10^{-2} \text{ W}}{\pi(5 \times 10^{-4})^2(\text{m}^2)} \\
&= -1.000 \times 10^5 \text{ W/m}^2.
\end{aligned}$$

Comparing the results for (i) and (ii), the heat transfer rate at the base is significantly increased (by a factor equal to $\text{Pe}_L = 40.92$). This is due to interception (and removal by convection) of the heat flow by the opposing fluid flow (as shown by the temperature distribution of Figure 5.3).

COMMENT:

Note that we have assumed that locally the fluid and solid have the same temperature T_f. This is the condition of negligible surface-convection heat transfer resistance between the fluid and solid. This resistance will be discussed in Chapter 7. Also, we have neglected the effect of added conductivity (this is called thermal dispersion) due to the nonuniformity of the velocity in the pores.

PROBLEM 5.8.FUN

GIVEN:

Capillary pumping (or wicking) refers to flow of liquid through and toward the porous solids by the force of surface tension (an intermolecular force imbalance at the liquid-gas interface). In capillary pumped evaporators, heat is also provided to the porous solid surface such that the liquid is completely evaporated on the surface. Figure Pr.5.8(a) shows three capillary-pumped evaporators, distinguished by the relative direction of the heat and liquid flows. These are used in heat pipes (to be discussed in Example 8.1) and in enhanced, surface evaporations.

Figure Pr.5.8(b) renders a counter heat-water capillary evaporator, where the liquid flow rate $\dot{M}_l$ flowing through the wick(distributed as attached, permeable cylinders) is evaporated at location L_1. Assume that the liquid is at $T_{f,1} = T_{f,2} = T_{lg}$ at $x = L_1$, such that $(Q_{k,u})_{1\text{-}2} = 0$. The temperature at $T(x = L_1)$ is the saturation temperature, so the heat for evaporation is provided by conduction in the region adjacent to the surface, i.e., $L_1 \leq x \leq L_1 + L_2$. Then $Q_{k,2\text{-}3}$ is determined from Table 3.2. This simple thermal circuit model is also shown in Figure Pr.5.8(b).

$R = 0.5$ mm, $L_2 = 150$ μm, $T_{lg} = 100$℃, $T_3 = 105$℃, $\langle k \rangle = 10$ W/m-K.

Determine saturated water properties from table C.27, at $T = 373.15$ K.

SKETCH:

Figure Pr.5.8(a) shows three different capillary-pumped evaporators and Figure Pr.5.8(b) shows the counter heat-liquid flow evaporator considered.

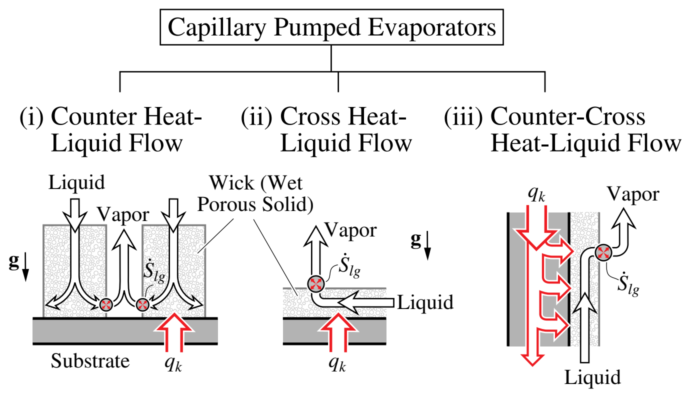


Figure Pr.5.8(a) Various capillary pumped evaporators, based on the relative direction of the heat and liquid flows.

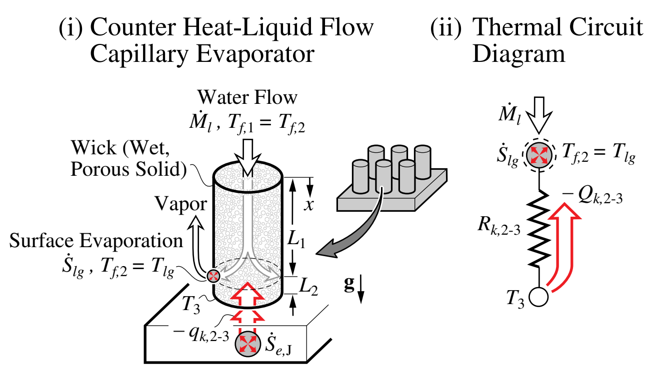


Figure Pr.5.8(b) A counter heat-liquid flow capillary evaporator with a simple conduction heat transfer model in the region adjacent to surface, i.e., $L_1 \leq x \leq L_1 + L_2$.

OBJECTIVE:

(a) Determine the liquid mass flow rates $\dot{M}_l$ and $\dot{m}_l = \dot{M}_l/A_k$, for the given conditions.

(b) Comment on the practical limit for the reduction of L_2.

SOLUTION:
(a) Since in the simple heat transfer model the heat supplied to the evaporation region is by conduction only, we have, from Figure Pr.5.8(b),

$$Q_{k,2\text{-}3} = \dot{S}_{lg}.$$

From Tables 2.1, for $\dot{S}_{lg}$, and 3.2, for $Q_{k,2\text{-}3}$, we have

$$\begin{aligned}\frac{T_{f,2} - T_3}{R_{k,2\text{-}3}} &= \frac{T_{lg} - T_3}{L_2/(A_k\langle k\rangle)} = \frac{T_{lg} - T_3}{L_2/(\pi R^2\langle k\rangle)} \\ &= -\dot{M}_l \Delta h_{lg}.\end{aligned}$$

From Table C.27, at $T = 373.15$ K, we have

$$\Delta h_{lg} = 2.257 \times 10^6 \text{ J/kg} \quad \text{Table C.27.}$$

Then using the numerical values, we have

$$\begin{aligned}\dot{M}_l &= -\frac{T_{lg} - T_3}{\dfrac{L_2 \Delta h_{lg}}{\pi R^2 \langle k\rangle}} = \frac{\pi R^2 \langle k\rangle (T_{lg} - T_3)}{L_2 \Delta h_{lg}} \\ &= -\frac{\pi \times (5 \times 10^{-4})^2 (\text{m}^2) \times 10(\text{W/m-°C})(100 - 105)(\text{°C})}{1.5 \times 10^{-4}(\text{m}) \times 2.257 \times 10^6 (\text{J/kg})} \\ &= 1.160 \times 10^{-7} \text{ kg/s} \\ \dot{m}_l &= \frac{\dot{M}_l}{A_k} = \frac{\dot{M}_l}{\pi R^2} = -\frac{\langle k\rangle (T_{lg} - T_3)}{L_2 \Delta h_{lg}} = 0.1477 \text{ kg/s-m}^2.\end{aligned}$$

(b) Reducing L_2 is limited by the fabrication technique. If sintered particles are used to make the wick, L_2 is limited to the diameter of a single particle.

COMMENT:
One advantage of the distributed wick stack region is that it allows for the passage of vapor in the areas between the stacks. This avoids the passage of both phases through the wick (the counter flow of the liquid and vapor) and allows for a larger $\dot{M}_l$ (for a given driving capillary pressure). The liquid flow will be ultimately limited by the formation of a vapor blanket on top of the stacks.

PROBLEM 5.9.FAM

GIVEN:

In order to protect a substrate from high temperatures resulting from intense irradiation, evaporation transpiration cooling is used. This is shown in Figure Pr.5.9(a). Liquid water is supplied under a porous layer and this liquid is evaporated by the heat reaching the liquid surface, which is at temperature $T_{lg} = T_{f,1}$. The heat flow to the liquid surface is only a fraction of the prescribed irradiation, because the water-vapor flow intercepts and carries away a fraction of this heat by convection.

$\rho_f = 0.596$ kg/m^3, $c_{p,f} = 2{,}029$ J/kg-K, $\Delta h_{lg} = 2.257 \times 10^6$ J/kg (water at 100℃), $\langle k \rangle = 15$ W/m-K, $L = 40$ cm, $w = 15$ cm, $L = 1.5$ cm, $T_{lg} = T_{f,1} = 100$℃, $\alpha_{r,2}$=0.9, $q_{r,i} = 10^5$ W/m^2, $\rho_l = 958$ kg/m^3.

Note that from conservation of mass across the liquid surface, $\rho_f u_f = \rho_l u_l$.

SKETCH:

Figure Pr.5.9(a) shows the evaporation transpiration cooling to protect a substrate from high temperatures resulting from intense irradiation.

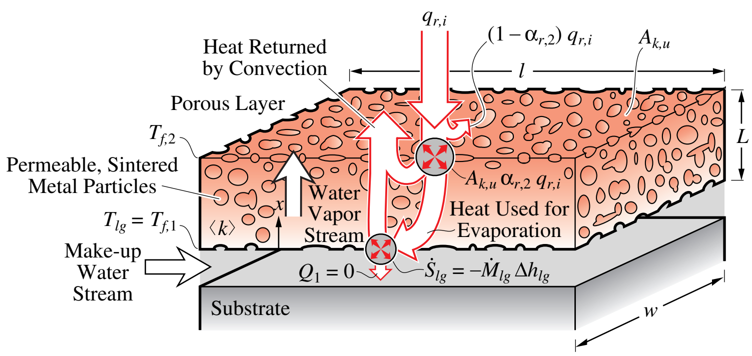


Figure Pr.5.9(a) Evaporation transpiration cooling.

OBJECTIVE:

(a) For the condition given in Figure Pr.5.9(a), draw the thermal circuit diagram.
(b) Determine $\dot{M}_{lg}$ and $T_{f,2}$.

SOLUTIONS:

(a) The thermal circuit diagram is shown in Figure Pr.5.9(b).

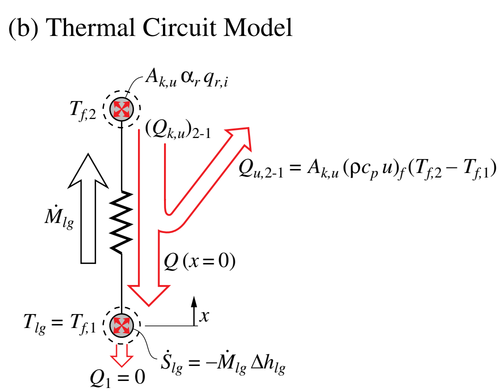


Figure Pr.5.9(b) Thermal circuit diagram.

(b) From Figure Pr.5.9(b), for node $T_{f,2}$, we have

$$Q|_{A,2} = (Q_{k,u})_{2\text{-}1} = A_{k,u}\alpha_{r,2}q_{1,i},$$

where from (5.13) and (5.14)

$$(Q_{k,u})_{2\text{-}1} = \frac{A_{k,u}\langle k\rangle}{L}\frac{\mathrm{Pe}_L e^{\mathrm{Pe}_L}}{e^{\mathrm{Pe}_L}-1}(T_{f,2}-T_{f,1}).$$

and for node $\mathrm{T}_{f,1}$, we have

$$\begin{aligned} Q|_{A,1} = -Q_1 &= \dot{S}_{lg} \\ &= -\dot{M}_{lg}\Delta h_{lg}, \end{aligned}$$

where from (5.23),

$$Q_1 = \frac{A_{k,u}\langle k\rangle}{L}\frac{\mathrm{Pe}_L}{e^{\mathrm{Pe}_L}-1}(T_{f,1}-T_{f,2})$$

$$\mathrm{Pe}_L = \frac{u_f L}{\langle\alpha\rangle}, \quad u_l = \frac{\dot{M}_{lg}}{A_{k,u}\rho_l}, \quad \langle\alpha\rangle = \frac{\langle k\rangle}{(\rho c_p)_f}, \quad A_{k,u} = lw, \quad \rho_l u_l = \rho_f u_f.$$

We have also used the continuity equation to equate the liquid and vapor from rates at the liquid surface. We need to solve these two energy equations simultaneously for $T_{f,2}$ and u_f (or $\dot{M}_{lg}$). Since the relation for u_f is nonlinear, a numerical solution is needed and a solver can be used.
Then thermal diffusivity is

$$\begin{aligned} \langle\alpha\rangle &= \frac{15(\mathrm{W/m\text{-}K})}{0.596(\mathrm{kg/m^3})\times 2029(\mathrm{J/kg\text{-}K})} \\ &= 1.240\times 10^{-2}\ \mathrm{m^2/s}. \end{aligned}$$

The two energy equations and the definition of Pe_L give

$$\begin{aligned} \mathrm{Pe}_L &= \frac{u_f(\mathrm{m/s})\times 0.015(\mathrm{m})}{1.240\times 10^{-2}(\mathrm{m^2/s})} \\ \frac{0.4(\mathrm{m})\times 0.15(\mathrm{m})\times 15(\mathrm{W/m\text{-}K})}{0.015(\mathrm{m})}\frac{\mathrm{Pe}_L e^{\mathrm{Pe}_L}}{e^{\mathrm{Pe}_L}-1}(T_{f,2}-100)(^\circ\mathrm{C}) &= 0.4(\mathrm{m})\times 0.15(\mathrm{m})\times 0.9\times 10^5(\mathrm{W/m^2}) \\ \frac{0.4(\mathrm{m})\times 0.15(\mathrm{m})\times 15(\mathrm{W/m\text{-}K})}{0.015(\mathrm{m})}\frac{\mathrm{Pe}_L}{e^{\mathrm{Pe}_L}-1}(100-T_{f,2})(^\circ\mathrm{C}) &= -0.4(\mathrm{m})\times 0.15(\mathrm{m})\times 0.596(\mathrm{kg/m^3})\times u_f\times 2.257\times 10^6(\mathrm{J/kg}). \end{aligned}$$

The results are

$$\begin{aligned} u_f &= 0.06207\ \mathrm{m/s} \\ \mathrm{Pe}_L &= 0.07508 \\ T_{f,2} &= 186.7^\circ\mathrm{C}. \end{aligned}$$

For $\dot{M}_{lg}$, we have

$$\begin{aligned} \dot{M}_{lg} &= A_{ku}\rho_f u_f \\ &= 0.4(\mathrm{m})\times 0.15(\mathrm{m})\times 0.596(\mathrm{kg/m^3})\times 0.06207(\mathrm{m/s}) = \times 10^{-3}\ \mathrm{kg/s} = 2.220\ \mathrm{g/s}. \end{aligned}$$

COMMENT:

Note that since Pe_L is small, nearly all the absorbed irradiation energy reaches the evaporation surface. Due to the large heat of evaporation, a small velocity is found for $q_{r,i} = 10^5\ \mathrm{W/m^2}$. For $q_{r,i} = 10^6\ \mathrm{W/m^2}$, we would have $u_f = 0.4083$ m/s. Also, the Péclet number is small due to the large $\langle k\rangle$.

PROBLEM 5.10.FUN

GIVEN:

Anesthetic drugs are supplied as liquid and are evaporated, mixed with gases (such as oxygen), and heated in portable vaporizer units for delivery to patients. This is shown in Figure Pr.5.10(a). The drugs, such as enfluorane, isofluorane, etc., have thermophysical properties similar to that of refrigerant R-134a (Table C.28). The drug is sprayed into the vaporizer. The heating of the gas mixture (drug and oxygen) is by surface convection and here it is prescribed by $\langle Q_{ku}\rangle_L$. This heat in turn is provided by Joule heating from a heater wrapped around the tube. Assume that droplets evaporate completely.

$T_{f,1} = 15°\text{C}$, $\dot{M}_l = 2\times10^{-5}$ kg/s, $\dot{M}_{\text{O}_2} = 2\times10^{-4}$ kg/s, $\langle Q_{ku}\rangle_L = -6$ W.

Use the specific heat capacity of oxygen (at $T = 300$ K, Table C.22) for the mixture, and use Δh_{lg} from Table C.28, at $p = 1$ atm.

SKETCH:

Figure Pr.5.10(a) shows the vaporizer. The oxygen (gas) and drug (liquid) streams enter and a gas mixture exits. The mixture is heated by surface convection.

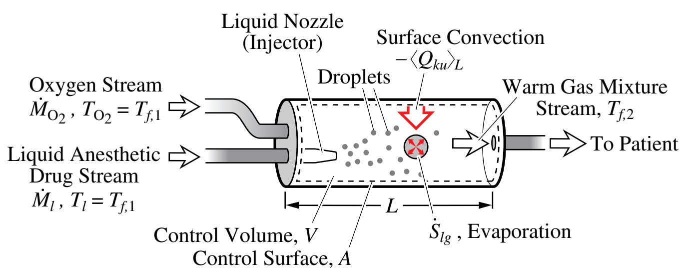


Figure Pr.5.10(a) A liquid anesthetic drug is evaporated, mixed with oxygen, and heated in a vaporizer. The drug is sprayed into the vaporizer.

OBJECTIVE:

(a) Draw the thermal circuit diagram for the control volume V.

(b) Determine the exit fluid stream temperature $T_{f,2}$, for the given conditions.

SOLUTION:

(a) Figure Pr.5.10(b) shows the thermal circuit diagram.

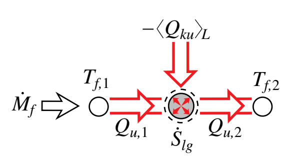


Figure Pr.5.10(b) Thermal circuit diagram.

(b) From continuity, we have

$$\dot{M}_f = \dot{M}_{O_2} + \dot{M}_l.$$

From Figure Pr.5.10(b), and from the energy equation, (5.17), we have

$$\begin{aligned} Q|_A &= \langle Q_{ku}\rangle_L + Q_{u,2} - Q_{u,1} = \dot{S}_{lg} \\ &= \langle Q_{ku}\rangle_L + \dot{M}_f c_{p,f}(T_{f,2} - T_{f,1}) = \dot{S}_{lg}. \end{aligned}$$

Solving for $T_{f,2}$, we have

$$\begin{aligned} T_{f,2} &= T_{f,1} + \frac{\dot{S}_{lg} - \langle Q_{ku}\rangle_L}{\dot{M}_f c_{p,f}} \\ &= T_{f,1} + \frac{-\dot{M}_l \Delta h_{lg} - \langle Q_{ku}\rangle_L}{(\dot{M}_{O_2} + \dot{M}_l) c_{p,f}}. \end{aligned}$$

From Table C.28, we have, for $p = 1$ atm = 0.1013 MPa,

$$\Delta h_{lg} = 2.172 \times 10^5 \text{ J/kg} \quad \text{Table C.28.}$$

From Table C.22, for oxygen at $T = 300$ K, we have

$$c_{p,f} = 920 \text{ J/kg-K} \quad \text{Table C.22.}$$

Using the numerical values, we have

$$\begin{aligned} T_{f,2} &= 15(℃) + \frac{-2 \times 10^{-5}(\text{kg/s}) \times 2.172 \times 10^5(\text{J/kg}) - [-6(\text{W})]}{(2 \times 10^{-4} + 2 \times 10^{-5})(\text{kg/s}) \times 920(\text{J/kg-℃})} \\ &= 15(℃) + 8.182(℃) \\ &= 23.18℃. \end{aligned}$$

COMMENT:

Since the vaporizer unit is portable, the surface convection heating $\langle Q_{ku} \rangle_L$, which is provided by Joule heating needs to be minimized and for an ideally insulated tube this would give $\dot{S}_{e,\mathrm{J}} = 5$ W.

Note that we did not addressed the sensible heat required to heat the droplet from $T_{f,1}$ to T_{lg} ($T_{lg} = 249.2$ K). This heat is not significant because $\dot{M}_{O_2}$ is much larger than $\dot{M}_l$. In Problem 8.2, a more complete analysis-description is made. In Section 6.9, we will discuss the heat and mass transfer resistances R_{ku} and R_{Du}, which influence the droplet evaporation rate.

PROBLEM 5.11.FUN

GIVEN:

In surface evaporation from permeable membranes, the heat for evaporation is partly provided by the ambient gas (by surface convection) and partly by the liquid reservoir (through the conduction-convection heat transfer through the membrane). This is shown in Figure Pr.5.11. The ambient gas may contain species other than the vapor produced by the evaporation. These other species are called the inert or noncondensables and provide a resistance to the vapor mass transfer. This will be discussed in Section 6.9 and here we do not address the mass transfer resistance and assume that the gas is made of the vapor only. Consider using superheated steam to evaporate water from a permeable membrane. We assume that the gas is moving and has a far-field temperature $T_{f,\infty}$ and that there is a surface-convection heat transfer resistance $R_{ku,2\text{-}\infty}$ between the surface and the gas stream. This is also shown in Figure Pr.5.11.

The surface temperature $T_{f,2}$ is equal to the saturation temperature $T_{lg}(p_g)$. Also, since $(R_{k,u})_{1\text{-}2}$ depends on $\dot{M}_l$, the liquid mass flow is determined such that it simultaneously satisfies $(Q_{k,u})_{1\text{-}2}$ and $\dot{S}_{lg}$.

$T_{f,1} = 100$℃, $T_{f,\infty} = 110$℃, $T_{lg} = 95$℃, $A_{k,a} = A_{ku} = 1$ m^2, $R_{ku,2\text{-}\infty} = 0.25$ K/W, $\langle k \rangle = 1$ W/m-K, $L = 1$ cm.

Determine the water properties at $T = 373.15$ K, from Table C.27.

SKETCH:

Figure Pr.5.11 shows the permeable membrane with the water flowing through it and evaporating on the steam-membrane interface.

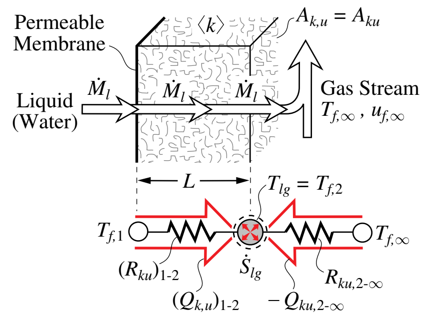


Figure Pr.5.11 Water supplied through a permeable membrane is evaporated on the gas-side interface. The thermal circuit diagram is also shown.

OBJECTIVE:

Determine $\dot{M}_l$ for the given conditions.

SOLUTION:

From Figure Pr.5.11, the energy equation is

$$Q|_A = -(Q_{k,u})_{1\text{-}2} + Q_{ku,2\text{-}\infty} = \dot{S}_{lg}$$

or

$$-\frac{T_{f,1} - T_{f,2}}{(R_{k,u})_{1\text{-}2}} + \frac{T_{f,2} - T_{f,\infty}}{(R_{ku})_{1\text{-}2}} = -\dot{M}_l \Delta h_{lg}.$$

From (5.14), we have

$$(R_{k,u})_{1\text{-}2} = \frac{L}{A_{k,u}\langle k \rangle}\frac{e^{\mathrm{Pe}_L} - 1}{\mathrm{Pe}_L e^{\mathrm{Pe}_L}},$$

$$\mathrm{Pe}_L = \frac{u_f L}{\langle \alpha \rangle}, \quad \langle \alpha \rangle = \frac{\langle k \rangle}{(\rho c_p)_f}, \quad u_f = \frac{\dot{M}_l}{\rho_f A_{k,u}}.$$

From Table C.27, we have

$$\rho_f = 958 \text{ kg/m}^3 \qquad \text{Table C.27}$$
$$c_{p,f} = 4{,}217 \text{ J/kg-K} \qquad \text{Table C.27}$$
$$\Delta h_{lg} = 2.257 \times 10^6 \text{ J/kg} \qquad \text{Table C.27.}$$

Then using the numerical values, we have

$$\langle \alpha \rangle = \frac{1(\text{W/m-K})}{958(\text{kg/m}^3) \times 4{,}217(\text{J/kg})} = 2.475 \times 10^{-7}\text{m}^2/\text{s}$$
$$\text{Pe}_L = \frac{\dot{M}_l \times 10^{-2}(\text{m})}{958(\text{kg/m}^3) \times 1(\text{m}^2) \times 2.475 \times 10^{-7}(\text{m}^2/\text{s})} = 42.17\dot{M}_l(\text{s/kg})$$
$$(R_{k,u})_{1\text{-}2} = \frac{10^{-2}(\text{m})}{1(\text{m}^2) \times 1(\text{W/m-K})} \frac{e^{42.17\dot{M}_l(\text{s/kg})} - 1}{42.17\dot{M}_l(\text{s/kg}) \times e^{42.17\dot{M}_l(\text{s/kg})}}.$$

The energy equation becomes

$$-\frac{(100-95)(^\circ\text{C})}{(R_{k,u})_{1\text{-}2}} + \frac{(95-110)(^\circ\text{C})}{0.25(^\circ\text{C/W})} = -\dot{M}_l(\text{s/kg}) \times 2.257 \times 10^6(\text{J/kg}).$$

Solving for $\dot{M}_l$, we have

$$\dot{M}_l = 0.5476 \text{ kg/s}.$$

COMMENT:

This liquid flow rate corresponds to $\text{Pe}_L = 23.09$ and $u_f = 0.5716$ mm/s, which are relatively large, This is because nearly all of the heat is arriving from the liquid side. The surface-convection resistance $R_{ku,2\text{-}\infty}$ corresponds to a laminar flow parallel to the interface. Means of reducing this resistance will be discussed in Chapter 6 (i.e., using perpendicular flow or turbulent flow, etc.)

PROBLEM 5.12.FAM

GIVEN:

Consider an adiabatic methane-air flame in a packed-bed of spherical alumina particles of diameter $D = 1$ mm and a bed porosity $\epsilon = 0.4$. Assume the average temperature of the bed to be $T = 1{,}300$ K. The inlet conditions are $T_{f,1} = 289$ K and $p_1 = 1$ atm and the mixture is stoichiometric.

OBJECTIVE:

(a) Determine the effective thermal conductivity for the bed. Use the radiant conductivity correlation for spheres given by

$$\langle k_r \rangle = 4D\sigma_{\rm SB}T^3 F_r = 4D\sigma_{\rm SB}T^3 \left\{ a_1 \epsilon_r \tan^{-1}\left[\frac{a_2}{\epsilon_r}\left(\frac{k_s}{4D\sigma_{\rm SB}T^3}\right)^{a_3}\right] + a_4 \right\}$$

correlation for radiant conductivity for packed bed of particle with $0.4 \le \epsilon \le 0.6$,

where F_r is the radiant exchange factor, $a_1 = 0.5756$, $a_2 = 1.5353$, $a_3 = 0.8011$, $a_4 = 0.1843$.

(b) Determine the effective radiant conductivity.

(c) Using the sum of these conductivities, at the above average temperature, determine the adiabatic flame speed. Use the results of Example 5.4, as needed.

SOLUTION:

(a) The effective thermal conductivity for a bed of spherical particles in random arrangement can be estimated from (3.28),

$$\frac{\langle k \rangle}{k_f} = \left(\frac{k_s}{k_f}\right)^{0.280 - 0.757\log(\epsilon) - 0.057\log(k_s/k_f)}.$$

For alumina at $T = 1300$ K, interpolating from Table C.14 gives $k_s = 6$ W/m-K. For air at $T = 1300$ K, interpolating from Table C.22 gives $k_f = 0.0791$ W/m-K. For a porosity $\epsilon = 0.4$, the effective thermal conductivity is

$$\begin{aligned}\langle k \rangle &= 0.0791(\text{W/m-K})\left[\frac{6(\text{W/m-K})}{0.0791(\text{W/m-K})}\right]^{0.280-0.757\log(0.4)-0.057\log\left[\frac{6(\text{W/m-K})}{0.0791(\text{W/m-K})}\right]} \\ &= 0.6158 \text{ W/m-K}.\end{aligned}$$

(b) The radiant thermal conductivity using the diffusion approximation can be estimated from the given relation

$$\langle k_r \rangle = 4D\sigma_{\rm SB}T^3 \left\{ a_1 \epsilon_r \tan^{-1}\left[\frac{a_2}{\epsilon_r}\left(\frac{k_f}{4D\sigma_{\rm SB}T^3}\right)^{a_3}\right] + a_4 \right\}.$$

For $D = 1$ mm and using $\epsilon_r = 0.78$ obtained from Table C.18 (a_1, a_2, a_3, and a_4 are constants) we have

$$\begin{aligned}\langle k \rangle_r &= 4 \times 0.001(\text{m}) \times 5.67\text{x}10^{-8}(\text{W/m}^2\text{-K}^4) \times (1300)^3(\text{K}^3) \\ &\left(0.5756 \times 0.78 \times \tan^{-1}\left\{\frac{1.5353}{0.78} \times \left[\frac{0.0791(\text{W/m-K})}{4 \times 0.001(\text{m}) \times 5.67\text{x}10^{-8}(\text{W/m}^2\text{-K}^4) \times (1300)^3(\text{K}^3)}\right]^{0.8011}\right\} + 0.1843\right) \\ &= 0.19 \text{ W/m-K}.\end{aligned}$$

(c) The flame speed, assuming a zeroth-order reaction, and constant properties is given by (5.55),

$$u_{f,1} = \left[\frac{2k_f}{\rho_{f,1}\rho_{\rm F,1}c_{p,f}} a_r \exp(\frac{-\Delta E_a}{R_g T_{f,2}}) \frac{R_g T_{f,2}^2}{\Delta E_a (T_{f,2} - T_{f,1})}\right]^{1/2}.$$

The inlet conditions are $T_{f,1} = 289$ K, $p = 1$ atm, and the mixture is stoichiometric. These are the conditions for Example 5.4 and the flame speed obtained there is $u_{f,1} = 0.4109$ m/s. Using the expression for $u_{f,1}$ from above, and noting from Section 5.4.6 that for a porous medium $k = \langle k \rangle + \langle k_r \rangle$, the ratio of the flame speeds with and without the porous medium is

$$\frac{(u_{f,1})_{\text{packed bed}}}{(u_{f,1})_{\text{plain medium}}} = \left(\frac{\langle k \rangle + \langle k \rangle_r}{k_f} \right)^{1/2}.$$

Therefore,

$$(u_{f,1})_{\text{packed bed}} = 0.4109(\text{m/s}) \left[\frac{0.63(\text{W/m-K}) + 0.19(\text{W/m-K})}{0.0824(\text{W/m-K})} \right]^{1/2} = 1.30 \text{ m/s}.$$

COMMENT:

An increase of the flame speed (and burning rate) is achieved by increasing the medium conductivity. The presence of the high-conductivity solid and the occurrence of the reaction in the gas phase only results in a local temperature difference between the solid and the gas. This leads to high local gas temperatures (a phenomenon called the local superadiabatic flame temperature).

PROBLEM 5.13.FAM

GIVEN:

To achieve the same flame speed that was obtained using the porous medium in Problem Pr.5.12, turbulent flow may be used. The turbulent intensity affects the flame speed.

The laminar flame speed is $u_{f,1} = 0.4109$ m/s and the packed-bed flame speed is 1.30 m/s.

OBJECTIVE:

Using the same adiabatic, stoichiometric methane-air flame, determine the needed turbulent intensity Tu to achieve the same flame speed.

SOLUTION:

The turbulent intensity is defined by (5.70) as

$$Tu = \frac{\overline{{u'_{f,1}}^2}}{\overline{u}^2_{f,1}}.$$

The ratio of the turbulent flame speed $\overline{u}_{f,1}$ to the laminar flame speed $u_{f,1}$ is correlated to the mean square of the velocity fluctuation $\overline{u'^2_{f,1}}$ by (5.71), i.e.,

$$\frac{\overline{u}_{f,1}}{u_{f,1}} = 1 + \frac{\overline{{u'_{f,1}}^2}}{u^2_{f,1}}.$$

Here the laminar flame speed is that obtained in Example 5.4, i.e., $u_{f,1} = 0.4109$ m/s and the packed-bed flame speed of Problem 5.12 is $u_{f,1} = \overline{u}_{f,1} = 1.30$ m/s.

If a turbulent flame speed equal to the flame speed obtained within the packed bed (from Problem 5.12) is desired, the required mean square of the velocity fluctuation is

$$\overline{{u'_{f,1}}^2} = u^2_{f,1}\left(\frac{\overline{u}_{f,1}}{u_{f,1}} - 1\right) = (0.4109)^2(\text{m/s})^2\left[\frac{1.30(\text{m/s})}{0.4109(\text{m/s})} - 1\right] = 0.3653\ (\text{m/s})^2.$$

For this mean square of the velocity fluctuation, the turbulent intensity is

$$Tu = \frac{\overline{{u'_{f,1}}^2}}{\overline{u}^2_{f,1}} = \frac{0.3653(\text{m/s})^2}{(1.30)^2(\text{m/s})^2} = 0.2162.$$

COMMENT:

The correlation (5.71) applies to low turbulent intensity and is an approximation.

PROBLEM 5.14.FUN

GIVEN:

In a premixed fuel-oxidant stream, as the fuel is oxidized, its density $\rho_{\rm F}$ decreases and this decrease influences the combustion and chemical kinetics. Consider a first-order chemical-kinetic model for reaction of methane and oxygen given by

$$\dot{n}_{r,{\rm F}} = -a_r \rho_{\rm F} e^{-\Delta E_a/R_g T_f}.$$

Start with the fuel-species conservation equation (B.51). Assume one-dimensional, incompressible flow, and negligible mass diffusion.

Use a constant, average temperature $T_f = (T_{f,0} + T_{f,L})/2$ to represent the average temperature over length L. The final expression is

$$\rho_{{\rm F},L} = \rho_{{\rm F},0}\, e^{(-a_r L/u_f) e^{-\Delta E_a/R_g T_f}}.$$

OBJECTIVE:

Derive the relation for the fuel density as it undergoes reaction over a length L with a velocity u_f, inlet density $\rho_{{\rm F},0}$, and temperature $T_{f,0}$.

SOLUTION:

From (B.51), for a steady-state condition, one-dimensional flow, and negligible diffusion, we have the following species F (fuel) conservation equation,

$$\frac{d}{dx}\rho_{\rm F} u_f = \dot{n}_{r,{\rm F}} = \dot{n}_{r,{\rm F}} = -a_r \rho_{\rm F} e^{-\dfrac{\Delta E_a}{R_g T_f}}.$$

Assuming an incompressible flow and using (B.50), the above equation becomes

$$u_f \frac{d\rho_{\rm F}}{dx} = -a_r \rho_{\rm F} e^{-\dfrac{\Delta E_a}{R_g T_f}}.$$

After re-arranging the above and integrating over the length L, we have,

$$\int_{\rho_{{\rm F},0}}^{\rho_{{\rm F},L}} \frac{d\rho_{\rm F}}{\rho_{\rm F}} = \int_0^L \frac{-a_r}{u_f} e^{-\dfrac{\Delta E_a}{R_g T_f}} dx$$

$$\ln \frac{\rho_{{\rm F},L}}{\rho_{{\rm F},0}} = \frac{-a_r L}{u_f} e^{-\dfrac{\Delta E_a}{R_g T_f}}.$$

Solving for $\rho_{{\rm F},L}$, the fuel density at location L, we have,

$$\rho_{{\rm F},L} = \rho_{{\rm F},0}\, e^{(-a_r L/u_f) e^{-\Delta E_a/R_g T_f}}.$$

COMMENT:

This chemical-kinetic model can be compared with (2.19), with $a_F = 1$ and $a_{\rm O} = 0$. This model is used in combustion in porous media. Note that the gas temperature used is the average over the length L and is considered constant. The above expression is a useful approximation for well mixed gas.

PROBLEM 5.15.FAM

GIVEN:

To estimate the flame speed of the atmospheric ($p_1 = 1$ atm) gasoline (assume it is octane) reaction with air (premixed), assume that the chemical kinetic model for this reaction can be approximated as being zeroth order and represented by a pre-exponential factor a_r and an activation energy ΔE_a given in Table 5.3 for zeroth-order reaction of methane and air. The reaction is represented by

$$C_8H_{18} + 12.5O_2 + 47.0N_2 \rightarrow 8CO_2 + 9H_2O + 47.0N_2.$$

Use Figure 5.9 for the adiabatic flame temperature, Table 5.2 for the heat of reaction, and evaluate the properties of air at the average flame temperature $\langle T_f \rangle_\delta = (T_{f,1} + T_{f,2})/2$. Determine the density $\rho_{f,1}$ from the ideal-gas law using $T_{f,1} = 16$℃.

OBJECTIVE:

Determine (a) the Zel'dovich number Ze, and (b) the flame speed $u_{f,1}$, for a premixed gasoline-air flame.

SOLUTION:

(a)The Zel'dovich number is given by (5.53),

$$\mathrm{Ze} = \frac{\Delta E_a (T_{f,2} - T_{f,1})}{R_g T_{f,2}^2}.$$

The adiabatic flow temperature $T_{f,2}$ is found from Figure 5.9 (for octane),

$$\begin{aligned} T_{f,2} &= 2{,}310℃ = 2{,}583.15 \text{ K} \\ T_{f,1} &= 16℃ = 289.15\text{K}. \end{aligned}$$

The chemical kinetic constants are assumed to be those given in Table 5.3 for methane oxidization (for a zeroth-order reaction),

$$\begin{aligned} a_r &= 1.3 \times 10^8 \text{ kg/m}^3\text{-s} \quad \text{Table 5.3} \\ \Delta E_a &= 2.1 \times 10^8 \text{ J/kmole} \quad \text{Table 5.3.} \end{aligned}$$

The Zel'dovich number is then

$$\begin{aligned} \mathrm{Ze} &= \frac{2.1 \times 10^8 (\text{J/kmole})(2{,}583.15 - 289.15)(\text{K})}{8{,}314(\text{J/kmole-K})(2{,}583.15)^2(\text{K})^2}. \\ &= 8.684. \end{aligned}$$

As Ze> 5, we can use the high activation energy approximation. Equation (5.55) can then be used for $u_{f,1}$.

(b) The flame speed is given by (5.55),

$$u_{f,1} = \left(\frac{2k_f a_r}{\rho_{f,1} c_{p,f} \rho_{\mathrm{F},1} \mathrm{Ze}} e^{-\dfrac{\Delta E_a}{R_g T_{f,2}}} \right)^{1/2},$$

where k_f, $\rho_{f,1}$, and $c_{p,f}$ need to be specified.
The average specific heat capacity is given by (5.35),

$$c_{p,f} = \frac{-\Delta h_{r,\mathrm{F}} (\rho_{\mathrm{F},1}/\rho_{f,1})}{T_{f,2} - T_{f,1}}.$$

The heat of reaction for octane is found in Table 5.2,

$$\Delta h_{r,\mathrm{F}} = -48.37 \times 10^6 \text{ J/kg} \quad \text{Table 5.2.}$$

For the given chemical reaction,

$$\frac{\rho_{\mathrm{F},1}}{\rho_{f,1}} = \frac{\nu_{C_8H_{18}} M_{C_8H_{18}}}{\nu_{C_8H_{18}} M_{C_8H_{18}} + \nu_{O_2} M_{O_2} + \nu_{N_2} M_{N_2}} = 0.06234.$$

The molecular masses are found in Table C.4.
The specific heat capacity is then

$$\begin{aligned} c_{p,f} &= \frac{-48.37 \times 10^6(\text{ J/kg}) \times 0.06234}{(2{,}310 - 16)(\text{K})} \\ &= 1{,}315 \text{ J/kg-K}. \end{aligned}$$

The average flame temperature is

$$\langle T_f \rangle_\delta = \frac{T_{f,1} + T_{f,2}}{2} = 1{,}436 \text{ K}.$$

From Table C.22, at $T = 1{,}436$ K, we have

$$k_f = 0.08447 \text{ W/m-K} \quad \text{Table C.22.}$$

The gas density at $T_{f,1}$ and $p_1 = 1$ atm is found from (3.18), i.e.,

$$\begin{aligned} \rho_{f,1} &= \frac{p}{R_g/M_1 T_{f,1}} = \frac{1.013 \times 10^5(\text{Pa}) \times 30.26(\text{kg/kmole})}{8.314 \times 10^3(\text{J/kmole-K}) \times 289.15(\text{K})} \\ &= 1.275 \text{ kg/m}^3, \end{aligned}$$

where

$$\begin{aligned} M_1 &= \frac{\nu_{C_8H_{18}} M_{C_8H_{18}} + \nu_{O_2} M_{O_2} + \nu_{N_2} M_{N_2}}{\nu_{C_8H_{18}} + \nu_{O_2} + \nu_{N_2}} \\ &= \frac{[1 \times (8 \times 12.011 + 18 \times 1.006) + 12.5 \times 2 \times 15.99 \times 5.999 + 47 \times 2 \times 14.007](\text{kg/kmole})}{1 + 12.5 + 47} \\ &= 30.26 \text{ kg/kmole}. \end{aligned}$$

The adiabatic flame speed is then

$$\begin{aligned} u_{f,1} &= \left\{ \frac{2 \times 0.08447 \times 1.3 \times 10^8(\text{kg/m}^3\text{-s}) \exp\left[-\frac{2.1 \times 10^8(\text{J/kgmole})}{8.314 \times 10^3(\text{J/kg mole-K}) \times 2{,}583.15(\text{K})} \right]}{1.275(\text{kg/m}^3) \times 1{,}315(\text{J/kg-K}) \times 0.06234 \times 1.275(\text{kg/m}^2) \times 8.684} \right\}^{1/2} \\ &= 1.037 \text{ m/s} = 103.7 \text{ cm/s}. \end{aligned}$$

COMMENT:

The chemical kinetic model used here is not a realistic representation of the gasoline-air reaction. The measured adiabatic flame speed at one atm pressure is given in table C.21(a) as $u_{f,1} = 0.38$ m/s. This is much lower than the predicted value. The use of a more realistic kinetic model and temperature dependent properties results in the need for a numerical solution of the energy equation (5.27) and the species conservation equation (5.29). This is commonly done to predict the flame speed.

PROBLEM 5.16.FUN

GIVEN:

In order to achieve higher flame temperatures, pure oxygen is used instead of air in burning hydrocarbons. Consider stoichiometric methane-oxygen premixed combustion.

$T_{f,1} = 16°\text{C}$, $c_{p,f} = 3{,}800$ J/kg-K.

Use the chemical kinetic constants for the zeroth-order reaction given in Table 5.3. Use the thermal conductivity of air at the average gas temperature $\langle T_f \rangle_\delta = (T_{f,1} + T_{f,2})/2$ for the mixture.

OBJECTIVE:

(a) Determine the adiabatic flame temperature $T_{f,2}$.
(b) Determine the laminar, adiabatic flame speed $u_{f,1}$.
(c) Compare this flame speed with the laminar, adiabatic flame speed of methane-air in Example 5.4 (i.e., $u_{f,1} = 0.4109$ m/s)

SOLUTION:

(a) The adiabatic flame temperature is given by (5.35), i.e.,

$$T_{f,2} = T_{f,1} - \frac{\Delta h_{r,\text{F}}}{c_{p,f}} \left(\frac{\rho_\text{F}}{\rho_f} \right)_1 .$$

We need to determine the fuel mass fraction (ρ_F/ρ_f). This is found from the stoichiometric reaction

$$\text{CH}_4 + 2\text{O}_2 \rightarrow 2\text{CO}_2 + 2\text{H}_2\text{O}.$$

Then

$$\begin{aligned}
\frac{\rho_{\text{F},1}}{\rho_{f,1}} &= \frac{\nu_{\text{CH}_4} M_{\text{CH}_4}}{\nu_{\text{CH}_4} M_{\text{CH}_4} + \nu_{\text{O}_2} M_{\text{O}_2}} \\
&= \frac{1 \times (12.011 + 1.008 \times 4)}{1 \times (12.011 + 1.008 \times 4) + 2 \times 2 \times 15.999} \\
&= \frac{16.04}{80.04} = 0.2004.
\end{aligned}$$

Then from Table 5.2, we have $-\Delta h_{r,\text{F}} = 5.553 \times 10^7$ J/kg and

$$\begin{aligned}
T_{f,2} &= 16(°\text{C}) - \frac{-5.553 \times 10^7 (\text{J/kg})}{3{,}800(\text{J/kg-K})} \times 0.2004 \\
&= 16(°\text{C}) + 2{,}928(°\text{C}) \\
&= 2{,}944°\text{C}.
\end{aligned}$$

(b) The laminar, adiabatic flame speed is given by (5.55), i.e.,

$$u_{f,1} = \left(\frac{2 k_f a_r}{\rho_f c_{p,f} \rho_{\text{F},1} \text{Ze}} e^{-\frac{\Delta E_a}{R_g T_{f,2}}} \right)^{1/2} .$$

The chemical-kinetic constants are given in Table 5.3. For methane, with a zeroth-order reaction, $a_r = 1.3 \times 10^8$ kg/m^3 and $\Delta E_a = 2.10 \times 10^8$ J/kmole.
The Zel'dovich number, Ze, is

$$\begin{aligned}
\text{Ze} &= \frac{\Delta E_a (T_{f,2} - T_{f,1})}{R_g T_{f,2}^2} \\
&= \frac{2.1 \times 10^8 (\text{J/kmole}) \times 2{,}928(\text{K})}{8.314 \times 10^3 (\text{J/kmole-K}) \times (2{,}944 + 273.15)^2 (\text{K}^2)} \\
&= 7.146.
\end{aligned}$$

As Ze> 5, (5.55) can be used to find $u_{f,1}$. The density $\rho_{f,1}$ is determined from the ideal-gas relation (3.18), i.e.,

$$\rho_{f,1} = \frac{p_1}{(R_g/M_1)T_{f,1}},$$

where

$$\begin{aligned} M_1 &= \frac{\nu_{CH_4} M_{CH_4} + \nu_{O_2} M_{O_2}}{\nu_{CH_4} + \nu_{O_2}} \\ &= \frac{[1 \times (12.011 + 1.008 \times 4) + 2 \times 2 \times 15.999](\text{kg/kmole})}{1+2} \\ &= \frac{80.04}{3} = 26.68 \text{ kg/kmole.} \end{aligned}$$

Then

$$\begin{aligned} \rho_{f,1} &= \frac{1.013 \times 10^5 (\text{Pa})}{\dfrac{8.314 \times 10^3 (\text{J/kmole-K})}{26.68(\text{kg/kmole})} \times (289.15)(\text{K})} \\ &= 1.124 \text{ kg/m}^3. \end{aligned}$$

The air thermal conductivity is found from Table C.22 at the average flame temperature

$$\begin{aligned} \langle T_f \rangle_\delta &= \frac{(16 + 2{,}944)(^\circ\text{C})}{2} + 273.15(^\circ\text{C}) \\ &= 1{,}753 \text{ K} \end{aligned}$$

From Table C.22, for air at $T = 1{,}753$ K, we have: $k_f = 0.09520$ W/m-K.
Then

$$\begin{aligned} u_{f,1} &= \left\{ \frac{2 \times 0.09520(\text{W/m-K}) \times 1.3 \times 10^8 (\text{kg/m}^3\text{-s})}{1.124(\text{kg/m}^3) \times 3{,}800(\text{J/kg-K}) \times 1.124 \times 0.2004(\text{kg/m}^3) \times 7.146} \times \right. \\ & \left. \exp\left[-\frac{2.10 \times 10^8 (\text{J/kg})}{8.314 \times 10^3 (\text{J/kg-K}) \times (2{,}944 + 273.15)(\text{K})} \right] \right\}^{1/2} \\ &= [3.600 \times 10^4 (\text{m}^2/\text{s}^2) \times 3.893 \times 10^{-4}]^{1/2} = 3.744 \text{ m/s.} \end{aligned}$$

(c) Comparing with the results of Example 5.4, with $T_{f,2} = 1{,}918^\circ$C, the adiabatic flame temperature is much higher for the methane burning in pure oxygen. The predicted laminar adiabatic flame speed is also much higher in pure oxygen, compared to the value of 0.4109 m/s which is predicted in air.

COMMENT:

The measured laminar flame speed is $u_{f,1} = 6.919$ m/s. The difference is due to the constant thermal conductivity used in the predictions, and due to the simplified chemical kinetic model used.

PROBLEM 5.17.FAM.S

GIVEN:

Surface-radiation drying of wet pulp in paper production uses permeable ceramic foams for both combustion and surface emission. This is shown Figure Pr.5.17(a). The premixed, gaseous fuel-air flows into the foam, and after an initial ignition, it undergoes steady combustion. The flue gas heats the foam and leaves while the foam radiates to the load (wet paper). The steady combustion requires a mixture flow rate that in turn is determined by the heat transfer. This flow rate, or specifically the velocity $u_{f,1}$, can be several times the laminar, adiabatic flame speed given by (5.55). Consider stoichiometric methane-air combustion with the mixture arriving at $T_{f,1}$ and at 1 atm pressure.

Assume that $F_{2\text{-}p} = 1$ and both the radiant-burner surface and the wet paper are blackbody surfaces.

$u_{f,1} = 0.2$ m/s, $T_{f,1} = 16$℃, $a = 25$ cm, $w = 60$ cm, $T_p = 60$℃, $c_{p,f} = 1{,}611$ J/kg-K, $\rho_{f,1} = 1.164$ kg/m^3, $(\rho_{\rm F}/\rho_f)_1 = 0.05519$.

SKETCH:

Figure Pr.5.17(a) shows the permeable ceramic foam with surface radiation to a wet-pulp sheet being dried.

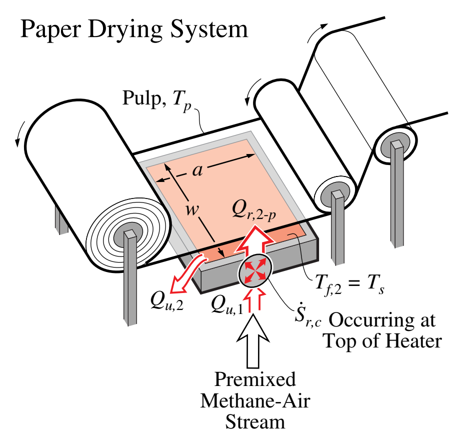


Figure Pr.5.17(a) A surface-radiation burner used for drying a wet-pulp sheet.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Assuming local thermal equilibrium between the gas and the permeable solid (i.e., the ceramic foam), determine the radiant-surface temperature $T_s = T_{f,2}$ and the radiation heat transfer rate $Q_{r,2\text{-}p}$.
(c) What is the radiant heat transfer efficiency of this heater?

SOLUTION:

(a) Figure Pr.5.17(b) shows the thermal circuit diagram. In addition to the surface-radiation heat transfer, the convection heat flow $Q_{u,2}$ is partly exchanged with the wet-pulp sheet by surface convection Q_{ku}(discussed in Chapter 6).

(b) From Figure Pr.5.17(b), the integral-surface energy equation becomes

$$Q_{u,2} + Q_{r,2} - Q_{u,1} = \dot{S}_{r,c},$$

where

$$\begin{aligned} Q_{u,2} - Q_{u,1} &= \rho_{f,1} c_{p,f} u_{f,1} A_u (T_{f,2} - T_{f,1}) \\ Q_{r,2} &= Q_{r,2\text{-}p} = \frac{E_{b,2} - E_{b,p}}{\dfrac{1}{A_{r,2} F_{2\text{-}p}}} = A_{r,2}\sigma_{\rm SB}(T_{f,2}^4 - T_p^4), \ \text{as} \epsilon_{r,2} = \epsilon_{r,p} = 1 \\ A_u &= A_{r,2} = aw \\ \dot{S}_{r,c} &= -\rho_{{\rm F},1} u_{f,1} A_u \Delta h_{r,{\rm F}} \ \text{[from (5.34)]}. \end{aligned}$$

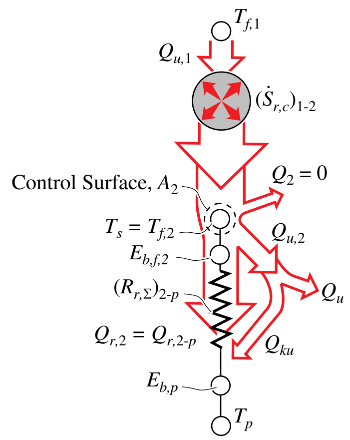


Figure Pr.5.17(b) Thermal circuit diagram.

Combining these, we will have the equation to solve for $T_{f,2}$, i.e.,

$$\rho_{f,1} c_{p,f} u_{f,1}(T_{f,2} - T_{f,1}) + \sigma_{\rm SB}(T_{f,2}^4 - T_p^4) = -\frac{\rho_{{\rm F},1}}{\rho_{f,1}} \rho_{f,1} u_{f,1} \Delta h_{r,{\rm F}}.$$

Then we can solve for $Q_{r,2\text{-}p}$. Alternatively, we can solve the above equations simultaneously using a software (such as SOPHT).
From Table 5.2, we have $\Delta h_{r,{\rm F}} = -5.553 \times 10^7$ J/kg. Then, using the numerical values , we have

$$1.164({\rm kg/m^3}) \times 1{,}611({\rm J/kg\text{-}K}) \times 0.2({\rm m/s}) \times [T_{f,2} - 289.15({\rm K})] + 5.67 \times 10^{-8}({\rm W/m^2\text{-}K^4}) \times$$
$$[T_{f,2}^4 - 333.15^4({\rm K^4})] = -1.164({\rm kg/m^3}) \times 0.05519 \times 0.2({\rm m/s}) \times (-5.553 \times 10^7)({\rm J/kg})$$

$$Q_{r,2\text{-}p} = 0.25({\rm m}) \times 0.6({\rm m}) \times 5.670 \times 10^{-8}({\rm W/m^2\text{-}K^4}) \times [T_{f,2}^4 - 333.15^4({\rm K^4})].$$

The solutions are

$$T_{f,2} = 1{,}476 \text{ K}.$$
$$Q_{r,2\text{-}p} = 40{,}259 \text{ W}.$$

(c) The surface-radiation efficiency η is defined as

$$\eta = \frac{Q_{r,2\text{-}p}}{-\rho_{{\rm F},1} u_{f,1} A_u \Delta h_{r,{\rm F}}} = 37.62\%.$$

COMMENT:

Figure Pr.5.17(c) shows the variation of η with respect to $u_{f,1}$. Note that η decreases as $u_{f,1}$ decreases. It is possible to avoid the low efficiency associated with the high velocities. This can be done by using a distributed fuel supply, instead of the premixed fuel-air considered here, along with an impermeable radiation surface.

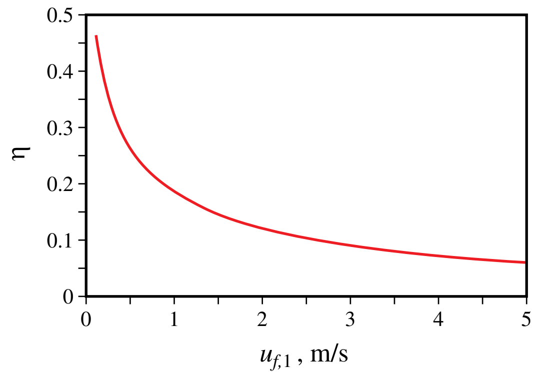


Figure Pr.5.17(c) Variation of surface-radiation efficiency with respect to the flame speed $u_{f,1}$.

PROBLEM 5.18.FUN

GIVEN:

In burning gaseous fuel in a tube or in a porous medium, it is possible to locally create gas temperatures above the adiabatic flame temperature. This is done by conduction of heat through the bounding solid (tube or solid matrix) from the high temperature region to the lower temperature region. This is called heat recirculation and the process of combustion with this local increase in the gas temperature is called the superadiabatic combustion. This is rendered for a premixed fuel (methane) and oxidant (air) in Figure Pr.5.18(a). In this idealized rendering, three different regions are identified, namely the gas-preheat, combustion, and solid-heating regions. The heat recirculation begins as surface convection from the flue gas to the solid, then it is conducted along the solid (flowing opposite to the gas flow), and is finally returned to the gas (premixed fuel-oxidant) by surface convection. The idealized gas temperature distribution (for the three regions) is also shown in the figure and can be represented by the internodal energy conservation equation and the temperatures labeled $T_{f,1}$ to $T_{f,4}$. The surface convection out of the solid-heating region is given per unit gas flow cross-sectional area, i.e., Q_{ku}/A_u.

$Q_{ku}/A_u = 5 \times 10^4$ W/m^2, $T_{f,1} = 20$°C.

Assume negligible heat loss in the combustion region (i.e., an adiabatic combustion region). Use the heat of combustion of methane $\Delta h_{r,\mathrm{F}}$ and the stoichiometric mass fraction of the fuel from Table C.21(a), and a constant specific heat capacity corresponding to air at $T = 1{,}500$ K.

SKETCH:

Figure Pr.5.18(a) shows the flow and reaction, and the anticipated gas temperature distribution along the tube.

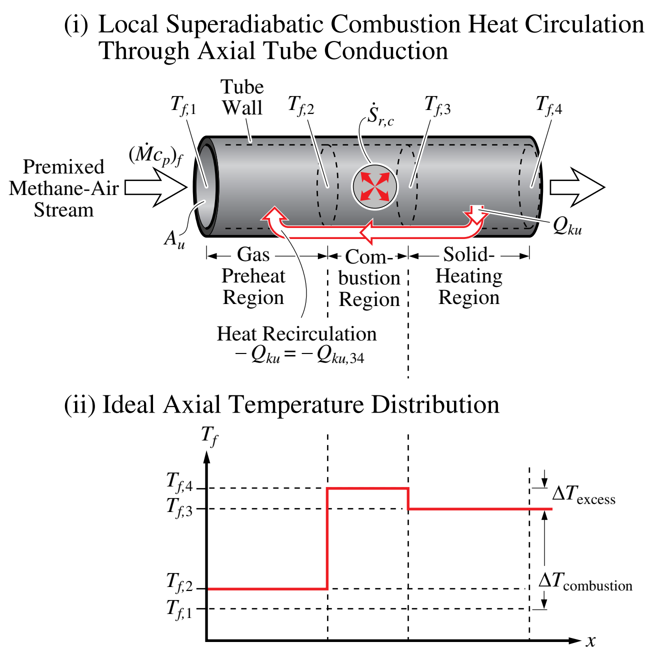


Figure Pr.5.18(a) Heat recirculation and local superadiabatic temperature in combustion in a tube.

OBJECTIVE:

(a) Draw the thermal circuit diagram. Determine (b) $T_{f,2}$, (c) $T_{f,3}$, (d) $T_{f,4}$, and (e) the excess temperature $\Delta T_{\text{excess}} = T_{f,4} - T_{f,3}$ for stoichiometric, premixed methane-air combustion.

(f) Comment on the effect of using temperature-dependent specific heat capacity on the predicted excess temperature.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.5.18(b). The surface-convection heat transfer out of the solid-heating region control volumes is equal in magnitude and opposite in sign to that entering the preheat region.

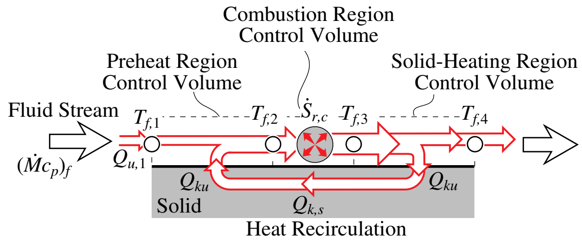


Figure Pr.5.18(b) Thermal circuit diagram.

(b) There are three regions and we write the energy equation (5.33) for each of them. The control volumes are shown in Figure Pr.5.18(b). The energy equations are

$$-Q_{u,1} + Q_{u,2} - Q_{ku} = 0 \qquad \text{gas-preheat region}$$
$$-Q_{u,2} + Q_{u,3} = \dot{S}_{r,c} \qquad \text{combustion region}$$
$$-Q_{u,3} + Q_{u,4} + Q_{ku} = 0 \qquad \text{solid-heating region.}$$

Now we use a constant and uniform $c_{p,f}$, similar to that used in (5.34), and the energy equations then become

$$(\rho c_p)_f(T_{f,2} - T_{f,1}) - \frac{Q_{ku}}{A_u} = 0$$
$$(\rho c_p)_f(T_{f,3} - T_{f,2}) = \frac{\dot{S}_{r,c}}{A_u}$$
$$(\rho c_p)_f(T_{f,4} - T_{f,3}) + \frac{Q_{ku}}{A_u} = 0.$$

Next, from (5.34), we replace $\dot{S}_{r,c}$ by

$$\dot{S}_{r,c} = A_u(\rho_{\rm F} u_f)_2 \Delta h_{r,\rm F}.$$

Then as in (5.38), we have

$$T_{f,3} = T_{f,2} - \frac{\Delta h_{r,\rm F}(\rho_{\rm F}/\rho_f)_2}{c_{p,f}}.$$

The density and the specific heat capacity of air at $T = 1{,}500$ K is found from Table C.22, i.e.,

$$\rho_f = 0.235 \text{ kg/m}^3 \qquad \text{Table C.22}$$
$$c_{p,f} = 1{,}202 \text{ J/kg-K} \qquad \text{Table C.22.}$$

From Table C.21(a), for the methane-air stoichiometric combustion, we have

$$\Delta h_{r,\rm F} = -5.553 \times 10^7 \text{ J/kg} \qquad \text{Table C.21(a)}$$
$$\left(\frac{\rho_{\rm F}}{\rho_f}\right)_2 = 0.0552 \qquad \text{Table C.21(a)}$$

For the gas-preheat region, we have

$$T_{f,2} = T_{f,1} + \frac{Q_{ku}/A_u}{(\rho c_p)_f}.$$

The second term on the right side will appear in all the energy equations, and we therefore calculate its value,

$$\frac{Q_{ku}/A_u}{(\rho c_p)_f} = \frac{5 \times 10^4 \text{ W/m}^2}{0.235(\text{kg/m}^3) \times 1{,}202(\text{J/kg-K})} = 177.0^\circ\text{C}.$$

The expression for $T_{f,2}$ then becomes

$$T_{f,2} = 20(℃) + 177.0(℃) = 197.0℃.$$

(c) In the combustion region, we have

$$\begin{aligned} T_{f,3} &= 197.0(℃) - \frac{-5.553 \times 10^7 (\text{J/kg}) \times 0.0552}{1{,}202(\text{J/kg-K})} \\ &= 191.0(℃) + 2{,}550(℃) = 2{,}747℃. \end{aligned}$$

(d) In the solid-heating region, we have

$$\begin{aligned} T_{f,4} &= T_{f,3} - \frac{Q_{ku}/A_u}{(\rho c_p)_f} \\ &= 2{,}747(℃) - 177.0(℃) = 2{,}569℃. \end{aligned}$$

(e) The excess temperature is

$$T_{f,4} - T_{f,3} = \frac{Q_{ku}/A_u}{(\rho c_p)_f} = 177.0℃.$$

(f) As shown in Example 5.4, the flue-gas specific heat capacity is larger than that of air, because it contains CO_2 and H_2O. The specific heat capacity of methane is also strongly temperature dependent [Figure 3.1(b)]. In principle, we should use a nonuniform mixture specific heat capacity. Using a larger mixture specific heat capacity results in a smaller excess and adiabatic flame temperatures.

COMMENT:

Note that due to the smaller $c_{p,f}$, the adiabatic flame temperature is larger than that found in Example 5.4. In chapter 6, we will determine Q_{ku} from the average gas and solid-surface temperatures. Surface-radiation heat transfer among the various regions can also be significant.

PROBLEM 5.19.FAM

GIVEN:

Consider nonadiabatic (i.e., lateral heat losses Q_{loss} not being negligible), one-dimensional flow and reaction of premixed methane-air gaseous mixture. There is a heat loss, per unit flow cross-sectional area, $Q_{loss}/A_u = q_{loss} = 10^5$ W/m^2. The mixture is stoichiometric and the initial temperature and pressure are $T_{f,1} = 298$ K and $p_1 = 1$ atm.

Use the Table C.21(a) data for adiabatic, stoichiometric flame ($\rho_{\rm F}/\rho_f$, $\Delta h_{r,F}$, and $u_{f,1}$), and for the average specific heat use $c_{p,f} = 1{,}611$ J/kg-K.

OBJECTIVE:

Determine the final temperature $T_{f,2}$.

SOLUTION:

The integral-volume energy equation applied to a control volume including the flame, under steady-state conditions, reduces to

$$Q|_A = \dot{S}_{r,c}.$$

The energy generation occurs by conversion from chemical-bond to thermal energy $\dot{S}_{r,c}$. The net heat flux leaving the control surface $Q|_A$ has contributions of convection heat transfer in and out of the flame and heat loss. The energy equation is (5.34), i.e.,

$$-\left(\rho_f c_{p,f} u_f T_f\right)_1 A_u + \left(\rho_f c_{p,f} u_f T_f\right)_2 A_u + Q_{loss} = -\dot{n}_{r,\rm F}\Delta h_{r,\rm F}.$$

Dividing the equation by A_u and noting that $\dot{n}_{r,\rm F} = \left(\rho_F u_f\right)_1 A_u$ we have

$$-\left(\rho_f c_{p,f} u_f T_f\right)_1 + \left(\rho_f c_{p,f} u_f T_f\right)_2 + q_{loss} = -\left(\rho_{\rm F} u_f\right)_1 \Delta h_{r,\rm F},$$

where $q_{loss} = Q_{loss}/A_u$ and A_u is the flow cross-section area.

From the conservation of mass equation (continuity) for a steady-state, uniform flow, we have

$$\rho_f u_f = \left(\rho_f u_f\right)_1 = \left(\rho_f u_f\right)_2 = \text{ constant},$$

and the energy equation is finally written as

$$-\left(c_{p,f} T_f\right)_1 + \left(c_{p,f} T_f\right)_2 + \frac{q_{loss}}{\left(\rho_f u_f\right)_1} = -\left(\frac{\rho_{\rm F}}{\rho_f}\right)_1 \Delta h_{r,\rm F}.$$

For the stoichiometric adiabatic reaction between methane and air, from Table C.21(a) we obtain $\Delta h_{r,\rm F} = -55.53$ MJ/kg$_F$, $\rho_{\rm F}/\rho_f = 0.0552$, and $u_{f,1} = 0.338$ m/s.

The temperature-average specific heat is assumed to be $c_{p,f} = 1{,}611$ J/kg-K. The temperature and pressure at the inlet are $T_{f,1} = 298$ K and $p = 1$ atm $= 101.3$ kPa. The density of the gas mixture at the inlet, assuming ideal-gas behavior and calculating the molecular weight for the gas mixture from the stoichiometric reaction (see Example 5.2), is given by

$$\rho_{f,1} = \frac{p}{\dfrac{R_g}{M_f} T_{f,1}} = \frac{1.013 \times 10^3 (\text{Pa})}{\dfrac{8{,}314(\text{J/kmole-K})}{27.63(\text{kmole})} 298(\text{K})} = 1.130 \text{ kg/m}^3.$$

Solving the energy equation for $T_{f,2}$ we have

$$\begin{aligned}
T_{f,2} &= T_{f,1} - \left(\frac{\rho_{\rm F}}{\rho_f}\right)_1 \frac{\Delta h_{r,\rm F}}{c_{p,f}} - \frac{q_{loss}}{\left(\rho_f u_f\right)_1 c_{p,f}} \\
&= 298(\text{K}) - 0.0552(\text{kg}_F/\text{kg}_f)\frac{-55.53 \times 10^6 (\text{J/kg}_F)}{1{,}611(\text{J/kg-K})} - \frac{10^5(\text{W/m}^2)}{1.130(\text{kg/m}^3) \times 0.338(\text{m/s}) \times 1{,}611(\text{J/kg-K})} \\
&= 298(\text{K}) + 1{,}903(\text{K}) - 162.5(\text{K}) = 2{,}039 \text{ K}.
\end{aligned}$$

COMMENT:

The increase in the heat loss will eventually cause the extinguishment of the flame. Note that the adiabatic flame temperature ($Q_{loss} = 0$) is $T_{f,2} = 2{,}187$ K and that this heat loss results in a reduction of 162.5 K. The heat loss also influences the flame speed $u_{f,1}$.

PROBLEM 5.20.FAM.S

GIVEN:

The adiabatic flame temperature and speed are referred to the condition of no lateral heat losses ($Q_{loss} = 0$) in combustion of a unidirectional premixed fuel-oxidant stream. The presence of such losses or gains decreases the flame temperature $T_{f,2}$, given by (5.35), and also the flame speed given by (5.55). This can continue until the flame temperature decreases below a threshold temperature required to sustain ignition and combustion.

$T_{f,1} = 16$℃, $\rho_{f,1} = 1.164$ kg/m^3, $(\rho_F/\rho_f)_1 = 0.05519$, $c_{p,f} = 1{,}611$ J/kg-K, $k_f = 0.07939$ W/m-K, $A_u = 10^{-2}$ m^2, $0 \leq Q_{loss} \leq 800$ W.

Use Tables 5.2 and 5.3 for $\Delta h_{r,F}$, a_r, and ΔE_a.

OBJECTIVE:

(a) Consider a laminar, stoichiometric premixed methane air combustion. For the conditions given above, plot the flame temperature $T_{f,2}$ and the flame $u_{f,1}$ speed for the given range of Q_{loss}.

(b) Comment on the quenching of the flame as Q_{loss} increases and $T_{f,2}$ decreases.

SOLUTION:

(a) The flame temperature $T_{f,2}$ is given by (5.35), and $\Delta h_{r,\mathrm{F}}$ is given in Table 5.2, i.e.,

$$\begin{aligned}
T_{f,2} &= T_{f,1} - \frac{\Delta h_{r,\mathrm{F}}}{c_{p,f}}\left(\frac{\rho_\mathrm{F}}{\rho_f}\right)_1 - \frac{Q_{loss}}{A_u \rho_{f,1} c_{p,f} u_{f,1}} \\
&= 16(℃) - \frac{-5.553\times10^7(\mathrm{J/kg})}{1{,}611(\mathrm{J/kg\text{-}K})}\times 0.05519 - \frac{Q_{loss}(\mathrm{W})}{10^{-2}(\mathrm{m}^2)\times 1.164(\mathrm{kg/m}^3)\times 1{,}611(\mathrm{J/kg\text{-}K})\times u_{f,1}(\mathrm{m/s})} \\
&= 289.15(\mathrm{K}) + 1{,}902(\mathrm{K}) - 5.333\times10^{-2}(\mathrm{m\text{-}K/s\text{-}W})\times\frac{Q_{loss}}{u_{f,1}}.
\end{aligned}$$

The flame speed is given by (5.55), and a_r and ΔE_a are given in Table 5.3, i.e.,

$$\begin{aligned}
u_{f,1} &= \left(\frac{2k_f a_r}{\rho_f c_{p,f}\rho_{\mathrm{F},1}\mathrm{Ze}} e^{-\frac{\Delta E_a}{R_g T_{f,2}}}\right)^{1/2} \\
\mathrm{Ze} &= \frac{\Delta E_a (T_{f,2} - T_{f,1})}{R_g T_{f,1}^2} \\
&= \frac{2.10\times10^8(\mathrm{J/kmole})\times(T_{f,2} - 289.15(\mathrm{K}))}{8.314\times10^3(\mathrm{J/kmole\text{-}K})\times T_{f,2}^2} \\
&= 2.526\times10^4(\mathrm{K})\times\frac{(T_{f,2} - 289.15(\mathrm{K}))}{T_{f,2}^2} \\
u_{f,1} &= \left[\frac{2\times0.07939(\mathrm{W/m\text{-}K})\times1.3\times10^8(\mathrm{kg/m^3\text{-}s})}{1.164(\mathrm{kg/m}^3)\times1{,}611(\mathrm{J/kg\text{-}K})\times0.05519\times1.164(\mathrm{kg/m}^3)\times\mathrm{Ze}} e^{-\frac{2.10\times10^8(\mathrm{J/kmole})}{8{,}314(\mathrm{J/kmole\text{-}K})\times T_{f,2}}}\right]^{1/2} \\
&= \left[\frac{1.713\times10^5(\mathrm{m^2/s^2})}{\mathrm{Ze}} e^{-\frac{2.526\times10^4(\mathrm{K})}{T_{f,2}}}\right]^{1/2}.
\end{aligned}$$

The solution to these three equations is fully defined by the specification of Q_{loss} and $T_{f,2}$. Thus, plots of $T_{f,2}$ and $u_{f,1}$ as functions of Q_{loss} can be made. These are shown in Figure Pr.5.20. While the decrease in $T_{f,2}$ is not very noticeable, $u_{f,1}$ decreases significantly with increase in Q_{loss}. This is because of the $T_{f,2}$ proportionality through Ze and the exponential relation dependence through the activation term.

(b) If we continue to increase Q_{loss} beyond 800 W, the flame may quench.

COMMENT:

If we continue to increase Q_{loss}, at $Q_{loss} \simeq 880$ W, the flame speed would tend to zero and no solution will be found. This can be defined as the theoretical quenching limit.

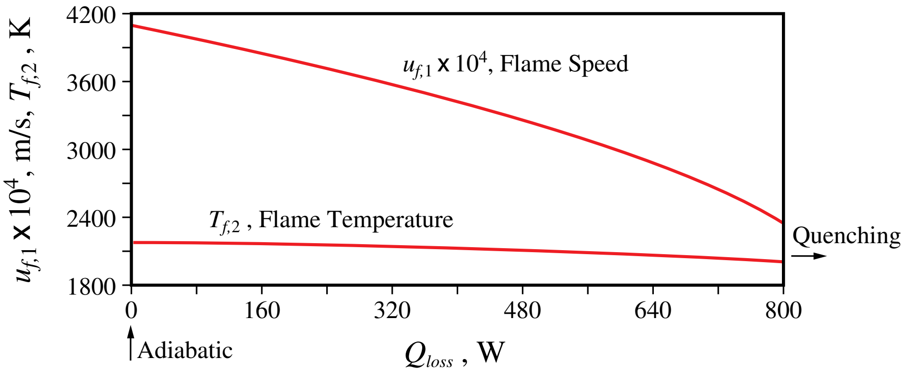


Figure Pr.5.20 Variation of flame temperature and speed with respect to the heat loss.

PROBLEM 5.21.FUN

GIVEN:

A surface-radiation burner, which uses distributed, direct fuel supply, is shown in Figure Pr.5.219a). The radiation is from the impermeable surface having a uniform temperature T_s and facing the radiation-load surface at temperature T_L. The oxidant stream (air) is at mass flow rate $\dot{M}_{\rm O}$ and temperature $T_{f,1}$.

The fuel stream (methane) is divided into three smaller streams with each flow rate designated as $\dot{M}_{{\rm F},i}$, $i = 1,2,3$. The product stream $\dot{M}_{\rm P}$ leaves at the exit port at temperature $T_{f,4}$. Here we assume that $T_{f,4} = T_{f,3} = T_{f,2} = T_s$. This is a design requirement that in practice is obtained by using more than three fuel stream ports and by taking the pressure drop in the fuel membrane and the combustion chamber into the account.

$\epsilon_{r,s} = 0.9$, $\epsilon_{r,L} = 1$, $F_{s\text{-}L} = 1$, $T_L = 700$ K, $T_{f,1} = 300$ K, $\dot{M}_{\rm O} = 0.013$ kg/s, $\dot{M}_{{\rm F},1} = 3 \times 10^{-4}$ kg/s, $\dot{M}_{{\rm F},2} = \dot{M}_{{\rm F},3} = 2 \times 10^{-4}$ kg/s.

Use an integral-volume energy equation for each of the three segments. Assume a constant specific heat capacity $c_{p,f} = 1{,}600$ J/kg-K.

SKETCH:

Figure Pr.5.21(a) shows the burner, the fuel and oxidant ports, and the heat transfer load surface.

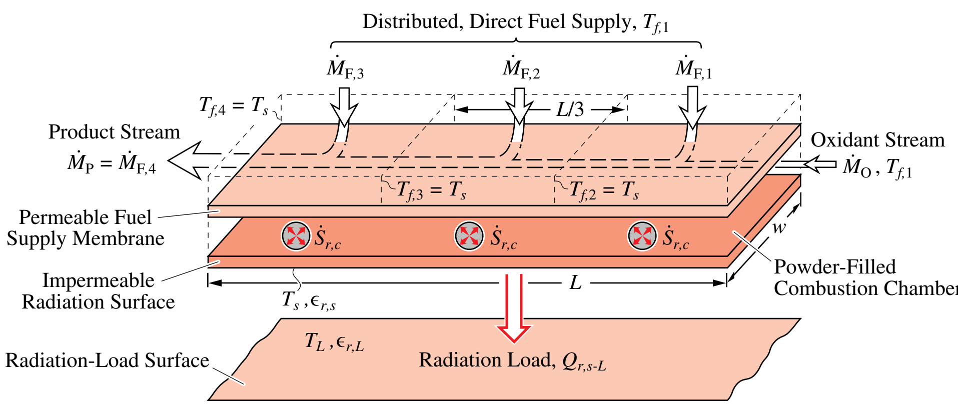


Figure Pr.5.21(a) A distributed, direct fuel supply surface-radiation burner. The radiation surface is impermeable.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine the surface temperature T_s.
(c) Determine the burner efficiency $\eta = Q_{r,s\text{-}L}/\dot{S}_{r,c}$, $\dot{S}_{r,c} = -\Delta h_{r,{\rm F}} \sum_i \dot{M}_{{\rm F},i}$.

SOLUTION:

(a) Figure Pr.5.21(b) shows the thermal circuit diagram. The three segments are connected by having the stream leaving one segment enter the next segment downstream, after fuel is added. The mass flow rate of the product $\dot{M}_{P,i}$ increases as more fuel is added.

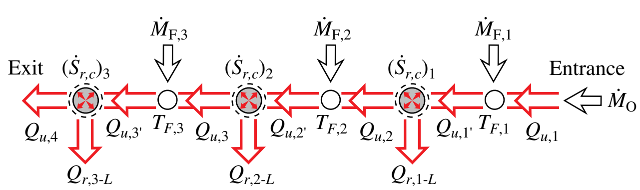


Figure Pr.5.21(b) Thermal circuit diagram.

(b) The energy equations for the three segments are written according to (5.33) as

$$\begin{aligned} Q_{u,2} - Q_{u,1'} + Q_{r,1\text{-}L} &= (\dot{S}_{r,c})_1 \\ Q_{u,3} - Q_{u,2'} + Q_{r,2\text{-}L} &= (\dot{S}_{r,c})_2 \\ Q_{u,4} - Q_{u,3'} + Q_{r,3\text{-}L} &= (\dot{S}_{r,c})_3. \end{aligned}$$

Now we note that the temperature for the oxidant and the fuel entering the burner is the same and equals $T_{f,1}$. By adding the three equations, we have

$$\begin{aligned} Q_{u,4} - Q_{u,1'} - (Q_{u,2'} - Q_{u,2}) - (Q_{u,3'} - Q_{u,3}) + \sum_{i=1}^{3} Q_{r,i\text{-}L} &= \sum_{i=1}^{3} (\dot{S}_{r,c})_i \\ &= -\Delta h_{r,\mathrm{F}} \sum_{i=1}^{3} \dot{M}_{F,i}. \end{aligned}$$

Now using (5.34) and (4.49), for a two-surface enclosure with $F_{s\text{-}L} = 1$ and $\epsilon_{r,L} = 1$, we have

$$\left(\dot{M}_{O_2} + \sum_{i=1}^{3} \dot{M}_{f,i}\right) c_{p,f} T_s - \left(\dot{M}_{O_2} + \sum_{i=1}^{3} \dot{M}_{f,i}\right) c_{p,f} T_{f,1} + \epsilon_{r,s} w L \sigma_{\mathrm{SB}} (T_s^4 - T_L^4) = -\Delta h_{r,\mathrm{F}} \sum_{i=1}^{3} \dot{M}_{f,i},$$

where we have used $T_{f,4} = T_s$ and for the exit product mass flow rate we have used the sum of the oxidant and the total fuel mass flow rate.

From Table C.21(a), we have for methane,

$$\Delta h_{r,\mathrm{F}} = -5.553 \times 10^7 \ \mathrm{J/kg} \quad \text{Table C.21(a).}$$

Using the numerical values, we then have

$$\begin{aligned} &(1.3 \times 10^{-2} + 7 \times 10^{-4})(\mathrm{kg/s}) \times 1{,}600(\mathrm{J/kg\text{-}K}) \times (T_s - 300)(\mathrm{K}) + 0.9 \times 0.5(\mathrm{m}) \times 1(\mathrm{m}) \times \\ &\quad 5.67 \times 10^{-8}(\mathrm{W/m^2\text{-}K^4})(T_s^4 - 700^4)(\mathrm{K}^4) = -(-5.553 \times 10^7)(\mathrm{J/kg}) \times 7 \times 10^{-4}(\mathrm{kg/s}). \end{aligned}$$

Solving for T_s, we get

$$T_s = 1{,}040 \ \mathrm{K}.$$

(c) With the given expression for η,

$$\begin{aligned} \eta &= \frac{Q_{r,s\text{-}L}}{\dot{S}_{r,c}} \\ &= \frac{\epsilon_{r,s} w L \sigma_{\mathrm{SB}} (T_s^4 - T_L^4)}{-\Delta h_{r,\mathrm{F}} \left(\sum_{i=1}^{3} \dot{M}_{F,i}\right)} \\ &= \frac{2.367 \times 10^4 (\mathrm{W})}{3.887 \times 10^4 (\mathrm{W})} \\ &= 0.6090 = 60.90\%. \end{aligned}$$

COMMENT:

This efficiency is rather low and can be increased by preheating the air and the fuel using heat exchangers. The burner can be made from ceramics such as zirconia and the thermal stress/strain during the cyclic use should be addressed. Also, by using pure oxygen instead of air, we can increase the surface temperature and the burner efficiency.

PROBLEM 5.22.FAM

GIVEN:

A water-cooled thermal plasma generator, used for spray coating and shown in Figure Pr.5.22(a), is to produce an argon gas plasma stream with an average exit temperature of $T_{f,2} = 5{,}000$ K. The Joule heating is by direct current (dc) and uses $J_e = 200$ A, and $\Delta\varphi = 150$ V provided by a power supply. The heat transfer to the water coolant is Q_{ku} and other heat losses are given by Q_{loss}. Assume $\dot{S}_{ij} = 0$.

$Q_{ku} = 5$ kW, $Q_{loss} = 3$ kW, $T_{f,1} = 300$ K.

Use a constant specific heat capacity for the ionized argon and use a value equal to five times that for argon at $T = 1{,}500$ K.

SKETCH:

Figure Pr.5.22(a) shows the plasma torch.

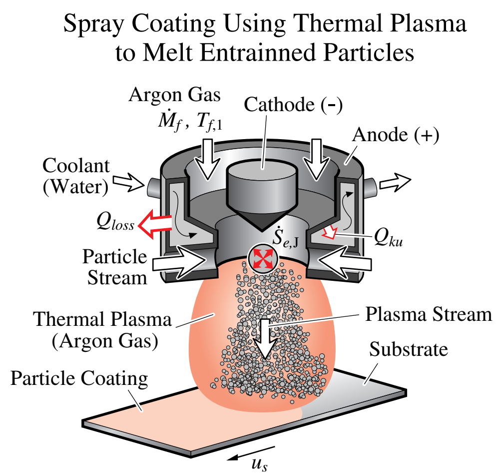


Figure Pr.5.22(a) Generation of an argon plasma stream using the Joule heating.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the argon gas flow rate $\dot{M}_f$.

SOLUTION:

(a) Figure Pr.5.22(b) shows the control volumes and the thermal circuit diagram.

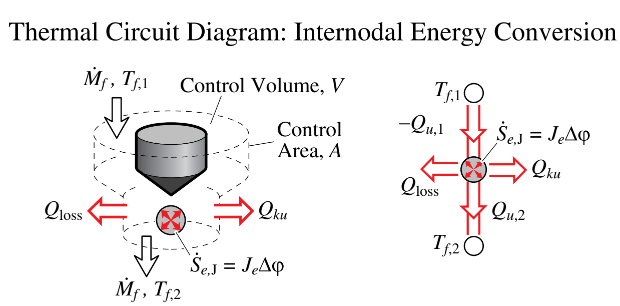


Figure Pr.5.22(b) Thermal circuit diagram.

(b) From Figure Pr.5.22(b), or from (5.73), we have the energy equation

$$\begin{aligned} Q|_A &= Q_{u,1} + Q_{u,2} + Q_{ku} + Q_{loss} = \dot{S}_{ij} + \dot{S}_{e,\mathrm{J}} \\ &= -\dot{M}_f c_{p,f}(T_{f,1} - T_{f,2}) + Q_{ku} + Q_{loss} = J_e\Delta\varphi. \end{aligned}$$

The specific heat capacity of argon at $T = 1{,}500$ K is given in Table C.22, and we use a value five times this, i.e.,

$$c_{p,f} = 5 \times 520(\text{J/kg-K}) = 2{,}600 \text{ J/kg-K} \quad \text{Table C.22.}$$

Then solving for $\dot{M}_f$, we have

$$\dot{M}_f = \frac{J_e \Delta\varphi - Q_{ku} - Q_{loss}}{c_{p,f}(T_{f,2} - T_{f,1})}.$$

Using the numerical values, we have

$$\begin{aligned} \dot{M}_f &= \frac{200(\text{A}) \times 150(\text{V}) - 5{,}000(\text{W}) - 3{,}000(\text{W})}{2{,}600(\text{J/kg-K}) \times (5{,}000 - 300)(\text{K})} \\ &= 1.800 \times 10^{-3} \text{ kg/s} = 1.800 \text{ g/s}. \end{aligned}$$

COMMENT:

The gas specific heat capacity is a strong function of temperature. Upon gas dissociation and ionization at high temperatures, it increases further.

PROBLEM 5.23.FAM

GIVEN:

An acetylene-oxygen torch has 70% by weight of its stoichiometric oxygen provided by a pressurized tank and this is called the primary oxygen. Due to the fast chemical reaction of C_2H_2 and O_2 and the fast diffusion of O_2, the remaining 30% of the oxygen is provided by entraining the ambient air and this is called the secondary oxygen. These are shown in Figure Pr.5.23(a). Along with the entrained oxygen, nitrogen is also entrained and this inert gas tends to lower the final temperature $T_{f,2}$.

The products of combustion (i.e., the flue gas) flows over a surface to be welded. Then a fraction of the sensible heat is transferred to the surface by surface-convection heat transfer Q_{ku}, and the remainder flows with the gas as $Q_{u,2}$.

$T_{f,1} = 20°\text{C}$, $p_1 = 1$ atm, $Q_{ku} = 0$.

Use a constant and uniform specific heat of $c_{p,f} = 3{,}800$ J/kg-K.

SKETCH:

An acetylene-oxygen torch, with primary (pure oxygen) and secondary oxygen (mixed with nitrogen) supplies, is shown in Figure Pr.5.23(a).

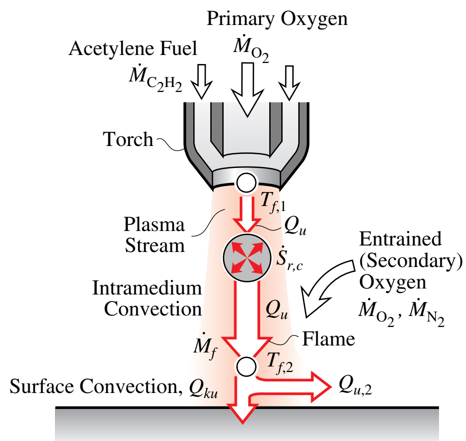


Figure Pr.5.23(a) An acetylene-oxygen torch used for welding of a surface. The oxygen is provided by a tank and by entraining air.

OBJECTIVE:

(a) Draw the thermal circuit diagram for this combustion and heat transfer.
(b) For the conditions given above, determine the flue gas temperature $T_{f,2}$.

SOLUTION:

(a) Figure Pr.5.23(b) shows the thermal circuit diagram with the internodal energy conversion and surface-convection heat transfer connected to node $T_{f,2}$.

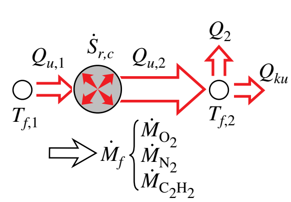


Figure Pr.5.23(b) Thermal circuit diagram.

(b) We need to determine $\rho_F/\rho_{f,1}$, in order to solve for $T_{f,2}$ from (5.35), i.e.,

$$T_{f,2} = T_{f,1} - \frac{\Delta h_{r,F}}{c_{p,f}} \frac{\rho_{F,1}}{\rho_{f,1}},$$

where we have assumed a constant $c_{p,f}$.
To determine $\rho_{F,1}/\rho_{f,1}$, we note that the two stoichiometric reactions are,

$$C_2H_2 + \frac{5}{2}O_2 \rightarrow 2CO_2 + H_2O$$
$$C_2H_2 + \frac{5}{2}O_2 + 9.40N_2 \rightarrow 2CO_2 + H_2O + 9.40N_2.$$

Then, similar to Example 5.4, we have

$$\frac{\rho_{F,1}}{\rho_{f,1}} = 0.70 \times \frac{\nu_{C_2H_2} M_{C_2H_2}}{\nu_{C_2H_2} M_{C_2H_2} + \nu_{O_2} M_{O_2}} + 0.30 \times \frac{\nu_{C_2H_2} M_{C_2H_2}}{\nu_{C_2H_2} M_{C_2H_2} + \nu_{O_2} M_{O_2} + \nu_{N_2} M_{N_2}}.$$

Using the numerical values, we have

$$\begin{aligned}
\frac{\rho_{F,1}}{\rho_{f,1}} &= 0.70 \times \frac{1 \times (12.011 \times 2 + 1.008 \times 2)}{1 \times (12.011 \times 2 + 1.008 \times 2) + 2.5 \times 2 \times 15.999} + \\
&\quad 0.30 \times \frac{1 \times (12.011 \times 2 + 1.008 \times 2)}{1 \times (12.011 \times 2 + 1.008 \times 2) + 2.5 \times 2 \times 15.999 + 9.40 \times 2 \times 14.007} \\
&= \frac{18.227}{106.03} + \frac{7.8114}{369.30} \\
&= 0.17190 + 0.021152 = 0.19305.
\end{aligned}$$

With $\Delta h_{r,F} = -4.826 \times 10^7$ J/kg from Table 5.2, [also listed in Table C.21(a)], we have

$$\begin{aligned}
T_{f,2} &= 20(℃) - \frac{-4.826 \times 10^7 (\text{J/kg})}{3{,}800(\text{J/kg-K})} \times 0.19305 \\
&= 20(℃) + 2{,}452(℃) = 2{,}472℃.
\end{aligned}$$

COMMENT:

From Figure 5.9, using pure oxygen gives an adiabatic flame temperature of about 3,100℃. Therefore, the nitrogen dilution should be avoided in order to achieve a higher adiabatic temperature. Note that high C-atom content and low H-atom content results in a high $(\rho_F/\rho_f)_1$. This makes acetylene a good fuel for achieving high adiabatic flame temperatures.

PROBLEM 5.24.FAM

GIVEN:

The acetylene-oxygen, flame-cutting torch is used with low carbon and low alloy irons. A simple mixer that requires pressurized oxygen and acetylene is shown in Figure Pr.5.24(a). In addition to the heat provided by the reaction

$$C_2H_2 + 2.5O_2 \rightarrow 2CO_2 + H_2O,$$

the excess oxygen provided by the torch reacts with the iron and releases heat. One of the reactions is

$$3Fe + 2O_2 \rightarrow Fe_3O_4.$$

This reaction is highly exothermic, i.e., $\Delta h_{r,\mathrm{F}} = -6.692 \times 10^6$ J/kg of Fe. Figure Pr.5.24(a) shows this iron oxidation. The rate of this reaction is controlled by the speed of the torch moving on the cutting surface, and with other variables.

Assume that this reaction will add an extra energy conversion such that we can approximate the contribution for this iron oxidation by adding 30% to the heat of reaction of C_2H_2. We also model the excess oxygen by adding 40%, by weight, to the stoichiometric oxygen needed to burn the acetylene.

$T_{f,1} = 20$℃, $c_{p,f} = 3{,}800$ J/kg-K.

SKETCH:

Figure Pr.5.24(a) shows the torch and the reacting-eroding workpiece.

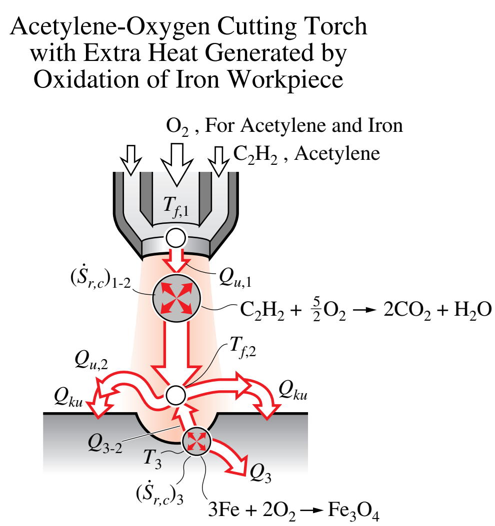


Figure Pr.5.24(a) A cutting torch with reacting workpiece.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the combustion product gas temperature $T_{f,2}$.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.5.24(b). The oxidization of Fe is shown as $(\dot{S}_{r,c})_3$ and a fraction of this heat generation rate is transferred to the gas. The gas in turn heats the solid in the regions away from where $(\dot{S}_{r,c})_3$ is generated, thus providing the preheating necessary for the iron oxidation.

(b) The combustion product gas temperature $T_{f,2}$ is given by (5.35), i.e.,

$$T_{f,2} = T_{f,1} - \frac{\Delta h_{r,\mathrm{F}}}{c_{p,f}} \frac{\rho_{\mathrm{F},1}}{\rho_{f,1}},$$

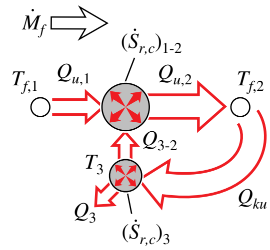


Figure Pr.5.24(b) Thermal circuit diagram.

where $\Delta h_{r,\mathrm{F}}$ is that given in Table 5.2 for C_2H_2, times 1.3.
To determine $(\rho_\mathrm{F}/\rho_f)_1$, we begin from the stoichiometric C_2H_2 oxidation,

$$C_2H_2 + 2.5O_2 \rightarrow 2CO_2 + H_2O.$$

Next we use the approximation given for the extra oxygen supplied for the iron oxidation, i.e.,

$$C_2H_2 + 1.4 \times 2.5O_2 \rightarrow 2CO_2 + H_2O + 0.4 \times 2.5O_2.$$

Then the density ratio $(\rho_\mathrm{F}/\rho_f)_1$ becomes

$$\begin{aligned}
\frac{\rho_{\mathrm{F},1}}{\rho_{f,1}} &= \frac{\nu_{C_2H_2} M_{C_2H_2}}{\nu_{C_2H_2} M_{C_2H_2} + \nu_{O_2} M_{O_2}} \\
&= \frac{1 \times (12.011 \times 2 + 1.008 \times 2)}{1 \times (12.011 \times 2 + 1.008 \times 2) + 1.4 \times 2.5 \times 2 \times 15.999} \\
&= \frac{26.04}{138.0} = 0.1886.
\end{aligned}$$

From Table 5.2, we have

$$C_2H_2: \ \Delta h_{r,\mathrm{F}} = -4.826 \times 10^7 \ \mathrm{J/kg} \qquad \text{Table 5.2.}$$

Then

$$\begin{aligned}
T_{f,2} &= 20(^\circ\mathrm{C}) - \frac{1.3 \times (-4.826 \times 10^7)(\mathrm{J/kg})}{3{,}800(\mathrm{J/kg\text{-}K})} \times 0.1886 \\
&= 20(^\circ\mathrm{C}) + 3{,}114(^\circ\mathrm{C}) = 3{,}134^\circ\mathrm{C}.
\end{aligned}$$

COMMENT:

From Figure 5.9, we note that the adiabatic flame temperature for the stoichiometric acetylene oxidation in pure oxygen is nearly 3,100°C. Thus the addition of 30% to the heat of combustion is nearly compensated by the excess oxygen that must be heated to the final temperature $T_{f,2}$.

PROBLEM 5.25.FUN

GIVEN:

The combustion product gas from a fireplace chamber enters its chimney at a flow rate $\dot{M}_f$ and a temperature $T_{f,1}$. This is shown in Figure Pr.5.25. Assume that the surface-convection heat transfer to the chimney wall is negligible.

$T_s = 120°C$, $T_{f,1} = 600°C$, $\dot{M}_f = 2.5 \times 10^{-3}$ kg/s, $R = 15$ cm, $L = 5$ m, $\epsilon_{r,f} = 0.1$ (for large soot concentration).

Use $c_{p,f}$ for air at $T = 600$ K. Use $\epsilon_{r,s} = \alpha_{r,s}$ for fireclay brick.

SKETCH:

Figure Pr.5.25 shows the chimney with the combustion product gas stream passing through it.

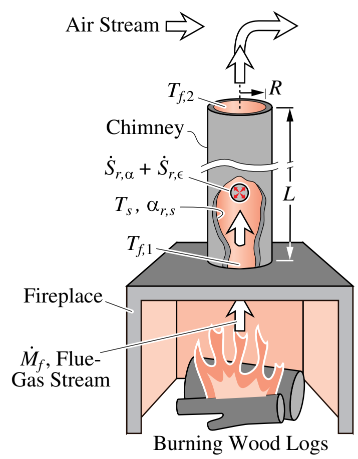


Figure Pr.5.25 A chimney bounding the combustion product gas stream exiting a fireplace chamber.

OBJECTIVE:

(a) Show that the surface radiation emitted by the chimney wall is negligible compared to that emitted by the combustion product gas stream.

(b) Determine the combustion product gas stream exit temperature $T_{f,2}$.

SOLUTION:

(a) The condition for neglecting the surface emission is given in (5.81), i.e.,

$$\frac{T_s^4}{T_{f,1}^4} \ll 1.$$

Here we have

$$\frac{(120 + 273.15)^4 (\text{K}^4)}{(600 + 273.15)^4 (\text{K}^4)} = 0.0410 \ll 1.$$

(b) Using (5.81), we have

$$\frac{1}{T_{f,2}^3} = \frac{1}{T_{f,1}^3} + \frac{A_r \epsilon_{r,f} \alpha_{r,s} \sigma_{\text{SB}}}{(\dot{M} c_p)_f}.$$

From Table C.22, for air at $T = 600$ K, we have

$$c_{p,f} = 1{,}038 \text{ J/kg-K} \quad \text{Table C.22.}$$

From Table C.18, we have for fireclay brick

$$\epsilon_{r,s} = \alpha_{r,s} = 0.75 \quad \text{Table C.18.}$$

Also,

$$A_r = 2\pi RL = 2 \times \pi \times 0.15(\text{m}) \times 5(\text{m}) = 4.713 \text{ m}^2.$$

Then

$$\begin{aligned}
\frac{1}{T_{f,2}^3} &= \frac{1}{(873.15)^3(\text{K}^3)} + \frac{4.713(\text{m}^3) \times 0.1 \times 0.75 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4)}{2.5 \times 10^{-3}(\text{kg/s}) \times 1{,}038(\text{J/kg-K})} \\
&= 1.502 \times 10^{-9}(1/\text{K}^3) + 7.723 \times 10^{-9}(1/\text{K}^3) \\
T_{f,2} &= 476.8 \text{ K} = 203.7^\circ\text{C}.
\end{aligned}$$

COMMENT:

Inclusion of the surface-convection heat transfer will further decrease $T_{f,2}$ and will be discussed in Chapter 7. Here the surface emission from the wall was neglected. When included, $T_{f,2}$ will increase by a small amount.

Chapter 6

Convection: Semi-Bounded Fluid Streams

PROBLEM 6.1.FUN

GIVEN:

Surface-convection heat transfer refers to heat transfer across the boundary separating a fluid stream and a condensed-phase (generally solid) volume, as rendered in Figure Pr.6.1(a) for a stationary solid.

Assume a uniform solid surface temperature T_s and a far-field temperature $T_{f,\infty} \neq T_s$.

SKETCH:

Figure Pr.6.1(a) renders the general surface-convection of a semi-bounded fluid stream.

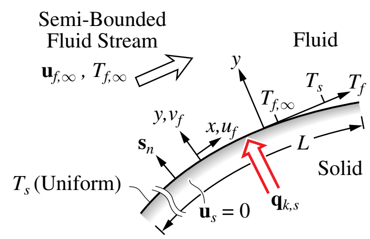


Figure Pr.6.1(a) A rendering of a semi-bounded fluid stream passing over a solid surface with $T_{f,\infty} \neq T_s$.

OBJECTIVE:

(a) Draw the heat flux vector tracking starting from $\mathbf{q}_{k,s}$ and ending with $\mathbf{q}_u$ away from the surface. At the surface use $\mathbf{q}_{k,f} = \mathbf{q}_{ku}$ and in the thermal boundary layer show both conduction and convection. Neglect radiation heat transfer.

(b) Quantitatively draw the fluid temperature distribution $T_f(y)$ at the location shown in Figure Pr.6.1(a).

(c) Show the thermal boundary layer thickness δ_α on the same graph. Show the viscous boundary layer thickness for Pr > 1 and Pr < 1.

(d) Draw the thermal circuit diagram for the solid surface and write the expression for the average heat transfer rate $\langle Q_{ku} \rangle_L$.

(e) What is the average surface-convection resistance $\langle R_{ku} \rangle_L$, if T_s is the same as $T_{f,\infty}$, i.e., for the solid surface temperature to be made equal to the fluid stream far-field temperature?

(f) If $T_{f,\infty} \neq T_s$ what should $\langle R_{ku} \rangle_L$ be for there to be no surface-convection heat transfer (ideal insulation)

SOLUTION:

(a) Figure Pr.6.1(b) shows the heat flux vector tracking. Note that as in Figures 6.3 and 6.7, at the surface $\mathbf{q}_{k,f} = \mathbf{q}_{ku}$, i.e., surface-convection heat transfer is the fluid conduction heat flux, since $\mathbf{u}_f = 0$ on the surface.

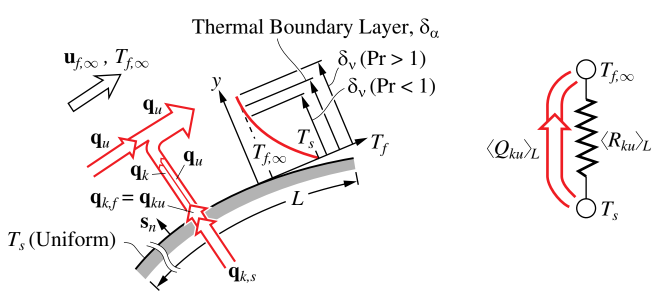


Figure Pr.6.1(b) Various features of the surface-convection heat transfer.

(b) The temperature distribution $T_f(y)$ is also shown in Figure Pr.6.1(b), starting from $T_f = T_s$ at $y = 0$ and having $T_f = T_{f,\infty}$, far away from the surface. Using the concept of the thermal boundary-layer thickness, the far-field conditions are applied at $y = \delta_\alpha$.

(c) The thermal boundary layer thickness δ_α is shown in Figure Pr.6.1(b) and this is where T_f reaches $T_{f,\infty}$ to within a small difference given by (6.20).

For Pr > 1, from (6.48), we have $\delta_\nu > \delta_\alpha$, and for Pr < 1 we have $\delta_\nu < \delta_\alpha$. These are shown in Figure Pr.6.1(b).

(d) The thermal circuit diagram is shown in Figure Pr.6.1(b). From this figure, or from (6.49), we have

$$\langle Q_{ku} \rangle_L = \frac{T_s - T_{f,\infty}}{\langle R_{ku} \rangle_L}.$$

(e) If $T_s = T_{f,\infty}$, there will be no surface convection heat transfer, and therefore there will be no thermal boundary layer. If there is no thermal boundary layer, then $\langle R_{ku} \rangle_L = 0$. This shows that the surface temperature approaches the fluid far-field temperature as $\langle R_{ku} \rangle_L \rightarrow 0$. This is a method for controlling (maintaining) the surface temperature.
(f) For an ideally insulated surface, we have

$$\langle Q_{ku} \rangle_L = 0 \quad \text{for} \quad \langle R_{ku} \rangle_L \rightarrow \infty$$

and this would require a large resistance (i.e., vacuum) for a finite difference between T_s and $T_{f,\infty}$. Similarly $\langle Q_{ku} \rangle_L = 0$ when $T_s = T_{f,\infty}$.

COMMENT:

Heat transfer between a semi-bounded fluid passing over a solid surface requires heat transfer by fluid conduction across the interface. This heat transfer is greatly influenced by the fluid motion, and other fluid properties, which in turn influence the gradient of temperature ∇T_f, and directly depends on the fluid conductivity k_f.

PROBLEM 6.2.FUN

GIVEN:

A surface, treated as a semi-infinite plate and shown in Figure Pr.6.2, is to be heated with a forced, parallel flow. The fluids of choice are (i) mercury, (ii) ethylene glycol (antifreeze), and (iii) air.

$T_s = 10$°C, $T_{f,\infty} = 30$°C, $u_{f,\infty} = 0.2$ m/s, $L = 0.2$ m.

Evaluate the properties at $T = 300$ K, from Tables C.22 and C.23.

SKETCH:

Figure Pr.6.2 shows a forced, parallel flow over a semi-infinite plate. The thermal boundary-layer thickness is also shown.

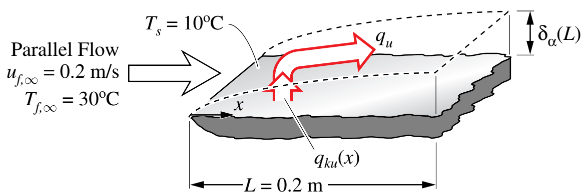


Figure Pr.6.2 A semi-bounded fluid stream exchanging heat with its semi-infinite plate bounding surface.

OBJECTIVE:

For the tailing edge of the plate $x = L$, do the following:

(a) Determine the local rate of heat transfer per unit area q_{ku}(W/m^2).

(b) Determine the thermal boundary-layer thickness δ_α(mm). Use the Nusselt number relation for Pr $\neq 0$.

(c) For mercury, also use the relation for Nusselt number for a zero viscosity (i.e., Pr = 0) and compare the results with that obtained from the nonzero viscosity relations.

SOLUTION:

(a) The Reynolds number is given by (6.45), i.e.,

$$\text{Re}_L = \frac{u_{f,\infty} L}{\nu_f}.$$

For $\text{Re}_L < 5 \times 10^5$, the flow regime is laminar. The local Nusselt number at $x = L$ for laminar, parallel flow over a flat plate is given by (6.44), i.e.,

$$\text{Nu}_L(x = L) = 0.332 \text{Re}_L^{1/2} \text{Pr}^{1/3}.$$

The local surface-convection heat flux, from (6.44), is

$$q_{ku} = \frac{\text{Nu}_L k_f}{L}(T_s - T_{f,\infty}).$$

(b) The thermal boundary-layer thickness for laminar flow at $x = L$ is given by (6.48)

$$\delta_\alpha(x = L) = \frac{5L}{\text{Re}_L^{1/2}} \frac{1}{\text{Pr}^{1/3}}$$

Table Pr.6.2 shows the thermophysical properties at $T = 300$ K for the three fluids and the numerical results obtained for δ_α and q_{ku}.

(c) With the zero viscosity (or Prandtl number) assumption, the Nusselt number is given by (6.30) and δ_α is given by (6.21), i.e.,

$$\text{Nu}_L(x = L) = \left(\frac{\text{Pe}_L}{\pi}\right)^{1/2}, \quad \delta_\alpha(x = L) = 3.6 \left(\frac{\alpha_f L}{u_{f,\infty}}\right)^{1/2}, \quad \text{for Pr} = 0,$$

Table Pr.6.2 Properties (from Tables C.22 and C.23) for the three fluids and the numerical results.

Fluid	ν_f, m^2/s	k_f, W/m-K	Pr	Re_L	$\delta_\alpha(x = L)$, mm	$Nu_L(x = L)$	q_{ku}, W/m^2
mercury	0.112×10^{-6}	8.86	0.0240	3.571×10^5	5.801	57.23	−50,703
ethylene glycol	18.09×10^{-6}	0.2515	193	2,211	3.680	90.22	−2,269
air	15.66×10^{-6}	0.0267	0.69	2,554	22.39	14.83	-39.59

where from (5.9), the Peclet number is

$$\mathrm{Pe}_L = \frac{u_{f,\infty} L}{\alpha_f}.$$

For mercury, from Table C.23 at $T = 300$ K, $\alpha_f = 4.70 \times 10^{-6}$ m^2/s. Then

$$\begin{aligned}
\mathrm{Pe}_L &= 8{,}511 \\
\mathrm{Nu}_L(x = L) &= 52.05 \\
q_{ku} &= -46{,}115 \text{ W/m}^2 \\
\delta_\alpha &= 7.805 \text{ mm.}
\end{aligned}$$

COMMENT:

Liquid metal flow makes for very effective surface-convection heat transfer. Also, liquids are more effective than gases in surface-convection heat transfer. Finally, treating mercury as a Pr = 0 fluid results in a q_{ku} which is within 10(hydrodynamic) boundary layer formed on the plate does not influence the surface-convection heat transfer when Pr is very small.

PROBLEM 6.3.FAM

GIVEN:

The top surface of a microprocessor chip, which is modeled as a semi-infinite plate, is to be cooled by forced, parallel flow of (i) air, or (ii) liquid Refrigerant-12. The idealized surface is shown in Figure Pr.6.3. The effect of the surfaces present upstream of the chip can be neglected.

Evaluate the properties at $T = 300$ K.

SKETCH:

Figure Pr.6.3 shows the surface of a microprocessor chip subjected to parallel flow. The thermal boundary-layer thickness is also shown.

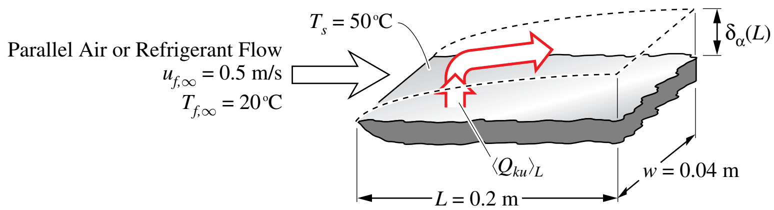


Figure Pr.6.3 Surface of a microprocessor is cooled by a semi-bounded fluid stream.

OBJECTIVE:

(a) Determine the surface-convection heat transfer rate $\langle Q_{ku}\rangle_L$(W).

(b) Determine the thermal boundary-layer thickness at the tail edge of the chip δ_α(mm).

SOLUTION:

(a) The Reynolds number is defined in (6.45) as

$$\mathrm{Re}_L = \frac{u_{f,\infty}L}{\nu_f}.$$

For $\mathrm{Re}_L < 5 \times 10^5$, the flow regime is laminar. The averaged Nusselt number (averaged over L) for laminar, parallel flow over a semi-infinite flat plate is given by (6.51), i.e.,

$$\langle \mathrm{Nu}\rangle_L = 0.664\mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/3}.$$

For the turbulent regime ($\mathrm{Re}_L > 5 \times 10^5$), the averaged Nusselt number (averaged over L) for parallel flow over a semi-infinite flat plate is given by (6.67), i.e.,

$$\langle \mathrm{Nu}\rangle_L = (0.037\mathrm{Re}_L^{4/5} - 871)\mathrm{Pr}^{1/3}.$$

(b) The thermal boundary-layer thickness for laminar flow, at $x = L$, is given by (6.48), i.e.,

$$\delta_\alpha(x = L) = 5\frac{L}{(\mathrm{Re}_L)^{1/2}}\frac{1}{\mathrm{Pr}^{1/3}}$$

and for turbulent flow δ_α is given by (6.66), i.e.,

$$\delta_\alpha(x = L) = 0.37\frac{L}{(\mathrm{Re}_L)^{1/5}}\frac{1}{\mathrm{Pr}^{1/3}}.$$

From (6.149), the average surface-convection resistance is

$$\langle R_{ku}\rangle_L = \frac{L}{A_{ku}k_f\langle \mathrm{Nu}\rangle_L},$$

where the surface area is $A_{ku} = wL$.

The averaged surface-convection heat transfer from the plate is given by (6.49), i.e.,

$$\langle Q_{ku}\rangle_L = \frac{(T_s - T_{f,\infty})}{\langle R_{ku}\rangle_L}.$$

The thermophysical properties are evaluated at $T = 300$ K from Tables C.22 and C.23.
Table Pr.6.3 lists the thermophysical properties for the fluids and the numerical results obtained for $\langle Q_{ku} \rangle_L$ and $\delta_\alpha(x = L)$,

Table Pr.6.3 Thermophysical properties for the fluids and numerical results.

Fluid	ν_f, m^2/s	k_f, W/m-K	Pr	Re_L	$\delta_\alpha(x = L)$, mm	$\langle Nu \rangle_L$	$\langle R_{ku} \rangle_L$, K/W	$\langle Q_{ku} \rangle_L$, W
air	15.66×10^{-6}	0.0267	0.69	6,386 (laminar)	14.16	46.89	19.97	1.502
R-12	0.195×10^{-6}	0.072	3.5	5.128×10^5 (turbulent)	3.515	754.9	0.4599	65.23

COMMENT:

Air has a larger boundary-layer thickness than liquid R-12. The Nusselt number and the heat transfer rate are larger for liquid R-12.

PROBLEM 6.4.FUN

GIVEN:

As discussed in Section 6.2.5, the stream function ψ, for a two-dimensional, laminar fluid flow (u_f, v_f), expressed in the Cartesian coordinate (x, y), is defined through

$$u_f \equiv \frac{\partial \psi}{\partial y}, \quad v_f \equiv -\frac{\partial \psi}{\partial x}.$$

OBJECTIVE:

Show that this stream function satisfies the continuity equation (6.37).

SOLUTION:

The continuity equation for laminar incompressible flow, in two dimensions, using the Cartesian coordinates, is given by (6.37), i.e.,

$$\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} = 0.$$

Substituting the above, we have

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0.$$

Thus, the definition of ψ given above automatically satisfies the continuity equation (6.37).

COMMENT:

With no need to include the continuity equation in the analysis, the momentum equation is used to determine ψ. This is investigated in the next problem.

PROBLEM 6.5.FUN

GIVEN:

As discussed in Section 6.2.5, the two-dimensional, (x, y), (u_f, v_f) , laminar steady viscous, boundary-layer momentum equation (6.36) can be reduced to an ordinary differential equation using a dimensionless similarity variable

$$\eta \equiv y \left(\frac{u_{f,\infty}}{\nu_f x} \right)^{1/2}$$

and a dimensionless stream function

$$\psi^* \equiv \frac{\psi}{(\nu_f u_{f,\infty} x)^{1/2}}, \; u_f \equiv \frac{\partial \psi}{\partial y}, \; v_f \equiv -\frac{\partial \psi}{\partial x}.$$

OBJECTIVE:

(a) Show that the momentum equation (6.36) reduces to

$$2\frac{d^3\psi^*}{d\eta^3} + \psi^* \frac{d^2\psi^*}{d\eta^2} = 0.$$

This is called the Blasius equation.

(b) Show that energy equation (6.35) reduces to

$$\frac{d^2 T_f^*}{d\eta^2} + \frac{1}{2}\mathrm{Pr}\psi^* \frac{dT_f^*}{d\eta} = 0, \quad T_f^* = \frac{T_f - T_{f,\infty}}{T_s - T_{f,\infty}}.$$

SOLUTION:

(a) We start with (6.36), written as

$$u_f \frac{\partial u_f}{\partial x} + v_f \frac{\partial u_f}{\partial y} - \nu_f \frac{\partial^2 u_f}{\partial y^2} = 0.$$

Using the stream function ψ, the dimensionless stream function ψ^* and the similarity variable η, we transform u_f and v_f into ψ^*, x and η. We start with

$$\begin{aligned} u_f \equiv \frac{\partial \psi}{\partial y} &= \frac{d\psi}{d\eta}\frac{\partial \eta}{\partial y} = \frac{d\psi^*}{d\eta}(\nu_f u_{f,\infty} x)^{1/2} \left(\frac{u_{f,\infty}}{\nu_f x} \right)^{1/2} \\ &= u_{f,\infty}\frac{d\psi^*}{d\eta} \end{aligned}$$

or

$$\frac{u_f}{u_{f,\infty}} = \frac{d\psi^*}{d\eta}.$$

Also,

$$\begin{aligned} v_f \equiv -\frac{\partial \psi}{\partial x} &= -\frac{\partial}{\partial x}[(\nu_f u_{f,\infty} x)^{1/2}\psi^*] = -\left[(\nu_f u_{f,\infty} x)^{1/2}\frac{\partial \psi^*}{\partial x} + \frac{1}{2}(\frac{\nu_f u_{f,\infty}}{x})^{1/2}\psi^* \right] \\ &= \frac{1}{2}\left(\frac{\nu_f u_{f,\infty}}{x} \right)^{1/2} \left(\eta \frac{d\psi^*}{d\eta} - \psi^* \right) \end{aligned}$$

or

$$\frac{v_f}{\left(\frac{\nu_f u_{f,\infty}}{x} \right)^{1/2}} = \frac{1}{2}\left(\eta\frac{d\psi^*}{d\eta} - \psi^* \right).$$

Next these velocity components are differentiated with respect to x and y, and we have

$$\begin{aligned}\frac{\partial u_f}{\partial x} &= -\frac{u_{f,\infty}}{2x}\eta\frac{d^2\psi^*}{d\eta^2}\\ \frac{\partial u_f}{\partial y} &= u_{f,\infty}\left(\frac{u_{f,\infty}}{\nu_f x}\right)^{1/2}\frac{d^2\psi^*}{d\eta^2}\\ \frac{\partial^2 u_f}{\partial y^2} &= \frac{u_{f,\infty}^2}{\nu_f x}\frac{d^3\psi^*}{d\eta^3}.\end{aligned}$$

Substituting these into the above momentum equation, we have

$$-u_{f,\infty}\frac{\partial\psi^*}{\partial\eta}\left(\frac{u_{f,\infty}}{2x}\eta\frac{d^2\psi^*}{d\eta^2}\right)+\frac{1}{2}\left(\frac{\nu_f u_{f,\infty}}{x}\right)^{1/2}\left(\eta\frac{d\psi^*}{d\eta}-\psi^*\right)u_{f,\infty}\left(\frac{u_{f,\infty}}{\nu_f x}\right)^{1/2}\frac{d^2\psi^*}{d\eta^2}-\nu_f\frac{u_{f,\infty}^2}{\nu_f x}\frac{d^3\psi^*}{d\eta^3}=0$$

or

$$\frac{u_{f,\infty}}{x}\left(-\frac{1}{2}\psi^*\frac{d^2\psi^*}{d\eta^2}-\frac{d^3\psi^*}{d\eta^3}\right)=0$$

or

$$2\frac{d^3\psi^*}{d\eta^3}+\psi^*\frac{d^2\psi^*}{d\eta^2}=0.$$

(b) Starting from (6.35), we have

$$u_f\frac{\partial T_f}{\partial x}+v_f\frac{\partial T_f}{\partial y}-\alpha_f\frac{\partial^2 T_f}{\partial y^2}=0.$$

We already have the expression for u_f and v_f from part (a), then

$$\begin{aligned}\frac{\partial T_f}{\partial x} &= \frac{dT_f}{d\eta}\frac{\partial\eta}{\partial x}=-\frac{y}{2}\left(\frac{u_{f,\infty}}{\nu_f x^3}\right)^{1/2}\frac{dT_f}{d\eta}=\frac{\eta}{2x}\frac{dT_f}{d\eta}\\ \frac{\partial T_f}{\partial y} &= \left(\frac{u_{f,\infty}}{\nu_f x}\right)^{1/2}\frac{dT_f}{d\eta}\\ \frac{\partial^2 T_f}{\partial y^2} &= \frac{u_{f,\infty}}{\nu_f x}\frac{d^2T_f}{d\eta^2}.\end{aligned}$$

Substituting these into the energy equation, and using T_f^*, we have

$$\begin{aligned}u_{f,\infty}\frac{d\psi^*}{d\eta}\left[-\frac{y}{2}\left(\frac{u_{f,\infty}}{\nu_f x^3}\right)^{1/2}\frac{dT_f^*}{d\eta}\right]+\frac{1}{2}\left(\frac{\nu_f u_{f,\infty}}{x}\right)^{1/2}\left(\eta\frac{d\psi^*}{d\eta}-\psi^*\right)\left(\frac{u_{f,\infty}}{\nu_f x}\right)^{1/2}\frac{dT_f^*}{d\eta}-\alpha_f\frac{u_{f,\infty}}{\nu_f x}\frac{d^2T_f^*}{d\eta^2}&=0\\ -\frac{1}{2}\frac{u_{f,\infty}}{x}\psi^*\frac{dT_f^*}{d\eta}-\alpha_f\frac{u_{f,\infty}}{\nu_f x}\frac{d^2T_f^*}{d\eta^2}&=0\\ \frac{d^2T_f^*}{d\eta^2}+\frac{1}{2}\mathrm{Pr}\psi^*\frac{dT_f^*}{d\eta}&=0.\end{aligned}$$

COMMENT:

Also note that outside the boundary layer, i.e., when $u_f = u_{f,\infty}$, we have

$$\frac{u_f}{u_{f,\infty}}=\frac{d\psi^*}{d\eta}=1,\quad d\psi^*=d\eta\quad\text{for}\quad u_f=u_{f,\infty}.$$

Then outside the boundary layer, we have

$$\begin{aligned}\frac{v_f}{\left(\frac{\nu_f u_{f,\infty}}{x}\right)^{1/2}} &= \frac{1}{2}\left(\eta\frac{d\psi^*}{d\eta}-\psi^*\right)\\ &= \frac{1}{2}(\eta-\psi^*)\end{aligned}$$

This would tend to a constant as η becomes large. This constant is

$$\frac{v_f}{\left(\frac{\nu_f u_{f,\infty}}{x}\right)^{1/2}} = 0.86054$$

Note that the momentum equation written in terms of velocity (u, v) is second order in y, while the use of the stream function results in a third-order differential equation.

PROBLEM 6.6.FUN

GIVEN:

The third-order, ordinary Blasius differential equation

$$2\frac{d^3\psi^*}{d\eta^3} + \psi^*\frac{d^2\psi^*}{d\eta^2} = 0,$$

subject to surface and far-field mechanical conditions

$$\text{at } \eta = 0: \quad \frac{d\psi^*}{d\eta} = \psi^* = 0$$
$$\text{for } \eta \to \infty: \quad \frac{d\psi^*}{d\eta} = 1.$$

Note that with an initial-value problem solver, such as SOPHT, the second derivative of ψ^* at $\eta = 0$ must be guessed. This guess is adjusted till $d\psi^*/d\eta$ becomes unity for large η.

Hint: $d^2\psi^*/d\eta^2(\eta = 0)$ is between 0.3 to 0.4.

OBJECTIVE:

Use a solver to integrate the dimensionless transformed boundary-layer momentum equation.

Plot ψ^*, $d\psi^*/d\eta = u_f/u_{f,\infty}$, and $d^2\psi^*/d\eta^2$, with respect to η.

SOLUTION:

The solver we choose is an initial-value solver, such as SOPHT, where the initial values (i.e., at $\eta = 0$) for ψ^*, $d\psi^*/d\eta$, and $d^2\psi^*/d\eta^2$ must be provided for this third-order, ordinary differential equation.
Therefore, in place of the condition for $\eta \to \infty$, we choose

$$\text{guess}: \frac{d^2\psi^*}{d\eta^2} = \text{constant at} \quad \eta = 0 \quad \text{such that} \quad \frac{d\psi^*}{d\eta} = 1 \quad \text{for } \eta \to \infty.$$

Note that we can write the Blausius equation as a set of first-order differential equations, i.e,

$$\begin{aligned} g' &= -\frac{1}{2}fg \\ z' &= g \\ f' &= z, \end{aligned}$$

here $g' = d^3\psi^*/d\eta^3, z' = d^2\psi^*/d\eta^2$, and $f' = d\psi/d\eta$.
The initial conditions are

$$\begin{aligned} f(\eta = 0) &= 0 \\ z(\eta = 0) &= 0 \\ g(\eta = 0) &= 0.332 \quad \text{after iterating to get } z(\eta \to \infty) = 1. \end{aligned}$$

The results are plotted in Figure Pr.6.6.

The results show that the streamwise velocity $u_f/u_{f,\infty}$ increases and reaches a value of unity at $\eta \simeq 5$. The stream function ψ^* increase monotonically, while $d^2\psi^*/d\eta^2$ decrease and vanishes at $\eta \simeq 5$.

COMMENT:

The results are sensitive to the initial choice for $d^2\psi^*/d\eta^2$ at $\eta = 0$. Because of the similarity between the momentum and energy equations, (6.35) and (6.36), this derivative is the same as the temperature derivative and therefore 0.332 is also the constant appearing in the solution (6.44) for the surface fluid conduction heat transfer rate (i.e., surface convection).

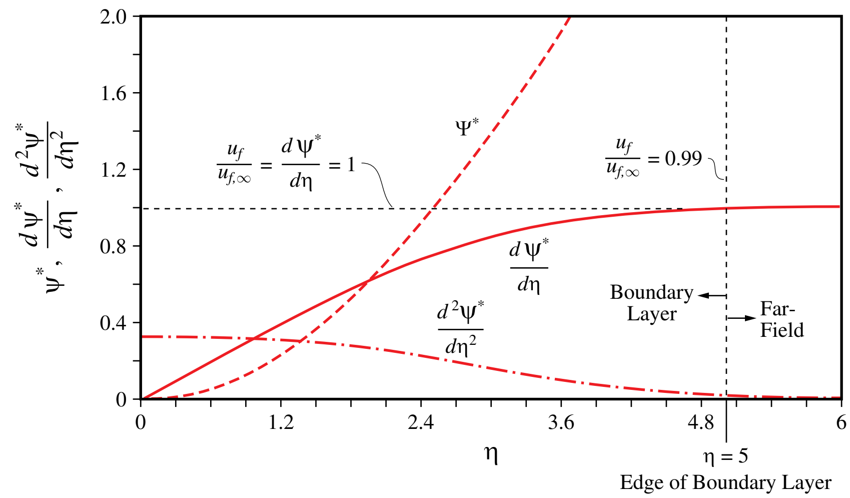


Figure Pr.6.6 Variations of ψ^*, $d\psi^*/d\eta$, and $d^2\psi^*/d\eta^2$ with respect to η.

PROBLEM 6.7.FAM

GIVEN:

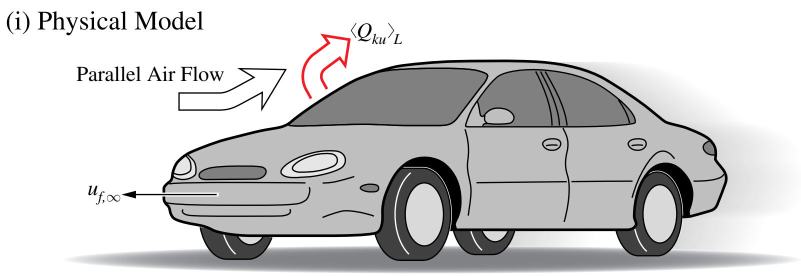
During part of the year, the automobile windshield window is kept at a temperature significantly different than that of the ambient air. Assuming that the flow and heat transfer over the windshield can be approximated as those for parallel flow over a semi-infinite, flat plate, examine the role of the automobile speed on the surface-convection heat transfer from the window. These are shown in Figure Pr.6.7.

The ambient air is at −10°C and the window surface is at 10°C. The window is 1 m long along the flow direction and is 2.5 m wide.

Use the average temperature between the air and the window surface to evaluate the thermophysical properties of the air.

SKETCH:

Figures Pr.6.7(i) and (ii) show an automobile windshield window and its idealization as a semi-infinite plate.



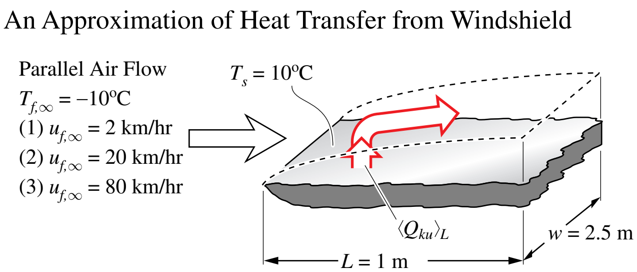


Figure Pr.6.7 (i) Fluid flow and heat transfer over an automobile windshield window.
(ii) Its idealization as parallel flow over a semi-infinite plate.

OBJECTIVE:

(a) To the end of automobile speed on the surface-convection heat transfer from the window, determine the average Nusselt number $\langle \text{Nu} \rangle_L$.

(b) Determine the average surface-convection thermal resistance $A_{ku}\langle R_{ku}\rangle_L$[°C/(W-m^2)].

(c) Determine the surface-averaged rate of surface-convection heat transfer $\langle Q_{ku}\rangle_L$(W).

Consider automobile speeds of 2, 20, and 80 km/hr. Comment on the effects of the flow-regime transition and speed on the surface-convection heat transfer.

SOLUTION:

(a) The Reynolds number is given by (6.45), i.e.,

$$\text{Re}_L = \frac{u_{f,\infty} L}{\nu_f}.$$

For $\text{Re}_L < 5 \times 10^5$ the flow regime is laminar. The average Nusselt number (averaged over L) for laminar, parallel flow over a flat plate is given by (6.51), i.e.,

$$\langle \text{Nu} \rangle_L = 0.664 \text{Re}_L^{1/2} \text{Pr}^{1/3}.$$

For $\mathrm{Re}_L > 5 \times 10^5$ the flow regime is turbulent and the averaged Nusselt number is given by (6.67), i.e.,

$$\langle \mathrm{Nu} \rangle_L = (0.037\mathrm{Re}_L^{4/5} - 871)\mathrm{Pr}^{1/3}.$$

(b) The average surface-convection thermal resistance is calculated from (6.49), i.e.,

$$A_{ku}\langle R_{ku} \rangle_L = \frac{L}{k_f \langle \mathrm{Nu} \rangle_L}.$$

(c) The averaged surface-convection heat transfer is then obtained from (6.49) as

$$\langle Q_{ku} \rangle_L = A_{ku}\frac{(T_s - T_{f,\infty})}{A_{ku}\langle R_{ku} \rangle_L}.$$

The thermophysical properties of air are obtained from Table C.22. For the average temperature $T_\delta = (T_s + T_{f,\infty})/2 = 273.15$ K, we have $k_f = 0.0251$ W/m-K, $\nu_f = 13.33 \times 10^{-6}$ m^2/s, and Pr = 0.69.
Table Pr.6.7 lists the numerical results obtained for the three vehicle different speeds.

Table Pr.6.7 Numerical results obtained for the three different speeds.

$u_{f,\infty}$, m/s	Re_L	flow regime	$\langle \mathrm{Nu} \rangle_L$ °C/(W/m^2)	$A_{ku}\langle R_{ku} \rangle_L$, W	$\langle Q_{ku} \rangle_L$,
0.5556	0.4168×10^5	laminar	119.8	0.3326	150.3
5.556	4.168×10^5	laminar	378.8	0.1052	475.4
22.22	16.67×10^5	turbulent	2,335	0.0171	2,930

COMMENT:

As the vehicle speed increases, the flow regime changes from laminar to turbulent. The Nusselt number for the turbulent regime is larger than that for the laminar regime. This effect, associated with the increase in the Reynolds number, causes the total heat transfer to increase by more than one order of magnitude, when the vehicle speed is changed only by a factor of four.

PROBLEM 6.8.FAM.S

GIVEN:

A square flat surface with side dimension $L = 40$ cm is at $T_s = 120$℃. It is cooled by a parallel air flow with far-field velocity $u_{f,\infty}$ and far-field temperature $T_{f,\infty} = 20$℃.

OBJECTIVE:

(a) Use a solver (such as SOPHT) to plot the variation of the averaged surface-convection heat transfer rate $\langle Q_{ku}\rangle_L$(W) with respect to $u_{f,\infty}$(m/s) from zero up to the sonic velocity. Use (3.20) to find the sonic velocity.
(b) Determine the air velocity needed to obtain $\langle q_{ku}\rangle_L = 1{,}200$ W/m^2.

SOLUTION:

(a) The average surface-convection heat transfer rate is given by (6.49) as

$$\langle Q_{ku}\rangle_L = \frac{T_s - T_{f,\infty}}{\langle R_{ku}\rangle_L},$$

where the average surface-convection resistance is also given by (6.49) as

$$\langle R_{ku}\rangle_L = \frac{L}{A_{ku} k_f \langle \text{Nu}\rangle_L}.$$

For $\text{Re}_L < 5 \times 10^5$ the flow regime is laminar. The averaged Nusselt number (averaged over L) for laminar, parallel flow over a flat plate is given by (6.51) and in Table 6.3 as

$$\langle \text{Nu}\rangle_L = 0.664\text{Re}_L^{1/2}\text{Pr}^{1/3}.$$

For the turbulent regime, the averaged Nusselt number (averaged over L) for parallel flow over a flat plate is given by (6.67) and in Table 6.3 as

$$\langle \text{Nu}\rangle_L = (0.037\text{Re}_L^{4/5} - 871)\text{Pr}^{1/3}.$$

The Reynolds number is given by (6.45) as

$$\text{Re}_L = \frac{u_{f,\infty} L}{\nu_f}.$$

The fluid properties are evaluated at the film temperature, given by $(T_s + T_{f,\infty})/2 = 70$℃. From Table C.22, interpolation gives $\nu_f = 19.66 \times 10^{-6}\text{m}^2/\text{s}, k_f = 0.0295$ W/m-K and Pr = 0.69. For the given conditions, the transition from laminar to turbulent flow then occurs at $u_{f,\infty} = 24.60$ m/s.
The speed of sound, assuming that air behaves as an ideal gas, is given by (3.20), i.e.,

$$a_s = \left(\frac{c_p}{c_v}\frac{R_u}{M_g}T_{f,\infty}\right)^{1/2} = (kRT_{f,\infty})^{1/2}$$

Where k for air is 1.4, $R = 287$J/kg-K and $T_{f,\infty}$ is in Kelvin. This gives

$$a_s = [1.4 \times 287(\text{J/kg-K}) \times 293.15(\text{K})]^{1/2} = 343 \text{ m/s}.$$

A plot of the surface-convection heat transfer rate as a function of the air speed is shown in Figure Pr.6.8.

(b) From the numerical data, for $\langle q_{ku}\rangle = \langle Q_{ku}\rangle_L / A_{ku} = 1200$ W/m^2, we obtain $u_{f,\infty} = 3.78$ m/s. For an analytic answer, the plot obtained could be used to identify that the desired heat flux lies in the laminar flow regime, and then the appropriate equation for $\langle Q_{ku}\rangle_L$ could be solved for $u_{f,\infty}$.

COMMENT:

Note the sudden rise in the rate of increase of the surface-convection heat transfer rate as the turbulent regime is entered. It is important to note that we have neglected any kind of transition region. In a real flow, the transition from laminar to turbulent flow would take place over a range of the Reynolds number. As the sonic speed is reached, the compressibility of the gas should be included in the $\langle \text{Nu}\rangle_l$ correlation.

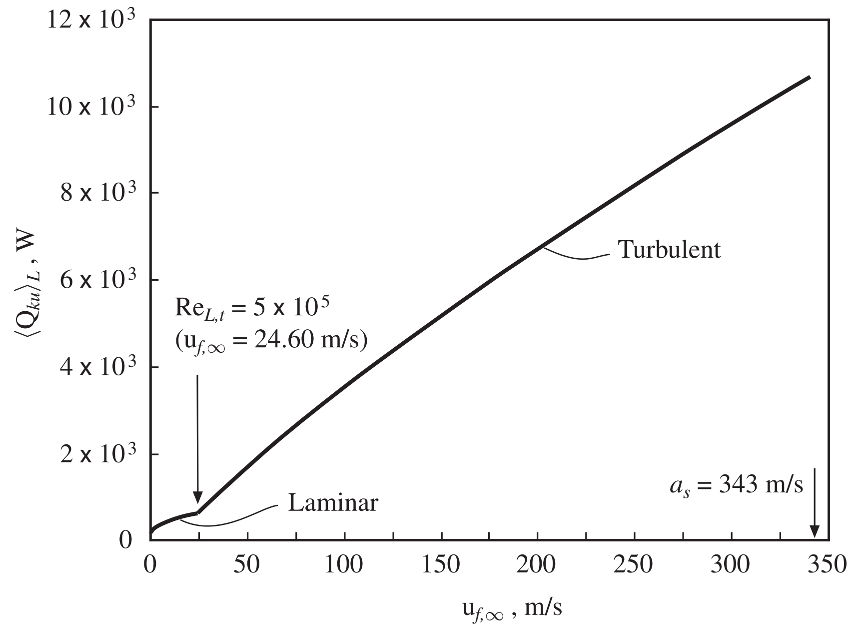


Figure Pr.6.8 Variation of the surface-convection heat transfer rate with respect to the far-field velocity.

PROBLEM 6.9.FAM

GIVEN:

On a clear night, a water layer formed on a paved road can freeze due to radiation heat losses to the sky. The water and the pavement are at the freezing temperature $T_1 = 0$°C. The water surface behaves as a blackbody and radiates to the deep sky at an apparent temperature of $T_{sky} = 250$ K. The ambient air flows parallel and over the water layer at a speed $u_{f,\infty} = 9$ m/s and temperature $T_{f,\infty}$, which is greater than the water temperature.

Assume that the surface convection is modeled using a surface that has a length $L = 2$ m along the flow and a width $w = 1$ m (not shown in the figure) perpendicular to the flow. These are shown in Figure Pr.6.9(a).

Neglect the heat transfer to the pavement and evaluate the air properties at $T = 273.15$ K (Table C.22).

SKETCH:

Figure Pr.6.9(a) shows the thin water layer exposed to a warm air stream and a cold radiation sink.

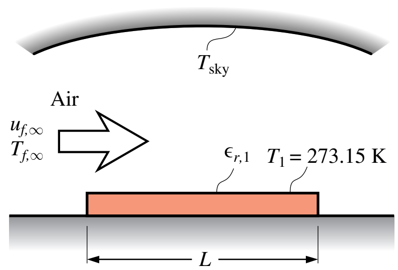


Figure Pr.6.9(a) Radiation cooling of a thin water film and its surface-convection heating.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the maximum ambient temperature below which freezing of the water layer occurs.

(c) When a given amount of salt is added to the water or ice, the freezing temperature drops by 10°C. If the water surface is now at -10°C, will freezing occur? Use the property values found for (b), the given $u_{f,\infty}$ and T_sky, and the freestream air temperature found in (b).

SOLUTION:

(a) To find the maximum $T_{f,\infty}$ above which melting will occur, we will assume that the water layer is at a uniform, lumped temperature. The thermal circuit is then shown in Figure Pr.6.9(b).

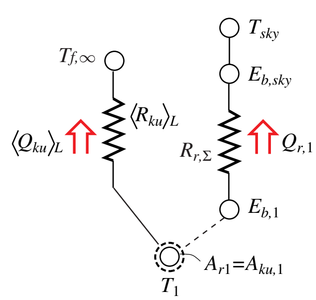


Figure Pr.6.9(b) Thermal circuit diagram.

(b) From Figure Pr.6.9(b), by applying the conservation of energy principle to node T_1 for steady-state conditions, we have

$$\begin{aligned} Q\mid_A &= 0 \\ \langle Q_{ku}\rangle_L + Q_{r,1} &= 0. \end{aligned}$$

We then evaluate each of the heat transfer terms as
(i) Heat Transfer By Surface Convection:
From Table C.22 for air at $T = 273.15$ K, $k_f = 0.0251$ W/m-K, $\nu_f = 13.33 \times 10^{-6}$ m^2/s, and Pr = 0.69. For parallel flow over a flat plate, we have

$$\mathrm{Re}_L = \frac{u_{f,\infty}L}{\nu_f} = \frac{9(\mathrm{m/s}) \times 2(\mathrm{m})}{13.33 \times 10^{-6}(\mathrm{m^2/s})} = 1.350 \times 10^6, \quad \text{turbulent flow regime.}$$

From Table 6.3, for combined laminar-turbulent flow,

$$\begin{aligned} \langle \mathrm{Nu}\rangle_L &= (0.037\mathrm{Re}_L^{4/5} - 871)\mathrm{Pr}^{1/3} = [0.037(1.350 \times 10^6)^{4/5} - 871](0.69)^{1/3} = 1{,}854 \\ \langle R_{ku}\rangle_L &= \frac{L}{A_{ku}\langle \mathrm{Nu}\rangle_L k_f} = \frac{2(\mathrm{m})}{2(\mathrm{m}) \times 1(\mathrm{m}) \times 1{,}854 \times 0.0251(\mathrm{W/m\text{-}K})} = 0.0215℃/\mathrm{W} \\ \langle Q_{ku}\rangle_L &= \frac{T_1 - T_{f,\infty}}{\langle R_{ku}\rangle_L}. \end{aligned}$$

(ii) Heat Transfer By Surface Radiation:
The water surface and the night sky are assumed to behave as black bodies, $\epsilon_{r,sky} = \epsilon_{r,1} = 1$, and the view factor from the water to the sky is $F_{1\text{-}sky} = 1$. Then

$$\begin{aligned} R_{r,\Sigma} &= \left(\frac{1-\epsilon_r}{\epsilon_r A_r}\right)_1 + \frac{1}{A_{r,1}F_{1\text{-}sky}} + \left(\frac{1-\epsilon_r}{\epsilon_r A_r}\right)_{sky} = 0 + \frac{1}{A_{r,1}F_{1\text{-}sky}} + 0 = \frac{1}{2(\mathrm{m^2})} = 0.5\ 1/\mathrm{m^2} \\ Q_{r,1} &= \frac{E_{b,1} - E_{b,sky}}{R_{r,\Sigma}} = \frac{\sigma_{\mathrm{SB}}(T_1^4 - T_{sky}^4)}{R_{r,\Sigma}}. \end{aligned}$$

Then from Figure Pr.6.9(b), the energy equation for node T_1 is (for no net heat transfer or for maximum $T_{f,\infty}$)

$$\frac{T_1 - T_{f,\infty}}{\langle R_{ku}\rangle_L} + \frac{E_{b,1} - E_{b,sky}}{R_{r,\Sigma}} = 0.$$

Solving for $T_{f,\infty}$

$$\begin{aligned} T_{f,\infty} &= T_1 + \langle R_{ku}\rangle_L \frac{\sigma_{\mathrm{SB}}(T_1^4 - T_{sky}^4)}{R_{r,\Sigma}} \\ &= 273.15 + (0.0215)(℃/\mathrm{W})\frac{5.67 \times 10^{-8}(\mathrm{W/m^2\text{-}K}) \times (273.15^4 - 250^4)(\mathrm{K^4})}{0.5(1/\mathrm{m^2})} \\ &= 277.20\ \mathrm{K} = 4.05℃. \end{aligned}$$

(c) For $T_{sl} = -10℃ = 263.15$ K, we repeat the above determinations of $\langle Q_{ku}\rangle_L$ and $Q_{r,1}$.

$$\begin{aligned} \langle Q_{ku}\rangle_L &= \frac{(263.15 - 277.20)(\mathrm{K})}{0.0215} = -653.5\ \mathrm{W}, \\ Q_{r,1} &= \frac{5.67 \times 10^{-8}(\mathrm{W/m^2\text{-}K^4}) \times (263.15^4 - 250^4)(\mathrm{K^4})}{0.5(1/\mathrm{m^2})} = 100.8\ \mathrm{W}. \end{aligned}$$

Since $\mid \langle Q_{ku}\rangle_L \mid > \mid Q_{r,w} \mid$, the ice will melt.

COMMENT:

It is possible to cool a body below the ambient temperature using radiation heat transfer.

PROBLEM 6.10.DES

GIVEN:

A square flat plate, with dimensions $a \times a$, is being heated by a thermal plasma (for a coating process) on one of its sides. To prevent meltdown and assist in the coating process, the other side is cooled by impinging air jets. This is shown in Figure Pr.6.10. In the design of the jet cooling, a single, large-diameter nozzle [Figure Pr.6.10(i)], or nine smaller diameter nozzles [Figure Pr.6.10(ii)] are to be used.

$a = 30$ cm, $T_s = 400$℃, $T_{f,\infty} = 20$℃.

Single nozzle: $D = 3$ cm, , $L_n = 6$ cm, $L = 15$ cm, $\langle u_f \rangle_A = 1$ m/s.

Multiple nozzles: $D = 1$ cm, $L_n = 2$ cm, $L = 5$ cm, $\langle u_f \rangle_A = 1$ m/s.

Use the average temperature between the air and the surface to evaluate the properties of the air.

SKETCH:

Figure Pr.6.10 shoes a single and a nine jet arrangement for cooling of a flat surface.

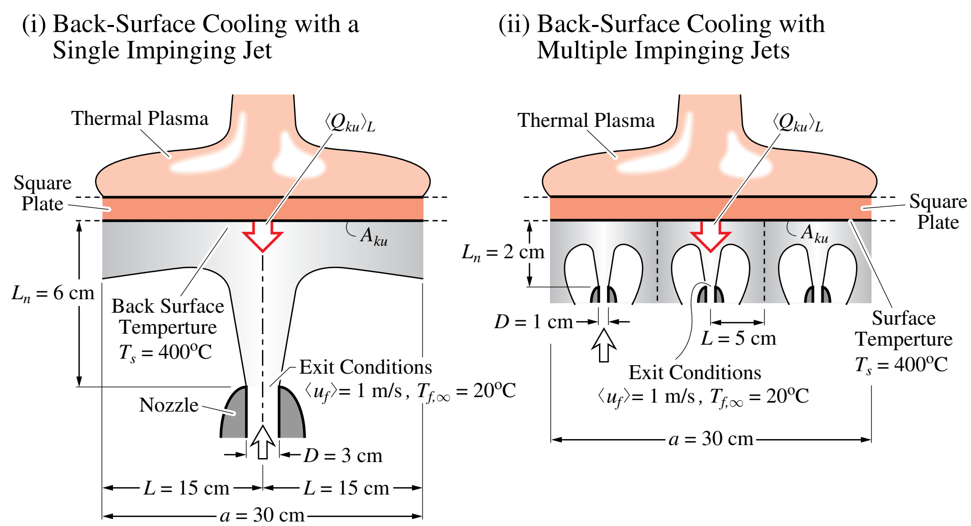


Figure Pr.6.10 (i) A single impinging jet used for surface cooling. (ii) Multiple impinging jets.

OBJECTIVE:

For each design, do the following:

(a) Determine the average Nusselt number $\langle \text{Nu} \rangle_L$.

(b) Determine the average surface-convection thermal resistance $A_{ku}\langle R_{ku} \rangle_L$[℃/(W/m^2)].

(c) Determine the rate of surface-convection heat transfer $\langle Q_{ku} \rangle_L$(W).

SOLUTION:

(i) Single Nozzle:

(a) The average Nusselt number for cooling with a single, round nozzle is given by (6.71), i.e.,

$$\langle \text{Nu} \rangle_L = 2\text{Re}_D^{1/2}\text{Pr}^{0.42}(1 + 0.005\text{Re}_D^{0.55})^{1/2}\frac{1 - 1.1D/L}{1 + 0.1(L_n/D - 6)D/L},$$

where the Reynolds number is based on the nozzle diameter and is given by (6.68), i.e.,

$$\text{Re}_D = \frac{\langle u_f \rangle_A D}{\nu_f}.$$

Equation (6.71) is valid for $L/D > 2.5$. For the nozzle given, $L/D = 15/3 = 5$, thus satisfying this constraint The properties for air at $T_\delta = (400 + 20)/2 = 210$℃ $= 483$ K are, from Table C.22, $k_f = 0.0384$ W/m-K, $\nu_f = 35.29 \times 10^{-6}$ m^2/s, and $\text{Pr} = 0.69$. The area for surface convection is $A_{ku} = a^2$.

Then Reynolds number becomes

$$\text{Re}_D = \frac{\langle u_f \rangle_A D}{\nu_f} = \frac{1(\text{m/s}) \times 0.03(\text{m})}{35.29 \times 10^{-6}(\text{m}^2/\text{s})} = 850.1.$$

The average Nusselt number, from (6.71) becomes

$$\langle \text{Nu} \rangle_L = 46.43.$$

(b) The average surface-convection thermal resistance is obtained from (6.49), i.e.,

$$A_{ku}\langle R_{ku}\rangle_L = \frac{L}{k_f \langle \text{Nu} \rangle_L} = \frac{0.15(\text{m})}{0.0384(\text{W/m-K}) \times 46.43} = 8.413 \times 10^{-2}\ ^\circ\text{C}/(\text{W/m}^2).$$

(c) The averaged surface-convection heat transfer is obtained from (6.49) as

$$\langle Q_{ku}\rangle_a = A_{ku}\frac{(T_s - T_{f,\infty})}{A_{ku}\langle R_{ku}\rangle_L} = (0.3)^2(\text{m})^2 \times \frac{400(^\circ\text{C}) - 20(^\circ\text{C})}{8.413 \times 10^{-2}[^\circ\text{C}/(\text{W/m}^2)]} = 406.5 \text{ W}.$$

(ii) Square Array of Multiple Nozzles:
(a) The average Nusselt number for cooling with a square array of round nozzles is given by (6.72), i.e.,

$$\langle \text{Nu} \rangle_L = \text{Re}_D^{2/3}\text{Pr}^{0.42}\frac{2L}{D}\left\{1 + \left[\frac{L_n/D}{\frac{0.6}{(1-\epsilon)^{1/2}}}\right]^6\right\}^{-0.05}(1-\epsilon)^{1/2}\frac{1 - 2.2(1-\epsilon)^{1/2}}{1 + 0.2(L_n/D - 6)(1-\epsilon)^{1/2}}.$$

This is valid for $L/D > 1.25$. For the nozzles, $L/D = 5/1 = 5$, thus satisfying this constraint.
The Reynolds number becomes

$$\text{Re}_D = \frac{u_{f,\infty}D}{\nu_f} = \frac{1(\text{m/s})0.01(\text{m})}{35.29 \times 10^{-6}(\text{m}^2/\text{s})} = 283.4.$$

The void fraction defined by (6.73) is

$$\epsilon = 1 - \frac{\pi D^2}{16L^2} = 0.9921.$$

The average Nusselt number becomes

$$\langle \text{Nu} \rangle_L = 28.35.$$

(b) The average surface-convection thermal resistance becomes

$$A_{ku}\langle R_{ku}\rangle_L = \frac{L}{k_f \langle \text{Nu} \rangle_L} = \frac{0.05(\text{m})}{0.0384(\text{W/m-K}) \times 28.35} = 4.593 \times 10^{-2}\,^\circ\text{C}/(\text{W/m}^2).$$

(c) The averaged surface-convection heat transfer can then be obtained from

$$\langle Q_{ku}\rangle_a = A_{ku}\frac{(T_s - T_{f,\infty})}{A_{ku}\langle R_{ku}\rangle_L} = (0.3)^2(\text{m})^2 \times \frac{400(^\circ\text{C}) - 20(^\circ\text{C})}{4.593 \times 10^{-2}[^\circ\text{C}/(\text{W/m}^2)]} = 744.6 \text{ W}.$$

COMMENT:

Note that under these conditions the single nozzle removes slightly more heat from the surface. However, the multiple nozzles results in a more uniform surface cooling.

PROBLEM 6.11.FUN

GIVEN:

Permanent damage occurs to the pulp of a tooth initially at $T(t=0) = 37°C$, when it reaches a temperature $T_p = 41°C$. Therefore, to prevent nerve damage, a water coolant must be constantly applied during many standard tooth drilling operations. In one such operation, a drill, having a frequency $f = 150$ Hz, a burr diameter $D_b = 1.2$ mm, a tooth contact area $A_c = 1.5 \times 10^{-7}$ m^2, and a coefficient of friction between the drill burr and the tooth $\mu_F = 0.4$, is used to remove an unwanted part of the tooth. The contact force between the drill burr and the tooth is $F = 0.05$ N. During the contact time, heat is generated by surface friction heating. In order to keep the nerves below their threshold temperature, the tooth surface must be maintained at $T_s = 45°C$ by an impinging jet that removes 80% of the generated heat. The distance between the jet and the surface L_n is adjustable.

Use the dimensions shown in Figure Pr.6.11(a)(ii).

$T_{f,\infty} = 20°C$, $\langle u_f \rangle = 0.02$ m/s, $D = 1.5$ mm, $L = 4$ mm, $\mu_F = 0.4$ Pa-s, $f = 150$ 1/s, $\Delta u_i = 2\pi f R_b$, $p_c = F_c/A_c$, $F_c = 0.05$ N, $A_c = 1.5 \times 10^{-7}$ m^2, $D_b = 2R_b = 1.2$ mm.

Use the same surface area for heat generation and for surface convection (so surface area A_{ku} will not appear in the final expression used to determine L_n).

Determine the water properties at $T = 293$ K.

SKETCH:

Figure Pr.6.11(a) shows the water-jet cooling of a tooth during drilling.

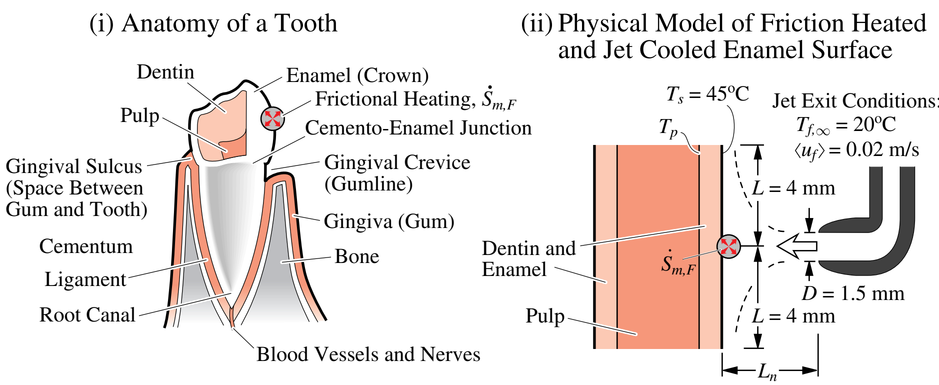


Figure Pr.6.11(a)(i)Cooling of tooth during drilling. (ii) Physical model of friction heated and jet cooled enamel surface.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Write the surface energy equation for the tooth surface.

(c) Determine the location L_n of the jet that must be used in order to properly cool the tooth.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.6.11(b).

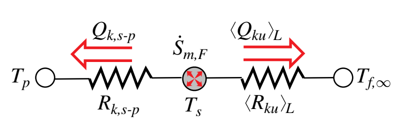


Figure Pr.6.11(b) Thermal circuit diagram.

(b) From Figure Pr.6.11(b), we have

$$Q|_A = Q_{ku} = A_{ku} q_{ku} = \sum_i \dot{S}_i = 0.8\dot{S}_{m,F}.$$

(c) The heat generated by surface friction heating, from (2.53), is

$$\dot{S}_{m,F}/A_{ku} = \mu_F p_c \Delta u_i, \quad \Delta u_i = 2\pi f R_b = 2\pi f \frac{D_b}{2}$$

where

$$\begin{aligned} p_c &= \frac{F_c}{A_c} = \frac{0.05(\text{N})}{1.5\times 10^{-7}(\text{m}^2)} = 3.333\times 10^5 \text{ N/m}^2 \\ \Delta u_i &= 2\pi f R_b = 2\pi f \frac{D_b}{2} = 2\pi \times 150(1/\text{s}) \times \frac{1.2\times 10^{-3}}{2}(\text{m}) \\ &= 0.5655 \text{ m/s}. \end{aligned}$$

Then,

$$\begin{aligned} \dot{S}_{m,F}/A_{ku} &= 0.4 \times 3{,}333 \times 10^6 (\text{N/m}^2) \times 0.5655(\text{m/s}) \\ &= 75{,}391 \text{ W/m}^2. \end{aligned}$$

From the energy equation, after dropping A_{ku}, we have

$$\begin{aligned} \langle q_{ku}\rangle_L &= 0.8 \times \frac{\dot{S}_{m,F}}{A_{ku}} = 0.8 \times 75{,}400(\text{W/m}^2) \\ &= 60{,}313 \text{ W/m}^2. \end{aligned}$$

Next, we use the Nusselt number for $\langle q_{ku}\rangle_L$ using (6.49), i.e.,

$$\langle Q_{ku}\rangle_L = A_{ku}\langle \text{Nu}\rangle_L \frac{k_f}{L}(T_s - T_{f,\infty})$$

$$\langle q_{ku}\rangle_L = \langle \text{Nu}\rangle_L \frac{k_f}{L}(T_s - T_{f,\infty}).$$

Properties (water, $T = 293$ K, Table C.23): interpolated values; $k_f = 0.595$ W/m-K, Pr = 7.528, $\nu_f = 106.7\times 10^{-8}$ $\text{m}^2/2$. Using $T_s = 45$℃, $T_{f,\infty} = 20$℃, $L = 4$ mm, we have

$$\begin{aligned} 60{,}313(\text{W/m-K}) &= \langle \text{Nu}\rangle_L \times \frac{0.595(\text{W/m-K})}{0.004(\text{m})} \times (45-20)(\text{K}) \\ \langle \text{Nu}\rangle_L &= 16.22. \end{aligned}$$

The Nusselt number relation is found from Table 6.3, i.e.,

$$\text{Nu}_L = 2\text{Re}^{1/2}\text{Pr}^{0.42}(1 + 0.005\text{Re}^{0.55})^{1/2} \frac{1 - 1.1\dfrac{D}{L}}{1 + 0.1\left(\dfrac{L_n}{D} - 6\right)\dfrac{D}{L}}$$

where,

$$\text{Re}_D = \frac{\langle u_f\rangle_A D}{\nu_f} = \frac{0.02(\text{m/s}) \times 0.0015(\text{m})}{106.7\times 10^{-8}(\text{m}^2/\text{s})} = 28.12.$$

Then

$$
\begin{aligned}
\mathrm{Nu}_L &= 2 \times (28.12)^{1/2} \times (7.528)^{0.42} \times [1 + 0.005 \times (28.12)^{0.55}]^{1/2} \times \\
&\quad \frac{1 - 1.1 \times \dfrac{1.5(\mathrm{cm})}{4(\mathrm{cm})}}{1 + 0.1 \times \left[\dfrac{L_n}{0.0015(\mathrm{m})} - 6\right] \times \dfrac{0.0015(\mathrm{m})}{0.004(\mathrm{m})}} \\
&= 25.14 \times \frac{0.5875}{1 + (666.7L_n - 6) \times 0.03750)} \\
&= 25.14 \times \frac{0.5875}{25L_n + 0.775} \\
&= \frac{14.77}{25L_n + 0.775}.
\end{aligned}
$$

Solving for L_n in the above relation, we have

$$
\begin{aligned}
16.22 &= \frac{14.77}{25L_n + 0.775} \\
405.5L_n + 12.57 &= 14.77 \\
L_n &= 0.005425 \text{ m} = 0.5425 \text{ cm}.
\end{aligned}
$$

COMMENT:

The presence of the drill in the impinging jet area is neglected. This is a reasonable nozzle to surface distance.

PROBLEM 6.12.FAM

GIVEN:

Heat-activated, dry thermoplastic adhesive films are used for joining surfaces. The adhesive film can be heated by rollers, hot air, radio-frequency and microwaves, or ultrasonics. Consider a flat fabric substrate to be coated with a polyester adhesive film with the film, heated by a hot air jet, as shown in Figure Pr.6.12(a). The film is initially at $T_1(t=0)$. The thermal set temperature is T_{sl}.

Assume that the surface-convection heat transfer results in the rise in the film temperature with no other heat transfer.

$L_n = 4$ cm, $L = 10$ cm, $D = 1$ cm, $\langle u_f \rangle = 1$ m/s, $l = 0.2$ mm, $T_{f,\infty} = 200$°C, $T_{sl} = 120$°C.

Determine the air properties at $T = 350$ K. For polyester, use Table C.17 and the properties of polystyrene.

SKETCH:

Figure Pr.6.12(a) shows the thin plastic film heated by a hot-air jet.

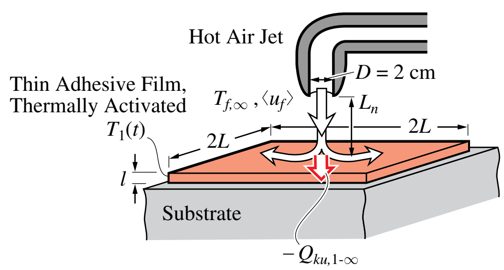


Figure Pr.6.12(a) A heat-activated adhesive film heated by a hot air jet.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the elapsed time needed to reach T_{sl}, for the conditions given above.

SOLUTION:

(a) Figure Pr.6.12(b) shows the thermal circuit diagram. The only heat transfer is assumed to be $Q_{ku,1\text{-}\infty}$, i.e., no heat losses are allowed.

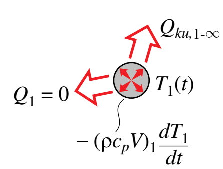


Figure Pr.6.12(b) Thermal circuit diagram.

(b) Assuming a uniform temperature for the film, the transient temperature is given by (6.156), i.e.,

$$T_1(t=0) = T_{f,\infty} + [T_1(t=0) - T_{f,\infty}]e^{-t/\tau_1} + a_1\tau_1(1 - e^{-t/\tau_1}),$$

where

$$a_1 = \frac{\dot{S}_1 - Q_1}{(\rho c_p V)_1} = 0, \quad \tau_1 = (\rho c_p V)_1 \langle R_{ku} \rangle_L.$$

The surface-convection resistance is given by (6.49) as

$$\langle R_{ku} \rangle_L = \frac{L}{A_{ku} \langle Nu \rangle_L k_f}$$

$$A_{ku} = 2L \times 2L$$

and $\langle \mathrm{Nu} \rangle_L$ is given in Table 6.3 as

$$\langle \mathrm{Nu} \rangle_L = 2\mathrm{Re}_D^{1/2}\mathrm{Pr}^{0.42}(1 + 0.005\mathrm{Re}_D^{0.55})^{1/2}\frac{1 - 1.1D/L}{1 + 0.1(L_n/D - 6)D/L},$$

$$\mathrm{Re}_D = \frac{\langle u_f \rangle D}{\nu_f}.$$

From Table C.22, for air at $T = 350$ K, we have

$$\nu_f = 2.030 \times 10^{-5} \quad \mathrm{m^2/s} \qquad \text{Table C.22}$$

$$k_f = 0.0300 \ \mathrm{W/m\text{-}K} \qquad \text{Table C.22}$$

$$\mathrm{Pr} = 0.69 \qquad \text{Table C.22.}$$

Then

$$\mathrm{Re}_D = \frac{1(\mathrm{m/s}) \times 0.02(\mathrm{m})}{2.030 \times 10^{-5}(\mathrm{m^2/s})} = 985.2$$

$$\langle \mathrm{Nu} \rangle_L = 2 \times (985.2)^{1/2} \times (0.69)^{0.42}(1 + 0.005 \times (985.2)^{0.55})^{1/2} \times \frac{1 - 1.1 \times 0.02(\mathrm{m})/0.1(\mathrm{m})}{1 + 0.1\{[0.04(\mathrm{m})/0.02(\mathrm{m})] - 6\} \times [0.02(\mathrm{m})/0.10(\mathrm{m})]},$$

$$= 2 \times 26.86 \times 1.105 \times 1.006 \times \frac{0.78}{0.92} = 50.33.$$

Then

$$\langle R_{ku} \rangle_L = \frac{0.10(\mathrm{m})}{(0.20)^2(\mathrm{m^2}) \times 50.33 \times 0.030(\mathrm{W/m\text{-}K})}$$

$$= 1.656 \ \mathrm{K/W}$$

$$V_1 = 2L \times 2L \times l = (2 \times 0.10)^2 \times 2 \times 10^{-4}(\mathrm{m^3}) = 8.0 \times 10^{-6} \ \mathrm{m^3}.$$

For polystyrene, from Table C.17, we have

$$\rho_1 = 1{,}050 \ \mathrm{kg/m^3} \qquad \text{Table C.17}$$

$$c_p = 1{,}800 \ \mathrm{J/kg\text{-}K} \qquad \text{Table C.17.}$$

Then

$$\tau_1 = 1{,}050(\mathrm{kg/m^3}) \times 1{,}800(\mathrm{J/kg\text{-}K}) \times 8 \times 10^{-6}(\mathrm{m^3}) \times 1.656(\mathrm{K/W})$$

$$= 25.04 \ \mathrm{s}.$$

We can solve the temperature $T_1(t)$ expression for t, since $a_1 = 0$, and we have, with $T_1(t) = T_{sl}$

$$t = \tau_1 \ln \left[\frac{T_1(t) - T_{f,\infty}}{T_1(t = 0) - T_{f,\infty}} \right]$$

$$= -25.04(\mathrm{s}) \times \ln \left[\frac{(120 - 200)(^\circ\mathrm{C})}{(20 - 200)(^\circ\mathrm{C})} \right] = 20.31 \ \mathrm{s}.$$

COMMENT:

Note that we have neglected the heat losses from the film to the substrate by conduction and to the surroundings by surface radiation. Inclusion of these would be increase the elapsed time. Note that we need slightly less than one time constant (τ_1) to reach the desired temperature.

PROBLEM 6.13.DES

GIVEN:

A pure aluminum plate is to be rapidly cooled from $T_1(t=0) = 40$℃ to $T_1(t) = 20$℃. The plate has a length $L = 12$ cm and thickness $w = 0.2$ cm. The plate is to be cooled using water by placing it at a distance $L_n = 10$ cm from a faucet with diameter $D = 2$ cm. The water leaves the faucet at a temperature $T_{f,\infty} = 5$℃ and velocity $\langle u \rangle_A = 1.1$ m/s. There are two options for the placement of the plate with respect to the water flow. The plate can be placed vertically, so the water flows parallel and on both sides of the plate. Then the water layer will have a thickness $l = 2$ mm on each side of the plate [shown in Figure Pr.6.13(a)(i)]. Alternately, it can be placed horizontally with the water flowing as a jet impingement [shown in Figure Pr.6.13(a)(ii)].

Assume that the results for impinging jets can be used here, even though the jet fluid (water) is not the same as the ambient (air) fluid. Use the water as the only fluid present. Assume a uniform plate temperature.

Estimate the parallel, far-field velocity $u_{f,\infty}$ using the mass flow rate out of the faucet. This approximate flux is assumed uniform over the rectangular flow cross section ($l \times 2L$) and is assumed to be flowing over a square surface ($2L \times 2L$).

SKETCH:

Figure Pr.6.13(a) shows the plate to be cooled by the water from a faucet. Two different plate orientations are considered.

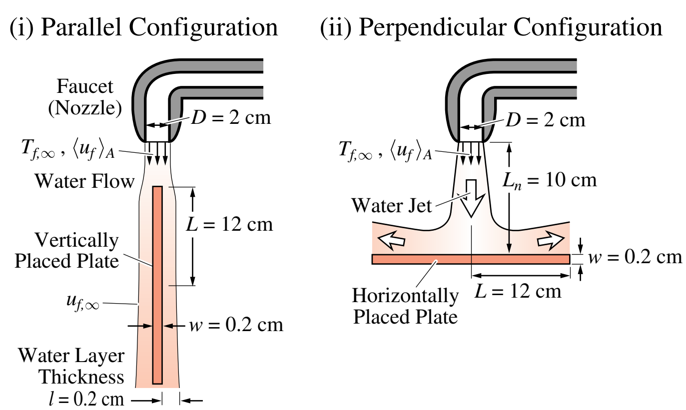


Figure Pr.6.13(a) Cooling of an aluminum plate under a faucet. (i) Parallel configuration. (ii) Perpendicular configuration.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine which of the orientations gives the shorter cooling time.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.6.13(b).

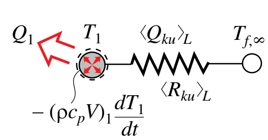


Figure Pr.6.13(b) Thermal circuit diagram.

(b) (i) Parallel Flow:
The mass flow rate of water out of the faucet is

$$\dot{M}_f = \rho_f \langle u_f \rangle A_u = \rho_f \langle u_f \rangle \frac{\pi D^2}{4}.$$

Properties: (water at $T = 280$ K, Table C.23): $k_f = 0.5675$ W/m-K, $c_p = 4{,}204$ J/kg-K, $\nu_f = 152 \times 10^{-8}$ m^2/s, Pr = 11.36; (pure aluminum at $T = 300$ K, Table C.16): $\rho_s = 2{,}702$ kg/m^3, $c_{p,s} = 903$ J/kg-K, $k_s = 237$ W/m-K.
This flow is approximately divided on both side of the plate, i.e.,

$$\dot{M}_f \equiv \rho_f 2 \times (l \times 2L) \times u_{f,\infty}.$$

Then, equating the two expressions for $\dot{M}_f$ gives,

$$\begin{aligned} u_{f,\infty} &= \langle u_f \rangle \frac{\pi D^2/4}{2(l \times 2L)} \\ &= 1.1(\text{m/s}) \times \frac{\pi (0.02)^2(\text{m}^2)/4}{2 \times (0.002)(\text{m}) \times 2 \times (0.12)(\text{m})} \\ &= 0.36 \text{ m/s}. \end{aligned}$$

we determine the elapsed time needed to cool the plate, using (6.156), i.e.,

$$\begin{aligned} T_1(t) - T_{f,\infty} &= [T_1(t=0) - T_{f,\infty}]e^{-t/\tau_1} + a_1\tau_1(1 - e^{-t/\tau_1}) \\ \tau_1 &= (\rho c_p V)_1 \langle R_{ku} \rangle_L, \quad a_1 = \frac{\dot{S}_1 - Q_1}{(\rho c_p V)_1}. \end{aligned}$$

Since there is no energy conversion and all other forms of heat transfer are assumed negligible, both $\dot{S}_1$ and Q_1 equal to zero. Then $a_1 = 0$, or,

$$T_1(t) = T_{f,\infty} = [T_1(t=0) - T_{f,\infty}]e^{-t/\tau_1}, \ \tau_1 = (\rho c_p V)_1 \langle R_{ku} \rangle_L.$$

The Reynolds number is

$$\text{Re}_L = \frac{u_{f,\infty} 2L}{\nu_f}, \quad 2L = 24 \text{ cm} = 0.24 \text{ m}$$

$$\text{Re}_L = \frac{0.36(\text{m/s}) \times (0.24)(\text{m})}{152 \times 10^{-8}(\text{m}^2/\text{s})} = 56{,}842 < \text{Re}_{L,t} = 5 \times 10^5.$$

The flow remains laminar over the plate.
From (6.49), we have

$$\langle R_{ku} \rangle_L = \frac{2L}{A_{ku} \langle \text{Nu} \rangle_L k_f},$$

where

$$A_{ku} = 2L^2 = 2(0.12)^2(\text{m}^2) = 0.0288 \text{ m}^2,$$

since there are two sides to the plate.
From Table 6.3, we have

$$\langle \text{Nu} \rangle_L = 0.664 \text{Re}_L^{1/2} \text{Pr}^{1/3} = 0.664(56{,}842)^{1/2}(11.36)^{1/3} = 355.9.$$

Then

$$\langle R_{ku} \rangle_L = \frac{0.24(\text{m})}{0.0288(\text{m}^2) \times 355.9 \times 0.5675(\text{W/m-K})} = 0.04126 \text{ K/W}.$$

Next,

$$\begin{aligned}
V_1 &= L^2 w, \quad w = 0.2 \text{ cm} = 0.002 \text{ m} \\
V_1 &= (0.12)^2(\text{m}^2) \times (0.002)(\text{m}) = 2.88 \times 10^{-5} \text{ m}^3 \\
\tau_1 &= (\rho c_p V)_1 \langle R_{ku} \rangle_L = [2{,}702(\text{kg/m}^2) \times 903(\text{J/kg-K}) \times 2.88 \times 10^{-5}(\text{m}^3)] \times 0.04126(\text{K/W}) \\
&= 2.899 \text{ s.}
\end{aligned}$$

Then

$$t = -\tau_1 \ln \left[\frac{T_1(t) - T_{f,\infty}}{T_1(t=0) - T_{f,\infty}} \right] = -2.899(\text{s}) \times \ln \left(\frac{20-5}{40-5} \right) = 2.456 \text{ s.}$$

(ii) Perpendicular Flow:
Again, we have

$$T_1(t) - T_{f,\infty} = [T_1(t=0) - T_{f,\infty}] e^{-t/\tau_1}, \quad \tau_1 = (\rho c_p V)_1 \langle R_{ku} \rangle_L$$

The nozzle Reynolds number is

$$\begin{aligned}
\text{Re}_D &= \frac{\langle u_f \rangle D}{\nu_f}, \quad D = 2 \text{ cm} = 0.02 \text{ m} \\
&= \frac{1.1(\text{m/s}) \times (0.02)(\text{m})}{152 \times 10^{-8}(\text{m}^2/\text{s})} = 14{,}474.
\end{aligned}$$

Also,

$$\begin{aligned}
\frac{L}{D} &= \frac{12(\text{cm})}{2(\text{cm})} = 6 \\
\langle R_{ku} \rangle_L &= \frac{L}{A_{ku} \langle \text{Nu} \rangle_L k_f} \\
A_{ku} &= L^2 = (0.12)^2(\text{m}^2) = 0.0144 \text{ m}^2.
\end{aligned}$$

From Table 6.3, we have

$$\begin{aligned}
\langle \text{Nu} \rangle_L &= 2\text{Re}_D^{1/2} \text{Pr}^{0.42} (1 + 0.005\text{Re}_D^{0.55})^{1/2} \frac{1 - \dfrac{1.1D}{L}}{1 + 0.1 \left(\dfrac{L_n}{D} - 6 \right) \dfrac{D}{L}} \\
&= 2 \times (14{,}474)^{1/2} \times (11.36)^{0.42} \times [1 + 0.005(14{,}474)^{0.55}]^{1/2} \times \\
&\quad \frac{1 - \dfrac{1.1 \times 0.02(\text{m})}{0.12(\text{m})}}{1 + 0.1 \times \left[\dfrac{0.1(\text{m})}{0.02(\text{m})} - 6 \right] \times \dfrac{0.02(\text{m})}{0.12(\text{m})}} \\
&= 667.7 \times 1.404 \times \frac{0.8167}{0.9833} = 778.6.
\end{aligned}$$

Then

$$\langle R_{ku} \rangle_L = \frac{L}{A_{ku} \langle \text{Nu} \rangle k_f} = \frac{0.12}{0.0144(\text{m}^2) \times (778.6) \times 0.5675(\text{W/m-K})} = 0.01189 \text{ K/W.}$$

Next

$$\begin{aligned}
\tau_1 &= (\rho c_p V)_1 \langle R_{ku} \rangle_L \\
&= (2702(\text{kg/m}^3) \times 903(\text{J/kg-K}) \times 2.88 \times 10^{-5}) \times 0.01189(\text{K/W}) = 1.325 \text{ s.}
\end{aligned}$$

Then

$$t = -\tau_1 \ln \left[\frac{T_1(t) - T_{f,\infty}}{T_1(t=0) - T_{f,\infty}} \right] = -1.325(\text{s}) \times \ln \left(\frac{20-5}{40-5} \right) = 1.123 \text{ s.}$$

This cooling time is less than that for the parallel arrangement, therefore the perpendicular orientation should be used for rapid cooling.

COMMENT:

In order to verify the lumped capacitance assumption, we must show that the Biot number is much less than one (for both flows). The Biot number is defined by (6.128) as

$$\text{Bi}_w = \frac{R_k}{\langle R_{ku} \rangle_L}.$$

For aluminum, from Table C.14, at $T = 300$ K, we have $k_s = 237$ W/m-K.
(i) Parallel Flow:

$$\begin{aligned}\text{Bi}_w &= \frac{w/A_k k_s}{\langle R_{ku} \rangle_L} \\ &= \frac{0.002(\text{m})/[0.0144(\text{m}^2) \times 237(\text{W/m-K})]}{0.04126(\text{K/W})} = 0.0142.\end{aligned}$$

(ii) Perpendicular Flow: The conduction resistance is the same, and therefore,

$$\text{Bi}_w = \frac{5.86 \times 10^{-4}(\text{K/W})}{0.001149(\text{K/W})} = 0.0493.$$

In both cases, the Biot number is much less than one and the lumped capacitance assumption is therefore valid.

PROBLEM 6.14.FAM

GIVEN:

A bottle containing a cold beverage is awaiting consumption. During this period, the bottle can be placed vertically or horizontally, as shown in Figure Pr.6.14. Assume that the bottle can be treated as a cylinder of diameter D and length L. We wish to compare the surface-convection heat transfer to the bottle when it is (i) standing vertically or (ii) placed horizontally. For the vertical position, the surface-convection heat transfer is approximated using the results of the vertical plate, provided that the boundary-layer thickness δ_α is much less than the bottle diameter D.

$D = 10$ cm, $L = 25$ cm, $T_s = 4$℃, $T_{f,\infty} = 25$℃.

Neglect the end areas. Use the average temperature between the air and the surface to evaluate the thermophysical properties of the air.

SKETCH:

Figure Pr.6.14 shows two positions of a beverage bottle.

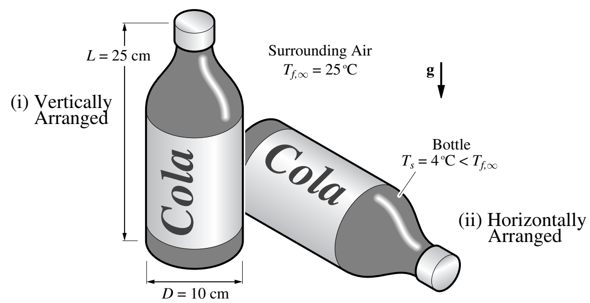


Figure Pr.6.14 Thermobuoyant flow and heat transfer from beverage bottles. (i) Standing vertically. (ii) Placed horizontally

OBJECTIVE:

(a) Determine the average Nusselt numbers $\langle \text{Nu} \rangle_L$ and $\langle \text{Nu} \rangle_D$.

(b) Determine the average surface-convection thermal resistances $A_{ku}\langle R_{ku}\rangle_L$ [℃/(W/m^2)] and $A_{ku}\langle R_{ku}\rangle_D$ [℃/(W/m^2)].

(c) Determine the rates of surface-convection heat transfer $\langle Q_{ku}\rangle_L$(W) and $\langle Q_{ku}\rangle_D$(W).

SOLUTION:

(i) Vertical Position:

(a) The Rayleigh number is given by (6.88) as

$$\text{Ra}_L = \frac{g\beta(T_s - T_{f,\infty})L^3}{\nu_f \alpha_f}.$$

The properties for air at $T_\delta = (T_s + T_{f,\infty})/2 = 288$ K are obtained from Table C.22: $k_f = 0.026$ W/m-K, $\nu_f = 14.60 \times 10^{-6}$ m^2/s, Pr = 0.69, and from (6.77) we have $\beta_f = 1/T_{ave} = 3.472 \times 10^{-3}$ 1/K.

With $\nu_f \alpha_f = \nu_f^2/\text{Pr}$, the Rayleigh number becomes

$$\text{Ra}_L = \frac{9.81(\text{m}^2/\text{s}) \times 3.472 \times 10^{-3}(1/\text{K}) \times (25-4)(\text{K}) \times (0.25)^3(\text{m})^3}{(14.60 \times 10^{-6})^2(\text{m}^2/\text{s})^2/(0.69)} = 3.618 \times 10^7.$$

Since $\text{Ra}_L < 10^9$, from (6.91) the flow is laminar. For thermobuoyant flow over a vertical flat plate, the average Nusselt number is obtained from (6.92) as

$$\begin{aligned}
a_1 &= \frac{4}{3}\frac{0.503}{\left[1+(\frac{0.492}{\text{Pr}})^{9/16}\right]^{4/9}} = 0.5131 \\
\text{Nu}_{L,l} &= \frac{2.8}{\ln\left[1+\frac{2.8}{a_1\text{Ra}_L^{1/4}}\right]} = 41.18 \\
\text{Nu}_{L,t} &= \frac{0.13\text{Pr}^{0.22}}{(1+0.61\text{Pr}^{0.81})^{0.42}}\text{Ra}_L^{1/3} = 33.88 \\
\langle\text{Nu}\rangle_D &= [(\text{Nu}_{L,l})^6 + (\text{Nu}_{L,t})^6]^{1/6} = 43.08.
\end{aligned}$$

(b) The average surface-convection thermal resistance is found from (6.49) as

$$A_{ku}\langle R_{ku}\rangle_L = \frac{L}{k_f\langle\text{Nu}\rangle_L} = \frac{0.25(\text{m})}{0.026(\text{W/m-K})\times 43.08} = 2.232\times 10^{-1}\,^\circ\text{C}/(\text{W/m}^2).$$

(c) The surface-averaged surface-convection heat transfer is found from (6.49) as

$$\langle Q_{ku}\rangle_L = A_{ku}\frac{T_s - T_{f,\infty}}{A_{ku}\langle R_{ku}\rangle_L} = \pi\times\ 0.1(\text{m})\times 0.25(\text{m})\times\frac{4(^\circ\text{C}) - 25(^\circ\text{C})}{2.232\times 10^{-1}[^\circ\text{C}/(\text{W/m}^2)]} = -7.390\text{ W}.$$

(ii) Horizontal Position:
(a) For the horizontal cylinder, the Rayleigh number is found from Table 6.4, i.e.,

$$\text{Ra}_L = \frac{g\beta(T_s - T_{f,\infty})D^3}{\nu_f\alpha_f} = \frac{9.81(\text{m}^2/\text{s})\times 3.472\times 10^{-3}(1/\text{K})\times[25(^\circ\text{C}) - 4(^\circ\text{C})](0.1)^3(\text{m})^3}{(14.60\times 10^{-6})^2(\text{m}^2/\text{s})^2/(0.69)} = 2.315\times 10^6.$$

A correlation for the average Nusselt number for a horizontal cylinder is given in Table 6.4. Using the values given

$$\begin{aligned}
a_1 &= \frac{4}{3}\frac{0.503}{\left[1+(\frac{0.492}{\text{Pr}})^{9/16}\right]^{4/9}} = 0.5131 \\
\text{Nu}_{D,l} &= \frac{1.6}{\ln\left[1+\frac{1.6}{0.772a_1\text{Ra}_D^{1/4}}\right]} = 16.24 \\
\text{Nu}_{D,t} &= \frac{0.13\text{Pr}^{0.22}}{(1+0.61Pr^{0.81})^{0.42}}\text{Ra}_D^{1/3} = 13.55 \\
\langle\text{Nu}\rangle_D &= [(\text{Nu}_{D,l})^{3.3} + (\text{Nu}_{D,t})^{3.3}]^{1/3.3} = 18.55.
\end{aligned}$$

(b) The average surface-convection thermal resistance is

$$A_{ku}\langle R_{ku}\rangle_D = \frac{D}{k_f\langle\text{Nu}\rangle_D} = \frac{0.10(\text{m})}{0.026(\text{W/m-K})\times 18.55} = 2.073\times 10^{-1}\,^\circ\text{C}/(\text{W/m}^2).$$

(c) The averaged surface-convection heat transfer is

$$\langle Q_{ku}\rangle_D = A_{ku}\frac{T_s - T_{f,\infty}}{A_{ku}\langle R_{ku}\rangle_D} = \pi\times 0.1(\text{m})\times 0.25(\text{m})\times\frac{4(^\circ\text{C}) - 25(^\circ\text{C})}{2.073\times 10^{-1}[^\circ\text{C}/(\text{W/m}^2)]} = -7.956\text{ W}.$$

COMMENT:
For the vertical plate, since $\text{Ra}_L < 10^9$, the flow regime is laminar. For the laminar thermobuoyant flow over a vertical flat plate, the average Nusselt number could also be determined from using the similar relation (6.89), i.e.,

$$\langle\text{Nu}\rangle_L = \frac{4}{3}\frac{0.503}{\left[1+(\frac{0.492}{\text{Pr}})^{9/16}\right]^{4/9}}\text{Ra}_L^{1/4} = 39.79$$

and the average surface-convection thermal resistance and heat transfer rate are

$$A_{ku}\langle R_{ku}\rangle_L = \frac{L}{k_f\langle \mathrm{Nu}\rangle_L} = \frac{0.25(\mathrm{m})}{0.026(\mathrm{W/m\text{-}K}) \times 39.79} = 2.416 \times 10^{-1}\,^\circ\mathrm{C}/(\mathrm{W/m^2})$$

$$\langle Q_{ku}\rangle_L = A_{ku}\frac{T_s - T_{f,\infty}}{A_{ku}\langle R_{ku}\rangle_L} = \pi \times 0.1(\mathrm{m}) \times 0.25(\mathrm{m}) \times \frac{4(^\circ\mathrm{C}) - 25(^\circ\mathrm{C})}{2.42 \times 10^{-1}[^\circ\mathrm{C}/(\mathrm{W/m^2})]} = -6.826\ \mathrm{W}.$$

These are considered close to the values obtained using the combined laminar-turbulent correlation. Note also that the horizontal position results in a slightly larger heat flow.
Also, note that determining $\delta_\alpha(L)$ from (6.90) we have

$$\begin{aligned}\delta_\alpha(L) &= 3.93L\left(\mathrm{Pr} + \frac{20}{21}\right)^{1/4}(\mathrm{Gr}_L^{1/2}\mathrm{Pr})^{-1/2} \\ &= 0.01574\mathrm{m},\end{aligned}$$

where Gr_L=Ra_L/Pr.
Then

$$\frac{\delta_\alpha(L)}{D} = \frac{0.01574(\mathrm{m})}{0.10(\mathrm{m})} = 0.1574,$$

which satisfies the needed constraint that $\delta_\alpha(L) \ll D$.

PROBLEM 6.15.FAM

GIVEN:

The fireplace can provide heat to the room through surface convection and surface radiation from that portion of the fireplace wall heated by the combustion products exiting through a chimney behind the wall. This heated area is marked on the fireplace wall in Figure Pr.6.15(a). Assume this portion of the wall (including the fireplace) is maintained at a steady, uniform temperature T_s. The surface convection is by a thermobuoyant flow that can be modeled as the flow adjacent to a heated vertical plate with length L. The surface radiation exchange is between this heated portion of the wall and the remaining surfaces in the room. Assume that all the remaining wall surfaces are at a steady uniform temperature T_w.

$T_s = 32$℃, $T_{f,\infty} = 20$℃, $T_w = 20$℃, $\epsilon_{r,s} = 0.8$, $\epsilon_{r,w} = 0.8$, $w = 3$ m, $L = 4$ m, $a = 6$ m.

Determine the air properties at 300 K (Table C.22).

SKETCH:

Figure Pr.6.15(a) shows the surface convection and radiation from a portion of the fireplace wall to the rest of the room.

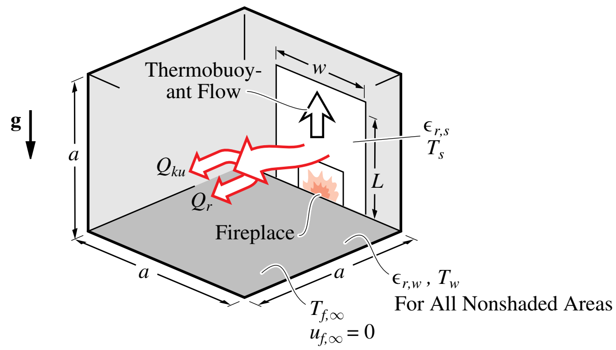


Figure Pr.6.15(a) Surface convection and radiation from a heated wall.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine the surface-convection heat transfer rate.
(c) Determine the surface-radiation heat transfer rate.
(d) Assume the fire provides an energy conversion rate due to combustion of $\dot{S}_{r,c} = 19{,}500$ W. (This would correspond to a large 5 kg log of wood burning at a constant rate to total consumption in one hour.) Determine the efficiency of the fireplace as a room-heating system (efficiency is defined as the ratio of the total surface heat transfer rate to the rate of energy conversion $\dot{S}_{r,c}$).

SOLUTION:

The thermal properties for air are evaluated at $T = 300$ K from Table C.22 and are $k_f = 0.0267$ W/m-K, $\nu_f = 15.66 \times 10^{-6}$ m^2/s, $\alpha_f = 22.57 \times 10^{-6}$ m^2/s, Pr $= 0.69$, and $\beta_f = 1/T = 1/300$ 1/K.

(a) The thermal circuit diagram is given in Figure Pr.6.15(b).

(b) The area for surface-convection is $A_{ku} = L \times w = 4(\text{m}) \times 3(\text{m}) = 12$ m^2. Then for surface convection from (6.49), we have

$$\langle Q_{ku}\rangle_L = \frac{T_s - T_\infty}{\langle R_{ku}\rangle_L}$$

where from (6.49)

$$\langle R_{ku}\rangle_L = \frac{1}{A_{ku}\langle \text{Nu}\rangle_L k_f/L}.$$

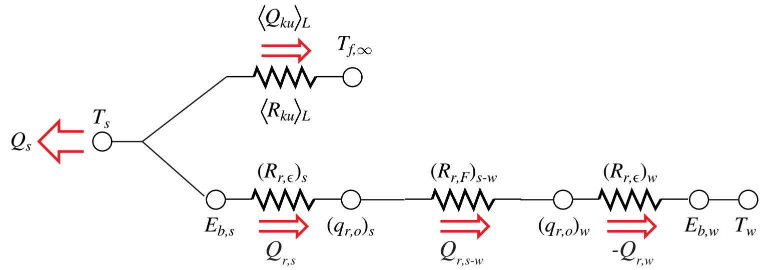


Figure Pr.6.15(b) Thermal circuit diagram.

The fluid flow for surface-convection is modeled as a thermobuoyant flow over a flat vertical plate. The $\langle \mathrm{Nu} \rangle_L$ is given in Table 6.4 as

$$\langle \mathrm{Nu} \rangle_L = [(\mathrm{Nu}_{L,l})^6 + (\mathrm{Nu}_{L,t})^6]^{1/6},$$

where

$$\begin{aligned} \mathrm{Nu}_{L,l} &= \frac{2.8}{\ln\left(1 + \dfrac{2.8}{a_1 \mathrm{Ra}_L^{1/4}}\right)} \\ \mathrm{Nu}_{L,t} &= \frac{0.13\mathrm{Pr}^{0.2}}{(1+0.61\mathrm{Pr}^{0.81})^{0.42}} \mathrm{Ra}_L^{1/3}. \end{aligned}$$

The Ra_L and a_1 are defined in Table 6.5 as

$$\begin{aligned} \mathrm{Ra}_L &= \frac{g\beta_f(T_s - T_\infty)L^3}{\nu_f \alpha_f} = \frac{9.81(\mathrm{m/s^2}) \times 1/300(1/\mathrm{K}) \times (32-20)(\mathrm{K}) \times (4)^3(\mathrm{m})^3}{15.66 \times 10^{-6}(\mathrm{m^2/s}) \times 22.57 \times 10^{-6}(\mathrm{m^2/s})} \\ &= 7.105 \times 10^{10} \\ a_1 &= \frac{4}{3}\frac{0.503}{[1+(0.492/\mathrm{Pr})^{9/16}]^{4/9}} \\ &= 0.5131. \end{aligned}$$

Then upon substitution, the $\mathrm{Nu}_{L,l}$ and $\mathrm{Nu}_{L,t}$ are

$$\begin{aligned} \mathrm{Nu}_{L,l} &= 266.31 \\ \mathrm{Nu}_{L,t} &= 424.33, \end{aligned}$$

Then $\langle \mathrm{Nu} \rangle_L$ then is

$$\begin{aligned} \langle \mathrm{Nu} \rangle_L &= [(266.31)^6 + (424.33)^6]^{1/6} \\ &= 428.55. \end{aligned}$$

Then $\langle R_{ku} \rangle_L$ is

$$\begin{aligned} \langle R_{ku} \rangle_L &= \frac{1}{12(\mathrm{m^2}) \times 428.55 \times 0.0267(\mathrm{W/m\text{-}^\circ C})/4(\mathrm{m})} \\ &= 0.0291^\circ\mathrm{C/W}. \end{aligned}$$

And finally, $\langle Q_{ku} \rangle_L$ is

$$\begin{aligned} \langle Q_{ku} \rangle_L &= \frac{(32-20)(^\circ\mathrm{C})}{0.0291(^\circ\mathrm{C/W})} \\ &= 411.9 \text{ W}. \end{aligned}$$

(c) The heated surface and the rest of the room constitute an enclosure and the surfaces are gray and diffuse. Then we apply the radiation enclosure analysis.
The wall area is $A_w = 5 \times a^2 = 5 \times (6^2)(\mathrm{m}^2) = 180\ \mathrm{m}^2$. Then for surface radiation exchange in a two-surface enclosure, we have from (4.47),

$$Q_{r,s\text{-}w} = \frac{E_{b,s} - E_{b,w}}{(R_{\Sigma,r})_{s\text{-}w}} = \frac{\sigma(T_s^4 - T_w^4)}{(R_{r,\epsilon})_s + (R_{r,F})_{s\text{-}w} + (R_{r,\epsilon})_w}.$$

Solving for the resistances, we have

$$(R_{r,\epsilon})_s = \frac{1-\epsilon_{r,s}}{A_s\epsilon_{r,s}} = \frac{1-0.8}{12(\mathrm{m}^2)\times 0.8} = 0.0208\ \frac{1}{\mathrm{m}^2}$$

$$F_{s\text{-}w} = 1 \qquad \text{by inspection}$$

$$(R_{r,F})_{s\text{-}w} = \frac{1}{A_sF_{s\text{-}w}} = \frac{1}{12(\mathrm{m}^2)\times 1} = 0.0833\ \frac{1}{\mathrm{m}^2}$$

$$(R_{r,\epsilon})_w = \frac{1-\epsilon_{r,w}}{A_w\epsilon_{r,w}} = \frac{1-0.8}{180(\mathrm{m}^2)\times 0.8} = 0.001389\ \frac{1}{\mathrm{m}^2}$$

$$R_{\Sigma,r} = (R_{r,\epsilon})_s + (R_{r,F})_{s\text{-}w} + (R_{r,\epsilon})_w = (0.0208 + 0.0833 + 0.001389)\ \frac{1}{\mathrm{m}^2} = 0.1055\ \frac{1}{\mathrm{m}^2}.$$

Therefore,

$$Q_{r,s\text{-}w} = \frac{5.67\times 10^{-8}(\mathrm{W/m^2\text{-}K^4})\times[305.15^4 - 293.15^4](\mathrm{K}^4)}{0.1055(\frac{1}{\mathrm{m}^2})} = 691.0\ \mathrm{W}.$$

(d) The efficiency η is defined as

$$\eta = \frac{Q|_A}{\dot{S}_{r,c}} = \frac{\langle Q_{ku}\rangle_L + Q_{r,s\text{-}w}}{\dot{S}_{r,c}}$$

$$= \frac{411.9(\mathrm{W}) + 691.0(\mathrm{W})}{19{,}500(\mathrm{W})} = \frac{1{,}102.9(\mathrm{W})}{19{,}500(\mathrm{W})} = 0.05656 = 5.656\%.$$

COMMENT:

Note that the surface-radiation heat transfer is larger than surface convection. Higher efficiencies are possible by heating a larger portion of the wall, allowing for convection directly into the room, and forcing an air stream around the fireplace.

PROBLEM 6.16.FUN

GIVEN:

As discussed in Section 6.5.1, the two-dimensional (x, y), (u_f, v_f), laminar viscous, thermobuoyant boundary layer (for vertical, uniform surface temperature plate) momentum equation (6.80) can be reduced to an ordinary differential equation using a dimensionless similarity variable

$$\eta = \frac{y}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4},$$

and a dimensionless stream function

$$\psi^* = \frac{\psi}{4\nu_f\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}}, \quad u_f = \frac{\partial \psi}{\partial y}, \quad v_f = -\frac{\partial \psi}{\partial x}, \quad \mathrm{Gr}_x = \frac{g\beta_f(T_s - T_{f,\infty})x^3}{\nu_f^2}.$$

OBJECTIVE:

(a) Show that the momentum equation (6.80) reduces to

$$\frac{d^3\psi^*}{d\eta^3} + 3\psi^*\frac{d^2\psi^*}{d\eta^2} - 2\left(\frac{d\psi^*}{d\eta}\right)^2 + T_f^* = 0 \quad T_f^* = \frac{T_f - T_\infty}{T_s - T_{f,\infty}}.$$

(b) Show that the energy equation (6.79) reduces to

$$\frac{d^2T_f^*}{d\eta^2} + 3\mathrm{Pr}\psi^*\frac{dT_f^*}{d\eta} = 0 \quad \mathrm{Pr} = \frac{\nu_f}{\alpha_f}.$$

SOLUTION:

We start from (6.80), written as

$$u_f\frac{\partial u_f}{\partial x} + v_f\frac{\partial u_f}{\partial y} - \nu_f\frac{\partial^2 u_f}{\partial y^2} - g\beta_f(T_f - T_{f,\infty}) = 0.$$

Using the stream function ψ, the dimensionless stream function ψ^* and the similarity variables η, we transform u_f and v_f into ψ^*, x and η. We start with

$$\begin{aligned} u_f \equiv \frac{\partial \psi}{\partial y} &= \frac{d\psi}{d\eta}\frac{d\eta}{dy} = \frac{d\psi^*}{d\eta}\frac{1}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/2} 4\nu_f\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4} \\ &= \frac{4\nu_f}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/2}\frac{d\psi^*}{d\eta} \end{aligned}$$

or

$$\frac{u_f}{\frac{4\nu_f}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/2}} = \frac{d\psi^*}{d\eta}.$$

Also,

$$\begin{aligned} v_f &= -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}4\nu_f\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}\psi^* \\ &= \frac{\nu_f}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}\left(\eta\frac{d\psi^*}{d\eta} - 3\psi^*\right). \end{aligned}$$

Next, these velocity components are differentiated, with respect to x and y, and we have

$$\begin{aligned}
\frac{\partial u_f}{\partial x} &= 4\nu_f\left[-\frac{\eta}{4x^2}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/2}\frac{d^2\psi^*}{d\eta^2}+\frac{1}{2x^2}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/2}\frac{d\psi^*}{d\eta}\right]\\
\frac{\partial u_f}{\partial y} &= \frac{4\nu_f}{x^2}\left(\frac{\mathrm{Gr}_x}{4}\right)^{3/4}\frac{d^2\psi^*}{d\eta^2}\\
\frac{\partial^2 u_f}{\partial y^2} &= \frac{4\nu_f}{x^2}\frac{\mathrm{Gr}_x}{4}\frac{d^3\psi^*}{d\eta^3}.
\end{aligned}$$

Substituting these in the above momentum equation and using

$$T_f^* = \frac{T_f - T_{f,\infty}}{T_s - T_{f,\infty}},$$

and after re-arranging the terms, we have

$$\frac{d^3\psi^*}{d\eta^3}+3\psi^*\frac{d^2\psi^*}{d\eta^2}-2\left(\frac{d\psi^*}{d\eta}\right)^2+T_f^*=0.$$

(b) We start from (6.79), rewritten as

$$u_f\frac{\partial T_f}{\partial x}+v_f\frac{\partial T_f}{\partial y}-\alpha_f\frac{\partial^2 T_f}{\partial y^2}=0.$$

We have

$$\begin{aligned}
\frac{\partial T_f}{\partial x} &= (T_s - T_{f\infty})\frac{\partial T_f^*}{\partial \eta}\frac{\partial \eta}{\partial x} = -\frac{T_s - T_{f,\infty}}{4x}\eta\frac{dT_f^*}{d\eta}\\
\frac{\partial T_f}{\partial y} &= (T_s - T_{f\infty})\frac{\partial T_f^*}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{T_s - T_{f,\infty}}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}\frac{dT_f^*}{d\eta}\\
\frac{\partial^2 T_f^2}{\partial y^2} &= (T_s - T_{f\infty})\frac{\partial}{\partial y}\left[\frac{1}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}\frac{dT_f^*}{d\eta}\right] = \frac{T_s - T_{f,\infty}}{x^2}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/2}\frac{d^2T_f^*}{d\eta^2}.
\end{aligned}$$

Substituting into the above energy equation, using the velocity results from part (a), and after re-arranging, we have

$$\frac{d^2T_f^*}{d\eta^2}+3\mathrm{Pr}\psi^*\frac{dT_f^*}{d\eta}=0.$$

COMMENT:

Note that for Pr $\rightarrow 0$ (liquid metals), the temperature distribution will be linear in η (because the second derivative in zero).

Also, note that from the above relation for $\partial T_f/\partial y$, we have the surface heat flux as

$$-k\frac{\partial T_f}{\partial y}\bigg|_{y=0} = q_{ku} = \frac{1}{x}\left(\frac{\mathrm{Gr}_x}{4}\right)^{1/4}\frac{dT_f^*}{d\eta}\bigg|_{\eta=0}.$$

PROBLEM 6.17.FUN

GIVEN:

The dimensionless, transformed coupled boundary-layer momentum and energy equations for thermobuoyant flow, are

$$\frac{d^3\psi^*}{d\eta^3} + 3\psi^*\frac{d^2\psi^*}{d\eta^2} - 2\left(\frac{d\psi^*}{d\eta}\right)^2 + T_f^* = 0$$

$$\frac{d^2T_f^*}{d\eta^2} + 3\mathrm{Pr}\psi^*\frac{dT_f^*}{d\eta} = 0,$$

subject to the surface and far-field thermal and mechanical conditions

$$\text{at} \qquad \eta = 0 : \frac{d\psi^*}{d\eta} = \psi^* = 0, \quad T_f^* = 1$$

$$\text{for} \qquad \eta \to \infty : \frac{d\psi^*}{d\eta} = 0, \quad T_f^* = 0.$$

Use $\mathrm{Pr} = 0.72$ and plot ψ^*, $d\psi^*/d\eta$, $d^2\psi^*/d\eta^2$, T_f^*, and $dT_f^*/d\eta$, with respect to η.

Note that with an initial-value problem solver such as SOPHT, the second derivative of ψ^* and first derivative of T_f^* at $\eta = 0$ must be guessed. The guesses are adjusted till $d^2\psi^*/d\eta^2$ becomes zero for large η.

Use $d^2\psi^*/d\eta^2(\eta = 0) = 0.6760$ and $dT_f^*/d\eta^*(\eta = 0) = -0.5064$.

OBJECTIVE:

Using a solver, integrate these coupled equations

SOLUTION:

The solver we choose is an initial-value-solver, such as SOPHT, where the initial values (i.e., at $\eta = 0$) for ψ^*, $d\psi * /d\eta$, $d^2\psi^*/d\eta^2$, T_f^*, and $dT_f^*/d\eta$ must be provided for these coupled third-and second-order, ordinary differential equations.

Note that using a set of arbitrary notations we can write these as five first-order, ordinary differential equations. These are

$$\begin{aligned} g' &= -3fg + 2z^2 - h \\ i' &= -3\mathrm{Pr}fi \\ h' &= i \\ z' &= g \\ f' &= z. \end{aligned}$$

The variations of ψ^*, $d\psi^*/d\eta$, $d^2\psi^*/d\eta^2$, T_f^*, and $dT_f^*/d\eta$, with respect to η, are plotted in Figure Pr.6.17.

The results show that for $\eta = 5.66$, the x-direction velocity represented by $d\psi^*/d\eta$ will have a magnitude 1/100 of its peak (or maximum value). This is designated as the edge of the boundary layer.

COMMENT:

Note that results are a strong function of Pr. In general, the derivatives are guessed until $T_f^* = d\psi^*/d\eta = 0$ far from the surface (large η).

Also note that from (6.90) we have

$$\frac{\delta_\alpha}{L}\left(\frac{\mathrm{Gr}L}{4}\right)^{1/4} = 3.804 \quad \text{for } \mathrm{Pr} = 0.72,$$

while the numerical results for $T_f^* = 0.01$ show that this is 4.4176. This is because (6.90) is an approximation to results over a large range of Pr.

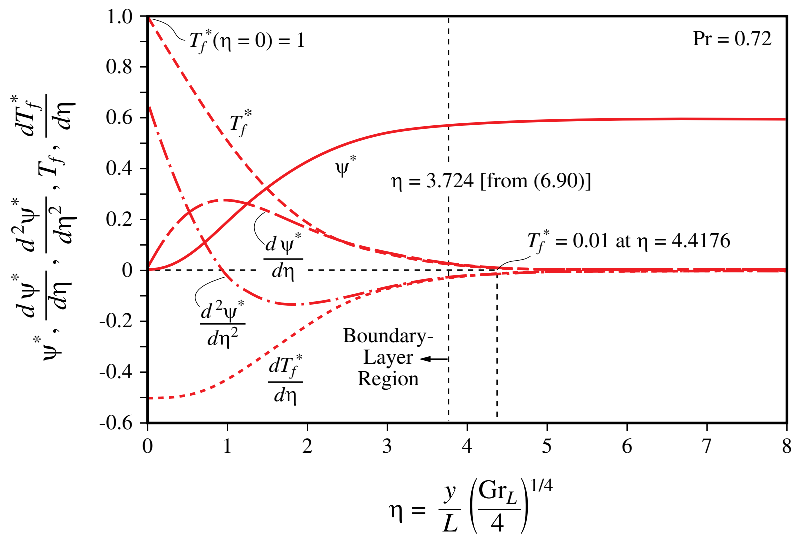


Figure Pr.6.17 Variation of the dimensionless, velocity and temperature variables with respect to the similarity variable.

PROBLEM 6.18.FAM

GIVEN:

An aluminum flat sheet, released from hot pressing, is to be cooled by surface convection in an otherwise quiescent air, as shown in Figure Pr.6.18. The sheet can be placed vertically [Figure Pr.6.18(a)(i)] or horizontally [Figure Pr.6.18(a)(ii)]. Both sides of the sheet undergo heat transfer and in treating the horizontal arrangement, treat the lower surface using the Nusselt number relations listed in Table 6.5 for the top surface.

$w = L = 0.4$ m, $T_{f,\infty} = 25$°C, $T_s = 430$°C.

Use air properties at $\langle T_f \rangle_\delta = (T_s + T_{f,\infty})/2$.

SKETCH:

Figure Pr.6.18(a) shows the two arrangements.

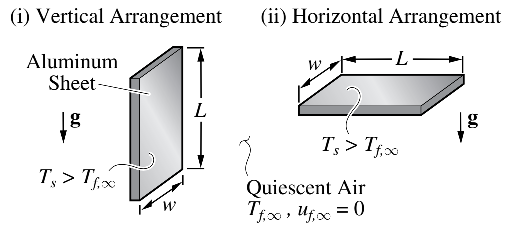


Figure Pr.6.18(a) A sheet of aluminum is cooled in an otherwise quiescent air. (i) Vertical placement. (ii) Horizontal placement.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the surface-convection heat transfer rate $\langle Q_{ku} \rangle_L$ for the two arrangements and for the conditions given above.

SOLUTION:

(a) Figure Pr.6.18(b) shows the thermal circuit diagram.

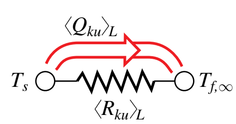


Figure Pr.6.18(b) Thermal circuit diagram.

(b) (i) For the vertical placement, from Table 6.5, we have

$$\begin{aligned}
\langle \mathrm{Nu} \rangle_L &= [(\mathrm{Nu}_{L,l})^6 + (\mathrm{Nu}_{L,t})^6]^{1/6} \\
\mathrm{Nu}_{L,l} &= \frac{2.8}{\ln\left(1 + \dfrac{2.8}{a_1 \mathrm{Ra}^{1/4}}\right)} \\
\mathrm{Nu}_{L,t} &= \frac{0.13\mathrm{Pr}^{0.22}}{(1 + 0.61\mathrm{Pr}^{0.81})^{0.42}} \mathrm{Ra}^{1/3} \\
a_1 &= \frac{4}{3} \frac{0.503}{\left[1 + \left(\dfrac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{4/9}} \\
\mathrm{Ra}_L &= \frac{g\beta_f (T_s - T_{f,\infty}) L^3}{\nu_f \alpha_f}.
\end{aligned}$$

From Table C.22, for air at

$$\langle T_f \rangle_\delta = \frac{T_s + T_{f,\infty}}{2} = \frac{(430+25)(^\circ\mathrm{C})}{2} + 273.15(\mathrm{K}) = 500.7 \text{ K},$$

the properties are

$$k_f = 0.0395 \text{ W/m-K} \qquad \text{Table C.22}$$
$$\nu_f = 3.730 \times 10^5 \text{ m}^2/\text{s} \qquad \text{Table C.22}$$
$$\alpha_f = 5.418 \times 10^{-5} \text{ m}^2/\text{s} \qquad \text{Table C.22}$$
$$\mathrm{Pr} = 0.69 \qquad \text{Table C.22,}$$

and treating air as an ideal gas, from (6.77),

$$\beta_f = \frac{1}{T_f} = \frac{1}{\langle T_f \rangle_\delta} = 1.997 \times 10^{-3} \text{ 1/K}$$

Then,

$$\begin{aligned}
\mathrm{Ra}_L &= \frac{9.81(\mathrm{m^2/s}) \times 1.997 \times 10^{-3}(1/\mathrm{K}) \times (430-25)(\mathrm{K}) \times (0.4)^3(\mathrm{m}^3)}{(3.730 \times 10^{-5})(\mathrm{m^2/s}) \times 5.418 \times 10^{-5}(\mathrm{m^2/s})} \\
&= 2.512 \times 10^8 \\
a_1 &= \frac{4}{3} \frac{0.503}{\left[1 + \left(\frac{0.492}{0.69}\right)^{9/16}\right]^{4/9}} = 0.5131 \\
\mathrm{Nu}_{L,l} &= \frac{2.8}{\ln[1 + \frac{2.8}{0.5131 \times (2.512 \times 10^8)^{1/4}}]} = 65.99 \\
\mathrm{Nu}_{L,t} &= \frac{0.13 \times (0.69)^{0.22}}{[1 + 0.61 \times (0.69)^{0.81}]^{0.42}} \times (2.512 \times 10^8)^{1/3} = 64.64 \\
\langle \mathrm{Nu} \rangle_L &= [(65.99)^6 + (64.64)^6]^{1/6} = 73.33.
\end{aligned}$$

From (6.124), and noting that $A_{ku} = 2wL$, we have

$$\begin{aligned}
\langle Q_{ku} \rangle_L &= A_{ku} \langle \mathrm{Nu} \rangle_L \frac{k_f}{L}(T_s - T_{f,\infty}) \\
&= 2 \times (0.4)^2(\mathrm{m}^2) \times 73.33 \times \frac{0.0395(\mathrm{W/m\text{-}K})}{0.4(\mathrm{m})} \times (430-25)(\mathrm{K}) \\
&= 938.5 \text{ W}.
\end{aligned}$$

(ii) For the horizontal placement, from Table 6.5, we have

$$\begin{aligned}
\langle \mathrm{Nu} \rangle_L &= [(\mathrm{Nu}_{L,l})^{10} + (\mathrm{Nu}_{L,t})^{10}]^{1/10} \\
\mathrm{Nu}_{L,l} &= \frac{1.4}{\ln\left(1 + \frac{1.4}{0.835 a_1 \mathrm{Ra}_L^{1/4}}\right)} \\
\mathrm{Nu}_{L,t} &= 0.14 \mathrm{Ra}_L^{1/3},
\end{aligned}$$

where a_1 is the same as in (i) and for the definition given for L in Table 6.5, we have

$$\begin{aligned}
L &= \frac{A_{ku}}{P_{ku}} = \frac{Lw}{2(L+w)} = \frac{L}{4} \quad \text{for each side} \\
&= \frac{0.4(\mathrm{m})}{4} = 0.1 \text{ m}.
\end{aligned}$$

Then using the results of (i), we have

$$\begin{aligned}
\mathrm{Ra}_L &= 2.512\times10^8\times\frac{1}{(4)^3}=3.925\times10^6\\
\mathrm{Nu}_{L,l} &= \frac{1.4}{\ln\left[1+\dfrac{1.4}{0.835\times0.5131\times(3.925\times10^6)^{1/4}}\right]}\\
&= 19.76\\
\mathrm{Nu}_{L,t} &= 0.14\times(3.925\times10^6)^{1/3}=22.08\\
\langle\mathrm{Nu}\rangle_L &= [(19.76)^{10}+(22.08)^{10}]^{1/10}\\
&= 22.72\\
\langle Q_{ku}\rangle_L &= 2\times(0.4)^2(\mathrm{m}^2)\times22.72\times\frac{0.0395(\mathrm{W/m\text{-}K})}{0.10(\mathrm{m})}\times405\mathrm{K}\\
&= 1{,}163\ \mathrm{W}.
\end{aligned}$$

COMMENT:

Note that although for the of horizontal placement $\langle\mathrm{Nu}\rangle_L$ is smaller, due to the smaller length used in the scaling of $\langle\mathrm{Nu}\rangle_L$, the heat transfer rate is larger for this arrangement.

The correlations valid for both laminar and turbulent flows trend to add uncertainties, compared to the correlations valid only for a given range of Ra_L [16]. For example using (6.89), for the vertical arrangement and for the $\mathrm{Ra}_L < 10^9$, we have

$$\begin{aligned}
\langle\mathrm{Nu}\rangle_L &= \frac{4}{3}a_1\mathrm{Ra}_L^{1/3}\\
&= \frac{4}{3}\times0.5131\times(2.512\times10^8)^{1/4}\\
&= 86.13.
\end{aligned}$$

This is to be compared to $\langle\mathrm{Nu}\rangle_L = 64.63$ in which we used in (b)(i).

PROBLEM 6.19.FAM

GIVEN:

Water, initially at $T = 12$℃, is boiled in a portable heater at one atm pressure, i.e., it has its temperature raised from 12℃ to 100℃. The heater has a circular, nickel surface with $D = 5$ cm and is placed at the bottom of the water, as shown in Figure Pr.6.19. The amount of water is 2 kg (which is equivalent to 8 cups) and the water is to be boiled in 6 min.

The properties for water are given in Table C.23.

SKETCH:

Figure Pr.6.19 shows boiling from the bottom surface of an electrical water heater.

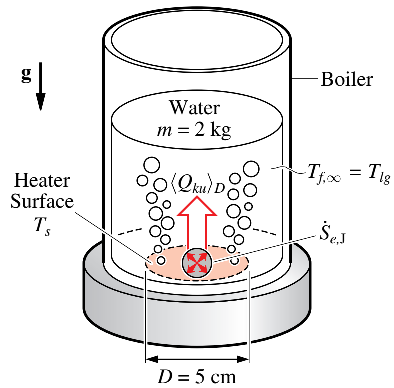


Figure Pr.6.19 An electric water heater using boiling surface-convection heat transfer.

OBJECTIVE:

(a) Determine the time-averaged (constant with time) electrical power needed $\dot{S}_{e,\text{J}}$(W) assuming no heat losses.

(b) Determine the critical heat flux $q_{ku,CHF}$(W/m^2) for this fluid and then comment on whether the required electrical power per unit area is greater or smaller than this critical heat flux. Note that the surface-convection heat transfer rate (or the electrical power) per unit area should be less than the critical heat flux; otherwise, the heater will burn out.

(c) Determine the required surface temperature T_s, assuming nucleate boiling. Here, assume that the effect of the liquid subcooling on the surface-convection heat transfer rate is negligible. When the subcooling is not negligible (i.e., the water is at a much lower temperature than the saturation temperature T_{lg}), the larger temperature gradient between the surface and the liquid and the collapse of the bubbles away from the surface, will increase the rate of heat transfer.

(d) Determine the average surface-convection thermal resistance $A_{ku}\langle R_{ku}\rangle_L$[℃/(W/m^2)] and the average Nusselt number $\langle \text{N}u\rangle_L$.

SOLUTION:

(a) The integral-volume energy equation (2.9) applied to a control volume containing the water, assuming constant properties and a uniform temperature (i.e., a lumped-capacitance analysis), is

$$Q|_A = -\rho c_p V \frac{dT}{dt}.$$

The heat losses to the ambient are neglected. One component of these heat losses is the energy leaving the water surface in the form of vapor. It is assumed that most of the vapor formed at the heater surface re-condenses as the bubbles rise in the liquid. Then, we have

$$-\langle Q_{ku}\rangle_D = -\rho c_p V \frac{dT}{dt}.$$

Integrating from $T_i = 12$℃ at $t = 0$ to $T_f = 100$℃ at $t = 6$ min gives

$$\langle Q_{ku}\rangle_D = \rho c_p V \frac{T_f - T_i}{t}.$$

For water at $T_{ave} = (T_f + T_i)/2 = 329$ K from Table C.23, $c_p = 4{,}183$ J/kg-K. Also, $\rho V = M = 2$ kg. Then

$$\langle Q_{ku}\rangle_D = 2(\text{kg}) \times 4{,}183(\text{J/kg-K})\frac{100(^\circ\text{C}) - 12(^\circ\text{C})}{360(\text{s})} = 2045 \text{ W}.$$

The integral-volume energy equation applied to the electrical heater gives

$$\langle Q_{ku}\rangle_D = \dot{S}_{e,\text{J}}$$

or

$$\dot{S}_{e,\text{J}} = 2{,}045 \text{ W}.$$

(b) The critical heat flux for pool boiling from a horizontal surface is given by (6.100)

$$\frac{q_{ku,CHF}}{\rho_g \Delta h_{lg}} \times \left(\frac{\rho_g^2}{g\sigma\Delta\rho_{lg}}\right)^{1/4} = 0.13.$$

The properties for saturated water and steam at a pressure of 1 atm are given in Table C.26, $\rho_l = 958.3$ kg/m^3, $\rho_g = 0.596$ kg/m^3, $\sigma = 0.05891$ N/m, $\Delta h_{lg} = 2.257 \times 10^6$ J/kg, $\mu_l = 277.53 \times 10^{-6}$ m^2/s, $\text{Pr}_l = 1.73$, $k_l = 0.6790$ W/m-K, and $c_{p,l} = 4{,}216$ J/kg-K.
Solving for $q_{ku,CHF}$ we have

$$q_{ku,CHF} = \frac{0.13\rho_g\Delta h_{lg}}{\left(\frac{\rho_g^2}{g\sigma\Delta\rho_{lg}}\right)^{1/4}} = \frac{0.13 \times 0.596(\text{kg/m}^3) \times 2.257 \times 10^6(\text{J/kg})}{\left[\frac{(0.596)^2(\text{kg/m}^3)^2}{9.81(\text{m}^2/\text{s}) \times 0.05891(\text{N/m}) \times 957.7(\text{kg/m}^3)}\right]^{1/4}} = 1.098 \times 10^6 \text{ W/m}^2.$$

The surface-convection heat transfer rate provided by the heater, at the critical heat flux condition, is

$$Q_{ku,CHF} = A_{ku}q_{ku,CHF} = \frac{\pi}{4} \times (0.05)^2(\text{m})^2 \times 1.098 \times 10^6(\text{W/m}^2) = 2{,}156 \text{ W}.$$

The critical heat flux is slightly larger than the required heat transfer rate. Therefore, for the desired heating rate, the electrical heater will operate at low surface temperatures, characteristic of nucleate boiling.

(c) The total surface-convection heat rate is given by (6.98)

$$\langle Q_{ku}\rangle_D = A_{ku}\frac{T_s - T_{lg}}{A_{ku}\langle R_{ku}\rangle_D}.$$

The average surface-convection resistance for pool nucleate boiling from a horizontal surface is determined from (6.98), i.e.,

$$A_{ku}\langle R_{ku}\rangle_D = a_s^3 \frac{\Delta h_{lg}^2}{\mu_l c_{p,l}^3 (T_s - T_{lg})^2}\left(\frac{\sigma}{g\Delta\rho_{lg}}\right)^{1/2}\text{Pr}_l^n = \frac{A_{ku}(T_s - T_{lg})}{\langle Q_{ku}\rangle_D}.$$

For a nickel surface, from Table 6.2, we have $a_s = 0.006$. For water, from (6.98), we have $n = 3$.
Solving the above for $T_s - T_{lg}$ gives

$$(T_s - T_{lg})^3 = \frac{a_s^3 \Delta h_{lg}^2 \langle Q_{ku}\rangle_D \text{Pr}_l^n}{A_{ku}\mu_l c_{p,l}^3} \times \left(\frac{\sigma}{g\Delta\rho_{lg}}\right)^{1/2},$$

and subbing in numerical values gives

$$T_s - T_{lg} = \left[\frac{6.84 \times 10^{-4}(\text{K}^3\text{-m}^2/\text{W}) \times 2{,}045(\text{W})}{\frac{\pi}{4}(0.05)^2(\text{m})^2}\right]^{1/3} = 8.93\text{K},$$

or, for $T_{lg} = 100^\circ$C, we have

$$T_s = 108.9^\circ\text{C}.$$

Table Pr.6.19 Summary of Nusselt numbers and average surface-convection thermal resistances in order of decreasing surface-convection thermal resistance.

Fluid Flow Arrangement with or without phase change	$\langle \mathrm{Nu} \rangle_L$ or $\langle \mathrm{Nu} \rangle_D$	$A_{ku}\langle R_{ku}\rangle_L$ or $A_{ku}\langle R_{ku}\rangle_D$, ℃/(W/m^2)
(6.7): laminar, parallel flow over a flat plate: $u_{f,\infty} = 0.5556$ m/s	119.8	3.326×10^{-1}
(6.14): laminar, thermobuoyant flow around a vertical cylinder	43.08	2.232×10^{-1}
(6.14): laminar, thermobuoyant flow around a horizontal cylinder	18.55	2.073×10^{-1}
(6.7): laminar, parallel flow over a flat plate: $u_{f,\infty} = 5.556$ m/s	378.8	1.052×10^{-1}
(6.10): perpendicular flow with a single, round impinging jet	46.43	8.413×10^{-2}
(6.10): perpendicular flow with an array of 9 round impinging jets	28.35	4.953×10^{-2}
(6.10): turbulent, parallel flow over a flat plate: $u_{f,\infty} = 22.22$ m/s	2,335	1.711×10^{-2}
(6.19): nucleate, pool-boiling on a horizontal flat surface	8,587	8.575×10^{-6}

(d) Using this T_s, the average surface-convection thermal resistance is

$$A_{ku}\langle R_{ku}\rangle_D = A_{ku}\frac{T_s - T_{lg}}{\langle Q_{ku}\rangle_D} = 8.75 \times 10^{-6}\,℃/(\mathrm{W/m^2}).$$

The average Nusselt number is determined from (6.99), i.e.,

$$\langle \mathrm{Nu}\rangle_D = \frac{D}{k_l A_{ku}\langle R_{ku}\rangle_D} = 8{,}587.$$

COMMENT:

Problems 6.7, 6.10, 6.14 and 6.19 present applications of four different fluid flow arrangements for surface-convection heat transfer. The choice of a certain process for convection heating or cooling of a surface depends initially on the desired rate of cooling or heating. Table Pr.6.19 summarizes the results obtained for the Nusselt number and average surface-convection thermal resistance in order of decreasing surface-convection thermal resistance.

The low speed laminar, parallel flow and the thermobuoyant flows have the largest surface-convection thermal resistances. As a consequence, they provide the lowest surface-convection cooling or heating power. However, the fluid propelling costs are minimal (or zero) making them attractive in situations where the desired heating/cooling powers can be achieved by an increase in the available surface-convection area. Residential applications such as climate control and cooling of the condenser of refrigerators are typical examples.

High-speed, parallel, laminar flows and perpendicular flows provide higher surface-convection heat transfer at a cost of fluid propelling power. For low conductance substrates, multiple jets provide a more uniform rate of heat transfer over the surface.

Finally, heat transfer with phase change gives the highest heat transfer rates. Modifications of the surface can increase the rates of heat transfer in the nucleate boiling regime (i.e., heat transfer enhancement).

The choice of a surface-convection heating/cooling mechanism depends on additional constraints, for example, available space, cost of fluid propelling power, weight, availability of fluids (e.g, situations were the necessary amount of a liquid is not available), whether liquids can be used (e.g., when the system has to be kept dry), continuous versus intermittent operation, reliability of the system (e.g., when the system has to operate by itself for long periods of time, or in a remote location), and environmental concerns related to chemical or thermal pollution (e.g., gas-pollutant emissions, hot-gas discharges in the atmosphere, and hot-water discharge into lakes and rivers).

PROBLEM 6.20.FAM

GIVEN:

Steam is produced by using the flue gas from a burner to heat a pool of water, as shown in Figure Pr.6.20(a). The water and the flue gas are separated by a plate. On the flue-gas side (modeled as air), the measurements show that the flue-gas, far-field temperature is $T_{f,\infty} = 977°C$ and flows parallel to the surface at $u_{f,\infty} = 2$ m/s, while the flue-gas side surface of the plate is at $T_{s,2} = 110°C$. The heat flows through the plate (having a length $L = 0.5$ m and a width w) into the water (water is at the saturated temperature $T_{lg} = 100°C$ and is undergoing nucleate boiling).

Evaluate the flue-gas properties as those of air at the flue-gas film temperature (i.e., at the average temperature between the flue-gas side surface temperature of the plate and the flue-gas, far-field temperature). For water, use the saturation liquid-vapor properties given in Table C.26.

SKETCH:

Figure Pr.6.20(a) shows the surface separating the flow from the boiling water.

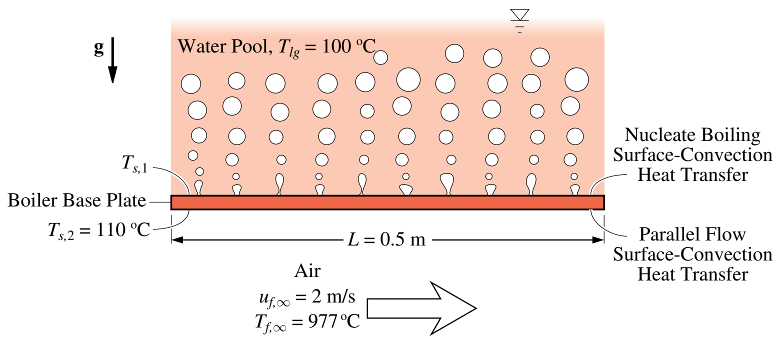


Figure Pr.6.20(a) A solid surface separating flue gas and water with a large difference between the far-field temperatures causing the water to boil and produce steam.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the surface temperature of the plate on the water side $T_{s,1}$. For the nucleate boiling Nusselt number correlation, use $a_s = 0.013$.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.6.20(b).

(b) The energy equations for nodes $T_{s,2}$ and $T_{s,1}$ gives

$$\langle Q_{ku}\rangle_{L,2} = Q_{k,2\text{-}1} = \langle Q_{ku}\rangle_{L,1}$$

where $\langle Q_{ku}\rangle_{L,2}$ is the surface-convection heat transfer rate on the parallel-flow, gas side, $Q_{k,2\text{-}1}$ is the conduction heat transfer rate through the plate, and $\langle Q_{ku}\rangle_{L,1}$ is the surface-convection heat transfer rate on the nucleate-boiling water side. Note that no information is provided about the material or thickness of the base plate. However, the above equation states that, under steady-state, the two surface-convection heat transfer rates are the same. We use this to determine $T_{s,1}$.

(i) For the parallel flow, the gas side, the surface-convection heat transfer rate is given by (6.49) as

$$\langle Q_{ku}\rangle_{L,2} = \frac{T_{f,\infty} - T_{s,2}}{\langle R_{ku}\rangle_{L,2}}.$$

The average surface-convection thermal resistance can be obtained from the conditions given. The properties for air at $T_\delta = (977+110)/2 = 544°C = 817$ K are obtained from Table C.22: $k_f = 0.0574$ W/m-K, $\nu_f = 84.16\times10^{-6}$

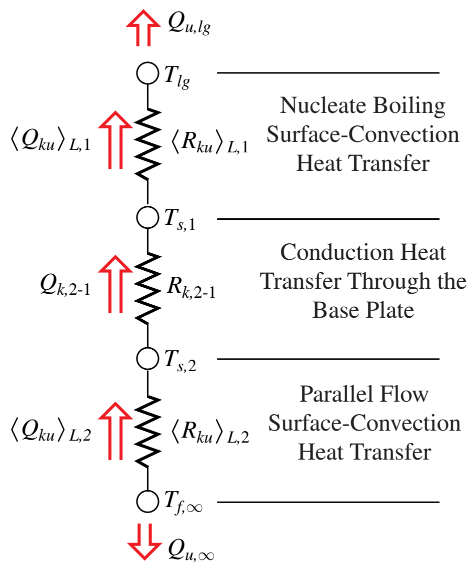


Figure Pr.6.20(b) Thermal circuit diagram.

m^2/s, and Pr= 0.70.
The Reynolds number is given by (6.45), i.e.,

$$\mathrm{Re}_L = \frac{u_{f,\infty}L}{\nu_f} = \frac{2(\mathrm{m/s}) \times 0.5(\mathrm{m})}{84.16 \times 10^{-6}} = 11{,}882.$$

For $\mathrm{Re}_L = 11{,}882 < \mathrm{Re}_{L,t} = 5\times10^5$ the flow is in the laminar regime. For the laminar regime the average Nusselt number is given in Table 6.3 as

$$\langle \mathrm{Nu}\rangle_{L,2} = 0.664\mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/3} = 0.664(11882)^{1/2}(0.70)^{1/3} = 64.27.$$

The average surface-convection thermal resistance is given by (6.49), i.e.,

$$A_{ku}\langle R_{ku}\rangle_{L,2} = \frac{L}{k_f\langle \mathrm{Nu}\rangle_{L,2}} = \frac{0.5(\mathrm{m})}{0.057(\mathrm{W/m\text{-}K}) \times 64.27} = 0.1355^\circ\mathrm{C}/(\mathrm{W/m^2}).$$

(ii) For the nucleate boiling, the liquid side, the surface-convection heat transfer rate is also given by (6.49) as

$$\langle Q_{ku}\rangle_{L,1} = \frac{T_{s,1} - T_{lg}}{\langle R_{ku}\rangle_{L,1}}.$$

The properties for water at $T_{lg} = 100^\circ$C are found from Table C.26: $k_l = 0.679$ W/m-K, $\mu_l = 277.53 \times 10^{-6}$ Pa.s, $\rho_l = 958.3$ kg/m^3, $\rho_g = 0.596$ kg/m^3, $c_{p,l} = 4{,}220$ J/kg-K, $\mathrm{Pr}_l = 1.73$, $\sigma = 0.05891$ N/m, and $\Delta h_{lg} = 2.257 \times 10^6$ J/kg.
The average Nusselt number is given Table 6.6 as

$$\langle \mathrm{Nu}\rangle_{L,1} = \frac{L}{k_l}\frac{\mu_l c_{p,l}^3(T_{s,1} - T_{lg})^2}{a_s^3\Delta h_{lg}^2}\left(\frac{g\Delta\rho_{lg}}{\sigma}\right)^{1/2}\mathrm{Pr}_l^{-n},$$

where $n = 3$ for water. The average surface-convection thermal resistance is given by (6.49) as

$$A_{ku}\langle R_{ku}\rangle_{L,1} = \frac{L}{k_l\langle \mathrm{Nu}\rangle_{L,1}} = \frac{a_s^3\Delta h_{lg}^2}{\mu_l c_{p,l}^3(T_{s,1} - T_{lg})^2}\left(\frac{\sigma}{g\Delta\rho_{lg}}\right)^{1/2}\mathrm{Pr}^n.$$

Note that $A_{ku}\langle R_{ku}\rangle_{L,1}$ cannot be determined, because $T_{s,1}$ is not known.
All the other properties are known and substituting their values we obtain

$$A_{ku}\langle R_{ku}\rangle_{L,1} = \frac{0.006957}{(T_{s,1} - T_{lg})^2}\mathrm{K^3/(W/m^2)}.$$

From the energy equations, we get an algebraic equation in $T_{s,1}$,

$$\frac{T_{f,\infty} - T_{s,2}}{0.1355} = \frac{T_{s,1} - T_{lg}}{\frac{0.006957}{(T_{s,1} - T_{lg})^2}}.$$

substituting for $T_{f,\infty}$ and $T_{s,2}$ and rearranging the right-hand side

$$44.51 = (T_{s,1} - T_{lg})^3.$$

Solving the equation above for $T_{s,1}$,

$$T_{s,1} = T_{lg} + (44.51)^{1/3} = 100 + 3.54 = 103.5^\circ\text{C}.$$

COMMENT:

Note that the difference in the surface and far-field temperatures for the flue gas side is $T_{f,\infty} - T_{s,2} = 867^\circ\text{C}$, while for the boiling water side it is $T_{s,1} - T_{lg} = 3.5^\circ\text{C}$. The temperature difference $T_{s,1} - T_{lg}$ is called the surface superheat.

PROBLEM 6.21.FAM

GIVEN:

A glass sheet is vertically suspended above a pan of boiling water and the water condensing over the sheet and raises its temperature. This is shown in Figure Pr.6.21(a). Filmwise condensation and uniform sheet temperature T_s are assumed. Note that the condensate is formed on both sides of the sheet. Also assume a steady-state heat transfer.

$l = 1$ mm, $L = 15$ cm, $w = 15$ cm, $T_{lg} = 100$°C, $T_s = 40$°C.

Use the saturated water properties at T_{lg}.

SKETCH:

Figure Pr.6.21(a) shows the glass sheet and the surrounding water vapor.

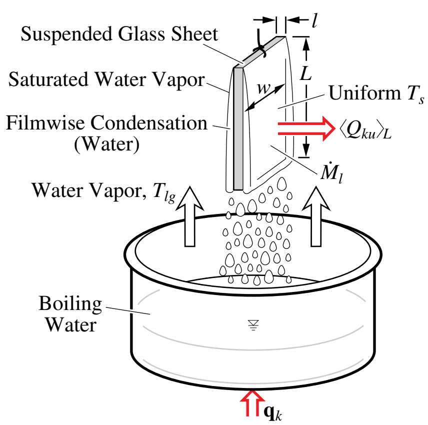


Figure Pr.6.21(a) A glass sheet is vertically suspended in a water-vapor ambient and the heat released by condensation raises the sheet temperature.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the heat transfer rate $\langle Q_{ku}\rangle_L$, for the conditions given above, for each side.

(c) Determine the condensate flow rate $\dot{M}_l = -\dot{M}_{lg}$, for each side.

(d) Is this a laminar film condensate flow?

SOLUTION:

(a) Figure Pr.6.21(b) shows the thermal circuit diagram.

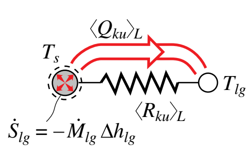


Figure Pr.6.21(b) Thermal circuit diagram.

(b) From (6.49) and Table 6.6, for filmwise condensation on vertical surfaces, we have

$$\langle Q_{ku}\rangle_L = A_{ku}\langle \mathrm{Nu}\rangle_L \frac{k_l}{L}(T_s - T_{lg})$$

$$\langle \mathrm{Nu}\rangle_L = 0.9428\left[\frac{g\Delta\rho_{lg}\Delta h_{lg}L^3}{k_l\nu_l(T_{lg}-T_s)}\right]^{1/4}.$$

From Table C.27, we have for saturated water,

$$\begin{aligned}
\rho_l &= 958 \text{ kg/m}^3 \\
\rho_g &= 0.596 \text{ kg/m}^3 \\
\Delta h_{lg} &= 2.257 \times 10^6 \text{ J/kg} \\
k_l &= 0.680 \text{ W/m-K} \\
\nu_l &= \frac{\mu_l}{\rho_l} = \frac{279 \times 10^{-6}(\text{Pa-s})}{958(\text{kg/m}^3)} = 2.91 \times 10^{-7} \text{ m}^2\text{/s}
\end{aligned}$$

Then using the numerical values, we have

$$\begin{aligned}
\langle \text{Nu} \rangle_L &= 0.9428 \times \left[\frac{9.81(\text{m/s}^2) \times (958 - 0.596)(\text{kg/m}^3) \times 2.257 \times 10^6(\text{J/kg}) \times (0.15)^3(\text{m}^3)}{0.680(\text{W/m-K}) \times 2.91 \times 10^{-7}(\text{m}^2\text{-s}) \times (100 - 40)(\text{K})} \right]^{1/4} \\
&= 1{,}480.
\end{aligned}$$

Then

$$\begin{aligned}
\langle Q_{ku} \rangle_L &= 0.15(\text{m}) \times 0.15(\text{m}) \times 1{,}480 \times \frac{0.680(\text{W/m-K})}{0.15(\text{m})} \times (40 - 100)(\text{K}) \\
&= \text{-9058 W.}
\end{aligned}$$

(c) Using the energy equation for the control volume shown in Figure Pr.6.21(b), we have

$$\langle Q_{ku} \rangle_L = \dot{S}_{lg} = -\dot{M}_{lg} \Delta h_{lg}$$

or

$$\begin{aligned}
\dot{M}_{lg} &= -\frac{\langle Q_{ku} \rangle_L}{\Delta h_{lg}} \\
&= -\frac{-9{,}058(\text{W})}{2.257 \times 10^6(\text{J/kg})} \\
&= 4.005 \times 10^{-3} \text{ kg/s} \\
&= 4.013 \text{ g/s}
\end{aligned}$$

(d) Using (6.114), we have

$$\begin{aligned}
\frac{4|\langle q_{ku} \rangle_L| L}{\mu_g \Delta h_{lg}} &= \frac{4 \times 9{,}058(\text{W}) \times 0.15(\text{m})}{(0.15)^2(\text{m}^2) \times 271 \times 10^{-6}(\text{Pa-s}) \times 2.257 \times 10^6(\text{J/kg})} \\
&= 394.9 < 1{,}800.
\end{aligned}$$

The film condensate flow is in the laminar regime, and the choice of $\langle \text{Nu} \rangle_L$ was correct.

COMMENT:

Note that we have assumed a steady state heat transfer, where in practice T_s increases and eventually reaches T_{lg}.

PROBLEM 6.22.FAM

GIVEN:

To boil water by electrical resistance heating would require a large electrical power per unit area of the heater surface. For a heater having a surface area for surface convection A_{ku}, this power from (2.28) is

$$\dot{S}_{e,\mathrm{J}} = \frac{\Delta\varphi^2}{R_e}, \quad A_{ku} = \pi D l,$$

where $\Delta\varphi$ is the applied voltage, R_e is the electrical resistance, and D and l are the diameter and length of the heater surface. Consider the water-boiler shown in Figure Pr.6.22(a). Using Figure 6.20(b), assume a surface superheat $T_s - T_{lg} = 10°\mathrm{C}$ is needed for a significant nucleate boiling. Then use the nucleate boiling correlation of Table 6.6.

$a_s = 0.01$, $D = 0.5$ cm, $l = 12$ cm, $T_{lg} = 100°\mathrm{C}$.

Use saturated water properties at $T = T_{lg}$.

SKETCH:

Figure Pr.6.22(a) shows the Joule heater in the water boiler.

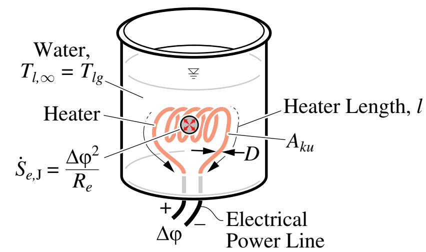


Figure Pr.6.22(a) A Joule heater is used to boil water.

OBJECTIVE:

(a) Draw the thermal circuit diagram for the heater.

(b) Determine the surface-convection heat transfer rate $\langle Q_{ku}\rangle_D$, for the conditions given above.

(c) For an electrical resistance of $R_e = 20$ ohm, what should be the applied voltage $\Delta\varphi$, and the electrical current J_e ?

SOLUTION:

(a) Figure Pr.6.22(b) shows the thermal circuit diagram for the heater. The energy equation is $Q|_A = \langle Q_{ku}\rangle_L = \dot{S}_{e,\mathrm{J}}$.

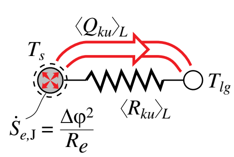


Figure Pr.6.22(b) Thermal circuit diagram.

(b) From Table 6.6, we have for the nucleate boiling regime and using (6.49),

$$\begin{aligned}\langle Q_{ku}\rangle_L &= A_{ku}\langle \mathrm{Nu}\rangle_L \frac{k_l}{L}(T_s - T_{lg}) \\ &= A_{ku}\frac{1}{a_s^3}\frac{\mu_l c_{p,l}^3 (T_s - T_{lg})^3}{\Delta h_{lg}}\left(\frac{g\Delta\rho_{lg}}{\sigma}\right)^{1/2}\mathrm{Pr}_l^{-3},\end{aligned}$$

where we have used $n = 3$ for water and L drops out of the relation.

Table C.26 list the water properties at $T_{lg} = 373.15$ K, i.e.,

$$
\begin{aligned}
\mu_l &= 277.5 \times 10^{-6} \text{ Pa-s} \\
c_{p,l} &= 4,220 \text{ J/kg-K} \\
\Delta h_{lg} &= 2{,}257 \times 10^6 \text{ J/kg} \\
\sigma &= 0.05891 \text{ N/m} \\
\Delta\rho_{lg} &= \rho_l - \rho_g = (958.3 - 0.596)(\text{kg/m}^3) = 957.7 \text{ kg/m}^3 \\
\text{Pr}_L &= 1.73 \\
\left(\frac{g\Delta\rho_{lg}}{\sigma}\right)^{1/2} &= \left[\frac{9.81(\text{m/s}) \times (957.7)(\text{kg/m}^3)}{0.05891(\text{N/m})}\right]^{1/2} = 399.3 \text{ 1/m} \\
\langle Q_{ku}\rangle_L &= \frac{\pi \times 0.005(\text{m}) \times 0.12(\text{m}) \times 2.775 \times 10^{-4}(\text{Pa-s}) \times (4{,}220)^3(\text{J/kg})^3 \times 10^3(\text{K})^3 \times 399.3(1/\text{m})}{(0.01)^3 \times (2.257 \times 10^6)^2(\text{J/kg})^2 \times (1.73)^3} \\
&= 595.1 \text{ W.}
\end{aligned}
$$

(c) The required electrical potential is

$$
\begin{aligned}
\Delta\varphi &= (\dot{S}_{e,\text{J}} R_e)^{1/2} \\
&= (\langle Q_{ku}\rangle_L R_e)^{1/2} \\
&= [595.1(\text{W}) \times 20(\text{ohm})]^{1/2} \\
&= 109.1 \text{ V.}
\end{aligned}
$$

From (2.32), we have

$$
J_e = \frac{\Delta\varphi}{R_e} = \frac{109.1 \text{ V}}{20(\text{ohm})} = 5.455 \text{ A.}
$$

COMMENT:

Boiling water requires a large electric power per unit area. For this reason, stored electric power, such as in batteries, can not be used. Also, note that from (2.32), we have

$$
R_e = \frac{\rho_e l}{A_e},
$$

where A_e is the electrical conductor cross-section area. To produce a large electrical resistance, a small A_e is used (see Example 2.10).

PROBLEM 6.23.FAM

GIVEN:

To reduce the air conditioning load, the roof of a commercial building is cooled by a water spray. The roof is divided into segments with each having a dedicated sprinkler, as shown in Figure Pr.6.23(a). Assume that the impinging-droplet film evaporation relation of Table 6.6 can be used here.

$L = 4$ m, $T_{f,\infty} = 30$℃, $T_s = 210$℃, $\langle D \rangle = 100$ μm, $\langle u_d \rangle = 2.5$ m/s, $\langle \dot{m}_d \rangle / \rho_{l,\infty} = 10^{-3}$ m/s.

Evaluate the water properties at $T = 373$ K.

SKETCH:

Figure Pr.6.23(a) shows the water sprinkler and the roof panel.

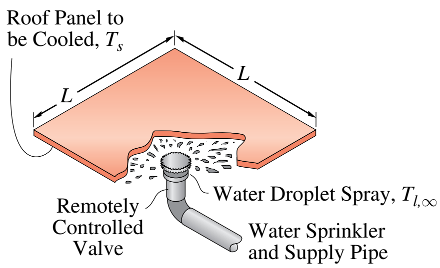


Figure Pr.6.23(a) Water-spray cooling of a roof panel.

OBJECTIVE:

(a) Draw the thermal circuit diagram for the panel surface.

(b) Using the conditions given above, determine the rate of surface-convection heat transfer $\langle Q_{ku} \rangle_L$ from the roof panel.

SOLUTION:

(a) Figure Pr.6.23(b) shows the thermal circuit diagram

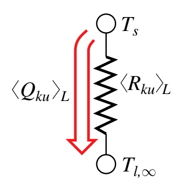


Figure Pr.6.23(b) Thermal circuit diagram.

(b) The rate of surface-convection heat transfer is given by (6.49) as

$$\langle Q_{ku} \rangle_L = \frac{T_s - T_{l,\infty}}{\langle R_{ku} \rangle_L} = A_{ku}(T_s - T_{l,\infty})\langle \mathrm{Nu} \rangle_L \frac{k_l}{L}, \quad A_{ku} = L^2.$$

From Table 6.5, we have

$$\begin{aligned}
\langle Q_{ku} \rangle_L &= A_{ku}\rho_{l,\infty}\Delta h_{lg,\infty}\left(\frac{\langle \dot{m}_d \rangle}{\rho_{l,\infty}}\right)\eta_d\left[1 - \frac{\langle \dot{m}_d \rangle/\rho_{l,\infty}}{(\langle \dot{m}_d \rangle/\rho_l)_\circ}\right] + \\
&\quad A_{ku} \times 1{,}720(T_s - T_{l,\infty})^{0.912}\langle D \rangle^{-1.004}\langle u_d \rangle^{-0.764}\frac{(\langle \dot{m}_d \rangle/\rho_{l,\infty})^2}{(\langle \dot{m}_d \rangle/\rho_l)_\circ}, \\
\left(\frac{\dot{m}_d}{\rho_{l,\infty}}\right)_o &= 5 \times 10^{-3} \text{ m/s} \\
\eta_d &= \frac{3.68 \times 10^4}{\rho_{l,\infty}\Delta h_{lg,\infty}}(T_s - T_{l,\infty})^{1.691}\langle D \rangle^{-0.062}.
\end{aligned}$$

From Tables C.4, and C.23, we have for water at $T = 373$ K

$$\Delta h_{lg} = 2.256 \times 10^6 \text{ J/kg} \qquad \text{Table C.4}$$
$$(c_{p,l})_\infty = 4{,}218 \text{ J/kg-K} \qquad \text{Table C.23}$$
$$\rho_{l,\infty} = 960.2 \text{ kg/m}^3 \qquad \text{Table C.23,}$$

Then

$$\begin{aligned}\Delta h_{lg,\infty} &= (c_{p,l})_\infty(T_{lg} - T_{l,\infty}) + \Delta h_{lg}\\ &= 4{,}218(\text{J/kg-K}) \times (100-30)(\text{K}) + 2.256 \times 10^6(\text{J/kg})\\ &= 2.551 \times 10^6 \text{ J/kg}\end{aligned}$$

Next,

$$\begin{aligned}\eta_d &= \frac{3.68 \times 10^4}{960.2(\text{kg/m}^3) \times 2.251 \times 10^6(\text{J/kg})} \times (210-30)^{1.691}(\text{K})^{1.691} \times (1 \times 10^{-4})^{-0.062}(\text{m})^{-0.062}\\ &= 0.1732.\end{aligned}$$

For $\langle Q_{ku}\rangle_L$, we have

$$\begin{aligned}\langle Q_{ku}\rangle_L &= (4)^2(\text{m})^2 \times 960.5(\text{kg/m}^3) \times 2.541 \times 10^6(\text{J/kg}) \times\\ &\quad 10^{-3}(\text{m/s}) \times 0.1732 \times \left[1 - \frac{10^{-3}(\text{m/s})}{5 \times 10^{-3}(\text{m/s})}\right] + (4)^2(\text{m})^2 \times 1{,}720 \times (210-30)^{0.912}(\text{K})^{0.912} \times\\ &\quad (10^{-4})^{-1.004}(\text{m})^{-1.004}(2.5)^{-0.746}(\text{m/s})^{-0.746}\frac{(10^{-3})^2(\text{m/s})^2}{5 \times 10^{-3}(\text{m/s})}\\ &= 5.432 \times 10^6(\text{W}) + 3.286 \times 10^6(\text{W})\\ &= 8.718 \times 10^6 \text{ W}.\end{aligned}$$

COMMENT:

This cooling rate is held only for a short time to reduce the roof panel temperature to a low, safe and desirable value. Note that water spray cooling is economical and when no high-humidity damage is expected, this cooling method can be effective.

PROBLEM 6.24.FAM

GIVEN:

In using water evaporation in surface-convection heat transfer, compare pool boiling by saturated water ($T_{l,\infty} = T_{lg}$), as shown in Figure 6.20(b), and droplet impingement by subcooled water droplets ($T_{l,\infty} < T_{lg}$) as shown in Figure 6.26. For pool boiling, the peak in $\langle Q_{ku} \rangle_L$ is given by the critical heat flux, i.e., (6.100), and the minimum is given by (6.101). For impinging droplets, the peak shown in Figure 6.26 is nearly independent of the droplet mass flux, in the high mass flux regime, and the minimum is approximately correlated by (6.116). The correlations are also listed in Table 6.6.

Pool boiling: $T_{lg} = 100$°C, $T_s = 300$°C.

Impinging droplets: $\langle \dot{m}_d \rangle = 1.43$ kg/m^2-s, $\langle u_d \rangle = 3.21$ m/s, $\langle D \rangle = 480$ μm, $T_{l,\infty} = 20$°C, $T_{lg} = 100$°C, $T_s = 300$°C, evaluate properties at 310 K.

Note that not all these conditions are used in every case considered.

SKETCH:

Figure Pr.6.24 shows the two surface cooling methods using water evaporation.

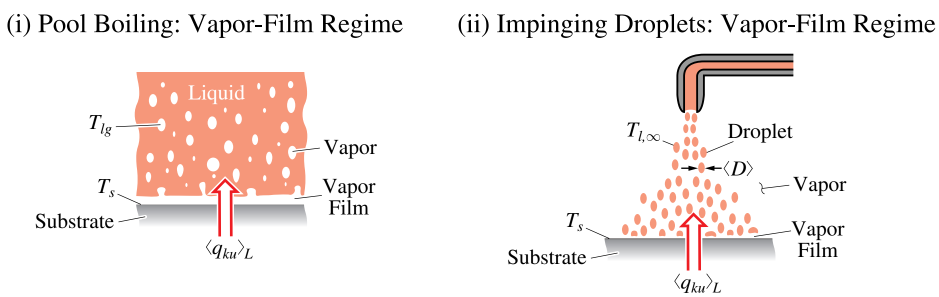


Figure Pr.6.24 Selection of a water evaporation surface cooling method, between (i) pool boiling and (ii) impingement droplets.

OBJECTIVE:

Select between water pool boiling [Figure Pr.6.24(i)] and droplet impingement [Figure Pr.6.24(ii)], by comparing (i) peak, and (ii) minimum, surface-convection heat flux $\langle q_{ku} \rangle_L$.

SOLUTION:

(a) Peak (or critical) heat flux:

(i) For pool boiling, from Table 6.6, we have

$$\langle q_{ku} \rangle_L = q_{ku,CHF} = 0.13 \rho_g \Delta h_{lg} \left(\frac{\sigma \Delta \rho_{lg}}{\rho_g^2} \right)^{1/4}.$$

From Table C.26, for water at $T = 373.15$ K, we have

$$\begin{aligned} \rho_l &= 958.3 \text{ kg/m}^3 \quad \text{Table C.26} \\ \rho_g &= 0.596 \text{ kg/m}^3 \quad \text{Table C.26} \\ \Delta h_{lg} &= 2.257 \times 10^6 \text{ J/kg} \quad \text{Table C.26} \\ \sigma &= 0.05891 \text{ N/m} \quad \text{Table C.26.} \end{aligned}$$

Then,

$$\begin{aligned} \langle q_{ku} \rangle_L &= q_{ku,CHF} = 0.13 \times 0.596(\text{kg/m}^3) \times 2.257 \times 10^6(\text{J/kg}) \times \\ & \left[\frac{0.05891(\text{N/m}) \times 9.81(\text{m/s}^2) \times (958.3 - 0.596)(\text{kg/m}^3)}{(0.596)^2(\text{kg/m}^3)^2} \right]^{1/4} \\ &= 1.099 \times 10^6 \text{ W/m}^2. \end{aligned}$$

This is close to the value shown in Figure 6.20(b).

(ii) For impinging droplets, reading from Figure 6.26 for $\langle \dot{m}_d \rangle = 1.43$ kg/m^2-s, we have

$$\langle q_{ku} \rangle_L = q_{ku,peak} = 2 \times 10^6 \text{ W/m}^2 \quad \text{Figure 6.26.}$$

Thus the impinging droplets gives a higher $\langle q_{ku} \rangle_L$.

(b) Minimum heat flux:
(i) For pool boiling, from Table 6.6, we have

$$\begin{aligned}
\langle q_{ku} \rangle_L &= q_{ku,min} = 0.09 \rho_g \Delta h_{lg} \left(\frac{\sigma g \Delta \rho_{lg}}{\rho_l + \rho_g} \right)^{1/4} . \\
&= 0.09 \times 0.596(\text{kg/m}^3) \times 2.257 \times 10^6 (\text{J/kg}) \times \\
& \left[\frac{0.0591(\text{N/m}) \times 9.81(\text{m/s}^2) \times (958.3 - 0.596)(\text{kg/m}^3)}{(958.3 + 0.596)^2 (\text{kg/m}^3)^2} \right]^{1/4} \\
&= (1.211 \times 10^5 \times 0.1567)(\text{W/m}^2) \\
&= 1.896 \times 10^4 \text{ W/m}^2.
\end{aligned}$$

(ii) For impinging droplets, we have from Table 6.6,

$$\begin{aligned}
\langle q_{ku} \rangle_L &= \rho_{l,\infty} \Delta h_{lg,\infty} \frac{\dot{m}_d}{\rho_{l,\infty}} \eta_d \left[1 - \frac{\langle \dot{m}_d \rangle / \rho_{l,\infty}}{(\langle \dot{m}_d \rangle / \rho_l)_\circ} \right] + \\
& 1{,}720 (T_s - T_{l,\infty})^{0.912} \langle D \rangle^{-1.004} \langle u_d \rangle^{-0.764} \frac{(\langle \dot{m}_d \rangle / \rho_{l,\infty})^2}{(\langle \dot{m}_d \rangle / \rho_l)_\circ} .
\end{aligned}$$

From Table C.23, at $T = 310$ K, we have

$$\begin{aligned}
\rho_{l,\infty} &= 995.3 \text{ kg/m}^3 \quad \text{Table C.23} \\
c_{p,l} &= 4{,}178 \text{J/kg-K} \quad \text{Table C.23.}
\end{aligned}$$

Then from Table 6.6, we have

$$\begin{aligned}
\Delta h_{lg,\infty} &= (c_{p,l})_\infty (T_s - T_{l,\infty}) + \Delta h_{lg} \\
&= 4{,}178(\text{J/kg-K}) \times (100 - 20)(\text{K}) + 2.257 \times 10^6 (\text{J/kg}) \\
&= 2.591 \times 10^6 \text{ J/kg}. \\
\eta_d &= \frac{3.68 \times 10^4}{\rho_{l,\infty} \Delta h_{lg,\infty}} (T_s - T_{l,\infty})^{1.691} \langle D \rangle^{-0.062} \\
&= \frac{3.68 \times 10^4}{995.3(\text{kg/m}^3) \times 2.591 \times 10^6 (\text{J/kg})} (300 - 20)^{1.691} (\text{K})^{1.691} (4.80 \times 10^{-4})^{-0.062} (\text{m})^{-0.062} \\
&= 0.3150.
\end{aligned}$$

Then

$$\begin{aligned}
\langle q_{ku} \rangle_L &= 995.3(\text{kg/m}^3) \times 2.591 \times 10^6 (\text{J/kg}) \times \\
& \frac{1.43(\text{kg/m}^2\text{-s})}{995.3(\text{kg/m}^3)} \times 0.3150 \times \left[1 - \frac{1.43(\text{kg/m}^2\text{-s})}{995.3(\text{kg/m}^3) \times 5 \times 10^{-3} (\text{m/s})} \right] + 1{,}720(300 - 20)^{0.912} (\text{K})^{0.912} \times \\
& (4.8 \times 10^{-4})^{-1.004} (\text{m})^{-1.004} (3.21)^{-0.746} (\text{m/s})^{-0.746} \frac{(1.43)^2 (\text{kg/m}^2\text{-s})^2}{(995.3)^2 (\text{kg/m}^3) \times 5 \times 10^{-3} (\text{m/s})} \\
&= (8.317 \times 10^5 + 1.090 \times 10^5)(\text{W/m}^2) \\
&= 9.407 \times 10^5 \text{ W/m}^2.
\end{aligned}$$

This is much larger than that for the vapor-film pool boiling.

COMMENT:
Here droplet impingement gives higher heat fluxes. Note that the correlation used for vapor-film regime of impingement droplets uses a temperature dependence that is much stronger than that found in the experimental results given in Figure 6.26.

PROBLEM 6.25.FUN

GIVEN:

A person caught in a cold cross wind chooses to curl up (crouching as compared to standing up) to reduce the surface-convection heat transfer from his clothed body. Figure Pr.6.25(a) shows two idealized geometries for the person while crouching [Figure Pr.6.25(a)(i)] and while standing up [Figure Pr.6.25(a)(ii)].

$D_s = 50$ cm, $D_c = 35$ cm, $L_c = 170$ cm, $T_1 = 12$°C, $T_{f,\infty} = -4$°C, $u_{f,\infty} = 5$ m/s, $k_i = 0.1$ W/m-K.

Use air properties (Table C.22) at $T = 300$ K.

SKETCH:

Figure Pr.6.25(a) shows the two positions.

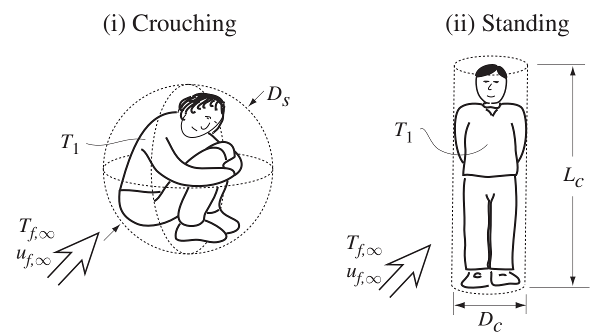


Figure Pr.6.25(a) Two positions by a person in a cold cross flow of air. (i) Crouching position. (ii) Standing position.

OBJECTIVE:

(a) Draw the thermal circuit diagram and determine the heat transfer rate for the idealized spherical geometry.
(b) Draw the thermal circuit diagram and determine the heat transfer rate for the idealized cylindrical geometry. Neglect the heat transfer from the ends of the cylinder.
(c) Additional insulation (with thermal conductivity k_i) is to be worn by the standing position to reduce the surface-convection heat transfer to that equal to the crouching position. Draw the thermal circuit diagram and determine the necessary insulation thickness L to make the two surface convection heat transfer rates equal. Assume that T_1 and the surface-convection resistance for the cylinder will remain the same as in part (b).
(d) What is the outside-surface temperature T_2 of the added insulation?

SOLUTION:

From Table C.22, the properties of air at $T = 300$ K are $\rho_f = 1.177$ kg/m^3, $\nu_f = 15.66 \times 10^{-6}$ m^2/s, Pr= 0.69, and $k_f = 0.0267$ W/m-K.

(a) Sphere: The thermal circuit diagram is shown in Figure Pr.6.25(b).

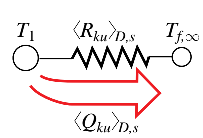


Figure Pr.6.25(b) Thermal circuit diagram.

The average surface-convection heat transfer rate from the sphere is then

$$\langle Q_{ku}\rangle_{D,s} = \frac{T_1 - T_{f,\infty}}{\langle R_{ku}\rangle_{D,s}},$$

where

$$\langle R_{ku}\rangle_{D,s} = \frac{D_s}{A_{ku,s}\langle \text{Nu}\rangle_{D,s} k_f}.$$

The surface area for convection is the area of the sphere, or

$$A_{ku,s} \quad = \quad \pi D_s^2 = \pi \times 0.5^2(\text{m}^2) = 0.7854 \text{ m}^2.$$

The Nusselt number for the sphere, from Table 6.4, is

$$\langle \text{Nu} \rangle_{D,s} = 2 + [0.4\text{Re}_{D,s}^{1/2} + 0.06\text{Re}_{D,s}^{2/3}]\text{Pr}^{0.4}.$$

The Reynolds number is found as

$$\text{Re}_{D,s} \quad = \quad \frac{u_{f,\infty} D_s}{\nu_f} = \frac{5(\text{m/s}) \times 0.5(\text{m})}{15.66 \times 10^{-6}(\text{m}^2/\text{s})} = 1.596 \times 10^5.$$

There was no Reynolds number restrictions given for the applicability of the Nusselt number correlation, so we will assume the correlation is valid.
Then the Nusselt number becomes

$$\langle \text{Nu} \rangle_{D,s} \quad = \quad 2 + (0.4(1.596 \times 10^5)^{1/2} + 0.06(1.596 \times 10^5)^{2/3})0.69^{0.4} = 291.9,$$

and the surface-convection heat transfer resistance becomes

$$\begin{aligned} \langle R_{ku} \rangle_{D,s} &= \frac{0.5(\text{m})}{0.7854(\text{m}^2) \times 291.9 \times 0.0267(\text{w/m-K})} \\ &= 0.08167^\circ\text{C/W}. \end{aligned}$$

The surface convection heat transfer from the sphere is then

$$\langle Q_{ku} \rangle_{D,s} \quad = \quad \frac{12^\circ\text{C} - (-4^\circ\text{C})}{0.08167^\circ\text{C/W}} = 195.9 \text{ W}.$$

(b) Cylinder: The thermal circuit diagram is shown in Figure Pr.6.25(c). The average surface-convection heat

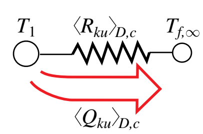


Figure Pr.6.25(c) Thermal circuit diagram.

transfer rate from the cylinder (neglecting the heat transfer from the ends) is then

$$\langle Q_{ku} \rangle_{D,c} = \frac{T_1 - T_{f,\infty}}{\langle R_{ku} \rangle_{D,c}},$$

where

$$\langle R_{ku} \rangle_{D,c} = \frac{D_c}{A_{ku,c} \langle \text{Nu} \rangle_{D,c} k_f}.$$

The surface area for convection is the area of the cylinder not including the end areas, or

$$A_{ku,c} \quad = \quad \pi D_c L_c = \pi \times 0.35(\text{m}) \times 1.70(\text{m}) = 1.869 \text{ m}^2.$$

The Nusselt number for a cylinder in cross flow, from Table 6.3, is Reynolds number dependent and has the form

$$\langle \text{Nu} \rangle_{D,c} = a_1 \text{Re}_{D,c}^{a_2} \text{Pr}^{1/3}.$$

The Reynolds number is found as

$$\text{Re}_{D,c} \quad = \quad \frac{u_{f,\infty} D_c}{\nu_f} = \frac{5(\text{m/s}) \times 0.35(\text{m})}{15.66 \times 10^{-6}(\text{m}^2/\text{s})} = 1.1175 \times 10^5.$$

For this Reynolds number, the constants are given in Table 6.3 as $a_1 = 0.027$ and $a_2 = 0.805$. Then the Nusselt number becomes

$$\langle \text{Nu} \rangle_{D,c} = 0.027 \times (1.1175 \times 10^5)^{0.805} \times (0.69)^{1/3} = 276.4,$$

and the surface-convection heat transfer resistance becomes

$$\langle R_{ku} \rangle_{D,c} = \frac{0.35(\text{m})}{1.869(\text{m}^2) \times 276.4 \times 0.0267(\text{w/m-K})} = 0.02538℃/\text{W}.$$

The surface convection heat transfer from the sphere is then

$$\langle Q_{ku} \rangle_{D,c} = \frac{12℃ - (-4℃)}{0.0254℃/\text{W}} = 630.5 \text{ W}.$$

Which is significantly higher than that for the sphere.

(c) Added insulation: The thermal circuit diagram is shown in Figure Pr.6.25(d). Insulation of thermal con-

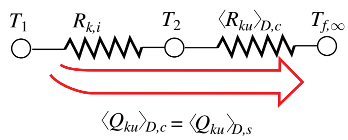


Figure Pr.6.25(d) Thermal circuit diagram.

ductivity $k_i = 0.1$ W/m-K and thickness L (to be determined) is added to the cylinder such that the average surface-convection heat transfer rate from the cylinder is equal to that from the sphere. The surface-convection resistance from the cylinder $\langle R_{ku} \rangle_{D,c}$ and the temperature T_1 are assumed the same as in Part (b). Therefore,

$$\langle Q_{ku} \rangle_{D,c} = \frac{T_1 - T_{f,\infty}}{R_{k,i} + \langle R_{ku} \rangle_{D,c}} = \langle Q_{ku} \rangle_{D,s} = \frac{T_1 - T_{f,\infty}}{\langle R_{ku} \rangle_{D,s}}.$$

Given that $\langle Q_{ku} \rangle_{D,c} = \langle Q_{ku} \rangle_{D,s}$, we then have

$$R_{k,i} + \langle R_{ku} \rangle_{D,c} = \langle R_{ku} \rangle_{D,s}$$

or

$$\begin{aligned} R_{k,i} &= \langle R_{ku} \rangle_{D,s} - \langle R_{ku} \rangle_{D,c} \\ &= 0.08167℃/\text{W} - 0.02538℃/\text{W} = 0.05629℃/\text{W}. \end{aligned}$$

For the cylindrical system, the conduction resistance in the radial direction across the insulation thickness $L = R_2 - R_1$ is

$$R_{k,i} = \frac{\ln(R_2/R_1)}{2\pi L_c k_i}$$

or

$$\begin{aligned} R_2/R_1 &= \exp(R_{k,i} 2\pi L_c k_i) \\ &= \exp[0.05629(℃/\text{W}) \times 2 \times \pi \times 1.7(\text{m}) \times 0.1(\text{W/m-K})] = 1.062. \end{aligned}$$

The inner radius is the uninsulated radius of the cylinder and is

$$R_1 = D_c/2 = 0.35(\text{m})/2 = 0.175 \text{ m},$$

and then

$$\begin{aligned} R_2 &= R_1 \times 1.062 \\ &= 0.175(\text{m}) \times 1.062 = 0.1858 \text{ m}. \end{aligned}$$

The thickness of insulation is then

$$\begin{aligned} L &= R_2 - R_1 \\ &= 0.186(\text{m}) - 0.175(\text{m}) \\ &= 0.0108 \text{ m} = 1.08 \text{ cm}. \end{aligned}$$

(d) The outside surface temperature of the insulation can be found by applying the energy equation between node T_2 and T_1 in Figure Pr.6.33(d) as

$$\langle Q_{ku} \rangle_{D,s} = \frac{T_1 - T_2}{\langle R_{k,i} \rangle}.$$

Then

$$\begin{aligned} T_2 &= T_1 - \langle Q_{ku} \rangle_{D,s} \times \langle R_{k,i} \rangle \\ &= 12(^\circ\text{C}) - 195.9(\text{W}) \times 0.05629(^\circ\text{C/W}) = 0.97^\circ\text{C}. \end{aligned}$$

COMMENT:

Note that the two Nusselt numbers are nearly the same, but the heat transfer rates are different to the surface area.

PROBLEM 6.26.FUN

GIVEN:

A thermocouple is placed in an air stream to measure the stream temperature, as shown in Figure Pr.6.26(a). The steady-state temperature of the thermocouple bead of diameter D is determined through its surface-convection (as a sphere in a semi-bounded fluid stream) and surface-radiation heat transfer rates.

$u_{f,\infty} = 2$ m/s, $T_{f,\infty} = 600$°C, $T_w = 400$°C, $\epsilon_{r,w} = 0.9$, $\epsilon_{r,s} = 0.8$, $D = 1$ mm.

Neglect the heat transfer to and from the wires and treat the surface-convection heat transfer to the thermocouple bead as a semi-bounded fluid flow over a sphere.

Assume the tube length L is large (i.e, $L \rightarrow \infty$). Evaluate the fluid properties at $T = 350$ K (Table C.22).

Numerical hint: The thermocouple bead temperature should be much closer to the air stream temperature than to the tube surface temperature. For iterations, start with a a guess of $T = 820$ K.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine the thermocouple bead temperature T_s.
(c) Comment on the difference between T_s and $T_{f,\infty}$. How can the difference (measurement error) be reduced?

SKETCH:

Figure Pr.6.26(a) shows the thermocouple placed in a tube to measure the fluid stream temperature at a location.

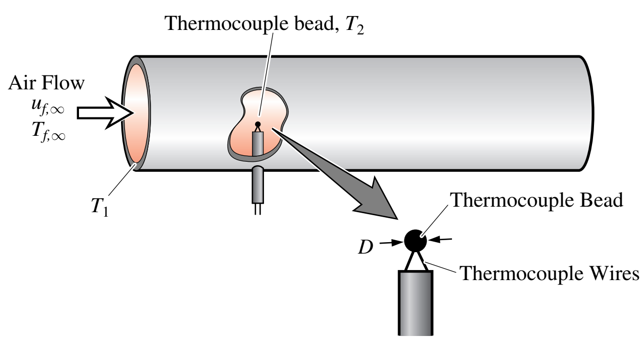


Figure Pr.6.26(a) A thermocouple used for measuring a fluid stream temperature.

SOLUTION:

From Table C.22, the properties of air at $T = 350$ K are $\nu_f = 20.30 \times 10^{-6}$ m^2/s, Pr$= 0.69$, and $k_f = 0.030$ W/m-K.

(a) The thermal circuit diagram is shown in Figure Pr.6.26(b).

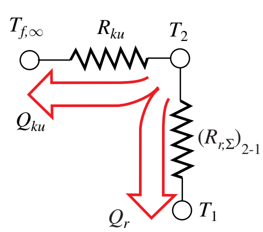


Figure Pr.6.26(b) Thermal circuit diagram.

(b) The conservation of energy applied to the spherical thermocouple bead gives

$$\begin{aligned} Q_r \quad &+ \quad Q_{ku} &= 0 \\ \frac{\sigma_{\rm SB}(T_2^4 - T_1^4)}{(R_{r,\Sigma})_{2\text{-}1}} &+ \frac{T_2 - T_{f,\infty}}{R_{ku}} &= 0. \end{aligned}$$

The radiation resistances are found as

$$(R_{r,\Sigma})_{2\text{-}1} = (R_{r,\epsilon})_2 + (R_{r,F})_{2\text{-}1} + (R_{r,\epsilon})_1.$$

Since $L \rightarrow \infty$, then we can assume $A_1 \rightarrow \infty$. Also, since L is very large compared to the tube diameter, we can neglect the tube ends on the radiation to the thermocouple bead. The view factor from the bead to the tube can then be assumed $F_{1\text{-}2} \approx 1$. The area of the bead is $A_2 = \pi D^2 = \pi \times (0.001)^2 = 3.142 \times 10^{-6}$ m^2. Thus we have

$$\begin{aligned}(R_{r,\Sigma})_{2\text{-}1} &= \frac{1-\epsilon_{r,2}}{A_2\epsilon_{r,2}} + \frac{1}{A_2F_{2\text{-}1}} + \frac{1-\epsilon_{r,1}}{A_1\epsilon_{r,1}} \\ &= \frac{1-0.8}{3.142\times10^{-6}(\text{m}^2)\times 0.8} + \frac{1}{3.142\times10^{-6}(\text{m}^2)\times(1)} + 0 \\ &= 7.957\times10^4\ \text{m}^{-2} + 3.183\times10^5\ \text{m}^{-2} = 3.978\times10^5\ \text{m}^{-2}.\end{aligned}$$

For the surface-convection resistance, we have

$$R_{ku} = \frac{1}{A_2\langle\text{Nu}\rangle_D k_f/D},$$

where the $\langle\text{Nu}\rangle_D$ is found from Table 6.4 as

$$\langle\text{Nu}\rangle_D = 2 + (0.4\text{Re}_D^{1/2} + 0.06\text{Re}_D^{2/3})\text{Pr}^{0.4}.$$

with the Re_D determined as

$$\text{Re}_D = \frac{u_{f,\infty}D}{\nu_f} = \frac{2(\text{m/s})\times0.001(\text{m})}{20.30\times10^{-6}(\text{m}^2/\text{s})} = 98.52.$$

The Nusselt number is then

$$\langle\text{Nu}\rangle_D = 2 + [0.4(98.52)^{1/2} + 0.06(98.52)^{2/3}]0.69^{0.4} = 6.526,$$

and the surface-convection resistance is

$$R_{ku} = \frac{1}{3.142\times10^{-6}(\text{m}^2)\times6.526\times0.030(\text{W/m-K})/0.001(\text{m})} = 1626^\circ\text{C/W}.$$

The energy balance cannot be solved explicitly for T_2, therefore we must use a solver or iterate. We can rearrange to facilitate iteration as

$$\begin{aligned}{}^{\text{new}}T_2 &= T_{f,\infty} - \frac{R_{ku}}{(R_{r,\Sigma})_{2\text{-}1}}\sigma_{\text{SB}}({}^{\text{old}}T_2^4 - T_1^4) \\ &= 873.15(\text{K}) - \frac{1626(^\circ\text{C/W})}{3.978\times10^5(\text{m}^{-2})}\times5.67\times10^{-8}(\text{W/m}^2\text{-K}^4)\times({}^{\text{old}}T_2^4 - 2.053\times10^{11})(\text{K}^4) \\ &= 873.15(\text{K}) - 2.3176\times10^{-10}(\text{K}^{-3})\times({}^{\text{old}}T_2^4 - 2.053\times10^{11})(\text{K}^4).\end{aligned}$$

To iterate, we guess an initial $T_2 = {}^{\text{old}}T_2$ and calculate ${}^{\text{new}}T_2$. Then the ${}^{\text{new}}T_2$ of the previous iteration becomes the ${}^{\text{old}}T_2$ of the next, and so on, until $|{}^{\text{new}}T_2 - {}^{\text{old}}T_2| <$ criterion (i.e., the same). Faster convergence can be achieved averaging the old and new values of the previous iteration to become the old value of the next iteration. Table Pr.6.26 shows the values at the iteration steps, using averaged guesses and a criterion of 0.1 K. The initial T_2 is taken to be closer to the fluid stream temperature $T_{f,\infty}$ at ${}^{\text{old}}T_2 = 820$ K.

Table Pr.6.26 Results of Numerical Iteration.

iteration	${}^{\text{old}}T_2$, K	${}^{\text{new}}T_2$, K	average	difference
1	820	815.95	817.97	4.05
2	817.97	816.98	817.43	0.99
3	817.43	817.25	817.34	0.18
4	817.34	817.30	817.32	0.04

Therefore, the bead temperature is $T_2 \simeq 817.3$ K.

(c) The temperature difference $T_2 - T_{f,\infty} = 55.8$ K is rather large and is due to the heat loss from the thermocouple bead. The heat loss is by radiation and conduction along the wires. At high temperatures, radiation heat transfer can become significant compared to the surface-convection heat transfer. To reduce the measurement error, the radiation losses from the bead must be reduced. This can be accomplished by placing a thin, cylindrical radiation shield around the bead that would be heated by the fluid stream to a temperature nearer to the thermocouple bead temperature and that would not greatly disturb the fluid flow around the bead. The shield increases the net sum radiation resistance between the bead and the tube wall. The measurement error can also be reduced by decreasing the bead diameter to reduce the area for radiation heat transfer.

COMMENT:

Note that errors due to the bead and wire surface-radiation heat transfer and conduction along the wire (where placed along a nonuniform temperature field) should be reduced for accurate measurements.

PROBLEM 6.27.FAM

GIVEN:

In order to prevent the flame from blow-off by a cross wind, a lighter is desired with a flame anchor (i.e., flame holder) in the from of a winding wire placed in the air-fuel stream undergoing combustion. This is shown in Figure Pr.6.27(a)(i). The wire retains (through its heat storage) a high temperature and will maintain the flame around it, despite a large, intermittent cross flow. Assume that the combustion of the n-butane in air is complete before the gas stream at temperature $T_{f,\infty}$ and velocity $u_{f,\infty}$ reach the flame holder. Treat the flame holder as a long cylinder with steady-state, surface-convection heating and surface-radiation cooling. The simplified heat transfer model is also shown, in Figure Pr.6.27(a)(ii).

$T_{f,\infty} = 1{,}300$℃, $D = 0.3$ mm, $T_{surr} = 30$℃, $\epsilon_{r,s} = 0.8$.

Use the adiabatic, laminar flame speed, for the stoichiometric n-butane in air, from Table C.21(a), for the far-field fluid velocity $u_{f,\infty}$.

You do not need to use tables or graphs for the view factors. Assume the properties of the combustion products are those of air at $T = 900$ K.

SKETCH:

Figure Pr.6.27(a) shows the flame holder and a simplified heat transfer model for the wire.

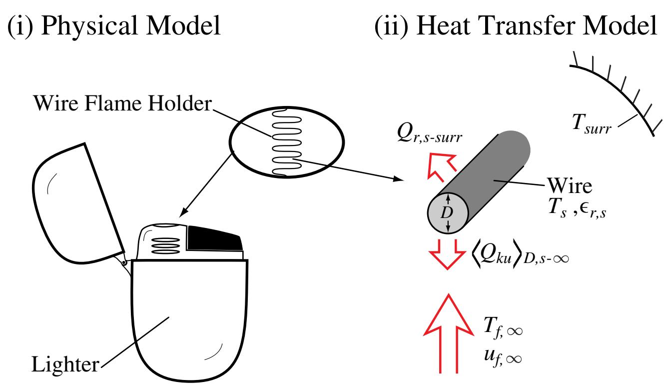


Figure Pr.6.27(a)(i) A winding-wire, flame holder used in a lighter. (ii) A simplified heat transfer model for the wire.

OBJECTIVE:

(a) Draw the thermal circuit for the flame holder.

(b) Determine the flame holder temperature T_s for the given conditions.

SOLUTION:

(a) Figure Pr.6.27(b) shows the thermal circuit diagrams. The heat is added by surface convection to the wire and is removed by radiation to the surroundings. The surface area for surface radiation and convection are the same i.e.,

$$A_{r,s} = A_{ku,s} = A_s.$$

(b) From Figure Pr.6.27(b), the energy equation is

$$Q|_{A,s} = \langle Q_{ku} \rangle_{D,s\text{-}\infty} + Q_{r,s\text{-}surr} = 0.$$

The surface convection is given by (6.124) as

$$\begin{aligned} \langle Q_{ku} \rangle_{D,s\text{-}\infty} &= \frac{T_s - T_{f,\infty}}{(R_{ku})_D} \\ &= A_{ku} \langle \mathrm{Nu} \rangle_D \frac{k_f}{D} (T_s - T_{f,\infty}), \end{aligned}$$

where $\langle \mathrm{Nu} \rangle_D$ is found from Table 6.3 for cross, forced flow over a cylinder, i.e.,

$$\langle \mathrm{Nu} \rangle_D = a_1 \mathrm{Re}_D^{a_2} \mathrm{Pr}^{1/3}, \quad \mathrm{Re}_D = \frac{u_{f,\infty} D}{\nu_f},$$

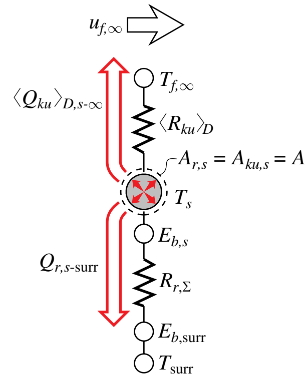


Figure Pr.6.27(b) Thermal circuit diagram.

and a_1 and a_2 depend on the magnitude of Re_D.

From Table C.21(a), for n-butane-air, we have

$$u_{f,\infty} = u_{f,1} = 0.379 \text{ m/s} \quad \text{Table C.21(a)}.$$

From Table C.22, for air at $T = 900$ K, we have

$$\begin{aligned} k_f &= 0.0625 \text{ W/m-K} && \text{Table C.22} \\ \nu_f &= 9.860 \times 10^{-5} \text{ m}^2\text{/s} && \text{Table C.22} \\ \text{Pr} &= 0.7 && \text{Table C.22.} \end{aligned}$$

Then

$$\begin{aligned} \text{Re}_D &= \frac{0.379(\text{m/s}) \times 3 \times 10^{-4}(\text{m})}{9.860 \times 10^{-5}(\text{m}^2\text{/s})} = 1.153 \\ a_1 &= 0.683, \quad a_2 = 0.466 \quad \text{Table 6.3} \\ \langle \text{Nu}\rangle_D &= 0.683 \times (1.153)^{0.466} \times (0.7)^{1/3} = 0.6481. \end{aligned}$$

The surface radiation for $A_{surr} \gg A_s$ and $F_{s\text{-}surr} = 1$, is given by (4.49), i.e.,

$$Q_{r,s\text{-}surr} = A_{r,s}\epsilon_{r,s}\sigma_{\text{SB}}(T_s^4 - T_{surr}^4).$$

Then the energy equation becomes

$$A_s\langle \text{Nu}\rangle_D \frac{k_f}{D}(T_s - T_{f,\infty}) + A_s\epsilon_{r,s}\sigma_{\text{SB}}(T_s^4 - T_{surr}^4) = 0$$

or

$$\langle \text{Nu}\rangle_D \frac{k_f}{D}(T_s - T_{f,\infty}) + \epsilon_{r,s}\sigma_{\text{SB}}(T_s^4 - T_{surr}^4) = 0.$$

Using the numerical values, we have

$$0.6481 \times \frac{0.0625(\text{W/m-K})}{3 \times 10^{-4}(\text{m})} \times (T_s - 1{,}573)(\text{m}) + 0.8 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}) \times [T_s^4 - (303.15)^4(\text{K}^4)] = 0$$

which gives an algebraic equation in T_s,

$$1.350 \times 10^2 \times [T_s - 1{,}573(\text{K})] + 4.536 \times 10^{-8} \times (T_s^4 - 8.446 \times 10^9) = 0.$$

Solving for T_s, we have

$$T_s = 1{,}094 \text{ K}.$$

COMMENT:

The wire can be made of tungsten to be resistive to oxidation at such high temperatures. Note that $\langle \text{Nu}\rangle_D$ is less than unity. This is because as $\text{Re}_D \to 0$, $\langle \text{Nu}\rangle_D$ for long cylinders tends to zero (unlike that for spheres for which $\langle \text{Nu}\rangle_D \to 2$ in the conduction limit). This is due to the available conduction area as $R_2 \to \infty$ in (3.61), as compared to (3.64).

PROBLEM 6.28.FUN

GIVEN:

Consider measuring the temperature $T_{f,\infty}$ of an air stream using a thermocouple. A thermocouple is a junction made of two dissimilar materials (generally metals, as discussed in Section 2.3.2), as shown in Figure Pr.6.28. The wires are electrically (and if needed, thermally) insulated. The wire may not be in thermal equilibrium with the stream. This can be due to the nonuniformity of temperature along the wire. Then the temperature of the thermocouple bead (i.e., its tip) $T_{s,L}$ may not be close enough to $T_{f,\infty}$, for the required accuracy. Consider an air stream with a far-field temperature $T_{f,\infty} = 27$°C and a far-field velocity $u_{f,\infty} = 5$ m/s. Assume that the bare (not insulated) end of the wire is at temperature $T_{s,o} = 15$°C. Consider one of the thermocouple wires made of copper, having a diameter $D = 0.2$ mm and a bare-wire length $L = 5$ mm.

Evaluate the properties of air at 300 K.

SKETCH:

Figure Pr.6.28 shows the thermocouple junction and the idealized model for heat transfer from one of its wires.

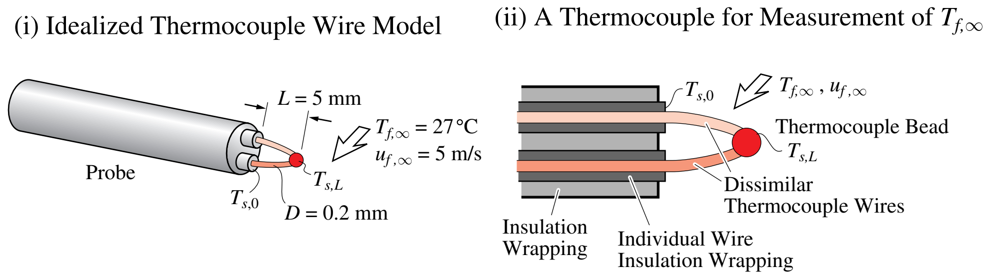


Figure Pr.6.28 A thermocouple junction used for temperature measurement in an air stream. (i) Idealized thermocouple wire model. (ii) Thermocouple for measurement of $T_{f,\infty}$.

OBJECTIVE:

Using the extended surface analysis, determine the expected uncertainty $T_{f,\infty} - T_{s,L}$.

SOLUTION:

The properties of air are determined from Table C.22 for $T = 300$ K and are $k_f = 0.0267$W/m-K, $\nu_f = 15.66 \times 10^{-6}$ m^2/s, Pr $= 0.69$. For pure copper from Table C.14 and $T = 300$ K, we have $k_s = 401$ W/m-K. We will use the extended surface (fin) results for the temperature distribution along the wire, i.e., from (6.143) we have,

$$\frac{T_s(x) - T_{f,\infty}}{T_{s,o} - T_{f,\infty}} = \frac{\cosh[m(L_c - x)]}{\cosh(mL_c)}.$$

We are interested in the end location along the thermocouple wire which is at the end or at $x = L_c$ (i.e., corrected length). Then we have

$$\begin{aligned}
\frac{T_s(L) - T_{f,\infty}}{T_{s,0} - T_{f,\infty}} &= \frac{\cosh[m(L_c - L_c)]}{\cosh(mL_c)} \\
\frac{T_s(L_c) - T_{f,\infty}}{T_{s,0} - T_{f,\infty}} &= \frac{\cosh(0)}{\cosh(mL_c)} \\
T_s(L_c) - T_{f,\infty} &= \frac{T_{s,0} - T_{f,\infty}}{\cosh(mL_c)}.
\end{aligned}$$

From (6.141), we have

$$\begin{aligned}
L_c &= L + \frac{D}{4} \\
L_c &= 0.005(\text{m}) + \frac{2 \times 10^{-4}(\text{m})}{4} = 0.00505 \text{ m}.
\end{aligned}$$

For m, from (6.144) we have

$$m = \left(\frac{P_{ku} \langle \mathrm{Nu} \rangle_D k_f}{A_k k_s D} \right)^{1/2}, \quad P_{ku} = \pi D, \quad A_k = \frac{\pi D^2}{4}.$$

For $\langle \mathrm{Nu} \rangle_D$, from Table 6.3 we have

$$\langle \mathrm{Nu} \rangle_D = a_1 \mathrm{Re}_D^{a_2} \mathrm{Pr}^{1/3},$$

where

$$\mathrm{Re}_D = \frac{u_{f,\infty} D}{\nu_f} = \frac{5(\mathrm{m/s}) \times (2 \times 10^{-4})(\mathrm{m})}{15.66 \times 10^{-6}(\mathrm{m^2/s})} = 63.86.$$

Then from Table 6.4, we have

$$\begin{aligned}
a_1 &= 0.683, \quad a_2 = 0.466 \\
\langle \mathrm{Nu} \rangle_D &= 0.683(63.8)^{0.466}(0.69)^{1/3} \\
&= 4.187 \\
m &= \left[\frac{\pi \times 2 \times 10^{-4}(\mathrm{m}) \times 4.187 \times 0.0267(\mathrm{W/m\text{-}K})}{\dfrac{\pi(2 \times 10^{-4})^2(\mathrm{m^2})}{4} \times 401(\mathrm{W/m\text{-}K}) \times 2 \times 10^{-4}} \right]^{1/2} \\
&= \left(\frac{7.3 \times 10^{-5}}{2.519 \times 10^{-4}}\right)^{1/2} \ 1/\mathrm{m} = 167.0 \ 1/\mathrm{m}.
\end{aligned}$$

The expected uncertainty is

$$T_{f,\infty} - T_{s,L} = \frac{27(^\circ\mathrm{C}) - 15(^\circ\mathrm{C})}{\cosh[167.0(1/\mathrm{m}) \times 0.00505(\mathrm{m})]} = 8.71^\circ\mathrm{C}.$$

COMMENT:

The uncertainty can be large for large $T_{f,\infty} - T_{s,o}$ and for small L. Here, the error is 8.7℃. In order to reduce this error, we can for example, triple the length and the error would be reduced to 1.9 ℃. With care, $T_{f,\infty} - T_{s,o}$ can also be reduced by placing the thermocouple wire along nearly isothermal paths in the flow field. Also note that $mL_c = \mathrm{Bi}_D^{1/2} = 0.8434$, or $\mathrm{Bi}_D = 0.7112$. This also shows that there is a significant temperature variation within the wire.

PROBLEM 6.29.FUN

GIVEN:

Shape-memory actuation devices are capable of recovering a particular shape upon heating above their transformation temperature. These alloys can be made of nickel and titanium and display two types of material properties. When at a temperature below their transformation temperature T_t, they display the properties of martensite and, when above this temperature they display the properties of austenite. The NiTi alloy shown in Figure Pr.6.29(a) is shaped as a spring and deforms from a compressed state [Figure Pr.6.29(a)(i)] to an extended state [Figure Pr.6.29(a)(ii)] when heated above its transformation temperature.

This spring is being tested for its suitability for use in the closing of heating ducts within a desired elapsed time. In order to close the duct, the spring must extend a lightweight-beam induced closing mechanism within 20 s. The air flow within the heating duct has a velocity $u_{f,\infty}$ and temperature $T_{f,\infty}$, near the spring. The spring has a length L, an outer diameter D_2, an inner diameter D_1, and an initial temperature $T_1(t = 0)$.

Assume that the entire spring is at a uniform temperature $T_1(t)$ and the dominant surface heat transfer is by surface convection.

Martensite: $\rho = 6{,}450$ kg/m^3, $k = 8.6$ W/m-℃, $c_p = 837.36$ J/kg-℃, $T_1(t = 0) = 21$℃, $T_t = 50$℃, $L = 4$ cm, $D_2 = 0.5$ cm, $D_1 = 0.4$ cm, $u_{f,\infty} = 5$ m/s, $T_{f,\infty} = 77$℃.

Evaluate the properties of air at 350 K.

SKETCH:

Figure Pr.6.29(a) shows the thermally actuated shape-memory spring.

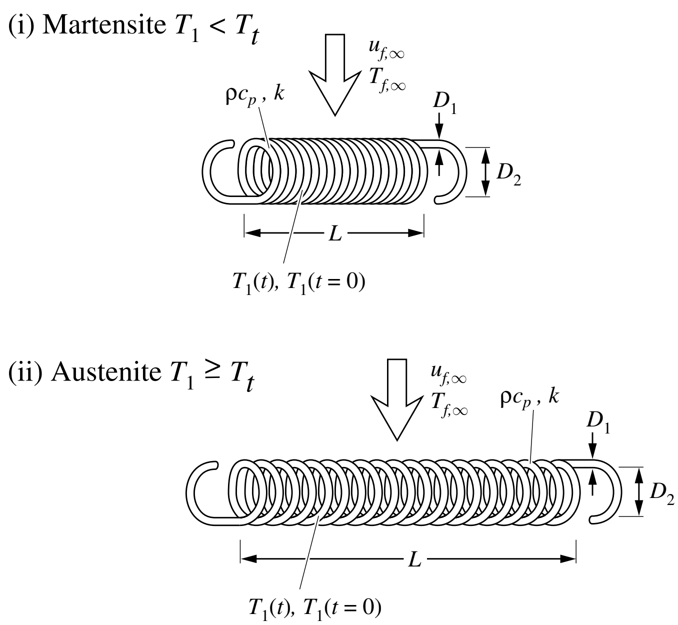


Figure Pr.6.29(a) Springs made of shape-memory alloy and used for thermal actuation.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Use the properties of the low-temperature form of NiTi listed above and determine if the spring will activate during the time allowed.

SOLUTION:

Properties (air, T = 350 K, Table C.22): k_f = 0.0300 W/m-K, ρ_f = 1.012 kg/m^3, $c_{p,f}$ = 1,007 J/kg-K, $\nu_f = 20.3 \times 10^{-6}$ m^2/s, $\alpha_f = 29.44 \times 10^{-6}$ m^2/s, Pr = 0.69.

(a) The thermal circuit diagram is shown in Figure Pr.6.29(b).

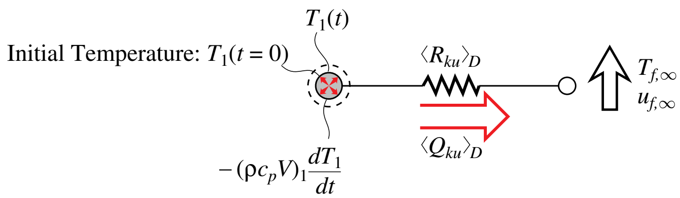


Figure Pr.6.29(b) Thermal circuit diagram.

(b) Assuming surface-convection heat transfer with transient, lumped capacitance treatment of the substrate, from (6.156) we have

$$T_1(t) - T_{f,\infty} = [T_1(t=0) - T_{f,\infty}]e^{(-t/\tau_1)} + a_1\tau_1[1 - \exp^{(-t/\tau_1)}]$$
$$\tau_1 = (\rho c_p V)_1 (R_{ku})_D \quad a_1 = \frac{(\dot{S} - Q)}{(\rho c_p V)_1}.$$

Since there is no energy conversion in the spring $a_1 = 0$.
Then, we have

$$T_1(t) - T_{f,\infty} = [T_1(t=0) - T_{f,\infty}] \exp^{-t/\tau_1}$$

The volume is

$$V_1 = \pi \times \frac{D_2^2 - D_1^2}{4} \times L = 2.827 \times 10^{-7} \text{ m}^3.$$

To determine τ_1, the surface-convection resistance $\langle R_{ku}\rangle_D$ is needed. Form (6.124), we have

$$\langle R_{ku}\rangle_D = \frac{D_2}{A_{ku}\text{Nu}_D k_f},$$

where

$$A_{ku} = \pi D_2 L$$
$$= \pi \times 0.005(\text{m}) \times 0.04(\text{m}) = 6.283 \times 10^{-4} \text{ m}^2.$$

The Nusselt number is found for a cylinder in cross flow as given in Table 6.4, i.e.,

$$\langle \text{Nu}_D\rangle = a_1 \text{Re}_D^{a_2} \text{Pr}^{1/3} \quad \text{Table 6.4},$$

where

$$\text{Re}_D = \frac{u_{f,\infty} D_2}{\nu_f}$$
$$= \frac{5(\text{m/s}) \times 0.005(\text{m})}{20.3 \times 10^{-6}(\text{m}^2/\text{s})}$$
$$= 1{,}231.5.$$

From Table 6.3,

$$a_1 = 0.683$$
$$a_2 = 0.466.$$

Then

$$\langle \text{Nu}\rangle_D = 0.683 \times (1{,}231.5)^{0.466} \times (0.69)^{1/3}$$
$$= 16.63$$
$$\langle R_{ku}\rangle_D = \frac{0.005(\text{m})}{6.28 \times 10^{-4}(\text{m}^2) \times 16.63 \times 0.0300(\text{W/m-K})} = 15.95 \text{ K/W}.$$

The time constant τ_1 is

$$\begin{aligned}\tau_1 &= 6{,}450(\mathrm{kg/m^3}) \times 837.36(\mathrm{J/kg\text{-}K}) \times 2.827 \times 10^{-7}(\mathrm{m^2}) \times 15.95(\mathrm{K/W}) \\ &= 24.36 \text{ s.}\end{aligned}$$

From (6.156), we solve for the time needed for the spring to reach $T_1 = 323$ K, i.e.,

$$\begin{aligned}(323.15 - 350.15)(\mathrm{K}) &= (294.15 - 350.15)(\mathrm{K})e^{(-t/24.36(\mathrm{s}))} \\ -27(\mathrm{K}) &= -56e^{[-t/24.36(\mathrm{s})]} \\ 0.48214 &= e^{[-t/24.26(\mathrm{s})]} \\ \ln(0.48214) &= -t/24.36(\mathrm{s}) \\ t &= 17.77 \text{ s.}\end{aligned}$$

This is less than 20 s and the device performs as required.

COMMENT:

This problem assumes that the temperature of the spring is uniform. To verify this, the Biot number given by (6.128) should be less than 0.1, i.e.,

$$\mathrm{Bi}_D = \frac{R_k}{\langle R_{ku} \rangle_D} < 0.1,$$

where

$$R_k = \frac{\ln(D_2/D_1)}{2\pi k_s L} = 1.032 \text{ K/W.}$$

Using this

$$\mathrm{Bi}_D = 0.06473.$$

Then the assumption of a uniform spring temperature is valid.

PROBLEM 6.30.FAM

GIVEN:

A steel cylindrical rod is to be cooled by surface convection using an air stream, as shown in Figure Pr.6.30. The rod can be placed perpendicular [Figure Pr.6.30(i)] or parallel [Figure Pr.6.30(ii)] to the stream. The Nusselt number for the parallel flow can be determined by assuming a flat surface. This is justifiable when the viscous boundary-layer thickness δ_ν is smaller than D.

$D = 1.5$ cm, $L = 40$ cm, $u_{f,\infty} = 4$ m/s, $T_{f,\infty} = 25$℃, $T_s = 430$℃.

Determine the air properties at $\langle T_f \rangle_\delta = (T_s + T_{f,\infty})/2$.

SKETCH:

Figure Pr.6.30(a) shows the two arrangement.

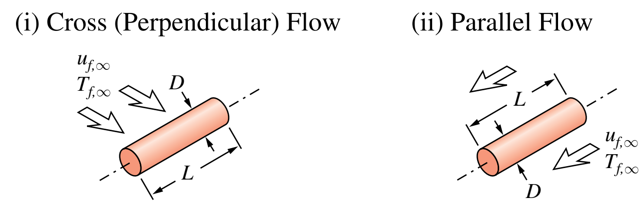


Figure Pr.6.30 A steel rod is cooled in an air stream with the choice of placing it perpendicular or parallel to the stream. (i) Cross (perpendicular) flow. (ii) Parallel flow.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the heat transfer rates $\langle Q_{ku} \rangle_D$ and $\langle Q_{ku} \rangle_L$, for the conditions given above (i.e., $\delta_\alpha < D$).

(c) Is neglecting the surface curvature and using a flat surface, for the parallel flow, justifiable?

SOLUTION:

(a) Figure Pr.6.30(b) shows the thermal circuit diagram, where for the cross flow the resistance is shown as $\langle Q_{ku} \rangle_D$ and for the parallel flow as $\langle Q_{ku} \rangle_L$.

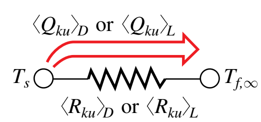


Figure Pr.6.30(b) Thermal circuit diagram.

(b) The surface-convection heat transfer is given by (6.124), i.e.,

$$\langle Q_{ku} \rangle_{D \text{ or } L} = A_{ku} \langle \text{Nu} \rangle_{D \text{ or } L} \frac{k_f}{\text{Nu}_{D \text{ or } L}(T_s - T_{f,\infty})}.$$

(i) The Nusselt number for the cross flow is given in Table 6.4 as

$$\langle \text{Nu} \rangle_D = a_1 \text{Re}^{a_2} \text{Pr}^{1/3},$$

where constants a_1 and a_2 depend on Re_D,

$$\text{Re}_D = \frac{u_{f,\infty} D}{\nu_f}.$$

From Table C.22, for air at temperature

$$\langle T_f \rangle_\delta = \frac{T_s + T_{f,\infty}}{2} = \frac{(430 + 25)(℃)}{2} + 273.15$$
$$= 500.7 \text{ K},$$

the properties are

$$k_f = 0.0395 \text{ W/m-K} \qquad \text{Table C.22}$$
$$\nu_f = 3.730 \times 10^5 \text{m}^2/\text{s} \qquad \text{Table C.22}$$
$$\text{Pr} = 0.69 \qquad \text{Table C.22.}$$

Then

$$\begin{aligned}
\text{Re}_D &= \frac{4(\text{m/s}) \times 0.015(\text{m})}{3.730 \times 10^{-5}(\text{m}^2/\text{s})} = 1{,}608.6 \\
a_1 &= 0.683, \quad a_2 = 0.466 \quad \text{Table 6.4} \\
\text{Nu}_D &= 0.683 \times (1608.6)^{0.466} \times (0.69)^{1/3} = 18.83 \\
\langle Q_{ku} \rangle_D &= \pi \times (0.015)(\text{m}) \times (0.4)(\text{m}) \times 18.83 \times \frac{0.0395(\text{W/m-K})}{0.015(\text{m})} \times (430 - 25)(\text{K}) \\
&= 378.6 \text{ W.}
\end{aligned}$$

(ii) The Nusselt number for the parallel flow us given in Table 6.3 and depends on the Reynolds number Re_L

$$\text{Re}_L = \frac{u_{f,\infty} L}{\nu_f} = \frac{4(\text{m/s}) \times 0.4(\text{m})}{3.730 \times 10^{-5}(\text{m}^2/\text{s})} = 4.290 \times 10^4 < \text{Re}_{L,t} = 10^5, \quad \text{laminar flow regime.}$$

Then from Table 6.3, we have

$$\begin{aligned}
\langle \text{Nu} \rangle_L &= 0.664 \text{Re}_L^{1/2} \text{Pr}^{1/3} \\
&= 0.664 \times (4.290 \times 10^4)^{1/2} \times (0.69)^{1/3} = 121.5 \\
\langle Q_{ku} \rangle_L &= \pi \times 0.015(\text{m}) \times 0.4(\text{m}) \times 121.5 \times \frac{0.0395(\text{W/m-K})}{0.40(\text{m})} \times 405\text{K} \\
&= 91.61 \text{ W,}
\end{aligned}$$

which is 24% of the result for cross flow.
(c) For laminar flow, the viscous boundary-layer thickness at the tailing edge is given by (6.48), i.e.,

$$\begin{aligned}
\delta_\nu &= 5(\frac{\nu_f L}{u_{f,\infty}})^{1/2} \\
&= 5 \times \left[\frac{3.730 \times 10^{-5}(\text{m}^2/\text{s}) \times 0.4(\text{m})}{4(\text{m/s})} \right]^{1/2} \\
&= 0.009657 \text{ m.}
\end{aligned}$$

Then

$$\frac{\delta_\nu}{D} = \frac{0.009657(\text{m})}{0.015(\text{m})} = 0.6438 < 1.$$

COMMENT:

The criterion $\delta_\alpha / D < 1$ is not a rigorously derived condition. However, for the conditions given, the cross flow results in a large heat transfer rate. The cross flow results in a relatively large q_{ku} in the front stagnation region. Also, at higher Reynolds numbers due to flow separation and flow vortices and turbulence, q_{ku} is large in the rear section of the cylinder.

PROBLEM 6.31.FUN

GIVEN:

An automobile brake rotor is idealized as a solid disc, as shown in Figure Pr.6.31(a). In a laboratory test the rotor is friction heated at a rate $\dot{S}_{m,\mathrm{F}}$, under steady-state heat transfer and its assumed uniform temperature becomes T_s. Assume that the heat transfer is by surface convection only and that the fluid (air) motion is only due to the rotation (rotation-induced motion) (Table 6.4).

$\omega = 130$ rad/s, $R = 30$ cm, $T_{f,\infty} = 20$°C, $\dot{S}_{m,\mathrm{F}} = 2 \times 10^4$ W.

Determine the air properties at $T = 400$ K.

SKETCH:

Figure Pr.6.31(a) shows the rotating disc.

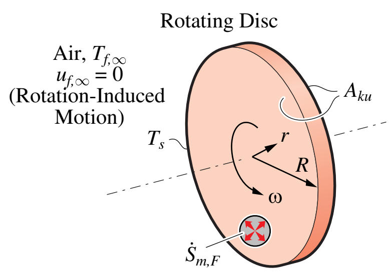


Figure Pr.6.31(a) A rotating disc is heated by friction and cooled by surface convection.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the radial location r_{tr} that the flow regime changes from laminar to turbulent ($\mathrm{Re}_{r,tr} = 2.4 \times 10^5$).

(c) Integrate the local surface convection over the entire surface area (two sides, neglect the edge).

(d) Determine the rotor temperature T_s.

SOLUTION:

(a) Figure Pr.6.31(b) shows the steady-state thermal circuit diagram. The surface averaged surface convection $\langle Q_{ku} \rangle_R$ is used.

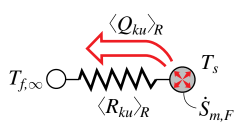


Figure Pr.6.31(b) Thermal circuit diagram.

(b) From Table C.22, for air at $T = 400$ K, we have

$$\begin{aligned} k_f &= 0.0331 \text{ W/m-K} \\ \nu_f &= 2.550 \times 10^{-5} \text{ m}^2\text{/s} \\ \mathrm{Pr} &= 0.69. \end{aligned}$$

From Table 6.4, we have

$$\mathrm{Re}_{r,tr} = \frac{\omega r_{tr}^2}{\nu_f} = 2.4 \times 10^5$$

or

$$\begin{aligned} r_{tr} &= \left(\frac{2.4 \times 10^5 \times \nu_f}{\omega} \right)^{1/2} = \left[\frac{2.4 \times 10^5 \times 2.550 \times 10^{-5} (\mathrm{m}^2/\mathrm{s})}{130 (\mathrm{rad/s})} \right]^{1/2} \\ &= 2.170 \times 10^{-1} \text{ m} = 21.70 \text{ cm}. \end{aligned}$$

(c) Then the inner portion, $0 \le r \le 21.70$ cm is subject to laminar flow regime, and the outer portion, 21.70 cm $\le r \le 35$ cm, is subject to turbulent flow regime. Then from (6.50), (6.49) and Table 6.4, we have

$$\begin{aligned}
\frac{\langle Q_{ku}\rangle_L}{T_s - T_{f,\infty}} &= \int_0^{r_{tr}} 2\pi r \frac{\mathrm{Nu}_r k_f}{r} dr + \int_{r_{tr}}^{R} 2\pi r \frac{\mathrm{Nu}_r k_f}{r} dr \\
&= 2\pi k_f \int_0^{r_{tr}} \frac{0.585 \left(\dfrac{\omega r^2}{\nu_f}\right)^{1/2}}{\left(\dfrac{0.6}{\mathrm{Pr}} + \dfrac{0.95}{\mathrm{Pr}^{1/3}}\right)} dr + 2\pi k_f \int_{r_{tr}}^{R} 0.021 \left(\frac{\omega r^2}{\nu_f}\right)^{0.8} \mathrm{Pr}^{1/3} dr \\
&= \frac{2\pi k_f \times 0.585}{\dfrac{0.6}{\mathrm{Pr}} + \dfrac{0.95}{\mathrm{Pr}^{1/3}}} \left(\frac{\omega}{\nu_f}\right)^{1/2} \frac{1}{2} r^2 \Big|_0^{r_{tr}} + 2\pi k_f \times 0.021 \times \left(\frac{\omega}{\nu_f}\right)^{0.8} \mathrm{Pr}^{1/3} \frac{1}{2.6} r^{2.6} \Big|_{r_{tr}}^{R} \\
&= \frac{2\pi \times 0.0331(\mathrm{W/m\text{-}K}) \times 0.585}{\dfrac{0.6}{0.69} + \dfrac{0.95}{(0.69)^{1/3}}} \left[\frac{130(\mathrm{rad/s})}{2.55 \times 10^{-5}(\mathrm{m^2/s})}\right]^{1/2} \times \frac{1}{2} \times (0.2170)^2(\mathrm{m}^2) + \\
&\quad 2\pi \times 0.0331(\mathrm{W/m\text{-}K}) \times 0.021 \times \left[\frac{130(\mathrm{rad/s})}{2.550 \times 10^{-5}(\mathrm{m^2/s})}\right]^{0.8} \times (0.69)^{1/3} \times \\
&\quad \frac{1}{2.6}[(0.35)^{2.6} - (0.2170)^{2.6}](\mathrm{m})^{2.6} \\
&= (3.33 + 16.00)(\mathrm{W/K}) \\
&= 19.33\ \mathrm{W/K} = 19.33\ \mathrm{W/^\circ C}.
\end{aligned}$$

(d) From Figure Pr.6.31(b), the energy equation is

$$\begin{aligned}
Q|_A &= \langle Q_{ku}\rangle_R = \dot{S}_{m,F} \\
&= 19.33(\mathrm{W/^\circ C}) \times (T_s - T_{f,\infty}) = \dot{S}_{m,F}
\end{aligned}$$

or

$$\begin{aligned}
T_s &= T_{f,\infty} + \frac{\dot{S}_{m,F}}{42.22(\mathrm{W/^\circ C})} \\
&= 20(^\circ\mathrm{C}) + \frac{2 \times 10^4(\mathrm{W})}{19.33(\mathrm{W/^\circ C})} \\
&= 1.055^\circ\mathrm{C}.
\end{aligned}$$

COMMENT:

The flow regime transition is similar to that occurring for the forced and thermobuoyant parallel flows. The material used for the disc should be chosen so as to withstand the resulting surface temperature which is very high.

PROBLEM 6.32.FAM

GIVEN:

A person remaining in a very cold ambient will eventually experience a drop in body temperature (i.e., experience hypothermia). This occurs when the body no longer converts sufficient chemical-bond energy to thermal energy to balance the heat losses. Consider an initial uniform temperature of $T_1(t = 0) = 31°\text{C}$ and a constant energy conversion of $\dot{S}_{r,c} = 400$ W. The body may be treated as a cylinder made of water with a diameter of 40 cm and a length of 1.7 m, as shown in Figure Pr.6.32(ii). Assume that the lumped-capacitance analysis is applicable.

Evaluate the properties at the average temperature between the initial temperature and the far-field fluid temperature.

SKETCH:

Figure Pr.6.32 shows a person undergoing surface-convection heat losses.

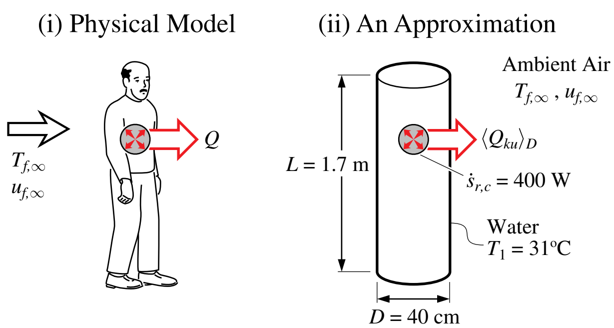


Figure Pr.6.32 (i) Surface-convection heat transfer from a person. (ii) Its geometric presentation.

OBJECTIVE:

Determine the elapsed time for a drop in the body temperature of $\Delta T_1 = 10°\text{C}$.

(a) Consider the ambient to be air with a temperature of $T_{f,\infty} = -10°\text{C}$, blowing at $u_{f,\infty} = 30$ km/hr across the body (i.e., in cross flow).

(b) Consider the ambient to be water with a temperature of $T_{f,\infty} = 0°\text{C}$ with a thermobuoyant motion ($u_{f,\infty} = 0$ km/hr) in the water along the length of the cylinder. For the thermobuoyant motion, use the results for a vertical plate and assume that the body temperature is the time-averaged body temperature (i.e., an average between the initial and the final temperature). This results in a constant surface-convection resistance.

SOLUTION:

(a) Air With Forced, Cross Flow:

This is a transient problem in which a lumped-capacitance analysis is to be used. The integral-volume energy equation (2.9) applied to the body gives

$$Q|_{A,1} = -(\rho c_p V)_1 \frac{dT_1}{dt} + \dot{S}_{r,c}.$$

The net heat transfer at the control surface (wrapped around the body) is due only to surface convection. Thus, we write

$$Q|_A = \langle Q_{ku}\rangle_D = \frac{T_1 - T_{f,\infty}}{\langle R_{ku}\rangle_D}.$$

Note that both the resistance and the heat transfer are averaged over D, because the air is in cross flow. The energy conversion is from chemical bond to thermal energy, $\dot{S}_{r,c}$. Substituting this into the energy equation we have

$$\frac{T_1 - T_{f,\infty}}{\langle R_{ku}\rangle_D} = -\rho_1 c_{p,1} V_1 \frac{dT_1}{dt} + \dot{S}_{r,c}$$

The solution to this is given by (6.156), and as in Example 6.15, solving for t we have

$$t = -\tau_1 \ln\left[\frac{T_1(t) - T_{f,\infty} - a_1\tau_1}{T_1(t=0) - T_{f,\infty} - a_1\tau_1}\right],$$

where

$$\tau_s = \rho_1 c_{p,1} V_1 \langle R_{ku} \rangle_D$$
$$a_1 = \frac{\dot{S}_r}{\rho_1 c_{p,1} V_1}.$$

The average surface-convection resistance is obtained from the Nusselt number using (6.49).
The properties for air, at $T_\delta = [(31+21)/2-10]/2 = 8.0$℃ $= 281.15$ K, are found from Table C.22, as $k_f = 0.0256$ W/m-K, $\nu_f = 14.01\text{x}10^{-6}$ m^2/s, Pr $= 0.69$.
The Reynolds number based on diameter is given in Table 6.4 as

$$\text{Re}_D = \frac{u_{f,\infty} D}{\nu_f} = \frac{8.333(\text{m/s}) \times 0.4(\text{m})}{14.01 \times 10^{-6}(\text{m}^2/\text{s})} = 2.379 \times 10^5.$$

From Table 6.4, the correlation for $\langle \text{Nu} \rangle_D$ for cross flow over a circular cylinder is found with $a_1 = 0.027$ and $a_2 = 0.805$. The Nusselt number is

$$\langle \text{Nu} \rangle_D = a_1 Re_D^{a_2} \text{Pr}^{1/3} = 0.027(2.379 \times 10^5)^{0.805} \times (0.69)^{1/3} = 507.74.$$

The average surface-convection resistance is given by (6.49), i.e.,

$$\langle R_{ku} \rangle_D = \frac{D}{k_f \langle \text{Nu} \rangle_D A_{ku}} = \frac{0.4(\text{m})}{0.0256(\text{W/m-K}) \times 507.74 \times \pi \times 0.4(\text{m}) \times 1.7(\text{m})} = 0.0144℃/\text{W}.$$

The properties for water at $T_\delta = (31+21)/2 = 26$℃ $= 299.15$ K, are found from Table C.23 as $\rho_l = 997.8$ kg/m^3 and $c_{p,l} = 4{,}182$ J/kg-K.
Thus, τ_1 and a_1 are

$$\tau_1 = 997.8(\text{kg/m}^3) \times 4{,}182(\text{J/kg-K}) \times \frac{\pi \times (0.4)^2(\text{m})^2}{4} \times 1.7(\text{m}) \times 0.0145(℃/\text{W}) = 12{,}837 \text{ s}$$

$$a_1 = \frac{400(\text{W})}{997.8(\text{kg/m}^3) \times 4{,}182(\text{J/kg-K}) \times \frac{\pi \times (0.4)^2(\text{m})^2}{4} \times 1.7(\text{m})} = 4.487 \times 10^{-4}\,℃/\text{s}.$$

Solving for t, we have

$$t = -12{,}837(\text{s}) \times \ln\left[\frac{21(℃) - [-10(℃)] - 4.487 \times 10^{-4}(℃/\text{s}) \times 12{,}837(\text{s})}{31(℃) - [-10(℃)] - 4.487 \times 10^{-4}(℃/\text{s}) \times 12{,}837(\text{ s})}\right] = 4{,}284 \ \ \text{s} \ = 71.41 \ \ \text{min}.$$

(b) Water With Thermobuoyant Flow:

The above energy equation and transient analysis remain the same. The average Nusselt number for the thermobuoyant flow over a vertical flat plate is given in Table 6.5.
The properties for water, at $T_\delta = [(31+21)/2+0]/2 = 286.15$ K, are obtained from Table C.23 as $k_f = 0.581$ W/m-K, $\nu_f = 1.28 \times 10^{-6}$ m^2/s, $\alpha_f = 0.139 \times 10^{-6}$ m^2/s, Pr $= 9.31$, and $\beta_f = 0.00018$ 1/K.
The time-averaged body temperature is $\overline{T}_1 = 26$℃.
The Rayleigh number is defined in Table 6.4 as

$$\text{Ra}_L = \frac{g\beta_f(\overline{T}_1 - T_{f,\infty})L^3}{\nu_f \alpha_f} = \frac{9.81(\text{m/s}^2) \times 0.00018(1/\text{K}) \times [26(℃) - 0(℃)] \times (1.7)^3(\text{m})^3}{1.28 \times 10^{-6}(\text{m}^2/\text{s}) \times 0.139 \times 10^{-6}(\text{m}^2/\text{s})} = 1.27 \times 10^{12}.$$

From Table 6.5,

$$a_1 = 0.6205$$
$$\text{Nu}_{L,l} = 659.8$$
$$\text{Nu}_{L,t} = 1{,}198.2$$
$$\langle \text{Nu} \rangle_L = 1{,}203.7.$$

The average surface-convection resistance is

$$\langle R_{ku} \rangle_L = \frac{L}{k_f \langle \text{Nu} \rangle_L A_{ku}} = \frac{1.7(\text{m})}{0.581(\text{W/m-K}) \times 1{,}203.7 \times \pi \times 0.4(\text{m}) \times 1.7(\text{m})} = 0.001137℃/\text{W}.$$

The properties for the body remain the same.
Then, τ_1 is

$$\tau_1 = 997.8(\text{kg/m}^3) \times 4182(\text{J/kg-K}) \times \frac{\pi \times (0.4)^2(\text{m})^2}{4} \times 1.7(\text{m}) \times 0.001137(^\circ\text{C/W}) = 1{,}013.5 \ \ \text{s}$$

and a_1 remains the same, i.e.,

$$a_1 = 4.487 \times 10^{-4}\,^\circ\text{C/s}.$$

Solving for t gives

$$t = -1{,}013.5(\text{s}) \times \ln\left[\frac{21(^\circ\text{C}) - 0(^\circ\text{C}) - 4.487 \times 10^{-4}(^\circ\text{C/s}) \times 1{,}013.5(\text{s})}{31(^\circ\text{C}) - 0(^\circ\text{C}) - 4.487 \times 10^{-4}(^\circ\text{C/s}) \times 1{,}013.5(\text{s})}\right] = 401.9 \ \ \text{s} \ = 6.7 \ \ \text{min}.$$

COMMENT:

Although the motion in the water is due to thermobuoyancy and of a smaller velocity (as compared to the forced flow), due to the larger thermal conductivity of water, the surface-convection heat transfer is larger (i.e., the surface-convection resistance is smaller) for water.

PROBLEM 6.33.FAM

GIVEN:

A methane-air mixture flows inside a tube where it is completely reacted generating a heating rate of $\dot{S}_{r,c} = 10^4$ W. This heat is removed from the tube by a cross flow of air, as shown in Figure Pr.6.33(a).

Evaluate the properties of air at $T = 300$ K.

SKETCH:

Figure Pr.6.33(a) shows energy conversion by combustion in a tube and heat removal by surface convection from the tube.

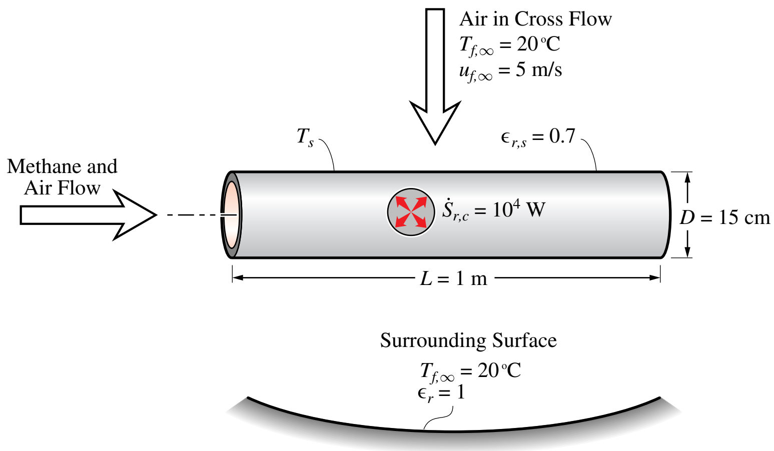


Figure Pr.6.33(a) Heat removal from a combustion tube.

OBJECTIVE:

(a) Draw the thermal circuit diagram and determine the tube surface temperature with no surface radiation.
(b) Repeat part (a) with surface radiation included.

SOLUTION:

(a) Neglecting radiation, the thermal circuit is shown in Figure Pr.6.33(b).

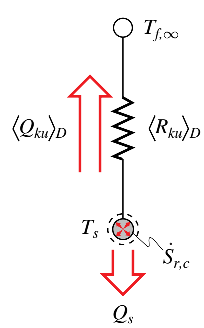


Figure Pr.6.33(b) Thermal circuit diagram for the case of no surface radiation heat transfer.

Applying the integral-volume energy equation (2.9), with $Q_s = 0$, to the tube we have

$$Q|_A = -(\rho c_p V)_s \frac{dT_s}{dt} + \dot{S}_{r,c},$$

where the lumped capacitance is assumed for into node T_s. As the only heat loss occurs by surface radiation and the energy conversion is by combustion, for this steady-state problem, the energy equation becomes

$$\frac{T_s - T_{f,\infty}}{\langle R_{ku}\rangle_D} = \dot{S}_{r,c}.$$

For air, from Table C.22, at $T = 300$ K, we have $\nu_f = 15.66 \times 10^{-6}$ m^2/s, $k_f = 0.0267$ W/m-K, Pr = 0.69. The Reynolds number is

$$\mathrm{Re}_D = \frac{u_{f,\infty} D}{\nu_f} = \frac{5(\mathrm{m/s}) \times 0.15(\mathrm{m})}{15.66 \times 10^{-6}(\mathrm{m^2/s})} = 47{,}893.$$

The Nusselt number for cross-flow over a circular cylinder, from Table 6.3, with $a_1 = 0.027$ and $a_2 = 0.805$, is

$$\langle \mathrm{Nu}\rangle_D = 0.027\mathrm{Re}_D^{0.805}\mathrm{Pr}^{1/3} = 0.027 \times (47{,}893)^{0.805} \times (0.69)^{1/3} = 139.7.$$

The average surface-convection resistance is given by (6.124), i.e.,

$$\langle R_{ku}\rangle_D = \frac{D}{A_{ku}\langle \mathrm{Nu}\rangle_D k_f} = \frac{0.15(\mathrm{m})}{\pi \times 0.15(\mathrm{m}) \times 1(\mathrm{m}) \times 139.7 \times 0.0267(\mathrm{W/m\text{-}K})} = 0.08532^\circ\mathrm{C/W}.$$

Then from the energy equation, we have

$$T_s = T_{f,\infty} + \langle R_{ku}\rangle_D \dot{S}_{r,c} = 20(^\circ\mathrm{C}) + 0.08532(^\circ\mathrm{C/W}) \times 10^4(\mathrm{W}) = 873.2^\circ\mathrm{C} = 1{,}146\mathrm{K}.$$

(b) With the inclusion of surface radiation, the thermal circuit is shown in Figure Pr.6.33(c).

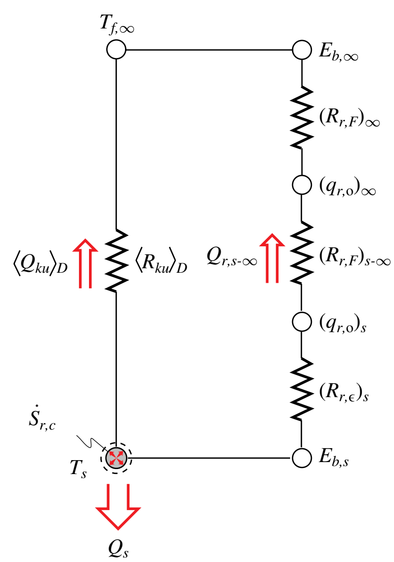


Figure Pr.6.33(c) Thermal circuit diagram for the case with surface-radiation heat transfer.

The energy equation then becomes

$$\frac{T_s - T_{f,\infty}}{\langle R_{ku}\rangle_D} + \frac{E_{b,s} - E_{b,\infty}}{(R_{\Sigma,r})_{s\text{-}\infty}} = \dot{S}_{r,c}$$

The thermal resistance for radiation is given by (4.47), i.e.,

$$\begin{aligned}(R_{\Sigma,r})_{s\text{-}\infty} &= \left(\frac{1-\epsilon_r}{\epsilon_r A_r}\right)_\infty + \frac{1}{A_{r,s}F_{s\text{-}\infty}} + \left(\frac{1-\epsilon_r}{\epsilon_r A_r}\right)_s \\ &= 0 + \frac{1}{\pi \times 0.15(\mathrm{m}) \times 0.1(\mathrm{m}) \times 1} + \frac{1-0.7}{(0.7) \times \pi \times 0.15(\mathrm{m}) \times 1(\mathrm{m})} \\ &= 3.032 \quad 1/\mathrm{m}^2.\end{aligned}$$

The surface-convection resistance and the energy conversion term remain the same. Then the energy equation becomes

$$\frac{T_s(\mathrm{K}) - 293.15(\mathrm{K})}{0.08532(^\circ\mathrm{C/W})} + \frac{\sigma_{\mathrm{SB}}[T_s^4(\mathrm{K}^4) - (293.15)^4(\mathrm{K}^4)]}{3.032(1/\mathrm{m}^2)} = 10^4 \ \mathrm{W}.$$

Using a solver (such as SOPHT), the results is $T_s = 722.6$ K

COMMENT:

Note the significant drop in T_s caused by the surface-radiation heat transfer. The assumption of a uniform tube temperature may not be valid if L/D is very large.

PROBLEM 6.34.FUN

GIVEN:

Consider surface-convection heat transfer from a sphere of radius R and initial temperature $T_s(t=0)$ as rendered in Figure Pr.6.34(a). The time dependence of the uniform sphere temperature, with surface convection, is given by (6.156) and is valid for $\text{Bi}_R < 0.1$. Also for $\text{Bi}_R > 10$ the surface-convection resistance becomes negligible and then the constant surface temperature, distributed transient temperature given in Figure 3.33(b)(ii) becomes valid. In the Biot number regime $10 < \text{Bi}_R < 0.1$, numerical or series, closed-form solutions are used. In an existing series solution, when the elapsed time is sufficiently large (i.e., $\text{Fo}_R > 0.2$) such that the penetration distance has reached and passed the center of the spheres, a single term from this series solution may be used to obtain $T_s = T_s(r,t)$. This solution for the center of the sphere, i.e., $r = 0$, is

$$T_s^*(r=0,t) = \frac{T_s(r=0,t) - T_{f,\infty}}{T_s(t=0) - T_{f,\infty}} = a_1 e^{-a_2^2 \text{Fo}_R}, \quad \text{Fo}_R = \frac{t\alpha_s}{R^2} > 0.2,$$

where the constants a_1 and a_2 depend on Bi_R and are listed for some values of Bi_R in Table Pr.6.34. From (6.128), we have

$$\text{Bi}_R = \frac{R_{k,s}}{\langle R_{ku}\rangle_D} = \frac{\langle \text{Nu}\rangle_D k_f/D}{k_s/R}.$$

Table Pr.6.34 The constants in the one-term solution.

	Bi_R	a_1	a_2	$(3\text{Bi}_R)^{1/2}$
	0	1.000	0	0
	0.01	1.003	0.1730	0.1732
Lumped	0.10	1.030	0.5423	0.5477
	1.0	1.273	1.571	1.414
	10	1.943	2.836	4.472
Constant Surface	100	1.999	3.110	14.14
Temperature	∞	2.000	$3.142 = \pi$	∞

OBJECTIVE:

(a) Show that (6.156) can be written as

$$T_s^*(t) = \frac{T_s(t) - T_{f,\infty}}{T_s(t=0) - T_{f,\infty}} = e^{-3\text{Fo}_R \text{Bi}_R}, \quad \text{Bi}_R < 0.1.$$

(b) Plot $T_s^*(t)$ with respect to Fo_R, for $0.01 \le \text{Fo}_R \le 1$, and for $\text{Bi}_R = 0.01$, 0.1, 1, 10, and 100.

(c) On the above graph, mark the center temperature $T_s(r=0,t)$ for $\text{Fo}_R = 0.2$ and 1.0, and for the Biot numbers listed in part (b).

(d) For $\text{Fo}_R = 0.2$ and 1.0, also mark the results found from Figure 3.33(b)(ii), noting that this corresponds to $\text{Bi}_R \to \infty$.

(e) Comment on the regime of a significant difference among the results of the lumped-capacitance treatment, the distributed-capacitance treatment with $\text{Bi}_R \to \infty$, and the single-term solution for distributed capacitance with finite Bi_R.

SKETCH:

Figure Pr.6.34(a) shows a sphere placed in a fluid stream with surface convection, and time-dependent temperature.

OBJECTIVE:

(a) Show that (6.156) can be written as

$$T_s^*(t) = \frac{T_s(t) - T_{f,\infty}}{T_s(t=0) - T_{f,\infty}} = e^{-3\text{Fo}_R \text{Bi}_R} \quad \text{Bi}_R < 0.1.$$

(b) Plot $T_s^*(t)$ with respect to Fo_R for $0.01 \le \text{Fo}_R < 1$, and for $\text{Bi}_R = 0.01, 0.1, 1, 10$ and 100.

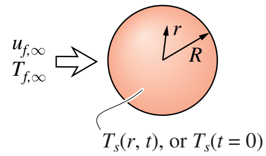


Figure Pr.6.34(a) A sphere of initial temperature $T_s(t = 0)$ is placed in a fluid stream with $T_{f,\infty}$, and $u_{f,\infty}$.

(c) On the above graph, mark the center temperature $T_s^*(r = 0, t)$ for $\mathrm{Fo}_R = 0.2$ and 1.0, and for the Biot numbers listed in part (b).
(d) For $\mathrm{Fo}_R = 0.2$ and 1.0, also mark the results found from Figure 3.33(b)(ii), noting that this graph corresponds to $\mathrm{Bi}_R \rightarrow \infty$.
(e) Comment on the regime a significant difference among the three solutions.

SOLUTION:
(a) Starting from (6.156), we write for $a_1 = 0$, and charging t to Fo_R using $\mathrm{Fo}_R = t\alpha_s/R^2$, we have

$$T_s^*(t) = \frac{T_s(t) - T_{f,\infty}}{T_s(t = 0) - T_{f,\infty}} = \exp(-\mathrm{Fo}_R R^2/\alpha_s \tau).$$

Next, using the definition of τ given by (6.157), we have

$$\frac{\mathrm{Fo}_R R^2}{\alpha_s \tau} = \frac{\mathrm{Fo}_R R^2}{\alpha_s (\rho c_p)_s V \langle R_{ku} \rangle_D}$$

Noting that $\langle R_{ku} \rangle_D = D/(A_{ku} \langle \mathrm{Nu} \rangle_D k_f)$, $(\alpha \rho c_p)_s = k_s$, $V = \pi D^3/6$ and that $A_{ku} = 4\pi R^2$,

$$\frac{\mathrm{Fo}_R R^2}{\alpha_s \tau} = 3\mathrm{Fo}_R \times \langle \mathrm{Nu} \rangle_D \frac{k_f}{k_s}.$$

From Example 6.15, we have

$$\mathrm{Bi}_D = \langle \mathrm{Nu} \rangle_D \frac{k_f}{4k_s}.$$

As $\mathrm{Bi}_x \propto x^2$, $\mathrm{Bi}_R = 4\mathrm{Bi}_D$, so that

$$\mathrm{Bi}_R = \langle \mathrm{Nu} \rangle_D \frac{k_f}{k_s},$$

and we can then write

$$\frac{\mathrm{Fo}_R R^2}{\alpha_s \tau} = 3\mathrm{Fo}_R \mathrm{Bi}_R.$$

Using this, we have

$$T_s^*(t) = e^{-3\mathrm{Fo}_R \mathrm{Bi}_R}.$$

(b) Figure Pr.6.34(b) shows the variation of $T_s^*(t)$ with respect to Fo_R for several values of Bi_R. For large Bi_R, $T_s(t)$ quickly (i.e., small Fo_R) becomes equal to $T_{f,\infty}$, i.e., $T_s^*(t = 0) \rightarrow 0$. For very small Bi_R, the sphere temperature does not change unless Fo_R is very large.

(c) The center temperature found from the one-term solution is marked (with closed circles) on Figure Pr.6.34(b). For $\mathrm{Bi}_R < 0.1$, the two results are identical. This can be also noticed from Table Pr.6.34, where $a_2^2 = 3\mathrm{Bi}_R$, for $\mathrm{Bi}_R \leq 0.1$. For larger Bi_R, there is a different between the two, especially at $\mathrm{Fo}_R = 0.2$. As Fo_R increases beyond 1, the difference decreases again, regardless of the value of Bi_R.

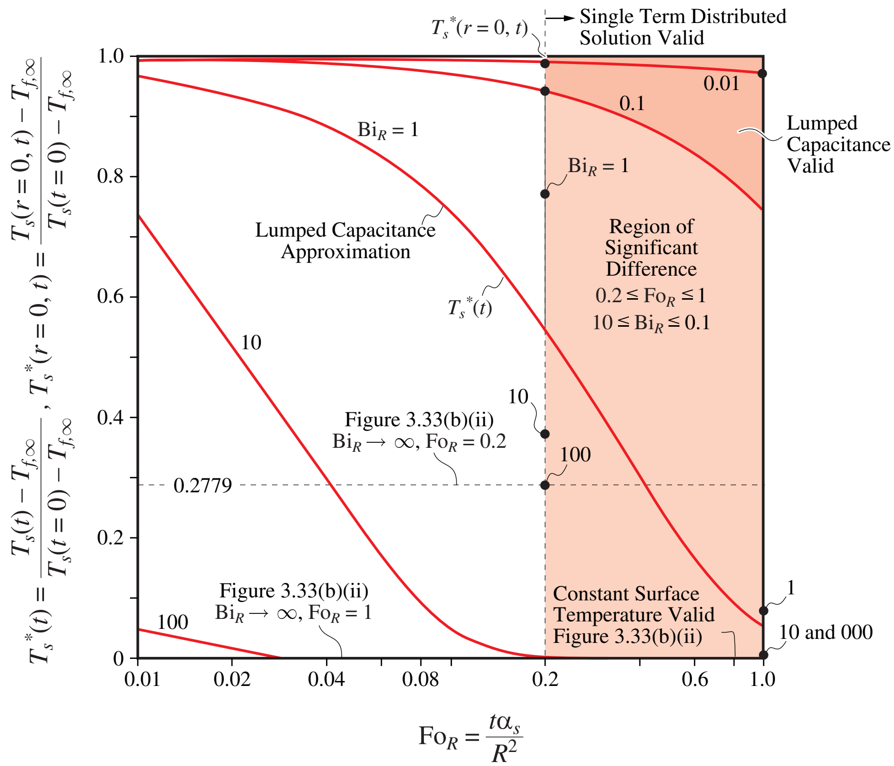


Figure Pr.6.34(b) Variation of dimensionless temperatures with respect to dimensionless time. The dots represent $T_s^*(r=0,t)$.

(d) From Figure 3.33(a)(ii), for $\text{Fo}_R = 0.2$, we have $T_s^*(r=0,t) = 1 - 0.72 = 0.28$ (the exact value is 0.2779 and is found by using the single-term solution for $\text{Bi}_R \to \infty$). For $\text{Fo}_R = 1$, we have $T_s^*(r=0,t) = 1 - 1 = 0$. These are also marked in Figure Pr.6.34(b).

(e) In the regime marked by

$$0.2 \leq \text{Fo}_R \leq 1, \quad 10 \leq \text{Bi}_R \leq 0,$$

the single-term solution gives more accurate results.

COMMENT:

When this region is encountered, the lumped-capacitance and constant surface-temperature approximations are not valid. But in practice, most problems fall outside this region and satisfy the requirements for the approximation lumped capacitance or constant surface-temperature solutions. The graphical Heisler results give the results for $\text{Fo}_R \geq 0.2$ and an arbitrary Bi_R for spheres, long cylinders, and finite slabs and can be found in Chapter 6, reference 19.

PROBLEM 6.35.FAM

GIVEN:

In a rapid solidification-coating process, a liquid metal is atomized and sprayed onto a substrate. The atomization is by gas injection into a spray nozzle containing the liquid-metal stream. The injected gas is small compared to the gas (assume air) entrained by the droplet spray stream. This entrained gas quickly cools the droplets such that at the time of impingement on the substrate the droplets contain a threshold amount of liquid that allows for them to adhere to each other and to the substrate surface. This is shown in Figure Pr.6.35(a), where a plastic balloon is coated with a tin layer and since the droplets are significantly cooled by surface convection, the balloon is unharmed. Assume that each droplet is independently exposed to a semi-bounded air stream.

$T_1(t=0) = 330$℃, $T_{f,\infty} = 40$℃, $u_{f,\infty}$(relative velocity) $= 5$ m/s, $D = 50\ \mu$m.

The Nusselt number can be determined (Table 6.4) using the relative velocity and the properties of tin are given in Tables C.5 and C.16. Determine the air properties at $T = 400$ K.

SKETCH:

Figure Pr.6.35(a) shows the flight of droplets.

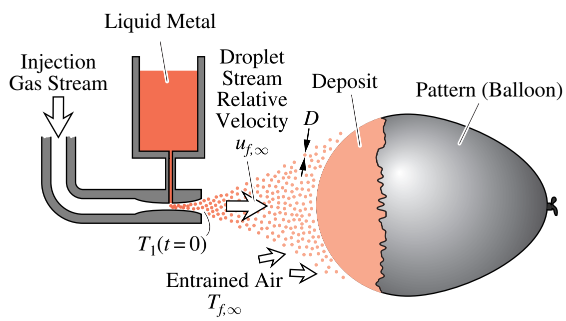


Figure Pr.6.35(a) A plastic balloon is spray coated with tin droplets solidifying on its surface.

OBJECTIVE:

(a) Draw the thermal circuit diagram for a tin droplet cooled from the initial temperature $T_1(t=0)$ to its solidification temperature T_{sl}. Assume a uniform temperature $T_1(t)$.

(b) By neglecting any motion within the droplet, determine if a uniform droplet temperature can be assumed; use $R_{k,s} = D/4A_{ku}k_s$.

(c) Determine the time of flight t, for the given conditions.

SOLUTION:

(a) Figure Pr.6.35(b) shows the thermal circuit diagram for the droplet cooling. The sensible heat (and not the phase change) is included.

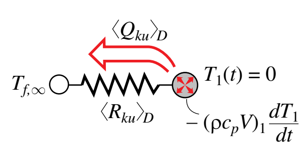


Figure Pr.6.35(b) Thermal circuit diagram.

(b) From (6.128) and(6.124), we have

$$\mathrm{Bi}_D = \frac{R_{k,1}}{\langle R_{ku}\rangle_D} = \frac{D/4A_{ku}k_s}{D/A_{ku}\langle \mathrm{Nu}\rangle_D k_f} = \langle \mathrm{Nu}_D\rangle \frac{k_f}{4k_s}.$$

From Tables C.16, we have for tin: $T_{sl} = 505$ K, $\rho_s = 7{,}310$ kg/m^3, $c_{p,s} = 227$ J/kg-K, $k_s = 66.6$ W/m-K. From Table C.22, we have for air at $T = 400$ K: $k_f = 0.0331$ W/m-K, $\nu_f = 2.55 \times 10^{-5}$ m^2/s, Pr $= 0.69$.

Then from (6.124) we have

$$\mathrm{Re}_D = \frac{u_{f,\infty} D}{\nu_f} = \frac{5(\mathrm{m/s}) \times 5 \times 10^{-5}(\mathrm{m})}{2.55 \times 10^{-5}(\mathrm{m^2/s})} = 9.804.$$

From Table 6.4, we have

$$\begin{aligned}\langle \mathrm{Nu}_D \rangle &= 2 + (0.4\mathrm{Re}_D^{1/2} + 0.06\mathrm{Re}_D^{2/3})\mathrm{Pr}^{0.4} \\ &= 2 + [0.4(9.804)^{1/2} + 0.06(9.804)^{2/3}](0.69)^{0.4} = 3.317.\end{aligned}$$

Then

$$\mathrm{Bi}_D = 3.317 \times \frac{0.0331(\mathrm{W/m\text{-}K})}{4 \times 66.6(\mathrm{W/m\text{-}K})} = 4.121 \times 10^{-4} < 0.1$$

and the variation of temperature within droplet is therefore negligible.

(c) From (6.156), with $Q_1 = \dot{S}_1 = 0$, $a_1 = 0$, and we have

$$\frac{T_1(t) - T_{f,\infty}}{T_1(t=0) - T_{f,\infty}} = e^{-t/\tau},$$

or

$$\begin{aligned}t &= -\tau_1 \ln\left[\frac{T_1(t) - T_{f,\infty}}{T_1(t=0) - T_{f,\infty}}\right] \\ \tau_1 &= (\rho c_p V)_1 \langle R_{ku} \rangle_D = (\rho c_p V)_1 \frac{D}{A_{ku} \langle \mathrm{Nu} \rangle_D k_f} \\ &= (\rho c_p)_1 \frac{VD}{A_{ku}} \frac{1}{\langle \mathrm{Nu} \rangle_D k_f} = (\rho c_p)_1 \frac{D^2}{6} \frac{1}{\langle \mathrm{Nu} \rangle_D k_f} \\ &= 7{,}310(\mathrm{kg/m^3}) \times 227(\mathrm{J/kg\text{-}K}) \times \frac{(5 \times 10^{-5})^2(\mathrm{m^2})}{6} \times \frac{1}{3.317 \times 0.0331(\mathrm{W/m\text{-}K})} \\ &= 6.297 \times 10^{-3}\ \mathrm{s} \\ t &= -6.297 \times 10^{-3}(\mathrm{s}) \times \ln \frac{505(\mathrm{K}) - (273.15 + 40)(\mathrm{K})}{(330 - 40)(\mathrm{K})} \\ &= 2.602 \times 10^{-3}\ \mathrm{s} \\ &= 2.602\ \mathrm{ms}.\end{aligned}$$

COMMENT:

Note that we have assumed that each droplet is independently exposed to a semi-infinite air stream. In practice, the cloud of droplets heat the stream and also reduces Nu_D through modifying the fluid flow and temperature distribution around each droplet. This results in a larger resistance and a larger time constant, requiring larger elapsed times to reach the desired temperature.

PROBLEM 6.36.FAM

GIVEN:

In order to enhance the surface-convection heat transfer rate, fins (i.e., extended surfaces) are added to a planar surface. This is shown in Figure Pr.6.36. The surface has a square geometry with dimensions $a = 30$ cm and $w = 30$ cm and is at $T_{s,o} = 80$℃. The ambient is air with a far-field velocity of $u_{f,\infty} = 1.5$ m/s and a temperature of $T_{f,\infty} = 20$℃ flowing parallel to the surface. There are $N = 20$ rectangular fins made of pure aluminum and each is $l = 2$ mm thick and $L = 50$ mm long.

Assume that the Nusselt number is constant and evaluate the properties at the average temperature between the plate temperature and the far-field fluid temperature.

SKETCH:

Figure Pr.6.36 shows the fins attached to the heat transfer surface.

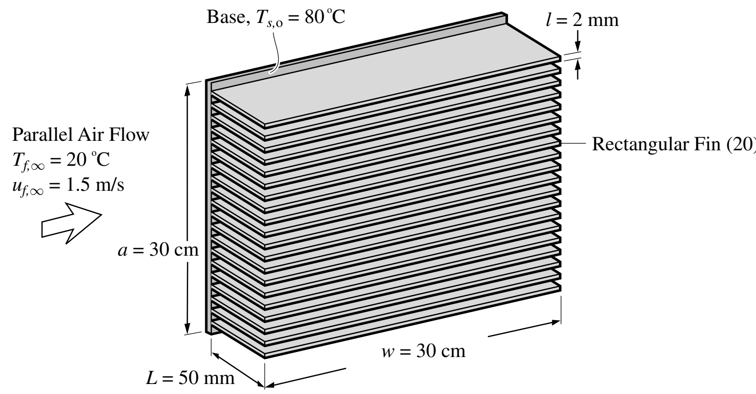


Figure Pr.6.36 An extended surface with parallel, forced flow.

OBJECTIVE:

(a) Determine the rate of heat transfer for the plate without the fins.

(b) Determine the rate of heat transfer for the plate with the fins. Treat the flow over the fins as parallel along the width w, thus neglecting the effect of the base and the neighboring fins on the flow and heat transfer.

SOLUTION:

(a) Without Fins:

The air is in parallel flow over the plate surface (along w).

For the properties for air, at $T_\delta = (80 + 20)/2$℃$= 323.15$ K, from Table C.22, we have: $k_f = 0.0283$ W/m-K, $\nu_f = 17.73 \times 10^{-6}$ m^2/s, Pr $= 0.69$.

The Reynolds number, based on the length w, is given by (6.45) as

$$\mathrm{Re}_w = \frac{u_{f,\infty} w}{\nu_f} = \frac{1.5(\mathrm{m/s}) \times 0.3(\mathrm{m})}{17.73 \times 10^{-6}(\mathrm{m^2/s})} = 25{,}381 < \mathrm{Re}_{w,t} = 10^5 \quad \text{laminar flow.}$$

From Table 6.3 for parallel flow with $\mathrm{Re}_w < 10^5$, the Nusselt number is

$$\langle \mathrm{Nu} \rangle_w = 0.664 \mathrm{Re}_w^{1/2} \mathrm{Pr}^{1/3} = 0.664 \times (25{,}295)^{0.5} \times (0.69)^{1/3} = 93.48.$$

The average surface-convection resistance is

$$\langle R_{ku} \rangle_w = \frac{w}{k_f \langle \mathrm{Nu} \rangle_w A_{ku}} = \frac{0.3(\mathrm{m})}{0.0283(\mathrm{W/m\text{-}K}) \times 92.48 \times 0.3(\mathrm{m}) \times 0.3(\mathrm{m})} = 1.26℃/\mathrm{W}.$$

The surface-convection heat transfer rate is given by (6.49) as

$$\langle Q_{ku} \rangle_w = \frac{T_s - T_{f,\infty}}{\langle R_{ku} \rangle_w} = \frac{80(℃) - 20(℃)}{1.26(℃/\mathrm{W})} = 47.62 \quad \mathrm{W}.$$

(b) With Fins:
The air is again in parallel flow over the fins. The fins surface is considered a flat semi-infinite (along the direction of the flow) surface. Then, the Nusselt number remains the same as the one used above. From Table C.14, for aluminum, $k_s = 238$ W/m-K.
The geometric parameters used in the fin efficiency, given by (6.147), are

$$L_c = L + l/2 = 0.05(\text{m}) + 0.002(\text{m})/2 = 0.051 \ \ \text{m}$$

$$P_{ku,f} = 2w + 2l = 0.604 \ \ \text{m}$$

$$A_k = wl = 0.3(\text{m}) \times 0.002(\text{m}) = 6.00 \times 10^{-4} \ \ \text{m}^2$$

$$A_{ku,f} = P_{ku} L_c = 0.604(\text{m}) \times 0.051(\text{m}) = 0.0308 \ \ \text{m}^2$$

$$A_b = aw - N_f A_k = (0.3)^2(\text{m})^2 - (20)6 \times 10^{-4}(\text{m}^2) = 0.078 \ \ \text{m}^2.$$

The fin parameter is given by (6.144), i.e.,

$$m = \left(\frac{P_{ku} \frac{\langle \text{Nu} \rangle_w k_f}{w}}{A_k k_s} \right)^{1/2} = \left[\frac{0.604(\text{m}) \times \frac{93.48 \times 0.0283(\text{W/m-K})}{0.3(\text{m})}}{6 \times 10^{-4}(\text{m}^2) \times 238(\text{W/m-K})} \right]^{1/2} = 6.107 \ \ 1/\text{m}.$$

Then the efficiency is

$$\eta_f = \frac{\tanh(mL_c)}{mL_c} = \frac{\tanh[6.107(\text{m}\times)0.051(\text{m})]}{6.107(\text{m}) \times 0.051(\text{m})} = 0.9689.$$

From (6.152) and (6.153), the average surface-convection resistances for the bare surface and fin are

$$\langle R_{ku} \rangle_{w,b} = \frac{w}{k_f \langle \text{Nu} \rangle_w A_b} = \frac{0.3(\text{m})}{0.0283(\text{W/m-K}) \times 93.48 \times 0.078(\text{m}^2)} = 1.454℃/\text{W}$$

$$\langle R_{ku} \rangle_{w,f} = \frac{w}{k_f \langle \text{Nu} \rangle_w A_{ku,f} \eta_f N_f} = \frac{0.3(\text{m})}{0.0283(\text{W/m-K}) \times 93.48 \times 0.0308(\text{m}^2) \times 0.97 \times 20} = 0.190℃/\text{W}.$$

From (6.151), the overall thermal resistance is

$$\begin{aligned} \frac{1}{R_\Sigma} &= \frac{1}{\langle R_{ku} \rangle_{w,b}} + \frac{1}{\langle R_{ku} \rangle_{w,f}} \\ R_\Sigma &= 0.1680℃/\text{W}. \end{aligned}$$

The surface-convection heat transfer rate is

$$\langle Q_{ku} \rangle_w = \frac{T_s - T_{f,\infty}}{R_\Sigma} = \frac{80(℃) - 20(℃)}{0.1680(℃/\text{W})} = 357.1 \ \ \text{W}.$$

COMMENT:
The use of the fins has increased the surface-convection heat transfer from the plate by a factor of 7.5.

PROBLEM 6.37.FAM

GIVEN:

An automobile disc-brake converts mechanical energy (kinetic energy) to thermal energy. This thermal energy is stored in the disc and is transferred to the ambient by surface convection and surface radiation and is also transferred to other mechanical components by conduction (e.g., the wheel, axle, suspension, etc). The rate of energy conversion decreases with time due to the decrease in the automobile speed. Here, assume that it is constant and is $(\dot{S}_{m,\mathrm{F}})_o = 6 \times 10^4$ W. Assume also that the heat loss occurs primarily by surface-convection heat transfer from the disc surface. The disc is made of carbon steel AISI 4130 (Table C.16) and its initial temperature is $T_1(t=0) = T_{f,\infty} = 27$℃. The disc surface-convection heat transfer is from the two sides of disc of diameter $D = 35$ cm, as shown in Figure Pr.6.37(a), and the disc thickness is $l = 1.5$ cm. The Nusselt number is approximated as that for parallel flow over a plate of length D and determined at the initial velocity. The average automobile velocity is $u_{f,\infty} = 40$ km/hr and the ambient air is at $T_{f,\infty}$.

Evaluate the air properties at $T_{f,\infty}$.

SKETCH:

Figure Pr.6.37(a) shows the physical and an approximation models of the disc brake.

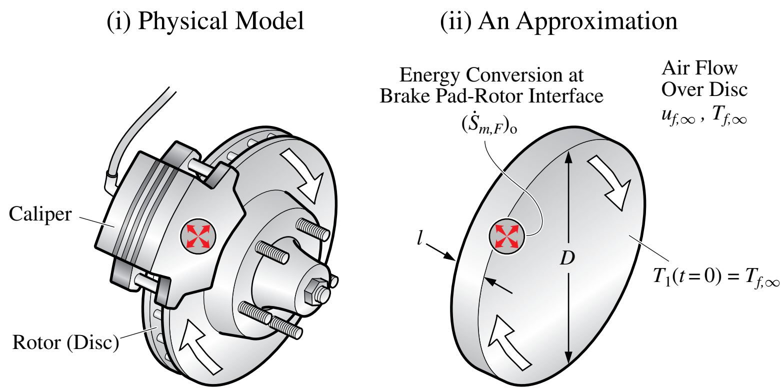


Figure Pr.6.37(a) An automobile brake cooled by parallel flow.
(i) Physical model. (ii) Approximate model.

OBJECTIVE:

(a) Assuming that the lumped-capacitance analysis is applicable, determine the temperature of the disc after 4 s [$T_1(t = 4$ s)].

(b) Using this temperature [i.e., $T_1(t = 4$ s)] as the initial temperature and setting the heat generation term equal to zero (i.e., the brake is released), determine the time it takes for the disc temperature to drop to $t_1 = 320$ K.

(c) Evaluate the Biot number and comment on the validity of the lumped-capacitance assumption. For the Biot number, the conduction resistance is based on the disc thickness l, while the surface convection resistance is based on the disc diameter D.

SOLUTION:

(a) This is a transient problem, with surface heating due to friction, and cooling by surface convection. The corresponding thermal circuit is shown in Figure Pr.6.37(b). Using a lumped-capacitance analysis, the integral-volume energy equation (2.9) becomes

$$.Q|_A = -(\rho c_p V)_1 \frac{dT_1}{dt} + (\dot{S}_{m,F})_o.$$

For surface-convection, heat transfer only, we have

$$Q|_A = \langle Q_{ku} \rangle_D = \frac{T_1 - T_{f,\infty}}{\langle R_{ku} \rangle_D} = -(\rho c_p V)_1 \frac{dT_1}{dt} + \dot{S}_{m,F}$$

The solution for this equation is given by (6.156), i.e.,

$$T_1(t) - T_{f,\infty} = [T_1(t=0) - T_{f,\infty}]e^{-t/\tau_1} + a_1\tau_1(1 - e^{-t/\tau_1}),$$

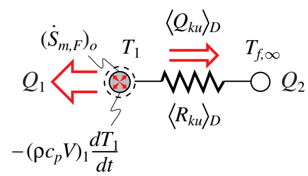


Figure Pr.6.37(b) Thermal circuit diagram.

where

$$\tau_1 = (\rho c_p V)_1 \langle R_{ku}\rangle_D, \qquad a_1 = \frac{\dot{S}_{m,F}}{(\rho c_p V)_1}$$

To determine the surface-convection resistance, the Nusselt number is needed. The properties for air, from Table C.22, evaluated at 300 K, are $\nu_f = 15.66 \times 10^{-6}$ m^2/s, $k_f = 0.0267$ W/m-K, and Pr = 0.69.
The Reynolds number for parallel flow over a flat plate with length D is given in Table 6.4 as

$$Re_D = \frac{u_{f,\infty} D}{\nu_f} = \frac{(40/3.6)(\text{m/s}) \times 0.35(\text{m})}{15.66 \times 10^{-6}(\text{m}^2/\text{s})} = 248{,}333 < \text{Re}_{D,t} = 5 \times 10^5$$

This Reynolds number is still in the laminar regime. The average Nusselt number, from Table 6.4, is given by

$$\langle\text{Nu}\rangle_D = 0.664\text{Re}_D^{1/2}\text{Pr}^{1/3} = 0.664 \times (248{,}333)^{1/2} \times (0.69)^{1/3} = 292.4.$$

Taking into account both sides of the plate, the average surface convection resistance is then given by (6.124) as

$$\langle R_{ku}\rangle_D = \frac{D}{\langle\text{Nu}\rangle_D k_f A_{ku}} = \frac{0.35(\text{m})}{292.4 \times 0.0267(\text{W/m-K}) \times 2 \times \frac{\pi \times (0.35)^2(\text{m}^2)}{4}} = 0.233^\circ\text{C/W}.$$

For carbon steel AISI 4130, from Table C.16, we have $\rho = 7{,}840$ kg/m^3, $c_p = 460$ J/kg-K, and $k_s = 43$ W/m-K. Then the parameters τ_1 (time constant) and a_1 are

$$\begin{aligned}
\tau_1 &= (\rho c_p V)_1 \langle R_{ku}\rangle_D = 7{,}840(\text{kg/m}^3) \times 460(\text{J/kg-K}) \times \frac{\pi \times (0.35)^2(\text{m}^2)}{4} \times 0.015(\text{m}) \times 0.233(^\circ\text{C/W}) \\
&= 1212.6\text{s} \\
a_1 &= \frac{(\dot{S}_{m,F})_o}{(\rho c_p V)_1} = \frac{6 \times 10^4}{7{,}840(\text{kg/m}^3) \times 460(\text{J/kg-K}) \times \frac{\pi(0.35)^2(\text{m}^2)}{4} \times 0.015(\text{m})} = 11.53^\circ\text{C/s}
\end{aligned}$$

Now the plate temperature after $t = 4$ s, for $T_1(t=0) = T_{f,\infty} = 300$ K, is

$$T_1(t = 4\,\text{s}) = 300.15(\text{K}) + 11.53(^\circ\text{C/s})] \times 1{,}212.6(\text{s})[1 - e^{-4(\text{s})/1212.6(\text{s})}] = 346.2 \text{ K}$$

(b) Once the break is released, the energy conversion due to friction stops. In this case, $a_1 = 0$. The time constant is still the same, because neither the disc properties nor the surface-convection resistance have changed. Then the time to cool the disc down to $T_1(t) = 320$ K is

$$t = -\tau_1 \ln\left[\frac{T_1(t) - T_{f,\infty}}{T_1(t = 4\,\text{s}) - T_{f,\infty}}\right] = -1{,}212.6(\text{s}) \times \ln\left[\frac{320(\text{K}) - 300.15(\text{K})}{346.2(\text{K}) - 300.15(\text{K})}\right] = 1020 \text{ s} = 17.0 \text{ min}$$

(c) The Biot number is given by the ratio of the conduction thermal resistance (through half the plate thickness) and the surface-convection resistance, i.e.,

$$\text{Bi}_l = \frac{A_k R_k}{A_{ku}\langle R_{ku}\rangle_D},$$

where $A_k = A_{ku} = \pi D^2/4$. Then we have

$$\text{Bi}_l = \frac{(l/2)/k_s}{D/(\langle\text{Nu}\rangle_D k_f)} = \frac{1.744 \times 10^{-4}[^\circ\text{C}/(\text{W/m}^2)]}{4.483 \times 10^{-2}[^\circ\text{C}/(\text{W/m}^2)]} = 3.89 \times 10^{-3}.$$

As $\text{Bi}_l \ll 1$, the lumped capacitance analysis can be used.

COMMENT:

Note that compared to the $t = 4$ s heat-up period, the cool-down period is very long. In Problem 6.39, the determination of $(\dot{S}_{m,F})_o$ is described.

PROBLEM 6.38.FAM

GIVEN:

In a portable, phase-change hand warmer, titanium bromide ($TiBr_4$, Table C.5) liquid is contained in a plastic cover (i.e., encapsulated) and upon solidification at the freezing temperature T_{sl} (Table C.5) releases heat. A capsule, which has a thin, rectangular shape and has a cross-sectional area $A_k = 0.04$ m^2, as shown in Figure Pr.6.38(a), is placed inside the pocket of a spectator watching an outdoor sport. The capsule has a planar surface area of $A_k = 0.04$ m^2.

The pocket has a thick insulation layer on the outside of thickness $L_o = 2$ cm (toward the ambient air) and a thinner insulation layer on the inside of thickness $L_i = 0.4$ cm (toward the body). The effective conductivity for both layers is $\langle k \rangle = 0.08$ W/m-K. The body temperature is $T_b = 32$℃. The outside layer is exposed to surface convection with a wind blowing as a cross flow over the body (diameter D) at a speed $u_{f,\infty} = 2$ m/s and temperature $T_{f,\infty} = 2$℃. For the surface-convection heat transfer, use cross flow over a cylinder of diameter $D = 0.4$ m. Assume that the heat flow is steady and that the temperatures are constant. Treat the conduction heat transfer as planar and through a cross-sectional surface area A_k.

Determine the air properties at $T = 300$ K from Table C.22.

For the surface-convection heat transfer, assume a cross flow over a cylinder of diameter $D = 0.4$ m. Assume that the heat flow is steady and that the temperatures are constant. Use planar geometry for the conduction resistances.

SKETCH:

Figure Pr.6.38(a) shows the phase-change material sandwiched between two insulation layers. The outside insulation layer is exposed to the surface convection.

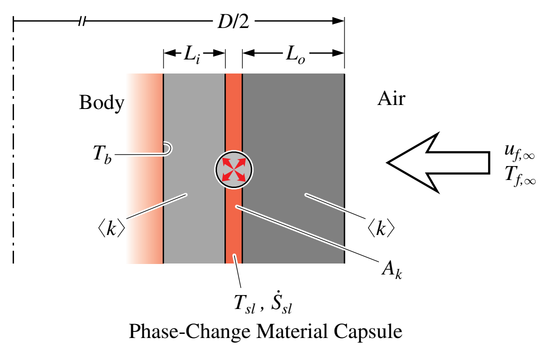


Figure Pr.6.38(a) Simplified, physical model for heat transfer from a hand warmer.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine all the thermal resistances that the heat flow from the capsule encounters. Use planar geometry for the conduction resistances.

(c) Determine the heat rate toward the body and toward the ambient air.

(d) Determine the total energy conversion rate $\dot{S}_{sl}$(W).

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.6.38(b).

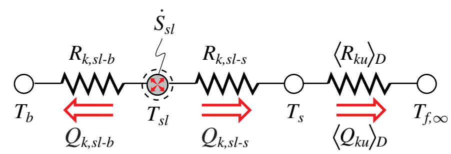


Figure Pr.6.38(b) Thermal circuit diagram.

(b) The thermal resistances are as follows.
(i) Internal conduction thermal resistance:

$$R_{k,sl\text{-}b} = \frac{L_i}{\langle k \rangle a_k} = \frac{0.04(\text{m})}{0.08(\text{W/m-K}) \times 0.04(\text{m}^2)} = 1.25^\circ\text{C/W}.$$

(ii) External conduction thermal resistance

$$R_{k,sl\text{-}s} = \frac{L_o}{\langle k \rangle a_k} = \frac{0.02(\text{m})}{0.08(\text{W/m-K}) \times 0.04(\text{m}^2)} = 6.25^\circ\text{C/W}.$$

(iii) Surface convection thermal resistance
The properties for air at $T = 300$ K, from Table C.22: $k_f = 0.0267$ W/m-K, $\nu_f = 15.66 \times 10^{-6}$ m^2/s, Pr = 0.69. For air in cross flow, the Reynolds number is given by (6.124) as

$$\text{Re}_D = \frac{u_{f,\infty} D}{\nu_f} = \frac{2(\text{m/s}) \times 0.4(\text{m})}{15.66 \times 10^{-6}(\text{m}^2/\text{s})} = 51{,}086.$$

From Table 6.4, with $a_1 = 0.027$ and $a_2 = 0.805$, the Nusselt Number is

$$\langle \text{Nu} \rangle_D = 0.027\text{Re}_D^{0.805}\text{Pr}^{1/3} = 0.027 \times (51{,}086)^{0.805} \times (0.69)^{1/3} = 147.2.$$

The area for convection relevant to the problem is the area over the pocket heater, or $A_{ku} = A_k$. Therefore, the surface convection thermal resistance is

$$\langle R_{ku} \rangle_D = \frac{D}{A_{ku} \langle \text{Nu} \rangle_D k_f} = \frac{0.4(\text{m})}{(0.04)(\text{m}^2) \times 147.2 \times 0.0267(\text{W/m-K})} = 2.54^\circ\text{C/W}.$$

(c) The fraction of heat flowing toward the body is (from Table C.5, for $TiBr_4$, $T_{sl} = 312.2$ K = 39.2℃)

$$Q_{k,sl\text{-}b} = \frac{T_{sl} - T_b}{R_{k,sl\text{-}b}} = \frac{(39.2 - 32)(^\circ\text{C})}{1.25(^\circ\text{C/W})} = 5.76 \text{ W}.$$

(d) The fraction of heat flowing toward the ambient air is

$$R_{\Sigma,sl\text{-}\infty} = R_{k,sl\text{-}s} + \langle R_{ku} \rangle_D = (6.25 + 2.54)(^\circ\text{C/w}) = 8.79^\circ\text{C/W}.$$

$$Q_{k,sl\text{-}\infty} = \frac{T_{sl} - T_{f,\infty}}{R_{\Sigma,sl\text{-}\infty}} = \frac{(39.2 - 2)(^\circ\text{C})}{8.79(^\circ\text{C/W})} = 4.23 \text{ W}.$$

(e) The energy conversion rate is then found from applying the integral-volume energy equation of energy to the T_{sl} node, i.e.,

$$\dot{S}_{sl} = Q_{k,sl\text{-}b} + Q_{k,sl\text{-}\infty} = (5.76 + 4.23)(\text{W}) = 9.99 \text{ W}.$$

COMMENT:
Note that although a thicker insulation layer was allowed on the outside, due to the low ambient air temperature a significant portion of the heat generated is still lost to the ambient air.

PROBLEM 6.39.FAM.S

GIVEN:

To analyze the heat transfer aspects of the automobile rear-window defroster, the window and the very thin resistive heating wires can be divided into identical segments. Each segment consists of an individual wire and an $a \times L \times l$ volume of glass affected by this individual wire/heater. Each segment has a uniform, transient temperature $T_1(t)$. This is shown in Figure Pr.6.41. In the absence of any surface phase change (such as ice or snow melting, or water mist evaporating), the Joule heating results in a temperature rise from the initial temperature $T_1(t = 0)$, and in a surface heat loss to the surroundings. The surface heat loss to the surroundings is represented by a resistance R_t. The surrounding far-field temperature is T_∞.

$T_1(t = 0) = -15$°C, $T_\infty = -15$°C, $l = 3$ mm, $a = 2$ cm, $L = 1.5$ m, $\dot{S}_{e,J} = 15$ W, $R_t = 2$°C/W.

Determine the glass plate properties from Table C.17.

SKETCH:

Figure Pr.6.39(a) shows a disc brake and its air flow and heat transfer characteristics.

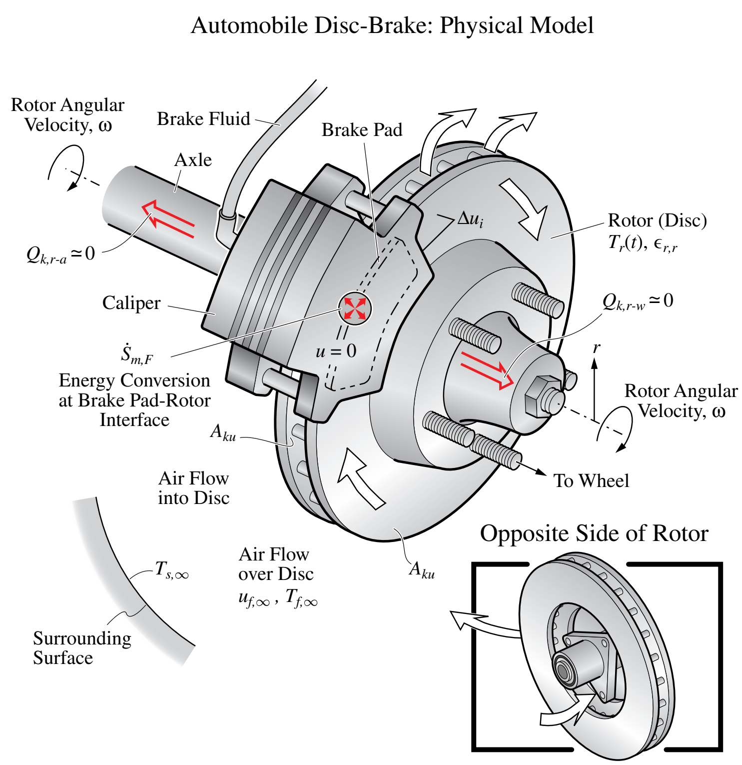


Figure Pr.6.39(a) An automobile disc brake showing the air flow around it.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Show that the lumped capacitance approximation is valid using l for the conduction resistance.
(c) Assuming no surface phase change occurs, determine the steady-state temperature of the glass.
(d) Still assuming no surface phase change occurs, determine the glass temperature after an elapsed time $t = 5$ min.

SOLUTION:

(a) The circuit diagram is shown in Figure Pr.6.39(b). The heat loss to the pad is negligible because of the small conductivity of the pad material ($\langle k \rangle \equiv 0.6$ W/m-K) which can be used as organic compound. The heat transfer to wheel and axle is also neglected.

(b) Thermal Circuit Model for Automobile Disc Brake

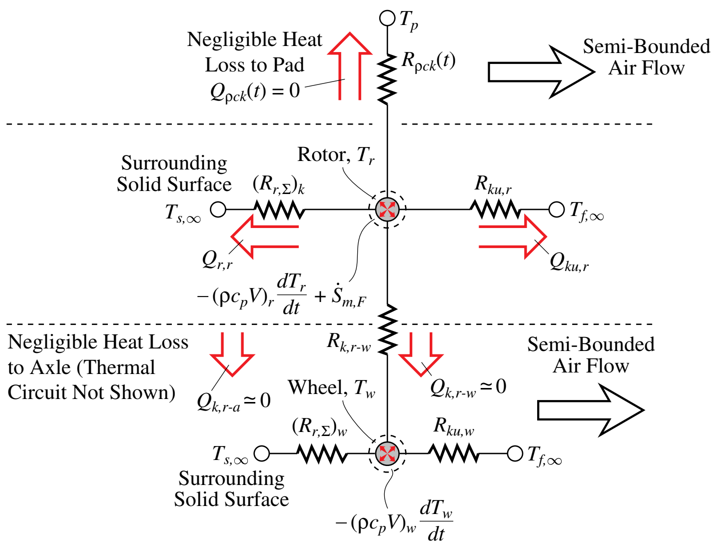


Figure Pr.6.39(b) Thermal circuit diagram.

(b) The integral-volume energy equation (6.155) is applicable, i.e.,

$$Q_{ku,r} + Q_{r,r} = -(\rho c_p V)_t \frac{dT_r}{dt} + \dot{S}_{m,F}(t).$$

Here $\dot{S}_{m,F}(t)$ is given and is zero for t $> \tau$.
The surface convection is given by (6.124) as

$$Q_{ku,r} = \frac{T_r - T_{f,\infty}}{\langle R_{ku}\rangle_D} \quad , \quad \langle R_{ku}\rangle_D = \frac{D}{A_{ku,r}\langle \mathrm{Nu}\rangle_D k_f}.$$

For $\langle \mathrm{Nu}\rangle_D$, we use parallel flow with $L = D$, i.e., we use Table 6.3, and we need to determine the magnitude of Re_D.

$$\mathrm{Re}_D = \frac{u_{f,\infty} D}{\nu_f}.$$

From Table C.22, for air we have,

$$\nu_f = 1.566 \times 10^{-5} \ \mathrm{m^2/s} \qquad \text{Table C.2}$$
$$k_f = 0.0267 \ \mathrm{W/m\text{-}K} \qquad \text{Table C.22}$$
$$\mathrm{Pr} = 0.69 \qquad \text{Table C.22.}$$

Then

$$\mathrm{Re}_D = \frac{22.22(\mathrm{m/s}) \times 0.35(\mathrm{m})}{1.566 \times 10^{-5}(\mathrm{m^2/s})} = 4.966 \times 10^5 \le \mathrm{Re}_{D,t} = 5 \times 10^5.$$

From Table 6.3, we use the laminar-regime correlation, i.e.,

$$\begin{aligned} \langle \mathrm{Nu}\rangle_D &= 0.664 \mathrm{Re}_D^{1/2} \mathrm{Pr}^{1/3} \\ &= 0.664 \times (4.966 \times 10^5)^{1/2} \times (0.69)^{1/3} \\ &= 413.5. \end{aligned}$$

The surface areas for the surface-convection and surface-radiation heat transfer are the two sides of the disc, i.e.,

$$\begin{aligned} A_{ku,r} = A_{r,r} &= 2(\pi D^2/4) \\ &= 0.1924 \ \mathrm{m^2} \end{aligned}$$

(c) Rotor Temperature

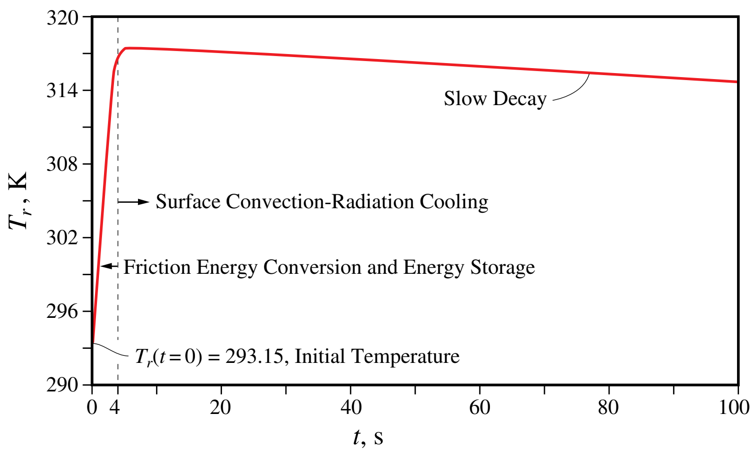


Figure Pr.6.39(c) Time variation of the disc (rotor) temperature during and after brake.

The surface-radiation heat transfer for $A_{r,\infty} \gg A_{r,r}$ and $\epsilon_{r,0} = 1.0$, is given by (4.49), i.e.,

$$\begin{aligned} Q_{r,r} &= A_{v,r}\epsilon_{r,r}\sigma_{\rm SB}(T_r^4 - T_{s,\infty}^4) \\ &= 0.1923(\text{m}^2) \times 0.4 \times 5.67 \times 10^{-8}(\text{W/m}^2\text{-K}^4) \times [T_r^4(t) - (293.15)(\text{K})]. \end{aligned}$$

From Table C.16 for carbon steel AISI 1010, we have,

$$\begin{aligned} \rho_r &= 7830 \text{ kg/m}^2 \\ c_{p,r} &= 434 \text{ J/kg-K} \\ k_r &= 64 \text{ W/m-K} \,. \end{aligned}$$

Also

$$V_r = \pi D^2 l/4 = 1.443 \times 10^{-3} \text{ m}^3.$$

Then

$$\begin{aligned} \dot{S}_{m,F} &= \frac{0.65}{2} \times 1500(\text{kg}) \times \frac{(22.22)^2(\text{m/s})^2}{4(\text{s})}\left[1 - \frac{t(s)}{4\text{s}}\right] \\ &= 6.017 \times 10^4(\text{W})\left[1 - \frac{t(s)}{4(\text{s})}\right]. \end{aligned}$$

(c) Using a software (such as SOPHT), Figure Pr.6.15(c) shows the time variation of T_r for $0 < t < 100$ s. Note that during the friction heating, there is a very rapid increase in T_r and during this period, the surface convection-radiation heat transfer is not significant. Using this assumption ($\langle Q_{ku}\rangle_D \simeq Q_{r,r} \simeq 0$) the energy equation can be integrated to find

$$T_r(t) = T_r(t=0) + \frac{0.65}{2}(\rho c_p V)_t M_a \frac{u_a^2}{\tau}\left(t - \frac{t^2}{2\tau}\right) \quad (\text{for } t \gg \tau).$$

which increases monotonically for $t \leq \tau$. From Figure Pr.6.15(c), note that even after 96 s of elapse time, T_r is still high.

COMMENT:

In order to examine the validity of the lumped-capacitance approximation for the rotor we need to show that the Biot Number is very small (i.e., less than 0.1). From (6.130), we have

$$\begin{aligned} \text{Bi}_D = \frac{R_{k,l}}{\langle R_{ku}\rangle_D} = \langle \text{Nu}\rangle_D \frac{l}{D}\frac{k_f}{k_s} &= 413.5 \times \frac{0.015(\text{m})}{0.35(\text{m})} \times \frac{0.0267(\text{W/m-K})}{64(\text{W/m-K})} \\ &= 7.393 \times 10^{-3} < 0.1. \end{aligned}$$

Note that we have used the thickness of the disk as the length for conduction.

PROBLEM 6.40.FAM

GIVEN:

A microprocessor chip generates Joule heating and needs to be cooled below a damage threshold temperature of 90°C. The heat transfer is by surface convection from its top surface and by conduction through the printed-circuit-board substrate from its bottom surface. The surface convection from the top surface is due to air flow from a fan that provides a parallel flow with a velocity of $u_{f,\infty}$. The conduction from the bottom surface is due to a temperature drop across the substrate of $T_p - T_s$.

The substrate is fabricated from a phenolic composite and has a thermal conductivity of k_s.

Neglect the contact resistance between the processor and the substrate.

Assume that the energy conversion occurs uniformly within the microprocessor chip.

Neglect the edge heat losses. Assume the processor is at a uniform temperature T_p.

$\dot{S}_{e,\mathrm{J}} = 35$ W, $T_{f,\infty} = 25$°C, $u_{f,\infty} = 0.5$ m/s, $w = 7$ cm, $a_s = 1.5$ mm, $k_s = 0.3$ W/m^2-K, $L = 3.5$ cm, $l = 1$ mm, $N_f = 16$.

Evaluate the properties of aluminum at $T = 300$ K. Evaluate the properties of air at $T = 300$ K.

SKETCH:

Figures Pr.6.40(a) and (b) show the microprocessor cooled by surface convection.

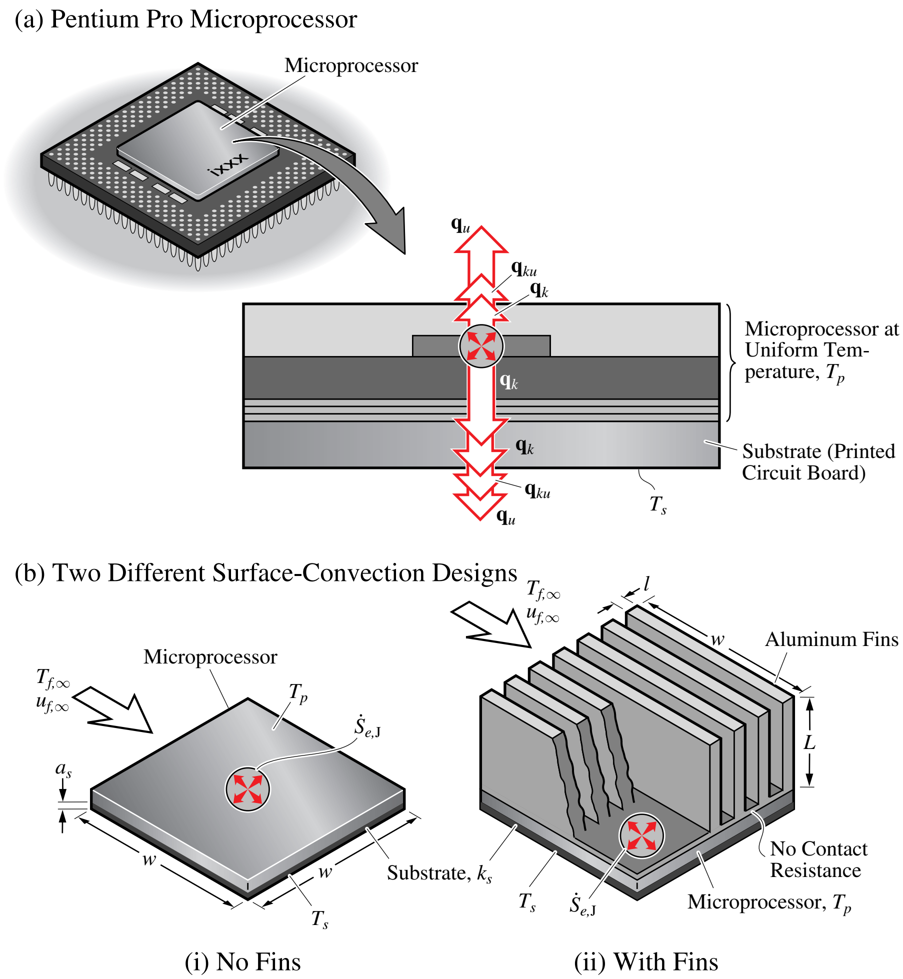


Figure Pr.6.40 Surface-convection cooling of a microprocessor. (a) Physical Model. (b) Two different surface-convection designs (i) without, and (ii) with back fins.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine T_p for the case with no fins and with $T_p - T_s = 10$℃.
(c) Determine T_p for the case with aluminum fins and with $T_p - T_s = 1$℃.
(d) Comment on the difference between the two cases with respect to the damage threshold temperature.

SOLUTION:
(a) The thermal circuit diagram for both cases is shown in Figure Pr.6.40(b).

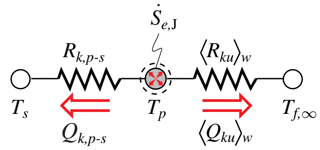


Figure Pr.6.40(b) Thermal circuit diagram.

Here T_p and $\dot{S}_{e,\mathrm{J}}$ are uniform within the volume of the processor chip. Therefore, we will consider the processor as lumped and model it with a single node T_p. From this node, we have surface-convection heat transfer from the top to the air, and conduction heat transfer from the bottom through a substrate. The conduction causes a temperature drop across the substrate of $T_p - T_s$, which is given.
For node T_p, for steady state conditions, we have from Figure Pr.6.40(b).

$$\begin{aligned} Q\mid_A = Q_{k,p\text{-}s} + \langle Q_{ku}\rangle_w &= \dot{S}_{e,\mathrm{J}} \\ \frac{T_p - T_s}{R_{k,p\text{-}s}} + \frac{T_p - T_{f,\infty}}{\langle R_{ku}\rangle_w} &= \dot{S}_{e,\mathrm{J}}, \end{aligned}$$

where

$$\begin{aligned} R_{k,p\text{-}s} &= \frac{a_s}{k_s A} = \frac{a_s}{k_s(w \times w)} = \frac{0.0015(\mathrm{m})}{0.3(\mathrm{W/m\text{-}K}) \times (0.07)^2(\mathrm{m})^2} = 1.02℃/\mathrm{W} \\ \langle R_{ku}\rangle_w &= \frac{w}{A_{ku}\langle \mathrm{Nu}\rangle_w k_f}. \end{aligned}$$

For the case with the fins, the fluid flow between the fins is assumed to be a parallel flow over a flat plate. Then, since w and $u_{f,\infty}$ are the same for both cases, we can use the same $\langle \mathrm{Nu}\rangle_w$ in parts (b) and (c).
From Table C.22 for air, at $T = 300$ K, we have $\nu_f = 15.66 \times 10^{-6}$ m^2/s, $k_f = 0.0267$ W/m-K and Pr $= 0.69$.
Then Re_w is

$$\mathrm{Re}_w = \frac{u_{f,\infty} w}{\nu_f} = \frac{0.5(\mathrm{m/s}) \times 0.07(\mathrm{m})}{15.66 \times 10^{-6}(\mathrm{m^2/s})} = 2235 < 5 \times 10^5, \text{ laminar flow.}$$

From Table 6.3, for parallel laminar flow over a flat plate, we have

$$\begin{aligned} \langle \mathrm{Nu}\rangle_w &= 0.664 \mathrm{Re}_w^{1/2} \mathrm{Pr}^{1/3} \\ \langle \mathrm{Nu}\rangle_w &= 0.664 \times (2235)^{1/2} \times (0.69)^{1/3} = 27.74. \end{aligned}$$

Then

$$\langle \mathrm{Nu}\rangle_w \frac{k_f}{w} = 27.74 \times \frac{0.0267(\mathrm{W/m\text{-}K})}{0.07(\mathrm{m})} = 10.58 \ \mathrm{W/m^2\text{-}K}.$$

The integral-volume of energy equation (2.9) for node T_p, then becomes

$$\begin{aligned} \frac{T_p - T_s}{R_{k,p\text{-}s}} + \frac{T_p - T_\infty}{R_{ku,p\text{-}\infty}} &= \dot{S}_{e,\mathrm{J}} \\ \frac{T_p - T_s}{1.02(℃/\mathrm{W})} + \frac{T_p - 25℃}{1/[A_{ku} \times 10.58(\mathrm{W/m^2\text{-}℃})]} &= 35 \ \mathrm{W}. \end{aligned}$$

(b) Case With No Fins, $T_p - T_s = 10$℃

For this case, A_{ku} is the top surface of the plate, i.e., $A_{ku} = w \times w = 0.0049$ m^2. The energy equation then becomes

$$\frac{10(℃)}{1.02(℃/\text{W})} + \frac{T_p - 25(℃)}{1/[0.0049(\text{m}^2) \times 10.58(\text{W/m}^2\text{-K})]} \quad = \quad 35 \text{ W}.$$

Solving for T_p, we have

$$T_p = 511℃.$$

(c) Case With Fins, $T_p - T_s = 1$℃

For this case, A_{ku} is the effective area of the fins and the base area (over which surface convection occurs). Then

$$A_{ku} = (A_b + N_f A_{ku,f} \eta_f),$$

where

$$\begin{aligned} A_b = A - N_f A_k &= w \times w - N_f \times (w \times l) \\ &= 0.0049(\text{m}^2) - 16 \times [0.07(\text{m}) \times 0.001(\text{m})] \\ &= 0.0049(\text{m}^2) - 16 \times [7 \times 10^{-5}(\text{m}^2)] = 0.00378 \text{ m}^2 \end{aligned}$$

and

$$\begin{aligned} A_{ku,f} &= P_{ku,f} \times L_c = 2(w + l) \times (L + l/2) \\ &= \{2 \times [0.07(\text{m}) + 0.001(\text{m})]\} \times [0.035(\text{m}) + 0.001(\text{m})/2] \\ &= 0.142(\text{m}) \times 0.0355(\text{m}) = 0.005041 \text{ m}^2. \end{aligned}$$

To find η_f we must first find the fin parameter m. The fins are fabricated from aluminum. From Table C.16, at $T = 300$ K, $k_{sl} = 237$ W/m-K. Then,

$$\begin{aligned} m &= \left(\frac{P_{ku,f} \langle \text{Nu} \rangle_w \frac{k_f}{w}}{k_{sl} A_k} \right)^{1/2} \\ &= \left[\frac{0.142(\text{m}) \times 10.58(\text{W/m}^2\text{-K})}{237(\text{W/m-K}) \times 7 \times 10^{-5}(\text{m}^2)} \right]^{1/2} = 9.516 \text{ m}^{-1}. \end{aligned}$$

Then, from (6.147) and (6.149), we have

$$\eta_f \quad = \quad \frac{\tanh(mL_c)}{mL_c} = \frac{\tanh[9.516(\text{m}^{-1}) \times 0.0355(\text{m})]}{9.516(\text{m}^{-1}) \times 0.0355(\text{m})} = \frac{\tanh(0.3378)}{0.3378} = 0.964.$$

$$\begin{aligned} A_{ku} = A_b + A_f \eta_f &= A_b + N_f A_{ku,f} \eta_f \\ &= [0.00378(\text{m}^2) + 16 \times 0.005041(\text{m}^2) \times 0.964] = 0.0815 \text{ m}^2. \end{aligned}$$

The energy equation becomes

$$\begin{aligned} \frac{T_p - T_s}{1.02(℃/\text{W})} + \frac{T_p - 25(℃)}{1/[A_{ku} \times 10.58(\text{W/m}^2\text{-K})]} &= 35 \text{ W} \\ \frac{1(℃)}{1.02(℃/\text{W})} + \frac{T_p - 25(℃)}{1/[0.0815(\text{m}^2) \times 10.58(\text{W/m}^2\text{-K})]} &= 35 \text{ W}. \end{aligned}$$

Solving for T_p, we obtain

$$T_p = 64.5℃.$$

(d) For the case with the fins, the calculated temperature is well above the damage threshold of $T_{p,max} = 90$℃. With the fins, there is a dramatic drop in the temperature to below this damage threshold temperature. This is due to the increased surface area, allowing for a much increased amount of surface-convection heat transfer to the fluid. Fins, or some equivalent heat transfer enhancement mechanism, are required for safe operation of this processor chip.

COMMENT:

Note that the fin effectiveness is

$$\Gamma_f = \frac{A_b + A_f \eta_f}{A} = \frac{0.00378 + 0.08066}{0.0049} = 17.23.$$

This shows a very effective fin attachment.

PROBLEM 6.41.FAM

GIVEN:

To analyze the heat transfer aspects of the automobile rear-window defroster, the window and the very thin resistive heating wires can be divided into identical segments. Each segment consists of an individual wire and an $a \times L \times l$ volume of glass affected by this individual heater. Each segment has a uniform, transient temperature $T_1(t)$. This is shown in Figure Pr.6.41(a). In the absence of any surface phase change, the Joule heating results in a temperature rise, from the initial temperature $T_1(t=0) = -15$℃, and a surface heat loss to the surroundings. The surface heat loss to the surroundings is represented by a resistance R_t.

The surrounding far-field temperature is T_∞. $T_1(t=0) = -15$℃, $T_\infty = -15$℃, $l = 3$ mm, $a = 2$ cm, $L = 1.5$ m, $\dot{S}_{e,\mathrm{J}} = 15$ W, $R_t = 2$℃/W.

Determine the glass plate properties from Table C.17.

SKETCH:

Figure Pr.6.41(a) shows a unit cell on a glass window, where a thin resistive heater heats the glass volume around it.

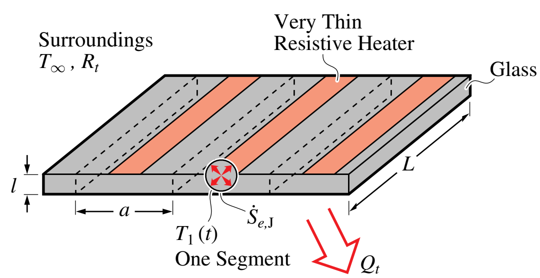


Figure Pr.6.41(a) Thin-film electric heaters on a glass surface.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Show that the lumped-capacitance approximation using l for the conduction resistance.

Assuming no surface phase change occurs, determine (c) the steady-state temperature of the glass, and (d) the glass temperature after an elapsed time $t = 5$ min.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.6.41(b).

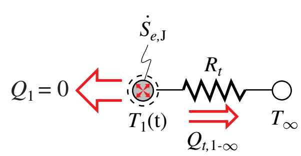


Figure Pr.6.41(b) Thermal circuit diagram.

(b) To show validity of the lumped capacitance assumption, we must show Bi $\ll 1$, or Bi < 0.1. For glass from Table C.17, at $T = 293$ K, we have $\rho = 2710$ kg/m^3, $c_p = 837$ J/kg-K, $k = 0.76$ W/m-K, $\alpha = 0.34 \times 10^{-6}$ m^2/s, and $V = a \times l \times L = 9 \times 10^{-5}$ m^3.

Then,

$$\begin{aligned}
\text{Bi}_l &= \frac{R_{k,l}}{R_k} \\
R_k &= \frac{l}{kA_k} = \frac{l}{kaL} = \frac{0.003(\text{m})}{(0.76)(\text{W/m-K}) \times (0.02)(\text{m}) \times (1.5)(\text{m})} = 0.1316℃/\text{W} \\
\text{Bi}_l &= \frac{0.1316}{2} = 0.0658 \ll 1 \quad \text{lumped assumption is valid.}
\end{aligned}$$

(c) The lumped-capacitance analysis, with a single external resistance heat transfer, results in (6.156), i.e.,

$$T_1(t) = T_\infty + [T_1(t=0) - T_\infty]e^{-t/\tau_1} + a_1\tau_1(1 - e^{-t/\tau_1}),$$

where

$$\begin{aligned}
\tau_1 &= (\rho c_p V)R_t = [(2710)(\text{kg/m}^3)(837)(\text{J/kg-K})(9 \times 10^{-5})(\text{m}^3)] \times (2)(℃/\text{W}) = 408.29 \text{ s} \\
a_1 &= \frac{\dot{S}_1 - Q_1}{(\rho c_p V)} = \frac{(15-0)(\text{W})}{(2710)(\text{kg/m}^3)(837)(\text{J/kg-K})(9 \times 10^{-5})(\text{m}^3)} = 0.0735℃/\text{s}.
\end{aligned}$$

Then

$$\begin{aligned}
T_1(t) &= -15(℃) + [(-15(℃) + 15(℃)] \times e^{-t/408.29(\text{s})} + 0.0735(℃/\text{s}) \times 408.29(\text{s})) \times [1 - e^{-t/408.29(\text{s})}] \\
T_1(t) &= -15(℃) + 0.0 + 30(℃)[1 - e^{-t/408.29(\text{s})}].
\end{aligned}$$

As $t \to \infty$, then $e^{-t/\tau_1} \to 0$ and

$$T_1(t \to \infty) = -15(℃) + 30(℃)(1 - 0) = 15℃ = 288.15\text{K}.$$

(d) At $t = 5$ min $= 300$ s, we have

$$T_1(t = 300 \text{ s}) = -15(℃) + 30(℃)[1 - e^{-300(\text{s})/408.29(\text{s})}] = 0.61℃ = 273.76\text{K}.$$

COMMENT:

Note that this heating rate is able to raise the glass temperature above 0℃ in 5 min. For faster response a higher heating rate is needed.

PROBLEM 6.42.FAM

GIVEN:

In particle spray surface coating using impinging-melting particles, prior to impingement the particles are mixed with a high temperature gas as they flow through a nozzle. The time of flight t (or similarly the nozzle-to-surface distance) is chosen such that upon arrival at the surface the particles are heated (i.e., their temperature is raised) close to their melting temperature. This is shown in Figure Pr.6.42(a). The relative velocity of the particle-gas, which is used in the determination of the Nusselt number, is Δu_p. Consider lead particles of diameter D flown in an air stream of $T_{f,\infty}$. Assume that the particles are heated from the initial temperature of $T_1(t=0)$ to the melting temperature T_{sl} with surface-convection heat transfer only (neglect radiation heat transfer).

$T_1(t=0) = 20°\text{C}$, $T_{f,\infty} = 1{,}500$ K, $D = 200\ \mu$m, $\Delta u_p = 50$ m/s.

Determine the air properties at $T = 1{,}500$ K (Table C.22), and the lead properties at $T = 300$ K (Table C.16).

SKETCH:

Figure Pr.6.42(a) shows the solid particles entrained in hot gas then after surface-convection heating arriving at the substrate for deposition.

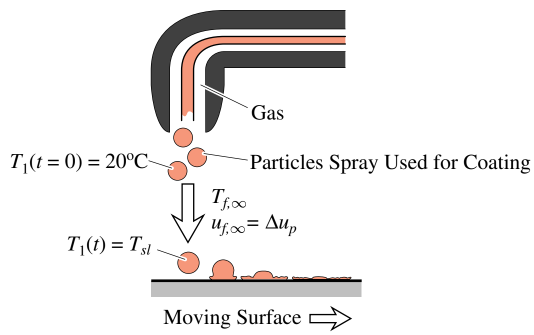


Figure Pr.6.42(a) A particle spray coating surface-coating process using impinging-melting particles.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the Biot number Bi_D, based on the particle diameter D. Can the particles be treated as lumped capacitance?

(c) Determine the time of flight t needed to reach the melting temperature T_{sl}.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.6.42(b).

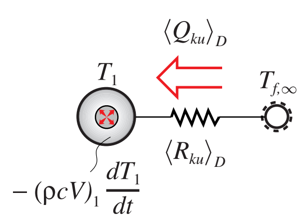


Figure Pr.6.42(b) Thermal circuit diagram.

(b) From Table C.16 for lead at 300 K, we have $\rho_s = 11{,}340$ kg/m^3, $c_{p,s} = 129$ J/kg-K, $k_s = 35.3$ W/m-K, $\alpha_s = 24.1 \times 10^{-6}$ m^2/s, and $T_{sl} = 601$ K.

From Table C.22 for air at 1500 K, we have $\rho_f = 0.235$ kg/m^3, $c_{p,f} = 1202$ J/kg-K, $k_f = 0.0870$ W/m-K, $\nu_f = 229 \times 10^{-6}$ m^2/s, and Pr = 0.7.

The Biot number based on D is defined as

$$\begin{aligned}\mathrm{Bi}_D &= \frac{R_{k,s}}{R_{k,u}}\\ &= \left(\frac{D}{4k_s}\right)\left(\mathrm{Nu}_D\frac{k_f}{D}\right) = \mathrm{Nu}_D\frac{k_f}{4k_s}.\end{aligned}$$

To determine the Nu_D, we first must determine the Re_D, given as

$$\mathrm{Re}_D = \frac{\Delta u_p D}{\nu_f} = \frac{50(\mathrm{m/s}) \times (200 \times 10^{-6})(\mathrm{m})}{229 \times 10^{-6}(\mathrm{m^2/s})} = 43.67.$$

Then from Table 6.4 for a sphere we have

$$\begin{aligned}\langle \mathrm{Nu}\rangle_D &= 2 + (0.4\mathrm{Re}_D^{1/2} + 0.06\mathrm{Re}_D^{2/3})\mathrm{Pr}^{0.4}\\ &= 2 + [0.4(43.67)^{1/2} + 0.06(43.67)^{2/3}](0.7)^{0.4} = 4.937.\end{aligned}$$

Then substituting in to the above, the Biot number is

$$\begin{aligned}\mathrm{Bi}_D &= \langle \mathrm{Nu}\rangle_D \frac{k_f}{4k_s}\\ &= (4.937)\frac{0.0870(\mathrm{W/m\text{-}K})}{4 \times 35.3(\mathrm{W/m\text{-}K})} = 3.042 \times 10^{-3}.\end{aligned}$$

(c) Since the $\mathrm{Bi}_D \ll 1$, we can analyze the lead droplets as lumped capacitance systems. Applying conservation of energy around the droplet gives

$$Q_{ku} = \frac{T_{f,\infty} - T_1}{\langle R_{ku}\rangle_D} = -(\rho c_p V)_1 \frac{dT_1}{dt}.$$

The droplet is a lumped system with a single resistive heat transfer. The solution to this is given by (6.156) as

$$T_1(t) = T_{f,\infty} + [T_1(t=0) - T_{f,\infty}]e^{-t/\tau_1} + a_1\tau_1(1 - e^{-t/\tau_1})$$

where

$$\tau_1 = (\rho c_p V)_1 \langle R_{ku}\rangle_D, \quad a_1 = \frac{\dot{S}_1 - Q_1}{(\rho c_p V)_1}.$$

For this case, $\dot{S}_1 = 0$ and $Q_1 = 0$, therefore $a_1 = 0$. Then

$$\begin{aligned}\tau_1 &= (\rho c_p V)_1 \langle R_{ku}\rangle_D\\ &= (\rho c_p V)_1 \frac{D}{A_{ku}\langle \mathrm{Nu}\rangle_D k_f}\\ &= \frac{(\rho c_p)_1 D}{\langle \mathrm{Nu}\rangle_D k_f}\left(\frac{V_1}{A_{ku}}\right) = \frac{(\rho c_p)_1}{\langle \mathrm{Nu}\rangle_D k_f}\left(\frac{D^2}{6}\right)\\ &= \frac{11{,}340(\mathrm{kg/m^3}) \times 129(\mathrm{J/kg\text{-}K}) \times (200 \times 10^{-6})^2(\mathrm{m^2})}{4.937 \times 0.0870(\mathrm{W/m\text{-}K}) \times 6}\\ &= 0.02270 \text{ s}.\end{aligned}$$

Now, solving for t, for $T_1 = T_{sl} = 601$ K, we have

$$\begin{aligned}T_{sl} &= T_{f,\infty} + [T_1(t=0) - T_{f,\infty}]e^{-t/\tau_1}\\ \frac{T_{sl} - T_{f,\infty}}{T_1(t=0) - T_{f,\infty}} &= e^{-t/\tau_1}\\ t &= -\tau_1 \ln\left[\frac{T_{sl} - T_{f,\infty}}{T_1(t=0) - T_{f,\infty}}\right]\\ &= -0.02270(\mathrm{s}) \times \ln\left[\frac{(601 - 1{,}500)(\mathrm{K})}{(293.15 - 1{,}500)(\mathrm{K})}\right] = 6.685 \times 10^{-3} \text{ s}.\end{aligned}$$

COMMENT:

Note that here about one-third of a time constant is needed to reach the desired particle temperature.

PROBLEM 6.43.FAM

GIVEN:

A rectangular (square cross section) metal workpiece undergoing grinding, shown in Figure Pr.6.43(a), heats up and it is determined that a surface-convection cooling is needed. The fraction of the energy converted by friction heating $\dot{S}_{m,\mathrm{F}}$, that results in this heating of the workpiece, is a_1. This energy is then removed from the top of the workpiece by surface convection. A single, round impinging air jet is used. Assume steady-state heat transfer and a uniform workpiece temperature T_s.

$\dot{S}_{m,\mathrm{F}} = 3{,}000$ W, $a_1 = 0.7$, $T_{f,\infty} = 35$°C, $\langle u_f \rangle = 30$ m/s, $D = 1.5$ cm, $L = 15$ cm, $L_n = 5$ cm.

Evaluate properties of air at $T = 300$ K.

SKETCH:

Figure Pr.6.43(a) shows the workpiece and surface convection cooling.

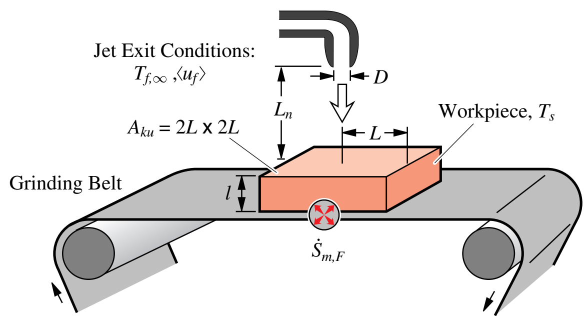


Figure Pr.6.43(a) Grinding of a metal workpiece.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the workpiece temperature T_s.

(c) What should the ratio of the workpiece thickness l to its conductivity k_s be for the uniform temperature assumption to be valid?

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.6.43(b).

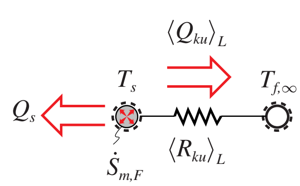


Figure Pr.6.43(b) Thermal circuit diagram.

(b) Applying the conservation of energy equation to the boundary node T_s at the interface of the workpiece and the grinder belt, and noting steady-state, we have

$$Q|_A = \langle Q_{ku} \rangle_L + Q_s = \dot{S}_{m,F}.$$

It is given that the fraction of energy conversion by friction heating $\dot{S}_{m,F}$ that results in heating of the workpiece is a_1. This is the same energy that must be removed for steady-state conditions to exist. So we then have

$$\begin{aligned} \langle Q_{ku} \rangle_L &= a_1 \dot{S}_{m,F} \\ \frac{T_s - T_{f,\infty}}{\langle R_{ku} \rangle_L} &= a_1 \dot{S}_{m,F}, \end{aligned}$$

where

$$\langle R_{ku}\rangle_L = \frac{1}{A_{ku}\langle \mathrm{Nu}\rangle_L \dfrac{k_f}{L}}.$$

Solving for $\langle \mathrm{Nu}\rangle_L$, we note that we have a single, round nozzle, impinging jet. Therefore, from Table 6.3,

$$\langle \mathrm{Nu}\rangle_L = 2\mathrm{Re}_D^{1/2}\mathrm{Pr}^{0.42}(1+0.005\mathrm{Re}_D^{0.55})^{1/2}\frac{1-1.1\dfrac{D}{L}}{1+0.1\left(\dfrac{L_n}{D}-6\right)\dfrac{D}{L}}.$$

From Table C.22 at $T = 300$ K, $\nu_f = 15.66\times 10^{-6}$ m^2/s, $k_f = 0.0267$ W/m-K, and Pr = 0.69. Then

$$\begin{aligned}
\mathrm{Re}_D &= \frac{\langle u_f\rangle D}{\nu_f} = \frac{30(\mathrm{m/s})\times 0.015(\mathrm{m})}{15.66\times 10^{-6}(\mathrm{m^2/s})} \\
&= 2.874\times 10^4 \\
\langle \mathrm{Nu}\rangle_L &= 2\times(2.874\times 10^4)^{1/2}\times(0.69)^{0.42}\times[1+0.005(2.874\times 10^4)^{0.55}]^{1/2} \\
&\quad \times\left\{\frac{1-1.1\times\left[\dfrac{0.015(\mathrm{m})}{0.15(\mathrm{m})}\right]}{1+0.1\left[\dfrac{0.05(\mathrm{m})}{0.015(\mathrm{m})}-6\right]\left[\dfrac{0.015(\mathrm{m})}{0.15(\mathrm{m})}\right]}\right\} \\
&= 412.3.
\end{aligned}$$

Then solving for $\langle R_{ku}\rangle_L$, we have

$$\begin{aligned}
A_{ku} &= 2L\times 2L = 4L^2 = 4\times(0.15)^2(\mathrm{m})^2 \\
&= 0.09\ \mathrm{m}^2 \\
\langle R_{ku}\rangle_L &= \frac{1}{0.09(\mathrm{m}^2)\times 412.3\times\dfrac{0.0267(\mathrm{W/m\text{-}K})}{0.15(\mathrm{m})}} = 0.1514^\circ\mathrm{C/W}.
\end{aligned}$$

From the conservation of energy equation, T_s is

$$\begin{aligned}
T_s &= T_{f,\infty} + a_1\langle R_{ku}\rangle_L \dot{S}_{m,F} \\
&= 35(^\circ\mathrm{C}) + (0.7)[0.1514(^\circ\mathrm{C/W})][3000(\mathrm{W})] \\
&= 352.9^\circ\mathrm{C} = 626.1\ \mathrm{K}.
\end{aligned}$$

(c) The lumped assumption is valid when Bi < 0.1. From (6.130) and noting for conduction across the thickness of the workpiece that $A_k = A_{ku}$, we have

$$\mathrm{Bi}_L = \frac{R_{k,s}}{\langle R_{ku}\rangle_L} = \frac{R_k A_k}{\langle R_{ku}\rangle_L A_{ku}} = \left(\frac{l}{k_s}\right)\left(\frac{1}{\langle \mathrm{Nu}\rangle_L k_f/L}\right)^{-1} < 0.1.$$

Solving for l/k_s we have

$$\frac{l}{k_s} < 0.1\frac{L}{\langle \mathrm{Nu}\rangle_L k_f} = 0.1\times\frac{0.15(\mathrm{m})}{412.3\times 0.0267(\mathrm{W/m\text{-}K})},$$

or

$$\frac{l}{k_s} < 1.36\times 10^{-3}\ ^\circ\mathrm{C}/(\mathrm{W/m^2}).$$

COMMENT:

This l/k_s can be easily achieved for metals ($k_s > 10$ W/m-K).

PROBLEM 6.44.FUN

GIVEN:

A microprocessor with the Joule heating $\dot{S}_{e,\mathrm{J}}$ is cooled by surface convection for one of its surfaces. An off-the-shelf surface attachment is added to this surface and has a total of N_f square-cross-sectional aluminum pin fins attached to it, as shown in Figure Pr.6.44. Air is blown over the fins and we assume that the Nusselt number can be approximated using the far-field air velocity $u_{f,\infty}$ and a cross flow over each square-cross-sectional cylinder fin (i.e., the Nusselt number is not affected by the presence of the neighboring fins). This is only a rough approximation.

$T_{f,\infty} = 35^\circ\mathrm{C}, u_{f,\infty} = 2$ m/s, $\dot{S}_{e,\mathrm{J}} = 50$ W, $D = 2$ mm, $a = 5$ cm, $N_f = 121, L = 2$ cm.

Evaluate the air and aluminum properties at $T = 300$ K. Assume that the $\langle \mathrm{Nu} \rangle_D$ correlation of Table 6.3 is varied.

SKETCH:

Figure Pr.6.44(a) shows the extended surface.

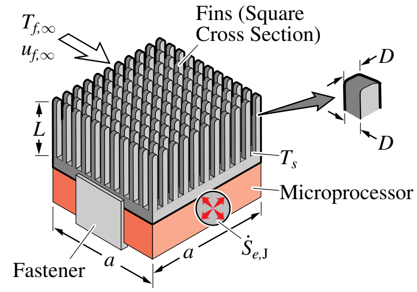


Figure Pr.6.44(a) A microprocessor with the Joule heating and a surface-convection cooling. There is an attached extended surface for reduction of the microprocessor temperature.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine the fin efficiency.
(c) Determine the steady-state surface temperature T_s.
(d) Determine the fin effectiveness.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.6.44(b). The steady-state, uniform surface temperature is T_s.

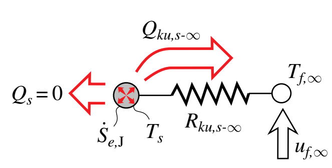


Figure Pr.6.44(b) Thermal circuit model.

(b) The fin efficiency is given by (6.147) as

$$\eta_f = \frac{\tanh(mL_c)}{mL_c}$$

$$m = \left(\frac{P_{ku,f} \langle \mathrm{Nu} \rangle_D k_f}{A_k k_s D} \right)^{1/2}.$$

We use Table 6.3 for $\langle \text{Nu} \rangle_D$, where D is the side length for the square cross section cylinder. Here

$$P_{ku,f} = 4D, \quad L_c = L + \frac{D}{4}, \quad A_k = D^2,$$

where we used (6.141) and a similarity to circular pin fins. From Table C.14, for aluminum, $k_s = 237$ W/m-K. From Table C.22, for air at $T = 300$ K, we have

$$\begin{aligned} \text{air:}\quad & \nu_f = 1.566 \times 10^{-5}\ \text{m}^2/\text{s} && \text{Table C.22} \\ & k_f = 0.0267\ \text{W/m-K} && \text{Table C.22} \\ & \text{Pr} = 0.69 && \text{Table C.22.} \end{aligned}$$

Then

$$\text{Re}_D = \frac{u_{f,\infty} D}{\nu_f} = \frac{2(\text{m/s}) \times 2 \times 10^{-3}(\text{m})}{1.566 \times 10^{-5}(\text{m}^2/\text{s})} = 255.4$$

This is outside the range of Re_D given in Table C.6.3, however, for lack of an alternative we will use the available results, i.e.,

$$\begin{aligned} \langle \text{Nu} \rangle_D &= a_1 \text{Re}^{a_2} \text{Pr}^{1/3} \\ &= 0.102 \text{Re}_D^{0.675} \text{Pr}^{1/3} \\ &= 0.102 \times (255.4)^{0.675} \times (0.69)^{1/3} \\ &= 3.800. \end{aligned}$$

Then

$$\begin{aligned} m &= \left(\frac{4\langle \text{Nu} \rangle_D}{D^2} \frac{k_f}{k_s} \right)^{1/2} \\ &= \left[\frac{4 \times 3.800}{(2 \times 10^{-3})^2 (\text{m}^2)} \frac{0.0267(\text{W/m-K})}{237(\text{W/m-K})} \right]^{1/2} = 20.69\ 1/\text{m} \end{aligned}$$

$$L_c = 0.02(\text{m}) + 0.002/4(\text{m}) = 0.0205\ \text{m}$$

$$mL_c = 20.69(1/\text{m}) \times 0.0205(\text{m}) = 0.4241.$$

Next, interpolating from Table 6.6, we have

$$\eta_f = \frac{\tanh(mL_c)}{mL_c} = \frac{\tanh(0.4241)}{0.4241} = \frac{0.3998}{0.4241} = 0.9426.$$

(c) The energy equation for the microprocessor volume is written, using Figure Pr.6.28(b), as

$$Q_{ku,s\text{-}\infty} = \dot{S}_{e,\text{J}}.$$

From (6.149), we have

$$Q_{ku,s\text{-}\infty} = (A_b + A_f \eta_f) \langle \text{Nu} \rangle_D \frac{k_f}{D} (T_s - T_{f,\infty}),$$

where we have used $\langle \text{Nu} \rangle_D$ for the base and the fin surfaces.
Here

$$\begin{aligned} A_b &= a^2 - N_f D^2 \\ &= (0.05)^2 (\text{m}^2) - 121 \times (0.002)^2 (\text{m}^2) = 2.016 \times 10^{-3}\ \text{m}^2 \\ A_f &= N_f \times 4 D L_c \\ &= 4 \times 121 \times 0.002(\text{m}) \times (0.0205)(\text{m}) = 1.984 \times 10^{-2}\ \text{m}^2. \end{aligned}$$

Then

$$\begin{aligned} \dot{S}_{e,\text{J}} = Q_{ku,s\text{-}\infty} &= (2.016 \times 10^{-3} + 1.984 \times 10^{-2} \times 0.9426)(\text{m}^2) \times \\ & 3.800 \times \frac{0.0267(\text{W/m-K})}{0.002(\text{m})} \times (T_s - T_{f,\infty}) \\ 50(\text{W}) &= 1.051(\text{W/°C})[T_s - 35(\text{°C})] \\ T_s &= 82.57\text{°C} \end{aligned}$$

(d) The fin effectiveness is defined in Section 6.8.2 as

$$\begin{aligned}\Gamma_f &= \frac{A_b + A_f\eta_f}{A} \\ &= \frac{(2.016\times10^{-3} + 1.984\times10^{-2}\times0.9426)(\text{m}^2)}{(0.05)^2(\text{m}^2)} = 8.287.\end{aligned}$$

COMMENT:

A fin effectiveness of $\Gamma_f = 8.287$ is high enough to allow for maintaining the microprocessor at a temperature below the damage threshold (which is around 100℃). Also note that we have used a $\langle\text{Nu}\rangle_D$ correlation that is a only an approximation for the collection of the fins used here. A more accurate value of $\tanh(0.4241) = 0.4004$ can be obtained from most pocket calculators.

PROBLEM 6.45.FUN

GIVEN:

Desiccants (such as silica gel) are porous solids that adsorb moisture (water vapor) on their large interstitial surface areas. The adsorption of vapor on the surface results in formation of an adsorbed water layer. This is similar to condensation and results in liberation of energy. The heat of adsorption, similar to the heat of condensation, is negative and is substantial. Therefore, during adsorption the desiccant heats up. The heat of adsorption for some porous solids is given in Table C.5(b). Consider a desiccant in the form of pellets and as an idealization consider a spherical pellet of diameter D in a mist-air stream with far-field conditions $T_{f,\infty}$ and $u_{f,\infty}$. Assume that the released energy is constant.

$\dot{S}_1 = \dot{S}_{ad} = \Delta h_{ad}\rho_{ad}V/t_o$, $D = 5$ mm, $\rho_{ad} = 200$ kg/m^3, $T_1(t=0) = 10$℃, $T_{f,\infty} = 10$℃, $(\rho c_p)_1 = 10^6$ J/m^3-K, $u_{f,\infty} = 3$ cm/s, $\Delta h_{ad} = 3.2 \times 10^6$ J/kg, $t_o = 1$ hr.

Evaluate properties of air at $T = 300$ K.

SKETCH:

Figure Pr.6.45(a) shows the desiccant pellet (porous zeolite) in cross, moist-air flow.

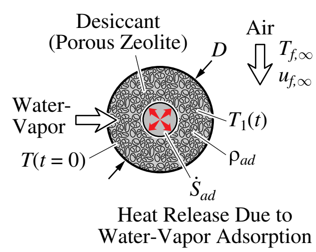


Figure Pr.6.45(a) A desiccant pellet in a cross, moist air flow.

OBJECTIVE:

(a) Draw the thermal circuit diagram for the pellet.

(b) Determine the pellet temperature after an elapsed time t_o.

SOLUTION:

(a) Figure Pr.6.45(b) shows the thermal circuit diagram.

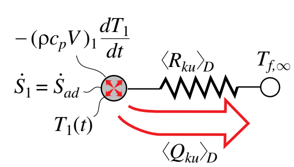


Figure Pr.6.45(b) Thermal circuit diagram.

(b)The temperature of the pellet is given by (6.156), i.e.,

$$T_1(t) = T_{f,\infty} + [T_1(t=0) - T_{f,\infty}]e^{-t/\tau_1} + a_1\tau_1(1 - e^{t/\tau_1})$$

$$\tau_1 = (\rho c_p V)_1 \langle R_{ku}\rangle_D, \quad a_1 = \frac{\dot{S}_1 - Q_1}{(\rho c_p V)_1}, \quad V_1 = \frac{\pi D^3}{6}.$$

The average surface-convection resistance $\langle R_{ku}\rangle_D$ is found from (6.124), i.e.,

$$\langle R_{ku}\rangle_D = \frac{D}{A_{ku}\langle \text{Nu}\rangle_D k_f}, \quad A_{ku} = \pi D^2.$$

The Nusselt number is found from Table 6.4, i.e.,

$$\langle \mathrm{Nu}\rangle_D = 2 + (0.4\mathrm{Re}_D^{1/2} + 0.06\mathrm{Re}_D^{2/3})\mathrm{Pr}^{0.4}, \mathrm{Re}_D = \frac{u_{f,\infty}D}{\nu_f}.$$

From Table C.22, at $T = 300\mathrm{K}$, we have for air

$$\begin{aligned} k_f &= 0.0267 \text{ W/m-K} && \text{Table C.22} \\ \nu_f &= 1.566 \times 10^{-5} \text{ m}^2/\text{s} && \text{Table C.22} \\ \mathrm{Pr} &= 0.69. \end{aligned}$$

Then

$$\begin{aligned}
\mathrm{Re}_D &= \frac{0.03(\mathrm{m/s}) \times 5 \times 10^{-3}(\mathrm{m})}{1.566 \times 10^{-5}(\mathrm{m}^2/\mathrm{s})} = 9.5785 \\
\langle \mathrm{Nu}\rangle_D &= 2 + [0.4(9.5785)^{1/2} + 0.06(9.5785)^{2/3}](0.69)^{0.4} = 3.300 \\
\langle R_{ku}\rangle_D &= \frac{5 \times 10^{-3}(\mathrm{m})}{\pi \times (5 \times 10^{-3})^2(\mathrm{m}^2) \times 3.300 \times 0.0267(\mathrm{W/m\text{-}K})} = 722.4^\circ\mathrm{C/W} \\
\tau_1 &= 10^6(\mathrm{J/m^3\text{-}K}) \times \frac{\pi}{6}(5 \times 10^{-3})^3(\mathrm{m}^3) \times 722.4(^\circ\mathrm{C/W}) = 47.28 \text{ s} \\
\dot{S}_1 &= \dot{S}_{ad} = \frac{3.2 \times 10^6(\mathrm{J/kg}) \times 200(\mathrm{kg/m^3}) \times \pi \times (5 \times 10^{-3})^3/6(\mathrm{m}^3)}{3{,}600(\mathrm{s})} = 1.164 \times 10^{-2} \text{ W} \\
a_1 &= \frac{1.164 \times 10^{-2}(\mathrm{W})}{10^6(\mathrm{J/m^3\text{-}K}) \times \pi \times (5 \times 10^{-3})^3/6(\mathrm{m}^3)} = 0.1778 \times 10^{-2}\,^\circ\mathrm{C/s} \\
T_1(t = t_o = 1 \text{ hr}) &= 10(^\circ\mathrm{C}) + 0.1778(^\circ\mathrm{C/s}) \times 47.28(\mathrm{s}) \times [1 - e^{-3{,}600(\mathrm{s})/47.28(\mathrm{s})}] \\
&= 18.41^\circ\mathrm{C}.
\end{aligned}$$

COMMENT:

Since the vapor is slow in diffusing into the porous pellet, the energy release rate is rather low. Also since the time constant τ_1, is much less than the elapsed time of interest, the above T_1 is the steady-state temperature during the heat release period t_o.

PROBLEM 6.46.FUN.S

GIVEN:

Consider the concept of the critical radius discussed in Example 6.13. An electrical-current conducting wire is electrically insulated using a Teflon layer wrapping, as shown in Figure Pr.6.46(a). Air flows over the wire insulation and removes the Joule heating. The thermal circuit diagram is also shown.

$L = 1$ m, $R_1 = 3$ mm, $u_{f,\infty} = 0.5$ m/s.

Evaluate the air properties at $T = 300$ K. Thermal conductivity of Teflon is given in Table C.17.

SKETCH:

Figure Pr.6.46(a) shows the insulated wire and the thermal circuit diagram.

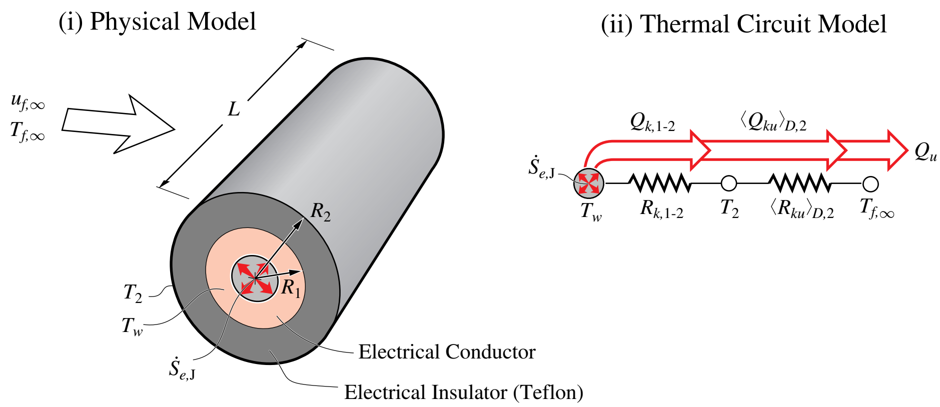


Figure Pr.6.46(a)(i) An electrical-current carrying wire is electrically insulated with a Teflon layer wrapping. (ii) Thermal circuit diagram.

OBJECTIVE:

(a) Plot the variation of $R_\Sigma = R_{k,1\text{-}2} + \langle R_{ku}\rangle_{D,2}$ with respect to R_2, for $R_1 \le R_2 \le 3R_1$.

(b) Determine $R_2 = R_c$ (where R_Σ is minimum).

(c) Show the contributions due to $R_{k,1\text{-}2}$ and $\langle R_{ku}\rangle_{D,2}$ at $R_2 = R_c$.

(d) Also determine R_c from the expression given in Example 6.13, i.e.,

$$R_c = \left(\frac{a_2 k_s}{2^{a_2-1} a_R}\right)^{1/a_2}.$$

SOLUTION:

(a) The total resistance to the heat flow is given in Example 6.13 as

$$R_\Sigma = \frac{\ln(R_2/R_1)}{2\pi L k_s} + \frac{1}{\pi L \langle \mathrm{Nu}\rangle_{D,2} k_f}.$$

From Table C.17, we have for Teflon

$$k_s = 0.26 \text{ W/m-K} \quad \text{Table C.17.}$$

From Table C.22 for air, at $T = 300$ K, we have

$$k_f = 0.0267 \text{ W/m-K} \qquad \text{Table C.22}$$
$$\nu_f = 1.566 \times 10^{-5} \text{ m}^2\text{/s} \qquad \text{Table C.22}$$
$$\mathrm{Pr} = 0.69 \qquad \text{Table C.22.}$$

The Reynolds number $\mathrm{Re}_{D,2}$ and $\langle \mathrm{Nu} \rangle_{D,2}$ are given in Table 6.3 as

$$\begin{aligned}
\mathrm{Re}_{D,2} &= \frac{u_{f,\infty} 2R_2}{\nu_f} \\
&= \frac{0.5(\mathrm{m/s}) \times 2}{1.566 \times 10^{-5}(\mathrm{m^2/s})} \times R_2 \\
&= 6.386 \times 10^4 (1/\mathrm{m}) \times R_2 \\
\langle \mathrm{Nu} \rangle_{D,2} &= a_1 \mathrm{Re}_{D,2}^{a_2} \mathrm{Pr}^{1/3}.
\end{aligned}$$

Since $R_2 \geq R_1$, we begin from $R_2 = R_1$. Then

$$\begin{aligned}
\mathrm{Re}_{D,2} &= 6.386 \times 10^4 (1/\mathrm{m}) \times 3 \times 10^{-3} (\mathrm{m}) \\
&= 191.6.
\end{aligned}$$

For $R_2 = 3R_1$, $\mathrm{Re}_{D,2} = 574.7$. From Table 6.3, we have for $191.6 < \mathrm{Re}_{D,2} < 574.7$,

$$a_1 = 0.683 \quad a_2 = 0.466 \quad \text{Table 6.3.}$$

Figure Pr.6.46(b) shows the variation of R_Σ with respect to R_2 for $R_1 \leq R_2 \leq 3R_1$. The numerical values are obtained and plotted using a solver (such as SOPHT).

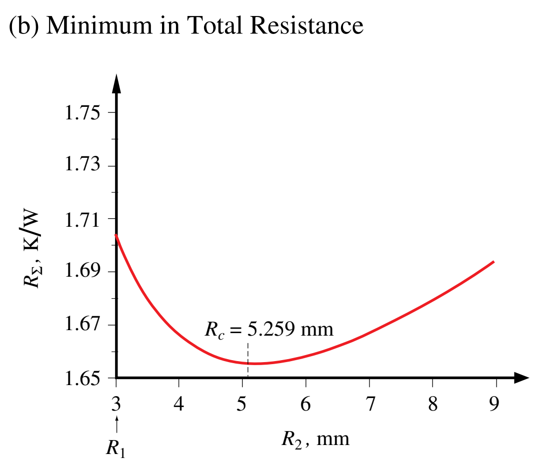


Figure Pr.6.46(b) Variation of R_Σ with respect to R_2 for $R_1 \leq R_2 \leq 3R_1$.

(b) The minimum in R_Σ occurs at $R_2 = R_c = 5.259$ mm.

(c) The value of the two resistances at $R_2 = R_c$ are

$$\begin{aligned}
R_{k,1\text{-}2} &= 0.3436℃/\mathrm{W} \\
\langle R_{ku} \rangle_{D,2} &= 1.314℃/\mathrm{W}.
\end{aligned}$$

Here the surface-convection resistance is much larger.

(d) The result of Example 6.13 for R_c for the above values for a_1, a_2, etc, is

$$
\begin{aligned}
R_c &= \left(\frac{a_2 k_s}{2^{a_2-1} a_R}\right)^{\frac{1}{a_2}} \\
a_R &= k_f a_1 \frac{u_{f,\infty}^{a_2}}{\nu_f^{a_2}} \mathrm{Pr}^{1/3} \\
&= 0.0267(\mathrm{W/m\text{-}K}) \times 0.683 \times \frac{(0.5)^{0.466}(\mathrm{m/s})^{0.466}}{(1.566\times 10^{-5})^{0.466}(\mathrm{m^2/s})^{0.466}} \times (0.69)^{1/3} \\
&= 2.024(\mathrm{W/m\text{-}K})/\mathrm{m}^{0.466} \\
R_c &= \left[\frac{0.466 \times 0.26(\mathrm{W/m\text{-}K})}{2^{0.466-1} \times 2.027(\mathrm{W/m\text{-}K})/\mathrm{m}^{0.466}}\right]^{\frac{1}{0.466}} \\
&= (0.08655)^{2.146}(\mathrm{m}) \\
&= 0.005259 \ \mathrm{m} \\
&= 5.259 \ \mathrm{mm}.
\end{aligned}
$$

As expected, this is equal to the numerical/graphical result of (b).

COMMENT:

From Figure Pr.6.46(b), note that R_Σ is rather independent of R_2 near R_c (nearly flat). Therefore a range of R_2 can be used with a nearly equal R_Σ.

PROBLEM 6.47.FUN

GIVEN:

In designing fins, from (6.149) we note that a combination of high fin surface area A_f and high fin efficiency η_f are desirable. Therefore, while high $\eta_f(\eta_f \to 1)$ is desirable, η_f decreases as A_f increases. From (6.149) the case of $\eta_f \to 1$ corresponds to $\mathrm{Bi}_w \to 0$.

Note that

$$\tanh(z) = \frac{\sinh(z)}{\cosh(z)}, \quad \sinh(z) = \frac{e^z - e^{-z}}{2}, \quad \cosh(z) = \frac{e^z + e^{-z}}{2}, \quad e^x = 1 + z + \frac{z^2}{2} + ... \,.$$

OBJECTIVE:

Show that in the limit of $mL_c \to 0$, the fin efficiency tends to unity.

SOLUTION:

We begin with

$$\begin{aligned} \tanh(z) &= \frac{\sinh(z)}{\cosh(z)}, \\ \sinh(z) &= \frac{e^z - e^{-z}}{2}, \quad \cosh(z) = \frac{e^z + e^{-z}}{2}. \end{aligned}$$

Then

$$\tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}.$$

Next we expand e^z using a Taylor series as

$$\begin{aligned} e^z &= 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + ... \\ e^{-z} &= 1 - z + \frac{z^2}{2!} - \frac{z^3}{3!} ..... \end{aligned}$$

Using these, we have

$$\begin{aligned} \tanh(z) &= \frac{1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} .. - 1 + z - \frac{z^2}{2!} + \frac{z^3}{3!} - ...}{1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} .. + 1 - z + \frac{z^2}{2!} - \frac{z^3}{3!} + ...} \\ &= \frac{2z + \frac{2z^3}{3!} + ...}{2 + \frac{2z^2}{2!} + ...}. \end{aligned}$$

We are interested in the fin efficiency, given by (6.147),

$$\eta_f = \frac{\tanh(z)}{z}, \quad z = mL_c = \mathrm{Bi}_w^{1/2}.$$

and the limit of $z = \mathrm{Bi}_w^{1/2} \to 0$. Then with $\tanh(z)$ from above,

$$\begin{aligned} \lim_{z \to 0}(\eta_f) &= \lim_{z \to 0} \left[\frac{\tanh(z)}{z} \right] \\ &= \lim_{z \to 0} \left(\frac{2 + \frac{2z^2}{3!} + ...}{2 + \frac{2z^2}{2!} + ...} \right) \\ &= 1. \end{aligned}$$

COMMENT:

When mL_c is small, i.e., from (6.147),

$$mL_c = \frac{R_{k,s}}{\langle R_{ku}\rangle_w} = \text{Bi}_w^{1/2} \rightarrow 0$$

then the temperature nonuniformity within the fin is not significant.

PROBLEM 6.48.FUN

GIVEN:

The body of the desert tortoise (like those of other cold-blooded animals) tends to have the same temperature as its ambient air. During daily variations of the ambient temperature, this body temperature also varies, but due to the sensible heat storage, thermal equilibrium (i.e., the condition of being at the same temperature) does not exist at all times. Consider the approximate model temperature variation given in Figure Pr.6.48(a)(i), which is based on the ambient temperature measured in early August, 1992, near Las Vegas, Nevada. Assume that a desert tortoise with a uniform temperature is initially at $T_1(t=0) = 55$℃. It is suddenly exposed to an ambient temperature $T_{f,\infty} = 35$℃ for 6 hours, after which the ambient temperature suddenly changes to $T_{f,\infty} = 55$℃ for another 12 hours before suddenly dropping back to the initial temperature. The heat transfer is by surface convection only (for accurate analysis, surface radiation, including solar radiation, should be included). The geometric model is given in Figure Pr.6.48(a)(ii), with surface convection through the upper (hemisphere) surface and no heat transfer from the bottom surface. For the Nusselt number, use that for forced flow over a sphere.

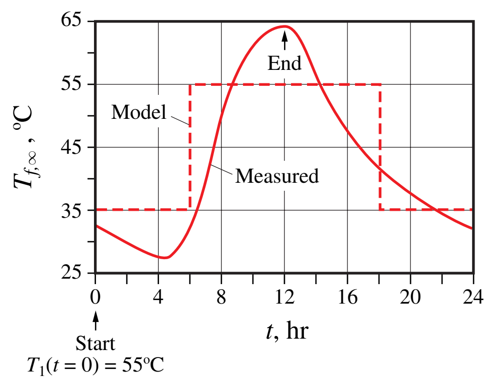
$\rho_1 = 1,000$ kg/m^3, $c_{p,1} = 900$ J/kg-K, $u_{f,\infty} = 2$ m/s.

Evaluate the air properties at $T = 320$ K.

SKETCH:

Figure Pr.6.48(a) shows the measured and model temperature variations and the geometric model for surface convection.

(i) Ambient Air Temperature



(ii) Geometric Model

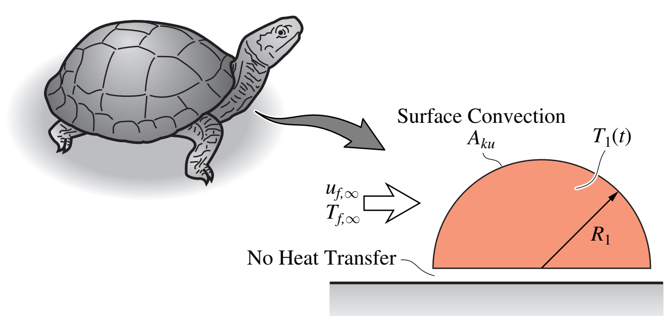


Figure Pr.6.48(a)(i) A measured daily ambient air temperature variation over 24 hr and an approximation (model) to the temperature variation. (ii) A geometric model for a tortoise in forced cross flow.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the body temperature after an elapsed time of 12 hours, i.e., $T_1(t = 12$ hr$)$ for $R_1 = 20$ cm.

(c) Repeat for $R_1 = 80$ cm.

SOLUTION:
(a) The thermal circuit diagram is shown in Figure Pr.6.48(b).

(b) From (6.156), for $Q_1 = \dot{S}_1 = 0$, we have

$$T_1(t) = T_{f,\infty} + [T_1(t=0) - T_{f,\infty}]e^{-t/\tau_1}, \quad \tau_1 = (\rho c_p V)_1 \langle R_{ku}\rangle_D.$$

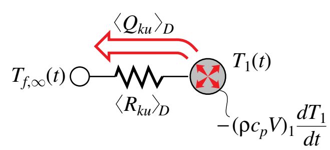


Figure Pr.6.48(b) Thermal circuit diagram.

From (6.124), we have

$$\langle R_{ku}\rangle_D = \frac{D}{A_{ku}\langle \mathrm{Nu}\rangle_D k_f}, \quad \mathrm{Re}_D = \frac{u_{f,\infty}D}{\nu_f}.$$

From Table C.22, we have for air at $T = 300$ K,

$$\begin{aligned}
k_f &= 0.0281 \text{ W/m-K}\\
\nu_f &= 17.44\times 10^{-6} \text{ m}^2/\text{s}\\
\mathrm{Pr} &= 0.69\\
\mathrm{Re}_D &= \frac{2(\text{m/s})\times 2\times R_1}{17.44\times 10^{-6}(\text{m}^2/\text{s})} = 2.294\times 10^5(\text{m}^{-1})R_1
\end{aligned}$$

$$\begin{aligned}
&\text{(i)} \quad \mathrm{Re}_D = 4.588\times 10^4\\
&\text{(ii)} \quad \mathrm{Re}_D = 1.835\times 10^5.
\end{aligned}$$

From Table 6.4, we have

$$\langle \mathrm{Nu}\rangle_D = 2 + (0.4\mathrm{Re}_D^{1/2} + 0.06\mathrm{Re}_D^{2/3})\mathrm{Pr}^{0.4}$$

$$\begin{aligned}
\text{(i)} \quad \langle \mathrm{Nu}\rangle_D &= 2 + [0.4(4.588\times 10^4)^{1/2} + 0.06(4.588\times 10^4)^{2/3}]\times (0.69)^{0.4}\\
&= 142.15
\end{aligned}$$

$$\begin{aligned}
\text{(ii)} \quad \langle \mathrm{Nu}\rangle_D &= 2 + [0.4(1.835\times 10^5)^{1/2} + 0.06(1.835\times 10^5)^{2/3}]\times (0.69)^{0.4}\\
&= 316.74.
\end{aligned}$$

Then

$$\begin{aligned}
\tau_1 &= (\rho c_p)_1 \frac{1}{2}\times\frac{4}{3}\pi R_1^3 \times \frac{2R_1}{\frac{1}{2}\times 4\pi R_1^2\langle \mathrm{Nu}\rangle_D k_f} = \frac{2(\rho c_p)_1 R_1^2}{3\langle \mathrm{Nu}\rangle_D k_f}\\
\text{(i)} \quad \tau_1 &= \frac{2\times 1{,}000(\text{kg/m}^3)\times 900(\text{J/kg-K})\times (0.2)^2(\text{m}^2)}{3\times 142.15\times 0.028(\text{W/m-K})}\\
&= 6{,}030 \text{ s} = 1.675 \text{ hr}\\
\text{(ii)} \quad \tau_1 &= \frac{2\times 1{,}000(\text{kg/m}^3)\times 900(\text{J/kg-K})\times (0.8)^2(\text{m}^2)}{3\times 316.74\times 0.028(\text{W/m-K})}\\
&= 4.330\times 10^4\text{s} = 12.03 \text{ hr}.
\end{aligned}$$

We need to first determine $T_1(t = 6 \text{ hr})$ and then use this to determine $T_1(t = 12 \text{ hr})$, i.e.,

$$\begin{aligned} T_1(t = 6 \text{ hr}) &= 35(^\circ\text{C}) + (55 - 35)(^\circ\text{C})e^{(-6/1.675)} \\ &= 35.56^\circ\text{C}. \end{aligned}$$

Using this, we have

$$\begin{aligned} T_1(t = 12 \text{ hr}) &= 55(^\circ\text{C}) + (35.56 - 55)(^\circ\text{C})e^{(-6/1.675)} \\ &= 54.46^\circ\text{C}. \end{aligned}$$

(c) For the larger tortoise, we have

$$\begin{aligned} T_1(t = 6 \text{ hr}) &= 35(^\circ\text{C}) + (55 - 35)(^\circ\text{C})e^{(-6/12.03)} \\ &= 47.15^\circ\text{C} \\ T_1(t = 12 \text{ hr}) &= 55(^\circ\text{C}) + (47.15 - 55)(^\circ\text{C})e^{(-6/12.03)} \\ &= 50.23^\circ\text{C}. \end{aligned}$$

COMMENT:

Due to its smaller thermal mass, the body of the smaller tortoise follows the ambient temperature more closely than that of the larger tortoise. Using numerical integration, the measured ambient air temperature can be used to predict the body temperature. Due to the blood circulation, the internal conduction resistance $R_{k,1}$ cannot be readily evaluated to examine the validity of the uniform-temperature (lumped-capacitance) treatment. Note that this circulation assists in creating a uniform body temperature.

PROBLEM 6.49.FUN

GIVEN:

When humans experience hypothermia (exposure to extreme low temperatures resulting in lower temperature over part or the entire body), the body provides intensified metabolic reactions to supply more heat. Glucose ($C_6H_{12}O_6$) is the primary body fuel and its heat of oxidation $\Delta h_{r,c}$ is rather large; therefore, the body prepares a less energetic fuel called ATP (a combination of adenine, ribose, and three phosphate radicals). Here for simplicity we assume that glucose oxidation results in thermal energy release as given by

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O.$$

During hypothermia, more energy conversion requires a larger oxygen consumption rate (in direct of energy voltage $\dot{S}_{r,c}$.

Consider a human fallen into a cold water pond as shown in Figure Pr.6.49(a). Assume steady-state heat transfer with surface convection due to the thermobuoyant motion (neglect end heat losses).

$D = 0.45$ m, $L = 1.70$ m, $T_s = 15$℃, $T_{f,\infty} = 15$℃, $T_{f,\infty} = 4$℃, $\Delta h_{r,c} = -1.6 \times 10^7$ J/kg.

Evaluate the water properties at $T = 290$ K.

SKETCH:

Figure Pr.6.49(a) shows the geometric model of the human body submerged in a cold water body.

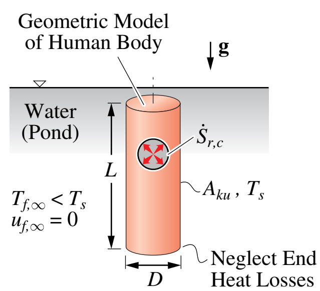


Figure Pr.6.49(a) Hypothermia resulting from excessive heat loss, as experienced during submergence in very cold water.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine $\dot{S}_{r,c}$.

(c) Determine the oxygen consumption rate $\dot{M}_{O_2}$.

(d) Compare the rate calculated in part (c) with that associated with the rest condition $\dot{S}_{r,c} = 45$ W [Figure Ex.1.3(d)].

SOLUTION:

(a) Figure Pr.6.49(b) shows the thermal circuit diagram.

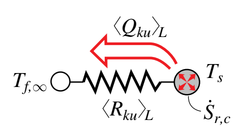


Figure Pr.6.49(b) Thermal circuit diagram.

(b) From Figure Pr.6.49(b), the energy equation is

$$\begin{aligned} Q|_A &= \langle Q_{ku} \rangle_L = \dot{S}_{r,c} \\ &= \frac{T_s - T_{f,\infty}}{\langle R_{ku} \rangle_L}. \end{aligned}$$

From (6.124), we have

$$\langle R_{ku}\rangle_L = \frac{L}{A_{ku}\langle \text{Nu}\rangle_L k_f}, \quad A_{ku} = \pi D L.$$

The Nusselt number is found from Table 6.5, and for vertical cylinders, we use the results for the vertical plates (subject to satisfying the stated criterion). Then

$$\begin{aligned}
\text{Nu}_L &= [(\text{Nu}_{L,l})^6 + (\text{Nu}_{L,t})^6]^{1/6} \\
\text{Nu}_{L,l} &= \frac{2.8}{\ln\left(1 + \dfrac{2.8}{a_1 \text{Ra}_L^{1/4}}\right)} \\
\text{Nu}_{L,t} &= \frac{0.13\text{Pr}^{0.22}}{(1+0.61\text{Pr}^{0.8})^{0.42}}\text{Ra}_L^{1/3} \\
a_1 &= \frac{4}{3}\frac{0.503}{\left[1+\left(\dfrac{0.492}{\text{Pr}}\right)^{9/16}\right]^{4/9}} \\
\text{Ra}_L &= \frac{g\beta_f(T_c - T_{f,\infty})L^3}{\nu_f \alpha_f}.
\end{aligned}$$

From Table C.23, we have for water at $T = 290$ K,

$$\begin{aligned}
k_f &= 0.590 \text{ W/m-K} \\
\nu_f &= 1.13 \times 10^{-6} \text{ m}^2\text{/s} \\
\alpha_f &= 1.41 \times 10^{-7} \text{ m}^2\text{/s} \\
\text{Pr} &= 8.02 \\
\beta_f &= 0.000203 \text{ 1/K}.
\end{aligned}$$

Then

$$\begin{aligned}
a_1 &= \frac{4}{3}\frac{0.503}{\left[1+\left(\dfrac{0.492}{8.02}\right)^{9/16}\right]^{4/9}} = 0.6166 \\
\text{Ra}_L &= \frac{9.81(\text{m/s}^2) \times 0.000203(1/\text{K}) \times (15-4)(\text{K}) \times (1.7)^3(\text{m})^3}{1.13 \times 10^{-6}(\text{m}^2/\text{s}) \times 1.41 \times 10^{-7}(\text{m}^2/\text{s})} = 6.755 \times 10^{11} \\
\text{Nu}_{L,l} &= \frac{2.8}{\ln\left[1 + \dfrac{2.8}{0.6166 \times (6.755 \times 10^{11})^{1/4}}\right]} = 560.4 \\
\text{Nu}_{L,t} &= \frac{0.13(8.02)^{0.22}}{[1+0.61(8.02)^{0.8}]^{0.42}}(6.755 \times 10^{11})^{1/3} = 984.4 \\
\text{Nu}_L &= [(560.4)^6 + (984.4)^6]^{1/6} = 989.9
\end{aligned}$$

From the energy equation

$$\begin{aligned}
\dot{S}_{r,c} &= A_{ku}\langle \text{Nu}\rangle_L \frac{k_f}{L}(T_s - T_{f,\infty}) \\
&= \pi \times 0.45(\text{m}) \times 1.7(\text{m}) \times 989.9 \times \frac{0.590(\text{W/m-K})}{1.7(\text{m})} \times (15-4)(\text{K}) \\
&= 9{,}082 \text{ W}.
\end{aligned}$$

(c) From (2.18), we have

$$\begin{aligned}
\dot{S}_{r,c} &= -\Delta h_{r,c}\dot{M}_F \\
&= -\Delta h_{r,c}\dot{M}_{O_2}\frac{\nu_{O_2}M_{O_2}}{\nu_F M_F},
\end{aligned}$$

where from the chemical reaction formula and from Table C.2, we have

$$\begin{aligned}
\nu_F &= 1, \quad \nu_{O_2} = 6, \quad M_F = (6 \times 12.011 + 12 \times 1.008 + 6 \times 15.999)(\text{kg/kmole}) = 180.16 \text{ kg/kmole}, \\
M_{O_2} &= 2 \times 15.999(\text{kg/kmole}) = 31.998 \text{ kg/kmole}.
\end{aligned}$$

Then

$$\begin{aligned}
\dot{M}_{O_2} &= -\frac{\dot{S}_{r,c}}{\Delta h_{r,c}} \frac{\nu_F M_F}{\nu_{O_2} M_{O_2}} \\
&= -\frac{9{,}082(\text{W})}{-1.6 \times 10^7 (\text{J/kg})} \times \frac{1 \times 180.16(\text{kg/kmole})}{6 \times 31.998(\text{kg/kmole})} \\
&= 5.327 \times 10^{-4} \text{ kg/s} \\
&= 0.5327 \text{ g/s}.
\end{aligned}$$

(d) For $\dot{S}_{r,c} = 45$ W, we have

$$\dot{M}_{O_2} = 0.002639 \text{ g/s}.$$

COMMENT:

The body is not capable of producing 9,082 W. From Figure Ex.1.3(c), we note that $\dot{S}_{r,c}$ close to 400 W is possible. Instead, the sensible heat of the body is used and the body temperature begins to drop. This creates the dangerous condition of hypothermia. Also note that body temperature drop results in decrease in $\langle Q_{ku} \rangle_L$ and $\dot{S}_{r,c}$.

PROBLEM 6.50.FUN.S

GIVEN:

In surface-convection evaporation cooling by water seepage of the evaporating liquid through a permeable wall, the heat required for evaporation is provided by the ambient gas and the liquid reservoir. This is shown in Figure Pr.6.50, which is similar to Figure 5.5, except here the gas surface convection is included.

$A_{ku}\langle R_{ku}\rangle_L = 5\times10^{-4}$ °C/(W/m^2), $(\rho_{f,w})_\infty = 0.0005$ kg/m^3, $T_{f,\infty} = 300$ K, $L = 20$ cm, $w = 20$ cm, $L_w = 6$ mm, $\langle k\rangle = 0.8$ W/m-K, $T_1 = 293$ K.

Evaluate all properties at $T = 300$ K [except for T_s and $(\rho_{f,w})_s$], and assume $\rho_{f,s} = \rho_{f,\infty}$.

SKETCH:

Figure Pr.6.50(a) shows the porous layer through which water permeates and evaporates on the surface. The heat for this evaporation is provided by the liquid reservoir and by ambient air.

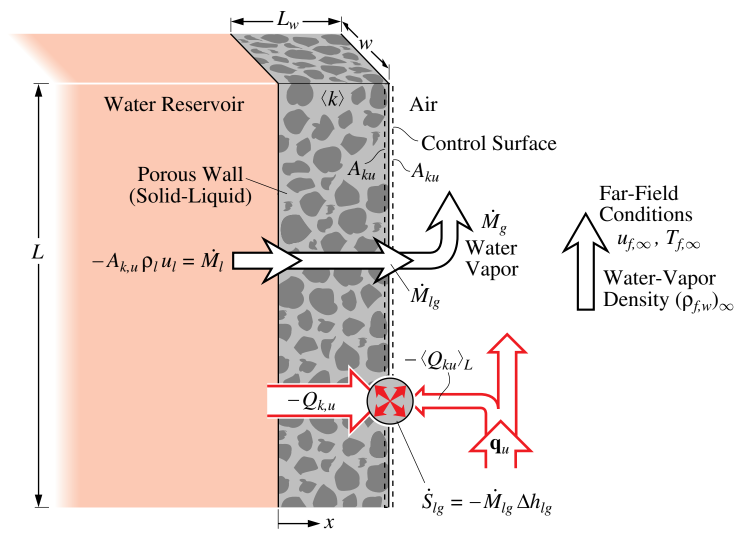


Figure Pr.6.50(a) Evaporation cooling of a surface. The heat is provided by the liquid reservoir and the ambient air.

OBJECTIVE:

(a) Sketch the qualitative temperature and water vapor density distributions, and draw the thermal and mass circuit diagrams.

(b) Write the energy equation for the evaporation surface, along with the relations for $Q_{k,u}$ (from Section 5.3), Q_{ku}, $\dot{S}_{lg}$, and $(\rho_{f,w})_s = (\rho_{f,w})_s(T_s)$.

(c) Write the water-vapor species mass conservation equation for the evaporation surface.

(d) Determine the surface temperature T_s for the conditions given below. Note that $u_l = \dot{m}_{lg}/\rho_l$ is unknown.

(e) Determine the rate of heat flowing to the evaporating surface from the gas stream Q_{ku} and from the liquid reservoir $Q_{k,u}$.

SOLUTION:

(a) The qualitative plot of the distribution of the temperature in the porous wall and in the gas stream, is shown in Figure Pr.6.50(b). The evaporation surface will have a temperature T_s which is lower than the gas stream temperature $T_{f,\infty}$. Under the proper seepage velocity and ambient conditions, T_s will also be lower than the liquid reservoir temperature T_1. This is the condition considered here and shown in Figure Pr.6.50(b).

The qualitative plot of distribution of the water-vapor density is also shown in this figure. The gas stream has a lower water vapor density, and therefore, water vapor is transferred from the surface to the ambient gas by surface-convection mass transfer.

The thermal and mass circuit diagram are shown in Figure Pr.6.50(c). The surface evaporation is shown as an energy conversion term $\dot{S}_{lg}$ and there are two surface heat transfer terms, $Q_{k,u}$ (from the reservoir) and Q_{ku} (to the gas stream). The mass transfer is shown with liquid supply to the surface $\dot{M}_l$ and surface-convection mass transfer $\dot{M}_{lg}$.

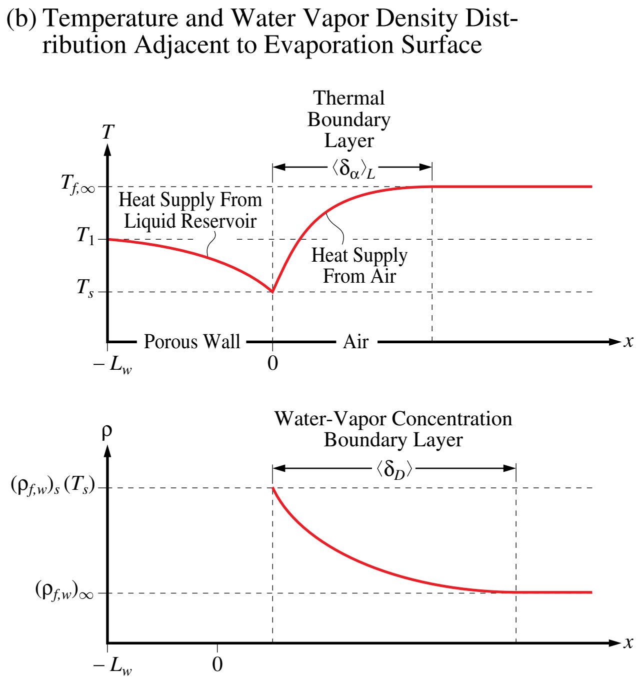


Figure Pr.6.50(b) Qualitative distributions of temperature and water-vapor density near the evaporation surface.

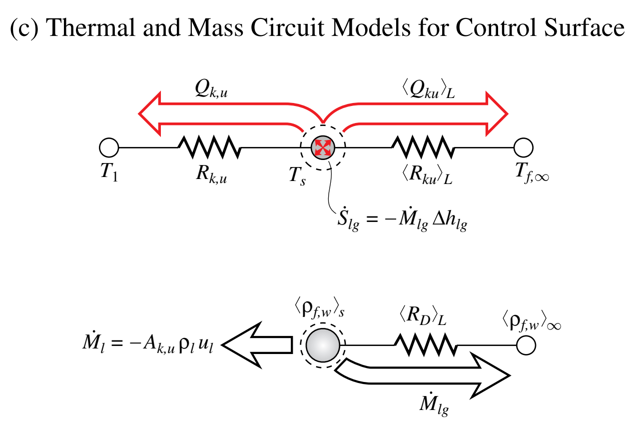


Figure Pr.6.50(c) Thermal and mass circuit diagrams.

(b) We denote the fluid properties with the subscript l, the vapour properties with the subscript f, w and the air stream properties with the subscript f, a. The energy equation for the evaporation surface is written from the thermal circuit diagram, Figure Pr.6.50(c), and is

$$Q_{k,u} + \langle Q_{ku} \rangle_L = \dot{S}_{lg}$$
$$\dot{S}_{lg} = -\dot{M}_{lg} \Delta h_{lg},$$

where,

$$\langle Q_{ku} \rangle_L = \frac{T_s - T_{f,\infty}}{\langle R_{ku} \rangle_L},$$

where, $A_{ku}\langle R_{ku}\rangle_L$ is given and $A_{ku} = wL = 0.04\ \mathrm{m}^2$ so that $\langle R_{ku}\rangle = 0.0125$ K/W. The conduction-convection heat transfer rate (for the permeable wall) is given by (5.19) as

$$\begin{aligned} Q_{k,u} &= \frac{T_s - T_1}{R_{k,u}} \\ &= \frac{A_{k,u}\langle k\rangle u_l}{\langle \alpha_l\rangle}\frac{e^{\mathrm{Pe}_{L_w}}}{e^{\mathrm{Pe}_{L_w}}-1}(T_s - T_1), \end{aligned}$$

where

$$\begin{aligned} A_{k,u} &= A_{ku} \\ \mathrm{Pe}_{L_w} &= \frac{u_l L_w}{\langle \alpha_l\rangle} \\ \langle \alpha_l\rangle &= \frac{\langle k\rangle}{(\rho c_p)_l}, \end{aligned}$$

so that

$$Q_{k,u} = A_{k,u}(\rho c_p)_l u_l \frac{e^{\mathrm{Pe}_{L_w}}}{e^{\mathrm{Pe}_{L_w}}-1}(T_s - T_1),$$

Finally we write the Clausius-Clapeyron relation (A.14) as

$$(\rho_{f,w})_s = \frac{(\rho_{f,w})_o T_{lg,o}}{T_s} e^{-\frac{M_w \Delta h_{lg}}{R_g}\left(\frac{1}{T_s}-\frac{1}{T_{lg,o}}\right)},$$

where $T_{lg,o} = 300$ K.

(c) Noting that there is no storage of water at the surface s, the water-vapor species mass conservation (6.181) is written at this surface as

$$\begin{aligned} \dot{M}_w|_A &= \dot{M}_l + \dot{M}_{lg} \\ 0 &= -\rho_l u_l A_{k,u} + \dot{M}_{lg} \end{aligned}$$

where from (6.179) we have

$$\dot{M}_{lg} = \langle \dot{M}_{Du}\rangle_L = \frac{(\rho_{f,w}/\rho_f)_s - (\rho_{f,w}/\rho_f)_\infty}{\langle R_{Du}\rangle_L}$$

and from (6.180),

$$\begin{aligned} \langle R_{Du}\rangle_L &= \langle R_{ku}\rangle_L c_{p,f,a}\mathrm{Le}^{-2/3} \\ \mathrm{Le} &= \frac{D_{m,w}}{\alpha_{f,a}}. \end{aligned}$$

As indicated, we assume that $\rho_{f,s} = \rho_{f,\infty}$. Furthermore, as the water vapor density will be much less than the air density, we take $\rho_{f,s} = \rho_{f,\infty} = \rho_{f,a}$.

(d) From Table C.22, for air at $T = 300$ K, we have

$\rho_{f,a} = 1.177\ \mathrm{kg/m^3}$	Table C.22
$c_{p,f,a} = 1{,}005$ J/kg-K	Table C.22
$\alpha_{f,a} = 2.257\times 10^{-5}\ \mathrm{m^2/s}$	Table C.22.

From Table C.20(a), for water vapor diffusing in air at 293 K, we have for the Lewis number

$$\begin{aligned} D_{m,w} &= 2.20\times 10^{-5}\ \mathrm{m^2/s} \\ \mathrm{Le} &= \frac{D_{m,w}}{\langle \alpha_f\rangle} = 0.9747. \end{aligned}$$

At the reference temperature $T_o = 300$ K, we also have from the Table C.27 for water

$$\Delta h_{lg} = 2.438 \times 10^6 \text{ J/kg} \qquad \text{Table C.27}$$
$$(\rho_{f,w})_o = 0.0256 \text{ kg/m}^3 \qquad \text{Table C.27}$$
$$\rho_l = 997 \text{ kg/m}^3 \qquad \text{Table C.27}$$
$$c_{p,l} = 4{,}179 \text{ J/kg-K} \qquad \text{Table C.27,}$$

so that,

$$\langle \alpha_l \rangle = \frac{0.8(\text{W/m-K})}{997(\text{kg/m}^3) \times 4{,}179(\text{J/kg-K})} = 1.920 \times 10^{-7} \text{ m}^2/\text{s}.$$

When the above equations (the two conservation equations and the Clausius-Clapeyron equation) are solved using a solver (such as a SOPHT) for T_s, $(\rho_{f,w})_s$, and u_l, we have

$$\begin{aligned} T_s &= 282.9 \text{ K} \\ (\rho_{f,w})_s &= 0.009365 \text{ kg/m}^3 \\ u_l &= 1.478 \times 10^{-5} \text{ m/s} \quad (\text{Pe}_{L_w} = 0.4618). \end{aligned}$$

(e) The rate of heat flowing to the surface from the liquid reservoir and from the gas stream are

$$\begin{aligned} Q_{k,u} &= -67.39 \text{ W} \\ \langle Q_{ku} \rangle_L &= -1{,}369.30 \text{ W}. \end{aligned}$$

COMMENT:

For most problems dealing with combined heat and mass transfer, numerical solutions are required as many of the relevant equations are non-linear.

PROBLEM 6.51.FAM.S

GIVEN:

A water droplet of initial diameter $D(t = 0)$ is in an air stream with a far-field velocity $u_{f,\infty}$, while the droplet is moving in the same direction with a velocity u_d. The air stream has a far-field temperature $T_{f,\infty}$ and water vapor density $(\rho_{f,w})_\infty$. This is shown in Figure Pr.6.519a). The droplet has a uniform temperature $T_s(t)$ at any time, which is determined from the spontaneous heat transfer and evaporation rates. Neglect the sensible heat storage/release in the droplet as T_s varies with time.

$D(t = 0) = 4$ mm, $T_{f,\infty} = 350°\text{C}$, $u_{f,\infty} = 0.5$ m/s, $u_d = 0.1$ m/s, $(\rho_{f,w})_\infty = 0$, $\rho_{f,\infty} = \rho_{f,s}$.

Evaluate the air properties at $T = 500$ K and water properties at $T = 300$ K.

SKETCH:

Figure Pr.6.51(a) shows the water-droplet with surface-convection heat and mass transfer.

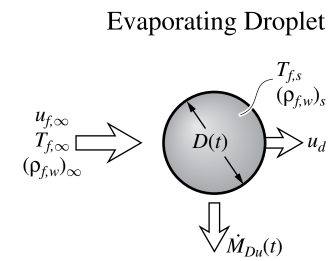


Figure Pr.6.51(a) An evaporating droplet with surface convection heat and mass transfer.

OBJECTIVE:

(a) Draw the thermal and mass circuit diagrams.

(b) For the conditions given above, plot the variation of the droplet diameter and volume as a function of time, up to the time the droplet vanishes. (c) Using the droplet velocity, determine the length of flight before the droplet vanishes.

SOLUTIONS:

(a) The thermal and mass circuit diagrams are shown in Figure Pr.6.51(b).

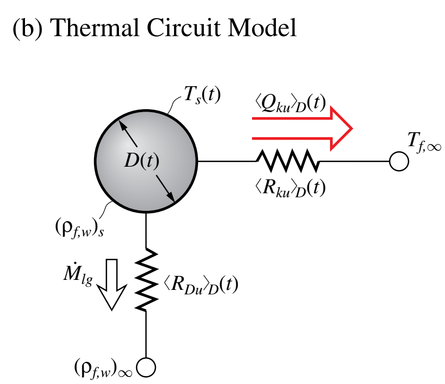


Figure Pr.6.51(b) Thermal and mass circuit diagrams.

(b) The energy equation (2.71) for the thermal node T_s is

$$Q|_A \quad = \quad \langle Q_{ku}\rangle_D(t) = -\rho c_p \frac{d}{dt}[V(t)T_s(t)] + \dot{S}_{lg}(t),$$

where

$$\begin{aligned}
\dot{S}_{lg}(t) &= -\dot{M}_{lg}(t)\Delta h_{lg} \\
\dot{M}_{lg}(t) &= \frac{(\rho_{f,w}/\rho_f)_s(t) - (\rho_{f,w}/\rho_f)_\infty}{\langle R_{Du}\rangle_D(t)} \\
\langle R_{Du}\rangle_D(t) &= \langle R_{ku}\rangle_D(t) c_{p,f} \mathrm{Le}^{-2/3} \\
\langle Q_{ku}\rangle_D(t) &= \frac{T_s(t) - T_{f,\infty}}{\langle R_{ku}\rangle_D(t)}.
\end{aligned}$$

The integral-volume mass conservation equation, (6.181), becomes

$$\dot{M}|_A = \dot{M}_{lg} = \frac{dM_w}{dt} = -\rho_l \frac{dV(t)}{dt}.$$

The Clausius-Clapeyron relation (A.14) is

$$\begin{aligned}
(\rho_{f,w})_s(t) &= \frac{(\rho_{f,\infty})_o T_{lg,o}}{T_s(t)} \exp\left\{\frac{-M_w \Delta h_{lg}}{R_g}\left[\frac{1}{T_{lg}(t)} - \frac{1}{T_{lg,o}}\right]\right\} \\
T_{lg,o} &= 300 \text{ K}.
\end{aligned}$$

The surface convection resistance $\langle R_{ku}\rangle_D$ is found from (6.124) and Table C.4 and is

$$\langle \mathrm{Nu}\rangle_D = 2 + (0.4\mathrm{Re}_D^{1/2} + 0.06\mathrm{Re}_D^{2/3})\mathrm{Pr}^{0.4},$$

where

$$\begin{aligned}
\mathrm{Re}_D(t) &= \frac{(u_{f,\infty} - u_d)D(t)}{\mu_f} \\
\langle R_{ku}\rangle_D(t) &= \frac{D(t)}{A_{ku}\langle \mathrm{Nu}\rangle_D(t) k_f} \\
A_{ku}(t) &= \pi D^2(t),
\end{aligned}$$

and we have used the relative velocity $u_{f,\infty} - u_d$.
The thermophysical properties for air at $T = 500$ K are found for Table C.22 and C.20(a), and for water at $T = 300$K from Table C.27, i.e.,

$\nu_f = 3.73 \times 10^{-5}$ m^2/s — Table C.22
$c_{p,f} = 1017$ J/kg-K — Table C.22
$k_f = 0.0395$ W/m-K — Table C.22
$\alpha_f = 5.418 \times 10^{-5}$ m^2/s — Table C.22
$\mathrm{Pr} = 0.69$ — Table C.22
$\rho_f = 0.706$ kg/m^3 — Table C.22
$D_{m,w} = 2.20 \times 10^{-5}$ m^2/s — Table C.20(a)
$\Delta h_{lg} = 2.438 \times 10^6$ J/Kg — Table C.27
$(\rho_{f,w})_o = 0.0256$ kg/m^3 — Table C.27
$\rho_l = 997$ kg/m^3 — Table C.27
$C_{p,l} = 4{,}179$ J/kg-K — Table C.27

$$\mathrm{Le} = \frac{D_{m,w}}{\alpha_f} = \frac{2.20 \times 10^{-5}(\mathrm{m^2/s})}{5.418 \times 10^{-5}(\mathrm{m^2/s})} = 0.4061$$

We now need to simultaneously solve the energy and mass conservation equations and the Clausius-Clapeyron relation for $T_s(t)$, $D(t)$ and $(\rho_{f,w})_s(t)$.

(c)The variation of D and V with respect to t is plotted in Figures Pr.6.51(c) and (d). At $t = 382$ s, the droplet vanishes.

(d) The distance the droplet travels is found as

$$L = u_d t \quad = \quad 0.1(\text{m/s}) \times 382(\text{s}) = 38.2 \text{ m}.$$

This is rather a long distance for the droplet to travel before complete evaporation.
The length can be shortened by increasing $T_{f,\infty}$ and or $u_{f,\infty}$.

COMMENT:

Note that T_s is independent of t, as shown in Figure 6.51(e). This is because it is determined from the energy equation and the Clausius-Clapeyron relation, where in the energy equation D cancels out.

Also note that $A_{ku}/V = 6\pi D^2/\pi D^3 = 6/D$ increases as D decreases, therefore, the rate of decrease in D increases with the elapsed time.

Also, note that as $D(t=0)$ decreases, the evaporation time decrease substantially. For $D(t=0) = 2$ mm, $t = 110$ s, for $D(t=0) = 0.2$ mm $= 200$ μm, $t = 1.46$ s, for $D(t=0) = 20$ μm, $t = 0.0165$ s $= 16.5$ ms, and for $D(t=0) = 2$ μm, $t = 0.170$ ms $= 170$ μs. Therefore, for rapid evaporation, very small droplets should be used.

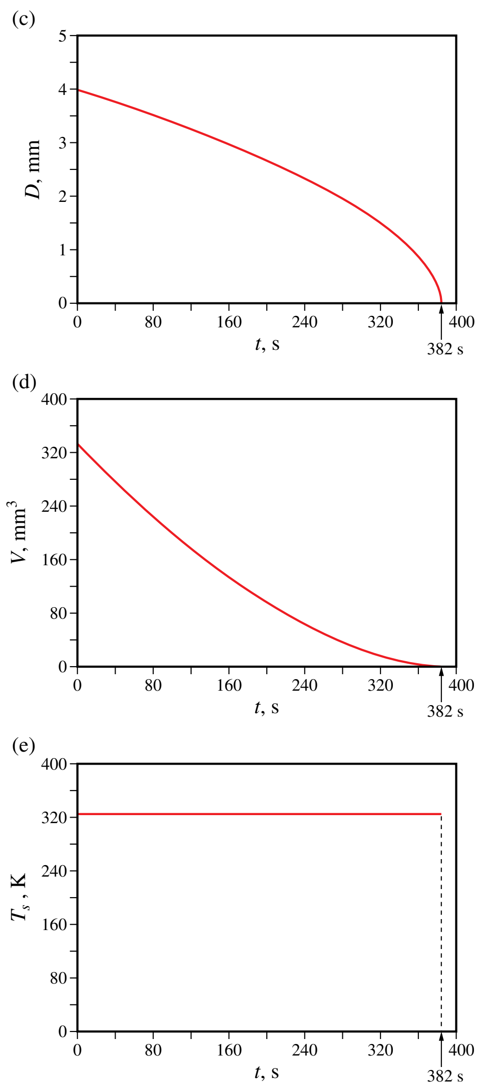


Figure Pr.6.51(c),(d) and(e) Time variation of droplet diameter, volume, and temperature.

PROBLEM 6.52.FAM.S

GIVEN:

Electric hand dryers provide hot air flow for the evaporation of thin water layers over human skin. The rate of evaporation, and hence the elapsed time for drying, depends on the air temperature and velocity and the water-layer thickness. This is shown in Figure Pr.6.52(a)(i). The hands can be modeled as a cylinder with the air flowing across it, as shown in Figure Pr.6.52(a)(ii).

$u_{f,\infty} = 0.8$ m/s, $T_{f,\infty} = 35°C$, $(\rho_{f,w})_\infty = 0$, $l = 0.06$ mm, $D = 6$ cm, $L = 16$ cm.

Determine all properties at $T = 300$ K. Assume $\rho_{f,s} = \rho_{f,\infty}$ and that all the heat for evaporation is provided by the air stream. Neglect the end surfaces of the cylinder.

SKETCH:

Figure Pr.6.52(a) shows the drying of wet hands by a hot, dry air stream and the simple geometric model for the hand, represented by a cylinder in cross flow.

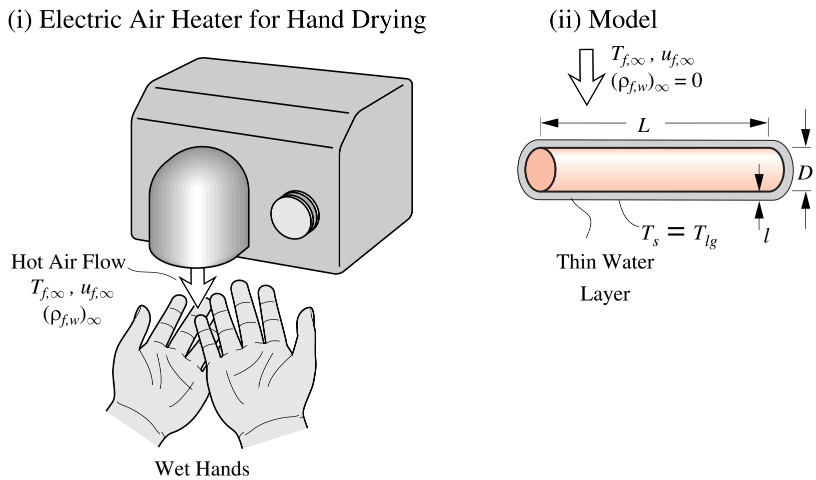


Figure Pr.6.52(a) Drying of wet hands by a hot air stream and its simple geometric model.

OBJECTIVE:

(a) Draw the thermal and mass circuit diagrams.

(b) Determine the evaporation rate $\dot{M}_{lg}$ and the water-surface temperature $T_s = T_{lg}$, for the conditions given above.

(c) Assuming that the mass transfer rate is constant, determine the elapsed time for the evaporation of a water layer with thickness l.

SOLUTION:

(a) Figure Pr.6.52(b) shows the thermal and mass circuit diagrams. The heat flow rate for the evaporation is assumed to be from the hot air stream only.

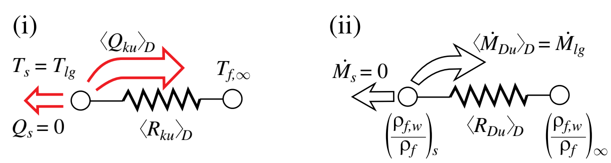


Figure Pr.6.52(b) (i)Thermal and (ii) mass circuit diagrams.

(b) The energy equation for the assumed uniform temperature surface T_s is given by (2.72) and here we have

$$Q_s + \langle Q_{ku}\rangle_D = \dot{S}_{lg}$$

$$\frac{T_s - T_{f,\infty}}{\langle R_{ku}\rangle_D} = -\dot{M}_{lg}\Delta h_{lg},$$

where we have used Table 2.1 for $\dot{S}_{lg}$.
The water-species conservation equation is (6.182), for the case of mass loss by surface convection only. Here we have

$$\begin{aligned}
\dot{M}_{lg} &= \dot{M}_w|_A = -\frac{d}{dt}M_w|_A \\
\frac{\left(\frac{\rho_{f,w}}{\rho_f}\right)_s - \left(\frac{\rho_{f,w}}{\rho_f}\right)_\infty}{\langle R_{Du}\rangle_D} &= -\frac{d}{dt}(\rho_w l\pi DL) \\
&= -\rho_w \pi DL\frac{dl}{dt},
\end{aligned}$$

where we have assumed that $l \ll D$, then set the volume of water as πDLl.

The heat and mass transfer surface-convection resistances are found from (6.180) and from Table 6.3, i.e.,

$$\begin{aligned}
\langle R_{Du}\rangle_D &= \langle R_{ku}\rangle_D c_{p,f}\mathrm{Le}^{-2/3}, \quad \mathrm{Le} = \frac{D_{m,w}}{\alpha_f} \\
\langle R_{ku}\rangle_D &= \frac{D}{A_{ku}\langle \mathrm{Nu}\rangle_D k_f} \\
\langle \mathrm{Nu}\rangle_D &= a_1 \mathrm{Re}_D^{a_2}\mathrm{Pr}^{1/3}, \quad \mathrm{Re}_D = \frac{u_{f,\infty}D}{\nu_f}.
\end{aligned}$$

We also need the Clausius-Clapeyron relation (A.14) to relate $T_s = T_{lg}$ and $(\rho_{f,w})_s$, i.e., as shown in Example 6.18, we have

$$(\rho_{f,w})_s = \frac{(\rho_{f,w})_o T_{lg,o}}{T_s} e^{-\frac{M_w \Delta h_{lg}}{R_g}\left(\frac{1}{T_{lg}} - \frac{1}{T_{lg,o}}\right)},$$

where for the reference state we will use $T_{lg,o} = 300$ K.
We now proceed with the numerical results starting from the evaluation of the properties.
For air and water we have, at $T = 300$ K,

$$\begin{aligned}
\text{air}: \quad k_f &= 0.0267 \text{ W/m-K} && \text{Table C.22} \\
\rho_f &= 1.177 \text{ kg/m}^3 && \text{Table C.22} \\
c_{p,f} &= 1{,}005 \text{ J/kg-K} && \text{Table C.22} \\
\nu_f &= 1.566 \times 10^{-5} \text{ m}^2/\text{s} && \text{Table C.22} \\
\alpha_f &= 2.257 \times 10^{-5} \text{ m}^2/\text{s} && \text{Table C.22} \\
\mathrm{Pr} &= 0.69 && \text{Table C.22} \\
\text{water}: \quad D_{m,w} &= 2.20 \times 10^{-5} \text{m}^2/\text{s} && \text{Table C.20(a)} \\
(\rho_{f,w})_o &= 0.0256 \text{ kg/m}^3 && \text{Table C.27} \\
\rho_w &= 997 \text{ kg/m}^3 && \text{Table C.27} \\
\Delta h_{lg} &= 2.438 \times 10^6 \text{ J/kg} && \text{Table C.27} \\
M &= 18 \text{ kg/kmole} \\
R_g &= 8{,}314 \text{ J/kmole-K} && \text{Table C.22} \\
\mathrm{Le} &= \frac{D_{m,w}}{\alpha_f} = \frac{2.20 \times 10^{-5}(\text{m}^2/\text{s})}{2.257 \times 10^{-5}(\text{m}^2/\text{s})} = 0.9747.
\end{aligned}$$

Then

$$\begin{aligned}
\mathrm{Re}_D &= \frac{0.8(\text{m/s}) \times 0.06(\text{m})}{1.566 \times 10^{-5}(\text{m}^2/\text{s})} \\
&= 3{,}065.
\end{aligned}$$

For this value of the Reynolds number, from Table 6.3, we have

$$\begin{aligned}
a_1 &= 0.683, \quad a_2 = 0.466 \\
\langle \mathrm{Nu}_D \rangle &= 0.683(3{,}065)^{0.466} \times (0.69)^{1/3} \\
&= 25.43 \\
\langle R_{ku} \rangle_D &= \frac{D}{\pi D L \mathrm{Nu}_D k_f} = \frac{1}{\pi L \langle \mathrm{Nu} \rangle_D k_f} \\
&= \frac{1}{\pi \times 0.16(\mathrm{m}) \times 25.43 \times 0.0267(\mathrm{W/m\text{-}K})} \\
&= 2.930 ^\circ\mathrm{C/W},
\end{aligned}$$

where we have assumed $l \ll D$.

Then

$$\begin{aligned}
\langle R_{Du} \rangle_D &= 2.930(^\circ\mathrm{C/W}) \times 1{,}005(\mathrm{J/kg\text{-}^\circ C}) \times (0.9747)^{-2/3} \\
&= 2{,}995 \ \mathrm{s/kg}.
\end{aligned}$$

Combining the energy and water-species conservation equations, we have

$$\frac{T_s - 308.15(\mathrm{K})}{2.930(\mathrm{K/W})} = -\frac{[(\rho_{f,w})_s/1.177(\mathrm{kg/m^2})] - 0}{2{,}995(\mathrm{s/kg})} \times 2.438 \times 10^6(\mathrm{J/kg}).$$

Then along with the Clausius-Clapeyron relation, we simultaneously solve for $T_s = T_{lg}$ and $(\rho_{f,w})_s$. Using a solver, we have

$$\begin{aligned}
T_s &= T_{lg} = 285.64 \ \mathrm{K} = 12.49^\circ\mathrm{C} \\
(\rho_{f,w})_s &= 0.01110 \ \mathrm{kg/m^3}.
\end{aligned}$$

Then

$$\begin{aligned}
\dot{M}_{lg} &= \frac{(0.01110/1.177) - 0}{2{,}995(\mathrm{s/kg})} \\
&= 3.120 \times 10^{-6} \ \mathrm{kg/s} \\
&= 3.120 \times 10^{-3} \ \mathrm{g/s}.
\end{aligned}$$

(c) The evaporation time is found for

$$\dot{M}_{lg} = -\rho_w \pi D L \frac{dl}{dt}$$

or upon integration, for a constant $\dot{M}_{lg}$, we have

$$\begin{aligned}
\Delta t &= \frac{\rho_w \pi D L l}{\dot{M}_{lg}} \\
&= \frac{997(\mathrm{kg/m^3}) \times \pi \times 0.06(\mathrm{m}) \times 0.16(\mathrm{m}) \times 6 \times 10^{-5}(\mathrm{m})}{3.120 \times 10^{-6}(\mathrm{kg/s})} \\
&= 578.2 \ \mathrm{s} \\
&= 9.637 \ \mathrm{min}.
\end{aligned}$$

COMMENT:

Note that the water surface cools down to 12.49°C. Under this condition, heat also flows from the body to the liquid surface. Also, as shown in Figure Pr.6.52(c), the mass flow rate increases nearly linearly with $T_{f,\infty} - T_s$. Then using a higher $T_{f,\infty}$, will allows for a shorter drying time.

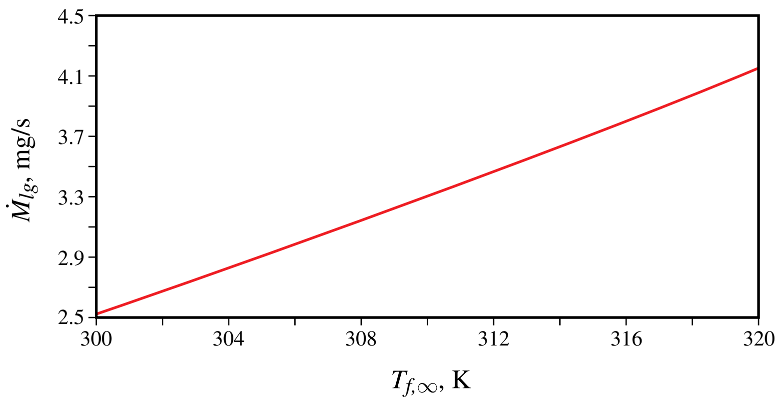


Figure Pr.6.52(c) Variation of mass flow rate with respect to $T_{f,\infty}$.

PROBLEM 6.53.FAM

GIVEN:

A well-insulated hot-beverage cup filled with hot water has its cap removed, thus allowing for evaporation and heat transfer from its top surface. This is shown in Figure Pr.6.53(a), where this evaporation and heat transfer result in a change in the water-cup temperature T_c, which is assumed to be uniform. Consider the instantaneous heat transfer at an elapsed time, when $T_c = T_s$ is known.

$R = 3.5$ cm, $V_c = 350$ cm^3, $(\rho c_p)_c = 2 \times 10^6$ J/m^3-K, $T_c = T_s = 80$℃, $T_{f,\infty} = 20$℃, $(\rho_{f,w})_\infty = 0.001$ kg/m^2.

Determine the properties of air at $T = 325$ K. Determine the water vapor properties at $T = 80$℃ from Table C.27.

SKETCH:

Figure Pr.6.53(a) shows the water-cup systems and the surface evaporation, surface-convection heat transfer, and change in the sensible heat of the system.

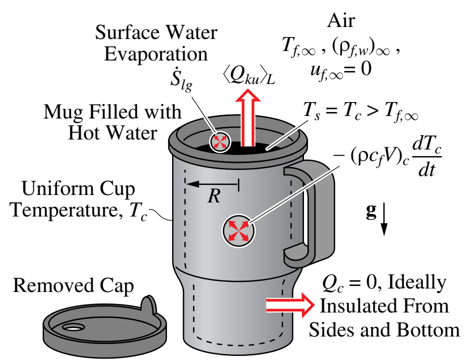


Figure Pr.6.53(a) A well-insulated, hot-water cup has its cap removed allowing for evaporation and heat transfer from its top surface. This heat transfer and evaporation result in a change in the water-cup temperature T_c.

OBJECTIVE:

(a) Draw the thermal and mass circuit diagrams for the water-cup volume.

(b) Determine the rate of surface-convection heat transfer $\langle Q_{ku}\rangle_L$ by thermobuoyant motion, for the given conditions.

(c) Determine the rate of water evaporation $\dot{M}_{lg}$ and the rate of phase-change energy conversion $\dot{S}_{lg}$.

(d) Determine the rate of change in the water-cup temperature dT_c/dt.

SOLUTION:

(a) The thermal and mass circuit diagrams are shown in Figure Pr.6.53(b) The integral-volume energy equation (2.73) becomes

$$Q|_{A,c} = \langle Q_{ku}\rangle_L + Q_c = -(\rho c_p V)_c \frac{dT}{dt} + \dot{S}_{lg},$$

where from (6.124), we have

$$\langle Q_{ku}\rangle_L \quad = \quad \frac{T_s - T_{f,\infty}}{\langle R_{ku}\rangle_L} = \frac{T_c - T_{f,\infty}}{\langle R_{ku}\rangle_L} = A_{ku}\langle \mathrm{Nu}\rangle_L \frac{k_f}{L}(T_c - T_{f,\infty})$$

and from (6.181), we have

$$\dot{M}_w|_{A,c} = \frac{(\rho_{f,w}/\rho_f)_s - (\rho_{f,w}/\rho_f)_\infty}{\langle R_{Du}\rangle_L} = -\frac{d}{dt}M_w|_V.$$

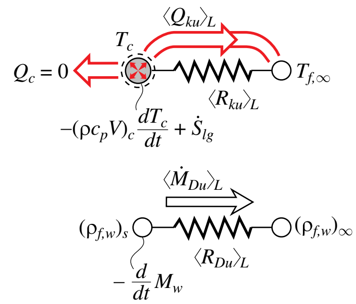


Figure Pr.6.53(b) Thermal and mass circuit diagrams.

(b) The Nusselt number is found from Table 6.5, i.e.,

$$\begin{aligned}
\mathrm{Nu}_L &= [(\mathrm{Nu}_{L,l})^{10} + (\mathrm{Nu}_{L,t})^{10}]^{1/10} \\
\mathrm{Nu}_{L,l} &= \frac{1.4}{\ln\left(1+\dfrac{1.4}{0.835a_1\mathrm{Ra}^{1/4}}\right)} \\
\mathrm{Nu}_{L,t} &= 0.14\mathrm{Ra}_L^{1/3} \\
a_1 &= \frac{4}{3}\frac{0.503}{\left[1+\left(\dfrac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{4/9}} \\
\mathrm{Ra}_L &= \frac{g\beta_f(T_c - T_{f,\infty})L^3}{\nu_f\alpha_f} \\
L &= \frac{A_{ku}}{P_{ku}} = \frac{\pi R^2}{2\pi R} = \frac{R}{2}.
\end{aligned}$$

The properties of air at $T = 325$ K are found from Table C.22, i.e.,

$k_f = 0.0284$ W/m-K	Table C.22
$\rho_f = 1.090$ kg/m^3	Table C.22
$c_{p,f} = 1{,}006$ J/kg-K	Table C.22
$\nu_f = 1.790 \times 10^{-5}$ m^2/s	Table C.22
$\alpha_f = 2.592 \times 10^{-5}$ m^2/s	Table C.22
$\mathrm{Pr} = 0.69$	Table C.22
$\beta_f = \dfrac{1}{T_f} = \dfrac{1}{325(\mathrm{K})} = 3.077 \times 10^{-3}$ 1/K	(6.77).

Then

$$\begin{aligned}
L &= 0.0175 \text{ m} \\
\mathrm{Ra}_L &= \frac{9.81(\mathrm{m/s^2}) \times 3.077 \times 10^{-3}(1/\mathrm{K}) \times (80-20)(\mathrm{K}) \times (0.0175)^3(\mathrm{m^3})}{1.790 \times 10^{-5}(\mathrm{m^2/s}) \times 2.592 \times 10^{-5}(\mathrm{m^2/s})} \\
&= 2.092 \times 10^4
\end{aligned}$$

$$
\begin{aligned}
a_1 &= \frac{4}{3}\frac{0.503}{\left[1+\left(\dfrac{0.492}{0.69}\right)^{9/16}\right]^{4/9}}\\
&= 0.5131\\
\mathrm{Nu}_{L,l} &= \frac{1.4}{\ln\left[1+\dfrac{1.4}{0.835\times 0.5131\times(2.092\times 10^4)^{1/4}}\right]} = 5.8246\\
\mathrm{Nu}_{L,t} &= 0.14(2.092\times 10^4)^{1/3} = 3.8576\\
\mathrm{Nu}_L &= [(5.8246)^{10}+(3.8576)^{10}]^{1/10} = 5.834\\
\langle Q_{ku}\rangle_L &= \pi(0.035)^2(\mathrm{m}^2)\times 5.834\times\frac{0.0284(\mathrm{W/m\text{-}K})(80-20)(\mathrm{K})}{0.0175(\mathrm{m})}\\
&= 2.186\ \mathrm{W}.
\end{aligned}
$$

(c) From (6.180), we have

$$
\langle R_{Du}\rangle_L = \langle R_{ku}\rangle_L c_{p,f}\mathrm{Le}^{-2/3},\quad \mathrm{Le}=\frac{D_{m,w}}{\alpha_f}.
$$

From Table C.20(a), we have

$$
\begin{aligned}
D_{m,w} &= 2.20\times 10^{-5}\ \mathrm{m^2/s}\quad \text{Table C.20(a)}\\
\mathrm{Le} &= \frac{2.20\times 10^{-5}(\mathrm{m^2/s})}{2.592\times 10^{-5}(\mathrm{m^2/s})} = 0.8488.
\end{aligned}
$$

Also from Table C.27, at $T_s = T_c = T_{lg} = (80+273.15)(\mathrm{K}) = 353.15$ K we have

$$
\begin{aligned}
(\rho_{f,w})_s &= 0.03057\ \mathrm{kg/m^3}\\
\Delta h_{lg} &= 2.309\times 10^6\ \mathrm{J/kg}.
\end{aligned}
$$

Then from (6.180), we have

$$
\begin{aligned}
\dot{M}_{lg} &= A_{ku}\langle \mathrm{Nu}\rangle_L\frac{k_f}{L}\frac{1}{c_{p,f}\mathrm{Le}^{-2/3}}[(\rho_{f,w})_s-(\rho_{f,w})_\infty]\\
&= \pi\times(0.035)^2(\mathrm{m}^2)\times 5.834\times\frac{0.0284(\mathrm{W/m\text{-}K})}{0.0175(\mathrm{m})}\times\frac{1}{1{,}006(\mathrm{J/kg\text{-}K})\times(0.8488)^{-2/3}}\times\frac{0.03057-0.001}{1.090}\\
&= 8.808\times 10^{-7}\ \mathrm{kg/s}.\\
\dot{S}_{lg} &= -\dot{M}_{lg}\Delta h_{lg} = -8.808\times 10^{-7}(\mathrm{kg/s})\times 2.309\times 10^6(\mathrm{J/kg})\\
&= -2.034\ \mathrm{W}.
\end{aligned}
$$

(d) From the energy equation, we have

$$
\begin{aligned}
\frac{dT_c}{dt} &= -\frac{\langle Q_{ku}\rangle_L-\dot{S}_{lg}}{(\rho c_p)_c V_c}\\
&= -\frac{2.186(\mathrm{W})-(-2.034)(\mathrm{W})}{2\times 10^6(\mathrm{J/m^3\text{-}K})\times 3.5\times 10^{-4}(\mathrm{m^3})}\\
&= -6.029\times 10^{-3}\,^\circ\mathrm{C/s}.
\end{aligned}
$$

COMMENT:

Note that the surface evaporation causes a similar cooling as that due to surface convection heat transfer. Therefore, preventing evaporation is as important as preventing heat transfer through the side walls and the top surface.

PROBLEM 6.54.FUN

GIVEN:

Equation (6.65) gives an empirical correlation for the local Nusselt number, $\overline{\mathrm{Nu}}_L$, for turbulent flow over a semi-infinite flat plate maintained at a constant temperature T_s. A prediction for $\overline{\mathrm{Nu}}_L$ can also be obtained by using a mixing length turbulence model and available empirical results. This problem and the one that follows outline the analysis procedure. In this problem, the conservation equations are derived and the velocity profile in the boundary layer is determined. In the following problem, the temperature profile in the boundary layer is determined and an expression for $\overline{\mathrm{Nu}}_L$ is found.

OBJECTIVE:

(a) Derive the time averaged forms of the continuity, momentum and energy equations under the boundary layer approximation.
(b) By assuming that $\overline{\mathbf{u}}_f = \overline{\mathbf{u}}_f(y)$ and $\overline{T}_f = \overline{T}_f(y)$, and using a no slip boundary condition at the fluid-solid interface, use the continuity equation to simplify the momentum and energy equations.
(c) Introduce the mixing length model and nondimensionalize the momentum equation to find a first order differential equation for the velocity profile in the boundary layer.
(d) Integrate the result of part (c) to find the velocity profile in the boundary layer.

SOLUTION:

(a) Before deriving the appropriate forms of the conservation equations, note the following properties of the time averaging operation, defined by (2.69),

$$\overline{a} \equiv \frac{1}{\tau}\int_0^\tau a dt,\ \overline{\overline{a}} = \overline{a},\ \overline{a'} = 0,\ \overline{\overline{a}b'} = 0,\ \overline{a+b} = \overline{a} + \overline{b},\ \overline{\frac{\partial a}{\partial x}} = \frac{\partial \overline{a}}{\partial x}.$$

The continuity equation for steady, incompressible two-dimensional flow is given by (6.37) as

$$\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} = 0.$$

From (6.54), we have $\mathbf{u}_f = \overline{\mathbf{u}}_f + \mathbf{u}'_f$, so that the velocity components u_f and v_f are

$$\begin{aligned} u_f &= \overline{u}_f + u'_f \\ v_f &= \overline{v}_f + v'_f. \end{aligned}$$

Substituting these into the continuity equation and time averaging term by term gives

$$\overline{\frac{\partial \overline{u}_f}{\partial x}} + \overline{\frac{\partial u'_f}{\partial y}} + \overline{\frac{\partial \overline{v}_f}{\partial x}} + \overline{\frac{\partial v'_f}{\partial y}} = 0.$$

By noting the properties of the time averaging operation, the second and fourth terms both equal zero, and the equation can be simplified to the required form of

$$\frac{\partial \overline{u}_f}{\partial x} + \frac{\partial \overline{v}_f}{\partial y} = 0.$$

For the momentum equation, start from (6.36), and noting that $\mu_f/\rho_f = \nu_f$,

$$u_f\frac{\partial u_f}{\partial x} + v_f\frac{\partial u_f}{\partial y} = \nu_f\frac{\partial^2 u_f}{\partial y^2}.$$

Using the expression for u_f and v_f from above, expanding, taking the time average and eliminating those terms which are the time average of a mean component and a fluctuating component leads to

$$\overline{u}_f\frac{\partial \overline{u}_f}{\partial x} + \overline{v}_f\frac{\partial \overline{u}_f}{\partial y} + \overline{u'_f\frac{\partial u'_f}{\partial x} + v'_f\frac{\partial u'_f}{\partial y}} = \nu_f\frac{\partial^2 \overline{u}_f}{\partial y^2}.$$

Consider the third and fourth terms on the left hand side:

$$\overline{u'_f\frac{\partial u'_f}{\partial x} + v'_f\frac{\partial u'_f}{\partial y}} = \overline{\frac{\partial}{\partial x}(u'^2_f) - u'_f\frac{\partial u'_f}{\partial x} + \frac{\partial}{\partial y}(u'_f v'_f) - u'_f\frac{\partial v'_f}{\partial x}}.$$

With the assumption that $\partial(\overline{u_f'^2})/\partial x \ll \partial(\overline{u_f'v_f'})/\partial y$ and noting that from the continuity equation, it can be shown that

$$\frac{\partial u_f'}{\partial x} + \frac{\partial v_f'}{\partial y} = 0,$$

this term simplifies to

$$\frac{\partial}{\partial y}(\overline{u_f'v_f'}),$$

which is modeled as

$$\frac{\partial}{\partial y}(\overline{u_f'v_f'}) = -\frac{\partial}{\partial y}(\nu_t \frac{\partial \overline{u}_f}{\partial y}).$$

Substituting this back into the momentum equation gives the required form of

$$\overline{u}_f \frac{\partial \overline{u}_f}{\partial x} + \overline{v}_f \frac{\partial \overline{u}_f}{\partial y} = \frac{\partial}{\partial y}[(\nu_f + \nu_t)\frac{\partial \overline{u}_f}{\partial y}].$$

For the energy equation, we start from (6.35),

$$u_f \frac{\partial T_f}{\partial x} + v_f \frac{\partial T_f}{\partial y} = \frac{k_f}{(\rho c_p)_f}\frac{\partial^2 T_f}{\partial y^2}.$$

Using the expressions for u_f and v_f from above, and noting that from (6.55), $T_f = \overline{T}_f + T_f'$, expanding, time averaging and eliminating those terms which are the time average of a mean component and a fluctuating component leads to

$$\overline{u}_f \frac{\partial \overline{T}_f}{\partial x} + \overline{v}_f \frac{\partial \overline{T}_f}{\partial y} + \overline{u_f' \frac{\partial T_f'}{\partial x} + v_f' \frac{\partial T_f'}{\partial y}} = \frac{k_f}{(\rho c_p)_f}\frac{\partial^2 \overline{T}_f}{\partial y^2}.$$

Consider the third and fourth terms on the left hand side:

$$\overline{u_f' \frac{\partial T_f'}{\partial x} + v_f' \frac{\partial T_f'}{\partial y}} = \overline{\frac{\partial}{\partial x}(u_f'T_f') - T_f'\frac{\partial u_f'}{\partial x} + \frac{\partial}{\partial y}(v_f'T_f') - T_f'\frac{\partial v_f'}{\partial x}}.$$

With the assumption that $\partial(\overline{u_f'T_f'})/\partial x \ll \partial(\overline{v_f'T_f'})/\partial y$ and using the continuity equation, this term simplifies to

$$\frac{\partial}{\partial y}(\overline{v_f'T_f'}),$$

which is modeled as

$$\frac{\partial}{\partial y}(\overline{v_f'T_f'}) = -\frac{1}{(\rho c_p)_f}\frac{\partial}{\partial y}(k_t \frac{\partial \overline{T}_f}{\partial y}).$$

Substituting this back into the energy equation gives the required form of

$$\overline{u}_f \frac{\partial \overline{T}_f}{\partial x} + \overline{v}_f \frac{\partial \overline{T}_f}{\partial y} = \frac{1}{(\rho c_p)_f}\frac{\partial}{\partial y}[(k_f + k_t)\frac{\partial \overline{T}_f}{\partial y}].$$

(b) If $\overline{\mathbf{u}}_f = \overline{\mathbf{u}}_f(y)$, then $\overline{u}_f = \overline{u}_f(y)$ and $\overline{v}_f = \overline{v}_f(y)$, and then $\partial\overline{u}_f/\partial x = 0$ and $\partial\overline{v}_f/\partial x = 0$, and the continuity equation becomes

$$\begin{aligned} \frac{\partial \overline{v}_f}{\partial y} &= 0 \\ \overline{v}_f &= \text{constant.} \end{aligned}$$

With a no slip boundary condition at the fluid-solid interface, $\overline{v}_f = 0$. The left side of the momentum equation is then equal to zero, giving

$$\frac{\partial}{\partial y}[(\nu_f + \nu_t)\frac{\partial \overline{u}_f}{\partial y}] = 0.$$

Similarly, if $\overline{T}_f = \overline{T}_f(y)$, then $\partial \overline{T}_f/\partial x = 0$. With $\overline{v}_f = 0$, the left side of the energy equation is zero, giving

$$\frac{1}{(\rho c_p)_f}\frac{\partial}{\partial y}[(k_f + k_t)\frac{\partial \overline{T}_f}{\partial y}] = 0.$$

(c) Integrating the momentum equation and replacing $\partial/\partial y$ with d/dy (as it has been assumed none of the dependent variables are x dependent) gives

$$(\nu_f + \nu_t)\frac{d\overline{u}_f}{dy} = a_1,$$

where a_1 is a constant. At the wall, where $y = 0$, $\nu_t = 0$, and therefore

$$\nu_f \frac{d\overline{u}_f}{dy}|_{y=0} = a_1.$$

However, it is given that $(d\overline{u}_f/dy)|_{y=0} = \overline{\tau}_s/\mu_f$ so that

$$a_1 = \frac{\nu_f \overline{\tau}_s}{\mu_f} = \frac{\overline{\tau}_s}{\rho_f},$$

and the equation for the velocity becomes

$$(\nu_f + \nu_t)\frac{d\overline{u}_f}{dy} = \frac{\overline{\tau}_s}{\rho_f}.$$

Now introduce the dimensionless parameters y^+ and $\overline{u}_f^+$, so that the equation for the velocity becomes

$$\begin{aligned}(\nu_f + \nu_t)\frac{d\overline{u}_f^+}{dy^+}\frac{(\overline{\tau}_s/\rho_f)^{1/2}}{\nu_f/(\overline{\tau}_s/\rho_f)^{1/2}} &= \frac{\overline{\tau}_s}{\rho_f} \\ (\frac{\nu_f + \nu_t}{\nu_f})\frac{d\overline{u}_f^+}{dy^+} &= 1.\end{aligned}$$

In the sublayer, $\nu_t = 0$, and this equation becomes

$$\frac{d\overline{u}_f^+}{dy^+} = 1.$$

In the turbulent part of the boundary layer, it is assumed that $\nu_t \gg \nu_f$, which gives

$$\frac{d\overline{u}_f^+}{dy^+} = \frac{\nu_f}{\nu_t}.$$

To find an expression for ν_t, start with the expression for k_t given in part (b) of the problem statement. With $\lambda_t = \kappa y$ from (6.63) and the nondimensionalization used previously,

$$\begin{aligned}k_t &= \mathrm{Pr}_t^{-1}(\rho c_p)_f \lambda_t^2 \frac{d\overline{u}_f}{dy} \\ \alpha_t &= \frac{\alpha_t}{\nu_t}\kappa^2 y^2 \frac{d\overline{u}_f}{dy} \\ \nu_t &= \kappa^2 {y^+}^2 \frac{\nu_f^2}{\overline{\tau}_s/\rho_f}\frac{d\overline{u}_f^+}{dy^+}\frac{(\overline{\tau}_s/\rho_f)^{1/2}}{\nu_f/(\overline{\tau}_s/\rho_f)^{1/2}} \\ \frac{\nu_t}{\nu_f} &= \kappa^2 {y^+}^2 \frac{d\overline{u}_f^+}{dy^+}.\end{aligned}$$

Substituting this into the equation for the velocity gives

$$\begin{aligned}\frac{d\overline{u}_f^+}{dy^+} &= \frac{1}{\kappa^2 {y^+}^2 \frac{d\overline{u}_f^+}{dy^+}} \\ \frac{d\overline{u}_f^+}{dy^+} &= \frac{1}{\kappa y^+}.\end{aligned}$$

(d) With a no slip boundary condition at the wall, $\overline{u}_f^+(y^+ = 0) = 0$. Separating the equation for the velocity in the sublayer and integrating leads to

$$\begin{aligned}\int_0^{\overline{u}_f^+} d\overline{u}_f^+ &= \int_0^{y^+} dy^+ \\ \overline{u}_f^+|_0^{\overline{u}_f^+} &= y^+|_0^{y^+} \\ \overline{u}_f^+ &= y^+.\end{aligned}$$

This equation is valid for $0 \leq y^+ \leq y^+_{\nu,crit} = 10.8$.
At the boundary between the sublayer and the turbulent portion of the boundary layer, $\overline{u}_f^+ = y^+ = 10.8$. Using this as the lower boundary condition for the turbulent portion, the equation for the velocity in that region can be separated and integrated to give

$$\begin{aligned}\int_{10.8}^{\overline{u}_f^+} d\overline{u}_f^+ &= \frac{1}{\kappa}\int_{10.8}^{y^+} \frac{dy^+}{y^+} \\ \overline{u}_f^+|_{10.8}^{\overline{u}_f^+} &= \frac{1}{\kappa}\ln y^+|_{10.8}^{y^+} \\ \overline{u}_f^+ &= \frac{1}{\kappa}\ln\frac{y^+}{10.8} + 10.8.\end{aligned}$$

With $\kappa = 0.41$,

$$\overline{u}_f^+ = 2.44 \ln y^+ + 5.00.$$

This equation is known as the Law of the Wall, and is valid for $y^+ \geq 10.8$. A plot of the velocity profile in the boundary layer is shown in Figure Pr.6.54.

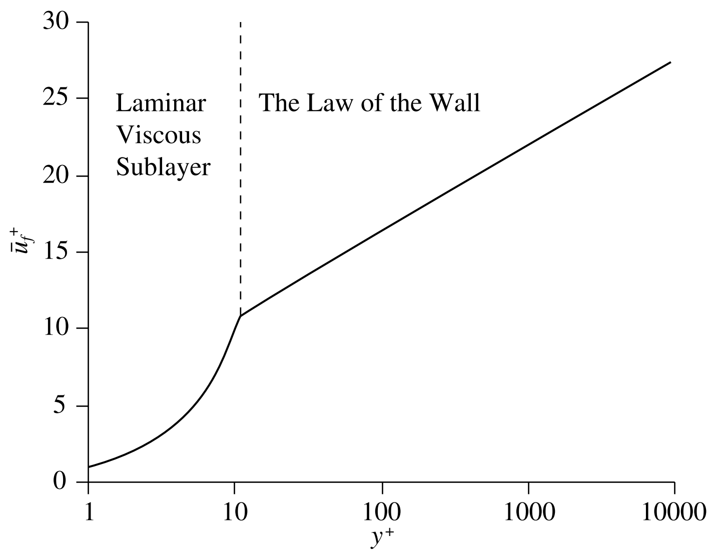


Figure Pr.6.54 Velocity profile in the turbulent boundary layer.

COMMENT:

The sublayer thickness used is an effective thickness, chosen that so experimental data fit the Law of the Wall. It has no physical significance. Molecular viscosity is found to play a significant role up to around $y^+ = 40$. While good agreement is thus found for the Law of the Wall and close to the wall, where turbulent fluctuations are negligible, in the transition between these two regions the agreement is poor. Other models for the mixing length can be chosen so as to get better agreement. For example, Van Driest assumes a continuous profile of the form

$$\lambda_t = \kappa y[1 - \exp(-y^+/A^+)],$$

where A^+ is an effective sublayer thickness and is found to be 25.0. Use of this model requires a numerical solution to the momentum equation, but gives a good fit to the experimental data in the sublayer and the Law of the Wall region. The Van Driest model is also in qualitative agreement with observations of the boundary layer structure. Turbulent transport from the wall is realized through fast, localized bursts of fluid from the sublayer into the turbulent portion of the boundary layer. While there are no turbulent fluctuations in the sublayer when it is undisturbed, these bursts are turbulent, and a statistical average of the behavior at any location in the sublayer would then yield a small but finite mixing length.

PROBLEM 6.55.FUN

GIVEN:
This problem is a continuation of the previous one. The next step is to find the temperature profile.

OBJECTIVE:
(a) Introduce the mixing length model to the energy equation, simplify, and nondimensionalize.
(b) Integrate the result of part (a) to find the temperature profile in the boundary layer.
(c) Evaluate the temperature profile using empirical data.
(d) Develop a relationship between the free stream velocity and temperature.
(e) Develop an expression for the Stanton number in terms of the friction coefficient and the Prandtl number.
(f) Develop an expression for the Nusselt number.

SOLUTION:
(a) Integrating the energy equation from part (b) of the previous problem, and noting that $\partial/\partial y$ can be replaced by d/dy, gives

$$(\alpha_f + \alpha_t)\frac{d\overline{T}_f}{dy} = \text{constant} = -\frac{q_s}{(\rho c_p)_f},$$

where $\alpha_f = k_f/(\rho c_p)_f$, $\alpha_t = k_t/(\rho c_p)_f$ and the definition of q_s has been used. Solving for $d\overline{T}_f$,

$$d\overline{T}_f = -\frac{q_s}{(\rho c_p)_f}\frac{dy}{\alpha_f + \alpha_t}.$$

From the definitions of y^+ and $\overline{T}_f^+$,

$$dy = \frac{\nu_f}{(\overline{\tau}_s/\rho_f)^{1/2}}dy \;\text{ and }\; d\overline{T}_f = -\frac{q_s/(\rho c_p)_f}{(\overline{\tau}_s/\rho_f)^{1/2}}d\overline{T}_f^+.$$

Substituting these into the equation for $d\overline{T}_f$ and dividing through by $\frac{q_s/(\rho c_p)_f}{(\overline{\tau}_s/\rho_f)^{1/2}}$ gives

$$d\overline{T}_f^+ = \frac{dy^+}{\dfrac{\alpha_f}{\nu_f} + \dfrac{\alpha_t}{\nu_f}}.$$

Noting that $\nu_f/\alpha_f = \text{Pr}$, and that $\overline{T}_f^+(y^+ = 0) = 0$, integration of the expression for $d\overline{T}_f^+$ gives

$$\overline{T}_f^+ = \int_0^{y^+}\frac{dy^+}{\dfrac{1}{\text{Pr}} + \dfrac{\alpha_t}{\nu_f}}.$$

(b) In the laminar thermal sublayer ($0 \le y^+ \le y^+_{\alpha,crit}$), $\alpha_t = 0$, and the temperature profile becomes

$$\overline{T}_f^+ = \text{Pr}\, y^+.$$

If it assumed that $\alpha_t \gg \alpha_f$ in the turbulent portion of the boundary layer, then in that region, $\alpha_t/\nu_f \gg 1/\text{Pr}$. For $y^+_{\alpha,crit} = 13.2$ the profile in the sublayer gives $\overline{T}_f^+ = 13.2\text{Pr}$, and integration of $d\overline{T}_f^+$ over the turbulent portion of the boundary layer leads to

$$\int_{13.2\text{Pr}}^{y^+} d\overline{T}_f^+ = \int_{13.2}^{y^+}\frac{dy^+}{\dfrac{\alpha_t}{\nu_f}}$$

$$\overline{T}_f^+ = 13.2\text{Pr} + \int_{13.2}^{y^+}\frac{\nu_f dy^+}{\alpha_t}.$$

(c) From the given expression for k_t, an expression for α_t/ν_f can be found.

$$\begin{aligned} k_t &= \mathrm{Pr}_t^{-1}(\rho c_p)_f \lambda_t^2 \frac{\partial \overline{u}_f}{\partial y} \\ \alpha_t &= \frac{\lambda_t^2}{\mathrm{Pr}_t} \frac{\partial \overline{u}_f}{\partial y}. \end{aligned}$$

From (6.63), $\lambda_t = \kappa y$. Substituting in, and dividing through by ν_f gives

$$\frac{\alpha_t}{\nu_f} = \frac{\kappa^2 y^2}{\nu_f \mathrm{Pr}_t} \frac{\partial \overline{u}_f}{\partial y}.$$

Nondimensionalizing with the given forms of y^+ and $\overline{u}_f^+$,

$$\begin{aligned} \frac{\alpha_t}{\nu_f} &= \frac{\kappa^2 \dfrac{{y^+}^2 \nu_f^2}{(\overline{\tau}_s/\rho_f)}}{\mathrm{Pr}_t \nu_f} \frac{\partial \overline{u}_f^+}{\partial y^+} \frac{(\overline{\tau}_s/\rho_f)^{1/2}}{\nu_f/(\overline{\tau}_s/\rho_f)^{1/2}} \\ &= \frac{\kappa^2 {y^+}^2}{\mathrm{Pr}_t} \frac{\partial \overline{u}_f^+}{\partial y^+}. \end{aligned}$$

From the result of part (c) of the previous problem, $\partial \overline{u}_f^+/\partial y^+ = 1/\kappa y^+$, so that

$$\frac{\alpha_t}{\nu_t} = \frac{\kappa y^+}{\mathrm{Pr}_t},$$

and the expression for the temperature profile becomes

$$\begin{aligned} \overline{T}_f^+ &= 13.2\mathrm{Pr} + \frac{\mathrm{Pr}_t}{\kappa} \int_{13.2}^{y^+} \frac{dy^+}{y^+} \\ &= 13.2\mathrm{Pr} + \frac{\mathrm{Pr}_t}{\kappa} \ln y^+ |_{13.2}^{y^+} \\ &= 13.2\mathrm{Pr} + \frac{\mathrm{Pr}_t}{\kappa} \ln \frac{y^+}{13.2}. \end{aligned}$$

For $\kappa = 0.41$ and $\mathrm{Pr}_t = 0.9$, the above expression evaluates to

$$\begin{aligned} \overline{T}_f^+ &= 13.2\mathrm{Pr} + \frac{0.9}{0.41} \ln \frac{y^+}{13.2} \\ &= 13.2\mathrm{Pr} + 2.20 \ln y^+ - 5.66. \end{aligned}$$

A plot of the temperature profile for Pr = 0.7 (typical for air over a wide range of temperatures) is shown in Figure Pr.6.55. Also plotted is the profile for Pr = 5.9 (water at room temperature). For this case, an effective thermal sublayer thickness of 7.55 is used. In general, this parameter is a functions of the Prandtl number. Note the difference in the magnitude of the temperature increase which occurs in the thermal sublayer for the two different fluids. Unlike the Law of the Wall, there is no universal temperature profile, and it therefore difficult to develop general heat transfer correlations. Similar comments to those discussed for the velocity profile can be made with respect to the agreement between these profiles and experimental data.

(d) From the results of part (d) of the previous problem and part (c) of the current problem, isolating for the $\ln y^+$ term leads to

$$\begin{aligned} \ln y^+ &= \frac{\overline{u}_f^+ - 5.00}{2.44} \\ \ln y^+ &= \frac{\overline{T}_f^+ - 13.2\mathrm{Pr} + 5.66}{2.20}. \end{aligned}$$

Eliminating $\ln y^+$ and evaluating the velocity and temperature at the edge of the boundary layer gives

$$\frac{u_{f,\infty}^+ - 5.00}{2.44} = \frac{T_{f,\infty}^+ - 13.2\mathrm{Pr} + 5.66}{2.20}.$$

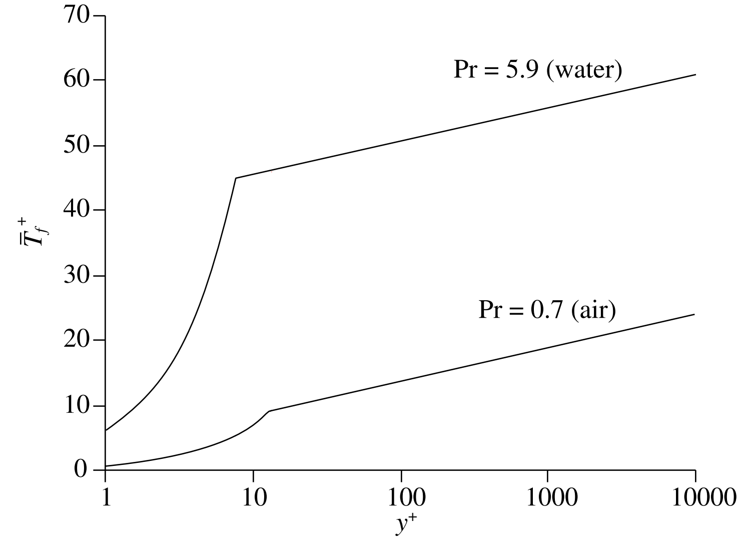


Figure Pr.6.55 Temperature profile in the turbulent boundary layer.

Where the overbar has been dropped as the far field conditions are independent of time. In order to equate the two expressions for $\ln y^+$, it has been assumed that the viscous and thermal boundary-layers have the same thickness. While in a laminar boundary-layer the relative magnitude of the two thicknesses is dependent on the Prandtl number, in a turbulent boundary layer, where the turbulent viscosity is the primary transport mechanism, it is not possible for the thermal boundary layer to have a significantly different thickness than the viscous boundary layer. Solving for $T^+_{f,\infty}$ leads to

$$T^+_{f,\infty} = 0.9u^+_{f,\infty} + 13.2\text{Pr} - 10.2.$$

(e) The expression for the mean Stanton number, $\overline{\text{St}}$, can be rearranged as

$$\begin{aligned}\overline{\text{St}} &= \frac{q_s}{(T_s - T_{f,\infty})(\rho c_p)_f(\overline{\tau}_s/\rho_f)^{1/2}}\frac{(\overline{\tau}_s/\rho_f)^{1/2}}{u_{f,\infty}} \\ &= \frac{1}{T^+_{f,\infty}}(\frac{0.5\overline{\tau}_s}{0.5\rho_f u^2_{f,\infty}})^{1/2} \\ &= \frac{(\overline{c}_f/2)^{1/2}}{T^+_{f,\infty}}.\end{aligned}$$

From the definition of $\overline{c}_f$, $u_{f,\infty} = (\overline{\tau}_s/0.5\rho_f\overline{c}_f)^{1/2}$, and thus

$$u^+_{f,\infty} = \frac{u_{f,\infty}}{(\overline{\tau}_s/\rho_f)^{1/2}} = \frac{(\overline{\tau}_s/\rho_f)^{1/2}(1/0.5\overline{c}_f)^{1/2}}{(\overline{\tau}_s/\rho_f)^{1/2}} = \frac{1}{(0.5\overline{c}_f)^{1/2}}.$$

The expression for $T^+_{f,\infty}$ can then be written as

$$T^+_{f,\infty} = 0.9\frac{1}{(0.5\overline{c}_f)^{1/2}} + 13.2\text{Pr} - 10.2.$$

Substituting this into the expression for the Stanton number and multiplying through by $(\overline{c}_f/2)^{1/2}$ gives

$$\overline{\text{St}} = \frac{(\overline{c}_f/2)^{1/2}}{T^+_{f,\infty}} = \frac{\overline{c}_f/2}{0.9 + (\overline{c}_f/2)^{1/2}(13.2\text{Pr} - 10.2)}.$$

(f) With $\overline{c}_f/2 = 0.0287\text{Re}_L^{-0.2}$ and $0.9 + (\overline{c}_f/2)^{1/2}(13.2\text{Pr} - 10.16) \simeq \text{Pr}^{0.4}$,

$$\overline{\text{St}} = \frac{0.0287\text{Re}_L^{-0.2}}{\text{Pr}^{0.4}}.$$

The Nusselt number is given by $\overline{\text{Nu}}_L = \overline{\text{St}}\text{Re}_L\text{Pr}$ so that

$$\begin{aligned}\overline{\text{Nu}}_L &= \frac{0.0287\text{Re}_L^{-0.2}}{\text{Pr}^{0.4}}\text{Re}_L\text{Pr} \\ &= 0.0287\text{Re}_L^{4/5}\text{Pr}^{3/5}.\end{aligned}$$

COMMENT:

The assumption of $\alpha_t = 0$ in the thermal sublayer, which, in light of the structure of the sublayer discussed in the previous solution, is better defined as assuming that $\alpha_t \ll \alpha_f$, will not be valid for fluids with high Prandtl numbers, such as thick oils. The assumption of $\alpha_t \gg \alpha_f$ in the turbulent portion of the boundary layer will not be valid for low Prandtl number fluids, such as liquid metals.

While the Nusselt number relation is similar to (6.65), it is limited to $0.5 \leq \text{Pr} \leq 1.0$ and $5 \times 10^5 \leq \text{Re}_L \leq 5 \times 10^6$. This range of the Prandtl number corresponds to gases. The fully empirical correlation covers a wider range of flow conditions ($0.6 \leq \text{Pr} \leq 60$ and $5 \times 10^5 \leq \text{Re}_L \leq 10^8$).

The mixing length model is an example of an algebraic (or zeroth-order) closure in turbulence modeling. It has advantages in that simple analytic predictions can be made, but it restricted by its simplicity. More involved methods include one- and two-equation models. The k-ϵ model is a well known two equation closure. Numerical solutions are generally required. For a more detailed discussion of turbulent transport, consult Kays, W.M., and Crawford, M.T., *Convective Heat and Mass Transfer*, Third Edition, McGraw-Hill, New York, 1993.

PROBLEM 6.56.FUN

GIVEN:

In the boundary-layer flow and heat transfer over a smooth, semi-infinite flat plate, the flow is initially laminar, but will transition to turbulence when the Reynolds number based on the location from the leading edge reaches a critical value, $\mathrm{Re}_{L,t}$. In order to use (6.50) to evaluate the average Nusselt number, $\langle \mathrm{Nu} \rangle_L$, over the plate, the laminar and turbulent regions must be considered separately.

OBJECTIVE:

(a) Using the expressions for the local Nusselt number, Nu_L, in the laminar and turbulent regions, given by (6.44) and (6.65) respectively, develop an expression for the average Nusselt number for a plate with length $L > L_t$.
(b) For a transition Reynolds number $\mathrm{Re}_{L,t} = 5 \times 10^5$, show that this expression reduces to (6.67).

SOLUTION:

(a) From (6.50), we have

$$\langle \mathrm{Nu} \rangle_L = \int_0^L \frac{\mathrm{Nu}_x}{x} dx,$$

which can be divided into a laminar and a turbulent region, i.e.,

$$\langle \mathrm{Nu} \rangle_L = \int_0^{L_t} \frac{(\mathrm{Nu}_x)_{lam}}{x} dx + \int_{L_t}^{L} \frac{(\mathrm{Nu}_x)_{turb}}{x} dx.$$

Using (6.44) and (6.65), we have

$$\langle \mathrm{Nu} \rangle_L = \int_0^{L_t} \frac{0.322\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}}{x} dx + \int_{L_t}^{L} \frac{0.0296\mathrm{Re}_x^{4/5}\mathrm{Pr}^{1/3}}{x} dx.$$

From (6.46) and using a variable location x, the Reynolds number is

$$\mathrm{Re}_x = \frac{u_{f,\infty} x}{\nu_f}.$$

Using this we have

$$\begin{aligned}
\langle \mathrm{Nu} \rangle_L &= 0.332\mathrm{Pr}^{1/3}(\frac{u_{f,\infty}}{\nu_f})^{1/2} \int_0^{L_t} x^{-1/2} dx + 0.0296\mathrm{Pr}^{1/3}(\frac{u_{f,\infty}}{\nu_f})^{4/5} \int_{L_t}^{L} x^{-1/5} dx \\
&= 0.332\mathrm{Pr}^{1/3}(\frac{u_{f,\infty}}{\nu_f})^{1/2}(2)L_t^{1/2} + 0.0296\mathrm{Pr}^{1/3}(\frac{u_{f,\infty}}{\nu_f})^{4/5}(\frac{5}{4})(L^{4/5} - L_t^{4/5}) \\
&= [0.664\mathrm{Re}_{L,t}^{1/2} + 0.037(\mathrm{Re}_L^{4/5} - \mathrm{Re}_{L,t}^{4/5})]\mathrm{Pr}^{1/3}.
\end{aligned}$$

(b) For $\mathrm{Re}_{L,t} = 5 \times 10^5$, we have

$$\begin{aligned}
\langle \mathrm{Nu} \rangle_L &= [(0.664)(5 \times 10^5)^{1/2} + 0.037(\mathrm{Re}_L^{4/5} - (5 \times 10^5)^{4/5})]\mathrm{Pr}^{1/3} \\
&= [0.037\mathrm{Re}_L^{4/5} - 871]\mathrm{Pr}^{1/3},
\end{aligned}$$

which is the same as (6.67).

COMMENT:

The above results are only valid when $L \geq L_t$. When $L < L_t$, the flow is fully laminar, and (6.51) can be used to determine the average Nusselt number. It has also been assumed that there is no significant transition region (i.e., the flow switches from laminar to turbulent at L_t). In a real flow, there is a finite transition-region length that would need to be separately modeled and included.

Chapter 7

Convection: Bounded Fluid Streams

PROBLEM 7.1.FAM

GIVEN:

Air is heated while flowing in a tube. The tube has a diameter $D = 10$ cm and a length $L = 4$ m. The inlet air temperature is $\langle T_f \rangle_0 = 20$℃ and the tube surface is at $T_s = 130$℃. The cross-sectional averaged air velocity is $\langle u_f \rangle = 2$ m/s. These are shown in Figure Pr.7.1(a).

Evaluate the properties of air at $T = 300$ K.

SKETCH:

Figure Pr.7.1(a) shows a uniform surface temperature tube with a bounded air stream flowing through it and being heated.

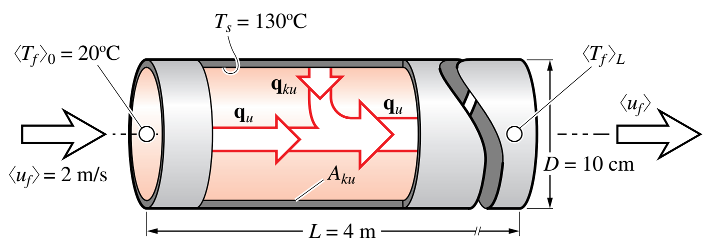


Figure Pr.7.1(a) Constant surface temperature tube heating bounded air stream.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the Nusselt number $\langle \mathrm{Nu} \rangle_D$.

(c) Determine the number of thermal units NTU.

(d) Determine the heat transfer effectiveness ϵ_{he}.

(e) Determine the average convection resistance $\langle R_u \rangle_L$(℃/W).

(f) Determine the convection heat transfer rate $\langle Q_u \rangle_{L-0}$(W).

(g) Determine the air exit temperature $\langle T_f \rangle_L$(℃).

SOLUTION:

(a) Figure Pr.7.1(b) shows the thermal circuit for a bounded air stream exchanging heat with its bounding tube wall. The fluid exchanges heat at a rate $\langle Q_u \rangle_{L\text{-}0}$(W) with the wall which is at T_s. As a consequence its temperature is raised from $\langle T_f \rangle_0$ to $\langle T_f \rangle_L$. The average convection resistance for this heat transfer is $\langle R_u \rangle_L$(℃/W).

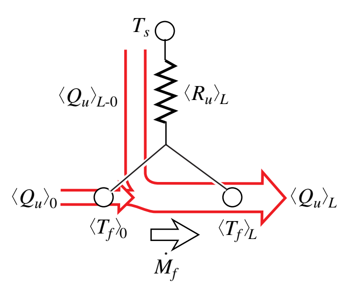


Figure Pr.7.1(b) Thermal circuit diagram.

(b) From Table C.22, the properties for air at $T = 300$ K are $k_f = 0.0267$ W/m-K, $\rho_f = 1.177$ kg/m^3, $c_{p,f} = 1{,}005$ J/kg-K, $\nu_f = 15.66 \times 10^{-6}$ m^2/s, and Pr = 0.69.

The Reynolds number based on diameter is given by (7.36) and is

$$\mathrm{Re}_D = \frac{\langle u_f \rangle D}{\nu_f} = \frac{2(\mathrm{m/s}) \times 0.10(\mathrm{m})}{15.66 \times 10^{-6}(\mathrm{m^2/s})} = 12{,}772.$$

Since $\mathrm{Re}_D > \mathrm{Re}_{D,t} = 2{,}300$, the flow regime is turbulent. For the turbulent regime the Nusselt number is obtained from Table 7.3. For $T_s > \langle T_f \rangle$ (i.e., the fluid is being heated), we have $n = 0.4$ and the Nusselt number is given by

$$\langle \mathrm{Nu} \rangle_D = 0.023\mathrm{Re}_D^{4/5}\mathrm{Pr}^{0.4} = 0.023 \times (12772)^{4/5} \times (0.69)^{0.4} = 38.22.$$

(c) For a circular tube, $A_u = \pi D^2/4$ and $A_{ku} = \pi DL$ The number of transfer units is given by (7.20) and is

$$NTU = \frac{A_{ku}\langle \mathrm{Nu} \rangle_D \; k_f}{D \; \dot{M}_f \; c_{p,f}}.$$

The mass flow rate $\dot{M}_f$ is given by (7.3), i.e.,

$$\dot{M}_f = A_u \rho_f \langle u_f \rangle = \frac{\pi D^2}{4}\rho_f \langle u_f \rangle = \frac{\pi \times (0.1)^2(\mathrm{m})^2}{4} \times 1.177(\mathrm{kg/m^3}) \times 2(\mathrm{m/s}) = 1.849 \times 10^{-2} \text{ kg/s}.$$

The number of transfer units NTU is then

$$NTU = \frac{\pi DL \langle \mathrm{Nu} \rangle_D \; k_f}{D \; \dot{M}_f \; c_{p,f}} = \frac{\pi \times \; 4(\mathrm{m}) \times 38.22 \times 0.0267(\mathrm{W/m\text{-}K})}{1.849 \times 10^{-2}(\mathrm{kg/m^3}) \times 1{,}005(\mathrm{J/kg\text{-}K})} = 0.6901.$$

(d) The heat transfer effectiveness ϵ_{he} is given by (7.22) and is

$$\epsilon_{he} = 1 - e^{-NTU} = 1 - e^{-0.6901} = 0.4985.$$

(e) The average convection resistance is given by (7.27), i.e.,

$$\langle R_u \rangle_L = \frac{1}{\epsilon_{he} \; \dot{M}_f \; c_{p,f}} = \frac{1}{0.4985 \times 1.849 \times 10^{-2}(\mathrm{kg/m^3}) \times 1{,}005(\mathrm{J/kg\text{-}K})} = 0.10795 \text{ °C/W}.$$

(f) From (7.25), the convection heat transfer rate is given by (7.25) as

$$\langle Q_u \rangle_{L\text{-}0} = \frac{T_s - \langle T_f \rangle_0}{\langle R_u \rangle_L} = \frac{130(\text{°C}) - 20(\text{°C})}{0.10795(\text{°C/W})} = 1{,}019 \text{ W}.$$

(g) The average fluid temperature at $x = L$ (exit) is determined from (7.22) and the result is

$$\langle T_f \rangle_L = \langle T_f \rangle_0 + \epsilon_{he}\left(T_s - \langle T_f \rangle_0 -\right) = 20(\text{°C}) + 0.4985\left[130(\text{°C}) - 20(\text{°C})\right] = 74.84\text{°C}.$$

COMMENT:

Note that $\langle Q_u \rangle_{L\text{-}0}$ is positive, because the net convection heat flow rate of the air has increased (there is more exiting compared to entering).

PROBLEM 7.2.FAM.S

GIVEN:

A thermoelectric cooler maintains the surface temperature of a metallic block at $T_s = 2$℃. The block is internally carved to form a connected, circular channel with diameter $D = 0.8$ cm and length $L = 40$ cm. This is shown in Figure Pr.7.2. Through this channel, a water stream flows and is cooled. The inlet temperature for the water is $\langle T_f \rangle_0 = 37$℃.

Evaluate the properties at the inlet temperature.

SKETCH:

Figure Pr.7.2 shows a metallic block with a water stream flowing through an internal channel.

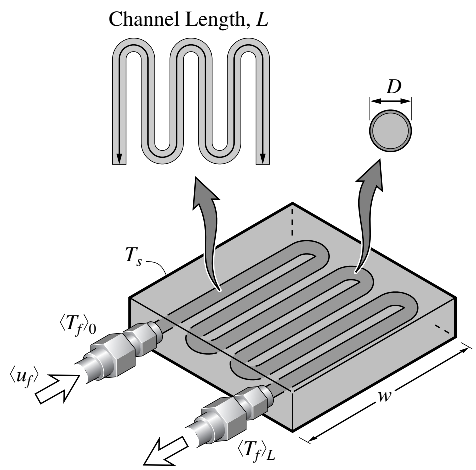


Figure Pr.7.2 A water-stream cooling block with internal channels.

OBJECTIVE:

(a) Determine and plot (i) the number of transfer units NTU, (ii) the thermal effectiveness ϵ_{he}, (iii) the water exit temperature $\langle T_f \rangle_L$(℃), and (iv) the convection heat transfer rate $\langle Q_u \rangle_{L-0}$(W), as a function of the water velocity for $0 < \langle u_f \rangle < 2$ m/s.

(b) At what water velocity is the exit temperature $\langle T_f \rangle_L$, 28℃ above T_s?

SOLUTION:

(a) From Table C.32, for water at 27℃= 310K,

$$\begin{aligned} \nu_f &= 7.11 \times 10^{-7} \text{ m}^2/\text{s} \\ \rho_f &= 995.3 \text{ kg/m}^3 \\ \text{Pr} &= 4.74 \\ k_f &= 0.023 \text{ W/m-K} \\ c_{p,f} &= 4{,}178 \text{ J/kg-K} \end{aligned}$$

For a circular tube, we have

$$\begin{aligned} A_u &= \frac{\pi D^2}{4} = \frac{\pi \times (0.008)^2 (\text{m})^2}{4} = 5.0625 \times 10^{-5} \text{ m}^2 \\ A_{ku} &= \pi D L = \pi \times 0.008(\text{m}) \times 0.40(\text{m}) = 0.1005 \text{ m}^2. \end{aligned}$$

The required quantities are calculated as follows:

(i) From (7.20),

$$NTU = \frac{A_{ku} \langle \text{Nu} \rangle_D k_f / D}{(\dot{M} c_p)_f}.$$

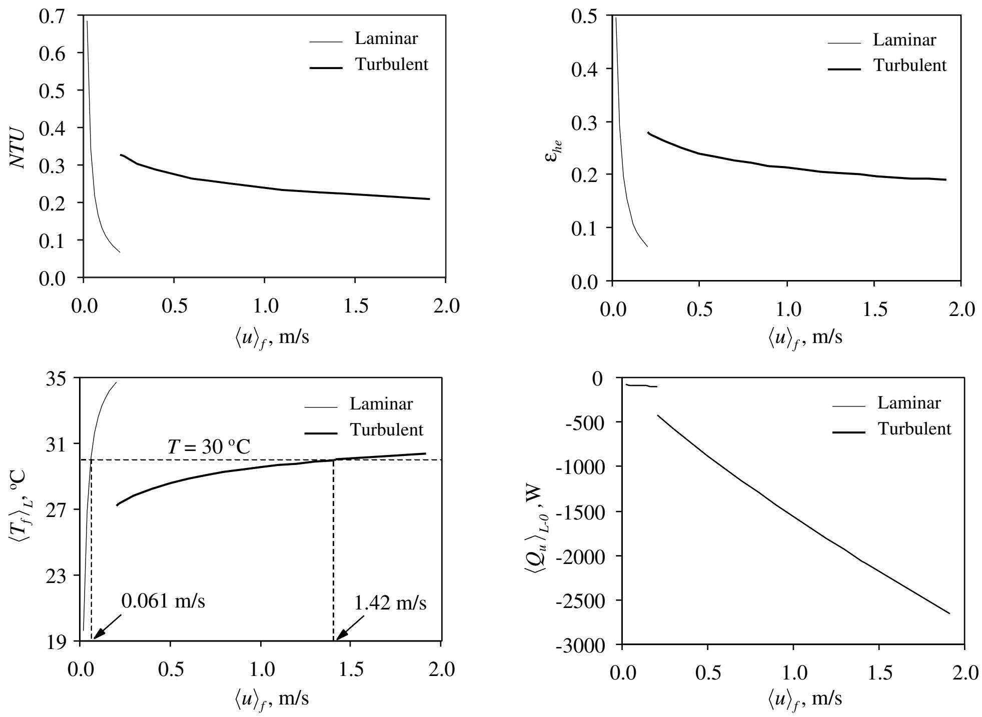


Figure Pr.7.2(b) SOPHT computer code and the resulting plots.

To specify $\langle \mathrm{Nu} \rangle_D$, the Reynolds number must be specified. It is given by (7.31) as

$$\mathrm{Re}_D = \frac{\langle u_f \rangle D}{\nu_f}.$$

For $\mathrm{Re}_D < 2300$, from Table 7.2 for constant wall temperature, $\langle \mathrm{Nu} \rangle_D = 3.66$.
Assuming that the transition to the turbulent regime takes place at a Reynolds number of 2300, for $\mathrm{Re}_D > 2300$, from Table 7.3, for $\langle T_f \rangle_0 > T_s$, $\langle \mathrm{Nu} \rangle_D = 0.023 \mathrm{Re}_D^{0.8} \mathrm{Pr}^{0.3}$.
(ii) From (7.22),

$$\epsilon_{he} = 1 - e^{-NTU}.$$

(iii) From (7.21),

$$\langle T_f \rangle_L = T_s + (\langle T_f \rangle_0 - T_s)\epsilon_{he}.$$

(iv) From (7.24),

$$\langle Q_u \rangle_{L\text{-}0} = (\dot{M} c_p)_f (\langle T_f \rangle_L - \langle T_f \rangle_0).$$

The required plots are shown in Figure Pr.7.2(b). Since $\langle \mathrm{Nu} \rangle_D$ correlations for transition regimes are complex and not very accurate, the text suggests the use of the turbulent regime correlation for Re> 2,300.

(b) From the plot for $\langle T_f \rangle_L$ and the resulting tabulated data (not shown), we have $\langle T_f \rangle_L$ = 30℃ = 303.15 K at velocities of 0.061 m/s for the laminar regime and 1.42 m/s for the turbulent. Had the transition regime been separately modeled, there would be a third velocity at which $\langle T_f \rangle_L$ = 30℃. Transition flows are unsteady, and for this reason, should be avoided when specifying operating conditions.

COMMENT:
Note that the large increase in $\langle \mathrm{Nu} \rangle_D$ for the turbulent flow regime can allow for the cooling of a larger mass flow rate (i.e., higher velocity stream).

PROBLEM 7.3.FUN

GIVEN:

A fluid enters a tube of uniform surface temperature $T_s$with temperature $\langle T_f\rangle_0$ and exits at $\langle T_f\rangle_L$. The tube has a length L, a cross-sectional area A_u, a surface-convection area A_{ku}, a surface temperature T_s, and a mass flow rate $\dot{M}_f$.

OBJECTIVE:

(a) Show that for $NTU \rightarrow 0$, $\langle T_f\rangle_L$ becomes a linear function of NTU.
(b) For a tube with a circular cross section, obtain an expression for NTU as a function of the tube diameter D and length L.
(c) How can the length L and the diameter D be changed such that $NTU \rightarrow 0$?

SOLUTION:

(a) The temperature at the end of a tube of uniform surface temperature T_s, with a inlet temperature $\langle T_f\rangle_0$ is given by (7.21), i.e.,

$$\langle T_f\rangle_L = T_s + (\langle T_f\rangle_0 - T_s)e^{-NTU}.$$

The exponential function can be expanded using the Taylor series expansion, as a power series in NTU, i.e.,

$$e^{-NTU} = 1 - NTU + \frac{1}{2}NTU^2 - \frac{1}{6}NTU^3 + \frac{1}{24}NTU^4 - \ldots$$

In the limit for $NTU \rightarrow 0$, the terms with the exponents larger than unity tend to zero much faster than the first two terms. Therefore, for a small NTU, the exponential function can be approximated as

$$e^{-NTU} \simeq 1 - NTU, \quad NTU \rightarrow 0.$$

Then

$$\begin{aligned}
\lim_{NTU\rightarrow 0}\langle T_f\rangle_L &= T_s + (\langle T_f\rangle_0 - T_s)\lim_{NTU\rightarrow 0} e^{-NTU} \\
&= T_s + (\langle T_f\rangle_0 - T_s)(1 - NTU) \\
&= \langle T_f\rangle_0 - NTU(\langle T_f\rangle_0 - T_s),
\end{aligned}$$

which is a linear relation between $\langle T_f\rangle_L$ and NTU.

(b) The NTU is defined by (7.20), i.e.,

$$NTU = \frac{1}{\langle R_{ku}\rangle_D(\dot{M}c_p)_f},$$

where from (7.19)

$$\langle R_{ku}\rangle_D = \frac{D}{\langle \mathrm{Nu}\rangle_D A_{ku} k_f},$$

and

$$\dot{M}_f = A_u \dot{m}_f.$$

Then

$$NTU = \frac{A_{ku}}{DA_u}\frac{\langle \mathrm{Nu}\rangle_D k_f}{\dot{m}_f c_{p,f}}.$$

For a circular tube we have

$$A_{ku} = \pi DL, \quad A_u = \pi D^2/4.$$

Then

$$NTU = \frac{4L}{D}\frac{\langle \mathrm{Nu}\rangle_D k_f}{\dot{m}_f c_{p,f}}.$$

(c) NTU decreases as L decreases or D increases.

COMMENT:

The linear relation between $\langle T_f \rangle_L$ and NTU is the solution of the energy equation

$$\dot{M}_f c_{p,f} \frac{d\langle T_f \rangle}{dx} = \frac{P_{ku} \langle \mathrm{Nu} \rangle_D k_f}{D} \left(T_s - \langle T_f \rangle_0 \right),$$

in which the heat transfer by surface convection is written as a function of the temperature difference $T_s - \langle T_f \rangle_0$. This becomes a valid approximation only in the limit of a very short tube ($NTU \rightarrow 0$), as shown above.

PROBLEM 7.4.FUN

GIVEN:

The blood flow through human tissues is by very small arteries called the arterioles, which have diameters in the range of 5 to 50 μm and a length of a few centimeter. These are fed by small arteries, which in turn are fed by the aorta. Each arteriole empties into 10 to 100 capillaries, which have porous walls and are the sites of the exchange between the blood and interstitial tissue fluid. These are shown in Figure Pr.7.4. There are about 10^{10} capillaries in peripheral tissue. This cascading of blood vessels results in a large increase in the total flow cross section A_u, as listed in Table Pr.7.4 along with the cross-sectional area and the time-area averaged blood velocity $\langle \overline{u_f} \rangle$.

Table Pr.7.4 Cross-sectional area (total) and time-area averaged blood velocity through various segments of blood pathways.

	A_u, cm^2	$\langle \overline{u_f} \rangle$, cm/s
aorta	2.5	33
small arteries	20	4.1
arterioles	40	2.1
capillaries	2,500	0.033
venues	250	0.33
small veins	80	1.0
venue cavao	8	10

As the total flow cross-sectional area A_u increases, $\langle \overline{u_f} \rangle$ decreases (because the mass flow rate is conserved).

In Example 7.4, we showed that for a very large specific surface area, i.e., $A_{ku}/V \to \infty$, we have $NTU \to \infty$ and that any fluid entering a porous solid with an inlet temperature $\langle T_f \rangle_0$ will leave with its exit temperature reaching the local solid temperature, i.e., $\langle T_f \rangle_L = T_s$.

$L_a = 2$ mm, $D_a = 50$ μm, $L_c = 30$ μm, $D_c = 3$ μm, $\rho_f = 1{,}000$ kg/m^3, $c_{p,f} = 3{,}000$ J/kg-K, $\mu_f = 10^{-3}$ Pa-s, $k_f = 0.6$ W/m-K.

SKETCH:

Figure Pr.7.4 shows the blood flowing through the capillaries for exchange with the interstitial tissue fluid.

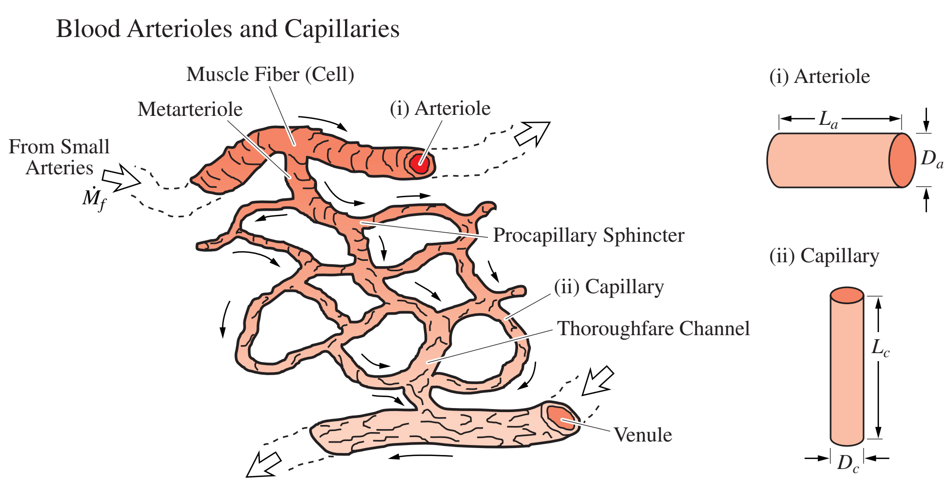


Figure Pr.7.4 Blood supply to tissue by arterioles feeding the capillaries.

OBJECTIVE:

(a) For the conditions given above and in Table Pr.7.4, determine NTU for (i) an arteriole, and (ii) a capillary. Address the entrance effect [note that from Table 7.2, for $(L/D)/\mathrm{Pe}_D > 0.06$, the entrance effect is negligible].

(b) Using (7.21), (7.22), and (7.24), comment on the ability of these blood streams to control the local tissue temperature T_s.

SOLUTION:

The number of transfer units is defined by (7.20) as

$$NTU = \frac{A_{ku}\langle \mathrm{Nu}\rangle_D k_f/D}{(\dot{M}c_p)_f},$$

where from (7.3), we have

$$\dot{M}_f = \rho_f A_u \langle u_f \rangle, \quad A_u = \pi D^2/4.$$

The Nusselt number for straight tubes with constant surface temperature is given in Table 7.2 for laminar flow and in Table 7.3 for turbulent flow. The flow regime is determined by calculating the Reynolds number (7.36), i.e.,

$$\mathrm{Re}_D = \frac{\langle u_f \rangle D}{\nu_f} = \frac{\rho_f \langle u_f \rangle D}{\mu_f}.$$

(i) Arteriole

$$\begin{aligned} \mathrm{Re}_D &= \frac{10^3(\mathrm{kg/m^3}) \times 2.1 \times 10^{-2}(\mathrm{m/s}) \times 50 \times 10^{-6}(\mathrm{m})}{10^{-3}(\mathrm{Pa\text{-}s})} \\ &= 1.050, \end{aligned}$$

and the flow is therefore laminar. Noting from Table 7.2 that $\mathrm{Re}_D\mathrm{Pr} = \mathrm{Pe}_D$, the Peclet number, and that $\mathrm{Pr}= \nu_f/\alpha_f = \mu_f c_{p,f}/k_f$, we have

$$\begin{aligned} \mathrm{Pe}_D &= \mathrm{Re}_D\mathrm{Pr} = \mathrm{Re}_D \frac{\mu_f c_{p,f}}{k_f} \\ &= 1{,}050 \times \frac{10^{-3}(\mathrm{Pa\text{-}s}) \times 3 \times 10^3(\mathrm{J/kg\text{-}K})}{0.6(\mathrm{W/m\text{-}K})} \\ &= 5.250 \end{aligned}$$

Then, the criteria for checking the length of the entrance region for laminar flow becomes

$$\frac{L/D}{\mathrm{Pe}_D} = \frac{2 \times 10^{-3}(\mathrm{m})/50 \times 10^{-6}(\mathrm{m})}{5.250} = 7.619 > 0.6 \quad \text{entrance effect is negligible.}$$

Assuming that the wall is at a constant temperature, the Nusselt number is found in Table 7.2 to be

$$\begin{aligned} \langle \mathrm{Nu}\rangle_D &= 3.66 \\ A_{ku} &= \pi D L \\ NTU &= \frac{\pi D L \langle \mathrm{Nu}\rangle_D k_f/D}{\frac{\pi D^2}{4}\rho_f \langle u_f \rangle c_{p,f}} = \frac{L\langle \mathrm{Nu}\rangle_D k_f}{D^2 \rho_f c_{p,f} \langle u_f \rangle} \\ &= 4 \times \langle \mathrm{Nu}\rangle_D \frac{k_f}{\rho_f c_{p,f}} \frac{L}{D^2} \frac{1}{\langle u_f \rangle} \\ &= 4 \times 3.66 \times \frac{0.6(\mathrm{W/m\text{-}K})}{10^3(\mathrm{kg/m^3}) \times 3 \times 10^3(\mathrm{J/kg\text{-}K})} \times \frac{2 \times 10^{-3}(\mathrm{m})}{(50 \times 10^{-6})^2(\mathrm{m^2})} \times \frac{1}{2.1 \times 10^{-2}(\mathrm{m/s})} \\ &= 115.5. \end{aligned}$$

(ii) Capillary

$$\begin{aligned}
\mathrm{Re}_D &= \frac{3.3 \times 10^{-4}(\mathrm{m/s}) \times 3 \times 10^{-6}(\mathrm{m}) \times 10^3(\mathrm{kg/m^3})}{10^{-3}(\mathrm{Pa\text{-}s})} \\
&= 9.9 \times 10^{-4} \\
\mathrm{Pe}_D &= 9.9 \times 10^{-4} \times \frac{10^{-3} \times 3 \times 10^3(\mathrm{J/kg\text{-}K})}{0.6(\mathrm{W/m\text{-}K})} \\
&= 4.950 \times 10^{-3} \\
\frac{L/D}{\mathrm{Pe}_D} &= \frac{30 \times 10^{-6}(\mathrm{m})/3 \times 10^{-6}(\mathrm{m})}{4.950 \times 10^{-3}} = 2{,}020 > 0.6 \quad \text{entrance effect is negligible.}
\end{aligned}$$

Then,

$$\begin{aligned}
NTU &= 4 \times 3.66 \times \frac{0.6(\mathrm{W/m\text{-}K})}{10^3(\mathrm{kg/m^3}) \times 3 \times 10^3(\mathrm{J/kg\text{-}K})} \times \frac{30 \times 10^{-6}(\mathrm{m})}{(3 \times 10^{-6})^2(\mathrm{m^2})} \times \frac{1}{3.34 \times 10^{-4}(\mathrm{m/s})} \\
&= 2.957 \times 10^4.
\end{aligned}$$

(b) The large NTU, used in (7.21), (7.22) and (7.24), gives

$$\begin{aligned}
\langle T_f \rangle_L &= T_s, \quad \epsilon_{he} = 1, \\
\langle Q_u \rangle_{L\text{-}0} &= (\dot{M} c_p)_f (T_s - \langle T_f \rangle_0).
\end{aligned}$$

This ensures that any difference $T_s - \langle T_f \rangle_0$, will results in the maximum heat transfer rate (since $\epsilon_{he} = 1$) which tends to bring T_s close to $\langle T_f \rangle_0$. Here $\langle T_f \rangle_0$ is the deep-body temperature $\langle T_f \rangle_0 = 37$℃.

COMMENT:

The mass flow rate per unit volume for the anteriors can be estimated using a volume fraction for these very small vessels. Assuming this fraction to be $\epsilon_a = 0.001$, we have

$$\begin{aligned}
\dot{n}_f &= \frac{(\dot{M}_f)_a}{V_a/\epsilon_a} = \frac{\epsilon_a \rho_f A_u \langle \overline{u_f} \rangle}{\dfrac{\pi D^2}{4} L} = \frac{\epsilon_a \rho_f \langle \overline{u_f} \rangle}{L} \\
&= \frac{0.001 \times 10^3(\mathrm{kg/m^3}) \times 2.1 \times 10^{-2}(\mathrm{m/s})}{2 \times 10^{-3}(\mathrm{m})} \\
&= 10.50\ \mathrm{kg/m^3\text{-}s}.
\end{aligned}$$

This is on the high side for the typical magnitudes of $\dot{n}_f$ listed in Example 7.4.

PROBLEM 7.5.FAM.S

GIVEN:

The thermally fully developed regime is defined in terms of dimensionless temperature T^* as $\partial T^*/\partial x \equiv \partial[(T_s - T_f)/(T_s - \langle T_f\rangle)]/\partial x = 0$, where x is along the tube and $T_f = T_f(r,x)$, $\langle T_f\rangle = \langle T_f\rangle(x)$, and $T_s(x)$ all change with x. For laminar, fully-developed temperature and velocity fields in a tube flow, the differential energy equation for the cylindrical coordinate system is the simplified form of (B.62). Due to the assumed angular symmetry, the ϕ dependence is omitted and because of the fully developed fields, the axial conduction and radial convection are omitted. Then for a steady-state heat transfer, we have (using the coordinates of Figure 7.1) from (B.62)

$$-k_f \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_f}{\partial r}\right) + \frac{\partial}{\partial x}(\rho c_p)_f u_f(r) T_f = 0,$$

where

$$u_f(r) = 2\langle u_f\rangle\left[1 - \left(\frac{2r}{D}\right)^2\right].$$

These are used to determine the fluid temperature $T_f(r,x)$, along with the condition of uniform heat flux q_s on the tube wall, which results in the combined integral-differential length energy equation (7.12), i.e.,

$$-P_{ku} q_s + A_u \frac{d}{dx}(\rho c_p)_f \langle u_f\rangle\langle T_f\rangle = 0.$$

Here q_s is taken to be positive when it flows into the fluid.

Then the Nusselt number is given by (7.19), i.e.,

$$\mathrm{Nu}_D = \langle \mathrm{Nu}\rangle_D = \frac{q_s D}{[T_s(x) - \langle T_f\rangle(x)]k_f}.$$

OBJECTIVE:

(a) Show that $\partial T_f/\partial x = d\langle T_f\rangle/dx$ is uniform along the tube.

(b) Derive the expression for $T_f = T_f(r,x)$, i.e.,

$$T_f(r,x) = T_s(x) - \frac{2q_s D}{k_f}\left[\frac{13}{6} + \frac{1}{16}\left(\frac{2r}{D}\right)^4 - \frac{1}{4}\left(\frac{2r}{D}\right)^2\right].$$

(c) Derive the expression for $T_s(x) - \langle T_f\rangle(x)$.

(d) Using (7.9) show that $\langle \mathrm{Nu}\rangle_D = 48/11 = 4.36$, for uniform q_s.

SOLUTION:

(a) We start with the definition of a fully-developed temperature field, in flow and heat transfer in a tube, i.e.,

$$\frac{\partial}{\partial x}\left(\frac{T_s - T_f}{T_s - \langle T_f\rangle}\right) = 0, \quad T_f = T_f(r,x), \quad T_s = T_s(x), \quad \langle T_f\rangle = \langle T_f\rangle(x),$$

We expand this to arrive at

$$\begin{aligned}
\frac{(T_s - \langle T_f\rangle)\left(\dfrac{dT_s}{dx} - \dfrac{\partial T_f}{\partial x}\right) - (T_s - T_f)\left(\dfrac{dT_s}{dx} - \dfrac{d\langle T_f\rangle}{dx}\right)}{(T_s - \langle T_f\rangle)^2} &= 0\\
\frac{1}{T_s - \langle T_f\rangle}\frac{dT_s}{dx} - \frac{1}{T_s - \langle T_f\rangle}\frac{\partial T_f}{\partial x} - \frac{T_s - T_f}{(T_s - \langle T_f\rangle)^2}\frac{dT_s}{dx} + \frac{T_s - T_f}{(T_s - \langle T_f\rangle)^2}\frac{d\langle T_f\rangle}{dx} &= 0.
\end{aligned}$$

Under the condition of uniform q_s and $\langle \mathrm{Nu}\rangle_D$, from the definition of the Nusselt number, we get

$$T_s - \langle T_f\rangle = \frac{q_s D}{\langle \mathrm{Nu}\rangle_D k_f} = \text{ constant},$$

so that

$$\frac{dT_s}{dx} = \frac{d\langle T_f\rangle}{dx}.$$

Substituting this into the expansion above leads to

$$\begin{aligned}\frac{1}{T_s - \langle T_f \rangle}\frac{dT_s}{dx} - \frac{1}{T_s - \langle T_f \rangle}\frac{\partial T_f}{\partial x} &= 0 \\ \frac{dT_s}{dx} &= \frac{\partial T_f}{\partial x}.\end{aligned}$$

Then comparing to the equality found from the Nusselt number expression gives

$$\frac{\partial T_f}{\partial x} = \frac{d\langle T_f \rangle}{dx}.$$

This allows us to replace $\partial T_f/\partial x$ in the energy equation by $d\langle T_f \rangle/dx$

(b) The differential-volume energy equation becomes

$$-k_f \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_f}{\partial r}\right) + (\rho c_p)_f u_f(r)\frac{d\langle T_f \rangle}{dx} = 0.$$

Now replacing $u_f(r)$ with the given profile, we have

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_f}{\partial r}\right) - \frac{2(\rho c_p)_f \langle u_f \rangle}{k_f}\left[1 - \left(\frac{2r}{D}\right)^2\right]\frac{d\langle T_f \rangle}{dx} = 0.$$

We now replace $d\langle T_f \rangle/dx$ from integral-differential length energy equation, i.e.,

$$\frac{d\langle T_f \rangle}{dx} = \frac{P_{ku} q_s}{A_u (\rho c_p)_f \langle u_f \rangle}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_f}{\partial r}\right) - \frac{2P_{ku} q_s}{A_u k_f}\left[1 - \left(\frac{2r}{D}\right)^2\right] = 0.$$

Upon two integrations, we get

$$T_f(r, x) = \frac{2P_{ku} q_s}{A_u k_f}\left(\frac{r^2}{4} - \frac{r^4}{4D^2}\right) + a_1 \ln r + a_2.$$

Using the continuity of temperature at the tube-wall surface, we have $T_f(r, x) = T_s(x)$ at $r = D/2$. Also since at $r = 0$ the fluid temperature has to be finite, we have $a_1 = 0$. Then solving for a_2, we have

$$\begin{aligned}a_2 &= T_s(x) - \frac{2P_{ku} q_s}{A_u k_f}\frac{3D^2}{64}, \quad P_{ku} = \pi D, \quad A_u = \pi D^2/4 \\ &= T_s(x) - \frac{3}{8}\frac{q_s D}{k_f}.\end{aligned}$$

Then

$$T_f(r, x) = T_s(x) - \frac{2q_s D}{k_f}\left[\frac{13}{6} + \frac{1}{16}\left(\frac{2r}{D}\right)^4 - \frac{1}{4}\left(\frac{2r}{D}\right)^2\right].$$

(c) The velocity-area average fluid temperature $\langle T_f \rangle$ is defined by (7.6), and upon using $T_f(r, x)$ and $u_f(r)$, we have

$$\begin{aligned}\langle T_f \rangle(x) &= \frac{1}{A_u \langle u_f \rangle}\int_o^r T_f(r, x) u_f(r) 2\pi r dr \\ &= T_s(x) - \frac{11}{48}\frac{q_s D}{k_f}\end{aligned}$$

or

$$T_s(x) - \langle T_f \rangle(x) = \frac{11}{48}\frac{q_s D}{k_f}.$$

(d) From the definition of the Nusselt number, given by (7.9), and restated as

$$\langle \mathrm{Nu} \rangle_D = \frac{q_s D}{[T_s(x) - \langle T_f \rangle(x)] k_f},$$

we have

$$\langle \mathrm{Nu} \rangle_D = \frac{48}{11} = 4.36 \quad \text{for uniform } q_s.$$

as listed in Table 7.2.

COMMENT:

Note that although $T_f(r, x)$, $\langle T_f \rangle(x)$ and $T_s(x)$ all change with respect to x, the dimensionless temperature $T^* \equiv (T_s - T_f)/(T_s - \langle T_f \rangle)$ is independent of x for a fully-developed thermal boundary layer.

PROBLEM 7.6.FAM

GIVEN:

A rectangular channel used for heating a nitrogen stream, as shown in Figure Pr.7.6(a), is internally finned to decrease the average convection resistance $\langle R_u \rangle_L$. The channel wall is at temperature T_s and nitrogen gas enters at a velocity $\langle u_f \rangle$ and temperature $\langle T_f \rangle_0$. The flow is turbulent (so the general hydraulic-diameter based Nusselt number of Table 7.3 can be used). The channel wall and the six fins are made of pure aluminum. Assume the same $\langle \mathrm{Nu} \rangle_{D,h}$ for channel wall and fin surfaces and assume a fin efficiency $\eta_f = 1$.

$\langle u_f \rangle = 25$ m/s, $\langle T_f \rangle_0 = -90$°C, $T_s = 4$°C, $a = 20$ mm, $w = 8$ mm, $L = 20$ cm, $L_f = 4$ mm, $l = 1$ mm.

Determine the nitrogen properties at $T = 250$ K.

SKETCH:

Figure Pr.7.6(a) shows the finned channel that heats a cold nitrogen stream.

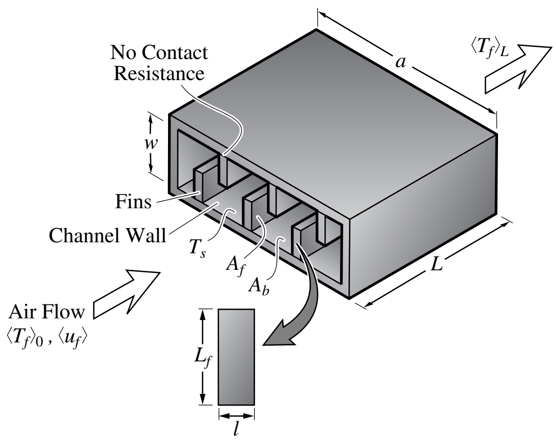


Figure Pr.7.6(a) Heat transfer between a bounded fluid stream and channel wall with extended surface.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) For the conditions given below, determine the exit temperature $\langle T_f \rangle_L$.

(c) Determine the heat flow rate $\langle Q_u \rangle_{L\text{-}0}$ for the same conditions.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.7.6(b). The surface-convection surface area A_{ku} is the sum of the bare surface area A_b and the product of the fin surface area A_f and the fin efficiency η_f.

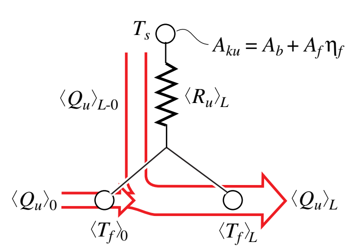


Figure Pr.7.6(b) Thermal circuit diagram.

(b) The fluid exit temperature is determined from (7.22), i.e.,

$$\frac{\langle T_f \rangle_0 - \langle T_f \rangle_L}{\langle T_f \rangle_0 - T_s} = 1 - e^{-NTU},$$

where from (7.20) and using (7.50), we have

$$\begin{aligned} NTU &= \frac{1}{(\dot{M}c_p)_f \langle R_{ku}\rangle_D} = \frac{A_{ku}\langle \mathrm{Nu}\rangle_{D,h} k_f}{(\dot{M}c_p)_f D_h} \\ A_{ku} &= A_b + A_f \eta_f . \end{aligned}$$

The hydraulic diameter D_h is defined by (7.40), i.e.,

$$D_h = \frac{4A_u}{P_{ku}},$$

where P_{ku} is the perimeter for the surface convection. Here

$$\begin{aligned} A_b &= [2(w+a) - 6l]L \\ A_f &= [6(2L_f + l)]L \\ A_u &= wa - 6lL_f \\ P_{ku} &= 2(w+a) + 6 \times 2L_f \\ D_h &= \frac{4(wa - 6lL_f)}{2w + 2a + 6 \times 2L_f}. \end{aligned}$$

We now determine the Nusselt Number. From Table 7.3 for turbulent flow, we have

$$\langle \mathrm{Nu}\rangle_{D,h} = 0.023 \mathrm{Re}_{D,h}^{4/5}\ \mathrm{Pr}^{0.4},$$

where we have used the condition $T_s < \langle T_f\rangle$, and the Reynolds number is

$$\mathrm{Re}_{D,h} = \frac{\langle u_f\rangle D_h}{\nu_f}.$$

From Table C.22 for nitrogen, at $T = 250$ K, we have for nitrogen

$$\begin{aligned} \nu_f &= 1.13 \times 10^{-5}\ \mathrm{m^2/s} && \text{Table C.22} \\ k_f &= 0.0234\ \mathrm{W/m\text{-}K} && \text{Table C.22} \\ c_{p,f} &= 1044\ \mathrm{J/kg\text{-}K} && \text{Table C.22} \\ \rho_f &= 1.366\ \mathrm{kg/m^3} && \text{Table C.22.} \end{aligned}$$

Using the numerical values, we have

$$\begin{aligned} D_h &= \frac{4 \times (0.008 \times 0.02 - 6 \times 0.001 \times 0.004)(\mathrm{m^2})}{2 \times (0.008 + 0.02)(\mathrm{m}) + 6 \times 2 \times 0.004(\mathrm{m})} \\ &= \frac{5.440 \times 10^{-4}(\mathrm{m^2})}{0.1040(\mathrm{m})} \\ &= 5.231 \times 10^{-3}\ \mathrm{m} \\ \mathrm{Re}_{D,h} &= \frac{25(\mathrm{m/s}) \times 5.231 \times 10^{-3}(\mathrm{m})}{1.13 \times 10^{-5}(\mathrm{m^2/s})} \\ &= 1.157 \times 10^4 > \mathrm{Re}_{D,t} = 2{,}300 \quad \text{turbulent regime flow.} \end{aligned}$$

Then correlation given above for $\langle \mathrm{Nu}\rangle_{D,h}$ is applicable, and

$$\langle \mathrm{Nu}\rangle_{D,h} = 0.023 \times (1.157 \times 10^4)^{4/5} \times (0.69)^{0.4} = 35.32.$$

Then

$$
\begin{aligned}
A_u &= (0.008 \times 0.02)(\text{m}^2) - 6 \times 0.001 \times 0.004(\text{m}^2) \\
&= 1.360 \times 10^{-4}\ \text{m}^2 \\
\dot{M}_f &= \rho_f \langle u_f \rangle A_u = 1.366(\text{kg/m}^3) \times 25(\text{m/s}) \times 1.360 \times 10^{-4}(\text{m}^2) \\
&= 4.644 \times 10^{-3}\ \text{kg/s} \\
A_b &= 2[0.008 + 0.02 - 3 \times 0.001](\text{m}) \times 0.2(\text{m}) = 0.01\ \text{m}^2 \\
A_f &= 6[2 \times (4 \times 10^{-3}) + 0.001](\text{m}) \times 0.2(\text{m}) = 0.0108\ \text{m}^2 \\
A_{ku} &= A_b + A_f \eta_f = 0.01(\text{m}^2) + 0.0108(\text{m}^2) \times 1.0 \\
&= 0.02080\ \text{m}^2 \\
NTU &= \frac{0.0208(\text{m}^2) \times 35.32 \times 0.0234(\text{W/m-K})}{4.644 \times 10^{-3}(\text{kg/m}^3) \times 1{,}044(\text{J/kg-K}) \times 5.231 \times 10^{-3}(\text{m})} \\
&= 0.6778.
\end{aligned}
$$

The exit temperature is

$$
\begin{aligned}
\langle T_f \rangle_L &= \langle T_f \rangle_0 + (T_s - \langle T_f \rangle_0)(1 - e^{-NTU}) \\
&= -90(^\circ\text{C}) + [4 - (-90)](^\circ\text{C}) \times (1 - e^{-0.6778}) = -43.73^\circ\text{C}.
\end{aligned}
$$

(c) The heat flow rate is given by (7.24), i.e.,

$$
\begin{aligned}
\langle Q_u \rangle_{L\text{-}0} &= (\dot{M}c_p)_f (\langle T_f \rangle_L - \langle T_f \rangle_0) \\
&= 4.644 \times 10^{-3}(\text{kg/s}) \times 1{,}044(\text{J/kg-K}) \times [-43.73 - (-90)](\text{K}) = 224.4\ \text{W}.
\end{aligned}
$$

COMMENT:

This is a small NTU (much less than 4), indicating that the nitrogen flow rate is possibly too high for this finned channel and that more fins should be used to increase A_{ku} and decrease D_h.

PROBLEM 7.7.FUN

GIVEN:

Many small devices, such as computer components, can malfunction when exposed to high temperatures. Fans and other such devices are used to cool these components constantly. To aid in the cooling process, extremely small heat exchangers are integrated with the parts being cooled. These heat exchangers are microfabricated through a co-extrusion process such that complex cross-sectional geometries can be formed using densely packed arrays. An example is shown in Figure Pr.7.7(a). A compressor is used to force air at temperature $\langle T_f \rangle_0 = 20°C$ and velocity $\langle u_f \rangle = 5$ m/s through a pure copper channel having a surface temperature $T_s = 90°C$, and the dimensions are as shown in Figure Pr.7.7.

Evaluate the air properties at $T = 300$ K.

SKETCH:

Figure Pr.7.7(a) shows the small cross section channel, with internal fins, used for heat removal from high heat flux surfaces.

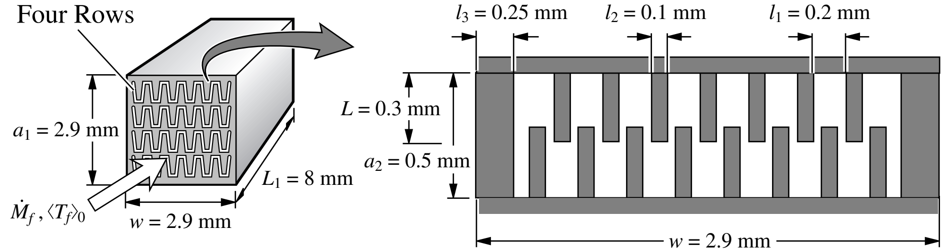


Figure Pr.7.7(a) (i) A small heat exchanger. (ii) Cross section of each of the four rows.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Assuming the flow can be approximated as flow through parallel plates, in which $D_h = 2l_1$ and $l_1 = 0.2$ mm, determine the heat transfer $\langle Q_u \rangle_{L\text{-}0}$ from the copper heat exchanger to the passing fluid.

(c) Now treating the flow as flow through a packed bed of particles, determine the heat transfer $\langle Q_u \rangle_{L\text{-}0}$ from the copper surface to the passing fluid. Comment on how the predicted Nusselt numbers obtained from the two treatments differ.

SOLUTION:

(a) The thermal circuit diagram for this channel flow is shown in Figure Pr.7.7(b).

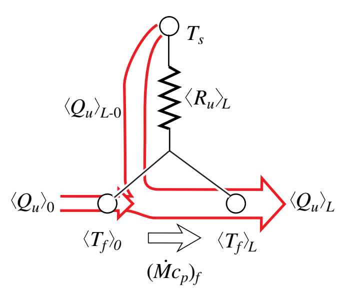


Figure Pr.7.7(b) Thermal circuit diagram.

(b) Properties (air, $T = 300$ K, Table C.22): $\rho_f = 1.177$ kg/m^3, $c_{p,f} = 1{,}005$ J/kg-K, $k_f = 0.0267$ W/m-K, Pr $= 0.69$, $\nu_f = 15.66 \times 10^{-6}$ m^2/s; (pure copper, Table C.16) $k_s = 401$ W/m-K.

The surface-convection heat transfer $\langle Q_u \rangle_{L\text{-}0}$ (W) is given by (7.32), i.e.,

$$\langle Q_u \rangle_{L\text{-}0} = \frac{T_s - \langle T_f \rangle_0}{\langle R_u \rangle_L},$$

where $\langle R_u \rangle_L$ is given by (7.27) as

$$\langle R_u \rangle_L = \frac{1}{\dot{M}_f \times c_{p,f}(1 - e^{-NTU})},$$

where

$$\begin{aligned}
\dot{M}_f &= A_u \times \rho_f \times \langle u_f \rangle \\
A_u|_{row} &= 2A_{u,1} + 15A_{u,2} + 14A_{u,3} \\
&= 2(l_2 \times a_2) + 15 \times \{l_1 \times [L - 1(\text{mm})]\} + 14 \times \{0.05 \times [L - 2(\text{mm})]\} \\
&= 2(0.1 \times 0.5)(\text{mm}^2) + 15 \times (0.2 \times 0.2)(\text{m}^2) + 14 \times (0.05 \times 0.1)(\text{mm}^2) \\
&= 7.7 \times 10^{-1}\ \text{mm}^2 = 7.7 \times 10^{-7}\ \text{m}^2 \\
A_u &= 7.7 \times 10^{-7}(\text{m}^2) \times 4(\text{rows}) = 3.08 \times 10^{-6}\ \text{m}^2 \\
\dot{M}_f &= 3.08 \times 10^{-6}(\text{m}^2) \times 1.177(\text{kg/m}^3) \times 5(\text{m/s}) = 1.813 \times 10^{-5}\ \text{kg/s} \\
NTU &= \frac{A_{ku}\langle \text{Nu} \rangle_{D_h} \dfrac{k_f}{D_h}}{\dot{M}_f \times c_{p,f}}
\end{aligned}$$

The areas $A_{u,1}, A_{u,2}$ and $A_{u,3}$ are shown in Figure Pr.7.7(c).

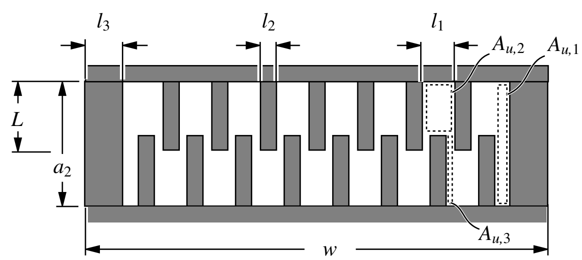


Figure Pr.7.7(c) Divisions of A_u.

From Table 7.2, for laminar flow (it will be shown that $\text{Re}_{D,h} < 2{,}300$), we have

$$\begin{aligned}
\langle \text{Nu} \rangle_{D_h} &= 7.54, \quad \text{for parallel plate flow, } a/b \to \infty \\
D_h &= 2l_1 = 0.4\ \text{mm}.
\end{aligned}$$

Determining A_{ku} from Figure Pr.7.7(a), we have

$$\begin{aligned}
A_{ku} &= A_b + A_f \eta_f \\
A_b|_{row} &= 2(a_2 \times L_1) + 15(l_1 \times L_1) + 2(l_2 \times L_1) \\
&= 2 \times [0.5(\text{mm}) \times 8(\text{mm})] + 15 \times [0.2(\text{mm}) \times 8(\text{mm})] + 2 \times [0.1(\text{mm}) \times 8(\text{mm})] \\
&= 8(\text{mm}^2) + 24(\text{mm}^2) + 1.6(\text{mm}^2) = 33.6\ \text{mm}^2 = 3.36 \times 10^{-5}\ \text{m}^2 \\
A_b &= 3.36 \times \text{m}^2 \times 4(\text{rows}) = 1.344 \times 10^{-4}\ \text{m}^2.
\end{aligned}$$

From (6.140), the corrected length L_c and the extended area are

$$\begin{aligned}
L_c &= L + \frac{l_2}{2} = 0.3(\text{mm}) + \frac{0.1}{2}(\text{mm}) = 0.35\ \text{mm} = 3.5 \times 10^{-4}\ \text{m} \\
A_f|_{row} &= 15(2L_c L_1) + 30(l_2 \times L_c) \\
&= 84(\text{mm}^2) + 1.05(\text{mm}^2) = 85.05\ \text{mm}^2 = 8.505 \times 10^{-5}\ \text{m}^2 \\
A_f &= 8.505 \times 10^{-5}(\text{m}^2) \times 4(\text{rows}) = 3.402 \times 10^{-4}\ \text{mm}^2.
\end{aligned}$$

The fin efficiency is given by (6.147), i.e.,

$$\begin{aligned}
\eta_f &= \frac{\tanh(mh_c)}{mL_c} \\
m &= \left[\frac{P_{ku}\langle \mathrm{Nu}\rangle_{D_h} \times k_f}{A_k \times k_s \times L_1}\right]^{1/2} \\
L_1 &= 0.008 \text{ m} \\
P_{ku} &= 2(l_2 + L_1) = 2 \times (0.1 + 8)(\text{mm}) = 0.162 \text{ m} \\
A_k &= l_2 L_1 = 0.1 \times 8(\text{mm})^2 = 8 \times 10^{-7} \text{ m}^2 \\
m &= \left[\frac{0.0162(\text{m}) \times 7.54 \times 0.0267(\text{W/m-K})}{8 \times 10^{-7}(\text{m}) \times 401(\text{W/m-K}) \times 0.004}\right]^{1/2} = 50.414 \text{ 1/m} \\
\eta_f &= \frac{\tanh[50.414(1/\text{m}) \times 3.5 \times 10^{-4}(\text{m})]}{50.414(1/\text{m}) \times 3.5 \times 10^{-4}(\text{m})} = 0.9999 \simeq 1.
\end{aligned}$$

Then, with $\eta_f = 1$,

$$\begin{aligned}
A_{ku} &= A_b + A_f \\
&= 1.344 \times 10^{-4}(\text{m}^2) + 3.402 \times 10^{-4}(\text{m}^2) = 4.746 \times 10^{-4} \text{ m}^2 \\
NTU &= \frac{4.75 \times 10^{-4}(\text{m}^2) \times 7.54 \times \dfrac{0.0267(\text{W/m-K})}{4 \times 10^{-4}(\text{m})}}{1.813 \times 10^{-5}(\text{kg/s}) \times 1{,}005(\text{J/kg-K})} = 13.12 \\
\langle R_u\rangle_L &= \frac{1}{(1.813 \times 10^{-5}(\text{kg/s}) \times 1{,}005(\text{J/kg-K})) \times (1 - e^{-13.12})} = 54.88 \text{ K/W} \\
\langle Q_u\rangle_{L\text{-}0} &= \frac{T_s - \langle T_f\rangle_0}{\langle R_u\rangle_L} = \frac{(90-20)(\text{K})}{54.88(\text{K/W})} = 1.276 \text{ W}.
\end{aligned}$$

(c) Now, treating the bounded solid as a packed bed of particle, from Table 7.5, we have

$$D_p = \frac{6V_s}{A_{ku}}, \ \mathrm{Re}_{D,p} = \frac{\langle u_f\rangle D_p}{\nu_f(1-\epsilon)}, \ \langle \mathrm{Nu}\rangle_{D,p} = 2 + (0.4\mathrm{Re}_{D,p}^{1/2} + 0.2\mathrm{Re}_{D,p}^{2/3})\mathrm{Pr}^{0.4} \quad \text{Table 7.5}$$

where

$$\begin{aligned}
V &= a_1 w L_1 = 2.9(\text{mm}) \times 2.9(\text{mm}) \times 8(\text{mm}) = 67.28(\text{mm})^3 \\
V_s &= V - A_u L_1 \\
&= a_1 w L_1 - A_u L_1 \\
&= 2.9(\text{mm}) \times 2.9(\text{mm}) \times 8(\text{mm}) - [3.08(\text{mm}^2) \times 8(\text{mm})] \\
&= 42.64 \text{ mm}^3 \\
D_p &= \frac{6V_s}{A_{ku}} = \frac{6 \times 42.64(\text{mm}^3)}{474.6(\text{mm}^2)} = 0.5391 \text{ mm} \\
\epsilon &= \frac{V_f}{V} = \frac{V - V_s}{V} = \frac{(67.28 - 42.64)(\text{mm}^2)}{67.28(\text{mm}^3)} = 0.3662 \\
\mathrm{Re}_{D,p} &= \frac{\langle u_f\rangle D_p}{\nu_f(1-\epsilon)} = \frac{5(\text{m/s}) \times 5.391 \times 10^{-4}(\text{m})}{15.66 \times 10^{-6}(\text{m}^2/\text{s}) \times (1 - 0.3662)} = 271.6 \\
\langle \mathrm{Nu}\rangle_{D,p} &= (2 + 0.4 \times (271.6)^{1/2} + 0.2 \times (271.6)^{2/3}) \times (0.69)^{0.4} = 14.64 \\
NTU &= \frac{A_{ku}\langle \mathrm{Nu}\rangle_{D,p}\dfrac{k_f}{D_p}}{(\dot{M}c_p)_f} = \frac{4.746 \times 10^{-4}(\text{m}^2) \times 14.64 \times \dfrac{0.0267(\text{W/m-K})}{5.391 \times 10^{-4}(\text{m})}}{1.813 \times 10^{-5}(\text{kg/s}) \times 1{,}005(\text{J/kg-K})} = 18.89 \\
\langle R_u\rangle_L &= \frac{1}{(\dot{M}c_p)_f(1 - e^{-NTU})} = \frac{1}{1.813 \times 10^{-5}(\text{kg/s}) \times 1{,}005(\text{J/kg-K}) \times 1} = 54.88 \text{ K/W} \\
\langle Q_u\rangle_{L\text{-}0} &= \frac{T_s - \langle T_f\rangle_0}{\langle R_u\rangle_L} = \frac{(90-20)(\text{K})}{54.88(\text{K/W})} = 1.276 \text{ W}.
\end{aligned}$$

COMMENT:

In order to use the laminar, parallel-plate Nusselt number correlation, it must be shown that the Reynolds number is less than 2,300. Here we have

$$\mathrm{Re}_{D,h} = \frac{\langle u_f \rangle D_h}{\nu} = \frac{5(\mathrm{m/s}) \times 2 \times 10^{-4}(\mathrm{m})}{15.66 \times 10^{-6}(\mathrm{m^2/s})} = 63.86.$$

Note that the two different Nusselt numbers have lead to the same heat flow rate, because of the large NTU obtained for both cases.

PROBLEM 7.8.FAM

GIVEN:

In a research nuclear (fission) reactor, a 17 channel element core, with each element being a rectangular cylinder of cross-sectional area $a \times w$ and length L, is used. This is shown in Figure Pr.7.8(a).

The nuclear fission energy conversion rate $\dot{S}_{r,fi}$ occurs in the channel walls. The coolant flow rate $\dot{M}_f$ per channel is designed for a desired channel wall temperature T_s. When for some reason, this flow rate is reduced, T_s can raise to hazardous levels.

$L = 100$ cm, $a = 2.921$ cm, $w = 7.5$ cm, $\dot{S}_{r,fi} = 4.5$ kW, $\dot{M}_f = 0.15$ kg/s, $\langle T_f \rangle_0 = 45$°C.

Determine the water properties at $T = 310$ K.

SKETCH:

Figure Pr.7.8(a) shows the cross section of the channels with energy conversion (nuclear fission) in the channel wall.

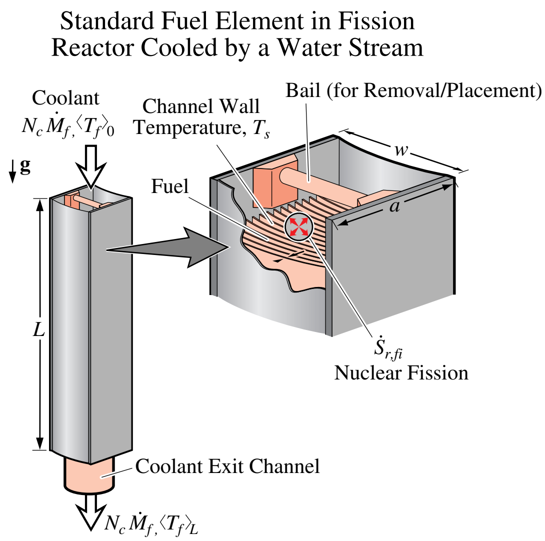


Figure Pr.7.8(a) A nuclear (fission) reactor cooled by a bounded water stream.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the fluid exit temperature $\langle T_f \rangle_L$ and the channel surface temperature T_s for the conditions given below.

(c) If the mass flow rate is reduced by one half, what will $\langle T_f \rangle_L$ and T_s be? Since $\langle T_f \rangle_0$ is below T_{lg}, for $T_s > T_{lg}$ any bubble formed will collapse as it departs from the surface (this is called subcooled boiling). Comment on how bubble nucleation affects T_s.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.7.8(b).

(b) From Figure Pr.7.8(b), the energy equation (2.9) for the T_s node becomes

$$Q|_{A,s} = \langle Q_u \rangle_{L\text{-}0} = \dot{S}_{r,fi},$$

where from (7.25), we have

$$\langle Q_u \rangle_{L\text{-}0} = \frac{T_s - \langle T_f \rangle_0}{\langle R_u \rangle_L}.$$

Solving for T_s, we have

$$T_s = \langle T_f \rangle_0 + \dot{S}_{r,fi} \langle R_u \rangle_L.$$

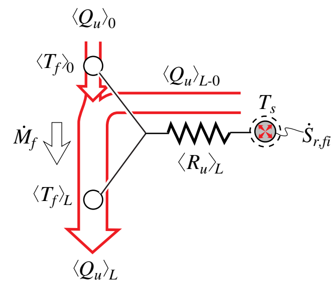


Figure Pr.7.8(b) Thermal circuit diagram.

The average convection resistance $\langle R_u\rangle_L$ is given by (7.27), i.e.,

$$\langle R_u\rangle_L = \frac{1}{(\dot{M}c_p)_f(1-e^{-NTU})},$$

where from (7.20) we have

$$NTU = \frac{A_{ku}\langle \text{Nu}\rangle_{D,h}k_f/D_h}{(\dot{M}c_p)_f}.$$

From the geometry,

$$A_{ku} = 2(a+w) = 2\times(0.02921+0.075)(\text{m}) = 0.2084\ \text{m}.$$

The Nusselt number is found from Table 7.2 or 7.3 and depends on the magnitude of the Reynolds number $\text{Re}_{D,h}$, where D_h is the hydraulic diameter. From (7.40), we have

$$\begin{aligned} D_h &= \frac{4A_u}{P_{ku}} = \frac{4aw}{4(a+w)} = \frac{2\times 0.02921(\text{m})\times 0.075(\text{m})}{(0.02921+0.075)\text{m}} \\ &= 0.04204\ \text{m} \end{aligned}$$

From Table C.23, we have for water at $T = 310$ K

$$\begin{aligned} k_f &= 0.623\ \text{W/m-K} && \text{Table C.23} \\ \rho_f &= 995.3\ \text{kg/m}^3 && \text{Table C.23} \\ c_{p,f} &= 4{,}178\ \text{J/kg-K} && \text{Table C.23} \\ \nu_f &= 7.11\times 10^{-7}\ \text{m}^2/\text{s} && \text{Table C.23} \\ \text{Pr} &= 4.74 && \text{Table C.23.} \end{aligned}$$

Using $\dot{M}_f = A_u\rho_f\langle u_f\rangle$, we write for $\text{Re}_{D,h}$,

$$\text{Re}_{D,h} = \frac{\rho_f\langle u_f\rangle D_h}{\mu_f} = \frac{\dot{M}_f D_h}{A_u\mu_f} = \frac{\dot{M}_f D_h}{A_u\rho_f\nu_f}.$$

Then using the numerical values, we have

$$\begin{aligned} \text{Re}_{D,h} &= \frac{0.15(\text{kg/s})\times 0.04204(\text{m})}{0.02921\times 0.075(\text{m}^2)\times 995.3(\text{kg/s})\times 7.11\times 10^{-7}(\text{m}^2/\text{s})} \\ &= 4{,}608 > (\text{Re}_{D,h})_t = 2{,}300 \quad \text{assume fully turbulent flow regime.} \end{aligned}$$

For the fully turbulent flow regime, we have from Table 7.3, we have

$$\langle \text{Nu}\rangle_{D,h} = 0.023\text{Re}^{0.8}\text{Pr}^{0.4} \qquad \text{Table 7.3,}$$

where for $T_s > \langle T_f \rangle_0$, we use $n = 0.4$. Then

$$\langle \mathrm{Nu} \rangle_{D,h} = 0.023 \times (4{,}068)^{0.8} \times (4.74)^{0.4} = 33.08.$$

Next using $A_{ku} = 2 \times (a + w)L = 0.2084$ m^2, we have

$$NTU = \frac{0.2084(\mathrm{m}^2) \times 33.08 \times 0.623(\mathrm{W/m\text{-}K})}{0.15(\mathrm{kg/s}) \times 4{,}178(\mathrm{J/kg\text{-}K}) \times 0.04204(\mathrm{m})} = 0.1630.$$

Next,

$$\begin{aligned}
\langle R_u \rangle_L &= \frac{1}{(\dot{M}c_p)_f(1 - e^{-NTU})} \\
&= \frac{1}{0.15(\mathrm{kg/s}) \times 4{,}178(\mathrm{J/kg\text{-}K}) \times (1 - e^{-0.1630})} = 1.061 \times 10^{-2}\mathrm{^\circ C/W}.
\end{aligned}$$

Now for $\langle T_f \rangle_L$ we use (7.24), i.e.,

$$\langle Q_u \rangle_{L\text{-}0} = (\dot{M}c_p)_f(\langle T_f \rangle_L - \langle T_f \rangle_0) = \dot{S}_{r,fi}$$

or

$$\begin{aligned}
\langle T_f \rangle_L &= \langle T_f \rangle_0 + \frac{\dot{S}_{r,f,i}}{(\dot{M}c_p)_f} \\
&= 45(\mathrm{^\circ C}) + \frac{4.5 \times 10^3(\mathrm{W})}{0.15(\mathrm{kg/s}) \times 4{,}178(\mathrm{J/kg\text{-}K})} \\
&= 45(\mathrm{^\circ C}) + 7.180(\mathrm{^\circ C}) = 52.18\mathrm{^\circ C}.
\end{aligned}$$

Solving for T_s, we have

$$\begin{aligned}
T_s &= 45(\mathrm{^\circ C}) + 4.5 \times 10^3(\mathrm{W}) \times 1.061 \times 10^{-2}(\mathrm{^\circ C/W}) \\
&= 45(\mathrm{^\circ C}) + 47.74(\mathrm{^\circ C}) \\
&= 92.74\mathrm{^\circ C} < T_{lg} = 100\mathrm{^\circ C} \qquad \text{Table C.27.}
\end{aligned}$$

(c) For $\dot{M}_f = 0.5 \times 0.15(\mathrm{kg/s}) = 0.075$ kg/s, we repeat the calculations, starting from

$$\mathrm{Re}_{D,h} = 2{,}034 < 2{,}300 \qquad \text{laminar flow regime.}$$

From Table 7.2, for $w/a = 2.58$, we have

$$\langle \mathrm{Nu} \rangle_{D,h} = 4.202.$$

Then

$$\begin{aligned}
NTU &= \frac{0.2084(\mathrm{m}^2) \times 4.202 \times 0.623(\mathrm{W/m\text{-}K})}{0.075(\mathrm{kg/s}) \times 4{,}178(\mathrm{J/kg\text{-}K}) \times 0.04204(\mathrm{m})} = 0.04141 \\
\langle R_u \rangle_L &= \frac{1}{0.075(\mathrm{kg/s}) \times 4{,}178(\mathrm{J/kg\text{-}K}) \times (1 - e^{-0.04141})} = 7.867 \times 10^{-2}\mathrm{^\circ C/W}.
\end{aligned}$$

Finally

$$\begin{aligned}
\langle T_f \rangle_L &= 45(\mathrm{^\circ C}) + \frac{4.5 \times 10^3(\mathrm{W})}{0.075(\mathrm{kg/s}) \times 4{,}178(\mathrm{J/kg\text{-}K})} \\
&= 45(\mathrm{^\circ C}) + 14.36(\mathrm{^\circ C}) = 59.36(\mathrm{^\circ C}) \\
T_s &= 45(\mathrm{^\circ C}) + 4.5 \times 10^3(\mathrm{W}) \times 7.867 \times 10^{-2}(\mathrm{^\circ C/W}) \\
&= 45(\mathrm{^\circ C}) + 354.0(\mathrm{^\circ C}) = 399.0\mathrm{^\circ C} > T_{lg} = 100\mathrm{^\circ C} \qquad \text{Table C.27}
\end{aligned}$$

COMMENT:

The lower flow rate results in laminar flow, which has a much lower Nusselt number. The small $\dot{M}_f$ and $\langle \mathrm{Nu} \rangle_{D,h}$ result in a resistance which is several time larger. In this case, then the water will boil. The boiling in turn will increase $\langle \mathrm{Nu} \rangle_{D,h}$ and decrease T_s. As long as $\langle T_f \rangle_L < T_{lg}$, it is possible to collapse the bubbles in this subcooled liquid. However, if special care is not taken, the vapor can block the water flow and cause a very large T_s resulting in a meltdown.

PROBLEM 7.9.FAM

GIVEN:

The surface-convection heat transfer from an automobile brake is by the flow induced by the rotor rotation. There is surface-convection heat transfer from the outside surfaces of the rotor and also from the vane between the two rotor surfaces (called the ventilation area or vent), as shown in Figure Pr.7.9(a).

The air flow rate through the vent is given by an empirical relation for the average velocity $\langle u_f \rangle_1$ as

$$\langle u_f \rangle (\text{m/s}) = rpm \times 0.0316(R_2^2 - R_1^2)^{1/2},$$

where R_2(m) and R_1(m) are outer and inner radii shown in Figure Pr.7.9(a). The air enters the vent at temperature $\langle T_f \rangle_0$ and the rotor is at a uniform temperature T_s. Assume a uniform flow cross-sectional area (although in practice it is tapered) $A_u = N_c a \times w$ and a total vent surface-convection area A_{ku}. Here a and w are for the rectangular cross section of each channel in the N_c vents.

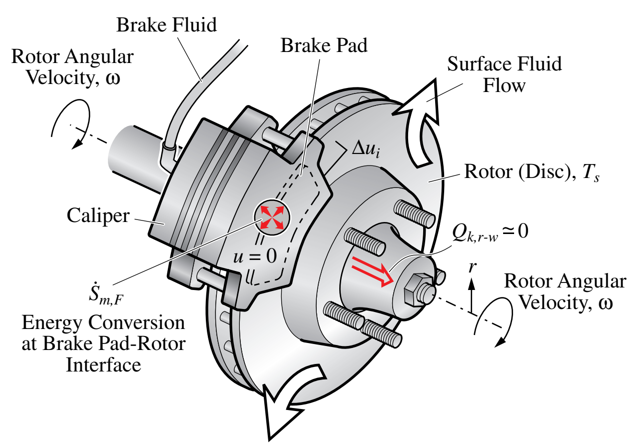
a = 0.5 cm, w = 1.5 cm, A_{ku} = 700 cm^2, rpm = 750, N_c = 36, R_2 = 10 cm, R_1 = 15 cm, $\langle T_f \rangle_0$ = 20°C, T_s = 400°C.

Evaluate air properties at T = 300 K.

SKETCH:

Figure Pr.7.9(a) shows the ventilated automobile brake with the rotation-induced vent air flow. The vent geometry is also shown.

(i) Automobile Disc-Brake: Physical Model



(ii) Rotation-Induced Flow Through Vanes (Ventilated Disc)

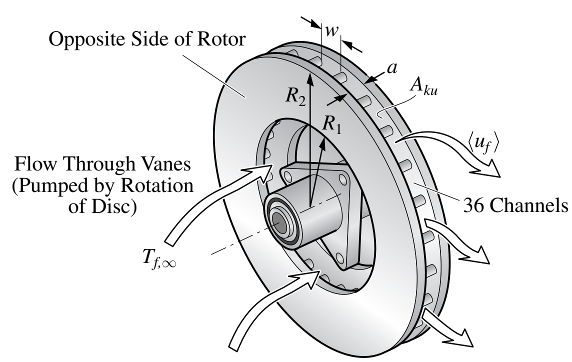


Figure Pr.7.9(a) A ventilated automobile brake with rotation induced vent air flow. (i) Physical model. (ii) Rotation-induced flow.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the vent surface heat transfer rate $\langle Q_u \rangle_{L\text{-}0}$.
(c) Determine the air exit temperature $\langle T_f \rangle_L$.

SOLUTION:
(a) The thermal circuit diagram is shown in Figure Pr.7.9(b). All other heat transfer mechanisms from the rotor are shown as Q_s.

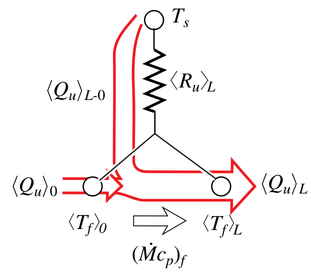


Figure Pr.7.9(b) Thermal circuit diagram.

(b) The rate of heat transfer is given by (7.25), (7.27), and (7.20), i.e.,

$$\langle Q_u \rangle_{L\text{-}0} = \frac{T_s - \langle T_f \rangle_0}{\langle R_u \rangle_L}$$

$$\langle R_u \rangle_L = \frac{1}{(\dot{M}c_p)_f (1 - e^{-NTU})}$$

$$\dot{M}_f = \rho_f A_u \langle u_f \rangle$$

$$NTU = \frac{1}{\langle R_u \rangle_D (\dot{M}c_p)_f}.$$

The surface convection resistance $\langle R_{ku} \rangle_D$ is determined for a rectangular cross-section channel from Table 7.2 or Table 7.3, depending on the Reynolds number $\mathrm{Re}_{D,h}$, where D_h is the hydraulic diameter. The Reynolds number and the hydraulic diameter are defined by (7.36) and (7.40), i.e,

$$\mathrm{Re}_{D,h} = \frac{\langle u_f \rangle D_h}{\nu_f}$$

$$D_h = \frac{4A_u}{P_{ku}} = \frac{4 \times a \times w}{2(a+w)} = \frac{2aw}{a+w}$$

$$= \frac{2 \times (0.005 \times 0.015)(\mathrm{m}^2)}{(0.005 + 0.015)(\mathrm{m})} = 0.00750 \text{ m}.$$

From Table C.22, for air at $T = 300$ K, we have

$k_f = 0.0267$ W/m-K Table C.22
$\rho_f = 1.177$ kg/m^3 Table C.22
$\nu_f = 1.566 \times 10^{-5}$ m^2/s Table C.22
$c_{p,f} = 1{,}005$ J/kg-K Table C.22
$\mathrm{Pr} = 0.69$ Table C.22

Then, from the given correlation,

$$\langle u_f \rangle = 750 \times 0.0316(0.15^2 - 0.10^2)^{1/2}(\mathrm{m/s})$$

$$= 2.650 \text{ m/s}$$

$$\mathrm{Re}_{D,h} = \frac{2.650(\mathrm{m/s}) \times 0.00750(\mathrm{m})}{1.566 \times 10^{-5}(\mathrm{m}^2/\mathrm{s})}$$

$$= 1{,}269 < (\mathrm{Re}_{D,L})_t = 2{,}300 \quad \text{laminar flow regime.}$$

We now to check to see if the flow is fully developed and choose an appropriate Nusselt number correlation. From Table 7.2, we have

$$\begin{aligned}\frac{L/D_h}{\mathrm{Pe}_{D,h}} &= \frac{L/D_h}{\mathrm{Pe}_{D,h}\mathrm{Pr}}\\ &= \frac{(R_2-R_1)/D_h}{\mathrm{Re}_{D,h}\mathrm{Pr}}\\ &= \frac{(0.15-0.10)(\mathrm{m})/0.00750(\mathrm{m})}{1{,}269\times 0.69}\\ &= 7.614\times 10^{-3}\simeq 0.03.\end{aligned}$$

Then from Table 7.2, we have

$$\begin{aligned}\langle \mathrm{Nu}_{D,h}\rangle &= 2.409\left(\frac{L/D_h}{\mathrm{Pe}_{D,h}}\right)^{-1/3}-0.7\\ &= 2.409\times(7.614\times 10^{-3})^{-1/3}-0.7\\ &= 11.54.\end{aligned}$$

The mass flow rate is

$$\begin{aligned}\dot{M}_f &= \rho_f A_u\langle u_f\rangle = \rho_f\times 36aw\times\langle u_f\rangle\\ &= 1.177(\mathrm{kg/m^3})\times 36\times 0.005(\mathrm{m})\times 0.015(\mathrm{m})\times 2.650(\mathrm{m/s})\\ &= 8.421\times 10^{-3}\ \mathrm{kg/s}.\end{aligned}$$

Then

$$\begin{aligned}\langle R_{ku}\rangle_D &= \frac{D_h}{A_{ku}\langle \mathrm{Nu}\rangle_{D,h}k_f}\\ &= \frac{0.00750(\mathrm{m})}{700\times 10^{-4}(\mathrm{m}^2)\times 11.54\times 0.0267(\mathrm{W/m\text{-}K})} = 0.3477\ \mathrm{K/W}\end{aligned}$$

$$\begin{aligned}NTU &= \frac{1}{\langle R_{ku}\rangle_D(\dot{M}c_p)_f}\\ &= \frac{1}{0.3477(\mathrm{K/W})\times 8.421\times 10^{-3}(\mathrm{kg/s})\times 1{,}005(\mathrm{J/kg\text{-}K})} = 0.3398\\ \langle R_u\rangle_L &= \frac{1}{8.421\times 10^{-3}(\mathrm{kg/s})\times 1{,}005(\mathrm{J/kg\text{-}K})(1-e^{-0.3398})} = 0.4102\ \mathrm{K/W}\\ \langle Q_u\rangle_{L\text{-}0} &= \frac{(400-20)(\mathrm{K})}{0.4102(\mathrm{K/W})} = 926.5\ \mathrm{W}.\end{aligned}$$

(c) The exit temperature is found from (7.24), i.e.,

$$\begin{aligned}\langle T_f\rangle_L &= \langle T_f\rangle_0 + \frac{\langle Q_u\rangle_{L\text{-}0}}{(\dot{M}c_p)_f}\\ &= 20(^\circ\mathrm{C}) + \frac{926.5(\mathrm{W})}{8.421\times 10^{-3}(\mathrm{kg/s})\times 1{,}005(\mathrm{J/kg\text{-}K})}\\ &= 20(^\circ\mathrm{C}) + 109.48(^\circ\mathrm{C})\\ &= 129.48^\circ\mathrm{C}.\end{aligned}$$

COMMENT:

Due to the small mass flow rate, here NTU is large enough to cause a significant increase in the air temperature, i.e., a large $\langle T_f\rangle_L - \langle T_f\rangle_0$ is found.

PROBLEM 7.10.FAM

GIVEN:

A refrigerant R-134a liquid-vapor stream is condensed, while passing thorough a compact condenser tube, as shown in Figure Pr.7.10(a). The stream enters at a mass flow rate $\dot{M}_f$, thermodynamic quality x_0, and a temperature $\langle T_f \rangle_0(p_{g,0})$ and here for simplicity assume that the exit conditions are the same as the inlet conditions. The condenser wall is at temperature T_s. Assume that the liquid (condensate) flow regime is annular and use the applicable correlation given in Table 7.6.

$N_c = 10$, $a = 1.5$ mm, $L = 15$ cm, $p_{g,0} = 1.681$ MPa, $x_0 = 0.5$, $T_s = 58$℃, $\dot{M}_f = 10^{-3}$ kg/s.

Use the saturation properties of Table C.28 at $p_{g,0}$.

SKETCH:

Figure Pr.7.10(a) shows the compact condenser tube and its channels.

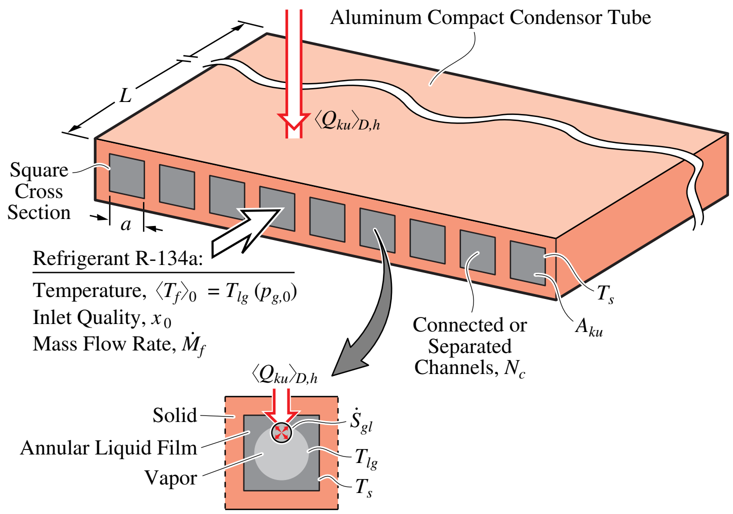


Figure Pr.7.10(a) A compact condenser tube is used to condense a refrigerant stream. The tube is a multichannel, extruded aluminum tube. There are N_c square cross-sectional channels in the tube.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Based on the constant x and $\langle T_f \rangle$ assumptions, determine the heat transfer rate, $\langle Q_{ku} \rangle_{D,L}$, for the conditions given below.

(c) From the results of (b), determine the condensation rate $\dot{M}_{gl}$ and estimate the exit quality x_L.

(d) Comment on how an iteration may be used to improve on the accuracy of these predictions.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.7.10(b).

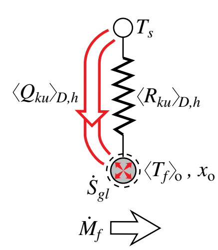


Figure Pr.7.10(b) Thermal circuit diagram.

Here we have assumed that $\langle T_f \rangle$ is constant and from (7.9), and using the hydraulic diameter D_h defined by

(7.40), we have

$$\langle Q_{ku}\rangle_{D,h} = A_{ku}\langle \text{Nu}\rangle_{D,h}\frac{k_l}{D_h}(T_s - \langle T_f\rangle_0).$$

Also, from Figure Pr.7.10(b), we have the energy equation

$$\begin{aligned} Q|_A &= -\langle Q_{ku}\rangle_{D,h} = \dot{S}_{gl} = -\dot{M}_{gl}\Delta h_{gl} \\ &= \dot{M}_{gl}\Delta h_{lg} \end{aligned}$$

(b) The Nusselt number is given in Table 7.6 as

$$\begin{aligned} \langle \text{Nu}\rangle_{D,h} &= a_1 \text{Re}_{l,eq}^{a_2}\text{Pr}^{1/3}, \\ \text{Re}_{l,eq} &= \frac{\dot{m}_f\left[(1-x) + x\left(\dfrac{\rho_l}{\rho_g}\right)^{1/2}\right]D_h}{\mu_l}, \end{aligned}$$

and a_1 and a_2 depend on the magnitude of $\text{Re}_{l,eq}$.

The saturation properties of R-134a from Table C.28 at $p_{g,0} = 1.681$ MPa, are

$$\begin{aligned} \rho_l &= 1{,}052 \text{ kg/m}^3 && \text{Table C.28} \\ \rho_g &= 87.26 \text{ kg/m}^3 && \text{Table C.28} \\ \Delta h_{lg} &= 1.386 \times 10^5 \text{ J/kg} && \text{Table C.28} \\ \mu_l &= 1.386 \times 10^{-4} \text{ Pa-s} && \text{Table C.28} \\ k_l &= 0.0658 \text{ W/m-K} && \text{Table C.28} \\ c_{p,l} &= 1{,}653 \text{ J/kg-K} && \text{Table C.28} \\ T_{lg} &= 333.2 \text{ K} = 60^\circ\text{C} && \text{Table C.28} \end{aligned}$$

$$\text{Pr} = \text{Pr}_l = \left(\frac{\mu c_p}{k}\right)_l = \frac{1.386\times10^{-4}(\text{Pa-s})\times 1{,}663(\text{J/kg-K})}{0.0658(\text{W/m-K})}$$

$$= 3.503$$

$$\nu_l = \frac{\mu_l}{\rho_l} = \frac{1.386\times10^{-4}(\text{Pa-s})}{1{,}052(\text{kg/m}^3)} = 1.317\times10^{-7} \text{ m}^2/\text{s}.$$

Then from (7.40), we have

$$D_h = \frac{4A_{ku}}{P_{ku}} = \frac{4a^2}{4a} = a.$$

Since $\dot{M}_f$ is divided between N_c channels, we have

$$\begin{aligned} \dot{m}_f &= \frac{\dot{M}_f}{N_c a^2} = \frac{10^{-3}(\text{kg/s})}{10\times(1.5\times10^{-3})^2(\text{m}^2)} \\ &= 44.44 \text{ kg/m}^2\text{-s} \\ \text{Re}_{l,eq} &= \frac{44.44(\text{kg/m}^2\text{-s})\left\{(1-0.5) + 0.5\times\left[\dfrac{1{,}052(\text{kg/m}^3)}{87.26(\text{kg/m}^3)}\right]^{1/2}\right\}\times(1.5\times10^{-3})(\text{m})}{1.386\times10^{-4}(\text{Pa-s})} \\ &= 1{,}075.4. \end{aligned}$$

Then from Table 7.6, we then have

$$\begin{aligned} \langle \text{Nu}\rangle_{D,h} &= 5.03\text{Re}_{l,eq}^{1/3}\text{Pr}^{1/3} \\ &= 5.03\times(1{,}075.4)^{1/3}\times(3.503)^{1/3} = 78.27 \\ A_{ku} &= N_c\times 4aL = 10\times4\times1.5\times10^{-3}(\text{m})\times0.15(\text{m}) = 9.0\times10^{-3}\text{m}^2 \\ \langle Q_{ku}\rangle_{D,h} &= 9.0\times10^{-3}(\text{m}^2)\times78.27\times\frac{0.0658(\text{W/m-K})(58-60)(\text{K})}{1.5\times10^{-3}(\text{m})} \\ &= -61.80 \text{ W}. \end{aligned}$$

(c) For the energy equation, we have

$$\begin{aligned}
\dot{M}_{gl} &= -\frac{\langle Q_{ku}\rangle_{D,h}}{\Delta h_{lg}} = -\frac{-61.80(\text{W})}{1.386\times 10^{5}(\text{J/kg})} \\
&= 4.458\times 10^{-4}\ \text{kg/s} \\
x_L &= \frac{\dot{M}_{g,L}}{\dot{M}_f} = \frac{\dot{M}_{g,0}-\dot{M}_{gl}}{\dot{M}_f} = \frac{x_0\dot{M}_f-\dot{M}_{gl}}{\dot{M}_f} \\
&= x_0 - \frac{\dot{M}_{gl}}{\dot{M}_f} = 0.5 - \frac{4.459\times 10^{-4}(\text{kg/s})}{10^{-3}(\text{kg/s})} \\
&= 0.5 - 0.4458 = 0.0541.
\end{aligned}$$

(d) Since x has changed significantly (and is nearly vanished at the end of the tube), we need to use an average x in the determination of $\text{Re}_{l,eq}$. We can repeat the calculations using $x = (x_o + x_L)/2$ and continue this until the predicted x_L no longer changes.

COMMENT:

The correlations used have some uncertainties and as the pressure drops along the tube, the liquid-vapor mixture temperature also drops. Also the liquid and vapor many not be in local thermal equilibrium.

PROBLEM 7.11.FAM

GIVEN:

A compact evaporation tube, made of N_c circular channels and placed vertically, is used for the evaporation of a stream mixture of liquid and vapor of refrigerant R-134a. This is shown in Figure Pr.7.11(a), where the inlet quality is x_0 and the inlet temperature $\langle T_f \rangle_0$ is determined from the inlet pressure $p_{g,0}$. The liquid-vapor mass flow rate is $\dot{M}_f$. For simplicity assume that along the evaporator $\langle T_f \rangle$ and x remain constant and use the correlation given in Table 7.6.

$N_c = 8$, $D = 2$ mm, $L = 15$ cm, $p_{g,0} = 0.4144$ MPa, $x_0 = 0.4$, $T_s = 12$°C, $\dot{M}_f = 10^{-3}$ kg/s.

SKETCH:

Figure Pr.7.11(a) shows the tube, the channels and the inlet conditions.

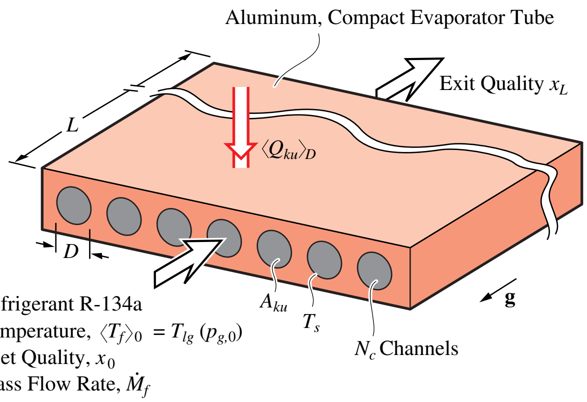


Figure Pr.7.11(a) A compact evaporation tube, having N_c circular channel, and placed vertically, is used to evaporate a refrigerant R-134a stream.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Based on contact x and $\langle T_f \rangle$ assumptions, determine the heat transfer rate, $\langle Q_{ku} \rangle_D$.
(c) From the results of (b), determine the evaporation rate $\dot{M}_{lg}$ and the exit quality x_L.
(d) Comment on how an iteration may be used to improve on the accuracy of predictions.

SOLUTION:

(a) Figure Pr.7.11(b) shows the thermal circuit diagram. Here a constant fluid stream temperature $\langle T_f \rangle$ is used and from (7.9) we have

$$\langle Q_u \rangle_{L\text{-}0} = \langle Q_{ku} \rangle_D = A_{ku} \langle \text{Nu} \rangle_D \frac{k_l}{D}(T_s - \langle T_f \rangle_0).$$

The energy equation for the stream is found from Figure Pr.7.11(b), i.e.,

$$Q|_A = -\langle Q_{ku} \rangle_D = \dot{S}_{lg} = -\dot{M}_{lg} \Delta h_{lg}.$$

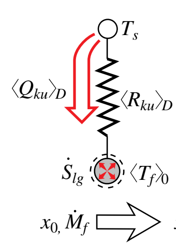


Figure Pr.7.11(b) Thermal circuit diagram.

(b) The Nusselt number for boiling in vertical tubes is given in Table 7.6 as

$$\begin{aligned}\langle \text{Nu}\rangle_D &= \langle \text{Nu}\rangle_{D,l}\left[1+3{,}000\text{Ku}^{0.86}+1.12\left(\frac{x}{1-x}\right)^{0.75}\left(\frac{\rho_l}{\rho_g}\right)^{0.41}\right]\\ \text{Ku} &= \frac{\langle q_{ku}\rangle_D}{\dot{m}_f\Delta h_{lg}},\end{aligned}$$

where $\langle \text{Nu}\rangle_{D,l}$ is found based on the magnitude of $\text{Re}_{D,l}$.

$$\text{Re}_{D,l}=\frac{\dot{m}_f D}{\mu_l},\quad \dot{m}_f=\frac{\dot{M}_f}{N_c\pi D^2/4}.$$

From Table C.28, for saturated refrigerant R-134a at $p_{g,0}=0.4144$ Mpa, we have

$$\rho_l = 1{,}260 \text{ kg/m}^3 \qquad \text{Table C.28}$$
$$\rho_g = 20.21 \text{ kg/m}^3 \qquad \text{Table C.28}$$
$$\Delta h_{lg} = 1.895\times10^5 \text{ J/kg} \qquad \text{Table C.28}$$
$$\mu_l = 2.543\times10^{-4} \text{ Pa-s} \qquad \text{Table C.28}$$
$$k_l = 0.0888 \text{ W/m-K} \qquad \text{Table C.28}$$
$$c_{p,l} = 1{,}367 \text{ J/kg-K} \qquad \text{Table C.28}$$
$$T_{lg} = 283.15 \text{ K} = 10^\circ\text{C} \qquad \text{Table C.28}$$

$$\begin{aligned}\text{Pr}=\text{Pr}_l &= \left(\frac{\mu c_p}{k}\right)_l=\frac{2.543\times10^{-4}(\text{Pa-s})\times1{,}367(\text{J/kg-K})}{0.0888(\text{ W/m-K})}\\ &= 3.915\\ \nu_l &= \frac{\mu_l}{\rho_l}=\frac{2.543\times10^{-4}(\text{Pa-s})}{1{,}260(\text{kg/m}^3)}=2.018\times10^{-7}\text{ m}^2/\text{s}.\end{aligned}$$

Then

$$\begin{aligned}\dot{m}_f &= \frac{4\times10^{-3}(\text{kg/s})}{8\times\pi\times(2\times10^{-3})^2(\text{m}^2)}=39.79\text{ kg/m}^2\text{-s}\\ \text{Re}_{D,l} &= \frac{4\dot{M}_f}{N_c\pi D\mu_l}=\frac{4\times10^{-3}(\text{kg/s})}{8\times\pi\times2\times10^{-3}(\text{m})\times2.543\times10^{-4}(\text{Pa-s})}\\ &= 312.9<\text{Re}_{D,t}=2{,}300\quad\text{laminar flow regime.}\end{aligned}$$

Assuming fully-developed temperature and velocity fields, and a constant heat flux, the Nusselt number from Table 7.2 is

$$\begin{aligned}\langle \text{Nu}\rangle_{D,l} &= 4.36\\ \text{Ku} &= \frac{\langle q_{ku}\rangle_D}{\dot{m}_f\Delta h_{lg}}=\frac{\langle q_{ku}\rangle_D}{39.79(\text{kg/m}^2\text{-s})\times1.895\times10^5(\text{J/kg})}\\ &= 1.326\times10^{-7}(\text{m}^2/\text{W})\langle q_{ku}\rangle_D\\ &= 1.326\times10^{-7}(1/\text{W})\frac{\langle Q_{ku}\rangle_D}{A_{ku}}=1.326\times10^{-7}(1/\text{W})\frac{\langle Q_{ku}\rangle_D}{8\pi DL}\\ &= 1.326\times10^{-7}(1/\text{W})\frac{\langle Q_{ku}\rangle_D}{8\times\pi\times2\times10^{-3}(\text{m})\times0.15(\text{m})}\\ &= 1.759\times10^{-5}(1/\text{W})\langle Q_{ku}\rangle_D.\\ \langle \text{Nu}\rangle_D &= 4.36\left\{1+3{,}000\times(1.759\times10^{-5}(1/\text{W})\langle Q_{ku}\rangle_D)^{0.86}+1.12\times\left(\frac{0.4}{1-0.4}\right)^{0.75}\left[\frac{1{,}260(\text{kg/m}^3)}{20.21(\text{kg/m}^3)}\right]^{0.41}\right\}\\ &= 4.36+1.065(\langle Q_{ku}\rangle_D(1/\text{W}))^{0.86}+19.61\\ \langle Q_{ku}\rangle_D &= 8\pi\times2\times10^{-3}(\text{m})\times0.15(\text{m})\times(23.97+1.065(\langle Q_{ku}\rangle_D(1/\text{W}))^{0.86})\times\frac{0.0888(\text{W/m-K})}{2\times10^{-3}(\text{m})}\times(12-10)(\text{K})\\ &= 16.05+0.71305(\langle Q_{ku}\rangle_D(1/\text{W}))^{0.86}.\end{aligned}$$

Solving this for $\langle Q_{ku}\rangle_D$, using a solver, we have

$$\langle Q_{ku}\rangle_D = 28.93 \text{ W}.$$

(c) From the energy equation, we have

$$\begin{aligned}
\dot{M}_{lg} &= \frac{\langle Q_{ku}\rangle_D}{\Delta h_{lg}} = \frac{28.93(\text{W})}{1.395\times 10^5(\text{J/kg})} \\
&= 2.074\times 10^{-4} \text{ kg/s} \\
x_L &= \frac{\dot{M}_{g,L}}{\dot{M}_f} = \frac{\dot{M}_{g,0}+\dot{M}_{lg}}{\dot{M}_f} = \frac{x_0\dot{M}_f+\dot{M}_{lg}}{\dot{M}_f} \\
&= x_0 + \frac{\dot{M}_{lg}}{\dot{M}_f} \\
&= 0.4 + 0.2074 = 0.6074.
\end{aligned}$$

(d) Since x has increased noticeably (from 0.4 to 0.6074), we should use an average quality$(x_o + x_L)/2$ in the determination of $\langle \text{Nu}\rangle_D$. This is iterated until $\langle \text{Nu}\rangle_D$ and x_L no larger change.

COMMENT:

The pressure p_g also decrease along the tube, resulting in a decrease in T_{lg} along the tube. Also, the liquid and vapor may not be in local thermal equilibrium.

PROBLEM 7.12.FAM

GIVEN:

A convection air heater is designed using forced flow through a square channel that has electrically heated wires running across it, as shown in Figure Pr.7.129a).

Evaluate the properties of air at 300 K.

SKETCH:

Figure Pr.7.12 shows the convection air heater with forced flow through a square channel and electrically heated wires.

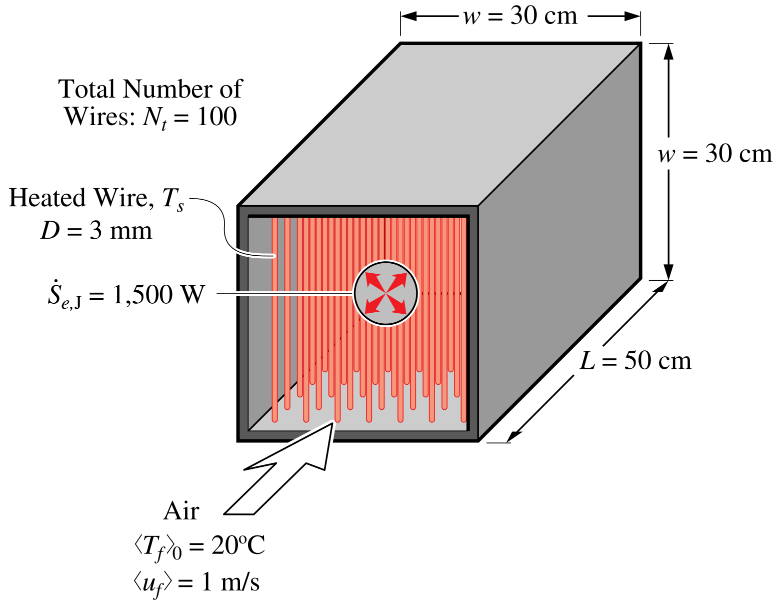


Figure Pr.7.12(a) An electric, air-stream heater.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the average wire temperature for a heating rate (i.e., Joule energy conversion) $\dot{S}_{e,J}$ = 1,500 W. Model the wires as a bank of tubes.

SOLUTION:

(a) The thermal circuit is shown in Figure Pr.7.12(b). All the wires are lumped into a single node T_s.

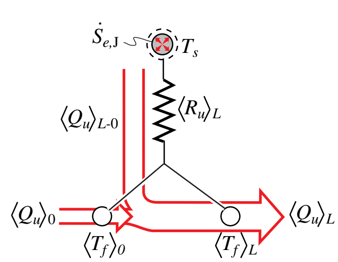


Figure Pr.7.12(b) Thermal circuit diagram.

(b) The integral-volume energy equation (2.9) applied to the T_s node is

$$Q|_A = -(\rho c_p V)_s \frac{dT_s}{dt} + \dot{S}_s.$$

The only heat loss from the wires occurs by surface convection to the bounded flow. The energy generation occurs by Joule heating. Therefore, for a steady-state condition the energy equation becomes

$$\frac{T_s - \langle T_f \rangle_o}{\langle R_u \rangle_L} = \dot{S}_{e,\mathrm{J}}.$$

The geometric parameters for the tube bundle are given by (7.44) and (7.41), i.e.,

$$D_p = \frac{6V_s}{A_{ku}} = 6\frac{(\pi D^2/4)w}{\pi D w} = \frac{3}{2}D = \frac{3}{2} \times 0.003(\mathrm{m}) = 0.0045 \ \mathrm{m}$$

$$\epsilon = \frac{V_f}{V_s + V_f} = \frac{w^2 L - N_t(\pi D^2/4)w}{w^2 L} = 1 - \frac{N_t(\pi D^2/4)}{wL} = 1 - \frac{100 \times \pi \times (0.003)^2(\mathrm{m}^2)}{4 \times 0.5(\mathrm{m}) \times 0.3(\mathrm{m})} = 0.9953.$$

For air, from Table C.22, at 300 K, we have $\nu_f = 15.66 \times 10^{-6}$ m^2/s, $k_f = 0.0267$ W/m-K, Pr= 0.69, $\rho_f = 1.177$ kg/m^3, $c_p = 1{,}005$ J/kg-K.
From (7.45), the Reynolds number is

$$\mathrm{Re}_{D,p} = \frac{\langle u_f \rangle D_p}{\nu_f(1-\epsilon)} = \frac{1(\mathrm{m/s}) \times 0.0045(\mathrm{m})}{15.66 \times 10^{-6}(\mathrm{m^2/s})(1 - 0.9953)} = 61{,}140.$$

The Nusselt number, From Table 7.5, is

$$\begin{aligned}
\langle \mathrm{Nu} \rangle_{D,P} &= 2 + (0.4\mathrm{Re}^{1/2} + 0.2\mathrm{Re}^{2/3})\mathrm{Pr}^{0.4} \\
&= 2 + [0.4 \times (61{,}140)^{1/2} + 0.2 \times (61{,}140)^{2/3}] \times (0.69)^{0.4} \\
&= 354.84.
\end{aligned}$$

The mass flow rate is given by (7.47), i.e.,

$$\dot{M}_f = \rho_f \langle u_f \rangle w^2 = 1.177(\mathrm{kg/m^3}) \times 1(\mathrm{m/s}) \times 0.3(\mathrm{m}) \times 0.3(\mathrm{m}) = 0.10593 \ \mathrm{kg/s}.$$

The surface-convection surface area is

$$A_{ku} = N_t \pi D w = 100 \times 0.003(\mathrm{m}) \times 0.3(\mathrm{m}) = 0.2827 \ \mathrm{m}^2.$$

The number of transfer units is given by (7.46), i.e.,

$$NTU = \frac{A_{ku}\langle \mathrm{Nu} \rangle_{D,p} k_f}{(\dot{M}c_p)_f D_p}\frac{1-\epsilon}{\epsilon} = \frac{0.2827(\mathrm{m}^2) \times 354.84 \times 0.0267(\mathrm{W/m\text{-}K})}{0.10593(\mathrm{kg/s}) \times 1{,}005(\mathrm{J/kg\text{-}K}) \times 0.0045(\mathrm{m})}\frac{1 - 0.9953}{0.9953} = 0.0264.$$

The heat transfer effectiveness is given by (7.22), i.e.,

$$\epsilon_{he} = 1 - e^{-NTU} = 1 - e^{-0.0264} = 0.02606.$$

Finally, the convection resistance is given by (7.27), i.e.,

$$\langle R_u \rangle_L = \frac{1}{(\dot{M}c_p)_f \epsilon_{he}} = \frac{1}{0.10593(\mathrm{kg/s}) \times 1{,}005(\mathrm{J/kg\text{-}K}) \times 0.02606} = 0.3605℃/\mathrm{W}.$$

Then, the wires surface temperature is found from the energy equation as

$$T_s = \langle T_f \rangle_0 + \langle R_u \rangle_L \dot{S}_{e,\mathrm{J}} = 20(℃) + 0.3605(℃/\mathrm{W}) \times 1{,}500(\mathrm{W}) = 560.8℃.$$

COMMENT:

Note that the wire temperature T_s depends on the surface-convection resistance and on $(\dot{M}c_p)_f$. To decrease T_s (the lowest possible value is $\langle T_f \rangle_0$) a very large flow rate can be used (while keeping ϵ_{he} large).

PROBLEM 7.13.FAM

GIVEN:

The monolith automobile catalytic converter is designated by the number of channels (square geometry) per square inch. Current designs are between 400 and 600 channels per square inch (62 and 93 channels per square centimeter). Each channel has dimensions $a \times a$ and the channel wall thickness is l. Assume that the converter has a square cross section of $w \times w$ and a length L. This is shown in Figure Pr.7.13(a).

The exhaust gas mass flow rate $\dot{M}_f$ (kg/s) is related to the rpm by

$$\dot{M}_f = \frac{1}{2}\frac{rpm}{60}\rho_{f,o}V_d\eta_V,$$

where the density $\rho_{f,o}$ is that of air at the intake condition, V_d is the total displacement volume, and η_V is the volumetric efficiency.

For simplicity, during the start-up, assume a uniform channel wall temperature T_s.

$\langle T_f\rangle_0 = 500°C$, $T_s = 30°C$, $w = 10$ cm, $L = 25$ cm, $l = 0.25$ mm, $V_d = 2.2 \times 10^{-3}$ m^3, $rpm = 2{,}500$, $\eta_V = 0.9$, $\rho_{f,o} = 1$ kg/m^3.

Evaluate the exhaust gas properties using air at $T = 700$ K.

SKETCH:

Figure Pr.7.13(a) shows the cross section of a monolith automobile catalytic converter with the geometric parameters for each channel (or cell).

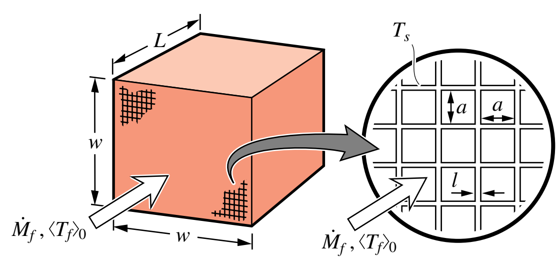


Figure Pr.7.13(a) Cross section of a monolith automobile catalytic converter.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine the exit temperature of the exhaust gas for both the 400 and 600 channels per square inch designs.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.3(b).

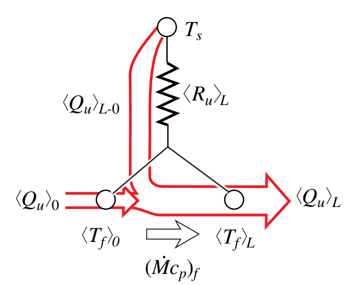


Figure Pr.7.13(b) Thermal circuit diagram.

(b) From Table C.22 for air at 700 K, we have $\rho_f = 0.507$ kg/m^3, $c_{p,f} = 1065$ J/kg-K, $k_f = 0.0513$ W/m-K, $\alpha_f = 93.1 \times 10^{-6}$ m^2/s, $\nu_f = 65.15 \times 10^{-6}$ m^2/s, and Pr = 0.7.

From the relation for $\dot{M}_f$, we have

$$\begin{aligned}
\dot{M}_f &= \frac{1}{2}\frac{\text{rpm}}{60}\rho_{f,o}V_d\eta_V \\
&= \frac{1}{2}\times\frac{2500}{60}\times 1(\text{kg/m}^3)\times 2.2\times 10^{-3}(\text{m}^3)\times 0.9 \\
&= 0.04125 \text{ kg/s}.
\end{aligned}$$

Now let us define $\dot{M}_{f,1}$ as the mass flow rate through a single channel of dimension a, and N as the number of channels per square inch. Then we have

$$\begin{aligned}
\dot{M}_{f,1} &= \frac{\dot{M}_f}{NA} \\
\dot{M}_{f,1} &= \frac{\dot{M}_f}{N\times[1(\text{in})/2.54(\text{cm})]^2\times w\times w} \\
\dot{M}_{f,1} &= \frac{0.04125(\text{kg/s})}{N(\text{channels/in}^2)\times 0.155(\text{in/cm})^2\times 10^2(\text{cm}^2)} \\
&= \frac{2.661\times 10^{-3}(\text{kg/s-in}^2)}{N(\text{channels/in}^2)} \\
&= 6.653\times 10^{-6}\text{ kg/s} \qquad (\text{for } N=400) \\
&= 4.435\times 10^{-6}\text{ kg/s} \qquad (\text{for } N=600).
\end{aligned}$$

The hydraulic diameter, defined by (7.40), for a square channel is

$$D_h = \frac{4A_u}{P_{ku}} = \frac{4a^2}{4a} = a.$$

The width/height of a channel a are found from the geometry:

$$\begin{aligned}
N(a+l)^2 &= 1\text{in}^2 \\
a &= N^{-1/2} - l \\
&= N^{-1/2}(\text{in/channels})\times 0.0254(\text{m/in}) - 0.00025(\text{m}) \\
&= 1.02\times 10^{-3}\text{ m} \qquad (\text{for } N=400) \\
&= 7.87\times 10^{-4}\text{ m} \qquad (\text{for } N=600).
\end{aligned}$$

Then noting that $\dot{M}_{f,1} = \rho_f\langle u_f\rangle a^2$, and $\text{Re}_{D,h} = \text{Re}_a$ is defined by (7.36) we have

$$\begin{aligned}
\text{Re}_a &= \frac{\langle u_f\rangle a}{\nu_f} = \frac{\dot{M}_{f,1}}{\nu_f\rho_f a} \\
&= \frac{\left(\dfrac{2.661\times 10^{-3}}{N}\right)(\text{kg/s})}{(0.507)(\text{kg/m}^3)\times[0.0254N^{-1/2}-0.00025](\text{m})\times(65.15\times 10^{-6})(\text{m}^2/\text{s})} \\
&= \frac{80.56}{N(0.0254N^{-1/2}-0.00025)} = 197.4 \qquad (\text{for } N=400) \\
\text{Re}_a &= 170.6 \qquad (\text{for } N=600).
\end{aligned}$$

For both cases, Re_a is laminar. The corresponding $\langle\text{Nu}\rangle_{D,h} = \langle\text{Nu}\rangle_a$ is found from Table 7.2, i.e., $\langle\text{Nu}\rangle_a = 2.98$ from Table 7.2. The thermal convection resistance is then

$$\begin{aligned}
\langle R_{ku}\rangle_a &= \frac{a}{A_{ku}\langle\text{Nu}\rangle_a k_f} = \frac{a}{(4aL)\langle\text{Nu}\rangle_a k_f} \\
&= \frac{1}{4L\langle\text{Nu}\rangle_a k_f} \\
&= \frac{1}{4\times 0.25(\text{m})\times 2.98\times 0.0513(\text{W/m-K})} = 6.541\text{ K/W}.
\end{aligned}$$

The NTU is defined by (7.20) and becomes

$$\begin{aligned} NTU &= \frac{1}{\langle R_{ku}\rangle \dot{M}_{f,1} c_{p,f}} \\ &= \frac{1}{6.541(℃/\text{W}) \times \left(\dfrac{2.661\times 10^{-3}}{N}\right)(\text{kg/s}) \times 1{,}065(\text{J/kg-K})} = \frac{N}{18.54} = 21.57 \qquad (\text{for } N = 400) \\ NTU &= 32.36 \qquad (\text{for } N = 600). \end{aligned}$$

This is a very high NTU and for both cases, $\exp(-NTU) \rightarrow 0$. Therefore, for both cases, the heat exchanger effectiveness is

$$\begin{aligned} \epsilon_{he} &= 1 - e^{-NTU} = 1 \\ \epsilon_{he} &= 1 = \frac{\langle T_f\rangle_L - \langle T_f\rangle_0}{T_s - \langle T_f\rangle_0}, \end{aligned}$$

and solving for $\langle T_f\rangle_L$ we have

$$\begin{aligned} \langle T_f\rangle_L &= (T_s - \langle T_f\rangle_0) + \langle T_f\rangle_0 = T_s \\ &= 30℃ \qquad (\text{for both } N = 400 \text{ and } 600). \end{aligned}$$

COMMENT:

The catalytic converter is designed for both heat and mass transfer. The mass transfer (and surface-mediated chemical reactions) is influenced by the deteriorating effect of poisoning compounds such as sulfur. Then the converter is overdesigned to allow for an extended life, after some portion (the entrance portion) of it becomes ineffective.

PROBLEM 7.14.DES

GIVEN:

In many cutting operations it is imperative that the cutting tool be maintained at an operating temperature T_s, well below the tool melting point. The cutting tool is shown in Figure Pr.7.14(a). Two designs for the cooling of the cutting tool are considered. The coolant is liquid nitrogen and the objective is to maintain the tool temperature at $T_s = 500°C$. The nitrogen stream is a saturated liquid at one atm pressure and flows with a mass flow rate of $\dot{M}_{N_2} = 1.6 \times 10^{-3}$ kg/s. These designs are also shown in Figure 7.14(a). The first design uses direct liquid nitrogen droplet impingement. The liquid nitrogen is at temperature $T_{lg,\infty} = 77.3$ K. The average droplet diameter is $\langle D \rangle = 200 \times 10^{-6}$ m, and the average droplet velocity is $\langle u_d \rangle = 3$ m/s. The second design uses the mixing of liquid nitrogen and air, where, after mixing, the nitrogen becomes superheated. Then the mixture is flown internally through the cutting tool. The air enters the mixer at a temperature of $T_a = 20°C$ and flows with a mass flow rate of $\dot{M}_a = 3.2 \times 10^{-3}$ kg/s. The mixture enters the permeable tool with a temperature $\langle T_f \rangle_0$ and flows through the cutting tool where a sintered-particle region forms a permeable-heat transfer medium. The average particle diameter is $D_p = 1$ mm and the porosity is $\epsilon = 0.35$, as depicted in Figure Pr.7.14(b).

Take the mixture conductivity to be $k_f = 0.023$ W/m-K, and evaluate the mixture specific heat using the average of the air specific heat at $T = 300$ K and the superheated nitrogen specific heat from Table C.26.

SKETCH:

Figures Pr.7.14(a) and (b) show the two designs considered. The permeable cutting tool has a large interstitial area formed by sintered spherical particles.

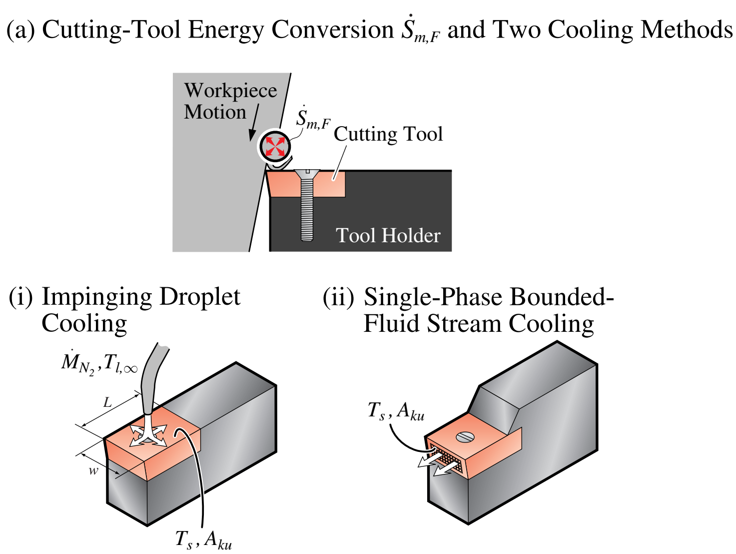


Figure Pr.7.14(a) Cooling of a cutting tool by semi-bounded and bounded coolant streams.

OBJECTIVE:

(a) Draw the thermal circuit diagram for the two designs.

(b) Assuming that both designs have the same liquid nitrogen mass flow rate and that (6.116) is valid, determine the amount of surface-convection cooling for surface droplet impingement cooling. Use the properties of air at $T = 300$ K.

(c) Making the same assumptions, determine the internal transpiration cooling. Use $\langle T_f \rangle_0$, determined from the energy equation for the adiabatic mixture, to determine the properties of air for the mixture.

SOLUTION:

(a) The physical model and the thermal circuit diagrams for the two cooling methods are shown in Figure Pr.7.14(c).

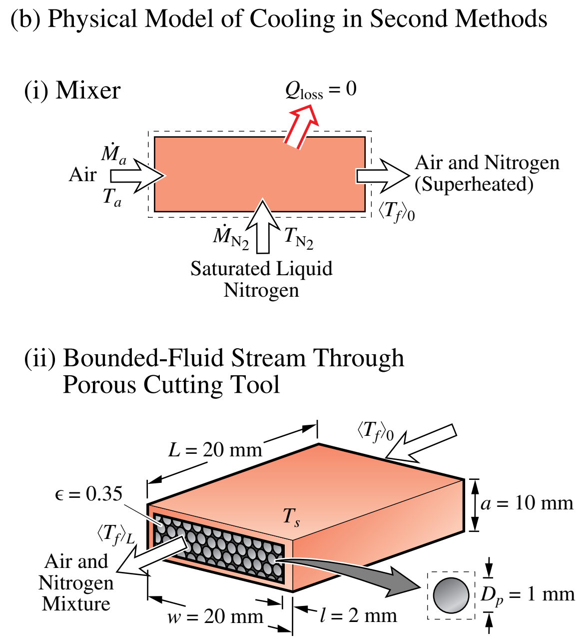


Figure Pr.7.14(b) The mixer and permeable cutting tool.

(b) (i) Surface Droplet Impingement Cooling:
From Table 6.6 and (6.124), for droplet impingement cooling, we have

$$\frac{\langle q_{ku}\rangle_L}{T_s - T_{l,\infty}} = (A_{ku}\langle R_{ku}\rangle_L)^{-1} = \rho_{l,\infty}\Delta h_{lg,\infty}\frac{\langle \dot{m}_d\rangle}{\rho_{l,\infty}}\eta_d\left[1 - \frac{\langle \dot{m}_d\rangle/\rho_{l,\infty}}{(\langle \dot{m}_d\rangle/\rho_{l,\infty})_0}\right]\frac{1}{T_s - T_{l,\infty}} +$$
$$1{,}720(T_s - T_{l,\infty})^{-0.088}\langle D\rangle^{-1.004}\langle u_d\rangle^{-0.764}\frac{(\langle \dot{m}_d\rangle/\rho_{l,\infty})^2}{(\langle \dot{m}\rangle_d/\rho_{l,\infty})_0},$$

where

$$\eta_d = \frac{3.68\times 10^4}{\rho_{l,\infty}\Delta h_{l,g,\infty}}(T_s - T_{l,\infty})^{1.691}\langle D\rangle^{-0.062}$$
$$\Delta h_{lg,\infty} = c_{p,l}(T_{lg} - T_{l,\infty}) + \Delta h_{lg}$$
$$\left(\frac{\langle \dot{m}_d\rangle}{\rho_{l,\infty}}\right)_o = 5\times 10^{-3}\ \text{m/s}.$$

For air at $T = 300$ K, from Table C.22, we have $c_{p,f} = 1{,}005$ J/kg-K, $\nu_f = 15.66\times 10^{-6}$ m^2/s, Pr = 0.69, $\rho_f = 1.177$ kg/m^3. For nitrogen from Table C.26, we have $T_{lg} = 77.35$ K, $\rho_{l,\infty} = 807.1$ kg/m^3, $\Delta h_{lg} = 1.976\times 10^5$ J/kg, $c_{p,g} = 1{,}123$ J/kg-K.
Using the numerical values, we have

$$\Delta h_{lg,\infty} = (c_{p,l})_\infty(T_{lg} - T_{l,\infty}) + \Delta h_{lg}$$
$$T_{lg} = T_{l,\infty}$$
$$\Delta h_{lg,\infty} = \Delta h_{lg} = 197.6\times 10^3\ \text{J/kg}$$
$$\eta_d = \frac{3.68\times 10^4}{807.1\times 197.6\times 10^3}(773 - 77.3)^{1.691}(200\times 10^{-6})^{-0.062}$$
$$= 25.06$$
$$\langle \dot{m}_d\rangle = \frac{\dot{M}}{A_{ku}},\quad A_{ku} = L\times w = (0.02)^2(\text{m}^2) = 4\times 10^{-4}\ \text{m}^2$$
$$\langle \dot{m}_d\rangle = \frac{1.6\times 10^{-3}(\text{kg/s})}{4\times 10^{-4}(\text{m}^2)} = 4.0\ \text{kg/m}^2\text{-s}.$$

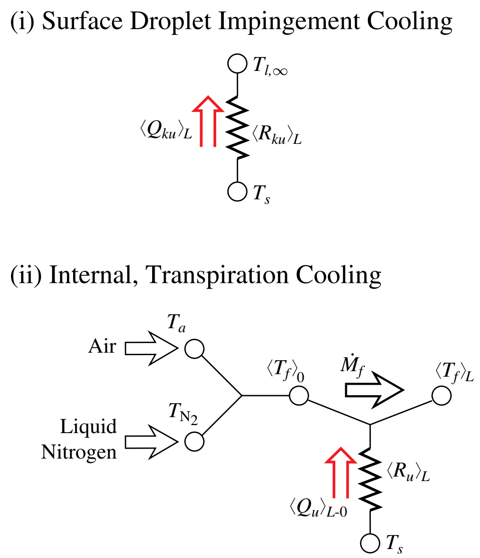


Figure Pr.7.14(c) Thermal circuit diagrams.

Then

$$\begin{aligned}
\frac{\langle q_{ku}\rangle_L}{(773-77.3)(\mathrm{K})} &= 807.1(\mathrm{kg/m^3}) \times 197.6\times 10^3(\mathrm{J/kg}) \times \frac{4.0\ \mathrm{kg/m^2\text{-}s}}{807.1(\mathrm{kg/m^3})} \times 25.06 \times \\
& \left[1 - \frac{4\ \mathrm{kg/m^2\text{-}s}/807.1(\mathrm{kg/m^3})}{5\times 10^{-3}(\mathrm{m/s})}\right] \frac{1}{(773-77.3)} + 1720 \times (773-77.3)^{-0.088}(\mathrm{K})^{-0.088} \\
& \times (200\times 10^{-6})^{-1.004}(\mathrm{m})^{-1.004} \times (3)^{-0.764}(\mathrm{m/s})^{-0.764} \frac{(4\ \mathrm{kg/m^2\text{-}s}/807.1(\mathrm{kg/m^3}))^2}{5\times 10^{-3}(\mathrm{m/s})} \\
\frac{\langle q_{ku}\rangle_L}{695.7(\mathrm{K})} &= [1.987\times 10^7 \times 1.265\times 10^{-5} + (966.9 \times 5{,}173 \times 0.432 \times 0.00491)](\mathrm{W/m^2\text{-}K}) \\
&= (250.56 + 1.061\times 10^4)(\mathrm{W/m^2\text{-}K}) = 10{,}861\ \mathrm{W/m^2\text{-}K} \\
\langle q_{ku}\rangle_L &= 7.556\times 10^6\ \mathrm{W/m^2}.
\end{aligned}$$

Using the surface area A_{ku}, we have

$$\langle Q_{ku}\rangle_L = q_{ku}A_{ku} = 3{,}022\ \mathrm{W}.$$

(ii) Internal, Transpiration Cooling:
To determine the temperature of the nitrogen mixture entering the permeable cutting tool, we use an integral-volume energy equation similar to (5.17). This gives, for the control volume shown in Figure Pr.7.14(b)(i),

$$\dot{M}_a c_{p,a}(\langle T_f\rangle_0 - T_a) + \dot{M}_{\mathrm{N_2}}[c_{p,\mathrm{N_2}}(\langle T_f\rangle_0 - T_{\mathrm{N_2}}) + \Delta h_{lg,\mathrm{N_2}}] = 0.$$

Solving for $\langle T_f\rangle_0$, we have

$$\begin{aligned}
3.2\times 10^{-3}(\mathrm{kg/s}) \times 1{,}005(\mathrm{J/kg\text{-}K}) \times (\langle T_f\rangle_0 - 293.15(\mathrm{K})) + 1.6\times 10^{-3}(\mathrm{kg/s}) & \\
\times[1{,}123(\mathrm{J/kg\text{-}K}) \times (\langle T_f\rangle_0 - 77.3(\mathrm{K})) + 1.976\times 10^5(\mathrm{J/kg})] &= 0 \\
3.216\langle T_f\rangle_0 - 942.3(\mathrm{K}) + 1{,}797\langle T_f\rangle_0 - 138.9(\mathrm{K}) + 316.16(\mathrm{K}) &= 0 \\
5.0128\langle T_f\rangle_0 - 765.04 &= 0 \\
\langle T_f\rangle_0 &= 152.6\ \mathrm{K}.
\end{aligned}$$

For the bounded fluid stream, we begin from (7.25), i.e.,

$$\langle Q_u\rangle_{L\text{-}0} = \frac{T_s - \langle T_f\rangle_0}{\langle R_u\rangle_L},$$

where from (7.27), we have

$$\langle R_u \rangle_L = \frac{1}{(\dot{M}c_p)_f(1 - e^{-NTU})}.$$

The mass flow rate $\dot{M}_f$ is

$$\dot{M}_f = \dot{M}_a + \dot{M}_{\mathrm{N}_2} = (3.2 \times 10^{-3} + 1.6 \times 10^{-3})(\mathrm{kg/s}) = 4.8 \times 10^{-3} \ \mathrm{kg/s}.$$

As indicated, the mixture $c_{p,f}$ is determined as a simple average

$$c_{p,f} = \frac{c_{p,a} + c_{p,\mathrm{N}_2}}{2} = \frac{(1{,}005 + 1{,}123)(\mathrm{J/kg\text{-}K})}{2} = 1{,}064 \ \mathrm{J/kg\text{-}K}.$$

The number of transfer units is given by (7.46), i.e.,

$$NTU = \frac{A_{ku}\langle \mathrm{Nu} \rangle_{D,p} \dfrac{k_f}{D_p} \dfrac{\epsilon}{1-\epsilon}}{(\dot{M}c_p)_f}.$$

To determine the interstitial surface area A_{ku}, we begin from (7.41), i.e.,

$$1 - \epsilon = \frac{V_s}{V}.$$

The volume is

$$V = L(w - 2l)(a - 2l) = 0.02(\mathrm{m}) \times 0.016(\mathrm{m}) \times 0.006(\mathrm{m}) = 1.92 \times 10^{-6} \ \mathrm{m}^3.$$

Then

$$\begin{aligned}
V_s &= V(1-\epsilon) \\
&= 1.92 \times 10^{-6}(\mathrm{m}^3) \times (1 - 0.35) \\
V_s &= 1.248 \times 10^{-6} \ \mathrm{m}^3 \\
A_{ku} &= \frac{6V_s}{D_p} = \frac{6 \times 1.248 \times 10^{-6}(\mathrm{m}^3)}{10^{-3}(\mathrm{m})} = 7.488 \times 10^{-3} \ \mathrm{m}^2.
\end{aligned}$$

The Nusselt number is determined from Table 7.5, i.e.,

$$\begin{aligned}
\langle \mathrm{Nu} \rangle_{D,p} &= 2 + (0.4\mathrm{Re}_{D,p}^{1/2} + 0.2\mathrm{Re}_{D,p}^{2/3})\mathrm{Pr}^{0.4}, \\
\mathrm{Re}_{D,p} &= \frac{\langle u_f \rangle D_p}{\nu_f(1-\epsilon)}.
\end{aligned}$$

The fluid velocity $\langle u_f \rangle$ is determined from (7.3), noting that ρ_f is evaluated at $T = 153$ K $\simeq 150$ K, i.e.,

$$\begin{aligned}
\dot{M}_f &= \rho_f \langle u_f \rangle A_u \\
A_u &= (w - 2l)(a - 2l) = 0.016(\mathrm{m}) \times 0.006(\mathrm{m}) = 9.6 \times 10^{-5} \ \mathrm{m}^2 \\
\rho_f &= 2.355 \ \mathrm{kg/m}^3 \\
\nu_f &= 4.52 \times 10^{-6} \ \mathrm{m}^2/\mathrm{s} \\
k_f &= 0.023 \ \mathrm{W/m\text{-}K} \\
\langle u_f \rangle &= \frac{\dot{M}_f}{\rho_f A_u} = \frac{4.8 \times 10^{-3}(\mathrm{kg/s})}{2.355(\mathrm{kg/m}^3) \times 9.6 \times 10^{-5}(\mathrm{m}^2)} = 21.23 \ \mathrm{m/s}.
\end{aligned}$$

Then,

$$\begin{aligned}
\mathrm{Re}_{D,p} &= \frac{21.23(\mathrm{m/s}) \times 0.001(\mathrm{m})}{4.52 \times 10^{-6}(\mathrm{m}^2/\mathrm{s})(1 - 0.35)} = 7{,}226.5 \\
\langle \mathrm{Nu} \rangle_{D,p} &= 2 + [0.4 \times (7{,}226.5)^{1/2} + 0.2 \times (7{,}226.5)^{2/3}] \times (0.69)^{0.4} \\
&= 95.76.
\end{aligned}$$

Using these, we have

$$NTU = \frac{7.048 \times 10^{-3}(\text{m}^2) \times 95.76 \times \dfrac{0.023(\text{W/m-K})}{0.001(\text{m})} \times \dfrac{1-0.35}{0.35}}{4.8 \times 10^{-3}(\text{kg/s}) \times 1{,}064(\text{J/kg-K})} = 5.997$$

$$\langle R_u \rangle_L = \frac{1}{(4.8 \times 10^{-3} \times 1{,}064)(1 - e^{-5.997})} = \frac{1}{5.065}(\text{K/W}) = 0.1963 \text{ K/W}$$

$$\langle Q_u \rangle_{L\text{-}0} = \frac{(773 - 153)(\text{K})}{0.1963(\text{K/W})} = 3{,}158 \text{ W}.$$

COMMENT:

The two methods give similar cooling power (about 3 kW). The internal transpiration cooling can be improve by increasing A_{ku} using a smaller particle diameter.

PROBLEM 7.15.FAM

GIVEN:

An electrical (Joule) heater is used for heating a stream of air $\dot{M}_f$ from temperature $\langle T_f\rangle_0$ to $\langle T_f\rangle_L$. The heater is made of a coiled resistance wire placed inside a tube of diameter D. The heater temperature is assumed uniform and at T_s. This is shown in Figure Pr.7.159a). The coiled wires can be represented as a porous medium of porosity ϵ with an equivalent particle diameter D_p.

$\dot{S}_{e,\text{J}} = 500$ W, $\dot{M}_f = 10^{-3}$ kg/s, $\langle T_f\rangle_0 = 20$℃, $D_p = 1$ mm, $\epsilon = 0.95$, $D = 1.9$ cm, $L = 30$ cm.

Evaluate air properties at $T = 500$ K.

SKETCH:

Figure Pr.7.15(a) shows an air heater using the Joule heating with the resistive wires forming a porous medium through which the air flows.

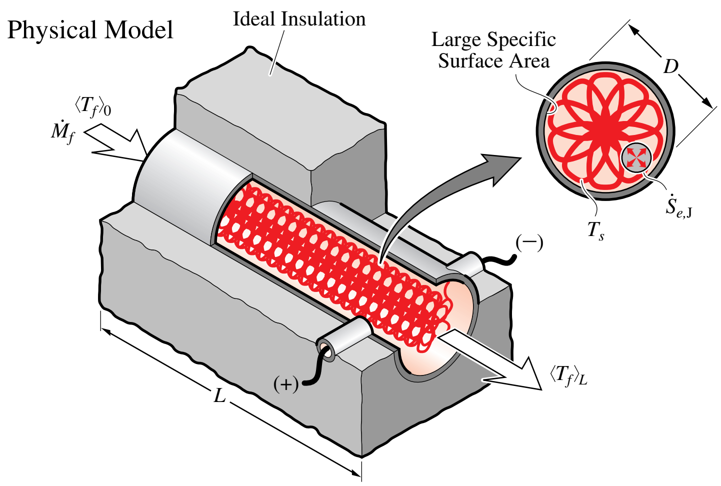


Figure Pr.7.15(a) An electric air-stream heater.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine the particle Nusselt number $\langle\text{Nu}\rangle_{D,p}$.
(c) Determine the number of transfer units, NTU.
(d) Determine the wire surface temperature T_s, as a function of $\dot{S}_{e,\text{J}}$ (W).
(e) Determine the air exit temperature as a function of $\dot{S}_{e,\text{J}}$ (W).
(f) Comment on the safety features that must be included to avoid heater meltdown.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.7.15(b). The Joule heating is transferred to the bounded fluid by surface convection and changes the fluid temperature.

(b) The particle Nusselt number is found form the correlation given in Table 7.5, i.e.,

$$\langle\text{Nu}\rangle_{D,p} = 2 + (0.4\text{Re}_{D,p}^{1/2} + 0.2\text{Re}_{D,p}^{2/3})\text{Pr}^{0.4},$$

where from (7.45), we have

$$\text{Re}_{D,p} = \frac{\rho_f\langle u_f\rangle D_p}{\mu_f(1-\epsilon)}$$

and for (7.47) we have

$$\langle u_f\rangle = \frac{\dot{M}_f}{A_u\rho_f} \quad , \quad A_u = \pi D^2/4$$

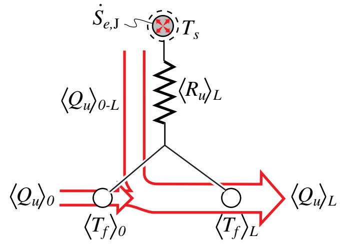


Figure Pr.7.15(b) Thermal circuit diagram.

or

$$\mathrm{Re}_{D,p} = \frac{4\dot{M}_f D_p}{\mu_f(1-\epsilon)\pi D^2}.$$

From Table C.22, for air at $T = 500$ K, we have

$$\begin{aligned}
\rho_f &= 0.706 \text{ kg/m}^3 && \text{Table C.22}\\
c_{p,f} &= 1017 \text{ J/kg-K} && \text{Table C.22}\\
\nu_f &= 3.7330 \times 10^{-5} \text{ m}^2\text{/s} && \text{Table C.22}\\
k_f &= 0.0395 \text{ W/m-K} && \text{Table C.22}\\
\mathrm{Pr} &= 0.69 && \text{Table C.22.}
\end{aligned}$$

Then (recalling that $\nu_f = \mu_f/\rho_f$)

$$\begin{aligned}
\mathrm{Re}_{D,p} &= \frac{4 \times [10^{-3}(\text{kg/s})] \times [10^{-3}(\text{m})]}{3.730 \times 10^{-5}(\text{m}^2/\text{s}) \times 0.706(\text{kg/m}^3) \times (1-0.95) \times \pi \times (1.9 \times 10^{-2})^2(\text{m}^2)}\\
&= 2{,}679.
\end{aligned}$$

Using this, we have

$$\begin{aligned}
\langle \mathrm{Nu} \rangle_{D,p} &= 2 + [0.4 \times (2{,}679)^{1/2} + 0.2 \times (2{,}679)^{2/3}] \times 0.69^{0.4}\\
&= 53.10.
\end{aligned}$$

(c) The number of transfer unit NTU is defined by (7.46), i.e.,

$$NTU = \frac{1}{(\dot{M}c_p)_f \langle \mathrm{R}_{ku} \rangle_D} = \frac{A_{ku}}{(\dot{M}c_p)_f}\frac{\langle \mathrm{Nu} \rangle_{D,p} k_f}{D_p}\frac{1-\epsilon}{\epsilon},$$

where from (7.42) we have

$$A_{ku} = \frac{6(1-\epsilon)V}{D_p} = \frac{6(1-\epsilon)\pi D^2 L}{4D_p}.$$

Then

$$\begin{aligned}
A_{ku} &= \frac{6 \times (1-0.95) \times \pi \times (1.9 \times 10^{-2})^2(\text{m}^2) \times 0.3\text{m}}{4 \times 10^{-3}(\text{m})}\\
&= 0.02552 \text{ m}^2\\
NTU &= \frac{0.02550\text{m}^2}{10^{-3}\text{kg/s} \times 1017(\text{J/kg-K})}\frac{53.10 \times 0.0395(\text{W/m-K})}{10^{-3}(\text{m})}\frac{(1-0.95)}{0.95}\\
&= 2.770.
\end{aligned}$$

(d) The energy equation written from node T_s in Figure Pr.7.15(b) gives

$$\langle Q_{ku} \rangle_{L\text{-}0} = \dot{S}_{e,\mathrm{J}},$$

where from (7.25) and (7.27), we have

$$\begin{aligned}\langle Q_{ku}\rangle_{L\text{-}0} &= \frac{T_s - \langle T_f\rangle_0}{\langle R_u\rangle_L}\\ \langle R_u\rangle_L &= \frac{1}{(\dot{M}c_p)_f\epsilon_{he}} = \frac{1}{(\dot{M}c_p)_f(1-e^{-NTU})}\end{aligned}$$

or

$$(T_s - \langle T_f\rangle_0)(\dot{M}c_p)_f(1-e^{-NTU}) = \dot{S}_{e,\mathrm{J}}.$$

Then

$$\begin{aligned}T_s &= \langle T_f\rangle_0 + \frac{\dot{S}_{e,\mathrm{J}}}{(\dot{M}c_p)_f(1-e^{-NTU})}.\\ T_s &= 20℃ + \frac{\dot{S}_{e,\mathrm{J}}}{10^{-3}(\mathrm{kg/s}) \times 1017(\mathrm{J/kg\text{-}K})(1-e^{-2.770})}\\ &= 20(℃) + 1.049\dot{S}_{e,\mathrm{J}}(℃/\mathrm{W}).\end{aligned}$$

(e) The air exit temperature is given by (7.21), i.e.,

$$\begin{aligned}\langle T_f\rangle_L &= T_s + (\langle T_f\rangle_0 - T_s)e^{-NTU}\\ &= T_s + 0.06266[20℃ - T_s]\\ &= 0.9373T_s + 1.309℃\\ &= 0.9373[20(℃) + 1.049\dot{S}_{e,\mathrm{J}}(℃/\mathrm{W})] + 1.2532(^\circ!\mathrm{C})\\ &= 20(℃) + 0.9832\dot{S}_{e,\mathrm{J}}(℃/\mathrm{W}).\end{aligned}$$

(f) As $\dot{S}_{e,\mathrm{J}}$ increases, T_s increases linearly. If a failure temperature is defined as $T_{s,max}$, then from the above equation for the maximum $\dot{S}_{e,\mathrm{J}}$ causing this threshold temperature, we have

$$T_{s,max} = 20(℃) + 1.049(\dot{S}_{e,\mathrm{J}})_{max}(℃/\mathrm{W})$$

or

$$(\dot{S}_{e,\mathrm{J}})_{max} = 0.9533(T_{s,max} - 20℃)(\mathrm{W}/℃).$$

For example, for $T_{s,max}$ = 1,200 K, (= 927℃), we have

$$(\dot{S}_{e,\mathrm{J}})_{max} = 0.9533(927-20)(℃)(\mathrm{W}/℃) = 864.6\ \mathrm{W}.$$

For this the exit air temperature is

$$\langle T_f\rangle_L = 0.9832(℃/\mathrm{W}) \times 864(\mathrm{W}) + 20℃ = 870.3℃.$$

COMMENT:

If care is not taken, and enough air does not flow through the tube, the failure temperature is reached very rapidly.

PROBLEM 7.16.FUN.S

GIVEN:

Sensible heat storage in packed beds is made by flow of a hot fluid stream through the bed and heat transfer by surface convection. A heat-storage bed of carbon-steel AISI spherical particles is shown in Figure Pr.7.16(a). Hot air is flowing through the bed with an inlet temperature $\langle T_f \rangle_0$ and a mass flow rate $\dot{M}_f$. The initial bed temperature is $T_s(t=0)$. When the assumption of $\mathrm{Bi}_L < 0.1$ is valid, then (6.156) can be used to predict the uniform, time-dependent bed temperature.

$T_s(t=0) = 30°\mathrm{C}, \langle T_f \rangle_0 = 190°\mathrm{C}, \dot{M}_f = 4$ kg/s, $L = 2$ m, $D_p = 8$ cm, $\epsilon = 0.40$.

Evaluate air properties at $T = 400$ K.

SKETCH:

Figure Pr.7.16(a) shows the storage medium, i.e., a packed bed of spherical particles, with a hot stream of air flowing through it.

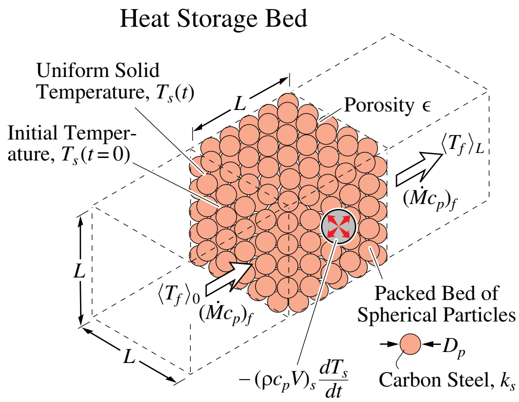


Figure Pr.7.16(a) Sensible heat storage/release in a packed bed of spherical particles.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine the bed effective thermal conductivity $\langle k \rangle$.
(c) Determine $\langle \mathrm{Nu} \rangle_{D,p}$, $\langle R_{ku} \rangle_L$, and NTU.
(d) Show that the Biot number $\mathrm{Bi}_L = R_{k,s}/\langle R_{ku} \rangle_L$ is not less than 0.1 (where $R_{k,s} = L/A_k k = 1/L\langle k \rangle$), and therefore, that assuming a uniform bed temperature is not justifiable (although that assumption makes the analysis here much simpler).
(e) Assume a uniform bed temperature and use (6.156) to plot $T_s(t)$ and $\langle T_f \rangle_L(t)$ for up to four time constants τ_s.
(f) Determine the amount of heat stored during this period.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.7.16(b).

(b) The effective thermal conductivity is given by (3.28), i.e.,

$$\frac{\langle k \rangle}{k_f} = \left(\frac{k_s}{k_f} \right)^{0.280 - 0.757 \log(\epsilon) - 0.057 \log(k_s/k_f)}.$$

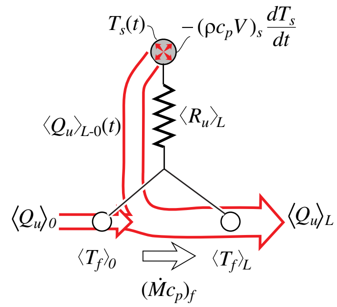


Figure Pr.7.16(b) Thermal circuit model.

The properties of air at $T = 400$ K, from Table C.22, are

$$\rho_f = 0.883 \text{ kg/m}^3 \qquad \text{Table C.22}$$
$$k_f = 0.0331 \text{ W/m-K} \qquad \text{Table C.22}$$
$$c_{p,f} = 1{,}009 \text{ J/kg-K} \qquad \text{Table C.22}$$
$$\nu_f = 2.55 \times 10^{-5} \text{ m}^2/\text{s} \qquad \text{Table C.22}$$
$$\text{Pr} = 0.69 \qquad \text{Table C.22.}$$

The properties of carbon steel AISI 1010, from Table C.16, are

$$\rho_s = 7{,}830 \quad \text{kg/m}^3 \qquad \text{Table C.16}$$
$$c_{p,s} = 434 \text{ J/kg-K} \qquad \text{Table C.16}$$
$$k_s = 64 \text{ W/m-K} \qquad \text{Table C.16.}$$

Then, with $k_s/k_f = 1933.5$,

$$\begin{aligned}
\langle k \rangle &= 0.0331(\text{W/m-K}) \times (1{,}933.5)^{0.280-0.757\log(0.40)-0.057\log(1{,}933.5)} \\
&= 0.0331(\text{W/m-K}) \times (1{,}934)^{0.3939} \\
&= 0.6521\text{W/m-K}.
\end{aligned}$$

(c) The Nusselt number is determine from the correlation in Table 7.5, i.e.,

$$\begin{aligned}
\langle \text{Nu} \rangle_{D,p} &= 2 + (0.4\text{Re}_{D,p}^{1/2} + 0.2\text{Re}_{D,p}^{2/3})\text{Pr}^{0.4} \\
\text{Re}_{D,p} &= \frac{\langle u_f \rangle D_P}{\nu_f} = \frac{\dot{M}_f D_p}{\rho_f A_u \nu_f} = \frac{\dot{M}_f D_P}{L^2 \nu_f \rho_f} \\
&= \frac{4(\text{kg/s}) \times 0.08(\text{m})}{(2)^2(\text{m}^2) \times 2.55 \times 10^{-5}(\text{m}^2/\text{s}) \times 0.883(\text{kg/m}^3)} = 3{,}553 \\
\langle \text{Nu} \rangle_{D,p} &= 2 + [0.4(3{,}553)^{1/2} + 0.2(3{,}553)^{2/3}] \times (0.69)^{0.4} = 62.70.
\end{aligned}$$

The surface-convection resistance $\langle R_{ku} \rangle_L$ is determined from (7.46), and A_{ku} is given by (7.42), i.e.,

$$\begin{aligned}
\langle R_{ku} \rangle_L &= \frac{D_p}{A_{ku} \langle \text{Nu} \rangle_{D,p} k_f} \frac{\epsilon}{1-\epsilon} \\
A_{ku} &= V \frac{6(1-\epsilon)}{D_p} = L^3 \frac{6(1-\epsilon)}{D_p},
\end{aligned}$$

so that

$$\langle R_{ku} \rangle_L = \frac{D_p^2 \epsilon}{6L^3(1-\epsilon)^2 \langle \text{Nu} \rangle_{D,p} k_f} = \frac{(0.08)^2(\text{m}^2) \times 0.4}{6 \times (2)^3(\text{m}^3) \times (1-0.4)^2 \times 62.70 \times 0.0331(\text{W/m-K})} = 7.138 \times 10^{-5}\,^\circ\text{C/W}.$$

The number of transfer units is given by (7.20), i.e.,

$$\begin{aligned} NTU &= \frac{1}{\langle R_{ku}\rangle_L \dot{M} c_p} \\ &= \frac{1}{7.140\times 10^{-4}(^\circ\text{C/W}) \times 4(\text{kg/s}) \times 1{,}009(\text{J/kg-K})} = 3.471 \end{aligned}$$

(d) The Biot number is defined by (6.128) as

$$\begin{aligned} \text{Bi}_L &= \frac{R_{k,s}}{\langle R_{ku}\rangle_L} \\ &= \frac{L/(L^2\langle k\rangle)}{\langle R_{ku}\rangle_L} = \frac{1}{7.138\times 10^{-5}(^\circ\text{C/W}) \times 2(\text{m}) \times 0.6522(\text{W/m-K})} = 1.075\times 10^4 \end{aligned}$$

This shows that temperature nonuniformity within the bed (along the flow direction) cannot be justifiably neglected. However, the inclusion of axial conduction, which can be readily done by dividing the bed length into small, uniform-temperature volumes (i.e., small-finite volumes), will add to the length of analysis.

(e) Using (6.156) for the uniform solid temperature, along with the average convection resistance $\langle R_u\rangle_L$, for Figure Pr.7.16(b), we have

$$\begin{aligned} T_s(t) &= \langle T_f\rangle_0 + [T_s(t=0) - \langle T_f\rangle_0]e^{-t/\tau_s} \\ \tau_s &= (\rho c_p V)_s \langle R_u\rangle_L, \end{aligned}$$

where $Q_s = \dot{S}_s = a_s = 0$.
From (7.27), we have

$$\begin{aligned} \langle R_u\rangle_L &= \frac{1}{(\dot{M} c_p)_f (1 - e^{-NTU})} \\ &= \frac{1}{4(\text{kg/s}) \times 1{,}009(\text{J/kg-K}) \times (1 - e^{-3.471})} = 2.5572\times 10^{-4}\,^\circ\text{C/W}. \end{aligned}$$

Then

$$\begin{aligned} \tau_s &= (\rho c_p)_s V(1-\epsilon)\langle R_u\rangle_L \\ &= 7{,}830(\text{kg/m}^3) \times 434(\text{J/kg-K}) \times (2)^3(\text{m}^3) \times (1-0.4) \times 2.5572\times 10^{-4}(\text{K/W}) \\ &= 4{,}171\text{ s} = 1.159\text{ hr}. \end{aligned}$$

The instantaneous air stream exit temperature $\langle T_f\rangle_L$ is found from (7.21), i.e.,

$$\langle T_f\rangle_L(t) = T_s(t) + [\langle T_f\rangle_0 - T_s(t)]e^{-NTU}.$$

The results for $T_s(t)$ and $\langle T_f\rangle_L(t)$ are plotted in Figure Pr.7.16(c), for $0 \le t \le 4\tau_s$. The results show that $\langle T_f\rangle_L(t)$ is only slightly larger than $T_s(t)$. After four time constants, $T_s(t)$ and $\langle T_f\rangle(t)$ approach $\langle T_f\rangle_0 = 190^\circ$C.

(f) The amount of heat stored in the bed is found from integrating the energy equation, i.e.,

$$\begin{aligned} \int_0^{4\tau_s} Q_{ku}\,dt &= (\rho c_p V)_s[T_s(t=4\tau_s) - T_s(t=0)] \\ &= 7{,}830(\text{kg/m}^3) \times 434(\text{J/kg-}^\circ\text{C}) \times 2^3(\text{m}^3) \times (1-0.4) \times (187.06 - 30)(^\circ\text{C}) \\ &= 2.562\times 10^9\text{ J}. \end{aligned}$$

COMMENT:

The assumption of uniform bed temperature can be relaxed by dividing the bed into small, finite volumes. Also we have used the instantaneous value of $T_s(t)$ to determine the exits temperature, i.e., no allowance is made for the residence time of the fluid particles. Note that since $(\dot{M} c_p)_f$ is much smaller than $(M c_p)_s$, a large elapsed time is needed to heat the solid.

Note that we could have used $\text{Bi}_L = R_{k,s}/\langle R_u\rangle_L$ and this would have given $\text{Bi}_L = 2{,}998$, which is still very high.

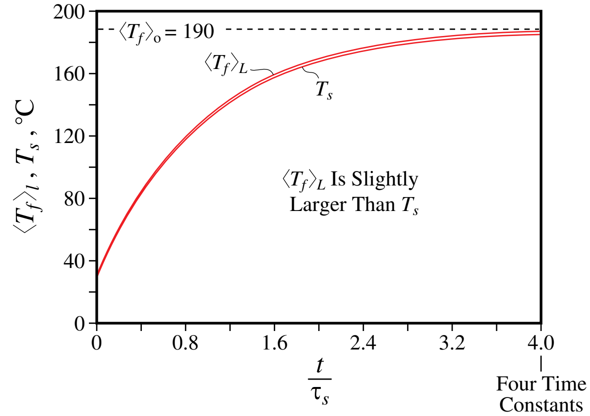


Figure Pr.7.16(c) Variation of the bed temperature and the fluid exit temperature with respect to time.

PROBLEM 7.17.FAM.S

GIVEN:

To improve the surface-radiation heat transfer from a fireplace, metallic chains are suspended above the flame, as shown in Figure Pr.7.17(a). The hot, thermobuoyant flue gas flows through and around the chains. The mass flow rate through the chains can be estimated from the fluid flow friction and the available thermobuoyant force and is assumed known here. In steady-state, this surface-convection heat transfer is balanced by the surface-radiation exchange with the surroundings, which is at T_∞. The radiating surface can be modeled as the surface area of a solid rectangle of dimension $w \times w \times L$ [as shown in Figure Pr.7.17(a)] scaled by the solid fraction term $(1 - \epsilon)$, where ϵ is the porosity. Only the surface radiation from the surface facing the surrounding is considered. The entire chain is assumed to have a uniform temperature T_s.

$\dot{M}_f = 0.007$ kg/s, $\langle T_f \rangle_0 = 600$℃, $\epsilon = 0.7$, $A_{ku} = 3$ m^3, $D_p = 4$ mm, $\epsilon_{r,s} = 1$, $w = 30$ cm, $L = 50$ cm, $T_\infty = 20$℃.

Treat the flue gas as air and evaluate the properties at $T = 500$ K.

SKETCH:

Figure Pr.7.17(a) shows the suspended chains.

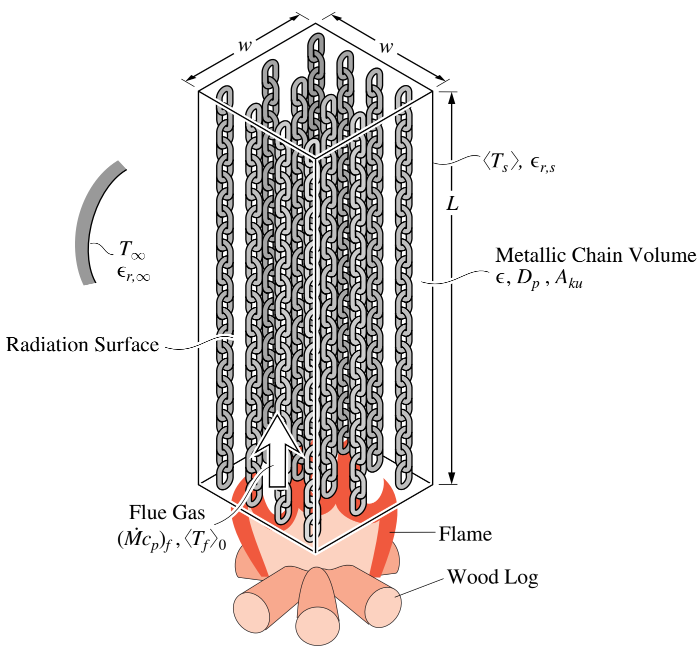


Figure Pr.7.17(a) Chains suspended above flame for enhanced surface radiation to the surroundings.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the chain temperature T_s.

(c) Determine the surface-radiation heat transfer rate $Q_{r,s\text{-}\infty}$.

SOLUTION:

(a) Figure Pr.7.17(b) shows the thermal circuit diagram.

(b) From the thermal circuit diagram, the energy equation is

$$\begin{aligned} Q|_A &= \langle Q_u \rangle_{L\text{-}0} + Q_{r,s\text{-}\infty} = 0 \\ &= \frac{E_{b,s} - E_{b,\infty}}{R_{r,\Sigma}} + \frac{T_s - \langle T_f \rangle_0}{\langle R_u \rangle_L}. \end{aligned}$$

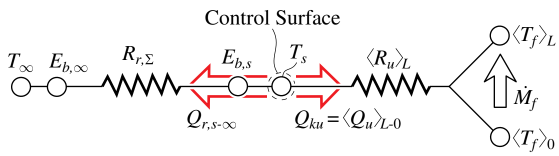


Figure Pr.7.17(b) Thermal circuit diagram.

The surface-radiation resistance $R_{r,\Sigma}$ is given by (4.48) as

$$\begin{aligned} R_{r,\Sigma} &= \frac{1-\epsilon_{r,s}}{\epsilon_{r,s}A_{r,s}} + \frac{1}{A_{r,s}F_{s\text{-}\infty}} + \frac{1-\epsilon_{r,\infty}}{\epsilon_{r,\infty}A_\infty} \\ &= \frac{1}{\epsilon_{r,s}A_{r,s}}, \quad \text{for} \quad A_\infty \gg A_{r,s} \quad \text{and} \quad F_{s\text{-}\infty} = 1. \end{aligned}$$

The surface area used for radiation is

$$A_{r,s} = wL(1-\epsilon).$$

The average-convection resistance $\langle R_u\rangle_L$ is defined in Table 7.1, i.e.,

$$\begin{aligned} \langle R_u\rangle_L &= \frac{1}{\dot{M}c_p(1-e^{-NTU})} \\ NTU &= \frac{1}{\langle R_{ku}\rangle_{D,p}(\dot{M}c_p)_f}. \end{aligned}$$

The surface-convection resistance $\langle R_{ku}\rangle_{D,p}$ is given in Table 7.5, for the porous medium, through

$$\begin{aligned} \langle \mathrm{Nu}\rangle_{D,p} &= \frac{D_p}{A_{ku}\langle R_{ku}\rangle_{D,p}k_f}\frac{\epsilon}{1-\epsilon} \\ &= 2 + (0.4\mathrm{Re}_{D,p}^{1/2} + 0.2\mathrm{Re}_{D,p}^{2/3})\mathrm{Pr}^{0.4}. \end{aligned}$$

The Reynolds number is defined by (7.45) as

$$\mathrm{Re}_D = \frac{\rho_f\langle u_f\rangle D_p}{\mu_f(1-\epsilon)} = \frac{\dot{M}D_p}{A_u\rho_f\nu_f(1-\epsilon)}, \quad A_u = w^2.$$

We now use the numerical values to determine $\langle R_u\rangle_L$. From Table C.22, at $T = 500$ K, we have

$\rho_f = 0.706\ \mathrm{kg/m^3}$	Table C.22
$k_f = 0.0395$ W/m-K	Table C.22
$c_{p,f} = 1{,}017$ J/kg-K	Table C.22
$\nu_f = 3.730 \times 10^{-5}\ \mathrm{m^2/s}$	Table C.22
$\mathrm{Pr} = 0.69$	Table C.22

Then

$$\begin{aligned} \mathrm{Re}_{D,p} &= \frac{0.007(\mathrm{kg/s}) \times (4\times 10^{-3})(\mathrm{m})}{(0.30)^2(\mathrm{m^2}) \times 0.706(\mathrm{kg/m^3}) \times 3.730 \times 10^{-5}(\mathrm{m^2/s}) \times (1-0.7)} = 39.38 \\ \langle \mathrm{Nu}\rangle_{D,p} &= 2 + [0.4(39.38)^{1/2} + 0.2(39.38)^{2/3}](0.69)^{0.4} = 6.160 \\ \langle R_{ku}\rangle_{D,p} &= \frac{D_p}{A_{ku}\langle \mathrm{Nu}\rangle_{D,p}k_f}\frac{\epsilon}{1-\epsilon} \\ &= \frac{4\times 10^{-3}(\mathrm{m})}{3(\mathrm{m^2}) \times 6.160 \times 0.0395(\mathrm{W/m\text{-}K})}\frac{0.7}{1-0.7} = 0.01279^\circ\mathrm{C/W} \\ NTU &= \frac{1}{0.01279(^\circ\mathrm{C/W}) \times 0.007(\mathrm{kg/s}) \times 1{,}017(\mathrm{J/kg\text{-}^\circ C})} = 10.99 \\ \langle R_u\rangle_L &= \frac{1}{0.007(\mathrm{kg/s}) \times 1{,}017(\mathrm{J/kg\text{-}^\circ C}) \times (1-e^{-10.99})} = 0.1405^\circ\mathrm{C/W}. \end{aligned}$$

Returning to the energy equation, we have

$$A_{r,s}\epsilon_{r,s}\sigma_{\rm SB}(T_s^4 - T_\infty^4) + \frac{T_s - \langle T_f\rangle_0}{\langle R_u\rangle_L} = 0$$

$$0.3({\rm m}) \times 0.5({\rm m}) \times (1-0.7) \times 1 \times 5.67 \times 10^{-8}({\rm W/m^2\text{-}K^4}) \times (T_s^4 - 293.15^4)({\rm K}^4) + \frac{(T_s - 873.15)({\rm K})}{0.1405({\rm K/W})} = 0.$$

Solving for T_s using a solver (or by iteration), we have

$$T_s = 757.7 \text{ K} = 484.6℃.$$

(c) The surface-radiation heat transfer is

$$Q_{r,s\text{-}\infty} = 0.3({\rm m}) \times 0.5({\rm m}) \times 0.3 \times 5.67 \times 10^{-8}({\rm W/m\text{-}K^4})(757.7^4 - 293.15^4) = 822.0 \text{ W}.$$

COMMENT:

The mass flow rate through the chains was estimated and perhaps is large. Lower $\dot{M}_f$ would still lead to a sufficiently large NTU for a significant surface-convection heating of the chains. But, $Q_{r,s\text{-}\infty}$ will be lower. Note that $\dot{M}c_p(\langle T_f\rangle_0 - T_\infty) = 4{,}129$ W, which indicates that the heating effectiveness is still rather low.

PROBLEM 7.18.FAM

GIVEN:

Two printed circuit boards containing Joule heating components are placed vertically and adjacent to each other for a surface-convection cooling by the thermobuoyant motion of an otherwise quiescent air. This is shown in Figure Pr.7.18(a). Consider surface-convection from both sides of each board.

$w = 10$ cm, $L = 15$ cm, $l = 4$ cm, $T_s = 65$°C, $T_{f,\infty} = 30$°C.

Determine the air properties at $\langle T_f \rangle_\delta = (T_s + T_{f,\infty})/2$.

SKETCH:

Figure Pr.7.18(a) shows the boards and the thermobuoyant motion.

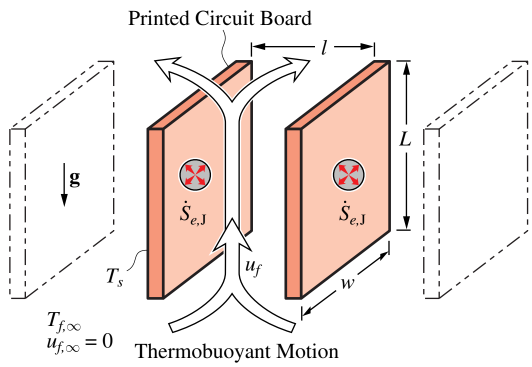


Figure Pr.7.18(a) Two printed circuit boards are placed vertically and adjacent to each other for a surface-convection cooling by the thermobuoyant air motion.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the Joule heating rate $\dot{S}_{e,\mathrm{J}}$, per board, for the following conditions.

SOLUTION:

(a) Figure Pr.7.18(b) shows the thermal circuit diagram for each of the boards. The energy equation for each board is

$$Q|_A = 2\langle Q_{ku} \rangle_l = \dot{S}_{e,J}.$$

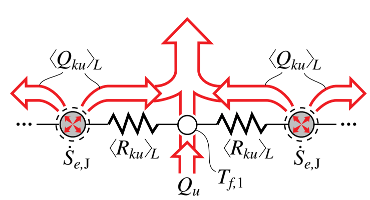


Figure Pr.7.18(b) Thermal circuit diagram.

(b) From (7.9), we have

$$\langle Q_{ku} \rangle_l \quad = \quad A_{ku} \langle \mathrm{Nu} \rangle_l \frac{k_f}{l} (T_s - T_{f,\infty}),$$

where $\langle \mathrm{Nu} \rangle_l$ is given in Table 7.4, i.e.,

$$\begin{aligned}
\mathrm{Nu}_l &= \left[\left(\frac{\mathrm{Ra}_l}{24} \right)^{-1.9} + (a_1 \mathrm{Ra}_l^{1/4})^{-1.9} \right]^{-1/1.9} \\
a_1 &= \frac{4}{3} \frac{0.503}{\left[1 + \left(\frac{0.492}{\mathrm{Pr}} \right)^{9/16} \right]^{4/9}} \\
\mathrm{Ra}_l &= \frac{g\beta_f (T_s - T_{f,\infty}) l^3}{\nu_f \alpha_f} \frac{l}{L}.
\end{aligned}$$

From Table C.22, for air at

$$\begin{aligned}
\langle T_f \rangle_\delta &= \frac{T_s + T_{f,\infty}}{2} = \left(\frac{65 + 30}{2} + 273.15 \right) (\mathrm{K}) \\
&= 320.65 \text{ K},
\end{aligned}$$

We have

$$\begin{aligned}
&k_f = 0.0281 \text{ W/m-K} && \text{Table C.22} \\
&\nu_f = 1.744 \times 10^{-5} \text{ m}^2/\text{s} && \text{Table C.22} \\
&\alpha_f = 2.526 \times 10^{-5} \text{ m}^2\text{-s} && \text{Table C.22} \\
&\mathrm{Pr} = 0.69 && \text{Table C.22} \\
&\beta_f = \frac{1}{T_f} = \frac{1}{320.65(\mathrm{K})} = 3.119 \times 10^{-3} \text{ 1/K} && (6.77).
\end{aligned}$$

Then

$$\begin{aligned}
\mathrm{Ra}_l &= \frac{9.81(\mathrm{m/s^2}) \times 3.119 \times 10^{-3}(1/\mathrm{K}) \times (65 - 30)(\mathrm{K}) \times (0.04)^3(\mathrm{m}^3)}{1.744 \times 10^{-5}(\mathrm{m^2/s}) \times 2.526 \times 10^{-5}(\mathrm{m^2/s})} \frac{(0.04)(\mathrm{m})}{0.15(\mathrm{m})} \\
&= 4.149 \times 10^4 \\
a_1 &= \frac{4}{3} \frac{0.503}{\left[1 + \left(\frac{0.492}{0.69} \right)^{9/6} \right]^{4/9}} \\
&= 0.5131 \\
\langle \mathrm{Nu}_l \rangle &= \left\{ \left(\frac{4.149 \times 10^4}{24} \right)^{-1.9} + \left[0.5131 \times \left(4.149 \times 10^4 \right)^{1/4} \right]^{-1.9} \right\}^{-1/1.9} \\
&= (7.052 \times 10^{-7} + 2.276 \times 10^{-2})^{-\frac{1}{1.9}} \\
&= 7.323.
\end{aligned}$$

For each surface, the surface-convection surface is $A_{ku} = 2wL$, then

$$\begin{aligned}
\langle Q_{ku} \rangle_l &= 2 \times 0.1(\mathrm{m}) \times 0.15(\mathrm{m}) \times 7.323 \times \frac{0.0281(\mathrm{W/m\text{-}K})}{0.04(\mathrm{m})} \times (65 - 30)(\mathrm{K}) \\
&= 5.402 \text{ W}.
\end{aligned}$$

From the energy equation

$$\dot{S}_{e,J} = 2 \langle Q_{ku} \rangle_l = 10.80 \text{ W}.$$

COMMENT:

Note that the laminar channel flow contribution is small and in this case negligible.

PROBLEM 7.19.FAM

GIVEN:

A solar collector is placed horizontally, and in order to reduce the heat losses to ambient air, a glass sheet is placed on top of it with the gap occupied by air. This is shown in Figure Pr.7.19(a). The energy absorbed by the collector surface $\dot{S}_{e,\alpha}$ is used to heat a water stream flowing underneath it, Q_c, or lost to air above it through $\langle Q_{ku}\rangle_L$. Assume a unit surface area of $A_c = 1$ m^2 and otherwise treat the collector surface as being infinity large in both directions. The heat transfer between the collector surface and the glass surface is by cellular thermobuoyant motion of the enclosed air. The Nusselt number correlation for this motion is given in Table 7.4 with the gap distance designated as L.

$\dot{S}_{e,\alpha}/A_c = \alpha_{r,c}(q_{r,i})_s = 400$ W/m^2, $L = 2$ cm, $T_{s,1} = 60$°C, $T_{s,2} = 35$°C.

Determine the air properties at $\langle T_f\rangle = (T_{s,1} + T_{s,2})/2$.

SKETCH:

Figure Pr.7.19(a) shows the collector, the glass cover, and a water (coolant) stream removing the heat for the collector at a rate per unit area Q_c/A_c.

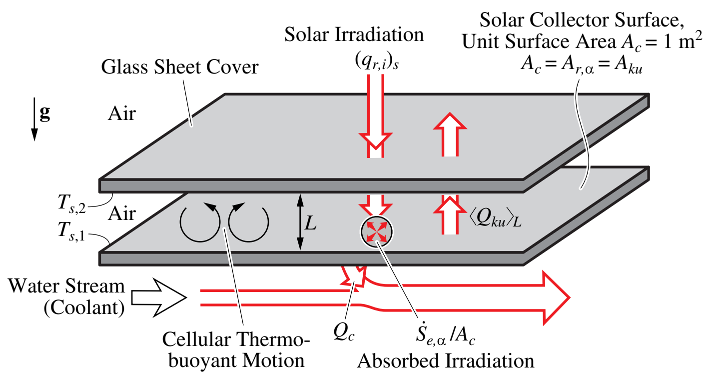


Figure Pr.7.19(a) A glass cover is used to reduce heat loss from a solar collector surface to ambient air. There is a cellular motion in the air gap between the two surfaces.

OBJECTIVE:

(a) Draw the thermal circuit diagram for the collector.

(b) Determine the rate of heat transfer per unit collector surface to the coolant (water) stream Q_c/A_c.

SOLUTION:

(a) Figure Pr.7.19(b) shows the thermal circuit diagram for the collector. The heat absorbed is lost to the air through the gap cellular motion, $\langle Q_{ku}\rangle_l$, and to the water stream, Q_c. Then

$$Q|_{A,c} = Q_c + \langle Q_{ku}\rangle_L = \dot{S}_{e,\alpha}$$

or

$$\frac{Q_c}{A_c} + \frac{\langle Q_{ku}\rangle L}{A_c} = \frac{\dot{S}_{e,\alpha}}{A_c},$$

here $A_c = A_{ku} = A_{r,\alpha}$.

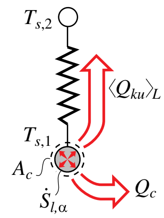


Figure Pr.7.19(b) Thermal circuit diagram.

(b) From Table 7.4, we have

$$\begin{aligned}
\langle Q_{ku}\rangle_L &= A_{ku}\langle \mathrm{Nu}\rangle_L \frac{k_f}{L}(T_{s,1}-T_{s,2}) \\
\langle \mathrm{Nu}\rangle_L &= 1+\left[1-\frac{1{,}708}{\mathrm{Ra}_L}\right]^* \left[a_1+2\left(\frac{\mathrm{Ra}_L^{1/3}}{a_2}\right)^{1-\ln\left(\mathrm{Ra}_L^{1/3}/a_2\right)}\right] + \left[\left(\frac{\mathrm{Ra}_L}{5{,}830}\right)^{1/3}-1\right]^* \\
a_1 &= \frac{1.44}{1+\dfrac{0.018}{\mathrm{Pr}}+\dfrac{0.00136}{\mathrm{Pr}^2}} \\
a_2 &= 75e^{1.5\mathrm{Pr}^{-1/2}} \\
\mathrm{Ra}_L &= \frac{g\beta_f(T_{s,1}-T_{s,2})L^3}{\nu_f\alpha_f}.
\end{aligned}$$

From Table C.22, for air at

$$\begin{aligned}
\langle T_f\rangle_\delta &= \left(\frac{60+35}{2}+273.15\right)(\mathrm{K}) \\
&= 320.65\ \mathrm{K},
\end{aligned}$$

we have

$$\begin{aligned}
k_f &= 0.0281\ \mathrm{W/m\text{-}K} && \text{Table C.22} \\
\nu_f &= 1.750\times 10^{-5}\ \mathrm{m^2/s} && \text{Table C.22} \\
\alpha_f &= 2.535\times 10^{-5}\mathrm{m^2\text{-}s} && \text{Table C.22} \\
\mathrm{Pr} &= 0.69 && \text{Table C.22} \\
\beta_f &= \frac{1}{\langle T_f\rangle} = \frac{1}{320.65(\mathrm{K})} = 3.119\times 10^{-3}\ 1/\mathrm{K} && (6.77).
\end{aligned}$$

Then

$$\begin{aligned}
\mathrm{Ra}_L &= \frac{9.81(\mathrm{m/s^2})\times 3.119\times 10^{-3}(1/\mathrm{K})\times(60-35)(\mathrm{K})\times(0.02)^3(\mathrm{m^3})}{1.750\times 10^{-5}(\mathrm{m^2/s})\times 2.535\times 10^{-5}(\mathrm{m^2/s})} \\
&= 1.373\times 10^4 \\
a_1 &= \frac{1.44}{1+\dfrac{0.018}{0.69}+\dfrac{0.00136}{(0.69)^2}} = 1.3995 \\
a_2 &= 75e^{1.5(0.69)^{-1/2}} = 456.36,
\end{aligned}$$

and so,

$$\begin{aligned}
\langle \mathrm{Nu} \rangle_L &= 1 + \left[1 - \frac{1{,}708}{1.373 \times 10^4}\right]^* \times \left\{1.3995 + 2 \times \left[\frac{(1.373 \times 10^4)^{1/3}}{456.36}\right]^{1 - \ln \frac{(1.373 \times 10^4)^{1/3}}{456.36}}\right\} \\
&\quad + \left[\left(\frac{1.373 \times 10^4}{5{,}830}\right)^{1/3} - 1\right]^* \\
&= 1 + 0.8752 \times \left[1.401 + 2 \times (5.243 \times 10^{-2})^{1+2.9483}\right] + [1.330 - 1]^* \\
&= 1 + 1.2262 + 0.3305 = 2.557.
\end{aligned}$$

Note that all terms inside [ ]* are positive and are therefore included.
Then,

$$\begin{aligned}
\langle Q_{ku} \rangle_L &= 1(\mathrm{m}^2) \times 2.557 \times \frac{0.0281(\mathrm{W/m\text{-}K})(60 - 35)(\mathrm{K})}{0.02(\mathrm{m})} \\
&= 89.81 \ \mathrm{W}.
\end{aligned}$$

Using the energy equation,

$$\begin{aligned}
\frac{Q_c}{A_c} &= \frac{\dot{S}_{e,\alpha}}{A_c} - \frac{\langle Q_{ku} \rangle_L}{A_c} \\
&= 400(\mathrm{W/m}^2) - 89.81(\mathrm{W/m}^2) \\
&= 310.2 \ \mathrm{W/m}^2.
\end{aligned}$$

COMMENT:

Note that about 25% of the heat absorbed is lost to the ambient. By reducing the width of the air gap this heat loss can be reduced.

PROBLEM 7.20.FUN

GIVEN:

In order to melt the winter ice formed on a concrete surface, hot-water carrying tubes are embedded in the concrete, near its surface. This is shown in Figure Pr.7.20(a). Assume a steady-state heat transfer. The heat flows from the hot-water stream, through the tube wall and through the concrete to the surface (at temperature T_c). There it melts the ice with the phase change energy conversion rate designated with $\dot{S}_{sl}$.

D = 3 cm, l = 5 cm, w = 10 cm, $\langle T_f \rangle_0$ = 25°C, T_c = 0°C, k_c(concrete) = 1.0 W/m-K, $\langle u_f \rangle$ = 0.5 m/s, $a = L = 5$ m.

Assume that the tubes have a negligible thickness. Determine the water properties at T = 290 K. The heat of melting for water is given in Table C.4. Note that for a uniform melting, a small NTU is used.

SKETCH:

Figure Pr.7.20(a) shows the buried pipes and its geometrical parameters.

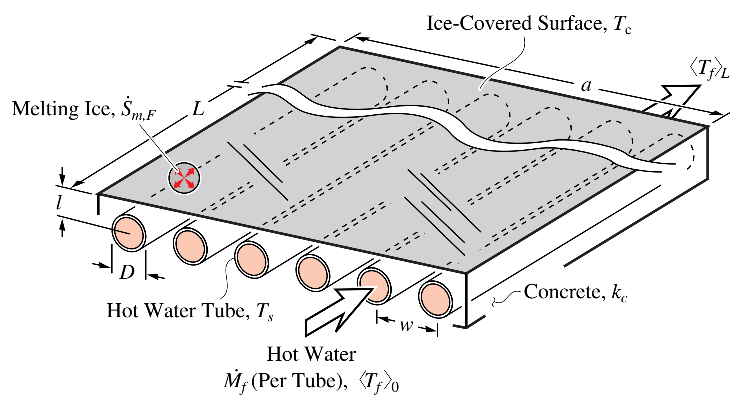


Figure Pr.7.20(a) Hot-water carrying tubes are placed near a concrete surface to melt the winter ice formed on the surface.

OBJECTIVE:

(a) Draw the thermal circuit diagram for the heat transfer between the hot-water stream and the concrete surface.
(b) Determine the Nusselt number $\langle \mathrm{Nu} \rangle_D$.
(c) Determine the surface-convection resistance $\langle R_{ku} \rangle_D$.
(d) Determine the concrete conduction resistance between the tube surface and concrete surface $R_{k,s\text{-}1}$, using Table 3.3(a). Divide the per-tube resistance by the number of tubes $N_t = a/w$ to obtain the total resistance.
(e) Determine the rate of heat transfer $\langle Q_u \rangle_{L\text{-}0}$ and the rate of ice melting $\dot{M}_{sl}$.

SOLUTION:

(a) Figure Pr.7.20(b) shows the thermal circuit diagram. The heat transfer from the hot-water stream to the tube surface, which is the same as that flowing through concrete, is labeled as $-\langle Q_{ku} \rangle_{L\text{-}0}$. The energy equation for concrete surface, from Figure Pr.7.20(b), is

$$\begin{aligned} Q|_A &= \langle Q_u \rangle_{L\text{-}0} = \dot{S}_{sl} \\ &= -\dot{M}_{sl} \Delta h_{sl}, \end{aligned}$$

where we have used Table 2.1 for $\dot{S}_{lg}$. The heat transfer rate is from, Figure Pr.7.20(b),

$$\langle Q_u \rangle_{L-0} = \frac{T_c - \langle T_f \rangle_o}{\langle R_u \rangle_L}.$$

Note that the top temperature node is T_c. The tube surface temperature T_s will be a function of the axial location in the tube. The resistance $\langle R_u \rangle_L$ must then take into account both convection and conduction effects. This will be seen in the form of R_Σ used in the calculation of NTU, and is a prelude to the heat exchanger problems discussed later in the chapter. Figure 7.12 illustrates the different resistances that exist in tube flow when the substrate is included. It is up to the heat transfer analyst to determine which of these resistances are important.

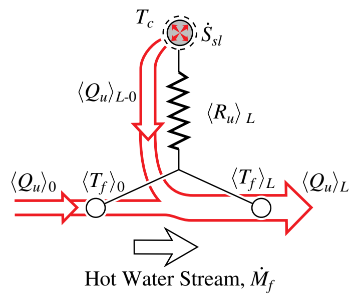


Figure Pr.7.20(b) Thermal circuit diagram.

(b) The Nusselt number is found from Tables 7.2 or 7.3, depending on the magnitude of Re_D. From (7.36), we have

$$\mathrm{Re}_D = \frac{\langle u_f \rangle D}{\nu_f}.$$

From Table C.23, at $T = 290$ K, for water we have

$$\begin{aligned}
k_f &= 0.590 \text{ W/m-K} && \text{Table C.23} \\
\rho_f &= 1{,}000 \text{ kg/m}^3 && \text{Table C.23} \\
c_{p,f} &= 4{,}186 \text{ J/kg-K} && \text{Table C.23} \\
\nu_f &= 1.13 \times 10^{-6} \text{m}^2\text{-s} && \text{Table C.23} \\
\mathrm{Pr} &= 8.02 && \text{Table C.23} \\
\mathrm{Re}_D &= \frac{0.5(\text{m/s}) \times 0.03(\text{m})}{1.13 \times 10^{-6}(\text{m}^2/\text{s})} = 13{,}274 > \mathrm{Re}_{D,t} = 2{,}300 && \text{turbulent-flow regime.}
\end{aligned}$$

From Table 7.3, we have

$$\begin{aligned}
\langle \mathrm{Nu} \rangle_D &= 0.023\mathrm{Re}^{4/5}\mathrm{Pr}^{0.3}, \quad T_s < \langle T_f \rangle_0 \\
&= 0.023 \times (13{,}274)^{4/5} \times (8.02)^{0.3} \\
&= 85.39.
\end{aligned}$$

(c) The surface-convection resistance $\langle R_{ku} \rangle_D$ is defined in (7.19), i.e.,

$$\begin{aligned}
\langle R_{ku} \rangle_D &= \frac{D}{A_{ku} \langle \mathrm{Nu} \rangle_D k_f} \\
&= \frac{D}{N_t \pi D L \langle \mathrm{Nu} \rangle_D k_f} \\
&= \frac{w}{a \pi L \langle \mathrm{Nu} \rangle_D k_f} \\
&= \frac{0.10(\text{m})}{5(\text{m}) \times \pi \times 5(\text{m}) \times 85.39 \times 0.590(\text{W/m-K})} \\
&= 2.527 \times 10^{-5} \text{ K/W}.
\end{aligned}$$

(d) The resistance per tube is given in Table 3.3(a), and for N_t tubes we have

$$
\begin{aligned}
R_{k,s\text{-}c} &= = \frac{1}{N_t}\frac{\ln\left[\frac{2w}{\pi D}\sinh\left(\frac{2\pi l}{w}\right)\right]}{2\pi k_c L} \\
&= \frac{w\ln\left[\frac{2w}{\pi D}\sinh\left(\frac{2\pi l}{w}\right)\right]}{a2\pi k_c L} \\
&= \frac{0.10(\text{m})\times\ln\left\{\frac{2\times 0.10}{\pi\times 0.03}\times\sinh\left[\frac{2\times\pi\times 0.05(\text{m})}{0.10(\text{m})}\right]\right\}}{5(\text{m})\times 2\pi\times 1(\text{W/m-K})\times 5(\text{m})} \\
&= 6.366\times 10^{-4}(\text{K/W})\times\ln[2.122\sinh(3.142)] \\
&= 6.366\times 10^{-4}(\text{K/W})\times\ln(2.122\times 11.55) \\
&= 2.036\times 10^{-3}\ \text{K/W}.
\end{aligned}
$$

(e) The average convection resistance is given by (7.27), i.e.,

$$
\langle R_u\rangle_L = \frac{1}{(\dot{M}c_p)_f(1-e^{-NTU})}
$$

where from (7.74), we have

$$
NTU = \frac{1}{R_\Sigma(\dot{M}c_p)_f},
$$

and here we use $R_\Sigma = \langle R_{ku}\rangle_D + R_{k,s\text{-}c}$ so that

$$
\begin{aligned}
NTU &= \frac{1}{(\langle R_{ku}\rangle_D + R_{k,s\text{-}c}(\dot{M}c_p)_f} \\
\dot{M}_f &= N_t\rho_f\frac{\pi D^2}{4}\langle u_f\rangle \\
&= \frac{a}{w}\rho_f\frac{\pi D^2}{4}\langle u_f\rangle \\
&= \frac{5(\text{m})}{0.1(\text{m})}\times 1{,}000(\text{kg/m}^3)\times\pi\times\frac{(0.03)^2(\text{m}^2)}{4}\times 0.5(\text{m/s}) \\
&= 17.67\ \text{kg/s} \\
NTU &= \frac{1}{(2.527\times 10^{-5}+2.036\times 10^{-3})(\text{K/W})\times 17.67(\text{kg/s})\times 4{,}186(\text{J/kg-K})} \\
&= 0.006559 \\
\langle R_u\rangle_L &= \frac{1}{17.67(\text{kg/s})\times 4{,}186(\text{J/kg-K})\times[1-\exp(-0.006559)]} \\
&= 2.068\times 10^{-3}\ \text{K/W} \\
\langle Q_u\rangle_{L\text{-}0} &= \frac{(0-25)(\text{K})}{2.068\times 10^{-3}(\text{K/W})} \\
&= -1.209\times 10^4\ \text{W}.
\end{aligned}
$$

From Table C.4, for water

$$
\Delta h_{sl} = 3.336\times 10^5\ \text{J/kg} \quad \text{Table C.4.}
$$

Then

$$
\begin{aligned}
\dot{M}_{sl} &= -\frac{\langle Q_u\rangle_{L\text{-}0}}{\Delta h_{sl}} = -\frac{-1.209\times 10^4(\text{W})}{3.336\times 10^5(\text{J/kg})} \\
&= 0.03621\ \text{kg/s} \\
&= 36.24\ \text{g/s}.
\end{aligned}
$$

COMMENT:

Note that $\langle R_{ku}\rangle_D \ll R_{k,sc}$, i.e., the surface-convection resistance is small such that T_s is very close to $\langle T_f\rangle$ Also note that $\langle T_f\rangle$ changes only slightly along the tube. The hot-water exit temperature is found from (7.24), i.e.,

$$\begin{aligned}\langle T_f\rangle_L &= \langle T_f\rangle_0 + \frac{\langle Q_u\rangle_{L\text{-}0}}{(\dot{M}c_p)_f}\\ &= 25(^\circ\mathrm{C}) + \frac{-1.209\times 10^4(\mathrm{W})}{17.67(\mathrm{kg/s})\times 4{,}186(\mathrm{J/kg\text{-}K})}\\ &= 24.84^\circ\mathrm{C}.\end{aligned}$$

A lower mass flow rate may be used although it will not result in a uniform melting rate.

PROBLEM 7.21.FUN

GIVEN:

In an internal combustion engine, for the analysis of the surface-convection heat transfer on the inner surface of the cylinder, $\langle \mathrm{Nu} \rangle_D$ must be determined for the cylinder conditions. Figure Pr.7.21 gives a rendering of the problem considered. The Woschni correlation for this Nusselt number uses the averaged cylinder velocity and cylinder pressure and is

$$\langle \mathrm{Nu} \rangle_D = 0.035 \mathrm{Re}_D^m, \quad \mathrm{Re}_D = \frac{\rho_f D \langle u_f \rangle}{\mu_f},$$

where the averaged fluid velocity in the cylinder $\langle u_f \rangle$ is given by

$$\langle u_f \rangle \;=\; a_1 (\overline{u_p^2})^{1/2} + a_2 \frac{V_f (p_f - p_o)}{M_f (R_g/M)}, \quad (\overline{u_p^2})^{1/2} = 2L_d (rpm) \frac{2\pi}{60(\mathrm{s/min})}, \quad p_o = \frac{M_f}{V_f} R_g T_o.$$

Here p_o is motored pressure used as a reference pressure, which is determined with the intake manifold air temperature T_o. In the second term in the averaged velocity $\langle u_f \rangle$ expression, the pressure rise $p_f - p_o$ is caused by combustion (and is used only during the combustion period).

$m = 0.8$, $D = 0.125$ m, $L_d = 0.14$ m, $rpm = 1{,}600$ rpm, $T_o = 329$ K, $T_s = 800$ K, $R_g/M = 290.7$ J/kg-K.

Note that the pressure and temperature given above are selected for a diesel engine.

SKETCH:

Figure Pr.7.21 shows the cylinder of the internal combustion engine.

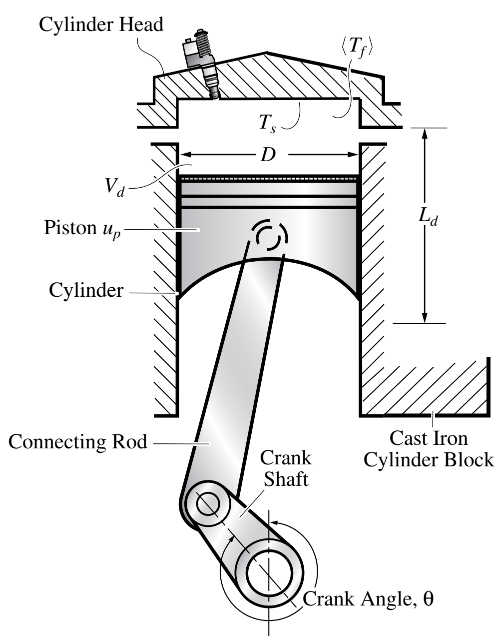


Figure Pr.7.21 Geometric parameters of the cylinder in a diesel internal combustion engine.

OBJECTIVE:

(a) Determine $\langle \mathrm{Nu}_D \rangle$ and $\langle q_{ku} \rangle_D$, for (i) the combustion period and (ii) the intake period.

(b) Comment on the magnitude of $\langle \mathrm{Nu} \rangle_D$ (i) during the combustion period ($a_1 = 2.28$, $a_2 = 3.24 \times 10^{-3}$ m/s, $M_f = 0.006494$ kg, $\langle T_f \rangle = 1{,}700$ K, $p_f = 23$ MPa, $V_f = 0.00013966$ m^3) and (ii) during the intake period ($a_2 = 0$, $p_f = 0.2$ MPa, $\langle T_f \rangle = 400$ K). Evaluate the properties of air for (i) at $T = \langle T_f \rangle = 1{,}700$ K, and for (ii) at $T = \langle T_f \rangle = 400$ K, using Table C.22. Use the ideal gas law and given pressure and temperature to evaluate the density of the gas (air).

SOLUTION:

In the internal combustion engine, the averaged Nusselt number $\langle \mathrm{Nu}\rangle_D$ for the surface-convection heat transfer is determined from the Woschni correlation using the average cylinder velocity and the cylinder pressure. This correlation for the $\langle \mathrm{Nu}\rangle_D$ is given as

$$\langle \mathrm{Nu}\rangle_D = 0.035\mathrm{Re}_D^m, \quad \mathrm{Re}_D = \frac{\rho_f D \langle u_f\rangle}{\mu_f}.$$

The average fluid velocity in the cylinder $\langle u_f\rangle$ is given by

$$\langle u_f\rangle = a_1(\overline{u_p^2})^{1/2} + a_2\frac{V_f(p_f - p_o)}{M_f(R_g/M)},$$

where

$$(\overline{u_p^2})^{1/2} = 2L_d(rpm)\frac{2\pi}{60(\mathrm{s/min})}, \quad p_m = \frac{M_f}{V_f}\frac{R_g}{M}T_o.$$

and during the combustion period, $a_1 = 2.28$, $a_2 = 3.24 \times 10^{-3}$ m/s, (ii) during the intake period, $a_2 = 0$. From Table C.22, we have for air at 1,700 K:

$$\begin{aligned}\nu_f &= 284.6 \times 10^{-6}\ \mathrm{m^2/s}\\ \rho_f &= 0.2114\ \mathrm{kg/m^3}\\ \mu_f &= \rho_f\nu_f = 60.16 \times 10^{-6}\\ k_f &= 0.09348\ \mathrm{W/m\text{-}K}.\end{aligned}$$

And then, as indicated, we recalculate the density using the ideal gas law:

$$\rho_f = \frac{p_f}{(R_g/M)\langle T_f\rangle} = \frac{23 \times 10^6(\mathrm{Pa})}{290.7(\mathrm{J/kg\text{-}K}) \times 1{,}700(\mathrm{K})} = 46.54\ \mathrm{kg/m^3}.$$

For air at 400 K,

$$\begin{aligned}\nu_f &= 25.50 \times 10^{-6}\ \mathrm{m^2/s}\\ \rho_f &= 0.883\ \mathrm{kg/m^3}\\ \mu_f &= \rho_f\nu_f = 22.52 \times 10^{-6}\\ k_f &= 0.0331\ \mathrm{W/m\text{-}K}.\end{aligned}$$

And then, as indicated, we recalculate the density using the ideal gas law:

$$\rho_f = \frac{p_f}{(R_g/M)\langle T_f\rangle} = \frac{0.2 \times 10^6(\mathrm{Pa})}{290.7(\mathrm{J/kg\text{-}K}) \times 400(\mathrm{K})} = 1.720\ \mathrm{kg/m^3}.$$

(a) (i) Combustion Period:
The averaged piston velocity $(\overline{u_p^2})^{1/2}$ is determined by

$$\begin{aligned}(\overline{u_p^2})^{1/2} &= 2L_d(\mathrm{m})(rpm)(1/\mathrm{min})\frac{2\pi}{60(\mathrm{s/min})}\\ &= 2 \times 0.14(\mathrm{m}) \times 1{,}600(1/\mathrm{min}) \times \frac{2\pi}{60(\mathrm{s/min})}\\ &= 46.91\ \mathrm{m/s}.\end{aligned}$$

and the motored pressure is determined as

$$\begin{aligned}p_o &= \frac{M_f}{V_f}R_gT_o\\ &= \frac{0.006494(\mathrm{kg})}{0.00013966(\mathrm{m^3})} \times 290.7(\mathrm{J/kg\text{-}K}) \times 329(\mathrm{K})\\ &= 4.447\ \mathrm{MPa}.\end{aligned}$$

Therefore, the effective fluid velocity $\langle u \rangle_f$ is

$$\begin{aligned}
\langle u_f \rangle &= a_1(\overline{u_p^2})^{1/2} + a_2 \frac{V_f(p_f - p_o)}{M_f(R_g/M)} \\
&= 2.28 \times 46.91(\text{m/s}) + 3.24 \times 10^{-3}(\text{m/s}) \times \frac{0.00013966(\text{m}^3) \times (23 - 4.447) \times 10^6(\text{Pa})}{0.006494(\text{kg}) \times 290.7(\text{J/kg-K})} \\
&= 106.96(\text{m/s}) + 4.45(\text{m/s}) \\
&= 111.4 \text{ m/s}.
\end{aligned}$$

The Reynolds number Re_D is

$$\begin{aligned}
\text{Re}_D &= \frac{\rho_f D \langle u_f \rangle}{\mu_f} \\
&= \frac{46.5(\text{kg/m}^3) \times 0.125(\text{m}) \times 111.4(\text{m/s})}{60.16 \times 10^{-6}(\text{kg/m-s})} \\
&= 10.763 \times 10^6.
\end{aligned}$$

Finally, the average Nusselt number $\langle \text{Nu} \rangle_D$ is

$$\begin{aligned}
\langle \text{Nu} \rangle_D &= 0.035 \text{Re}_D^{0.8} \\
&= 14{,}778.
\end{aligned}$$

The surface-convection heat flux $\langle q_{ku} \rangle_D$ is given by

$$\begin{aligned}
\langle q_{ku} \rangle_D &= \frac{\langle Q_{ku} \rangle_D}{A_{ku}} = \frac{\langle \text{Nu} \rangle_D k_f}{D}(\langle T_f \rangle - T_s) \\
&= \frac{14{,}778 \times 0.09348(\text{W/m-K})}{0.125(\text{m})} \times (1{,}700 - 800)(\text{K}) \\
&= 9{,}946.5 \text{ kW/m}^2.
\end{aligned}$$

(ii) Intake Period:
During the intake period, the pressure and temperature in the cylinder are $p_f = 0.2$ MPa, and $\langle T_f \rangle = 400$ K respectively.
Similarly to the above, the average fluid velocity $\langle u_f \rangle$ is

$$\begin{aligned}
\langle u_f \rangle &= 2.28 \times 46.91(\text{m/s}) \\
&= 106.96 \text{ m/s}.
\end{aligned}$$

The Reynolds number Re_D is

$$\begin{aligned}
\text{Re}_D &= \frac{1.720(\text{kg/m}^3) \times 0.125(\text{m}) \times 106.96(\text{m/s})}{22.52 \times 10^{-6}(\text{m}^2\text{-s})} \\
&= 1.022 \times 10^6.
\end{aligned}$$

Finally, the average Nusselt number $\langle \text{Nu} \rangle_D$ is

$$\begin{aligned}
\langle \text{Nu} \rangle_D &= 0.035 \text{Re}_D^{0.8} \\
&= 2{,}246.
\end{aligned}$$

The surface-convection heat flux $\langle q_{ku} \rangle_D$ is given by

$$\begin{aligned}
\langle q_{ku} \rangle_D &= \frac{\langle Q_{ku} \rangle_D}{A_{ku}} = \frac{\langle \text{Nu} \rangle_D k_f}{D}(\langle T_f \rangle - T_s) \\
&= \frac{2{,}246 \times 0.0331(\text{W/m-K})}{0.125(\text{m})} \times (400 - 800)(\text{K}) \\
&= -237.9 \text{ kW/m}^2.
\end{aligned}$$

(b) The Reynolds number for the combustion period is much higher than that for the intake period, and therefore, $\langle \mathrm{Nu} \rangle_D$ is much higher.

COMMENT:

Note that for the combustion period, the heat flux is positive, i.e., there is a heat loss through cylinder wall. For the intake, the heat flux is negative, i.e., the hot cylinder wall heats the cold intake air inside the cylinder, and this surface-convection heat transfer affects the volume efficiency during the intake period.

Note that the coefficients (a_1 and a_2) in the Woschni relation are determined by a calibration against engine test results. The coefficients also have different magnitudes for the other engine cycle periods, such as the intake, exhaust and compression periods.

PROBLEM 7.22.FAM.S

GIVEN:

Placing an air gap between brick walls can reduce the heat transfer across the composite, when the thermobuoyant motion in the air gap is not significant. Consider the one-dimensional heat flow through a composite of two brick walls and an air gap between them, as shown in Figure Pr.7.22(a).

$T_1 = 40$°C, $T_2 = 15$°C, $l_1 = l_2 = 10$ cm, $w = 6$ m, $L = 3$ m, $\langle k_1 \rangle = \langle k_2 \rangle = 0.70$ W/m-K.

Evaluate the air properties at $T = 300$ K and use $\beta_f = 2/[(T_1 + T_2)(\text{K})]$. Since $T_{s,1}$ and $T_{s,2}$ depend on $Q_{1\text{-}2}$, and in turn the overall resistance $R_{\Sigma,1\text{-}2}$ depends on $T_{s,1}$ and $T_{s,2}$, a solver should be used.

SKETCH:

Figure Pr.7.22(a) shows the composite and the thermobuoyant motion in the air gap.

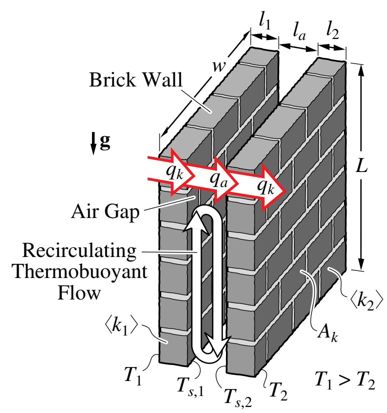


Figure Pr.7.22(a) An air gap placed between two brick walls to reduce heat transfer. The thermobuoyant flow in the air gap enclosure is also shown.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the steady-state heat flow rate $Q_{1\text{-}2}$ for the case of l_a equal to (i) 1 cm, (ii) 2 cm, and (iii) 4 cm.

(c) Comment on the minimum $Q_{1\text{-}2}$ and its corresponding air gap size l_a.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.7.22(b). The air gap is treated as a cavity and the surface-convection heat transfer across the cavity is shown by $Q_{ku,s1\text{-}s2}$ and the resistance as $R_{ku,s1\text{-}s2}$.

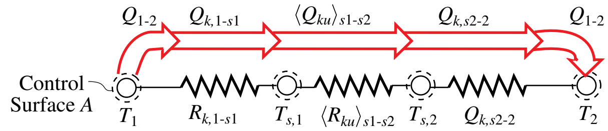


Figure Pr.7.22(b) Thermal circuit diagram.

(b) We note that $T_{s,1}$ and $T_{s,2}$ cannot be prescribed and are determined along with $Q_{1\text{-}2}$. We also note that $\langle R_{ku} \rangle_{s1\text{-}s2}$ depends on $T_{s,1} - T_{s,2}$ and is also determined along with $T_{s,1}$ and $T_{s,2}$. By examining Figure 7.22(b), we write the surface energy equation for nodes $T_1, T_{s,1}, T_{s,2}$ and T_2 as

$$\begin{aligned}
\text{node} \quad T_1 &: -Q_{1\text{-}2} + Q_{k,1\text{-}s1} = 0 \\
\text{node} \quad T_{s,1} &: -Q_{k,1\text{-}s1} + \langle Q_{ku}\rangle_{s1\text{-}s2} = 0 \\
\text{node} \quad T_{s,2} &: -\langle Q_{ku}\rangle_{s1\text{-}s2} + Q_{k,s1\text{-}2} = 0 \\
\text{node} \quad T_2 &: -Q_{k,s2\text{-}2} + Q_{1\text{-}2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
Q_{k,1\text{-}s1} &= \frac{T_1 - T_{s,1}}{R_{k,1\text{-}s1}} \\
\langle Q_{ku}\rangle_{s1\text{-}s2} &= \frac{T_{s,1} - T_{s,2}}{\langle R_{ku}\rangle_{s1\text{-}s2}} \\
Q_{k,s2\text{-}2} &= \frac{T_{s,2} - T_2}{R_{k,s2\text{-}2}} \\
R_{k,1\text{-}s1} &= R_{k,s2\text{-}2} = \frac{l_1}{A_k \langle k\rangle}, \ A_k = Lw \\
\langle R_{ku}\rangle_{s1\text{-}s2} &= \frac{l_a}{A_{ku}\langle \text{Nu}\rangle_{l_a} k_f}, \ A_{ku} = Lw.
\end{aligned}$$

From Table 7.5, we have for air (Pr = 0.7)

$$\begin{aligned}
\langle \text{Nu}\rangle_{l_a} &= (\text{Nu}_{l_a,k\text{-}t}, \text{Nu}_{l_a,l}, \text{Nu}_{l_a,t})_{max} \\
\text{Nu}_{l_a,k\text{-}t} &= \left\{1 + \left[\frac{0.104\text{Ra}_{l_a}^{0.293}}{1 + (6{,}310/\text{Ra}_{l_a})^{1.36}}\right]^3\right\}^{1/3} \\
\text{Nu}_{l_a,l} &= 0.242\left(\text{Ra}_{l_a}\frac{l_a}{L}\right)^{0.273} \\
\text{Nu}_{l_a,t} &= 0.0605\text{Ra}_{l_a}^{1/3} \\
\text{Ra}_l &= \frac{g\beta_f (T_{s,1} - T_{s,2}) l_a^3}{\nu_f \alpha_f}.
\end{aligned}$$

For the air properties, at $T = 300$ K, from Table C.22 we have

$$\begin{aligned}
\nu_f &= 1.566 \times 10^{-5} \ \text{m}^2/\text{s} && \text{Table C.22} \\
\alpha_f &= 2.257 \times 10^{-5} \ \text{m}^2/\text{s} && \text{Table C.22} \\
k_f &= 0.0267 \ \text{W/m-K} && \text{Table C.22.}
\end{aligned}$$

From (6.77) $\beta_f = 1/T_f$, $T_f = (T_1 + T_2)/2$, i.e.,

$$\begin{aligned}
T_f &= \frac{[(273.15 + 40) + (273.15 + 15)](\text{K})}{2} \\
&= 300.65 \ \text{K} \\
\beta_f &= 3.326 \times 10^{-3} \ 1/\text{K}.
\end{aligned}$$

Then

$$\text{Ra}_l = \frac{9.81(\text{m/s}^2) \times 3.326 \times 10^{-3}(1/\text{K}) \times (T_{s,1} - T_{s,2}) \times l_a^3(\text{m}^3)}{1.566 \times 10^{-5}(\text{m}^2/\text{s}) \times 2.257 \times 10^{-5}(\text{m}^2/\text{s})}.$$

We can combine the four energy equation giving us the unknowns

$$Q_{1\text{-}2} = Q_{k,1\text{-}s1} = Q_{ku,s1\text{-}s2} = Q_{k,s2\text{-}2},$$

and $T_{s,1}$ and $T_{s,2}$.
Using a solver, such as SOPHT, we have

$$\begin{array}{rlcll}
\text{(i)} & l_a = 1 \text{ cm} & : & T_{s,1} = 34.59^\circ\text{C}, \quad T_{s,2} = 20.41^\circ\text{C}, & Q_{1\text{-}2} = 681.7 \text{ W} \\
\text{(ii)} & l_a = 2 \text{ cm} & : & T_{s,1} = 35.67^\circ\text{C}, \quad T_{s,2} = 19.33^\circ\text{C}, & Q_{1\text{-}2} = 545.2 \text{ W} \\
\text{(iii)} & l_a = 4 \text{ cm} & : & T_{s,1} = 35.50^\circ\text{C}, \quad T_{s,2} = 19.50^\circ\text{C}, & Q_{1\text{-}2} = 567.1 \text{ W.}
\end{array}$$

(c) The minimum in $Q_{1\text{-}2}$ occurs here for $l_a = 2$ cm. With this air gap size, the gap is large enough to cause a significant resistance, but not large enough to have a large thermobuoyant motion that tends to decrease this resistance.

COMMENT:

Also note that for case (ii), we have

$$R_{k,1\text{-}s1} = 7.937 \times 10^{-3} \text{ K/W}, \quad \text{Ra}_l = 1.207 \times 10^4, \quad \langle \text{Nu} \rangle_l = 1.388, \quad R_{ku,s1\text{-}s2} = 0.02998 \text{ K/W}.$$

Here the Nusselt number corresponds to the third term in the $\langle \text{Nu} \rangle_l$ expression (i.e., it gives the maximum Nusselt number). For l_a values of 0.01 m and 0.04 m, it is the first expression that gives the maximum Nusselt number

PROBLEM 7.23.FUN

GIVEN:
In arriving at the axial temperature distribution of a bounded fluid stream (entering at a temperature of $\langle T_f \rangle_0$ and at a surface temperature of T_s), i.e., (7.22), the axial fluid conduction was neglected. This is valid for high Péclet number ($\text{Pe}_L = \langle u_f \rangle L/\alpha_f$) streams where L is the length along the flow. For low Pe_L, i.e., for low velocities or high α_f, this axial conduction may become significant.

The axial conduction can be added to the energy equation (7.11) for flow through a tube of diameter D, and this gives

$$-P_{ku} q_{ku} + A_{ku} \frac{d}{dx} q_u + A_{ku} \frac{d}{dx} q_k = 0$$

or

$$-P_{ku} q_{ku} + A_{ku} (\rho c_p)_f \langle u_f \rangle \frac{d \langle T_f \rangle}{dx} - A_{ku} k'_f \frac{d^2 \langle T_f \rangle}{dx} = 0,$$

where k'_f is the sum of fluid conduction and a contribution due to averaging of the nonuniform fluid temperature and velocity over tube cross-sectional area. This contribution is called the dispersion (or Taylor dispersion).

Here constant thermophysical properties are assumed. This axial conduction-convection and lateral surface-convection heat transfer in a bounded fluid stream is shown in Figure Pr.7.23.
Using the surface-convection resistance R_{ku} and the Nusselt number, we can write this energy equation as

$$-\frac{d^2 \langle T_f \rangle}{dx^2} + \frac{\langle u_f \rangle}{\alpha_f} \frac{d \langle T_f \rangle}{dx} + \frac{P_{ku} \langle \text{Nu} \rangle_D k_f}{A_{ku} D k_f} (\langle T_f \rangle - T_s) = 0 \qquad \alpha_f = \frac{k'_f}{(\rho c_p)_f},$$

where T_s is assumed constant.

The fluid thermal conditions at $x = 0$ and $x = L$ are

$$\langle T_f \rangle (x = 0) = \langle T_f \rangle_0$$
$$\langle T_f \rangle (x = L) = \langle T_f \rangle_L.$$

SKETCH:
Figure Pr.7.23 shows the axial conduction-convection in a fluid stream flowing through a tube with surface convection.

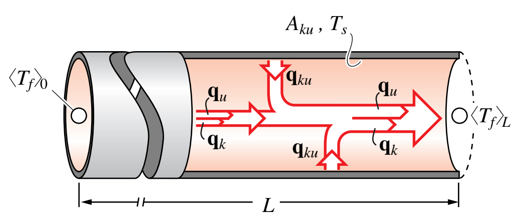


Figure Pr.7.23 Axial conduction-convection and lateral surface-convection heat transfer in a bound fluid stream.

OBJECTIVE:
(a) Using the length L and the temperature difference $T_s - \langle T_f \rangle_0$, show that the energy equation becomes

$$\frac{d^2 \langle T_f^* \rangle}{dx^{*2}} - \text{Pe}_L \frac{d \langle T_f^* \rangle}{dx^*} - \text{Pe}_L NTU \frac{D}{4L} \langle T_f^* \rangle = 0,$$

$$x^* = \frac{x}{L}, \quad \langle T_f^* \rangle = \frac{\langle T_f \rangle - T_s}{\langle T_f \rangle_0 - T_s}, \quad \text{Pe}_L = \frac{\langle u_f \rangle L}{\alpha_f},$$

$$NTU = \frac{A_{ku} \langle \text{Nu} \rangle_D k_f}{(\dot{M} c_p)_f D}, \quad (\dot{M} c_p)_f = A_u \rho_f \langle u_f \rangle c_{p,f},$$

$$A_{ku} = P_{ku} L, \ \langle T_f^* \rangle (x^* = 0) = 1, \ \langle T_f^* \rangle (x^* = 1) = \langle T_f^* \rangle_L.$$

(b) Show that the solution to this ordinary second-order differential equation gives the fluid axial temperature distribution as

$$\langle T_f^* \rangle(x^*) = \frac{\langle T_f \rangle - T_s}{\langle T_f \rangle_0 - T_s} = \frac{\langle T_f^* \rangle_L (e^{m_2 x^*} - e^{m_1 x^*}) + e^{m_2} e^{m_1 x^*} - e^{m_1} e^{m_2 x^*}}{e^{m_2} - e^{m_1}}.$$

Note that the solution to

$$\frac{d^2 \langle T_f^* \rangle}{dx^{*2}} - b \frac{d \langle T_f^* \rangle}{dx^*} - c \langle T_f^* \rangle = 0$$

is

$$\langle T_f^* \rangle(x^*) = a_1 e^{m_1 x*} + a_2 e^{m_2 x*},\ m_{1,2} = \frac{b \pm (b^2 + 4c)^{1/2}}{2}.$$

SOLUTION:
(a) We start with the first term and write

$$\frac{d^2 \langle T_f \rangle}{dx^2} = \frac{\langle T_f \rangle_0 - T_s}{L^2} \frac{d^2 \langle T_f^* \rangle}{dx^{*2}},$$

where we have used the given T_s = constant. Following this for the remaining terms, we have

$$\frac{\langle T_f \rangle_0 - T_s}{L^2} \frac{d^2 \langle T_f^* \rangle}{dx^{*2}} - \frac{\langle u_f \rangle (\langle T_f \rangle_0 - T_s)}{\alpha_f L} \frac{d \langle T_f \rangle}{dx^*} - \frac{P_{ku} \langle \mathrm{Nu} \rangle_D k_f (\langle T_f \rangle_0 - T_s)}{A_{k,u} D k_f^{'}} \langle T_f^* \rangle = 0.$$

or

$$\frac{d^2 \langle T_f^* \rangle}{dx^{*2}} - \frac{\langle u_f \rangle L}{\alpha_f} \frac{d \langle T_f^* \rangle}{dx^*} - \frac{P_{ku} L^2 \langle \mathrm{Nu} \rangle_D k_f}{A_{ku} D k_f^{'}} \langle T_f^* \rangle = 0.$$

From (7.20), NTU is defined as

$$NTU = \frac{A_{ku} \langle \mathrm{Nu} \rangle_D k_f}{(\dot{M} c_p)_f D}, \quad (\dot{M} c_p)_f = \rho_f A_u \langle u_f \rangle c_{p,f}, \quad \alpha_f = \frac{k_f^{'}}{(\rho c_p)_f},$$

then

$$\frac{P_{ku} L \langle \mathrm{Nu} \rangle_D k_f L}{A_{ku} D k_f^{'}} = \frac{A_{ku} \langle \mathrm{Nu} \rangle_D k_f \langle u_f \rangle (\rho c_p)_f L}{A_{ku} \langle u_f \rangle (\rho c_p)_f D k_f^{'}} = \mathrm{Pe}_L NTU \frac{D}{4L}.$$

Finally, we have

$$\frac{d^2 \langle T_f^* \rangle}{dx^{*2}} - \mathrm{Pe}_L \frac{d \langle T_f^* \rangle}{dx^*} - \mathrm{Pe}_L NTU \frac{D}{4L} \langle T_f^* \rangle = 0.$$

(b) From the roots given in the problem statement and by comparing the above with the coefficients b and c, we have

$$m_1 = \frac{1}{2} \left[\mathrm{Pe}_L + (\mathrm{Pe}_L + \mathrm{Pe}_L NTU \frac{D}{L})^{1/2} \right]$$
$$m_2 = \frac{1}{2} \left[\mathrm{Pe}_L - (\mathrm{Pe}_L + \mathrm{Pe}_L NTU \frac{D}{L})^{1/2} \right]$$
$$\langle T_f \rangle^* = a_1 e^{m_1 x^*} + a_2 e^{m_2 x^*}.$$

Then using

$$\langle T_f \rangle^* (x^* = 0) = 1, \quad \text{and} \quad \langle T_f^* \rangle (x^* = 1) = \frac{\langle T_f \rangle_L - T_s}{\langle T_f \rangle_0 - T_s} = \langle T_f^* \rangle_L,$$

we have

$$1 = a_1 + a_2 \quad \text{or} \quad a_1 = 1 - a_2$$

$$\langle T_f^* \rangle_L = a_1 e^{m_1} + a_2 e^{m_2} \quad \text{or} \quad a_2 = \frac{\langle T_f^* \rangle_L - a_1 e^{m_1}}{e^{m_2}}.$$

Then

$$a_1 = 1 - \frac{\langle T_f^* \rangle_L - a_1 e^{m1}}{e^{m_2}} \quad \text{or} \quad a_1 = \frac{e^{m_2} - \langle T_f^* \rangle}{e^{m_2} - e^{m_1}}$$

and

$$a_2 = \frac{\langle T_f^* \rangle_L - \dfrac{e^{m_2} - \langle T_f^* \rangle_L}{e^{m_2} - e^{m_1}} e^{m_1}}{e^{m_2}} = \langle T_f^* \rangle_L e^{-m_2} - \frac{e^{m_2} - \langle T_f^* \rangle_L}{e^{m_2} - e^{m_1}} e^{m_1 - m_2}.$$

Using these for a_1 and a_2, we have

$$\begin{aligned}
\langle T_f^* \rangle &= \frac{e^{m_2} - \langle T_f^* \rangle_L}{e^{m_2} - e^{m_1}} e^{m_1 x^*} + \langle T_f^* \rangle_L e^{-m_2} e^{m_2 x^*} - \frac{e^{m_2} - \langle T_f^* \rangle_L}{e^{m_2} - e^{m_1}} e^{m_1 - m_2} e^{m_2 x^*} \\
\langle T_f^* \rangle &= \frac{(e^{m_2} - \langle T_f^* \rangle_L) e^{m_1 x^*} + \langle T_f \rangle_L e^{-m_2} (e^{m_2} - e^{m_1}) e^{m_2 x^*} - (e^{m_2} - \langle T_f^* \rangle_L) e^{m_1 - m_2} e^{m_2 x^*}}{e^{m_2} - e^{m_1}} \\
\langle T_f^* \rangle (x^*) &= \frac{\langle T_f^* \rangle_L (e^{m_2 x^*} - e^{m_1 x^*}) + e^{m_2} e^{m_1 x^*} - e^{m_1} e^{m_2 x^*}}{e^{m_2} - e^{m_1}}.
\end{aligned}$$

COMMENT:

Note that for $NTU = 0$, we have $m_1 = \mathrm{Pe}_L$, $m_2 = 0$ and for a prescribed $\langle T_f^* \rangle_L = 0$, we have the result of (5.12), i.e.,

$$\langle T_f^* \rangle (x^*) = \frac{e^{\mathrm{Pe}_L x^*} - e^{\mathrm{Pe}_L}}{1 - e^{\mathrm{Pe}_L}} = 1 - \frac{e^{\mathrm{Pe}_L x^*} - 1}{e^{\mathrm{Pe}_L} - 1}, \quad \text{for } NTU = 0.$$

For $\mathrm{Pe}_L \to \infty$, we drop the second derivative term and will have the temperature $\langle T_f^* \rangle_L$ given by (7.21).

The contribution due to dispersion is proportional to Pe_D^2 and becomes significant for high $\mathrm{Pe}_D = \langle u_f \rangle D / \alpha_f$.

PROBLEM 7.24.FAM

GIVEN:

Plate-type heat exchangers are corrugated thin metallic plates held together in a frame, as shown in Figure Pr.7.24(a). Gasket or welding is used for sealing. The flow arrangement is counterflow with the periodic alternation shown in the figure.

Consider an assembly of N plates, each having a surface area $w \times L$ (making a total surface area NwL). The heat exchanger transfers heat between a combustion flue-gas stream with an inlet temperature $\langle T_{f,h}\rangle_0$ and a room temperature air stream with $\langle T_{f,c}\rangle_L$ and a mass flow rate $\dot{M}_{f,c} = \dot{M}_{f,h}$. The rate of heat transfer is $\langle Q_u\rangle_{L\text{-}0}$ and the overall resistance is R_Σ.

$\langle T_{f,h}\rangle_0 = 900$°C, $\langle T_{f,c}\rangle_L = 20$°C, $\dot{M}_{f,c} = \dot{M}_{f,h} = 70$ g/s, $\langle Q_u\rangle_{L\text{-}0} = 45$ kW, $R_\Sigma = (10^{-3}/NwL)$°C/W, $w = 0.08$ m, $L = 0.35$ m.

Evaluate the thermophysical properties using air at $T = 700$ K.

SKETCH:

Figure Pr.7.24(a) shows the plate-type heat exchanger. The required number of plates are added, as needed.

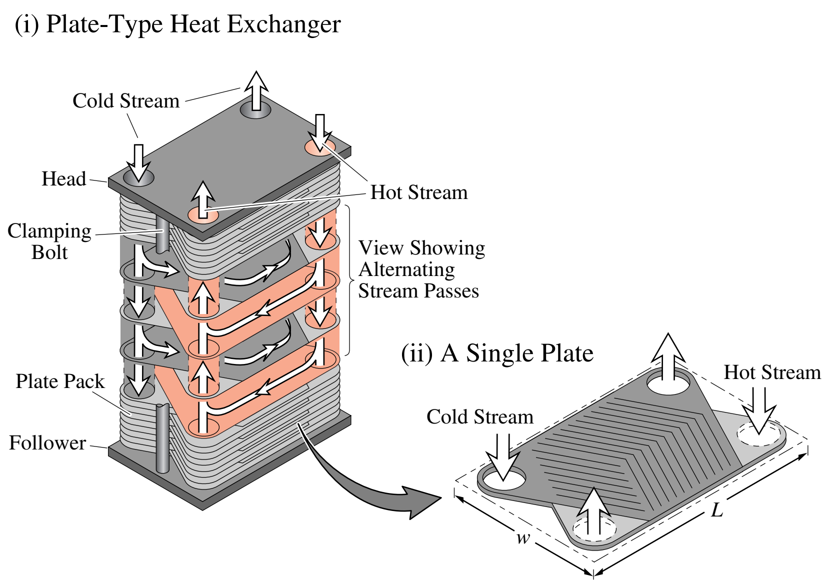


Figure Pr.7.24(a)(i) A plate-type heat exchanger. (ii) A single plate in a plate-type heat transfer.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the cold fluid exit temperature.

(c) Determine the number of plates N required.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.7.24(b).

(b) For air at $T = 700$ K, from Table C.22, we have

$$c_p = 1{,}065 \text{ J/kg-K} \quad \text{Table C.22.}$$

Then from (7.70), we have

$$\begin{aligned}
\langle Q_u\rangle_{L\text{-}0} &= (\dot{M}c_p)_c(\langle T_{f,c}\rangle_0 - \langle T_{f,c}\rangle_L) \\
45{,}000(\text{W}) &= 7 \times 10^{-2}(\text{kg/s}) \times 1{,}065(\text{J/kg-°C}) \times [\langle T_{f,c}\rangle_0 - 20(\text{°C})] \\
\langle T_{f,c}\rangle_0 &= 623.6\text{°C}.
\end{aligned}$$

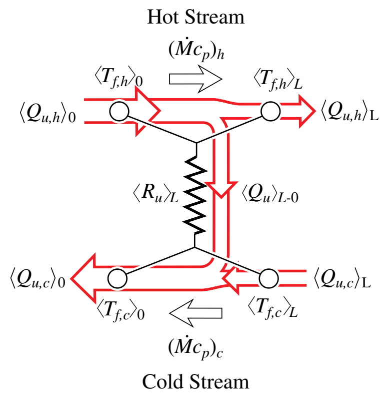


Figure Pr.7.24(b) Thermal circuit diagram.

(c) The effectiveness $\epsilon_{h,e}$ is given by (7.72), and since $(\dot{M}c_p)_{min} = (\dot{M}c_p)_c$, we have

$$\begin{aligned}\epsilon_{h,c} &= \frac{\Delta T_c}{\langle T_{f,h}\rangle_0 - \langle T_{f,h}\rangle_L} = \frac{\langle T_{f,c}\rangle_0 - \langle T_{f,c}\rangle_L}{\langle T_{f,h}\rangle_0 - \langle T_{f,c}\rangle_L} \\ &= \frac{(623.6 - 20)(^\circ\mathrm{C})}{(900 - 20)(^\circ\mathrm{C})} = 0.6859.\end{aligned}$$

The $\epsilon_{he} - NTU$ relations for the counter-flow heat exchangers are given in Table 7.7. As $(\dot{M}c_p)_c = (\dot{M}c_p)_h$, $C_r = 1$. Then from Table 7.7, we have

$$\epsilon_{he} = \frac{NTU}{1 + NTU}$$

so that

$$NTU = \frac{\epsilon_{he}}{1 - \epsilon_{he}} = 2.184.$$

From (7.74), we have

$$NTU = \frac{1}{R_\Sigma (\dot{M}c_p)_{min}}$$

or

$$\begin{aligned}R_\Sigma &= \frac{1}{NTU(\dot{M}c_p)_{min}} \\ &= \frac{1}{2.184 \times 0.070(\mathrm{kg/s}) \times 1{,}065(\mathrm{J/kg\text{-}K})} \\ &= 6.142 \times 10^{-3}\,^\circ\mathrm{C/W}.\end{aligned}$$

Then

$$6.142 \times 10^{-3}(^\circ\mathrm{C/W}) = \frac{10^{-3}}{Nwl}(^\circ\mathrm{C}/(\mathrm{W/m^2})) = \frac{10^{-3}[^\circ\mathrm{C}/(\mathrm{W/m^2})]}{N \times 0.08(\mathrm{m}) \times 0.35(\mathrm{m})}$$

or

$$N = 5.81 \simeq 6.$$

Note that by using 6 plates, the operating conditions will be different than the given values.

COMMENT:

Note that $A_{ku}R_\Sigma = NwlR_\Sigma = 10^{-3}[^\circ\mathrm{C}/(\mathrm{W/m^2})]$ is rather a very small resistance. This is achieved using high gas-stream speeds and roughness (corrugated) plates surfaces.

PROBLEM 7.25.FAM

GIVEN:

An auxiliary, fuel-fired automobile heater uses diesel fuel and heats the water circulating through the radiator or the water circulating through the heater core (i.e., the heat exchanger for heating the air flowing through the passenger compartment). This heat exchanger is shown in Figure Pr.7.25(a) along with the heat flux vector track showing the direction of heat transfer.

The flue gas (products of the combustion of the diesel-air mixture) flows through a counterflow heat exchanger with the flue gas flowing through the inner annulus and the water flowing through the outer annulus. The overall thermal resistance is $R_\Sigma = 0.5$°C/W and the heat exchanger is $L = 25$ cm long. The mass flow rates of the fuel, air, and water are $\dot{M}_F = 0.03$ g/s, $\dot{M}_a = 0.75$ g/s and $\dot{M}_w = \dot{M}_c = 5$ g/s. The water inlet temperature is $\langle T_{f,c}\rangle_0 = 15$°C and the fuel and air combustion chamber inlet temperature is $T_{f,\infty} = 20$°C. Assume complete combustion. The heat of combustion is found in Table 5.2.

This is a counterflow heat exchanger. Use (5.34) to determine $\langle T_{f,h}\rangle_L$. Use $\langle T_{f,c}\rangle_0$, and $\langle T_{f,h}\rangle_L$ as the inlet temperatures. For the gas, use the properties of air at 1,000 K and for the water, evaluate the properties at $T = 310$ K.

SKETCH:

Figure Pr.7.25(a) shows a combustible auxiliary fuel-fired water heater.

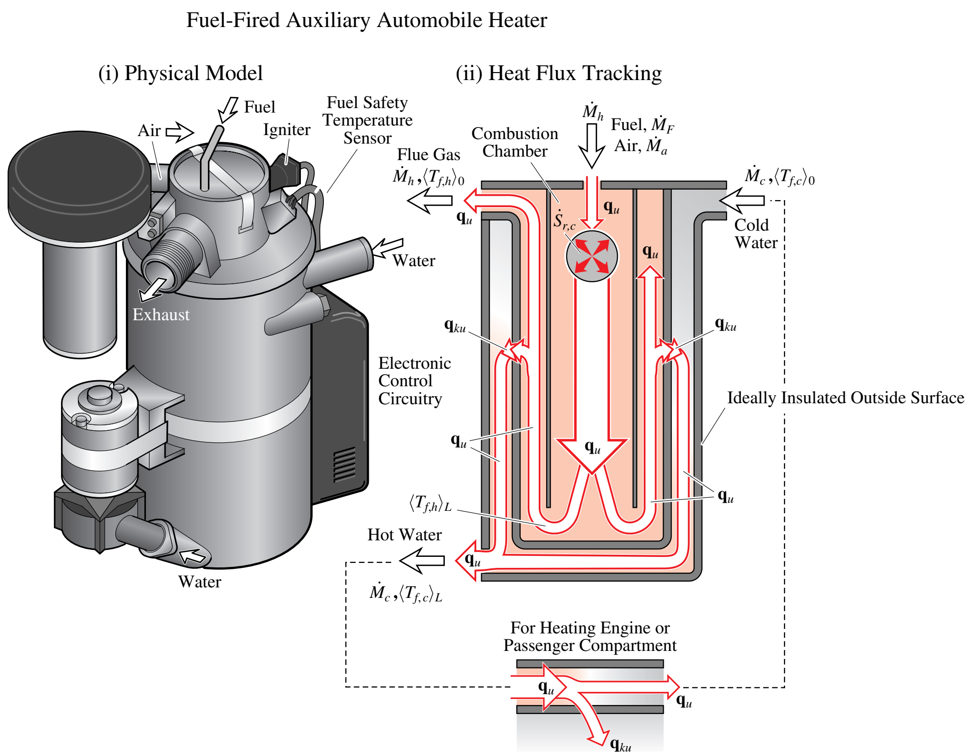


Figure Pr.7.25(a) An combustible auxiliary fuel-fired water heater.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the flue gas temperature leaving the combustion chamber $\langle T_{f,h}\rangle_L$.

(c) Determine the exit temperatures for the flue gas $\langle T_{f,h}\rangle_0$ and water $\langle T_{f,c}\rangle_L$

(d) Determine the amount of heat exchanged between the two streams and the heater efficiency, defined as $\langle Q_u\rangle_{L\text{-}0}/\dot{S}_{r,c}$.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.7.25(b).

(b) The flue-gas temperature is found by writing the energy equation for the internodal energy conservation, i.e., (5.33)

$$\langle Q_u \rangle_{h,\infty} + \langle Q_u \rangle_{h,L} + Q_{loss} = \dot{S}_{r,c}.$$

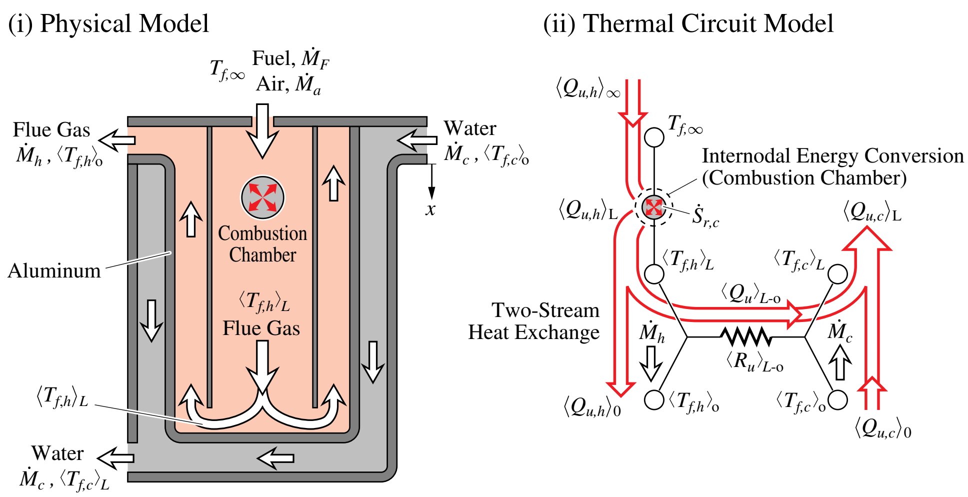


Figure Pr.7.25(b) A simplified physical model and the corresponding thermal circuit diagram.

Here, for the combustion chamber we assume $Q_{loss} = 0$, and from (5.34) we have

$$\langle Q_u \rangle_{h,\infty} + \langle Q_u \rangle_{h,L} = (\dot{M}c_p)_h(\langle T_{f,h} \rangle_L - T_{f,\infty}),$$

where for the flue-gas mass flow rate we have

$$\dot{M}_h = \dot{M}_F + \dot{M}_a = 3 \times 10^{-5}(\text{kg/s}) + 7.5 \times 10^{-4}(\text{kg/s}) = 7.8 \times 10^{-4} \text{ kg/s}.$$

From Table 5.2, we have for the diesel fuel, $\Delta h_{r,F} = -43.31 \times 10^6$ J/kg. Then from (5.34), we have

$$\begin{aligned} \dot{S}_{r,c} &= -\dot{M}_F \Delta h_{r,F} \\ &= -3 \times 10^{-5}(\text{kg/s}) \times (-43.31 \times 10^6)(\text{J/kg}) \\ &= 1{,}299 \text{ W}. \end{aligned}$$

The specific heat capacity of air is found from Table C.22, at $T = 1{,}000$ K, as

$$c_p = 1{,}130 \text{ J/kg-K}, \quad \text{at } T = 1{,}000 \text{ K} \qquad \text{Table C.22.}$$

The the flue gas temperature leaving the combustion chamber is found from above as

$$\begin{aligned} \langle T_{f,h} \rangle_L &= T_{f,\infty} + \frac{\dot{S}_{r,c}}{(\dot{M}c_p)_h} \\ &= 293.15(\text{K}) + \frac{1{,}299(\text{W})}{7.8 \times 10^{-4}(\text{kg/s}) \times 1{,}130(\text{J/kg-K})} = 1{,}767 \text{ K}. \end{aligned}$$

(c) To determine the exit temperatures, we use (i) the ϵ_{he}-NTU relation from Table 7.7, (ii) the ϵ_{he}-ΔT relation (7.82), and (iii)the heat transfer rates from (7.83) and (7.84).

(i) The ϵ_{he}-NTU relation for counter-flow, coaxial heat exchanger from Table 7.7 is

$$\epsilon_{he} = \frac{1 - e^{-NTU(1-C_r)}}{1 - C_r e^{-NTU(1-C_r)}}.$$

The specific heat capacity of water is found from Table C.23, at $T = 310$ K, as

$$c_p = 4{,}178 \text{ J/kg-K}, \quad \text{at } T = 310 \text{ K} \qquad \text{Table C.23.}$$

The $(\dot{M}c_p)_f$ are

$$\begin{aligned} \text{flue gas:} \qquad (\dot{M}c_p)_h &= 7.8 \times 10^{-4}(\text{kg/s}) \times 1{,}130(\text{J/kg-K}) = 0.8814 \text{ W/°C} \\ \text{water:} \qquad (\dot{M}c_p)_c &= 5.0 \times 10^{-3}(\text{kg/s}) \times 4{,}178(\text{J/kg-K}) = 20.89 \text{ W/°C}. \end{aligned}$$

Then from (7.75), we have

$$\begin{aligned} (\dot{M}c_p)_{min} &= (\dot{M}c_p)_h \\ C_r &= \frac{(\dot{M}c_p)_h}{(\dot{M}c_p)_c} = \frac{0.8814 \text{ W/°C}}{20.89 \text{ W/°C}} = 0.04219. \end{aligned}$$

The NTU relationship (7.75) is

$$NTU = \frac{1}{R_\Sigma(\dot{M}c_p)_{min}} = \frac{1}{0.5(\text{W/°C}) \times 0.8814(\text{°C/W})} = 2.269.$$

Then we determine ϵ_{he}-NTU as

$$\epsilon_{he} = \frac{1 - e^{-2.269(1-0.04219)}}{1 - 0.04219e^{-2.269(1-0.04219)}} = 0.8905.$$

(ii) The ϵ_{he}-$\Delta\langle T_f\rangle$ relation is given by (7.82), i.e.,

$$\begin{aligned} \epsilon_{he} &\equiv \frac{\Delta\langle T_f\rangle\,|_{(\dot{M}c_p)_{min}}}{\Delta T_{max}} \\ &= \frac{\langle T_{f,h}\rangle_L - \langle T_{f,h}\rangle_0}{\langle T_{f,h}\rangle_L - \langle T_{f,c}\rangle_0} \\ \epsilon_{he} &= 0.8905 = \frac{1{,}767(\text{K}) - \langle T_{f,h}\rangle_0}{1{,}767(\text{K}) - 288.15(\text{K})}. \end{aligned}$$

Solving for $\langle T_{f,h}\rangle_0$, we have

$$\langle T_{f,h}\rangle_0 = 1{,}767(\text{K}) - 1{,}317(\text{K}) = 450 \text{ K}.$$

(iii) From division of (7.83) by (7.84), after modification for counterflow arrangement, we have

$$\frac{\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0}{\langle T_{f,h}\rangle_L - \langle T_{f,h}\rangle_0} = \frac{(\dot{M}c_p)_h}{(\dot{M}c_p)_c} = C_r.$$

Solving for $\langle T_{f,c}\rangle_L$, we have

$$\begin{aligned} \langle T_{f,c}\rangle_L &= \langle T_{f,c}\rangle_0 + C_r(\langle T_{f,h}\rangle_L - \langle T_{f,h}\rangle_0) \\ &= 288.15(\text{K}) + 0.04219 \times 1{,}317(\text{K}) \\ &= 343.6 \text{ K}. \end{aligned}$$

(d) The heat exchange rate is

$$\begin{aligned} \langle Q_u\rangle_{L\text{-}0} &= (\dot{M}c_p)_c(\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0) \\ &= 20.89(\text{W/°C}) \times 55.6(\text{°C}) \\ &= 1{,}161 \text{ W}. \end{aligned}$$

The efficiency is defined as

$$\begin{aligned} \eta &= \frac{\langle Q_u\rangle_{L\text{-}0}}{\dot{S}_{r,c}} = \frac{1{,}161(\text{W})}{1{,}299(\text{W})} \\ &= 0.8936 = 89.36\%. \end{aligned}$$

COMMENT:

The rather large overall resistance R_Σ is due to the small surface-convection area available for such a compact heater. However, due to the large NTU, the effectiveness is noticeably large. This is considered a fairly good efficiency for such a heater.

PROBLEM 7.26.FAM

GIVEN:

During cardiopulmonary bypass, in open-heart surgery, the blood is cooled by an external heat exchanger to lower the body temperature. This lowering of the body temperature (called whole-body hypothermia) reduces metabolic demand and protects the vital organs during the operation. The heat exchanger is part of the extracorporeal circulation circuit shown in Figure Pr.7.26(a)(i). Special pumps (e.g., roller pumps, which use compression of elastic tubes to move the liquid) are used to protect the blood cells from mechanical damage. A shell and tube heat exchanger (with one shell pass) is used to cool (and later heat) the bloodstream, using a water stream, as shown in Figure Pr.7.26(a)(ii), to the hypothermic temperature $\langle T_{f,h}\rangle_L$.

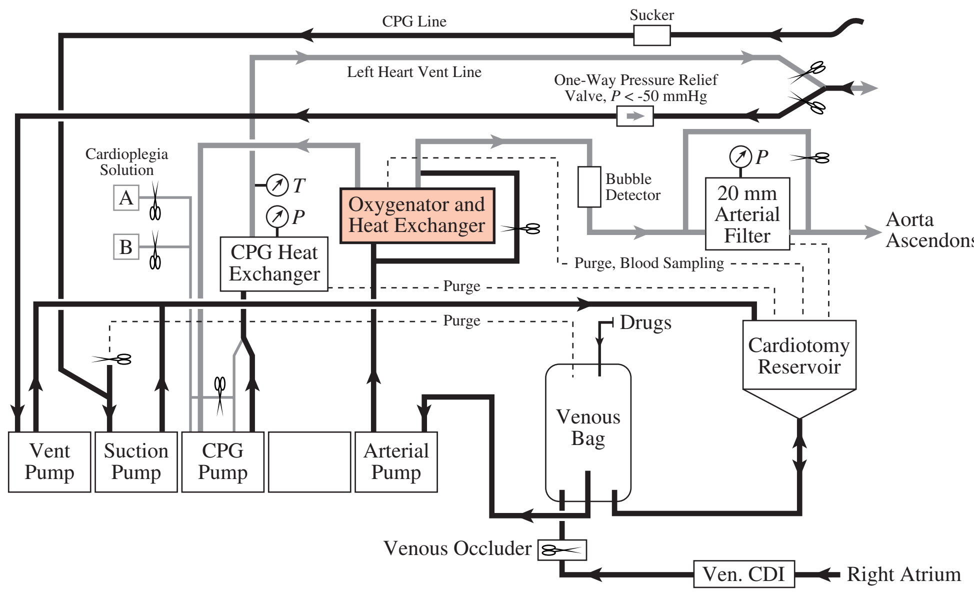
$\langle T_{f,h}\rangle_0 = 37°\text{C}$, $\langle T_{f,c}\rangle_0 = 15°\text{C}$, $(\dot{M}_f/\rho_f)_h = 250$ ml/min, $(\dot{M}_f/\rho_f)_c = 200$ ml/min, $R_\Sigma = 5\times10^{-2}$ °C/W.

Use properties of water at $T = 300$ K, for both blood and water.

SKETCH:

Figure Pr.7.26(a) shows the extracorporeal circulation circuit and the tube and shell blood heat exchanger.

(i) Extracorporeal Circulation Circuit Used in Open-Heart Surgery



(ii) Oxygenator and Blood Heat Exchanger

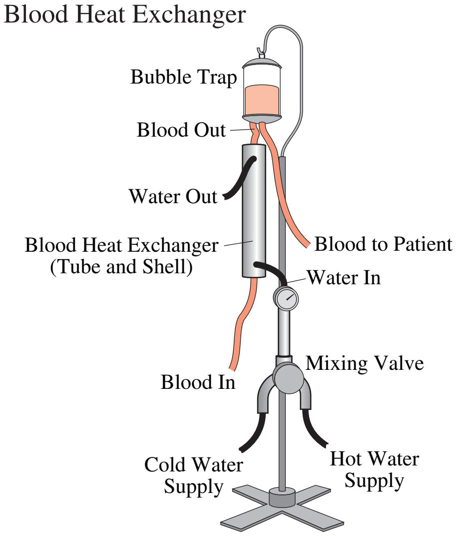


Figure Pr.7.26(a)(i) An extracorporeal circulation circuit used in open-heart surgery. (ii) A tube and shell heat exchanger used for cooling (and heating) the blood stream.

OBJECTIVE:
(a) Draw the thermal circuit diagram.
(b) Determine the bloodstream exit temperature $\langle T_{f,h}\rangle_L$.
(c) Determine $\langle Q_u\rangle_{L\text{-}0}$.

SOLUTION:
(a) Figure Pr.7.26(b) shows the thermal circuit diagram.

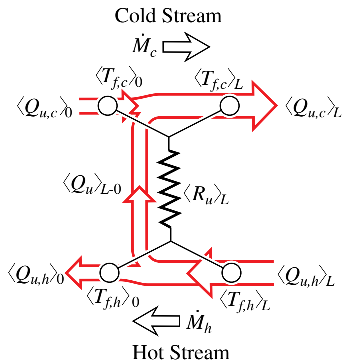


Figure Pr.7.26(b) Thermal circuit diagram.

(b) To determine the blood stream exit temperature $\langle T_{f,h}\rangle_L$, we note that from $(\dot{M}c_p)_{min}$ and R_Σ, we can determine NTU and then from Table 7.7 we can determine ϵ_{he}, and finally $\langle T_{f,h}\rangle_L$. The $\dot{M}c_p$ for the two streams are determined first by evaluating ρ_f and $c_{p,f}$ from Table C.23.

From Table C.23, at $T = 300$ K, we have

$$c_{p,f} = 4{,}182 \text{ J/kg-K} \qquad \text{Table C.23}$$
$$\rho_f = 997.7 \text{ kg/m}^3 \qquad \text{Table C.23.}$$

Then

$$\begin{aligned}
(\dot{M}c_p)_h &= \frac{250(\text{ml/min}) \times 10^{-6}(\text{m}^3/\text{ml}) \times 997.7(\text{kg/m}^3)}{60(\text{s/min})} \times 4{,}182(\text{J/kg-K}) \\
&= 17.38 \text{ W/K} \\
(\dot{M}c_p)_c &= \frac{200(\text{ml/min}) \times 10^{-6}(\text{m}^3/\text{ml}) \times 997.7(\text{kg/m}^3)}{60(\text{s/min})} \times 4{,}182(\text{J/kg-K}) \\
&= 13.91 \text{ W/K.}
\end{aligned}$$

Then using (7.75), we have

$$\begin{aligned}
(\dot{M}c_p)_{min} &= (\dot{M}c_p)_c = 13.91 \text{ W/K} \\
C_r &= \frac{13.91(\text{W/K})}{17.38(\text{W/K})} = 0.8.
\end{aligned}$$

Now using (7.74), we have

$$\begin{aligned}
NTU &= \frac{1}{R_\Sigma(\dot{M}c_p)_{min}} = \frac{1}{5 \times 10^{-2}(\text{K/W}) \times 13.91(\text{W/K})} \\
&= 1.438.
\end{aligned}$$

From Table 7.7, we have for the shell and tube heat exchanger, with one shell pass,

$$\begin{aligned}
\epsilon_{he} &= 2\left[1 + C_r + (1 + C_r^2)^{1/2}\frac{1 + e^{-NTU(1+C_r^2)^{1/2}}}{1 - e^{-NTU(1+C_r^2)^{1/2}}}\right]^{-1} \\
&= 2\left\{1 + 0.8 + [1 + (0.8)^2]^{1/2} \times \frac{1 + e^{-1.438(1+0.8^2)^{1/2}}}{1 - e^{-1.438(1+0.8^2)^{1/2}}}\right\}^{-1} \\
&= 2\left(1.8 + 1.281 \times \frac{1 + 0.1585}{1 - 0.1585}\right)^{-1} \\
&= 0.5612.
\end{aligned}$$

Now noting that (7.82) becomes

$$\epsilon_{he} = \frac{\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0}{\langle T_{f,h}\rangle_0 - \langle T_{f,c}\rangle_0},$$

we solve for $\langle T_{f,c}\rangle_L$, i.e.,

$$\begin{aligned}
\langle T_{f,c}\rangle_L &= \langle T_{f,c}\rangle_o + \epsilon_{he}(\langle T_{f,h}\rangle_o - \langle T_{f,c}\rangle_c) \\
&= 15(℃) + 0.5612(37 - 15)(℃) \\
&= 27.35℃.
\end{aligned}$$

Next we use (7.83) and (7.84) to find that

$$\frac{\langle T_{f,h}\rangle_L - \langle T_{f,h}\rangle_0}{\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0} = \frac{(\dot{M}c_p)_c}{(\dot{M}c_p)_h} = C_r$$

and solving for $\langle T_{f,h}\rangle_L$, we have

$$\begin{aligned}
\langle T_{f,h}\rangle_L &= \langle T_{f,h}\rangle_0 - C_r(\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0) \\
&= 37(℃) - 0.8(27.35 - 15)(℃) \\
&= 27.12℃.
\end{aligned}$$

(c) The heat transfer rate is

$$\begin{aligned}
\langle Q_u\rangle_{L\text{-}0} &= (\dot{M}c_p)_h(\langle T_{f,h}\rangle_0 - \langle T_{f,h}\rangle_L) \\
&= 17.38(\mathrm{W/K}) \times (37 - 27.12)(\mathrm{K}) \\
&= 171.7\ \mathrm{W}.
\end{aligned}$$

COMMENT:

The heat exchanger is generally made with metallic tubes (stainless steel) with a polymeric (PVC) shell.

PROBLEM 7.27.FAM

GIVEN:

Gray whales have counterflow heat exchange in their tongues to preserve heat. The tip of the tongue is cooled with the cold sea water. The heat exchange is between the incoming warm bloodstream (entering with the deep-body temperature) flowing through the arteries and the outgoing cold bloodstream (leaving the tongue surface region) flowing through the veins. This is shown in Figure Pr.7.27(a). In each heat exchanger unit, nine veins of diameter D_c completely encircle (no heat loss to the surroundings) the central artery of diameter D_h. The length of the heat exchange region is L.

$\langle T_{f,c}\rangle_0 = 2°\text{C}$, $\langle T_{f,h}\rangle_L = 36°\text{C}$, $L = 55$ cm, $D_h = 3$ mm, $D_c = 1$ mm, $R_{k,h\text{-}c} = 5°\text{C/W}$, $\langle u_{f,h}\rangle = 1$ mm/s, $\langle u_{f,c}\rangle = 1$ mm/s.

Use water properties at $T = 290$ K for blood. Note that for $C_r = 1$, (7.78) should be used for the counterflow heat exchanger. Also use uniform q_s results for the Nusselt numbers.

SKETCH:

Figure Pr.7.27(a) shows the counter-flow heat exchanger within the gray whale tongue.

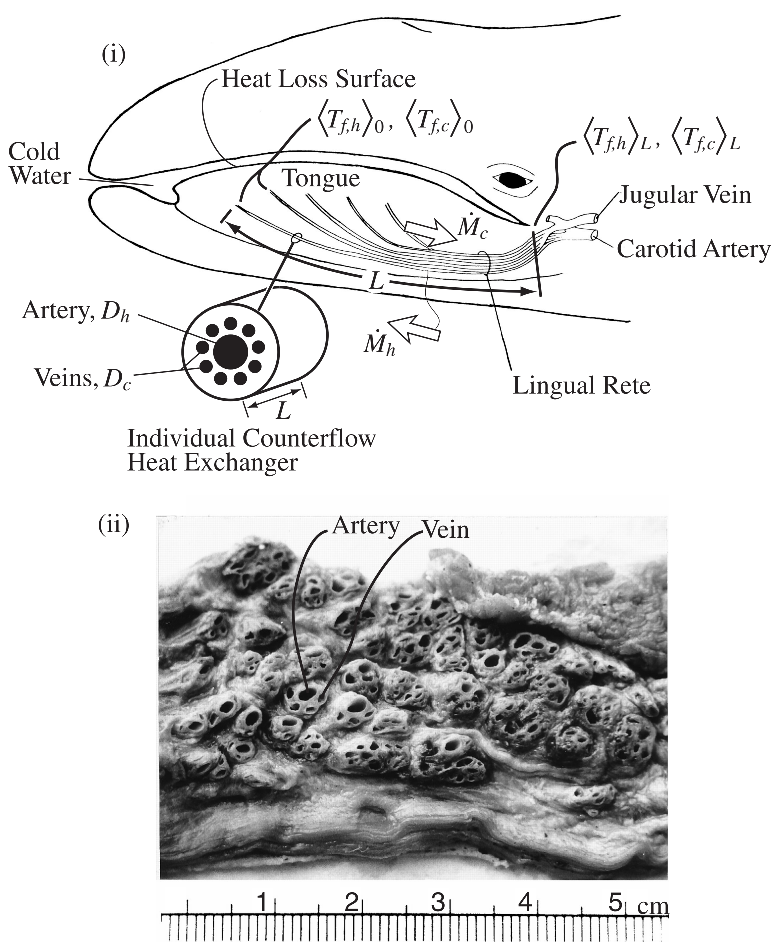


Figure Pr.7.27(a)(i) A schematic of the vascular heat exchanger in the gray whale tongue. (ii) The cross section of the lingual rete showing several vascular heat exchangers.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine $R_{ku,c}$ and $R_{ku,h}$.
(c) Determine the exit temperature of the cold bloodstream $\langle T_{f,c}\rangle_L$.

SOLUTION:
(a) Figure Pr.7.27(b) shows the thermal circuit diagram. Note that due to the counter-flow arrangement, $\langle T_{f,h}\rangle_L$ is the inlet temperature of the hot stream.

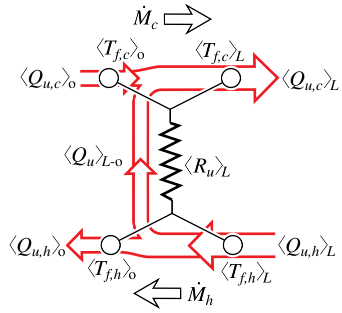


Figure Pr.7.27(b) Thermal circuit diagram.

(b) To determine $R_{ku,c}$ and $R_{ku,h}$ we use (7.88), i.e.,

$$R_{ku,c}^{-1} = A_{ku,c}\langle \text{Nu}\rangle_{D,c}\frac{k_f}{D_L}$$
$$R_{ku,h}^{-1} = A_{ku,h}\langle \text{Nu}\rangle_{D,h}\frac{k_f}{D_L},$$

where since there are no extended surfaces, we have used $\eta_f = 1$.

Here

$$A_{ku,c} = 9\pi D_c L, \quad A_{ku,L} = \pi D_h L.$$

To determine the Nusselt numbers, we first evaluate the Reynolds numbers, i.e.,

$$\text{Re}_{D,c} = \frac{\langle u_{f,c}\rangle D_c}{\nu_f}, \quad \text{Re}_{D,h} = \frac{\langle u_{f,h}\rangle D_h}{\nu_f}.$$

From Table C.23, for water $T = 290$ K, we have

$$\rho_f = 1{,}000 \text{ kg/cm}^3 \quad \text{Table C.23}$$
$$k_f = 0.590 \text{ W/m-K} \quad \text{Table C.23}$$
$$\nu_f = 1.13 \times 10^{-6} \text{ m}^2\text{/s} \quad \text{Table C.23}$$
$$c_{p,f} = 4{,}186 \text{ J/kg-K} \quad \text{Table C.23.}$$

Then

$$\text{Re}_{D,c} = \frac{10^{-3}(\text{m/s}) \times 10^{-3}(\text{m})}{1.13 \times 10^{-7}(\text{m}^2\text{/s})} = 0.885 < \text{Re}_{D,t} = 2{,}300$$
$$\text{Re}_{D,h} = \frac{10^{-3}(\text{m/s}) \times 3 \times 10^{-3}(\text{m})}{1.13 \times 10^{-7}(\text{m}^2\text{/s})} = 2.655 < \text{Re}_{D,t} = 2{,}300.$$

Both of these fluid streams are smaller than the transition Reynolds number given in (7.37).

Then from Table 7.2, we have (for a constant q_s which represents the counter-flow heat exchanger more accurately).

$$\langle \text{Nu}\rangle_{D,c} = \langle \text{Nu}\rangle_{D,h} = 4.36.$$

Next

$$R_{ku,c}^{-1} = 9\pi \times (10^{-3})(\text{m}) \times 0.55(\text{m}) \times 4.36 \times \frac{0.590(\text{W/m-K})}{10^{-3}(\text{m})}$$
$$R_{ku_c} = 0.0250 \text{ K/W},$$
$$R_{ku,h}^{-1} = \pi \times 3 \times 10^{-3}(\text{m}) \times 0.55(\text{m}) \times 4.36 \times \frac{0.590(\text{W/m-K})}{3 \times 10^{-3}(\text{m})}$$
$$R_{ku,h} = 0.2250 \text{ K/W}.$$

Then

$$R_\Sigma = (0.02499 + 5 + 0.2250)(\text{K/W}) = 5.250 \text{ K/W}.$$

(c) The exit temperature of the cold stream is found from (7.82), where we note the counter-flow arrangement and write

$$\epsilon_{he} = \frac{\Delta T|_{min}}{\langle T_{f,h}\rangle_L - \langle T_{f,c}\rangle_0}.$$

From (7.3), for each stream, we have

$$\dot{M}_c = 9\rho_f \frac{\pi D_c^2}{4}\langle u_{f,c}\rangle$$
$$\dot{M}_c = \rho_f \frac{\pi D_h^2}{4}\langle u_{f,h}\rangle$$

$$\begin{aligned}
\dot{M}_c &= 9 \times 10^3(\text{kg/m}^3) \times \frac{\pi \times (10^{-3})^2(\text{m})^2}{4} \times 10^{-3}(\text{m/s}) \\
&= 7.070 \times 10^{-6} \text{ kg/s} \\
\dot{M}_c &= 10^3(\text{kg/m}^3)\frac{\pi \times (3 \times 10^3)^2(\text{m}^2)}{4} \times 10^{-3}(\text{m/s}) \\
&= 7.070 \times 10^{-6} \text{ kg/s}
\end{aligned}$$

$$\begin{aligned}
(\dot{M}c_p)_c &= (\dot{M}c_p)_h = 7.070 \times 10^{-6}(\text{kg/s}) \times 4{,}186(\text{J/kg-K}) \\
&= 2.959 \times 10^{-2} \text{ W/K}.
\end{aligned}$$

Then from (7.75), we have

$$C_r = \frac{(\dot{M}c_p)_{min}}{(\dot{M}c_p)_{max}} = 1.$$

From Table 7.7, we have

$$\epsilon_{he} = \frac{NTU}{1+NTU} \quad \text{Table 7.7.}$$

From (7.74), we have

$$\begin{aligned}
NTU &= \frac{1}{R_\Sigma(\dot{M}c_p)_{min}} = \frac{1}{R_\Sigma(\dot{M}c_p)_c} \\
&= \frac{1}{5.25(\text{K/W}) \times 2.959 \times 10^{-2}(\text{W/K})} \\
&= 6.437.
\end{aligned}$$

Then

$$\epsilon_{he} = \frac{6.437}{1+6.437} = 0.8656.$$

Since $C_r = 1$, we have

$$\epsilon_{he} = \frac{\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0}{\langle T_{f,h}\rangle_L - \langle T_{f,c}\rangle_0}$$

or

$$\begin{aligned}
\langle T_{f,c}\rangle_L &= \langle T_{f,c}\rangle_0 + \epsilon_{he}(\langle T_{f,h}\rangle_L - \langle T_{f,c}\rangle_0) \\
&= 2(^\circ\text{C}) + 0.8656 \times (36-2)(^\circ\text{C}) \\
&= (2 + 29.43)(^\circ\text{C}) \\
&= 31.43^\circ\text{C}.
\end{aligned}$$

COMMENT:

Note that for $C_r = 1$, even for a large NTU, ϵ_{he} is less than unity (unless $NTU \rightarrow \infty$, which gives $\epsilon_{he} \rightarrow 1$).

PROBLEM 7.28.FAM

GIVEN:

The fan-coil furnace used in domestic air heaters uses a crossflow heat exchange (both fluids unmixed) between a stream of products of combustion and a stream of air, as shown in Figure Pr.7.28(a). The thermal resistance on the air side is $R_{ku,c}$, that on the combustion products side is $R_{ku,h}$, and the conduction resistance of the separating wall is negligible.

$\langle T_{f,c}\rangle_0 = 15$℃, $\langle T_{f,h}\rangle_0 = 800$℃, $\dot{M}_c = 0.1$ kg/s, $\dot{M}_h = 0.01$ kg/s, $R_{ku,c} = 2 \times 10^{-2}$ ℃/W, $R_{ku,h} = 3 \times 10^{-2}$ ℃/W, $\dot{S}_{r,c} = 7{,}300$ W.

Treat the combustion products as air and determine the air properties at $T = 300$ K (Table C.22).

The ϵ_{he}-NTU relation for this heat exchanger is given in Table 7.7.

SKETCH:

Figure Pr.7.28(a) shows a cross-flow heat exchanger used in a furnace.

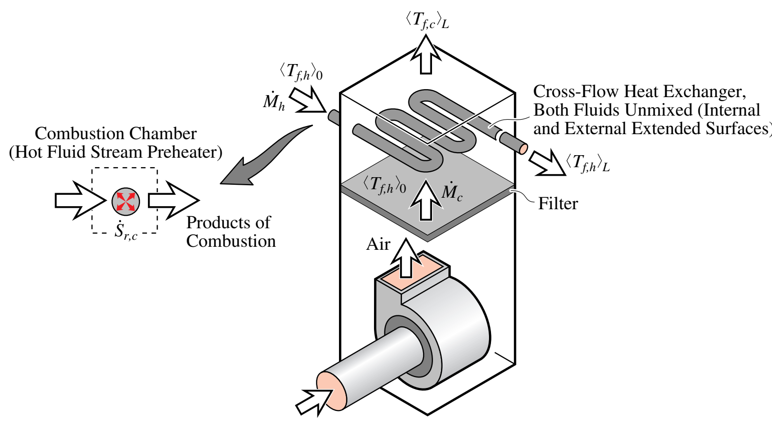


Figure Pr.7.28(a) A gas-gas, crossflow heat exchanger in a fan-coil furnace.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the rate of heat exchange between the two streams.

(c) Determine the efficiency of this air heater (defined as the ratio of the rate of heat exchange through the heat exchanger $\langle Q_u\rangle_{L-0}$ to the rate of energy conversion by combustion in the hot fluid stream preheater $\dot{S}_{r,c}$).

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.7.28(b).

(b) From Table c.22 for air, $c_{p,c} = c_{p_h} = 1{,}005$. To find the heat exchange between the two fluid streams $\langle Q_u\rangle_{L\text{-}0}$, we must determine NTU and ϵ_{he}. From (7.75) and (7.52), determining C_r and R_Σ, we have, as $\dot{M}_h < \dot{M}$, $(\dot{M}c_p)_{min} = (\dot{M}c_p)_h$,

$$
\begin{aligned}
C_r &= \frac{(\dot{M}c_p)_{min}}{(\dot{M}c_p)_{max}} \\
&= \frac{0.01(\text{kg/s}) \times 1{,}005(\text{J/kg-K})}{0.1(\text{kg/s}) \times 1{,}005(\text{J/kg-K})} = 0.1 \\
R_\Sigma &= R_{ku,c} + R_{k,h\text{-}c} + R_{ku,h} \\
&= 2 \times 10^{-2}(\text{℃/W}) + 0 + 3 \times 10^{-2}(\text{℃/W}) = 5 \times 10^{-2}\ \text{℃/W}.
\end{aligned}
$$

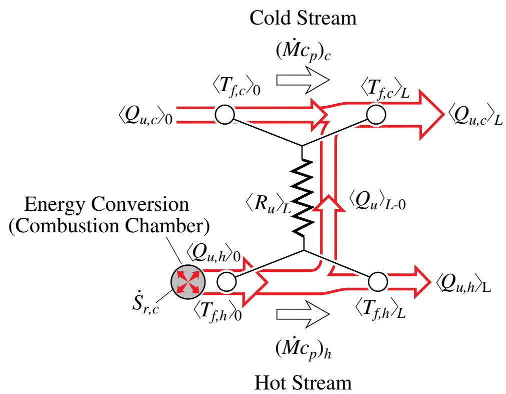


Figure Pr.7.28(b) Thermal circuit diagram.

Then, from (7.74) and from Table 7.6 (ϵ_{he}-NTU relations), we have

$$\begin{aligned}
NTU &= \frac{1}{R_\Sigma(\dot{M}c_p)_{min}} \\
&= \frac{1}{5\times 10^{-2}(^\circ\text{C/W})\times 0.01(\text{kg/s})\times 1{,}005(\text{J/kg-K})} = 1.990 \\
\epsilon_{he} &= 1-\exp\left\{\frac{NTU^{0.22}}{C_r}\exp\left[\left(-C_rNTU^{0.78}\right)-1\right]\right\} \\
&= 1-\exp\left\{\frac{(1.990)^{0.22}}{0.1}\times\exp\left[\left(-0.1\times 5\times(1.990)^{0.78}\right)-1\right]\right\} = 0.8395.
\end{aligned}$$

The effectiveness is given by (7.82) i.e.,

$$\begin{aligned}
\epsilon_{he} &= \frac{\Delta\langle T_f\rangle\mid_{(\dot{M}c_p)_{min}}}{\Delta T_{max}} \\
&= \frac{\langle T_{f,h}\rangle_0-\langle T_{f,h}\rangle_L}{\langle T_{f,h}\rangle_0-\langle T_{f,c}\rangle_0} \\
&= \frac{800^\circ\text{C}-\langle T_{f,h}\rangle_L}{800^\circ\text{C}-15^\circ\text{C}} = 0.8395.
\end{aligned}$$

Solving for the hot fluid stream exit temperature $\langle T_{f,h}\rangle_L$ gives

$$\langle T_{f,h}\rangle_L = 141.0^\circ\text{C}.$$

The heat exchange rate is given by (7.84) and is

$$\begin{aligned}
\langle Q_u\rangle_{L\text{-}0} &= (\dot{M}c_p)_h(\langle T_{f,h}\rangle_0-\langle T_{f,h}\rangle_L) \\
&= [0.01(\text{kg/s})\times 1{,}005(\text{J/kg-K})]\times[800^\circ\text{C}-141.0^\circ\text{C}] = 6{,}623\ \text{W}.
\end{aligned}$$

(c) The efficiency is defined as the achieved heat exchange rate divided by the required energy consumption rate. Then, the efficiency is

$$\eta = \frac{\langle Q_u\rangle_{L\text{-}0}}{\dot{S}_{r,c}} = \frac{6{,}623(\text{W})}{7{,}300(\text{W})} = 0.9073 = 90.73\%.$$

COMMENT:

Note that the heat exchanger effectiveness is $\epsilon_{he} = 0.8395$. A further increase in the NTU will give a higher effectiveness. This can be achieved by a higher surface area or a larger Nusselt number.

PROBLEM 7.29.FAM

GIVEN:

The automobile passenger compartment heater uses a crossflow heat exchanger (called the heater core), as shown in Figure Pr.7.29(a). Due to the presence of fins, the air (cold stream) is unmixed as it flows through the heater exchanger. The water (hot stream) flowing through flat tubes is also unmixed.

$\dot{M}_c = 0.03$ kg/s, $\dot{M}_h = 0.10$ kg/s,$\langle T_{f,c}\rangle_0 = 4$℃, $\langle T_{f,h}\rangle_0 = 50$℃, $R_\Sigma = 3 \times 10^{-2}$℃/W.

Evaluate the properties at $T = 300$ K.

SKETCH:

Figure Pr.7.29(a) shows a cross-flow heat exchanger.

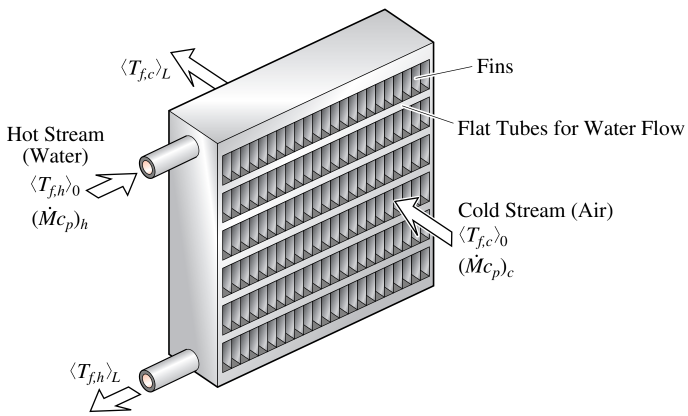


Figure Pr.7.29(a) A cross-flow heat exchanger.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine the number of thermal units NTU.
(c) Determine the ratio of thermal capacitance C_r.
(d) Determine the effectiveness ϵ_{he}.
(e) Determine the exit temperature of the cold stream (air) $\langle T_{f,c}\rangle_L$.
(f) Determine the exit temperature of the hot stream (water) $\langle T_{f,h}\rangle_L$.
(g) Determine the amount of heat exchanged $\langle Q_u\rangle_{L\text{-}0}$.

SOLUTION:

(a) The thermal circuit diagram, using the average convection resistance $\langle R_u\rangle_L$, is shown in Figure Pr.7.29(b).

(b) The number of thermal units is given by (7.74), i.e.,

$$NTU = \frac{1}{R_\Sigma(\dot{M}c_p)_{min}}.$$

From Tables C.22 and C.23, and at $T = 300$ K and $T = 310$ K, we have

$$\begin{aligned} \text{air:}\quad T &= 300\text{ K},\quad c_p = 1{,}005\text{ J/kg-K}\quad \text{Table C.22} \\ \text{water:}\quad T &= 300\text{ K},\quad c_p = 4{,}182\text{ J/kg-K}\quad \text{Table C.23.} \end{aligned}$$

Then

$$\begin{aligned} (\dot{M}c_p)_c &= 0.03(\text{kg/s}) \times 1{,}005(\text{J/kg-K}) \\ &= 30.15\text{ W/K} \\ (\dot{M}c_p)_h &= 0.10(\text{kg/s}) \times 4{,}182(\text{J/kg-K}) \\ &= 418.2\text{ W/K}. \end{aligned}$$

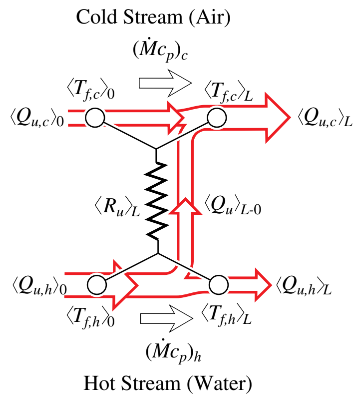


Figure Pr.7.29(b) Thermal circuit diagram.

And therefore, we have

$$(\dot{M}c_p)_{min} = (\dot{M}c_p)_c = 30.15 \text{ W/K}.$$

Then

$$\begin{aligned} NTU &= \frac{1}{0.03(\text{K/W}) \times 30.15(\text{W/K})} \\ &= 1.106. \end{aligned}$$

(c) The ratio C_r is defined by (7.75) as

$$C_r = \frac{(\dot{M}c_p)_{min}}{(\dot{M}c_p)_{max}} = \frac{(\dot{M}c_p)_h}{(\dot{M}c_p)_c} = \frac{30.15(\text{W/K})}{418.2(\text{W/K})} = 0.0721.$$

(d) The ϵ_{he}-NTU reaction is given in Table 7.7. For cross-flow heat exchanger with both fluids unmixed, we have

$$\begin{aligned} \epsilon_{he} &= 1 - \exp\left\{\frac{NTU^{0.22}}{C_r}[\exp(-C_r NTU^{0.78}) - 1]\right\} \\ &= 1 - \exp\left\{\frac{(1.106)^{0.22}}{0.0721}[\exp(-0.0721 \times 1.106^{0.78}) - 1]\right\} \\ &= 1 - \exp[14.17 \times (-0.075)] = 1 - \exp(-1.064) = 0.6549. \end{aligned}$$

(e) The cold stream is the stream with $(\dot{M}c_p)_{min}$, from the definition of ϵ_{he} given by (7.82), we have

$$\begin{aligned} \epsilon_{he} &= \frac{\Delta\langle T_f\rangle|_{(\dot{M}c_p)_{min}}}{\Delta T_{max}} \\ &= \frac{\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0}{\langle T_{f,h}\rangle_0 - \langle T_{f,c}\rangle_0} \\ 0.6549 &= \frac{\langle T_{f,c}\rangle_L - 4(^\circ\text{C})}{50(^\circ\text{C}) - 4(^\circ\text{C})} \\ \langle T_{f,c}\rangle_L &= 34.13^\circ\text{C}. \end{aligned}$$

(f) The hot water exit temperature is found from division of (7.83) by (7.84), i.e.,

$$\frac{\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0}{\langle T_{f,h}\rangle_0 - \langle T_{f,h}\rangle_L} = \frac{(\dot{M}c_p)_h}{(\dot{M}c_p)_c} = \frac{1}{C_r}$$

or

$$\begin{aligned} \langle T_{f,h}\rangle_L &= \langle T_{f,h}\rangle_0 - C_r(\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0) \\ &= 50^\circ\text{C} - 0.0721 \times (34.13 - 4)(^\circ\text{C}) = 47.83^\circ\text{C}. \end{aligned}$$

(g) The heat exchange rate is given by (7.83) as,

$$\begin{aligned}\langle Q_u\rangle_{L\text{-}0} &= (\dot{M}c_p)_c\Delta T_c\\ &= 30.15(\text{W/℃}) \times (34.13-4)(\text{℃}) = 908.4\ \text{W}.\end{aligned}$$

COMMENT:

Note that the hot stream, having a much larger $(\dot{M}c_p)_f$, undergoes a smaller change in temperature, $\langle T_{f,h}\rangle_0 - \langle T_{f,h}\rangle_L = 2.17$℃.

PROBLEM 7.30.FAM

GIVEN:

In a shell and tube heat exchanger, two fluid streams exchange heat. This is shown in Figure Pr.7.30. The hot stream is a saturated steam at $\langle T_{f,h}\rangle_0 = 400$ K ($p_g = 0.2455$ MPa, Table C.27) and it loses heat and condenses. The cold stream is the subcooled liquid Refrigerant R-134a.

$\dot{M}_c = 3$ kg/s, $\langle T_{f,h}\rangle_0 = \langle T_{f,h}\rangle_L = 400$ K, $\langle T_{f,c}\rangle_0 = 300$ K, $R_\Sigma = 3 \times 10^{-3}$ °C/W.

Evaluate the refrigerant R-134a saturation properties at $T = 303.2$ K (Table C.28).

SKETCH:

Figure Pr.7.30(a) shows the shell and tube heat exchanger.

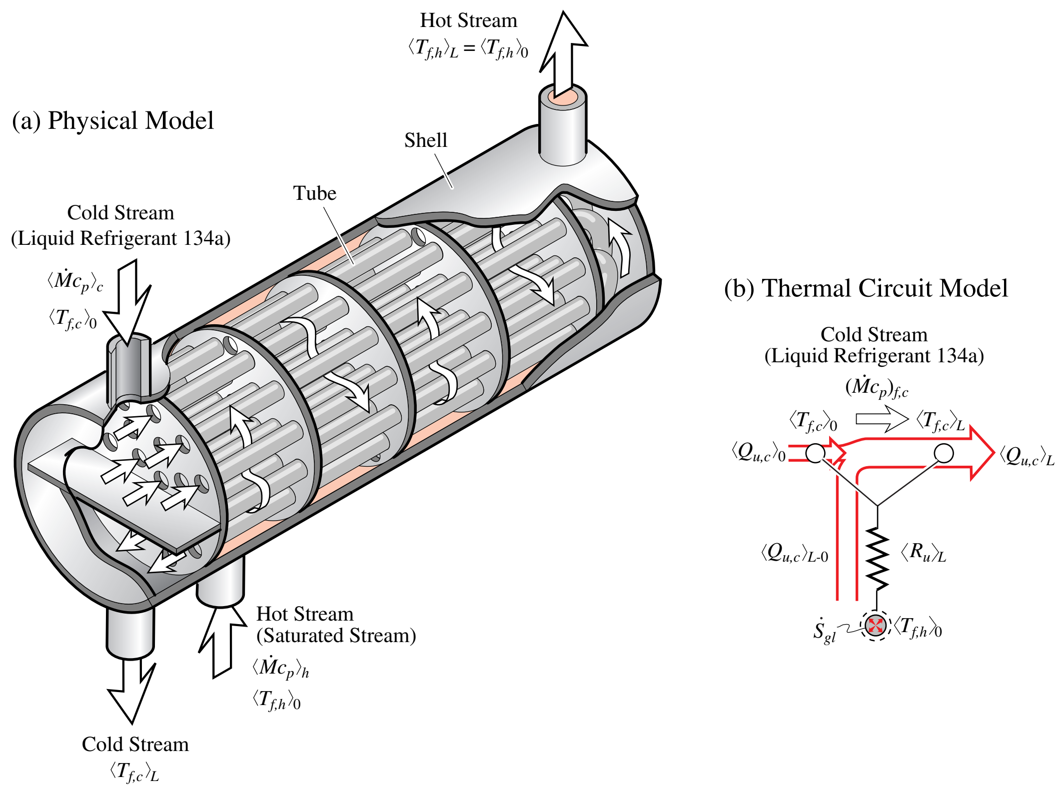


Figure Pr.7.30 A tube and shell heat exchanger.

OBJECTIVE:

(a) Draw the thermal circuit diagram.
(b) Determine the number of thermal units NTU.
(c) Determine the effectiveness ϵ_{he}.
(d) Determine the exit temperature of the cold stream $\langle T_{f,c}\rangle_L$.
(e) Determine the heat exchange rate $\langle Q_u\rangle_{L\text{-}0}$.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.7.30(b). The hot stream does not undergo any temperature change as there is phase change occurring and no pressure drop is assumed.

(b) The number of thermal units NTU is defined by (7.74), i.e.,

$$NTU = \frac{1}{R_\Sigma (\dot{M}c_p)_{min}}.$$

Here the hot fluid will undergo a zero temperature change, and therefore, has a apparent thermal capacitance of infinity, i.e.,

$$(\dot{M}c_p)_h \rightarrow \infty.$$

Then

$$(\dot{M}c_p)_c = (\dot{M}c_p)_{min}$$
$$C_r = \frac{(\dot{M}c_p)_c}{(\dot{M}c_p)_h} = 0.$$

From Table C.28, and at $T = 303.2$ K, we have

$$\text{liquid R-134a:} \quad c_{p,c} = 1{,}447 \text{ J/kg-K} \qquad \text{Table C.28.}$$

Then

$$\begin{aligned}
(\dot{M}c_p)_{min} &= (\dot{M}c_p)_c = 3(\text{kg/s}) \times 1{,}447(\text{J/kg-K}) \\
&= 4341 \text{ W/K.} \\
NTU &= \frac{1}{3 \times 10^{-3}(\text{K/W}) \times 4{,}341(\text{W/K})} = 0.07679.
\end{aligned}$$

(c) The effectiveness for all heat exchangers with $C_r = 0$, is given by (7.79), and also listed in Table 7.7, i.e.,

$$\epsilon_{he} = 1 - e^{-NTU} = 1 - e^{-0.7679} = 0.07391.$$

(d) The exit temperature of the cold fluid is found from the definition of ϵ_{he}, given by (7.84), i.e.,

$$\begin{aligned}
\epsilon_{he} &= \frac{\Delta T_f|_{(\dot{M}c_p)_{min}}}{\Delta T_{max}} = \frac{\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0}{\langle T_{f,h}\rangle_0 - \langle T_{f,c}\rangle_0} \\
0.07391 &= \frac{\langle T_{f,c}\rangle_L - 300(\text{K})}{400(\text{K}) - 300(\text{K})}
\end{aligned}$$

or

$$\langle T_{f,c}\rangle_L = 307.4 \text{ K.}$$

(e) The rate of heat exchange is given by (7.83), i.e.,

$$\begin{aligned}
\langle Q_u\rangle_{L\text{-}0} &= (\dot{M}c_p)_c(\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0) \\
&= 4341(\text{W/K}) \times (307.4 - 300)(\text{K}) = 32{,}086 \text{ W} = 32.09 \text{ kW.}
\end{aligned}$$

COMMENT:

Note that this heat exchanger is not designed to give a cold stream exit temperature close to the uniform hot stream temperature. This occurs in applications where the available hot stream has a higher temperature than needed for the cold stream, and therefore, a smaller heat exchanger, i.e., smaller NTU, is used. Also, note that from the ϵ_{he}-NTU relation of Table 7.7, we have for $C_r = 0$

$$\begin{aligned}
\epsilon_{he} &= 2\left(1 + \frac{1 + e^{-NTU}}{1 - e^{-NTU}}\right)^{-1} \\
&= 2\left(\frac{2}{1 - e^{-NTU}}\right)^{-1} = 1 - e^{-NTU},
\end{aligned}$$

as expected.

PROBLEM 7.31.FAM

GIVEN:

Many hot-water heaters consist of a large reservoir used to store the hot water. This may have a batch-type processing that results in a low efficiency, because the hot water must be constantly heated to make up for heat losses. Alternatively, no-storage, on-demand, high-efficiency crossflow heat exchangers can provide the hot water needed. One such design, along with its dimensions, is shown in Figure Pr.7.31(a). In this design a mixture of air and propane, initially at a temperature $T_{f,\infty} = 25°\mathrm{C}$ and with a fuel mass fraction of $(\rho_{\rm F}/\rho_f)_1 = 0.015$, undergoes combustion with no heat loss (i.e., $Q_{loss} = 0$) with a generation of $\dot{S}_{r,c} = 12{,}900$ W. The flue gas then flows over a tube of diameter $D = 1.3$ cm that is curved as shown in Figure Pr.7.31(a), such that the total length is $5w$. The tube contains water with an inlet temperature of $\langle T_{f,c}\rangle_0 = 20°\mathrm{C}$ and a volumetric flow rate of 1(gal/min) $= 6.3 \times 10^{-5}(\mathrm{m^3/s})$. In order to increase the heat transfer, the tube is surrounded by fins with a density of 8 fins per cm and a fin efficiency $\eta_f = 1$.

Use the properties of water at $T = 310$ K and treat the combustion products as air with the properties evaluated at $T = 300$ K.

SKETCH:

Figure Pr.7.31(a) shows the exchanger and its physical features and dimensions.

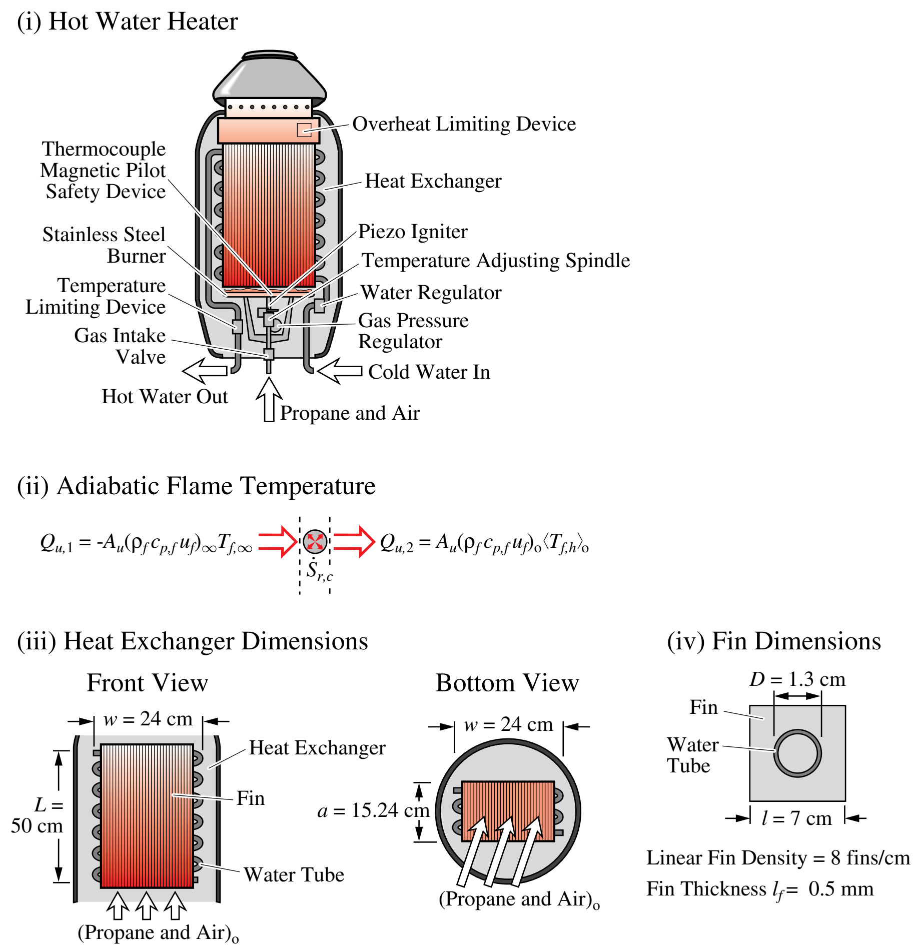


Figure Pr.7.31(a) (i)A wall-mounted, on-demand, hot-water heater. (ii) Adiabatic flame temperature. (iii) Heat exchanger dimensions. (iv) Fin dimensions.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the overall efficiency of the heat exchanger, defined as $\eta = \langle Q_u\rangle_{L\text{-}0}/\dot{S}_{r,c}$.

SOLUTION:
(a) Figure Pr.7.31(b) shows the thermal circuit diagram. The flue gas temperature is determined from the energy equation applied to the internodal energy conversion.

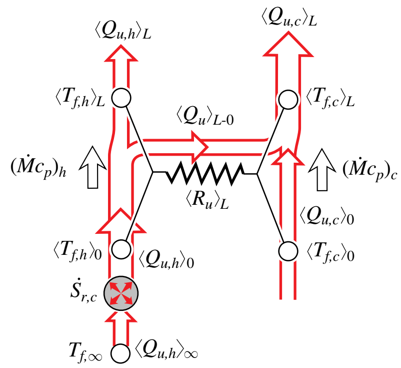


Figure Pr.7.31(b) Thermal circuit diagram.

(b) The process efficiency is defined as

$$\eta = \frac{\langle Q_u \rangle_{L\text{-}0}}{\dot{S}_{r,c}}, \quad \dot{S}_{r,c} = 12{,}900 \text{ W}.$$

We need to determine $\langle Q_u \rangle_{L\text{-}0}$. From (7.83), we have

$$\langle Q_u \rangle_{L\text{-}0} = (\dot{M}c_p)_c(\langle T_{f,c} \rangle_L - \langle T_{f,c} \rangle_0).$$

In order to determine $\langle T_{f,c} \rangle_L$ we use (7.82), i.e.,

$$\epsilon_{he} = \frac{\Delta \langle T_f \rangle |(\dot{M}c_p)_{min}}{\langle T_{f,h} \rangle_0 - \langle T_{f,c} \rangle_0}.$$

To determine ϵ_{he}, we need to evaluate NTU, given by (7.74), i.e.,

$$NTU = \frac{1}{R_\Sigma (\dot{M}c_p)_{min}},$$

and from Table 7.7, we choose the $\epsilon_{he} - NTU$ relation for the cross-flow heat exchanger (both fluids unmixed), i.e.,

$$\epsilon_{he} = 1 - e^{\frac{NTU^{0.22}}{C_r}\left[e^{-C_r NTU^{0.78}} - 1\right]} \qquad \text{Table 7.6.}$$

The inlet temperature of flue gas, $\langle T_{f,L} \rangle_0$ is found from (5.35) with Q_{loss} i.e.,

$$c_{p,f} T_{f,1} - \Delta h_{r,F} \left(\frac{\rho_{\text{F}}}{\rho_f} \right)_1 = c_{p,f} T_{f,2},$$

where we are given

$$\left(\frac{\rho_{\text{F}}}{\rho_f} \right)_1 = 0.015.$$

For air from Table C.22 at $T = 300$ K, we have $c_{p,f} = 1{,}005$ J/kg-K and for propane from Table C.21(a), we have $\Delta h_{r,\text{F}} = -50.4 \times 10^6$ J/kg. Then

$$\begin{aligned} 1{,}005(\text{J/kg-K}) \times 298.15(\text{K}) - [-50.4 \times 10^6(\text{J/kg}) \times 0.015] &= 1{,}005(\text{J/kg-K}) T_{f,2} \\ 1{,}055{,}640(\text{K}) &= 1{,}005 \times T_{f,2} \\ T_{f,2} &= \langle T_{f,h} \rangle_0 = 1{,}050 \text{ K} = 777^\circ\text{C}. \end{aligned}$$

The overall resistance R_Σ is given by (7.88), i.e.,

$$R_\Sigma = R_{ku,c} + R_k + R_{ku,h}, \quad R_k = 0.$$

For $R_{ku,c}$, we begin from (7.19), i.e.,

$$R_{ku,c} = \langle R_{ku} \rangle_D = \frac{D}{A_{ku} \langle \text{Nu} \rangle_D k_f}.$$

From Table C.23, for $T = 310$ K, for water, we have $k_f = 0.623$ W/m-K, $\nu_f = 711 \times 10^{-9}$ m^2/s and Pr $= 4.74$. The surface area is

$$A_{ku} \quad = \quad \pi D \times 5w = \pi \times 0.013 \times 1.2 = 0.049 \text{ m}^2.$$

From (7.36), we have

$$\text{Re}_D = \frac{\langle u_f \rangle D}{\nu_f}.$$

The average water velocity is found from

$$\begin{aligned} \frac{\dot{M}_c}{\rho_{f,c}} &= \langle u_f \rangle A_u \\ 6.3 \times 10^{-5} (\text{m}^3/\text{s}) &= \langle u_f \rangle \frac{\pi D^2}{4} = \langle u_f \rangle \ times \frac{\pi \times (0.013)^2 (\text{m})^2}{4} \\ \langle u_f \rangle &= 0.4746 \text{ m/s}. \end{aligned}$$

Then

$$\text{Re}_D \quad = \quad \frac{0.4746(\text{m/s}) \times 0.013(\text{m/s})}{711 \times 10^{-9} (\text{m}^2/\text{s})} = 8{,}678 > \text{Re}_{D,t} = 2{,}300 \quad \text{turbulent flow regime.}$$

From Table 7.3, we have

$$\begin{aligned} \langle \text{Nu} \rangle_D &= 0.023 \text{Re}_D^{4/5} \text{Pr}^n, \quad n = 0.4 \text{ for fluid cooling} \\ &= 0.023 \times (8{,}681)^{4/5} \times (4.24)^{0.4} = 60.60. \end{aligned}$$

Then

$$R_{ku,c} = \frac{0.013}{0.44 \times 60.65 \times 0.623} = 0.007 \text{ K/W}.$$

For the external (semi-bounded) flow over the fins, from (6.151), (6.152) and (6.153), we have

$$\begin{aligned} \frac{1}{R_{ku,h}} &= \frac{1}{R_{ku,b}} + \frac{1}{R_{ku,f}} \\ \frac{1}{R_{ku,b}} &= A_b \langle \text{Nu} \rangle_D \frac{k_w}{D}, \quad k_f = 0.0267 \text{ W/m-K} \\ A_b &= A - N_f A_k \\ N_f &= 8(\text{fins/cm}) \times 120(\text{cm}) = 960 \text{ fins} \\ A_k &= \pi D l_f = \pi (0.013)(\text{m}) \times (5 \times 10^{-4})(\text{m}) = 2.042 \times 10^{-5} \text{ m}^2 \\ A &= \pi D (5w) = \pi (0.013)(\text{m})(1.2)(\text{m}) = 0.04901 \text{ m}^2 \\ A_b &= 0.049(\text{m}^2) - (2.042 \times 10^{-5} \times 960)(\text{m}^2) = 0.02941 \text{ m}^2. \end{aligned}$$

From Table C.22, for air at $T = 300$ K, we have $\nu_f = 15.66 \times 10^{-6}$ m^2/s and Pr$= 0.69$. Also, $l = 0.07$ m. To determine, $u_{f,\infty}$ we have from (5.34)

$$\begin{aligned} \dot{S}_{r,c} &= \dot{M}_\text{F} \Delta h_{r,\text{F}} \\ 12{,}900(\text{W}) &= \dot{M}_\text{F} \times 50.4 \times 10^6 (\text{J/kg}) \\ \dot{M}_\text{F} &= 2.560 \times 10^{-4} \text{ kg/s} \quad \text{fuel mass flow rate} \\ \dot{M}_f &= \frac{\dot{M}_\text{F}}{(\rho_\text{F}/\rho_f)_1} = \frac{2.56 \times 10^4}{0.015} = 0.01706 \text{ kg/s} \quad \text{total flow rate.} \end{aligned}$$

Also,

$$\dot{M}_f = A_u u_{f,\infty} \rho_{f,\infty},$$

where from Table C.22, for air at $T = 300$ K, we have

$$\rho_{f,\infty} = 1.177 \text{ kg/m}^3$$

Using Figure 7.14(a), we have

$$A_u = wa = 0.24(\text{m}) \times 0.1524(\text{m}) = 0.03658 \text{ m}^2.$$

Then

$$\begin{aligned} 0.01706(\text{kg/s}) &= 0.03658(\text{m})^2 \times 1.177(\text{kg/m}) \times u_{f,\infty} \\ u_{f,\infty} &= 0.3963 \text{ m/s}. \end{aligned}$$

The flow over the base area will be modeled as flow over a cylinder with a characteristic length of the diameter D (the wall thickness is ignored). Then, from (6.45), the Reynolds number is

$$\text{Re}_D = \frac{u_{f,\infty} D}{\nu_f} = \frac{0.3963(\text{m/s}) \times 0.013(\text{m})}{15.66 \times (\text{m}^2/\text{s})} = 329.0$$

The Nusselt number correlation is obtained from Table 6.4, where for the given Reynolds number,

$$\begin{aligned} \langle \text{Nu} \rangle_D &= 0.683 \text{Re}_D^{0.466} \text{Pr}^{1/3} \\ &= 0.683 \times (329.0)^{0.466} \times (0.69)^{1/3} \\ &= 8.990. \end{aligned}$$

And so,

$$\frac{1}{R_{ku,b}} = 0.02941(\text{m})^2 \times 8.990 \times \frac{0.0267(\text{W/m-K})}{0.013(\text{m})}) = 0.5869 \text{ W/K}.$$

The flow over the fins is modeled as that over a flat plate of length l. The Reynolds number is

$$\text{Re}_l = \frac{0.3963(\text{m/s}) \times 0.07(\text{m})}{15.66 \times 10^{-6}(\text{m}^2/\text{s})} = 1{,}781 < \text{Re}_{l,t} = 5 \times 10^5, \quad \text{laminar flow regime.}$$

From Table 6.3, we have

$$\begin{aligned} \langle \text{Nu} \rangle_l &= 0.664 \text{Re}_l \text{Pr}^{1/3} \\ \langle \text{Nu} \rangle_l &= 0.664 \times (1{,}781)^{1/2} \times (0.69)^{1/3} = 24.70 \end{aligned}$$

From (6.153), we have

$$\frac{1}{R_{ku,f}} = N_f A_{ku} \eta_f \langle \text{Nu} \rangle_l \frac{k_f}{l},$$

where $\eta_f = 1$, $k_f = 0.0267$ W/m-K, and $l = 0.07$ m.
Then,

$$\begin{aligned} A_{ku,fin} &= 2(l^2 - \pi D^2/4) + 4 l l_f \\ &= 2[(0.07)^2(\text{m})^2 - \pi \times (0.013)^2/4(\text{m})^2] + 4 \times 0.07(\text{m}) \times 0.0005(\text{m}) \\ &= 9.6745 \times 10^{-3}(\text{m})^2 \\ A_{ku} &= N_f A_{ku,fin} = 960 \times 9.6745 \times 10^{-3}(\text{m})^2 \\ &= 9.2876 \text{ m}^2 \\ \frac{1}{R_{ku,f}} &= 9.2876 \times 24.70 \times \frac{0.0267(\text{W/m-K})}{0.07(\text{m})}) = 87.50\text{W/K} \\ \frac{1}{R_{ku,h}} &= (87.50 + 0.5869)(\text{W/K}) \\ R_{ku,h} = \frac{1}{88.09(\text{W/K})} &= 0.01135 \text{ K/W}. \end{aligned}$$

From Table C.23, for water at $T = 300$ K, we have $\rho_{f,c} = 995.3$ kg/m^3 and $c_{p,c}$ =4,178 J/kg-K. Then,

$$\begin{aligned}
\dot{M}_c &= \left(\frac{\dot{M}}{\rho_f}\right)_c \rho_{f,c} = 6.3\times10^{-5}(\text{m}^3/\text{s})\times 995.3(\text{kg/m}^3) = 0.06270 \text{ kg/s} \\
(\dot{M}c_p)_c &= 0.0627\times 4178 = 262.0 \text{ W/K} \\
\dot{M}_{f,h} &= 0.01076 \text{ kg/s} \\
(\dot{M}c_p)_h &= (\dot{M}c_p)_{min} = 0.01706(\text{kg/s})\times 1{,}005(\text{J/kg-K}) = 17.145 \text{ W/K.}
\end{aligned}$$

Also

$$R_\Sigma = R_{ku,h} + R_{ku,c} = (0.01135 + 0.007342)(\text{K/W}) = 0.018695 \text{ K/W.}$$

Next

$$\begin{aligned}
NTU &= \frac{1}{R_\Sigma(\dot{M}c_p)_{min}} = \frac{1}{0.018695 \text{ K/W}\times 17.145 \text{ K/W}} = 3.120 \\
\epsilon_{he} &= 1 - e^{\frac{NTU^{0.22}}{c_r}\left[e^{-c_r NTU^{0.78}}-1\right]} \\
c_r &= \frac{(\dot{M}c_p)_{min}}{(\dot{M}c_p)_{max}} = \frac{17.145}{262} = 0.06544 \\
\epsilon_{he} &= 1 - e^{19.628\times(-0.1470)} = 0.9442.
\end{aligned}$$

Then

$$\begin{aligned}
\epsilon_{he} &= \frac{\langle T_{f,h}\rangle_0 - \langle T_{f,h}\rangle_L}{\langle T_{f,h}\rangle_0 - \langle T_{f,c}\rangle_0} \\
0.9442 &= \frac{(777.24 - \langle T_{f,h}\rangle_L)(^\circ\text{C})}{(777.24-20)(^\circ\text{C})} \\
714.96 &= 777.24 - \langle T_{f,h}\rangle_L \\
\langle T_{f,h}\rangle_L &= 62.28^\circ\text{C.}
\end{aligned}$$

Finally

$$\begin{aligned}
\langle Q_u\rangle_{L\text{-}0} &= (\dot{M}c_p)_h\Delta T_h = 17.145(\text{W/K})\times 714.96(\text{K}) = 12{,}258 \text{ W} \\
\eta &= \frac{\langle Q_u\rangle_{L\text{-}0}}{\dot{S}_{r,c}} = \frac{12{,}258}{12{,}900} = 95.0\%.
\end{aligned}$$

COMMENT:

It should be noted that in order to have a large NTU, the overall resistance R_Σ must be low. This can be achieved with a large surface area for surface-convection heat transfer. This is accomplished here using the fins. If the gas-side (with low k_f) was not finned, the heat transfer would have been minimal and the efficiency would have dropped significantly. For more accuracy, the properties of the flue gas should be determined at a higher average temperature. The Nusselt number correlations used are approximations. The real flow will be very complex due to the geometry of the tube and the fins. Neither the flat plate flow or cylinder crossflow will be a representation of the real flow, but are the most appropriate correlations available. Experiments would need to be run on the apparatus or a scale model to accurately determine a Nusselt number correlation.

PROBLEM 7.32.FAM

GIVEN:

Water is heated in a heat exchanger where the hot stream is a pressurized fluid undergoing phase change (condensing). The pressure of the hot stream is regulated such that $\Delta T_{max} = \langle T_{f,h}\rangle_0 - \langle T_{f,c}\rangle_0$ remains constant. The parallel-flow heat exchanger is shown in Figure Pr.7.32(a). The heat exchanger is used for the two cases where the average cold stream temperature (i) $\langle T_{f,c}\rangle = 290$ K, and (ii) $\langle T_{f,c}\rangle = 350$ K. These influence the thermophysical properties.

$\langle T_{f,h}\rangle_0 = \langle T_{f,h}\rangle_L = 100°\text{C}$, $\Delta T_{max} = 20°\text{C}$, $\dot{M}_{f,c} = 0.5$ kg/s, $L = 5$ m, $D_i = 2$ cm, $\Delta h_{lg} = 2.2 \times 10^6$ J/kg.

Evaluate the water properties (Table C.23), for the cold stream, at the cold stream average temperature for each case. Neglect the effect of coiling (bending) of the tube on the Nusselt number. Neglect the wall and hot stream thermal resistances.

SKETCH:

Figure Pr.7.32(a) shows the coaxial heat exchanger.

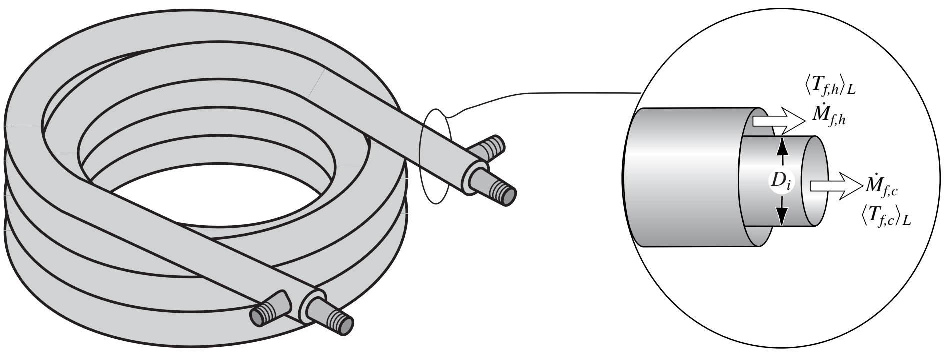


Figure Pr.7.32(a) A coaxial heat exchanger.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the NTU for cases (i) and (ii) and briefly comment on the effect of cold stream average temperature on the capability of the heat exchanger.

(c) Determine the heat transfer rates (W) for cases (i) and (ii).

(d) Determine the rates of condensation [i.e., mass flow rates of condensed fluid in the hot stream (kg/s)] for cases (i) and (ii) assuming Δh_{lg} remains constant.

SOLUTION:

(a) The thermal circuit is shown in Figure Pr.7.32(b).

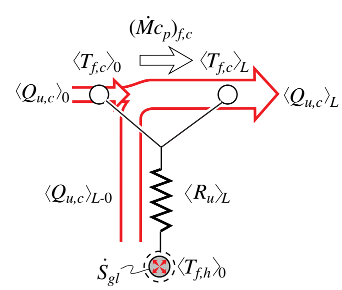


Figure Pr.7.32(b) Thermal circuit diagram.

(b) From Table C.23, the water properties are listed in Table Pr.7.32.

Table Pr.7.32 Properties of water

	$\langle T_{f,c}\rangle = 290$ K	$\langle T_{f,c}\rangle = 350$ K
ρ_f, kg/m^3	1,000	975.7
ν_f, m^2/s	113×10^{-8}	381×10^{-9}
k_f, W/m-K	0.59	0.665
$c_{p,f}$, J/kg-K	4,186	4,194
Pr	8.02	2.34

Case (i) $\langle T_{f,c}\rangle = 290$ K:
The NTU is defined from (7.74) as

$$NTU = \frac{1}{(\dot{M}_f c_p)_{min} R_\Sigma}.$$

Since the hot stream experiences phase change, the $R_{ku,h}$ is negligible and $(\dot{M}_f c_p)_h \to \infty = (\dot{M}_f c_p)_{max}$. Also neglecting the conduction resistance in the separating wall, we then have $R_\Sigma = R_{ku,h} + R_k + R_{ku,c} = R_{ku,c}$, or

$$R_\Sigma = R_{ku,c} = \frac{1}{A_{ku}\langle \text{Nu}\rangle_{D,i} k_f/D_i},$$

where $A_{ku} = \pi D_i L = \pi \times 0.02(\text{m}) \times 5(\text{m}) = 0.3142$ m^2. The Nusselt number for the cold stream depends on $\text{Re}_{D,i}$, which is

$$\text{Re}_{D,i} = \frac{\langle u_f\rangle_c D_i}{\nu_{c,i}} = \frac{\dot{M}_{f,c} D_i}{A_u \mu_c} = \frac{\dot{M}_{f,c} D_i}{\frac{\pi D_i^2}{4}\mu_f} = \frac{4\dot{M}_{f,c}}{\pi D_i \mu_c}$$

$$= \frac{4 \times 0.5(\text{kg/s})}{113 \times 10^{-8}(\text{m}^2/\text{s}) \times 1{,}000(\text{kg/m}^3) \times \pi \times 0.02(\text{m})} = 28{,}169.$$

The Nusselt number is then (assuming turbulent fluid flow and for $\text{Re}_{D,i} \geq 10^4$) found from Table 7.3 as

$$\begin{aligned}\langle \text{Nu}\rangle_{D,i} &= 0.023\text{Re}_{D,i}^{4/5}\text{Pr}^{0.4}\\ &= 0.023 \times (28{,}169)^{4/5} \times (8.02)^{0.4} = 191.96,\end{aligned}$$

and the thermal resistance is

$$R_\Sigma = \frac{1}{0.3142(\text{m}^2) \times 191.96 \times 0.59(\text{W/m-K})/0.02(\text{m})} = 5.620 \times 10^{-4}\,^\circ\text{C/W}.$$

The NTU is then

$$NTU = \frac{1}{0.5(\text{kg/s}) \times 4186(\text{J/kg-K}) \times 5.620 \times 10^{-4}(^\circ\text{C/W})} = 0.8501$$

Case (ii) $\langle T_{f,c}\rangle = 350$ K:
The $\text{Re}_{D,i}$ is

$$\text{Re}_{D,i} = \frac{4\dot{M}_{f,c}}{\pi D_i \mu_c}$$

$$= \frac{4 \times 0.5(\text{kg/s})}{381 \times 10^{-9}(\text{m}^2/\text{s}) \times 975.7(\text{kg/m}^3) \times \pi \times 0.02(\text{m})} = 85{,}627.$$

The Nusselt number is then (assuming turbulent fluid flow and for $\text{Re}_{D,i} \geq 10^4$)

$$\begin{aligned}\langle \text{Nu}\rangle_{D,i} &= 0.023\text{Re}_{D,i}^{4/5}\text{Pr}^{0.4}\\ &= 0.023 \times (85{,}627)^{4/5} \times (2.34)^{0.4} = 285.43,\end{aligned}$$

and the thermal resistance is

$$R_\Sigma = \frac{1}{0.3142(\text{m}^2) \times 285.43 \times 0.665(\text{W/m-K})/0.02(\text{m})} = 3.354 \times 10^{-4}\,^\circ\text{C/W}.$$

The NTU is then

$$NTU = \frac{1}{0.5(\text{kg/s}) \times 4194(\text{J/kg-K}) \times 3.354 \times 10^{-4}(^\circ\text{C/W})} = 1.422$$

The lower cold stream inlet temperature of Case (i) results in an NTU nearly 60% of that of Case (ii). Thus at higher temperatures, the effect of temperature on the properties allows for an increased effectiveness of the heat exchanger.

(c) Case (i):
The heat exchanger effectiveness for this heat exchanger ($C_r = 1$) is found from Table 7.7 as

$$\epsilon_{he} = 1 - e^{-NTU} = 1 - e^{-0.8501} = 0.5726.$$

The heat transfer rate is then found from

$$\begin{aligned}\langle Q_u \rangle_{L\text{-}0} &= (\dot{M}c_p)_{f,c}(\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0) \\ &= (\dot{M}c_p)_{f,c}\epsilon_{he}(\langle T_{f,h}\rangle_0 - \langle T_{f,c}\rangle_0) \\ &= 0.5(\text{kg/s}) \times 4186(\text{J/kg-K}) \times 0.5726 \times 20(\text{K}) = 23{,}970 \text{ W}\end{aligned}$$

Case (ii):
The heat exchanger effectiveness is

$$\epsilon_{he} = 1 - e^{-NTU} = 1 - e^{-1.422} = 0.7588.$$

The heat transfer rate is then

$$\begin{aligned}\langle Q_u \rangle_{L\text{-}0} &= (\dot{M}c_p)_{f,c}\epsilon_{he}(\langle T_{f,h}\rangle_0 - \langle T_{f,c}\rangle_0) \\ &= 0.5(\text{kg/s}) \times 4194(\text{J/kg-K}) \times 0.7588 \times 20(\text{K}) = 31{,}823 \text{ W}.\end{aligned}$$

(d) Case (i):
The rate of condensation is found from the heat transfer rate as

$$\langle Q_u \rangle_{L\text{-}0} = \dot{M}_{lg}\Delta h_{lg},$$

or

$$\dot{M}_{lg} = \frac{\langle Q_u \rangle_{L\text{-}0}}{\Delta h_{lg}} = \frac{23{,}970(\text{W})}{2.2 \times 10^6(\text{J/kg})} = 0.01090 \text{ kg/s}.$$

Case (ii):
The rate of condensation is

$$\dot{M}_{lg} = \frac{\langle Q_u \rangle_{L\text{-}0}}{\Delta h_{lg}} = \frac{31{,}823(\text{W})}{2.2 \times 10^6(\text{J/kg})} = 0.01446 \text{ kg/s}.$$

COMMENT:
The bending of the tube results in an increase of the Nusselt number. Note that the NTU is significantly different between the two cases indicating the sensitivity of the predictions to the fluid temperature used to evaluate the assumed constant thermophysical properties.

PROBLEM 7.33.FAM.S

GIVEN:

A water boiler using natural gas combustion is shown in Figure Pr.7.33. Consider the evaporator section, where the water temperature is assumed constant and at $T_{lg}(p_g)$. The conduction through the walls separating the combustion flue gas and the water stream is assumed negligible. Also, assume that that water-side surface convection resistance $R_{ku,c}$ is negligibly small.

$\dot{M}_{f,h} = 0.02$ kg/s, $\dot{S}_{r,c} = 30$ kW, $D = 15$ cm, $\langle T_{f,h}\rangle_0 = 1{,}200$°C, $\langle T_{f,c}\rangle = T_{lg} = 100$°C.

Evaluate flue gas properties using air at $T = 1{,}000$ K.

SKETCH:

Figure Pr.7.33 shows the boiler with the water stream flowing through a coil wrapped around the combustor.

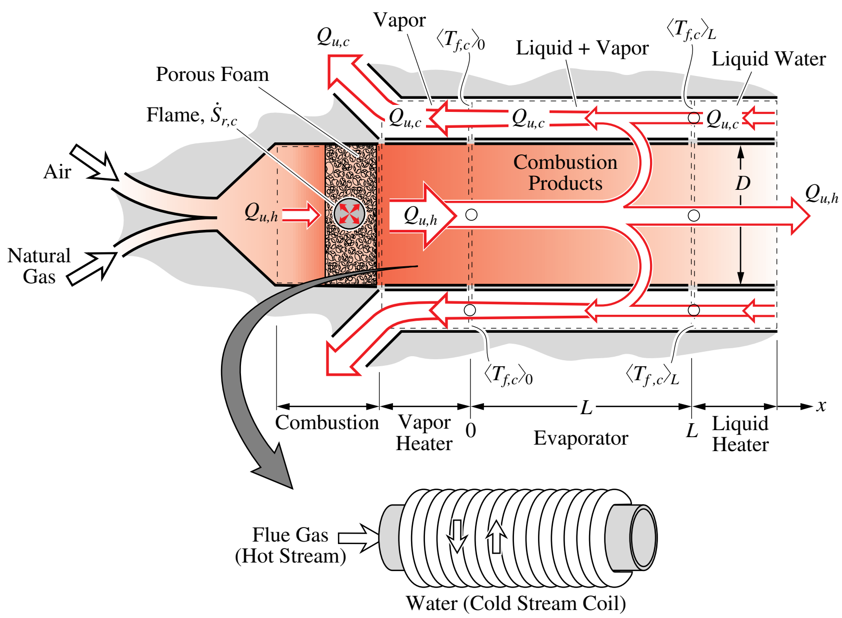


Figure Pr.7.33 A water boiler using the flue gas from the combustion of natural gas.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the length L needed to transfer 70% of $\dot{S}_{r,c}$ to the water.

SOLUTION:

(a) Figure Pr.7.33(b) shows the thermal circuit diagram.

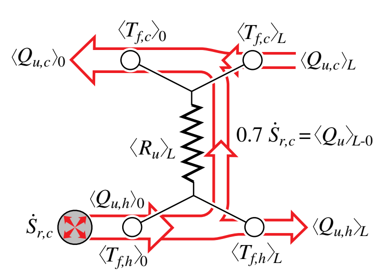


Figure Pr.7.33(b) Thermal circuit diagram.

(b) From (7.85), we have

$$\langle Q_u\rangle_{L\text{-}0} = \frac{\langle T_{f,h}\rangle_0 - \langle T_{f,c}\rangle_0}{\langle R_u\rangle_L} = 0.7\dot{S}_{r,c},$$

where from (7.86), we have

$$\langle R_u\rangle_L = \frac{1}{(\dot{M}c_p)_h\epsilon_{he}}.$$

From Table 7.7, for a constant $\langle T_{f,c}\rangle$ (i.e., $C_r = 0$), we have

$$\epsilon_{he} = 1 - e^{-NTU}.$$

From (7.74), we have

$$NTU = \frac{1}{R_\Sigma(\dot{M}c_p)_h}.$$

From (7.87), based on given simplifications, we have

$$R_\Sigma = R_{ku,h}.$$

To determine $R_{ku,h}$ from Table 7.2 or 7.3, we begin with the Reynolds number, i.e.,

$$\text{Re}_D = \frac{\langle u_f\rangle D}{\nu_f} = \frac{\dot{M}_{f,h}D}{A_u\mu_f} = \frac{\dot{M}_{f,h}D}{\dfrac{\pi D^2}{4}\mu_f} = \frac{4\dot{M}_f}{\pi D\mu_f}.$$

From Table C.22, for air at $T_o = 1{,}000$K, We have

$$\begin{aligned}
\text{air}: c_p &= 1{,}130 \text{ J/kg-℃} && \text{Table C.22}\\
\rho_f &= 0.354 \text{ kg/m}^3 && \text{Table C.22}\\
k_f &= 0.0672 \text{ W/m-K} && \text{Table C.22}\\
\nu_f &= 1.173\times10^{-4} \text{ m}^2\text{/s} && \text{Table C.22}\\
\text{Pr} &= 0.70 && \text{Table C.22.}
\end{aligned}$$

Then

$$\begin{aligned}
\text{Re}_D &= \frac{4\times0.02(\text{kg/s})}{\pi\times0.15(\text{m})\times0.3554(\text{kg/m}^3)\times1.173\times10^{-4}(\text{m}^2/\text{s})}\\
&= 4{,}088 > \text{Re}_t = 2{,}300 \quad \text{turbulent flow regime}
\end{aligned}$$

From Table 7.3, we have, assuming fully-developed turbulent flow,

$$\begin{aligned}
\langle\text{Nu}\rangle_D &= 0.023\text{Re}_D^{0.8}\text{Pr}^{0.3}\\
&= 0.023\times(4{,}088)^{0.8}\times(0.7)^{0.3} = 16.01.
\end{aligned}$$

From (7.88), we have

$$\begin{aligned}
R_{ku,h}^{-1} &= \frac{A_{ku}\langle\text{Nu}\rangle_D k_f}{D}, \quad A_{ku} = \pi DL\\
&= \pi L\langle\text{Nu}\rangle_D k_f\\
&= \pi\times16.01\times0.0672(\text{W/m-K})\times L\\
&= 3.381L(\text{W/m℃}).
\end{aligned}$$

Now combining the relations, we have

$$0.7\dot{S}_{r,c} = (\langle T_{f,h}\rangle_0 - T_{lg})(\dot{M}c_p)_h(1-e^{-NTU}),$$

where

$$NTU = \frac{1}{R_{ku,h}(\dot{M}c_p)_L} = \frac{3.381L(\text{W/m℃})}{0.02(\text{kg/s})\times1{,}130(\text{J/kg-℃})} = 0.1496(1/\text{m})\times L.$$

Then

$$
\begin{aligned}
0.7 \times 3 \times 10^4 (\text{W}) &= (1{,}200 - 100)(^\circ\text{C}) \times 0.02(\text{kg/s}) \times \\
&\quad 1{,}130(\text{J/kg-}^\circ\text{C}) \times (1 - e^{-0.1496L}) \\
2.1 \times 10^4 (\text{W}) &= 2.486 \times 10^4 (\text{W})(1 - e^{-0.1496L})
\end{aligned}
$$

or

$$L = 12.45 \text{ m}.$$

COMMENT:

Since no fins (extended surface)are used, the length of the heat exchanger is rather large. By adding fins which can have a fin effectiveness near 10, the length can be significantly reduced.

PROBLEM 7.34.FAM

GIVEN:

Air and nitrogen (gas) stream exchange heat in a parallel-flow, coaxial tube heat exchanger. The inner diameter is $D_i = 3$ cm and the outer diameter is $D_o = 5$ cm. This is shown in Figure Pr.7.34. Nitrogen flows through the inside tube at an average velocity of $\langle u_f \rangle_c = 1$ m/s and enters at $\langle T_{f,c} \rangle_0 = 4$℃. Air flows in the outside tube at an average velocity of $\langle u_f \rangle_h = 2$ m/s and enters at $\langle T_{f,h} \rangle_0 = 95$℃. Neglect the tube wall thickness and assume heat exchange only between these streams.

For simplicity, evaluate the air and nitrogen properties at their inlet temperatures. Use the constant surface temperature condition to determine the Nusselt number.

SKETCH:

Figure Pr.7.34 shows a coaxial, parallel-flow heat exchanger.

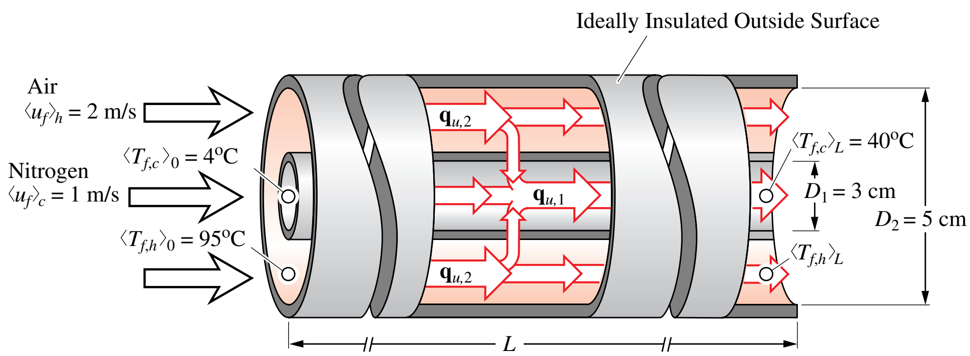


Figure Pr.7.34 A parallel-flow heat exchanger.

OBJECTIVE:

(a) For a nitrogen exit temperature of 40℃, determine the amount of heat transferred $\langle Q_u \rangle_{L\text{-}0}$.
(b) Also, determine the length of heat exchanger L needed.
(c) Assuming that the average velocities and inlet temperatures remain the same, what would be the maximum increase in the nitrogen stream temperature $\Delta \langle T_{f,c} \rangle_{max}$ that could be achieved?

SOLUTION:

(a) The inlet and outlet temperatures for the nitrogen, which is the cold stream, are known. Then the heat transferred between the two fluids can be calculated from the difference in the convection heat flow for the nitrogen between the inlet and exit. Using (7.85) we have

$$\langle Q_u \rangle_{L\text{-}0} = (\dot{M} c_p)_c (\langle T_{f,c} \rangle_L - \langle T_{f,c} \rangle_0).$$

The properties for nitrogen at $T = 4$℃$= 277.15$ K, from Table C.22, are $k_f = 0.0252$ W/m-K, $\rho_f = 1.243$ kg/m^3, $c_{p,f} = 1043.5$ J/kg-K, $\nu_f = 13.57 \times 10^{-6}$ m^2/s, and Pr $= 0.69$.
The mass flow rate is

$$\dot{M}_c = (A_u \rho_f \langle u_f \rangle)_c = \frac{\pi D_1^2}{4} (\rho_f \langle u_f \rangle)_c = \frac{\pi (0.03)^2 (\text{m})^2}{4} \times 1.243 (\text{kg/m}^3) \times 1 (\text{m/s}) = 8.786 \times 10^{-4} \text{ kg/s}.$$

Then the heat transfer rate is

$$\langle Q_u \rangle_{L\text{-}0} = 8.786 \times 10^{-4} (\text{kg/m}^3) \times 1043.5 (\text{J/kg-K}) \times [40(℃) - 4(℃)] = 33.01 \text{ W}.$$

(b) To calculate the required length of the heat exchanger needed, we note that the internal surface-convection areas $A_{ku,h}$ and $A_{ku,c}$ depend on L. The surface-convection area for the cold fluid $A_{ku,c}$ is

$$A_{ku,c} = \pi D_1 L$$

and, for the hot fluid, as the thickness of the pipe wall for the internal pipe is very thin, the surface-convection area is

$$A_{ku,h} = A_{ku,c} = \pi D_1 L.$$

These influence the the overall heat exchanger thermal resistance (7.88), i.e.,

$$R_\Sigma = \langle R_{ku,c}\rangle_{D_h,h} + R_{k,c\text{-}h} + \langle R_{ku,h}\rangle_{D_h,h}$$

Using (7.88) for the surface-convection resistances $R_{ku,h}$, and neglecting the conduction resistance through the pipe wall, we have

$$R_\Sigma = \frac{D_{h,c}}{A_{ku,c}\ \langle \text{Nu}\rangle_{D_h,c}\ k_{f,c}} + \frac{D_{h,h}}{A_{ku,h}\ \langle \text{Nu}\rangle_{D_h,h}\ k_{f,h}}.$$

The Nusselt numbers are obtained from the correlations in Tables 7.2 and 7.3. The overall thermal resistance R_Σ is related to the number of transfer units NTU through (7.74), i.e.,

$$NTU = \frac{1}{R_\Sigma(\dot{M}c_p)_{min}},$$

where $(\dot{M}c_p)_{min}$ is the $\dot{M}c_p$ for the fluid with the smallest $\dot{M}c_p$. For a parallel-flow heat exchanger, the number of transfer units is related to the heat exchanger effectiveness ϵ_{he} as given in Table 7.7, i.e.,

$$\epsilon_{he} = \frac{1 - e^{-NTU(1+C_r)}}{1 + C_r}.$$

The heat exchanger effectiveness ϵ_{he} can be obtained from the temperatures through (7.72), i.e.,

$$\epsilon_{he} = \frac{\Delta\langle T_f\rangle_{(\dot{M}c_p)_{min}}}{\langle T_{f,h}\rangle_0 - \langle T_{f,c}\rangle_0}.$$

Solving the equations above will lead to the determination of L.
First, the properties for air are needed. From Table C.22, at $T = 95$℃$= 368.15$ K, we have $k_f = 0.0312$ W/m-K, $\rho_f = 0.961$ kg/m^3, $c_{p,f} = 1{,}008$ J/kg-K, $\nu_f = 22.13 \times 10^{-6}$ m^2/s, and Pr $= 0.69$.
The mass flow rate of air is

$$\begin{aligned}\dot{M}_h &= (A_u \rho_f \langle u_f\rangle)_h = \frac{\pi(D_o^2 - D_i^2)}{4}(\rho_f\langle u_f\rangle)_h \\ &= \frac{\pi \times (0.05^2 - 0.03^2)(\text{m})^2}{4} \times 0.961(\text{kg/m}^3) \times 2(\text{m/s}) = 2.415 \times 10^{-3} \text{ kg/s}.\end{aligned}$$

The streams thermal capacities are

$$\begin{aligned}(\dot{M}c_p)_c &= 8.786 \times 10^{-4}(\text{kg/s}) \times 1{,}043.5(\text{J/kg-K}) = 0.9168 \text{ W/℃} \\ (\dot{M}c_p)_h &= 2.415 \times 10^{-3}(\text{kg/s}) \times 1{,}008(\text{J/kg-K}) = 2.434 \text{ W/℃}.\end{aligned}$$

Then $(\dot{M}c_p)_{min} = (\dot{M}c_p)_c$. The heat exchanger effectiveness ϵ_{he} becomes

$$\epsilon_{he} = \frac{\langle T_{f,c}\rangle_L - \langle T_{f,c}\rangle_0}{\langle T_{f,h}\rangle_0 - \langle T_{f,c}\rangle_0} = \frac{40(℃) - 4(℃)}{95(℃) - 4(℃)} = 0.3956.$$

The ratio of the thermal capacitances is

$$C_r = \frac{(\dot{M}c_p)_{min}}{(\dot{M}c_p)_{max}} = \frac{0.917(\text{W/℃})}{2.449(\text{W/℃})} = 0.3767.$$

Solving (7.74) for NTU, we have

$$NTU = \frac{-\ln\left[1 - \epsilon_{he}\,(1 + C_r)\right]}{1 + C_r} = \frac{-\ln\left[1 - 0.3956\,(1 + 0.3767)\right]}{1 + 0.3767} = 0.5714.$$

The overall thermal resistance is

$$R_\Sigma = \frac{1}{NTU\left(\dot{M}c_p\right)_{\min}} = \frac{1}{0.572 \times 0.9168(\text{W/}^\circ\text{C})} = 1.909^\circ\text{C/W}$$

We now need to determine the average Nusselt numbers.
For the nitrogen, the Reynolds number is

$$\text{Re}_{D,h} = \frac{\langle u_f \rangle D_{h,c}}{\nu_f} = \frac{1(\text{m/s}) \times 0.03(\text{m})}{13.57 \times 10^{-6}(\text{m}^2/\text{s})} = 2{,}211.$$

Since $\text{Re}_{D,h} < \text{Re}_{D,t} = 2300$, the flow regime is laminar. For the laminar regime the Nusselt number is obtained from Table 7.2. Assuming that T_s is constant, the Nusselt number is

$$\langle \text{Nu} \rangle_{D_h,c} = 3.66.$$

For the air, the Reynolds number is

$$\text{Re}_{D,h} = \frac{\langle u_f \rangle_c D_{h,h}}{\nu_f} = \frac{2(\text{m/s}) \times (0.05 - 0.03)(\text{m})}{22.13 \times 10^{-6}(\text{m}^2/\text{s})} = 1{,}808,$$

where the hydraulic diameter D_h, from Table 7.2, is $D_{h,h} = D_2 - D_1$.
Since $\text{Re}_{D,h} < \text{Re}_{D,t} = 2300$, the flow regime is laminar. For the laminar regime the Nusselt number is obtained from Table 7.2. For $D_1/D_2 = 0.6$, the Nusselt number is

$$\langle \text{Nu} \rangle_{D_h,h} = 5.912.$$

The surface-convection area for hot and cold streams is the same (zero wall thickness) and equal to $\pi D_1 L$. Solving for L from the expression for R_Σ gives

$$\begin{aligned} L &= \frac{1}{R_\Sigma \pi D_1}\left(\frac{D_{h,c}}{\langle \text{Nu} \rangle_{D_h,c} k_{f,c}} + \frac{D_{h,h}}{\langle \text{Nu} \rangle_{D_h,h}\ k_{f,h}}\right) \\ &= \frac{1}{\pi \times 1.909(^\circ\text{C/W}) \times 0.03(\text{m})}\left[\frac{0.03(\text{m})}{3.66 \times 0.0252(\text{W/m-K})} + \frac{0.02(\text{m})}{5.912 \times 0.0312(\text{W/m-K})}\right]. \\ &= 2.4 \text{ m}. \end{aligned}$$

(c) The maximum increase in the nitrogen temperature is achieved when $R_\Sigma \rightarrow 0$. From (7.74), this gives $NTU \rightarrow \infty$. For counter-flow heat exchangers, from Table 7.7, we have

$$(\epsilon_{he})_{max} = \lim_{NTU \to \infty} \epsilon_{he} = \lim_{NTU \to \infty}\left[\frac{1 - e^{-NTU(1+C_r)}}{1 + C_r}\right] = \frac{1}{1 + C_r}.$$

For $C_r = 0.3767$, we have

$$(\epsilon_{he})_{\max} = \frac{1}{1 + C_r} = \frac{1}{1 + 0.3767} = 0.7264.$$

Now using (7.72) and solving for $\langle T_{f,c} \rangle_L - \langle T_{f,c} \rangle_0$ from (7.72), we have

$$\Delta \langle T_{f,c} \rangle_{max} = (\epsilon_{he})_{max}(\langle T_{f,h} \rangle_0 - \langle T_{f,c} \rangle_0) = 0.7264 \times [95(^\circ\text{C}) - 4(^\circ\text{C})] = 66.10^\circ\text{C}.$$

COMMENT:

Using $\langle Q_u \rangle_{L\text{-}0} = 33.01$ W in (7.84), the outlet temperature of the air is $\langle T_{f,h} \rangle_L = 81.44^\circ\text{C}$. Note that here the mass flow rates for both fluids is low enough to result in a laminar flow in both streams.

PROBLEM 7.35.FUN

GIVEN:

The flow is said to be thermally fully-developed, when

$$\frac{\partial}{\partial x}\left(\frac{T_s-\overline{T}_f}{T_s-\langle\overline{T}_f\rangle}\right)=0,$$

and the energy equation for turbulent flow in a tube becomes

$$\overline{u}_f\frac{\partial\overline{T}_f}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left[r(\alpha_f+\alpha_t)\frac{\partial\overline{T}_f}{\partial r}\right].$$

OBJECTIVE:

(a) Show that for the thermally fully-developed turbulent flow with uniform wall temperature T_s, we have

$$\frac{\partial\overline{T}_f}{\partial x}=\frac{T_s-\overline{T}_f}{T_s-\langle\overline{T}_f\rangle}\frac{d\langle\overline{T}_f\rangle}{dx}.$$

(b) Show that the mean fluid temperature distribution can be expressed as

$$1-T^*=\frac{\overline{T}_f-\overline{T}_f(r=0)}{T_s-\overline{T}_f(r=0)}=\frac{R^2\langle\overline{u}_f\rangle(d\langle\overline{T}_f\rangle/dx)}{T_s-\langle\overline{T}_f\rangle}\int_0^{r^*}\frac{\phi(r^*)}{r^*(\alpha_f+\alpha_t)}dr^*,$$

where

$$\phi(r^*)=\int_0^{r^*}\frac{\overline{u}_f}{\langle\overline{u}_f\rangle}T^*r^*dr^*,\qquad T^*=\frac{T_s-\overline{T}_f}{T_s-\overline{T}_f(r=0)}\qquad\text{and}\qquad r^*=\frac{r}{R}.$$

Hint: For part (b), integrate the energy equation over $r=0$ to $r=r$, using the result from part (a) to eliminate $\partial\overline{T}_f/\partial x$.

SOLUTION:

(a) By differentiating the expression for thermally fully-developed flow, we have

$$\frac{1}{T_s-\langle\overline{T}_f\rangle}\frac{\partial T_s}{\partial x}-\frac{1}{T_s-\langle\overline{T}_f\rangle}\frac{\partial\overline{T}_f}{\partial x}+\frac{T_s}{(T_s-\langle\overline{T}_f\rangle)^2}\frac{\partial\langle\overline{T}_f\rangle}{\partial x}-\frac{T_s}{(T_s-\langle\overline{T}_f\rangle)^2}\frac{\partial T_s}{\partial x}+$$
$$+\frac{\overline{T}_f}{(T_s-\langle\overline{T}_f\rangle)^2}\frac{\partial T_s}{\partial x}-\frac{T_s}{(T_s-\langle\overline{T}_f\rangle)^2}\frac{\partial\langle\overline{T}_f\rangle}{\partial x}=0$$

or

$$\frac{1}{T_s-\langle\overline{T}_f\rangle}\frac{\partial T_s}{\partial x}-\frac{1}{T_s-\langle\overline{T}_f\rangle}\frac{\partial\overline{T}_f}{\partial x}+\frac{\overline{T}_f-T_s}{(T_s-\langle\overline{T}_f\rangle)^2}\left[\frac{\partial T_s}{\partial x}-\frac{\partial\langle\overline{T}_f\rangle}{\partial x}\right]=0.$$

Since $\partial T_s/\partial x=0$, we have

$$-\frac{1}{T_s-\langle\overline{T}_f\rangle}\frac{\partial\overline{T}_f}{\partial x}+\frac{\overline{T}_f-T_s}{(T_s-\langle\overline{T}_f\rangle)^2}\left[-\frac{\partial\langle\overline{T}_f\rangle}{\partial x}\right]=0\ \text{, which gives}\quad\frac{\partial\overline{T}_f}{\partial x}=\frac{T_s-\overline{T}_f}{T_s-\langle\overline{T}_f\rangle}\frac{d\langle\overline{T}_f\rangle}{dx}.$$

(b) Substituting the above equation into the energy equation and integrating from $r=0$ to $r=r$, we have

$$\frac{d\langle\overline{T}_f\rangle}{dx}\int_0^r\overline{u}\left(\frac{T_s-\overline{T}_f}{T_s-\langle\overline{T}_f\rangle}\right)rdr=\int_0^r d\left[r(\alpha_f+\alpha_t)\frac{\partial\overline{T}_f}{\partial r}\right].$$

Multiplying the left-hand side by

$$1=\frac{T_s-\overline{T}_f(r=0)}{T_s-\overline{T}_f(r=0)}\frac{\langle\overline{u}_f\rangle}{\langle\overline{u}_f\rangle}\frac{R^2}{R^2},$$

we have

$$\frac{R^2\langle\overline{u}_f\rangle(d\langle\overline{T}_f\rangle/dx)}{T_s-\langle\overline{T}_f\rangle}[T_s-\overline{T}_f(r=0)]\int_0^{r^*}\frac{\overline{u}_f}{\langle\overline{u}_f\rangle}T^*r^*dr^*=r^*(\alpha_f+\alpha_t)\frac{\partial\overline{T}_f}{\partial r^*}.$$

Now, expressing the integral as a function of $\phi(r^*)$, we have

$$\frac{R^2\langle\overline{u}_f\rangle(d\langle\overline{T}_f\rangle/dx)}{T_s-\langle\overline{T}_f\rangle}[T_s-\overline{T}_f(r=0)]\frac{\phi(r^*)}{r^*(\alpha_f+\alpha_t)}=\frac{\partial\overline{T}_f}{\partial r^*}.$$

Upon integration, we have

$$\frac{R^2\langle\overline{u}_f\rangle(d\langle\overline{T}_f\rangle/dx)}{T_s-\langle\overline{T}_f\rangle}[T_s-\overline{T}_f(r=0)]\int_0^{r^*}\frac{\phi(r^*)}{r^*(\alpha_f+\alpha_t)}dr^*=\int_{\overline{T}_f(r=0)}^{\overline{T}_f}d\overline{T}$$

or

$$\overline{T}_f-\overline{T}_f(r=0)=\frac{R^2\langle\overline{u}_f\rangle(d\langle\overline{T}_f\rangle/dx)}{T_s-\langle\overline{T}_f\rangle}[T_s-\overline{T}_f(r=0)]\int_0^{r^*}\frac{\phi(r^*)}{r^*(\alpha_f+\alpha_t)}dr^*.$$

or

$$1-T^*=\frac{\overline{T}_f-\overline{T}_f(r=0)}{T_s-\overline{T}_f(r=0)}=\frac{R^2\langle\overline{u}_f\rangle(d\langle\overline{T}_f\rangle/dx)}{T_s-\langle\overline{T}_f\rangle}\int_0^{r^*}\frac{\phi(r^*)}{r^*(\alpha_f+\alpha_t)}dr^*,$$

COMMENT:

The evaluation of $\phi(r^*)$ requires the prescription of the dimensionless fluid temperature distribution T^*. Therefore, an iterative procedure must be employed, in which experimental results may be used as a first approximation for T^*. This is done in the following problem.

PROBLEM 7.36.FUN

GIVEN:

Assume that the turbulent Prandtl number $\mathrm{Pr}_t = \nu_t/\alpha_t$ is equal to unity, and obtain the radial variation of the turbulent thermal diffusivity, using the measured velocity distributions. The turbulent momentum diffusivity is related to the shear stress and the mean velocity gradient as

$$\frac{\overline{\tau}}{\rho_f} = -(\nu_f + \nu_t)\frac{\partial \overline{u}_f}{\partial r}.$$

OBJECTIVE:

Derive expressions for the radial distribution of turbulent kinematic viscosity ν_t for (a) the region very close to the wall (laminar sublayer), (b) the region far away from the wall (turbulent buffer zone), and (c) the turbulent core region, using the measured mean axial velocity distributions

$$u^+ = y^+ \qquad 0 \le y^+ \le 5, \text{ laminar sublayer}$$
$$u^+ = -3.05 + 5.0 \ln y^+ \qquad 5 \le y^+ \le 30, \text{ turbulent buffer zone}$$
$$u^+ = 5.5 + 2.5 \ln y^+ \qquad y^+ > 30, \text{ turbulent core,}$$

where the dimensionless variables are

$$y^+ = \frac{y(\overline{\tau}_s/\rho_f)^{1/2}}{\nu_f} \quad , \qquad u^+ = \frac{\overline{u}_f}{(\overline{\tau}_s/\rho_f)^{1/2}}.$$

Note that the shear stress varies linearly with the radius as $\overline{\tau} = \overline{\tau}_s r/R$ ($\overline{\tau}_s$ is the wall shear stress), and that $r = R - y$, where y is the distance from the wall.
(d) Plot the velocity distribution in the forms (u^+ vs y^+) and ($\overline{u}_f$ vs r^*), and the turbulent kinematic viscosity distribution (ν_t/ν_f vs r^*), for Re = 10,000 and Pr = 0.01. Comment on the results.

SOLUTION:

Since $r = R - y$, the shear stress is expressed as

$$\overline{\tau} = \overline{\tau}_s \left(1 - \frac{y}{R}\right).$$

Using this in the shear stress expression, with $dr = -dy$, we have

$$\overline{\tau}_s \left(1 - \frac{y}{R}\right) = \rho(\nu_f + \nu_t)\frac{\partial \overline{u}_f}{\partial y}.$$

Solving for $\nu_t(y)$, we have

$$\nu_t = \frac{\overline{\tau}_s/\rho_f(1 - y/R)}{\partial \overline{u}_f/\partial y} - \nu_f.$$

(a) In the laminar sublayer, there are no turbulent fluctuations and

$$\nu_t = 0 \qquad \text{laminar sublayer region.}$$

(b) Far away from the wall, both molecular and turbulent shear stresses are important, and from the velocity distribution we have

$$\frac{\partial u^+}{\partial y^+} = \frac{5}{y^+} \qquad \text{or} \qquad \frac{\partial \overline{u}_f}{\partial y} = \frac{5(\overline{\tau}_s/\rho_f)^{1/2}}{y}.$$

Substituting this into the expression for the turbulent kinematic viscosity and using $y = R - r$, we have

$$\nu_t = \frac{R(\overline{\tau}_s/\rho_f)^{1/2}}{5.0}(1 - r^*)r^* - \nu_f \qquad \text{turbulent buffer region.}$$

(c) In the turbulent core region, the molecular viscous stress can be neglected, i.e., $\nu_f = 0$, and the velocity distribution becomes

$$\frac{\partial u^+}{\partial y^+} = \frac{2.5}{y^+} \qquad \text{or} \qquad \frac{\partial \overline{u}_f}{\partial y} = \frac{2.5(\overline{\tau}_s/\rho_f)^{1/2}}{y}.$$

Substituting this result into the expression for the turbulent kinematic viscosity and applying $y = R - r$, we obtain

$$\nu_t = \frac{R(\overline{\tau}_s/\rho_f)^{1/2}}{2.5}(1-r^*)r^* \qquad \text{turbulent core region.}$$

(d) The fluid velocity and fluid kinematic viscosity distributions are presented in Figure Pr.7.36. Plot (i) was obtained from the measured velocity distributions

$$u^+ = y^+ \qquad 0 \le y^+ \le 5, \text{ laminar sublayer}$$
$$u^+ = -3.05 + 5.0 \ln y^+ \qquad 5 \le y^+ \le 30, \text{ turbulent buffer zone}$$
$$u^+ = 5.5 + 2.5 \ln y^+ \qquad y^+ > 30, \text{ turbulent core.}$$

Plot (ii) was obtained by replacing the dimensionless variable

$$y^+ = \frac{y(\overline{\tau}_s/\rho_f)^{1/2}}{\nu_f} \ ,$$

into the above equations. The mean fluid velocity distribution is then expressed as

$$u^+ = \frac{R(1-r^*)(\overline{\tau}_s/\rho_f)^{1/2}}{\nu_f} \qquad 0 \le y^+ \le 5,$$
$$u^+ = -3.05 + 5.0 \ln\left[\frac{R(1-r^*)(\overline{\tau}_s/\rho_f)^{1/2}}{\nu_f}\right] \qquad 5 \le y^+ \le 30,$$
$$u^+ = 5.5 + 2.5 \ln\left[\frac{R(1-r^*)(\overline{\tau}_s/\rho_f)^{1/2}}{\nu_f}\right] \qquad y^+ > 30,$$

where

$$r^* = 1 - \frac{y^+ \nu_f}{R(\overline{\tau}_s/\rho_f)^{1/2}}$$

The wall shear stress is given by

$$\overline{\tau}_s = \frac{R}{2}\frac{\Delta p}{L}, \quad \text{where} \quad \Delta P = \frac{c_f \rho_f L \langle \overline{u}_f \rangle^2}{4R}.$$

Therefore

$$\overline{\tau}_s = \frac{c_f \rho_f \langle \overline{u}_f \rangle^2}{8}, \quad \text{where} \quad \langle \overline{u}_f \rangle = \frac{\nu_f \text{Re}}{2R}.$$

From Table C.24, Hg was selected as the fluid with Pr approximately 0.01. The properties used are $\rho_f = 13270$ kg/m^3, $\nu_f = 85.8 \times 10^{-9}$ m^2/s, $\alpha_f = 6.31 \times 10^{-6}$ m^2/s, and Pr $= 0.0136$. The friction coefficient c_f is obtained from the equation given in Table 7.3,

$$\frac{1}{c_f^{1/2}} = -2.0 \log\left(\frac{\langle \delta^2 \rangle^{1/2}}{3.7D} + \frac{2.51}{\text{Re}_{D,h} c_f^{1/2}}\right)$$

where for a smooth tube, the surface roughness δ is equal to zero. Using SOPHT to solve the above equation, we obtain $c_f = 0.0301$.

Plot (iii) was obtained form the equations derived in (a), (b) and (c).

It can be seen that in the buffer region, ν_t and ν_f are of the same order of magnitude, and that ν_t is much larger than ν_f in the turbulent core. The discontinuity in ν_t at the edge of the buffer layer results from the discontinuity in the slope of the velocity profiles at that point, as observed in plots (i) and (ii). It is not true that ν_t is zero at the center of the tube, as shown in plot (iii). The zero value of ν_t at $r = 0$ results from the fact that the shear stress becomes zero at the center of the tube, but the slope of the empirical logarithmic velocity distribution does not approach zero at the center, as indicated in plot (ii). Actually, the velocity gradient is zero at the center and the actual turbulent kinematic viscosity is not zero. This inconsistency is due to the erroneous approximation of the velocity distribution in the core region.

COMMENT:

In the next problem, the turbulent thermal diffusivity $\alpha_t(r) = \nu_t(r)$, for Pr$_t = 1$, is used to determine the radial mean fluid temperature distribution from the energy equation derived in the previous problem.

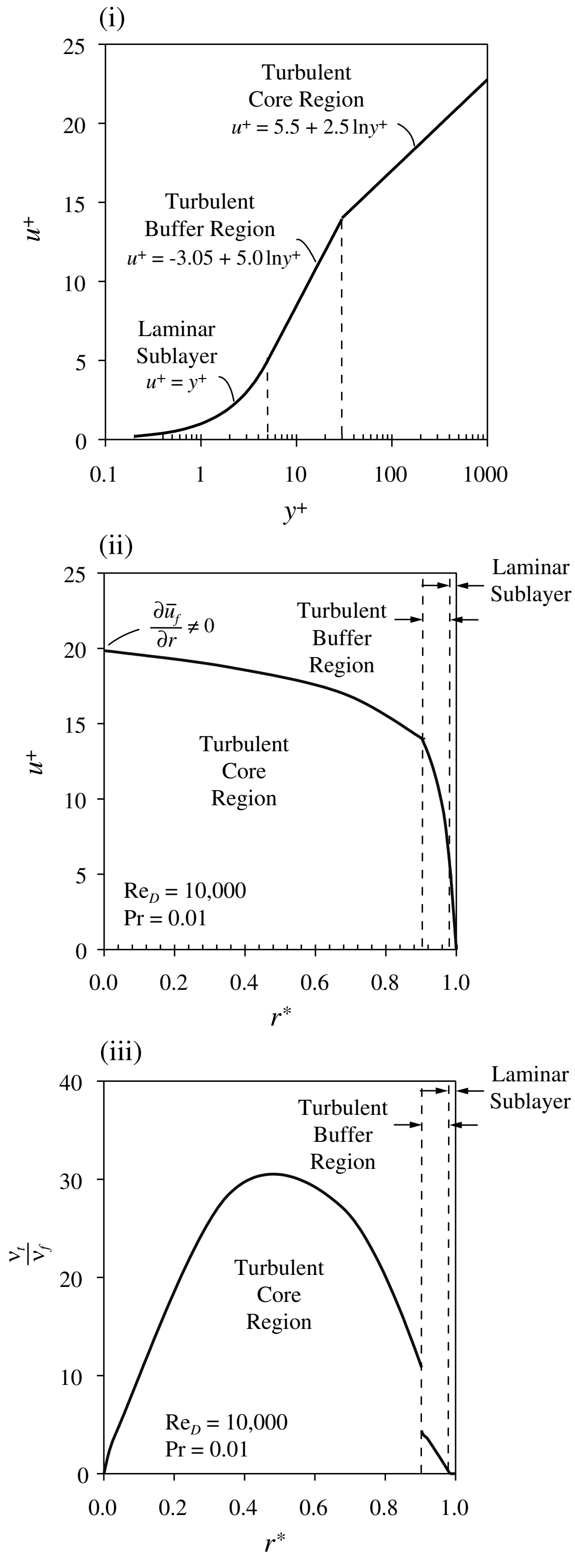


Figure Pr.7.36 Fluid velocity and turbulent kinematic viscosity distributions. (i) Dimensionless velocity distribution u^+ as a function of y^+; (ii) Dimensionless velocity distribution u^+ as a function of r^* for Re = 10,000 and Pr = 0.01; (iii) Dimensionless kinematic viscosity distribution ν_t as a function of r^* for Re = 10,000 and Pr = 0.01.

PROBLEM 7.37.FUN

GIVEN:

The expression derived for the radial distribution of the fluid mean temperature is simplified as

$$\frac{\overline{T}_f - \overline{T}_f(r=0)}{T_s - \overline{T}_f(r=0)} = \frac{R^2 \langle \overline{u}_f \rangle (d\langle \overline{T}_f \rangle / dx)}{T_s - \langle \overline{T}_f \rangle} \frac{\psi(r^*)}{\nu_f}.$$

The dimensionless temperature is defined as

$$T^* = \frac{T_s - \overline{T}_f}{T_s - \overline{T}_f(r=0)} = 1 - \frac{\overline{T}_f - \overline{T}_f(r=0)}{T_s - \overline{T}_f(r=0)}.$$

OBJECTIVE:

(a) Show that, after the first iteration, the dimensionless temperature distribution is given by

$$T^* = 1 - \frac{\psi(r^*)}{\psi(1)}.$$

(b) Show that the Nusselt number is given by

$$\overline{\text{Nu}}_D = \frac{\nu_f}{\alpha_f} \frac{1}{\psi(1)}.$$

Begin by writing the combined integral-differential length energy equation for the fluid control volume.

(c) The Nusselt numbers predicted by the expression derived in (b), is curve fitted with an accuracy of ±6 %, as

$$\overline{\text{Nu}}_D = \frac{q_{ku} D}{(T_s - \langle \overline{T}_f \rangle) k_f} = 5.0 + 0.025 \text{Re}^{0.8} \text{Pr}^{0.8}.$$

for Prandtl number less than 0.1 (liquid metals).

Compare this correlation with the Nusselt number correlation obtained experimentally by Dittus and Boelter (correlation presented in Table 7.3 for uniform T_s and $0.7 < \text{Pr} < 160$), and the correlation suggested by Sleicher and Rouse (correlation presented in Table 7.3 for uniform T_s and $\text{Pr} < 0.1$). Comment on the results.

SOLUTION:

(a) At $r = R$, $\psi(r^*) = \psi(1)$ and $\overline{T}_f = T_s$. Substituting this boundary condition into the expression for the radial distribution of fluid temperature, we have

$$\frac{R^2 \langle \overline{u}_f \rangle (d\langle \overline{T}_f \rangle / dx)}{T_s - \langle \overline{T}_f \rangle} \frac{1}{\nu_f} = \frac{1}{\psi(1)}.$$

Substituting this relation back into the expression for the radial distribution of the fluid temperature, we have

$$\frac{\psi(r^*)}{\psi(1)} = \frac{\overline{T}_f - \overline{T}_f(r=0)}{T_s - \overline{T}_f(r=0)} \quad \text{or,} \quad 1 - \frac{\overline{T}_f - \overline{T}_f(r=0)}{T_s - \overline{T}_f(r=0)} = 1 - \frac{\psi(r^*)}{\psi(1)} \quad \text{or,} \quad \frac{T_s - \overline{T}_f}{T_s - \overline{T}_f(r=0)} = 1 - \frac{\psi(r^*)}{\psi(1)}.$$

This is the dimensionless radial temperature distribution T^*.

(b) The axial variation of the cross-sectional averaged fluid temperature, is caused by the surface convection heat transfer between the tube wall and the fluid stream. This can be expressed by (7.12), i.e.,

$$-P_{ku} q_{ku} + A_u \frac{d}{dx} q_u = 0$$

$$-\pi D q_{ku} + (\pi D^2/4) \frac{d}{dx} (\rho c_p)_f \langle \overline{u}_f \rangle d\langle \overline{T}_f \rangle = 0.$$

Then

$$\frac{\langle \overline{u}_f \rangle (d\langle \overline{T}_f \rangle / dx)}{T_s - \langle \overline{T}_f \rangle} = \frac{4 q_{ku}}{D (\rho c_p)_f (T_s - \langle \overline{T}_f \rangle)}.$$

Substituting the previous relation into the radial temperature distribution derived as a function of $\psi(r^*)$, we obtain

$$\frac{Dq_{ku}}{\nu_f(\rho c_p)_f(T_s - \langle \overline{T}_f \rangle)} = \frac{\overline{T}_f - \overline{T}_f(r=0)}{T_s - \overline{T}_f(r=0)} \frac{1}{\psi(r^*)}.$$

Noting that $(\rho c_p)_f = k_f/\alpha_f$, and substituting the dimensionless temperature distribution derived as a function of $\psi(r^*)$ and $\psi(1)$ into the previous relation, we obtain

$$\overline{\mathrm{Nu}}_D = \langle \overline{\mathrm{Nu}} \rangle_D = \frac{q_{ku} D}{(T_s - \langle \overline{T}_f \rangle) k_f} = \frac{\nu_f}{\alpha_f} \frac{1}{\psi(1)}.$$

This is the expression for the Nusselt number as a function of $\psi(1)$.

(c) Figure Pr.7.37(a) shows the predicted and measured Nusselt number correlations, as functions of the Reynolds and Prandtl numbers, over $10^3 \le \mathrm{Re}_D \le 10^6$ and for Pr = 0.01, 0.1, 1 and 10. The results obtained by Seban and Shimazaki, and the correlation suggested by Sleicher, are in agreement for all the cases presented. Both correlations are said to be accurate for $\mathrm{Pr} < 0.1$.

The correlation proposed by Dittus and Boelter is reasonably accurate for $0.7 < \mathrm{Pr} < 120$. The predictions of Seban and Shimazaki are not as close for $\mathrm{Pr} > 1$.

We also can observe that for very low Prandtl number, as shown in Figure Pr.7.37(a)(i), the dependence of the Nusselt number on the Reynolds number is rather weak as the Reynolds number decreases (compared with the other cases for higher Prandtl numbers). This behavior is expected, since for low Pr, $\alpha_f >> \nu_f$, there is a masking the viscous effect from the heat transfer within the fluid.

Note that the Dittus and Boelter correlation does not reproduce this effect for low Pr numbers, which shows that this correlation should not be used for fluids with high thermal diffusivity and low kinematic viscosity, such as liquid metals.

COMMENT:

The iterative method described in the last three problems was employed to obtain the Nusselt number of a turbulent flow through a circular tube with walls at uniform temperature. MAPLE was used as the mathematical tool. The equations and boundary conditions are presented next.

Initially, $\phi(r^*)$ was calculated for the laminar sublayer ($r_{1-inf} \le r^* \le r_{1-sup}$), turbulent buffer ($r_{2-inf} \le r^* \le r_{2-sup}$) and core ($r_{3-inf} \le r^* \le r_{3-sup}$) regions as a function of the temperature and velocity distributions in those regions.

$$\phi(r^*) = \int_0^{r^*} \frac{\overline{u}_f}{\langle \overline{u}_f \rangle} T^* r^* dr^* = \int_0^{r^*} f(r^*) dr^*,$$

or,

$$\phi_3(r^*) = \int_{r_{3-inf}}^{r^*} f_3(r^*) dr^* \quad , \quad (r_{3-inf} \le r^* \le r_{3-sup}),$$

$$\phi_2(r^*) = \int_{r_{3-inf}}^{r_{3-sup}} f_3(r^*) dr^* + \int_{r_{2-inf}}^{r^*} f_2(r^*) dr^* \quad , \quad (r_{2-inf} \le r^* \le r_{2-sup}),$$

$$\phi_1(r^*) = \int_{r_{3-inf}}^{r_{3-sup}} f_3(r^*) dr^* + \int_{r_{2-inf}}^{r_{2-inf}} f_2(r^*) dr^* + \int_{r_{1-inf}}^{r^*} f_1(r^*) dr^* \quad , \quad (r_{1-inf} \le r^* \le r_{1-sup}),$$

The integral named $\psi(r^*)$ was then calculated as a function of $\phi(r^*)$ and $\alpha_t(r^*)$.

$$\psi(r^*) = \int_0^{r^*} \frac{\phi(r^*)}{r^*(\alpha_f + \alpha_t)/\nu_f} dr^* = \int_0^{r^*} g(r^*) dr^*$$

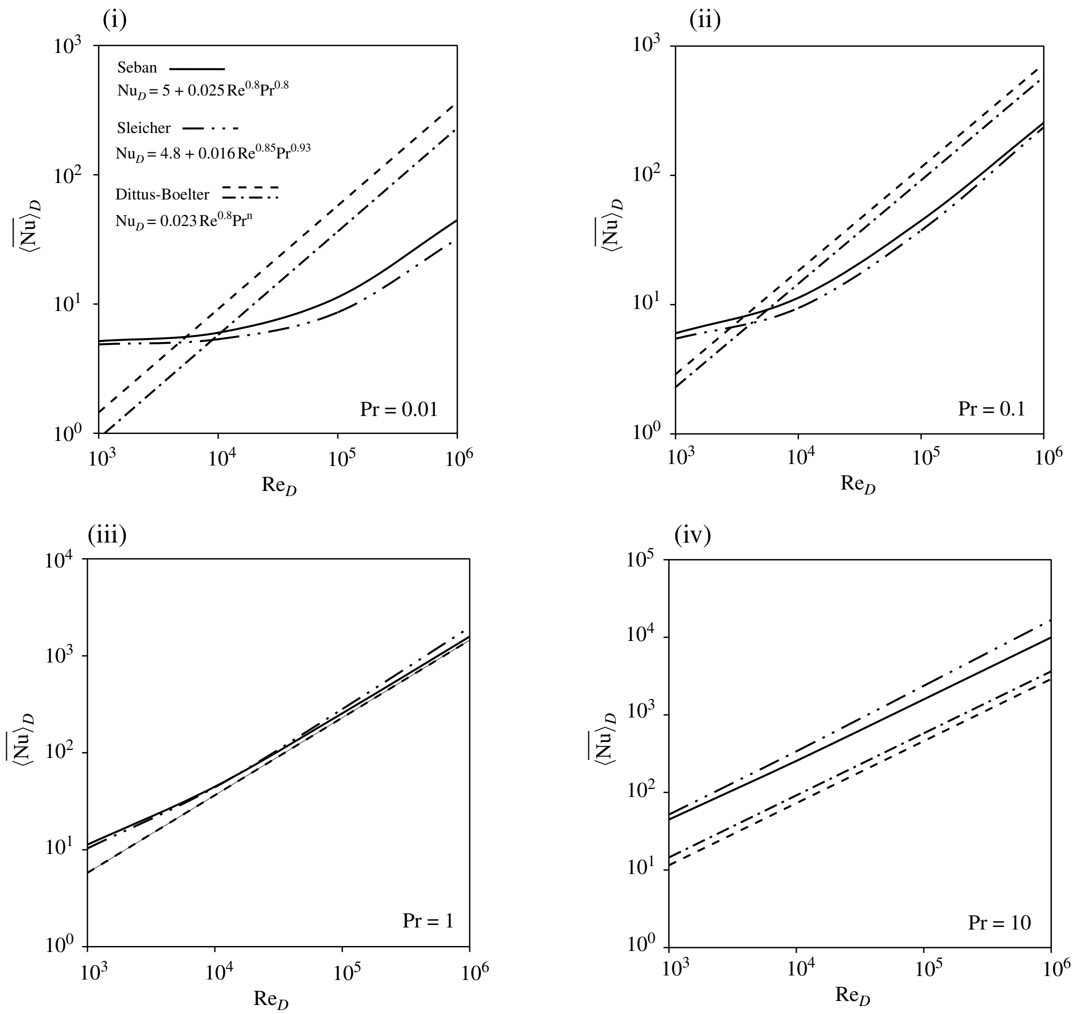


Figure Pr.7.37(a) Variations of Nusselt number $\langle\overline{\mathrm{Nu}}\rangle_D$ for turbulent flow inside circular tubes, with respect to Reynolds number Re_D. (i) Pr = 0.01; (ii) Pr = 0.1; (iii) Pr = 1; (iv) Pr = 10.

or,

$$\psi_3(r^*) = \int_{r_{3-inf}}^{r^*} g_3(r^*)dr^* \quad , \quad (r_{3-inf} \le r^* \le r_{3-sup}),$$

$$\psi_2(r^*) = \int_{r_{3-inf}}^{r_{3-sup}} g_3(r^*)dr^* + \int_{r_{2-inf}}^{r^*} g_2(r^*)dr^* \quad , \quad (r_{2-inf} \le r^* \le r_{2-sup}),$$

$$\psi_1(r^*) = \int_{r_{3-inf}}^{r_{3-sup}} g_3(r^*)dr^* + \int_{r_{2-inf}}^{r_{2-inf}} g_2(r^*)dr^* + \int_{r_{1-inf}}^{r^*} g_1(r^*)dr^* \quad , \quad (r_{1-inf} \le r^* \le r_{1-sup}).$$

Since at this point, $\psi(r^*)$ was a known function, we could calculate

$$\psi(1) = \psi(r^* = r_{1-sup}),$$

i.e.,

$$\psi(1) = \int_{r_{3-inf}}^{r_{3-sup}} g_3(r^*)dr^* + \int_{r_{2-inf}}^{r_{2-inf}} g_2(r^*)dr^* + \int_{r_{1-inf}}^{r_{1-inf}} g_1(r^*)dr^*.$$

The temperature distribution was then calculated as

$$T_3^* = 1 - \frac{\psi_3(r^*)}{\psi(1)} \quad , \quad (r_{3-inf} \leq r^* \leq r_{3-sup}),$$
$$T_2^* = 1 - \frac{\psi_2(r^*)}{\psi(1)} \quad , \quad (r_{2-inf} \leq r^* \leq r_{2-sup}),$$
$$T_1^* = 1 - \frac{\psi_1(r^*)}{\psi(1)} \quad , \quad (r_{1-inf} \leq r^* \leq r_{1-sup}).$$

These temperature distributions were used as the next approximation in the expression for $\phi(r^*)$. The Nusselt number was calculated in each of the iterations as a function of $\psi(1)$ as shown previously. The iterations ended when the temperature profile converged.

Mercury (Pr = 0.0135) was used as the working fluid and the Nusselt number, for Reynolds number 10^4 and 10^5, was calculated and compared with the results presented by Seban and Shimazaki.

The temperature distributions, starting from the temperature profile from constant heat flux case and used as the first approximation, up to the temperature profile approaching convergence, are shown in Figure Pr.7.37(b) (i) and (ii). We can note that, after the second iteration, the temperature distributions are very close to each other and Nusselt number changes less than 5%. Comparing the converged temperature distributions in graphs (i) and (ii), we observe that for higher Reynolds number, the laminar sublayer and turbulent buffer region decrease, since the increase of the fluid velocity causes the turbulent eddies to start closer to the wall. As the Reynolds number decreases, the temperature distribution approaches the parabolic profile obtained in laminar flows.

A comparison between the results obtained in the present analysis using MAPLE and the results obtained (also analytically) by Seban and Shimazaki is presented in Figure Pr.7.37(b)(iii). The agreement between the results comproves the applicability of the iterative method and correlation proposed by the referenced authors.

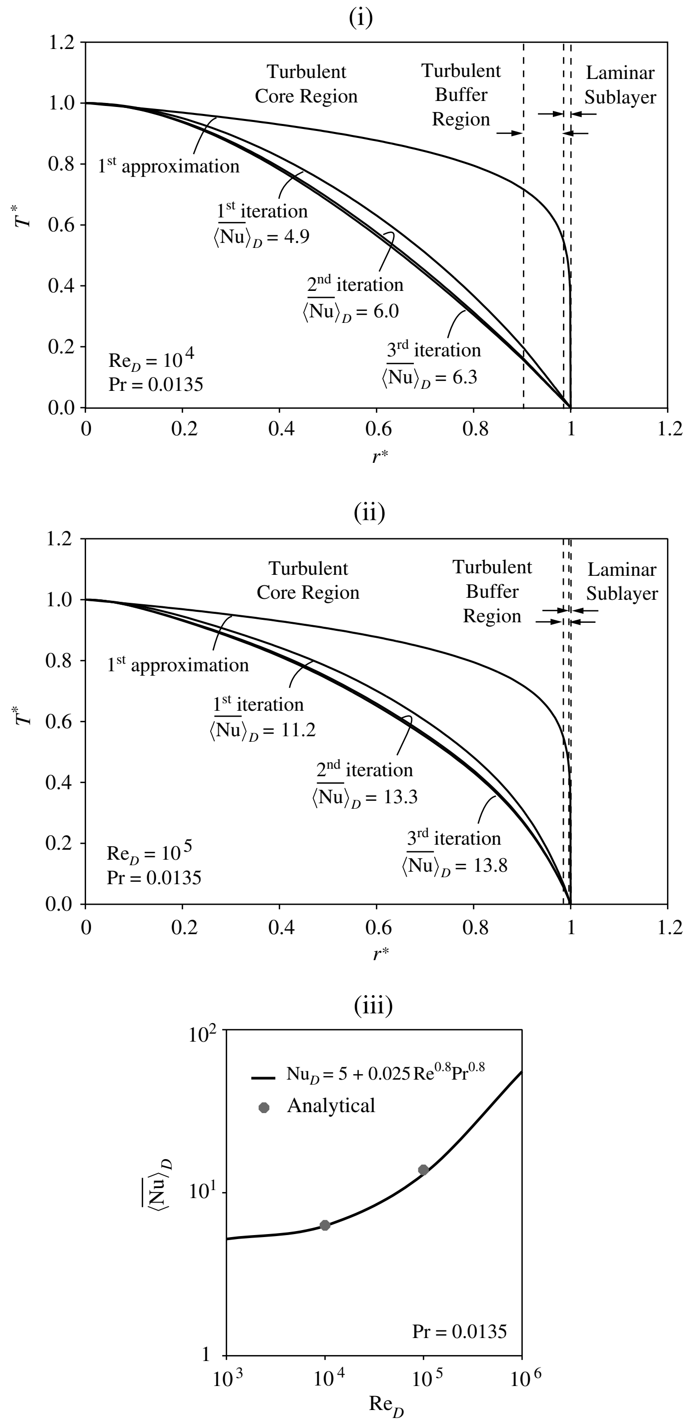


Figure Pr.7.37(b) Dimensionless temperature profiles and Nusselt numbers obtained analytically by means of the iterative method. (i) Variation of the temperature distribution along the iterative method up to convergence, for $\mathrm{Re}_D = 10^4$. (ii) Variation of the temperature distribution, for $\mathrm{Re}_D = 10^5$. (iii) Nusselt number obtained analytically compared with correlation proposed by Seban and Shimazaki.

Chapter 8

Heat Transfer in Thermal Systems

PROBLEM 8.1.DES.S

GIVEN:

To harvest the automobile exhaust-gas heat, a shell of bismuth-telluride thermoelectric elements is placed around the tailpipe. This is shown in Figure Pr.8.1(a). In principle, placement of the elements closer to the exhaust manifold, where higher gas temperatures are available, is more beneficial. However, high temperature thermoelectric materials are needed.

The local tailpipe wall temperature is $\langle T_{f,h} \rangle = T_h$ and the elements are placed between the pipe and an outer-ceramic shell. The contact resistance between the thermoelectric elements and the pipe is $(R_{k,c})_h$ and that with the ceramic is $(R_{k,c})_c$. The ceramic is cooled with surface convection using a crossflow air stream with a far-field temperature $T_{f,\infty}$. The ceramic surface is also cooled by surface radiation to the surrounding solid surfaces (all surfaces are diffuse, gray opaque) with the surrounding solid surfaces at $T_{s,\infty}$. The emissivity of the ceramic surface is $\epsilon_{r,c}$ and that of surrounding surfaces is $\epsilon_{r,\infty}$.

The electrical power generated is maximized with respect to the external electrical resistance $R_{e,o}$, by choosing $R_{e,o} = R_e$.

$N_{TE} = 1{,}000$, $a_{TE} = 2$ mm, $L_{TE} = 3$ mm, $L = 30$ cm, $D = 4$ cm, $T_h = 300$℃, $T_{f,\infty} = 20$℃, $T_{s,\infty} = 20$℃, $u_{f,\infty} = 50$ km/hr, $\epsilon_{r,c} = 0.90$, $\epsilon_{r,\infty} = 0.85$, $(R_{k,c})_c = (R_{k,c})_h = 0$.

Determine the thermophysical properties of air at $T = 350$ K.

SKETCH:

Figure Pr.8.1(a) shows the harvesting of the exhaust-gas heat by a thermoelectric power generator.

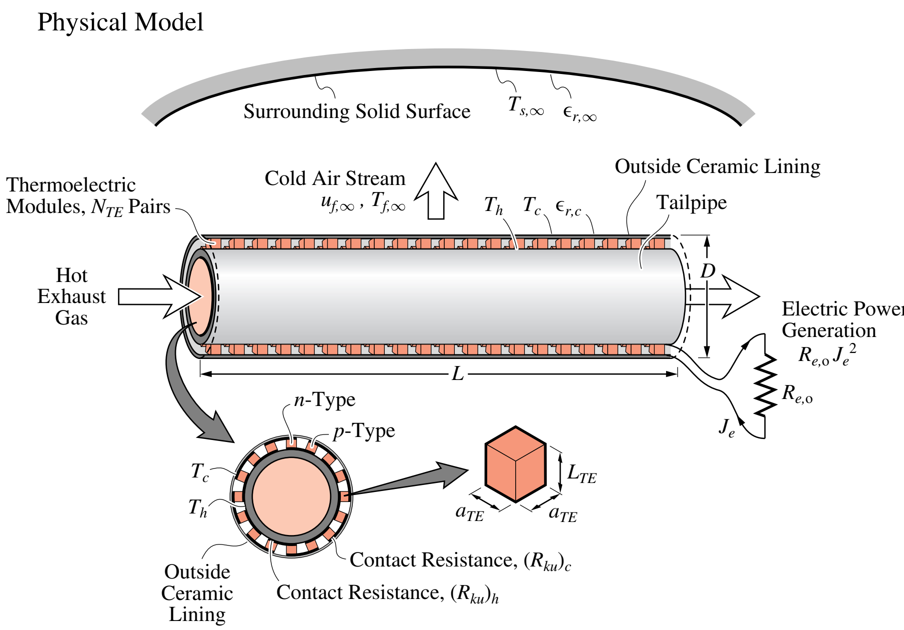


Figure Pr.8.1(a) A thermoelectric power generator using the automobile exhaust gas heat content.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) For the conditions above, determine the power output $J_e^2 R_{e,o}$.

(c) Plot the variation of the electrical power generation with respect to the surface convection resistance R_{ku}. Comment on how this resistance can be reduced.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.8.1(b).

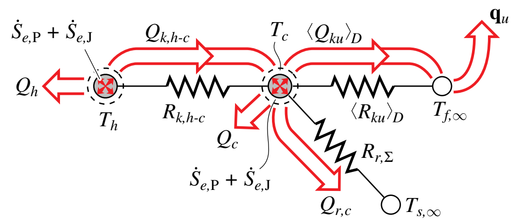


Figure Pr.8.1(b) Thermal circuit diagram.

(b) The electrical power generation is given by (2.40) and when written for N_{TE} pairs, we have

$$J_e^2 R_{e,o} = \frac{N_{TE}^2 \alpha_{\mathrm{S}}^2 (T_h - T_c)^2 R_{e,o}}{(R_{e,o} + R_e)^2}.$$

For optimal performance,

$$R_{e,o} = R_e$$

or

$$J_e^2 R_{e,o} = \frac{N_{TE}^2 \alpha_{\mathrm{S}}^2 (T_h - T_c)^2}{4R_{e,o}}.$$

Here we need to determine T_c and R_e.
For bismuth telluride, the electrical properties are given in Table C.9(a). These are

$$\alpha_{\mathrm{S},p} = 230 \times 10^{-6} \text{ V/°C} \qquad \text{Table C.9(a)}$$
$$\alpha_{\mathrm{S},n} = -210 \times 10^{-6} \text{ V/°C} \qquad \text{Table C.9(a)}$$
$$k_p = 1.70 \text{ W/m-K} \qquad \text{Table C.9(a)}$$
$$k_n = 1.45 \text{ W/m-K} \qquad \text{Table C.9(a)}$$
$$\rho_e = 1.0 \times 10^{-5} \text{ ohm-m} \qquad \text{Table C.9(a)}$$
$$\alpha_{\mathrm{S}} = \alpha_{\mathrm{S},p} + \alpha_{\mathrm{S},n} = (2.3 + 2.1) \times 10^{-4}(\text{V/°C}) = 4.4 \times 10^{-4} \text{ V/°C}.$$

The electrical resistance for each pair is given by (3.116), i.e.,

$$\begin{aligned} R_e|_{each} &= \left(\frac{\rho_e L_{TE}}{A_k}\right)_p + \left(\frac{\rho_e L_{TE}}{A_k}\right)_n \\ &= \frac{2\rho_e L_{TE}}{A_k} \\ A_k &= a_{TE}{}^2. \end{aligned}$$

Using the numerical values, we have

$$\begin{aligned} R_e|_{each} &= \frac{2 \times 10^{-5}(\text{ohm-m}) \times 3 \times 10^{-3}(\text{m})}{(2 \times 10^{-3})^2(\text{m})^2} \\ &= 0.015 \text{ ohm} \\ R_{e,o} &= R_e = N_{TE} R_e|_{each} = 15 \text{ ohm}. \end{aligned}$$

The temperature of the ceramic is determined from the energy conservation equation. From Figure Pr.8.1(b), we have

$$Q|_{A,C} = Q_{k,c\text{-}h} + \langle Q_{ku}\rangle_D + Q_{r,c} = \dot{S}_{e,\mathrm{J}} + \dot{S}_{e,\mathrm{P}}.$$

Now we use the two-surface enclosure radiations and note that heat is released at the cold junction and that as given by (3.111) the Joule heating is equally split between the two ends. Then, we have,

$$\frac{T_c - T_h}{R_{k,c\text{-}h}} + \frac{T_c - T_{f,\infty}}{\langle R_{ku}\rangle_D} + \frac{\sigma_{\rm SB}(T_c^4 - T_{s,\infty}^4)}{R_{r,\Sigma}} = \frac{1}{2}R_{e,o}J_e^2 + N_{TE}\alpha_{\rm S}J_e^2 T_c.$$

The conduction resistance is given by (3.116), i.e.,

$$\begin{aligned}
R_{k,c\text{-}h}^{-1} &= N_{TE}\left[\left(\frac{A_k k}{L_{TE}}\right)_p + \left(\frac{A_k k}{L_{TE}}\right)_n\right] \\
&= N_{TE}\frac{A_k}{L_{TE}}(k_p + k_n) \\
&= N_{TE}\frac{a_{TE}^2}{L_{TE}}(k_p + k_n) \\
&= 1{,}000 \times \frac{(2\times 10^{-3})^2(\text{m}^2)}{(3\times 10^{-3})(\text{m})} \times (1.70 + 1.45)(\text{W/m-K}) = 4.20\ \text{W/K}.
\end{aligned}$$

or

$$R_{k,c\text{-}h} = 0.238\ \text{K/W}.$$

The surface-convection resistance is determined using the Nusselt number correlation of Table 6.4, i.e.,

$$\begin{aligned}
\langle \text{Nu}\rangle_D &= a_1 \text{Re}_D^{a_2}\text{Pr}^{1/3} \\
\text{Re}_D &= \frac{u_{f,\infty}D}{\nu_f} \\
\langle R_{ku}\rangle_D &= \frac{D}{A_{ku}\langle \text{Nu}\rangle_D k_f} \\
A_{ku} &= \pi D L.
\end{aligned}$$

The thermophysical properties of air, at $T = 350$K, are found from table C.22, i.e.,

$k_f = 0.0300$ W/m-K Table C.22

$\upsilon_f = 2.030 \times 10^{-5}$ m^2/s Table C.22

$\text{Pr} = 0.69$ Table C.22.

Then

$$\text{Re}_D = \frac{u_{f,\infty}D}{\nu_f} = \frac{13.89(\text{m/s}) \times 4\times 10^{-2}(\text{m})}{2.030\times 10^{-5}(\text{m}^2/\text{s})} = 2.737\times 10^4.$$

From Table 6.4, we have

$$a_1 = 0.193 \quad , \quad a_2 = 0.618.$$

Then

$$\begin{aligned}
\langle \text{Nu}\rangle_D &= 0.193 \times (2.737\times 10^4)^{0.618} \times (0.69)^{1/3} = 94.22 \\
\langle R_{ku}\rangle_D &= \frac{4\times 10^{-2}(\text{m})}{\pi \times 4\times 10^{-2}(\text{m}) \times 0.3(\text{m}) \times 94.22 \times 0.0300(\text{W/m-K})} = 0.3756\ \text{K/W}.
\end{aligned}$$

The radiation resistances are determined from (4.49), for the case of $A_{r,\infty} \gg A_{r,c}$, i.e.,

$$R_{r,\Sigma} = \frac{1}{A_{r,c}\epsilon_{r,c}}$$

$$A_{r,c} = A_{ku} = \pi D L$$

or

$$R_{r,\Sigma} = \frac{1}{\pi D L \epsilon_{r,c}} = \frac{1}{\pi \times 4 \times 10^{-2}(\text{m}) \times 0.3(\text{m}) \times 0.90} = 29.49\ \text{m}^{-2}.$$

We now solve the electrical power relation and the T_c node energy equation for J_e and T_c. This is done using a solver (e.g., SOPHT). The results are

$$T_c = 490.2\ \text{K}$$
$$J_e = 1.216\ \text{A}$$
$$J_e^2 R_{e,o} = 22.18\ \text{W}.$$

(c) The variation of the electrical power produced, with respect to $\langle R_{ku}\rangle_D$ is shown in Figure Pr.8.1(c). The results show that by reducing $\langle R_{ku}\rangle_D$ to less than 0.1 K/W, a significant increase in the power generation is found. This is due to the corresponding decrease in the ceramic surface temperature T_c.

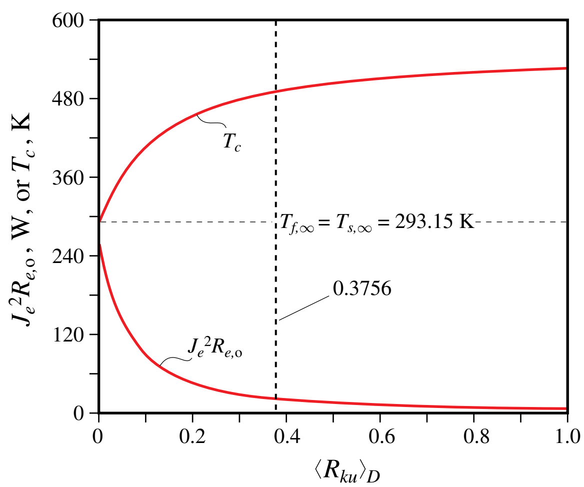


Figure Pr.8.1(c) Variation of electrical power generated and the ceramic temperature with respect to the surface-convection resistance.

COMMENT:

Note that by decreasing $\langle R_{ku}\rangle_D$ and $R_{r,\Sigma}$, the ceramic temperature T_c can be further reduced, thus increasing the electrical power generated. By increasing the air flow speed $u_{f,\infty}$ and the emissivities $\epsilon_{r,c}$ and $\epsilon_{r,\infty}$ the ceramic temperature will be reduced.

PROBLEM 8.2.DES

GIVEN:

Anesthetic drugs are available in liquid form and are evaporated, heated, and mixed with other gases (e.g., oxygen) in a vaporizer tube. This is shown in Figure Pr.8.2(a). The heat is supplied through the Joule heating and the tube is ideally insulated on the outside. The primary and secondary air mix with the evaporated drug, and initially the temperature of the gas mixture drops from T_i (inlet condition) to $\langle T_f \rangle_0$ (after the assumed complete evaporation).

$D = 2.3$ cm, $L = 18$ cm, $T_i = 20$℃, $\dot{M}_{O_2} = \dot{M}_p + \dot{M}_s = 2.17 \times 10^{-4}$ kg/s, $\dot{M}_l = 1.66 \times 10^{-5}$ kg/s.

Evaluate the thermophysical properties of the drug (assume that they are the same as those for Refrigerant 134a) at $T = 253.2$ K, and the properties of oxygen at $T = 300$ K. Use the properties of oxygen for the evaluation of the Nusselt number and $(\dot{M}c_p)_f$.

SKETCH:

Figure Pr.8.2(a) shows the evaporation and heating of anesthetic liquid in a vaporizer tube.

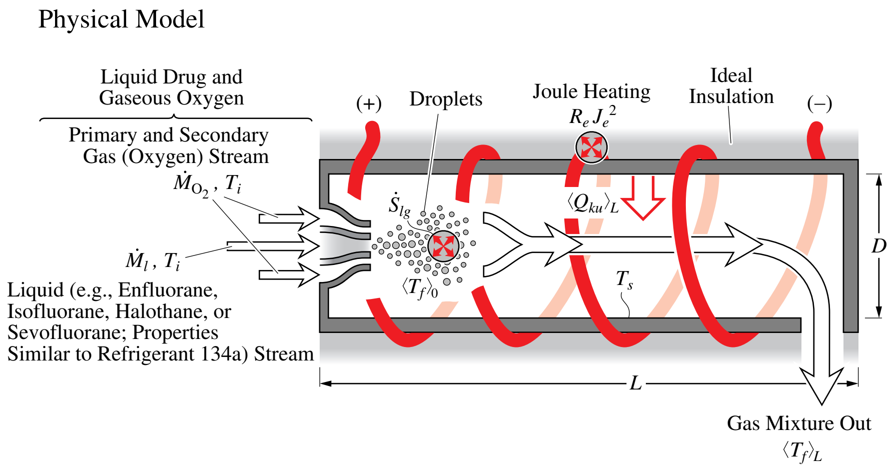


Figure Pr.8.2(a) An anesthetic liquid evaporator and vapor heater.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Determine the gas mixture temperature after the assumed complete evaporation and before the surface convection begins.

(c) Determine the required Joule heating rate $\dot{S}_{e,J}$ for an exit temperature $\langle T_f \rangle_L = 30$℃.

(d) Determine the tube surface temperature T_s needed to have an exit temperature $\langle T_f \rangle_L = 30$℃.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure.Pr.8.2(b). The combined primary and secondary oxygen gas stream $\dot{M}_{O_2} = \dot{M}_p + \dot{M}_s$ mixes with the liquid droplets and complete evaporation is assumed leading to a gas mixture stream temperature $\langle T_f \rangle_0$. Then this stream is heated by surface convection, with a uniform tube surface temperature T_s, leading to an exit temperature $\langle T_f \rangle_0$.

(b) The energy equation for the phase change energy conversion $\dot{S}_{lg}$, from Figure 8.2(b), and given similarly by (5.17), is

$$(\dot{M}_{O_2} c_{p,O_2} + \dot{M}_l c_{p,d})(\langle T_f \rangle_0 - T_i) = -\dot{M}_l \Delta h_{lg}$$
$$\dot{M}_{O_2} = \dot{M}_p + \dot{M}_s.$$

Here we have represented the sensible heat change for the liquid (from T_i to T_{lg}) and vapor (from T_{lg} to $\langle T_f \rangle_0$), with a simple expression $\dot{M}_l c_{p,d}$ $(\langle T_f \rangle_0 - T_i)$ and we will use the specific heat capacity of the vapor for $c_{p,d}$. This approximation is expected to be valid considering that the liquid and gas specific heat capacities for R-134a, are

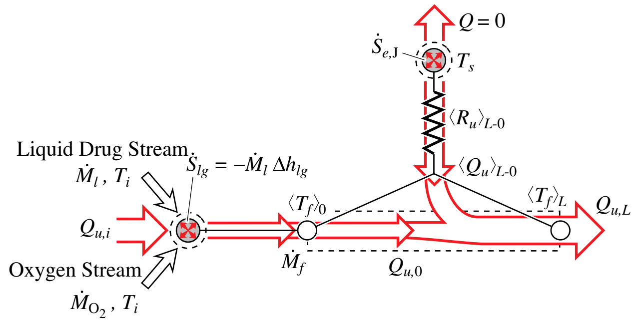


Figure Pr.8.2(b) Thermal circuit diagram.

not greatly different.
The anesthetic drugs have thermophysical properties similar to that of Refrigerant 134a (Table C.28). Then from Tables C.22 and C.28, we have:
oxygen at $T = 300$ K:

$$k_f = 0.0274 \text{ W/m-K} \qquad \text{Table C.22}$$
$$\rho_f = 1.299 \text{ kg/m}^3 \qquad \text{Table C.22}$$
$$c_{p,O_2} = 920 \text{ J/kg-K} \qquad \text{Table C.22}$$
$$\nu_f = 2.61 \times 10^{-5} \text{ m}^2/\text{s} \qquad \text{Table C.22}$$
$$\text{Pr} = 0.69 \qquad \text{Table C.22}$$

R-134a at $T = 253.2$ K:

$$\Delta h_{lg} = 2.110 \times 10^5 \text{ J/kg} \qquad \text{Table C.28}$$
$$c_{p,d} = 805 \text{ J/kg-K} \qquad \text{Table C.28.}$$

Then

$$\begin{aligned}
\langle T_f \rangle_0 &= T_i - \frac{\dot{M}_l \Delta h_{lg}}{(\dot{M}_p + \dot{M}_s) c_{p,O_2} + \dot{M}_l c_{p,d}} \\
&= 20(^\circ\text{C}) - \frac{1.66 \times 10^{-5}(\text{kg/s}) \times 2.110 \times 10^5(\text{J/kg})}{2.17 \times 10^{-4}(\text{kg/s}) \times 920(\text{J/kg-K}) + 1.66 \times 10^{-5}(\text{kg/s}) \times 805(\text{J/kg-K})} \\
&= 20(^\circ\text{C}) - \frac{3.502(\text{W})}{0.2130(\text{W/K})} = 20(^\circ\text{C}) - 16.44(^\circ\text{C}) = 3.559^\circ\text{C}.
\end{aligned}$$

(c) The energy equation for the convection stream, between $\langle T_f \rangle_0$ and $\langle T_f \rangle_c$, gives, as shown in Figure Pr.8.2(b),

$$\langle Q_u \rangle_{L\text{-}0} (\dot{M} c_p)_f (\langle T_f \rangle_L - \langle T_f \rangle_0) = \dot{S}_{e,\text{J}}$$
$$\dot{M}_f = \dot{M}_p + \dot{M}_s + \dot{M}_l$$
$$c_{p,f} = c_{p,O_2}.$$

Then

$$\dot{S}_{e,\text{J}} \quad = \quad (2.17 \times 10^{-4} + 1.66 \times 10^{-5})(\text{kg/s}) \times 920(\text{J/kg-K}) \times (30 - 3.559)(\text{K}) = 5.683 \text{ W}.$$

(d) To determine the tube wall temperature T_s, we use (7.22), i.e.,

$$\frac{\langle T_f \rangle_L - \langle T_f \rangle_0}{T_s - \langle T_f \rangle_0} = 1 - e^{-NTU}$$

or

$$T_s = \langle T_f \rangle_0 + \frac{\langle T_f \rangle_L - \langle T_f \rangle_0}{1 - e^{-NTU}},$$

where NTU is given by (7.20) as

$$NTU = \frac{A_{ku}\langle \mathrm{Nu}_D\rangle k_f}{(\dot{M}c_p)_f D}$$
$$A_{ku} = \pi DL.$$

Then Nusselt number is determined knowing the range of the Reynolds number.From (7.36) and (7.3), we have

$$\mathrm{Re}_D = \frac{\langle u_f\rangle D}{\nu_f}$$
$$\langle u_f\rangle = \frac{\dot{M}_f}{\rho_f A_u} = \frac{4\dot{M}_f}{\rho_f \pi D^2}$$

or

$$\begin{aligned}\mathrm{Re}_D &= \frac{4\dot{M}_f}{\rho_f \pi D^2} = \frac{4 \times (2.17 \times 10^{-4} + 1.66 \times 10^{-5})(\mathrm{kg/s})}{1.299(\mathrm{kg/m^3}) \times 2.61 \times 10^{-5}(\mathrm{m^2/s}) \times \pi \times 2.3 \times 10^{-2}(\mathrm{m})} \\ &= 381.6 < \mathrm{Re}_{D,t} = 2{,}300, \quad \text{laminar flow.}\end{aligned}$$

For laminar flow, from Table 7.2, we use the developing field correlations for $\langle \mathrm{Nu}\rangle_D$. We need to determine the Graetz number (note that $\mathrm{Pe}_D = \mathrm{Re}_D\mathrm{Pr}$).

$$\frac{L/D}{\mathrm{Re}_D\mathrm{Pr}} = \frac{0.18(\mathrm{m})/0.023(\mathrm{m})}{381.6 \times 0.69} = 0.02972 < 0.03.$$

Then from Table 7.2, we have

$$\langle \mathrm{Nu}\rangle_D = 2.409\left(\frac{L/D}{\mathrm{Re}_D Pr}\right)^{-1/3} - 0.7 = 7.077$$

Then

$$\begin{aligned}NTU &= \frac{\pi L\langle \mathrm{Nu}\rangle_D k_f}{(\dot{M}c_p)_f} \\ &= \frac{\pi \times 0.18(\mathrm{m}) \times 7.077 \times 0.0274(\mathrm{W/m\text{-}K})}{(2.17 \times 10^{-4} + 1.66 \times 10^{-5})(\mathrm{kg/s}) \times 920(\mathrm{J/kg\text{-}K})} = 0.5099.\end{aligned}$$

Using this, we have for T_s

$$\begin{aligned}T_s &= 3.559(^\circ\mathrm{C}) + \frac{(30 - 3.559)(^\circ\mathrm{C})}{1 - e^{-0.5099}} \\ &= 3.559(^\circ\mathrm{C}) + 66.19(^\circ\mathrm{C}) = 69.75^\circ\mathrm{C}.\end{aligned}$$

COMMENT:

This heater surface temperature is rather high. One method of reducing this is to increase A_{ku} (e.g., by using fins or increasing L).

PROBLEM 8.3.DES

GIVEN:

A condenser-chemical analyzer uses a stream of cold water, where the temperature of the stream entering the condenser is controlled to within a small deviation.

An off-the-shelf thermoelectric cooler-heat exchange unit is used to provide this cold water stream. This is shown in Figure Pr.8.3(a). The heat exchange surface is assumed to be at a uniform temperature T_s with the water entering at $\langle T_f \rangle_0$ and exiting at $\langle T_f \rangle_L$. The heat exchange surface is the cold side of a thermoelectric module (i.e., $T_s = T_c$), where there are N_{TE} pairs of bismuth-telluride thermoelectric elements with the specifications given below. The hot side of the thermoelectric module is connected to an extended surface (fins or heat sink) cooled by air blown over its surfaces with far-field conditions $T_{f,\infty}$ and $u_{f,\infty}$. The fins have an efficiency of unity. The dominant resistances between $T_{f,\infty}$ and $\langle T_f \rangle_0$ are due to the surface convection, conduction in thermoelectric materials, and the average convection resistance.

$\langle T_f \rangle_0 = 20$℃, $\dot{M}_{f,w} = 0.01$ kg/s, $l = 5$ mm, $L = 35$ cm, $a_{TE} = 1.5$ mm, $L_{TE} = 3.5$ mm, $N_{TE} = 400$, $A_{ku} = 0.05$ m^2, $u_{f,\infty} = 1$ m/s, $T_{f,\infty} = 25$℃, $w = 9$ cm.

Use properties of air at $T = 300$ K and water at $T = 290$ K, and determine $\langle \mathrm{Nu} \rangle_w$ using w, the fin width.

SKETCH:

The thermoelectric unit with surface-convection heat removal from the hot surface and convection heat removal from the cold surface, is shown in Figure Pr.8.3(a).

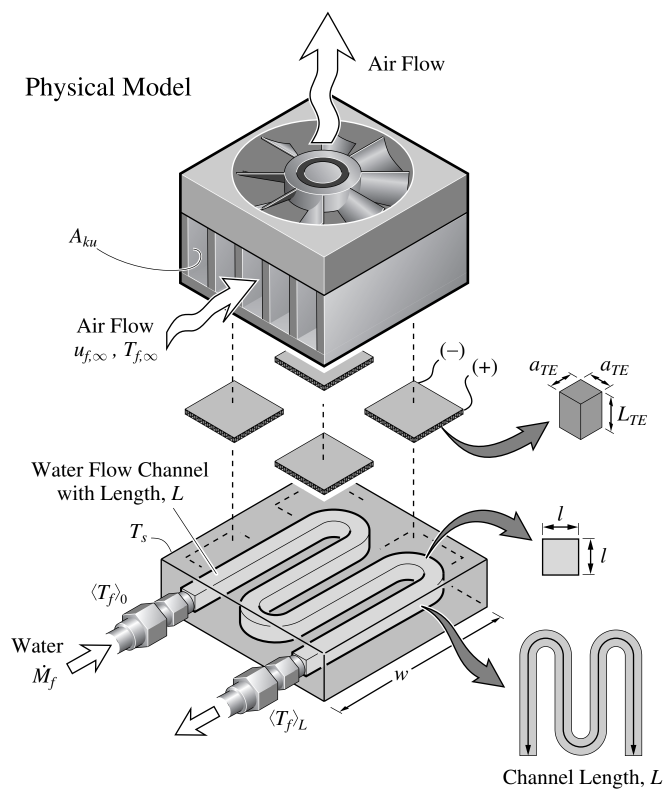


Figure Pr.8.3(a) A thermoelectric cool-unit used to cool a bounded water stream.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) Plot the heat removal rate from the water stream $\langle Q_u \rangle_{L\text{-}0}$, as function of the current for $0.2 \le J_e \le 1.4$ A.

(c) Comment on the optimum current.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.8.3(b). The three major resistances, $\langle R_{ku}\rangle_w$, $R_{k,h\text{-}c}$, and $\langle R_u\rangle_L$ are shown. In accordance with the developments of Section 3.3.7, and as shown in Figure 3.28(d), the Peltier cooling/heating and Joule heating are shown at the cold and hot junctions.

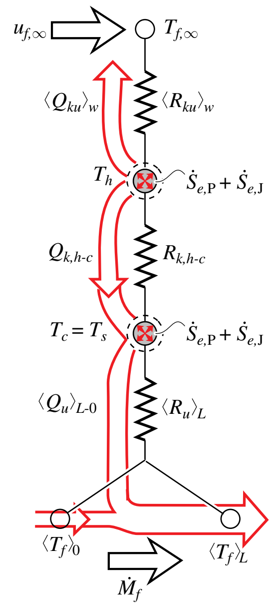


Figure Pr.8.3(b) Thermal circuit diagram.

(b) As shown in Figure Pr.8.3(b), the energy equations for nodes T_h and $T_c = T_s$ are

$$T_h \quad \text{node} \quad : \quad \langle Q_{ku}\rangle_w + Q_{k,h\text{-}c} = (\dot{S}_{e,\text{P}})_h + (\dot{S}_{e,\text{J}})_h$$
$$T_c = T_s \quad \text{node} \quad : \quad -Q_{k,h\text{-}c} + \langle Q_u\rangle_{L\text{-}0} = (\dot{S}_{e,\text{P}})_c + (\dot{S}_{e,\text{J}})_c,$$

where

$$\langle Q_{ku}\rangle_w = \frac{T_h - T_{f,\infty}}{\langle R_{ku}\rangle_w}$$
$$\langle R_{ku}\rangle_w^{-1} = A_{ku}\langle \text{Nu}\rangle_w \frac{k_{f,a}}{w} \quad \text{from (6.149)}$$
$$Q_{k,h\text{-}c} = \frac{T_h - T_c}{R_{k,h\text{-}c}}$$
$$R_{k,h\text{-}c}^{-1} = \left(\frac{A_k k}{L_{TE}}\right)_p + \left(\frac{A_k k}{L_{TE}}\right)_n \quad \text{from (3.116)}$$
$$= N_{TE}\frac{a_{TE}^2}{L_{TE}}(k_p + k_n)$$
$$\langle Q_u\rangle_{L\text{-}0} = \frac{T_c - \langle T_f\rangle_0}{\langle R_u\rangle_L}$$
$$= (\dot{M}c_p)_{f,w}(\langle T_f\rangle_L - \langle T_f\rangle_0)$$
$$\langle R_u\rangle_L^{-1} = (\dot{M}c_p)_{f,w}(1 - e^{-NTU}) \quad \text{from (7.27)}$$
$$NTU = \frac{4L\langle \text{Nu}\rangle_{D,h} k_{f,w}}{(\dot{M}c_p)_f} \quad \text{from (7.20)}.$$

The air-side Nusselt number is found from Table 6.3. The Reynolds number is

$$\text{Re}_w = \frac{u_{f,\infty} w}{\nu_{f,a}}.$$

The thermophysical problems for air at T = 300 K from Table C.22, and for water at T = 290 K from Table C.23, are

$$\text{air}: \quad k_{f,a} = 0.0267 \text{ W/m-K} \qquad \text{Table C.22}$$
$$\nu_{f,a} = 1.566 \times 10^{-5} \text{ m}^2/\text{s} \qquad \text{Table C.22}$$
$$\text{Pr}_a = 0.69 \qquad \text{Table C.22}$$

$$\text{water}: \quad k_{f,w} = 0.590 \text{ W/m-K} \qquad \text{Table C.23}$$
$$\rho_{f,w} = 1{,}000 \text{ kg/m}^3 \qquad \text{Table C.23}$$
$$c_{p,w} = 4.186 \text{ J/kg-K} \qquad \text{Table C.23}$$
$$\nu_{f,w} = 1.13 \times 10^{-6} \text{ m}^2/\text{s} \qquad \text{Table C.23}$$
$$\text{Pr}_w = 8.02. \qquad \text{Table C.23.}$$

Then

$$\text{Re}_w = \frac{1(\text{m/s}) \times 0.09(\text{m})}{1.566 \times 10^{-5}(\text{m}^2/\text{s})} = 5{,}747 < \text{Re}_{w,t} = 5 \times 10^5 \quad \text{laminar flow regime.}$$

Then from Table 6.3, we have

$$\langle \text{Nu} \rangle_w = 0.664 \text{Re}_D^{1/2} \text{Pr}^{1/3} = 0.664 \times (5{,}747)^{1/2} \times (0.69)^{1/3} = 44.49.$$

The water side Nusselt number is found from Table 7.2 or Table 7.3, depending on $\text{Re}_{D,h}$. The hydraulic diameter is

$$\begin{aligned}
D_h &= \frac{4A_u}{P_{ku}} = \frac{4 \times l^2}{4l} = l \\
\text{Re}_{D,h} &= \frac{\langle u_f \rangle l}{\nu_{f,w}} \\
\langle u_f \rangle &= \frac{\dot{M}_{f,w}}{\rho_{f,w} A_u} = \frac{\dot{M}_{f,w}}{\rho_{f,w} l^2} \\
\text{Re}_{D,h} &= \frac{\dot{M}_{f,w}}{\rho_{f,w} \nu_{f,w} l} = \frac{0.01(\text{kg/s})}{1{,}000(\text{kg/m}^3) \times 1.13 \times 10^{-6}(\text{m}^2/\text{s}) \times 5 \times 10^{-3}(\text{m})} \\
&= 1{,}770 < \text{Re}_{D,t} = 2{,}300 \quad \text{laminar flow regime.}
\end{aligned}$$

Next, from Table 7.2, we have

$$\text{Nu}_{D,h} = 2.98 \quad \text{for square channels with } T_s \text{ uniform.}$$

The energy conversion terms are

$$\begin{aligned}
(\dot{S}_{e,\text{J}})_c &= (\dot{S}_{e,\text{J}})_h = \frac{1}{2} N_{TE} J_e^2 R_e \quad \text{from (3.115)} \\
R_e &= (\frac{\rho_e L_{TE}}{A_k})_p + (\frac{\rho_e L_{TE}}{A_k})_n \quad \text{from (3.116)} \\
&= \frac{L_{TE}}{a_{TE}^2} (\rho_{e,p} + \rho_{e,n}) \\
(\dot{S}_{e,\text{P}})_h &= N_{TE} \alpha_\text{S} J_e T_h \quad \text{from (3.112)} \\
(\dot{S}_{e,\text{P}})_c &= -N_{TE} \alpha_\text{S} J_e T_c \quad \text{from (3.112)} \\
\alpha_\text{S} &= \alpha_{\text{S},p} - \alpha_{\text{S},n} \quad \text{from (3.112).}
\end{aligned}$$

From Table C.9(a), we have for bismuth telluride

$$\begin{aligned}
\alpha_{\text{S},p} &= 2.30 \times 10^{-4} \text{ V/℃} \\
\alpha_{\text{S},n} &= 2.10 \times 10^{-4} \text{ V/℃} \\
\rho_{e,p} &= \rho_{e,n} = 10^{-5} \quad \text{ohm-m} \\
k_p &= 1.70 \text{ W/m-K} \\
k_u &= 1.45 \text{ W/m-K.}
\end{aligned}$$

We need to determine T_h, $T_c = T_s$, and $\langle T_f \rangle_L$ and the three needed equations are the T_h-node and T_c-node energy equations and the expression for $\langle Q_u \rangle_{L\text{-}0}$ (which is used for $\langle T_f \rangle_L$).
The variation of $\langle Q_u \rangle_{L\text{-}0}$ with respect to the current is shown in Figure Pr.8.3(c).

(c) The largest heat removal rate, $\langle Q_u \rangle_{L-0} = -6.47$ W, occurs at a current of $J_e = 0.8$ A. At this current, we have

$$\begin{aligned} T_c &= T_s = 290.5 \text{ K} \\ T_h &= 333.3 \text{ K} \\ \langle T_f \rangle_L &= 293.0 \text{ K}. \end{aligned}$$

The three resistances are

$$\langle R_{ku} \rangle_w = 1.515 \text{ K/W}, \quad R_{k,h\text{-}c} = 1.235 \text{ K/W}, \quad \text{and} \quad \langle R_u \rangle_L = 0.4183 \text{ K/W}.$$

At this optimum current, we also have

$$Q_{k,h\text{-}c} = 36.65 \text{ W}, (\dot{S}_{e,\text{P}})_c = -46.02 \text{ W}, \quad \text{and} \quad (\dot{S}_{e,\text{J}})_c = 5.04 \text{ W}.$$

As the current increases, T_h increases and $Q_{k,h\text{-}c}$ and $(\dot{S}_{e,\text{J}})_c$ increase faster than $(\dot{S}_{e,\text{P}})_c$, and $\langle Q_u \rangle_{L\text{-}0}$ begins to drop.

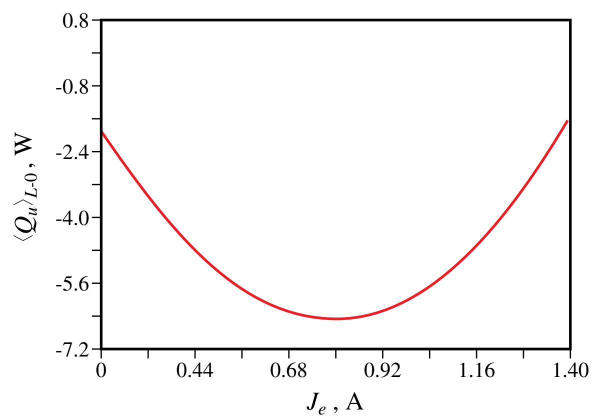


Figure Pr.8.3(c) Variation of the water stream convection heat transfer rate as a function of the electrical current.

COMMENT:
Here $NTU = 0.05880$ and is rather small. The water stream Nusselt number $\langle \text{Nu} \rangle_{D,h}$ can be increased by adding an extending surface.

PROBLEM 8.4.DES

GIVEN:

The automobile exhaust pollutants (NO, CO, and unburned hydrocarbons) escape conversion in the catalytic converter during converter warm-up (when the automobile first starts). This accounts for a significantly large fraction of the automobile pollution produced. In order to remedy this, a two-segment converter is used, where the first segment (or stage) has a smaller mass (and surface area) and heats up faster. This is shown in Figure Pr.8.4(a).

Assume that each segment is at a uniform, but time-dependent, temperature and each is heated by surface convection. The fluid enters the first segment at temperature $\langle T_f \rangle_0$ and exits at temperature $\langle T_f \rangle_1$, and arrives at the second segment and then exits that at temperature $\langle T_f \rangle_2$. Initially we have temperatures $T_{s,1}(t=0)$ and $T_{s,2}(t=0)$ for the two segments.

$\langle T_f \rangle_0 = 400$℃, $T_{s,1}(t=0) = T_{s,2}(t=0) = 20$℃, $D_{p,1} = 3$ mm, $A_{ku,1} = 8$ m^2, $\epsilon_1 = 0.9$, $(\rho c_p V)_1 = 50$ J/K, $D_{p,2} = 2$ mm, $A_{ku,2} = 10$ m^2, $\epsilon_2 = 0.65$, $(\rho c_p V)_2 = 500$ J/K, $c_{p,f} = 1{,}100$ J/kg-K, $\langle \text{Nu} \rangle_{D,p,1} = 150$, $\langle \text{Nu} \rangle_{D,p,2} = 150$, $k_f = 0.03$ W/m-K.

The exhaust gas mass flow rate for $\dot{M}_f$ is estimated closely using

$$\dot{M}_f = \frac{1}{2}\frac{rpm}{60}\rho_{f,o} V_d,$$

where rpm is the engine rpm, V_d is the total displacement volume (assuming ideal volumetric efficiency, $\eta_V = 1$), and $\rho_{f,o}$ is the cylinder inlet gas density (for a nonturbocharged engine, it will be air at ambient temperature and nearly one atm pressure). Here an ideal volume efficiency is assumed allowing complete filling of the displacement volume with fresh air. Then for a typical 2.2 liter engine, $\dot{M}_f = 0.03667$ kg/s.

SKETCH:

Figure Pr.8.4(a) shows the two-segment automobile catalytic converter. The segment with smaller mass is encountered first.

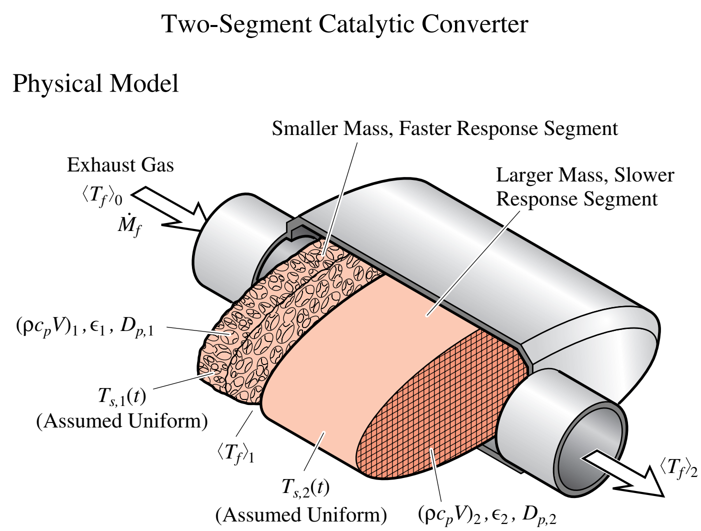


Figure Pr.8.4(a) A two-segment, automobile catalytic converter.

OBJECTIVE:

(a) Draw the thermal circuit diagram.

(b) For the conditions given above, plot the temperatures $\langle T_f \rangle_1, \langle T_f \rangle_2, T_{s,1}(t)$, and $T_{s,2}(t)$ up to $t = 100$ s.

SOLUTION:

(a) The thermal circuit diagram is shown in Figure Pr.8.4(b). The fluid stream temperatures $\langle T_f \rangle_1(t)$ and $\langle T_f \rangle_2(t)$ are assumed to be quasi-steady when they are treated as bounded fluid streams with surface convection heat transfer.

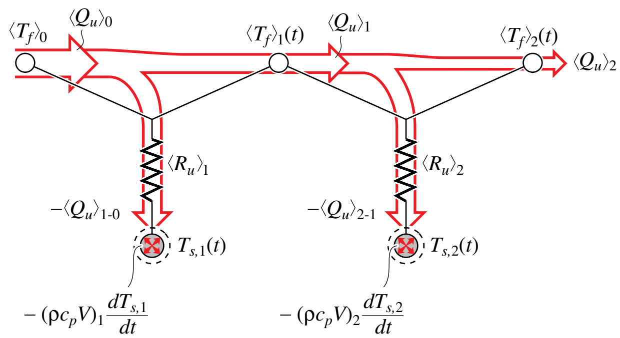


Figure Pr.8.4(b) Thermal circuit diagram.

(b) The energy equations for nodes $T_{s,1}$, and $T_{s,2}$ [from Figure 8.4(b)], the convection heat transfer, the surface-convection heat transfer, and the associated resistances are from Section 7.4.3, i.e.,

$$\begin{aligned}
\langle Q_u\rangle_{1\text{-}0} &= -(\rho c_p V)_1 \frac{dT_{s,1}}{dt} \\
\langle Q_u\rangle_{2\text{-}1} &= -(\rho c_p V)_2 \frac{dT_{s,2}}{dt} \\
\langle Q_u\rangle_{1\text{-}0} = (\dot{M}c_p)_f(\langle T_f\rangle_1 - \langle T_f\rangle_0) &= \frac{T_{s,1} - \langle T_f\rangle_0}{\langle R_u\rangle_1} \\
\langle Q_u\rangle_{2\text{-}1} = (\dot{M}c_p)_f(\langle T_f\rangle_2 - \langle T_f\rangle_1) &= \frac{T_{s,2} - \langle T_f\rangle_1}{\langle R_u\rangle_2} \\
\langle R_u\rangle_1^{-1} &= (\dot{M}c_p)_f(1 - e^{-NTU_1}), \quad \langle R_u\rangle_2^{-1} = (\dot{M}c_p)_f(1 - e^{-NTU_2}) \\
NTU_1 &= \frac{1}{(\dot{M}c_p)_f\langle R_{ku}\rangle_1}, \quad NTU_2 = \frac{1}{(\dot{M}c_p)_f\langle R_{ku}\rangle_2} \\
\langle R_{ku}\rangle_1^{-1} &= A_{ku,1}\langle \text{Nu}\rangle_{D,p_1} \frac{k_f}{D_{p,1}} \frac{1-\epsilon_1}{\epsilon_1} \\
\langle R_{ku}\rangle_2^{-1} &= A_{ku,2}\langle \text{Nu}\rangle_{D,p_2} \frac{k_f}{D_{p,2}} \frac{1-\epsilon_2}{\epsilon_2}.
\end{aligned}$$

Using the numerical values, a solver such as SOPHT is used to determine the unknowns.
The results are plotted in Figure Pr.8.4(c). The first segment ($T_{s,1}(t)$) heats up to the maximum temperature $\langle T_f\rangle_0$ in about 3 s. The second segment, having a much larger mass, $(\rho c_p V)_1$, takes much longer to heat up. The fluid stream temperature $\langle T_f\rangle_1(t)$ is very close to $T_{s,1}(t)$. Also, $\langle T_f\rangle_2$ is close to $T_{s,2}(t)$. This is due to the large values of NTU ($NTU_2 = 4.131, NTU_1 = 100.1$). Other values are: $\langle R_u\rangle_1 = 0.02520$ K/W and $\langle R_u\rangle_2 = 2.2476\times10^{-4}$ K/W.

COMMENT:

Here we have assumed that the at any time a temperature distribution given by (7.21) exists for the fluid. This, along with a uniform solid temperature allow us to represent each segment with only one thermal node. For accurate results, each segment can be divided into many nodes.

For yet a faster response, the mass of the first segment can be decreased, along with its surface-convection resistance.

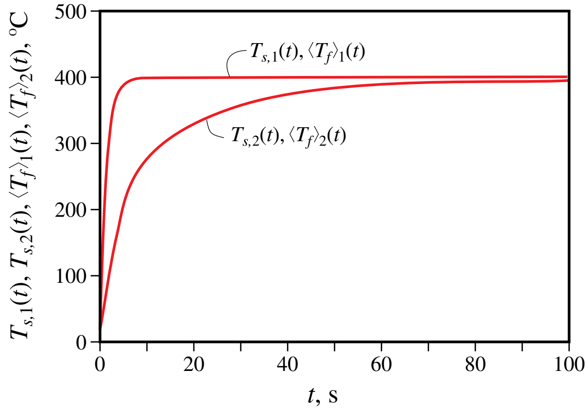


Figure Pr.8.4(c) Time variation of the temperature of the two segments and the two gas stream temperatures.