Hydrogen Atom Spectrum

CONTENTS:

In this lecture for Hydrogen Atom, we have to study about;

White light spectra

Absorption spectra

Emission spectra

Hydrogen spectra: series, Lymen, Balmer Paschen, Bracket, P-Fund ......

Therefore, Wave number = $R_{\infty} Z^2 / (1/n_1^2 - 1/n_2^2)$

Bohr theory of Hydrogen Atom

Radius of orbit: $\mathbf{r_n= \{h^2\epsilon_0/\pi me^2\}\{n^2/Z\}}$, (H-atom, Z=1, n=1)

$r=0.53A^0$. which is known as Bohr Radius of Hydrogen atom.

Velocity: $v_n= \{e^2/2h\epsilon_0\}\{Z/n\}$, for H-atom , $v= 2.18\times10^6$ m/s

Total energy: $En= - \{me^4/8\epsilon_0^2h^2\}\{Z^2/n^2\}$

So quantization of orbital angular momentum of the electron leads to quantization of its total energy. Therefore, E1=-13.6 eV, E2= -3.39eV, …. etc.

Rydberg's constant: R= 'me^4/8ϵ_0^2h^{3}c'=1.09×10^7 /m

Rydberg's Equation

$$\text{wavenumber} = \bar{\nu} = \frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\text{Energy} = E = hc\bar{\nu} = hcRZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Spectrum

Spectroscopy is the branch of physics which deals with the interaction of electromagnetic radiation with matter and the interaction is associated with the exchange of energy between the two. Historically, spectroscopy dates back to Newton's discovery in 1666 – white light coming from the sun dispersed by a prism to seven colours (Fig 1).

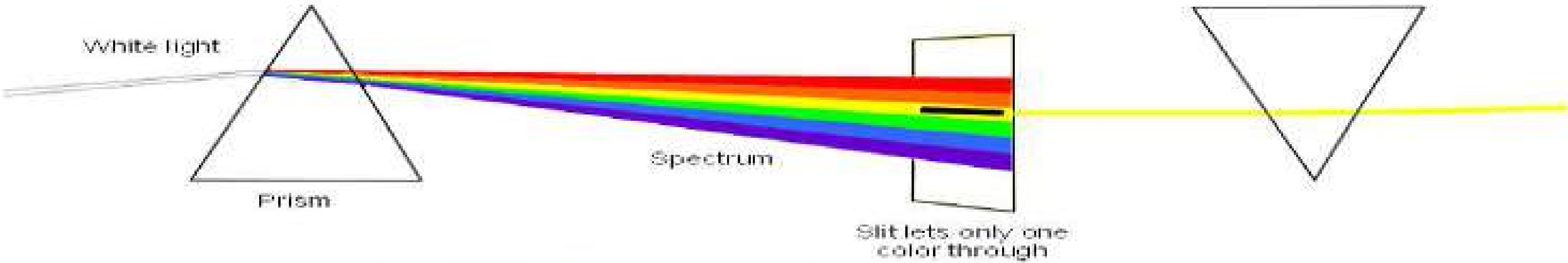


Fig. 1 Dispersion of white light through Prism

The observation of seven colours is the characteristic of the light and not that of the prism and for that matter of any dispersing object like crystal, thin films, grating or other material which on placing in direct light may give rise to colours. For example, rainbow in the sky, colours from the thin oily film on the road that arises due to the reflection of sun light and so on so forth.

Spectrum

In 1814 William von Fraunhofer, on repeating the experiment undertook earlier by William Wollaston in 1802, had established that the dispersed colours of sun light did not merge smoothly into each other but had many dark lines (absorption lines) – among large number of dark lines, eight prominent lines labeled after the first eight letters of the alphabets and commonly known as Fraunhofer lines (Fig. 2).

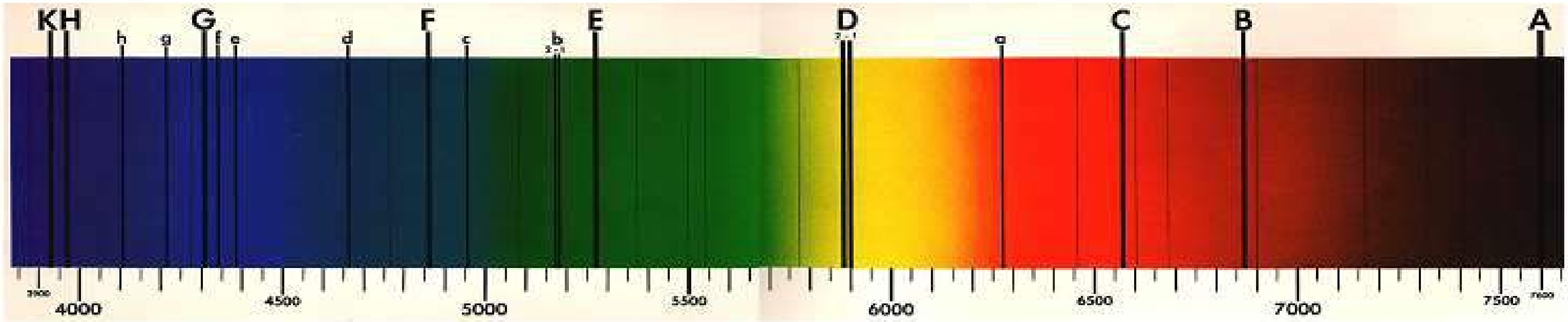


Fig. 2 Fraunhofer lines in the spectrum of Sun light.

Spectrum

To summarize, by the end of 19^{th} century, it was well established that the spectra of almost all elements have hundreds of discrete lines. Hydrogen atom is not only the simplest atom but also has the simplest spectra, mostly confined to the visible and ultra violet region (Fig. 3).

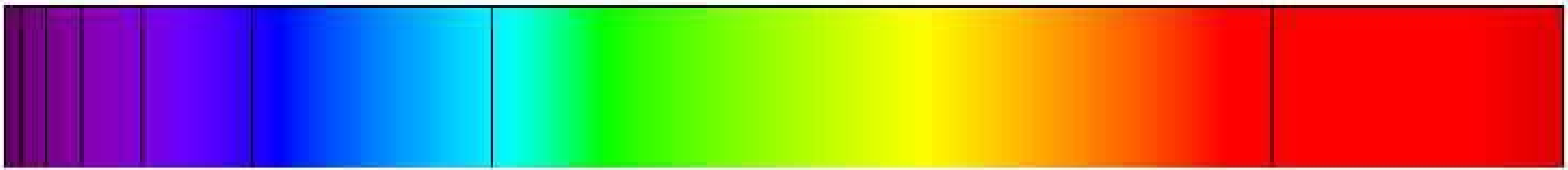
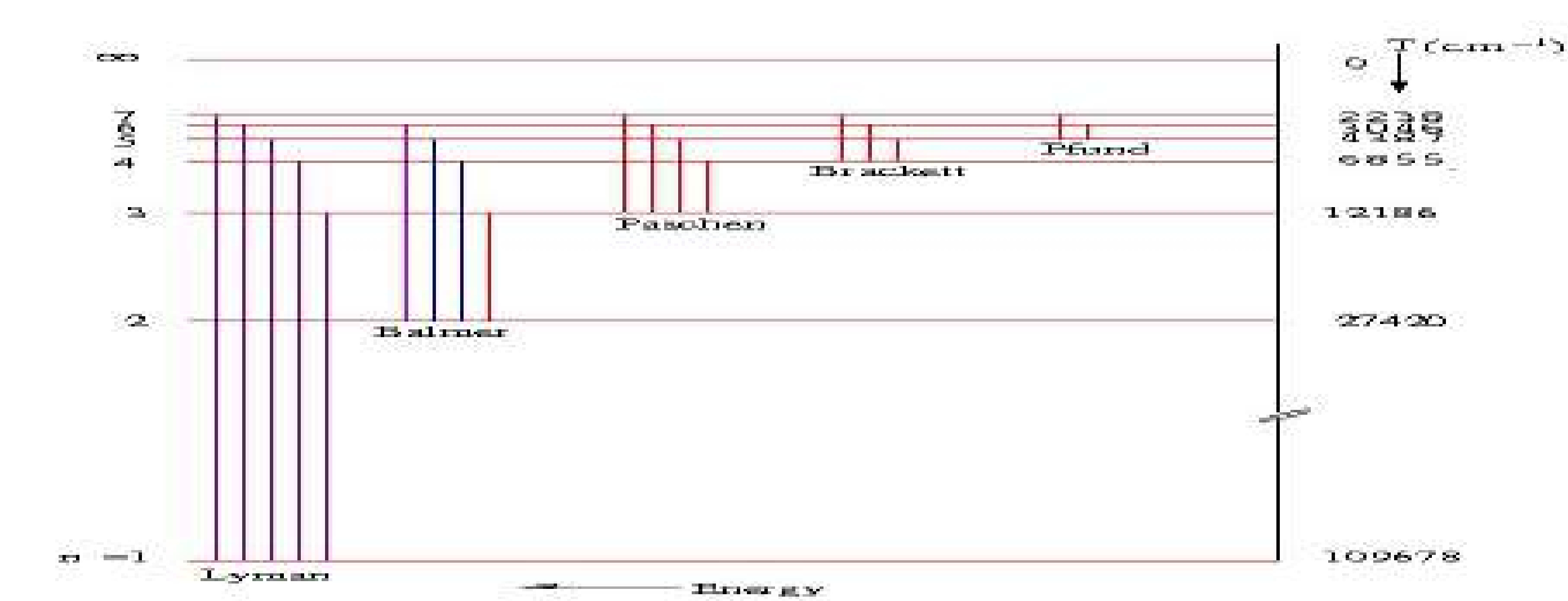


Fig. 3 Hydrogen spectrum in Visible & UV region.

Spectrum

Rydberg's formula related inverse wavelength (known as "wavenumber") Wave number = $1/\lambda = R_H\{1/n_f^2 - 1/n_i^2\}$

Where R_H is equal to 1.097×10^7 m^{-1} and n_f and n_i are integers (1,2,3,....) that describe the initial and final states of the electron.



Hydrogen Spectrum

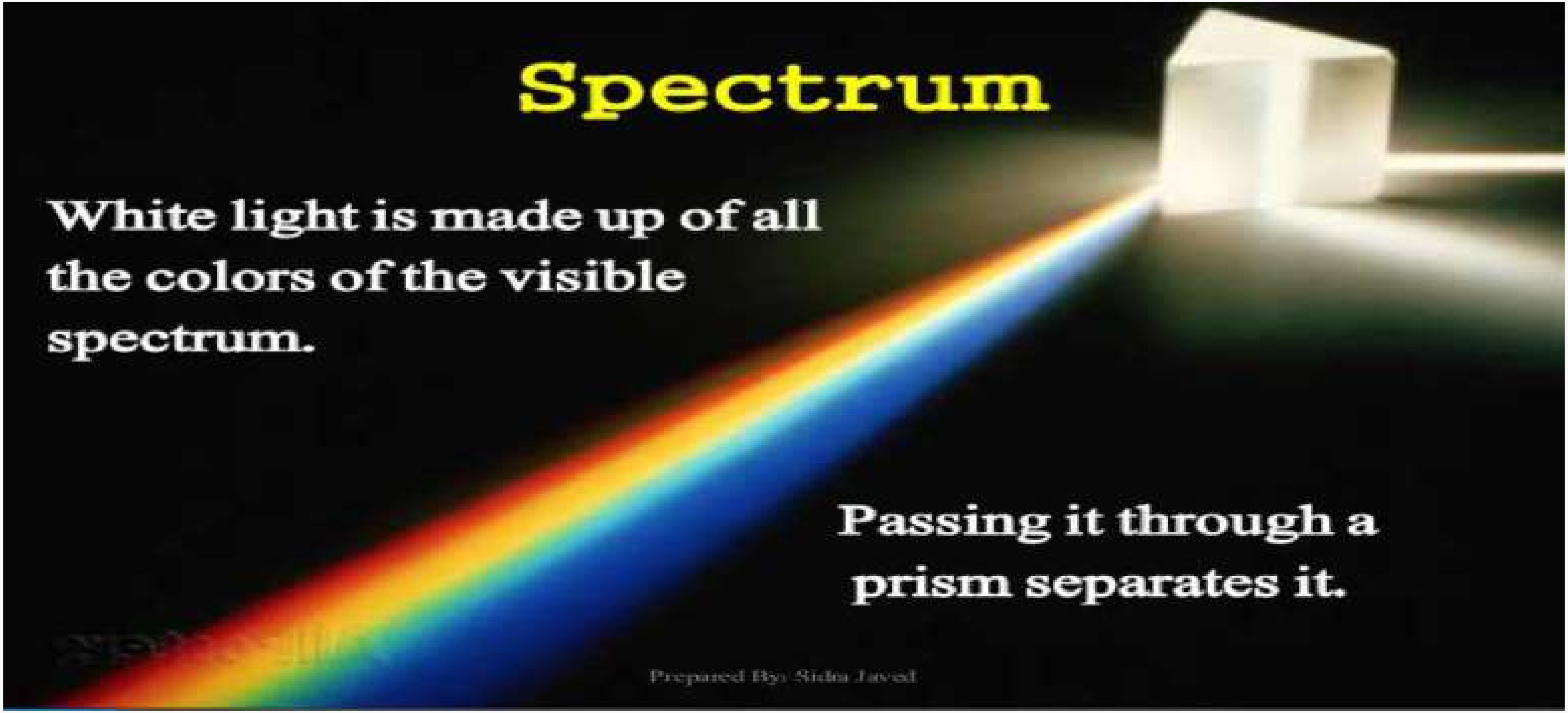
Hydrogen Spectra

Using Rutherford model of the atom that the atom consists of positively charged heavy nucleus surrounded by electrons moving in circular orbits around the nucleus and by analogy with Planck's assumption of the quantization of the energy, Bohr postulated that

1. The electrons move in circular orbit but don't lose energy so long they remain in one and the same orbit. The latter part of the postulate is completely in disagreement with the law of classical mechanics which predict that the charge accelerated in circular motion will release the electromagnetic radiation and gradually spiral inwards releasing energy continuously; this corresponds to continuous spectrum instead of discrete lines. Bohr countered this situation proposing that the accelerated electrons in atoms don't release energy.

2. Arguing from correspondence principle which states that predictions of quantum theory approach those of classical physics in the limit of large quantum numbers (or one may put it as classical mechanics can be understood as a limiting case of quantum mechanics), he postulated that the angular momentum of the electrons can have only certain discrete values in units of $\hbar$ - quantization of the angular momentum as a consequence of which restricted number of non-radiating orbits are permissible. This condition was later interpreted by de Broglie as a standing wave condition (see section 5).

3. Electromagnetic radiation energy is emitted or absorbed only when electron jumps from one permissible orbit to the other. It amounts to say that discrete energy, corresponding to the separation between the permissible orbits, can be emitted or absorbed.


Spectrum
White light is made up of all the colors of the visible spectrum.
Passing it through a prism separates it.
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Spectrum


If the Light is not white
By heating a gas or with electricity we can get it to give off colors.
Passing this light through a prism does something different.
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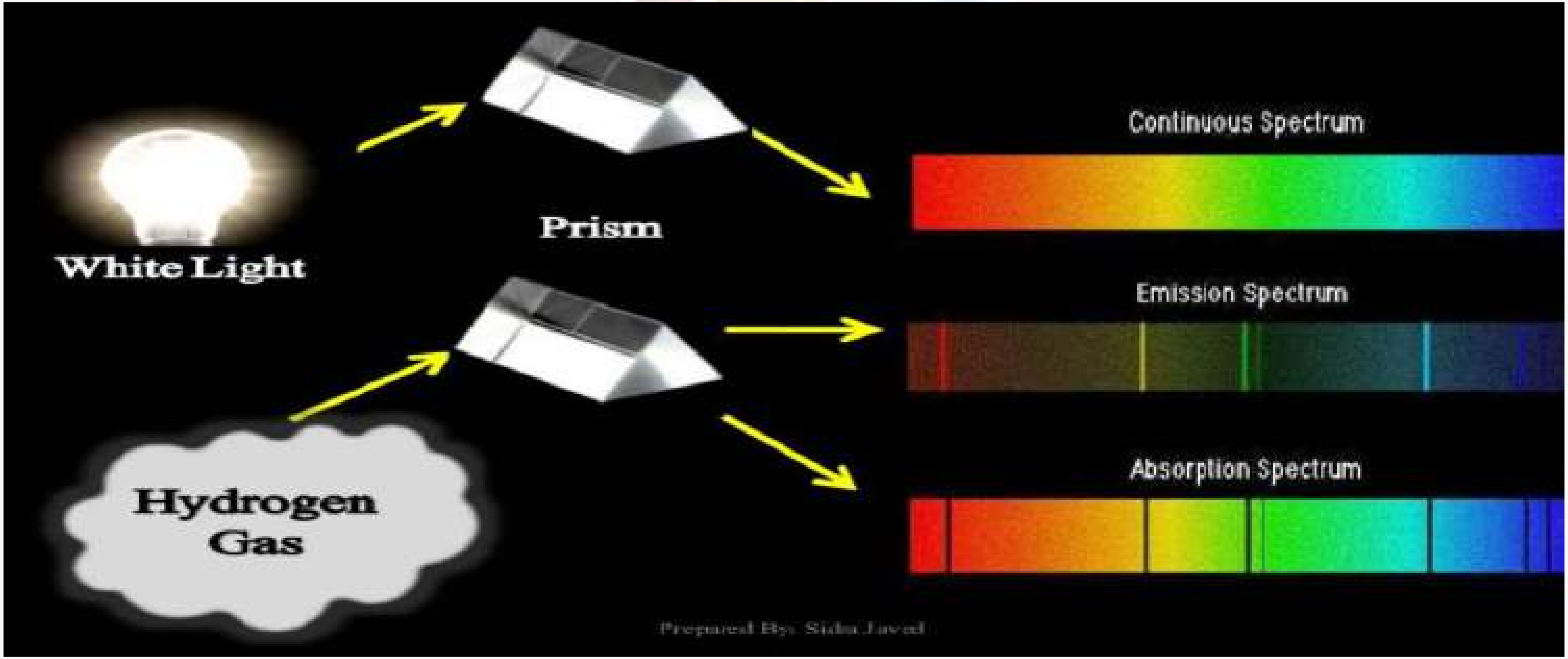
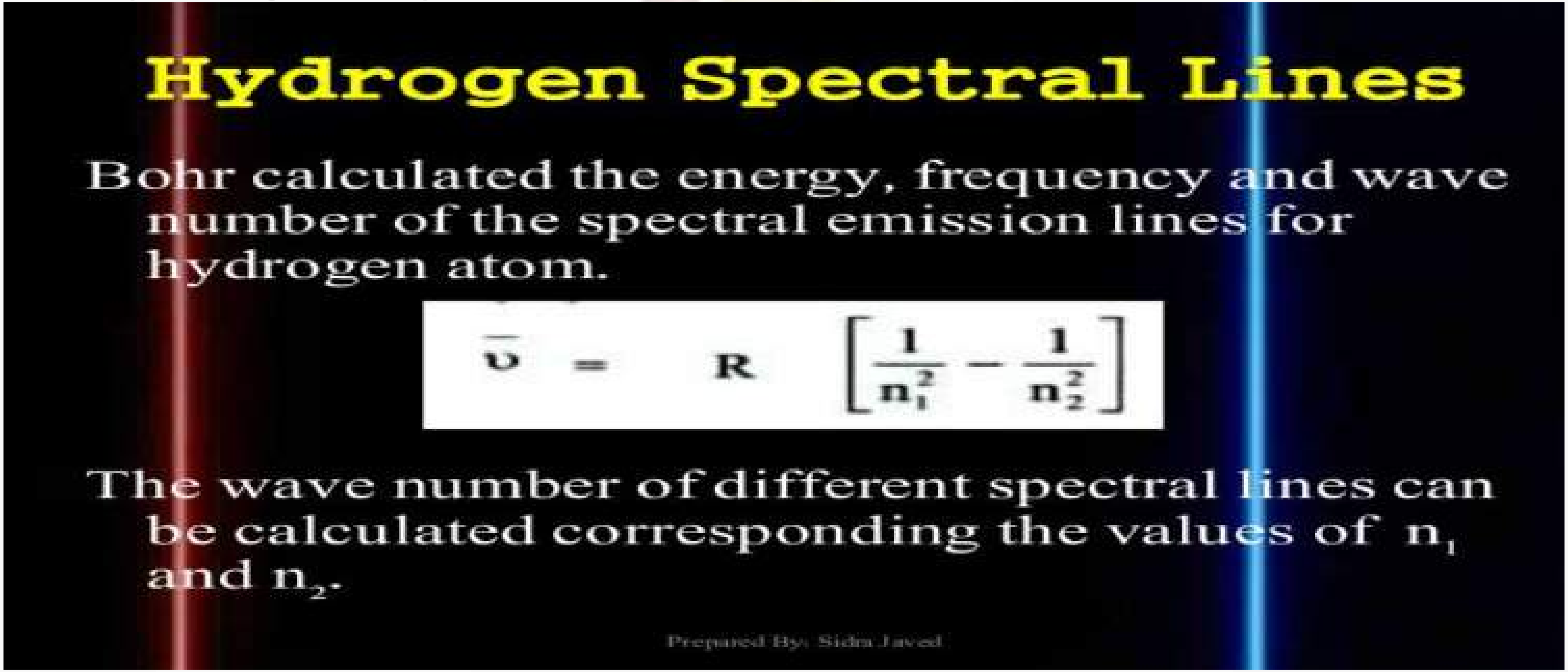
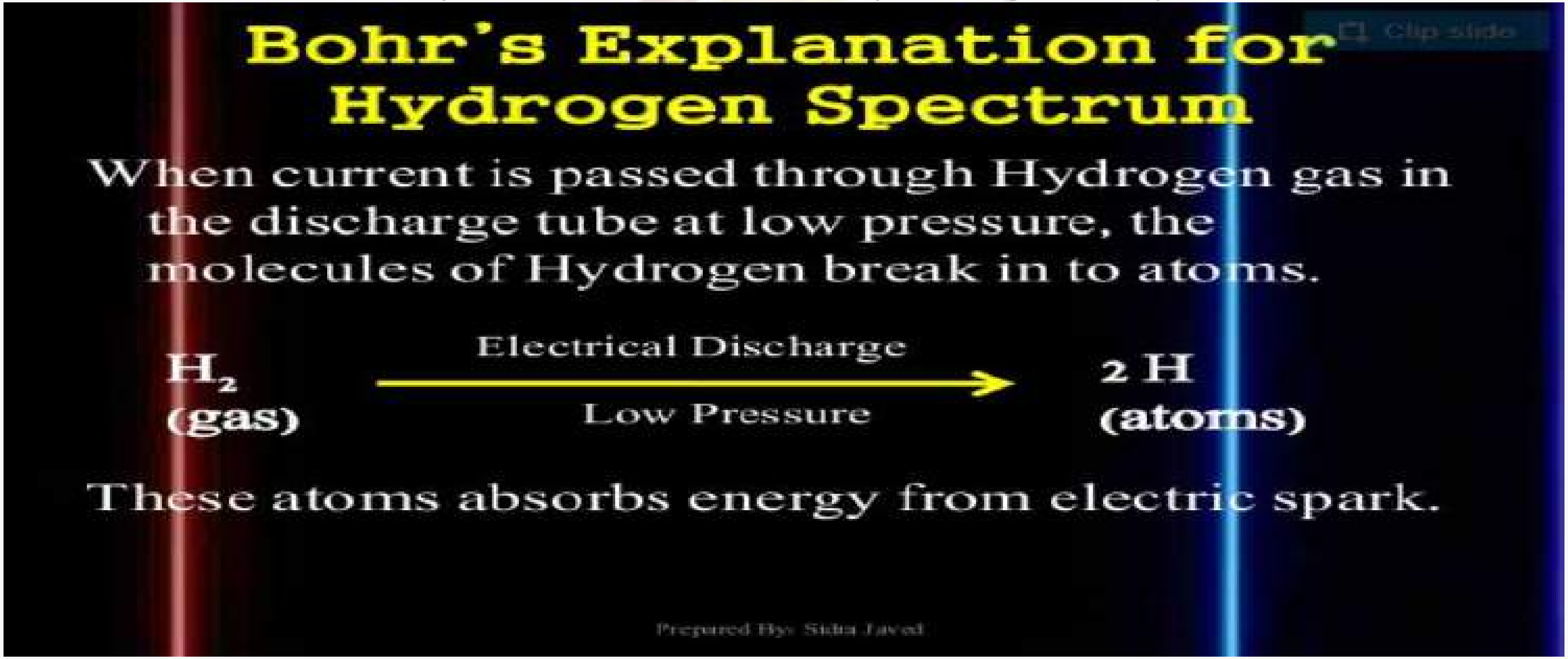
Emission and Absorption spectra



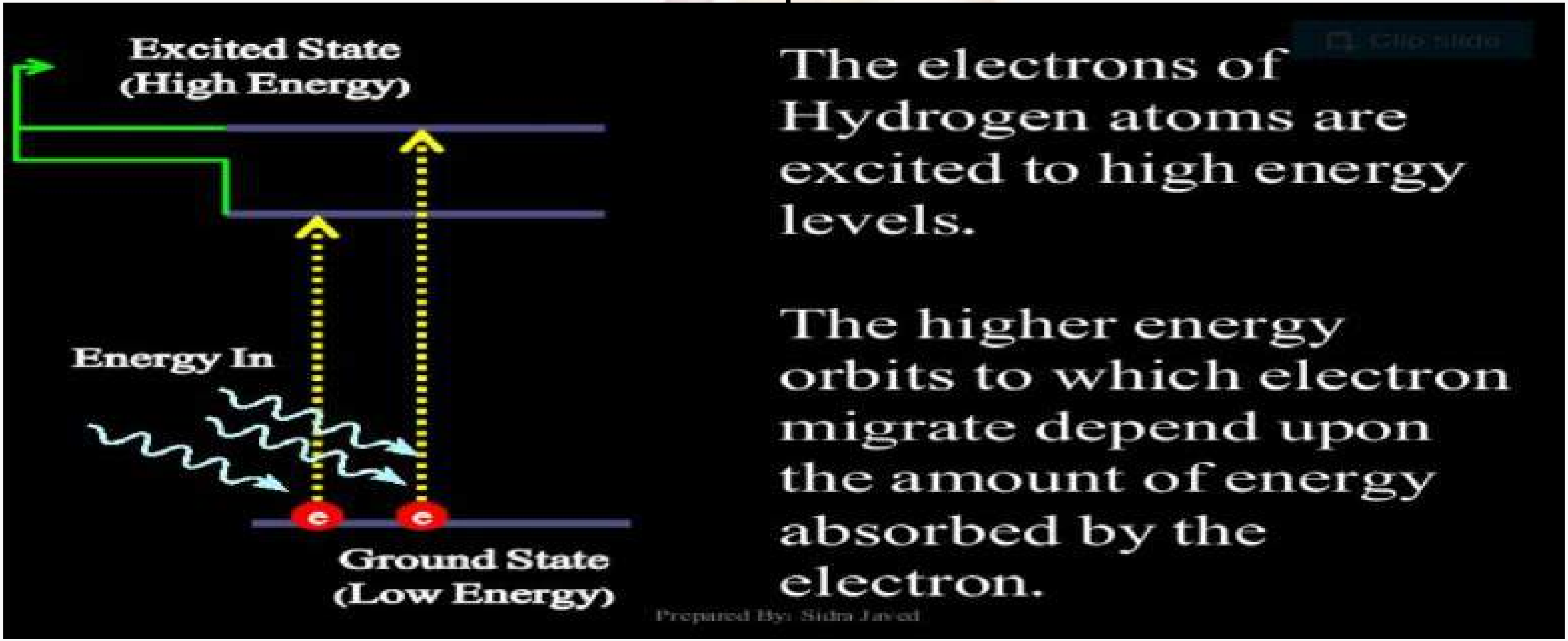
Hydrogen Spectral Lines



Bohr’s Explanation for hydrogen spectra



Absorption


Excited State
(High Energy)
Energy In
Ground State
(Low Energy)
The electrons of Hydrogen atoms are excited to high energy levels.
The higher energy orbits to which electron migrate depend upon the amount of energy absorbed by the electron.
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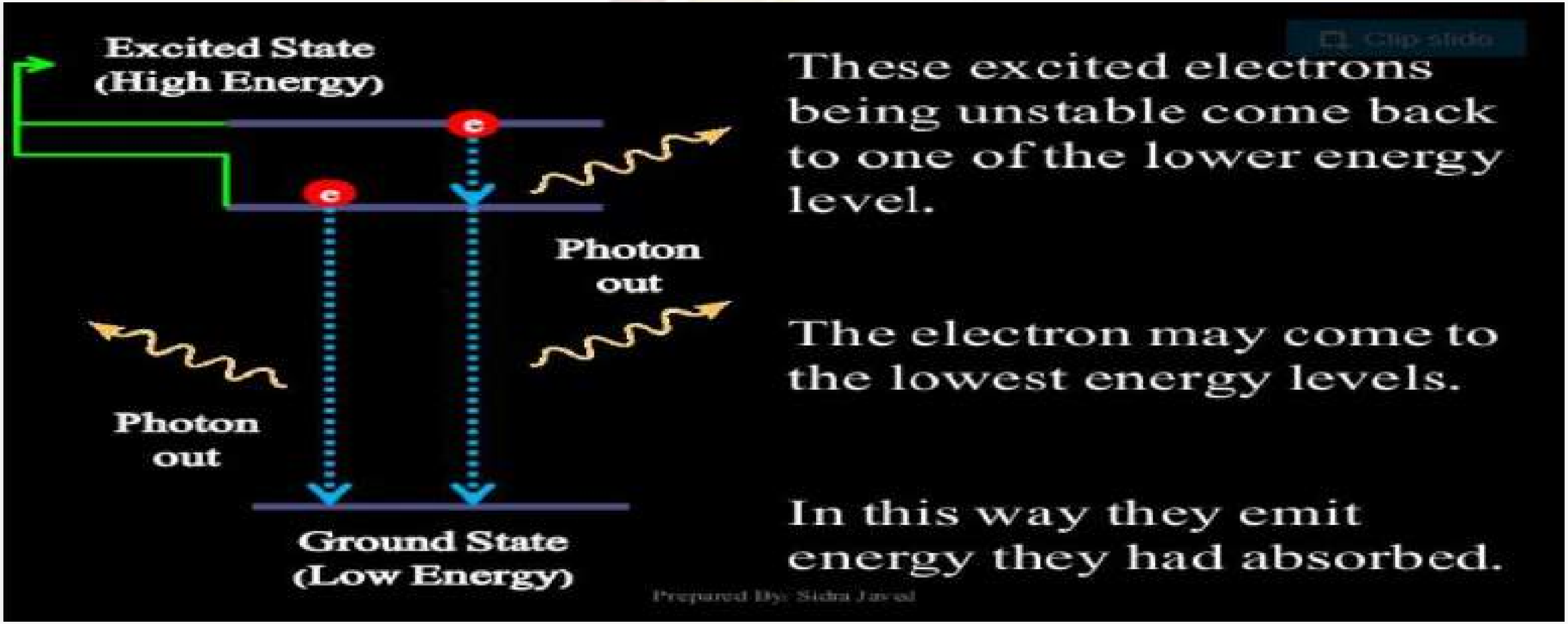
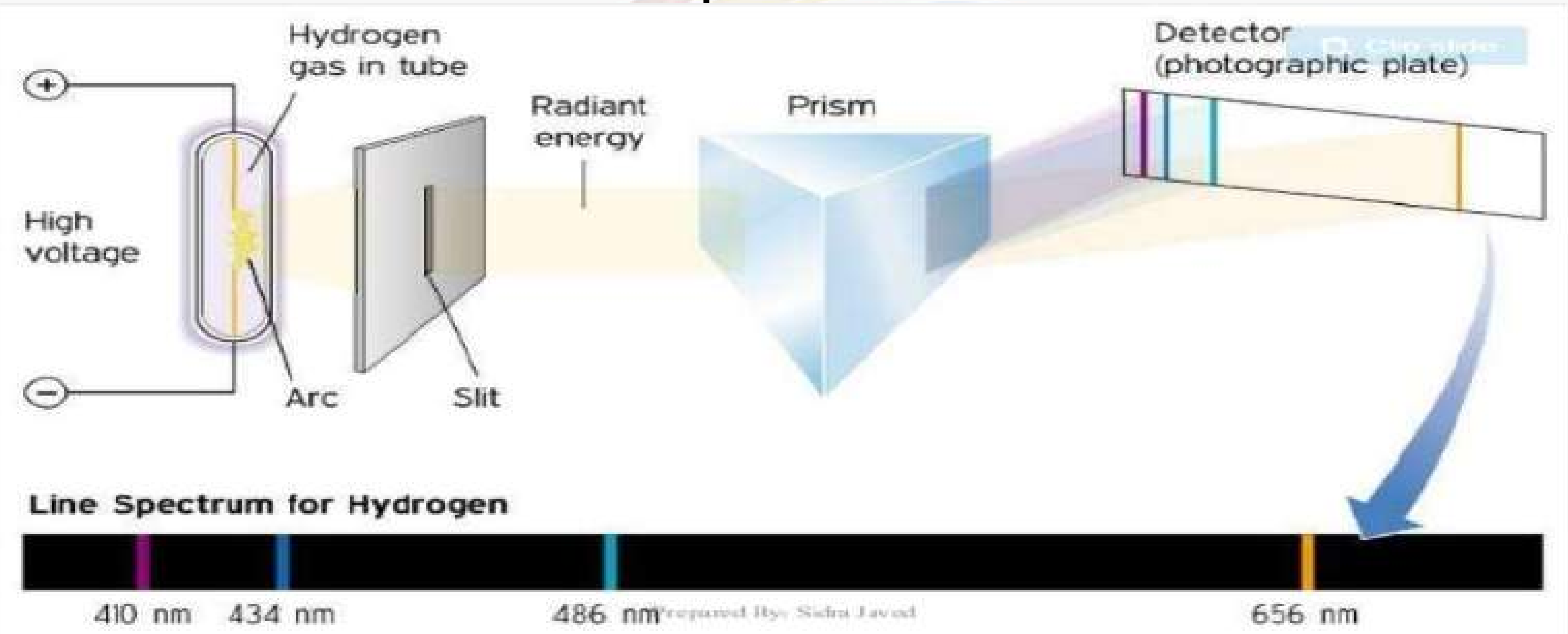
Emission


Excited State
(High Energy)
Photon
out
Photon
out
Ground State
(Low Energy)
These excited electrons being unstable come back to one of the lower energy level.
The electron may come to the lowest energy levels.
In this way they emit energy they had absorbed.

Line Spectrum



Wave Number of Spectral Lines

Wave number of spectral lines

Different spectral series are produced depending upon the excited and ground energy level of electron.

Wave number of these spectral lines can be calculated by Bohr's equation.

$$\bar{v} = 1.09678 \times 10^{+7}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] m^{-1}$$

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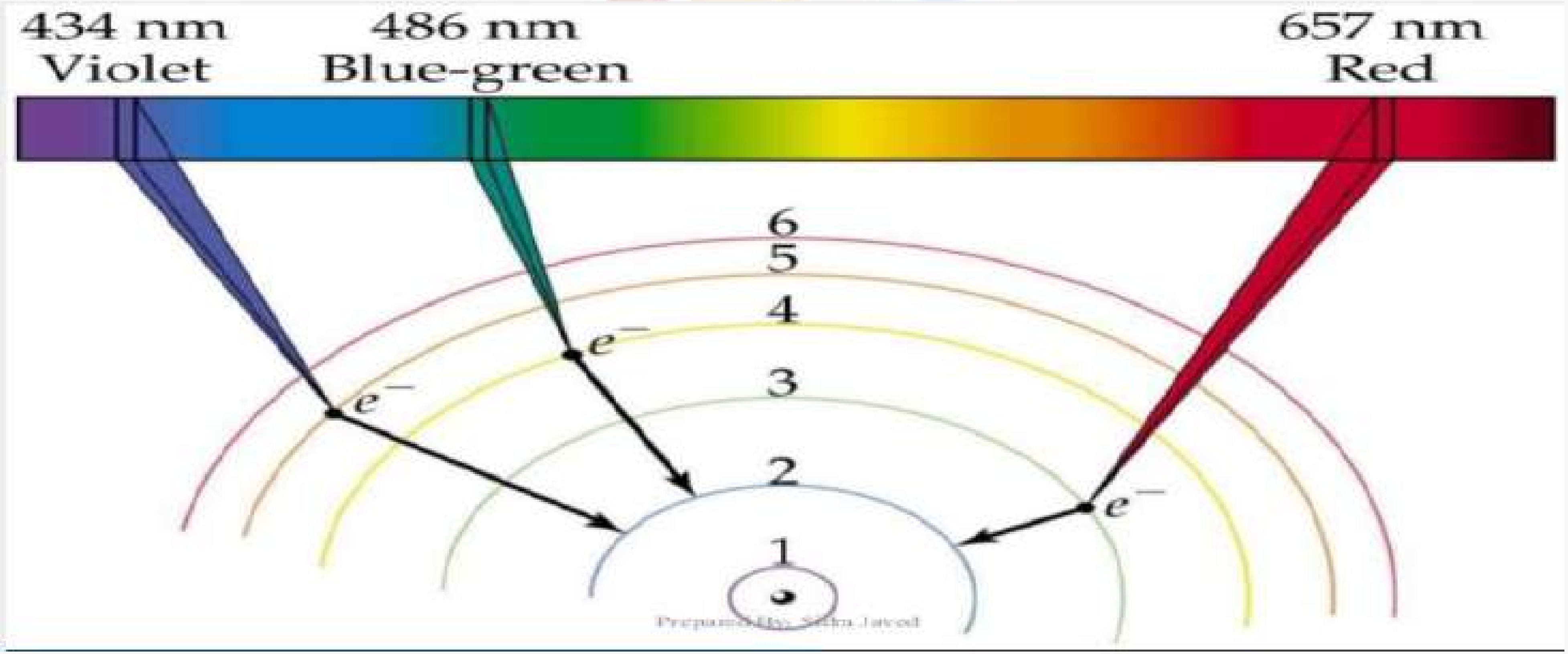
Various series in Hydrogen Spectra



Lymen Series

The spectral lines in Lymen Series are explained by considering that the electron falls back to n=1 from higher levels

$$\bar{\upsilon} = 1.09678 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{n_2{}^2} \right)$$

where,

$$n_2 = 2,3,4,\ldots$$

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Lymen Series (Ultra–violet Region)

First Line : $n_1 = 1, n_2 = 2$

$$\bar{\upsilon} = 1.09678 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = 82.26 \times 10^5 m^{-1}$$

Second Line : $n_1 = 1, n_2 = 3$

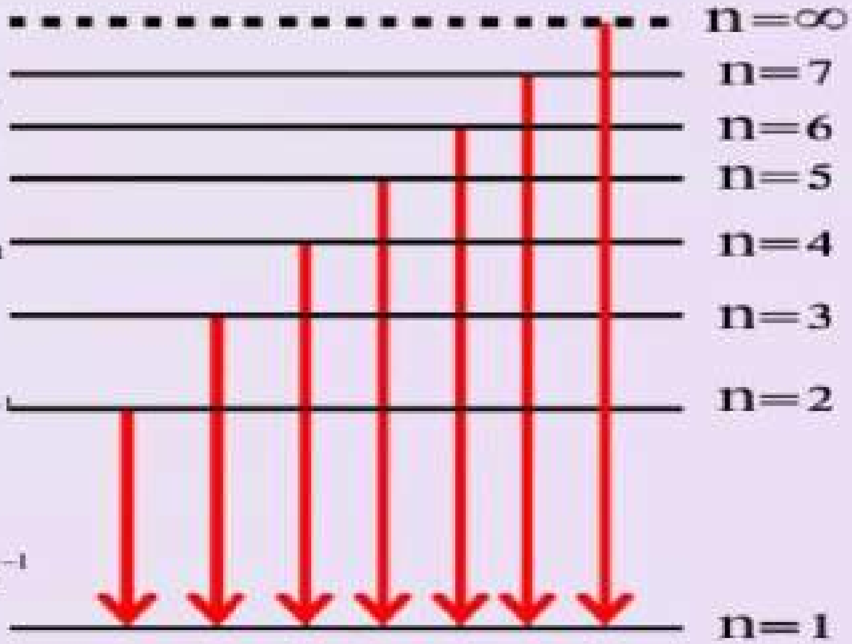
$$\bar{\upsilon} = 1.09678 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{3^2} \right) = 97.60 \times 10^5 m^{-1}$$

Third Line : $n_1 = 1, n_2 = 4$

$$\bar{\upsilon} = 1.09678 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{4^2} \right) = 102.70 \times 10^5 m^{-1}$$

Limiting Line : $n_1 = 1, n_2 = \infty$

$$\bar{\upsilon} = 1.09678 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right) = 1.09678 \times 10^7 m^{-1}$$



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Balmer Series (Visible Region)

First Line (Hα Line): $n_1 = 2, n_2 = 3$

$$\bar{\upsilon} = 1.09678 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = 15.234 \times 10^5 m^{-1}$$

Second Line (Hβ Line): $n_1 = 2, n_2 = 4$

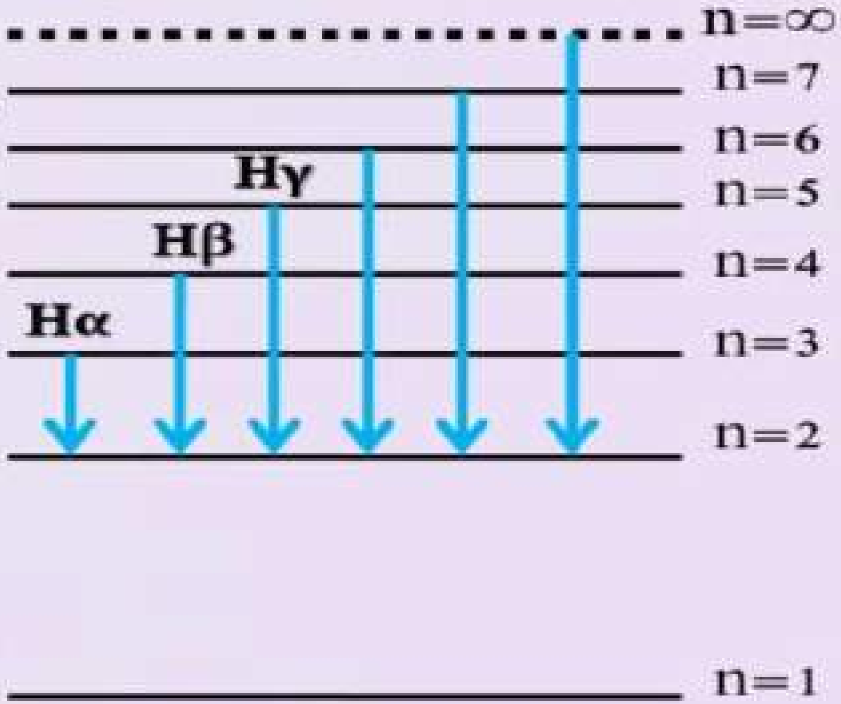
$$\bar{\upsilon} = 1.09678 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = 20.566 \times 10^5 m^{-1}$$

Third Line (Hγ Line): $n_1 = 2, n_2 = 5$

$$\bar{\upsilon} = 1.09678 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{5^2} \right) = 23.05 \times 10^5 m^{-1}$$

Limiting Line: $n_1 = 2, n_2 = \infty$

$$\bar{\upsilon} = 1.09678 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right) = 27.421 \times 10^5 m^{-1}$$



Paschen Series
(Infra Red Region)

First Line : $n_1 = 3, n_2 = 4$

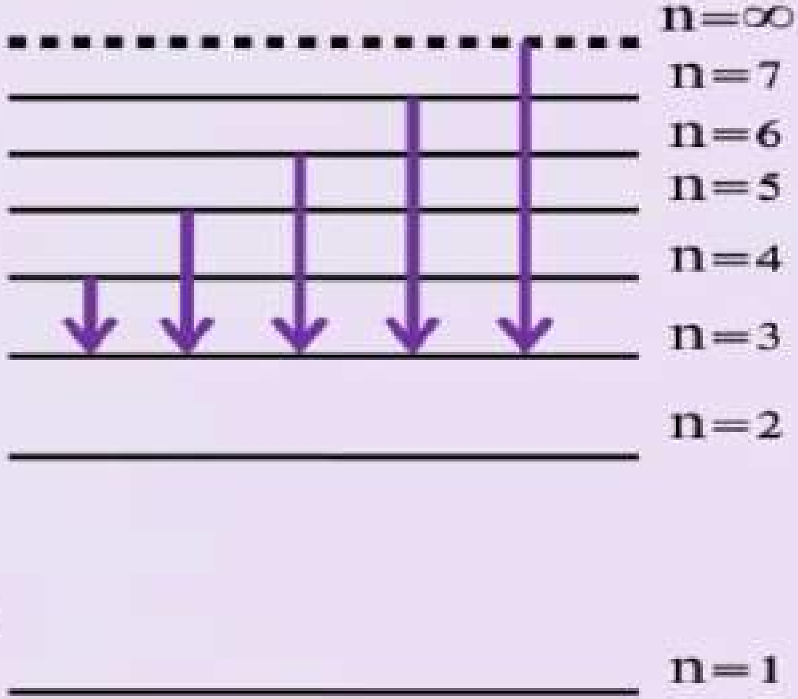
$$\bar{\nu} = 1.09678\times10^7\left(\frac{1}{3^2}-\frac{1}{4^2}\right) = 5.3310\times10^5 m^{-1}$$

Second Line : $n_1 = 3, n_2 = 5$

$$\bar{\nu} = 1.09678\times10^7\left(\frac{1}{3^2}-\frac{1}{5^2}\right) = 7.99\times10^5 m^{-1}$$

Limiting Line : $n_1 = 3, n_2 = \infty$

$$\bar{\nu} = 1.09678\times10^7\left(\frac{1}{3^2}-\frac{1}{\infty^2}\right) = 12.187\times10^5 m^{-1}$$



Brackett Series

First Line: $n_1 = 4, n_2 = 5$

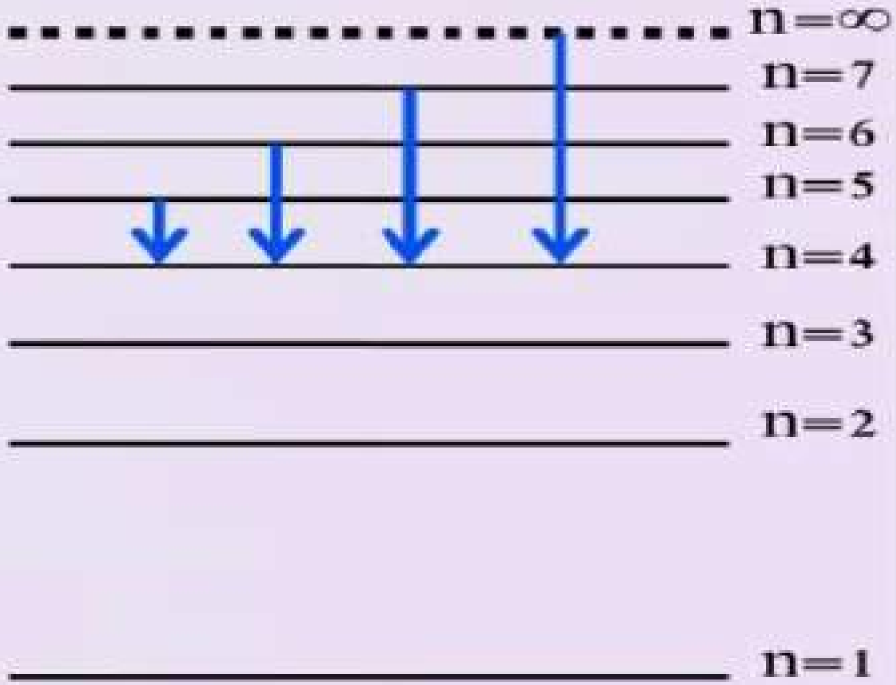
$$\bar{\upsilon} = 1.09678\times10^7\left(\frac{1}{4^2}-\frac{1}{5^2}\right) = 2.45\times10^5 m^{-1}$$

SecondLine : $n_1 = 4, n_2 = 6$

$$\bar{\upsilon} = 1.09678\times10^7\left(\frac{1}{4^2}-\frac{1}{6^2}\right) = 3.808\times10^5 m^{-1}$$

Limiting Line : $n_1 = 4, n_2 = \infty$

$$\bar{\upsilon} = 1.09678\times10^7\left(\frac{1}{4^2}-\frac{1}{\infty^2}\right) = 6.855\times10^5 m^{-1}$$



Pfund Series

First Line: $n_1 = 5, n_2 = 6$

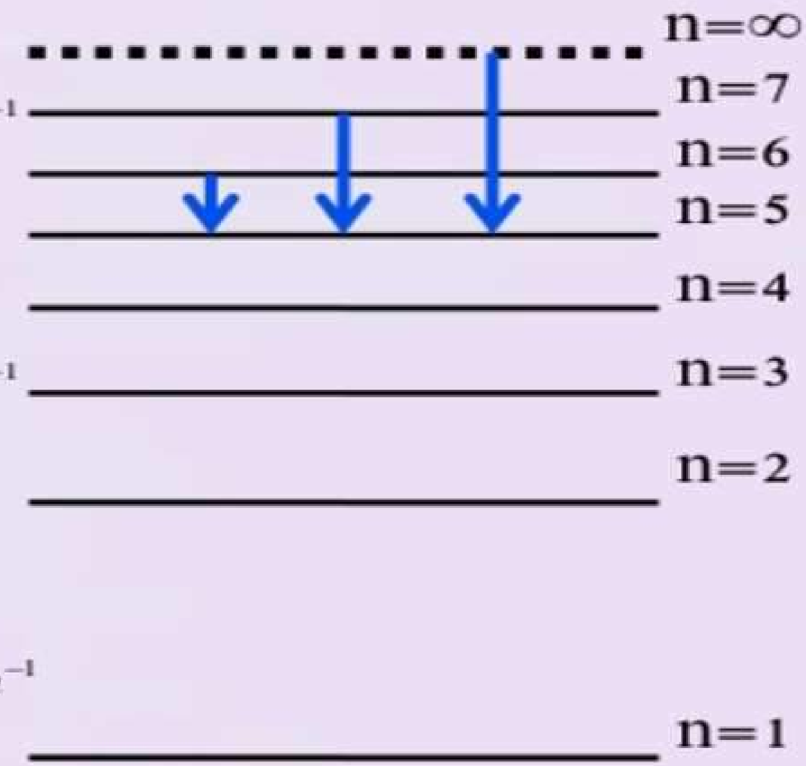
$$\bar{\upsilon} = 1.09678 \times 10^7 \left(\frac{1}{5^2} - \frac{1}{6^2} \right) = 1.340 \times 10^5 m^{-1}$$

SecondLine: $n_1 = 5, n_2 = 7$

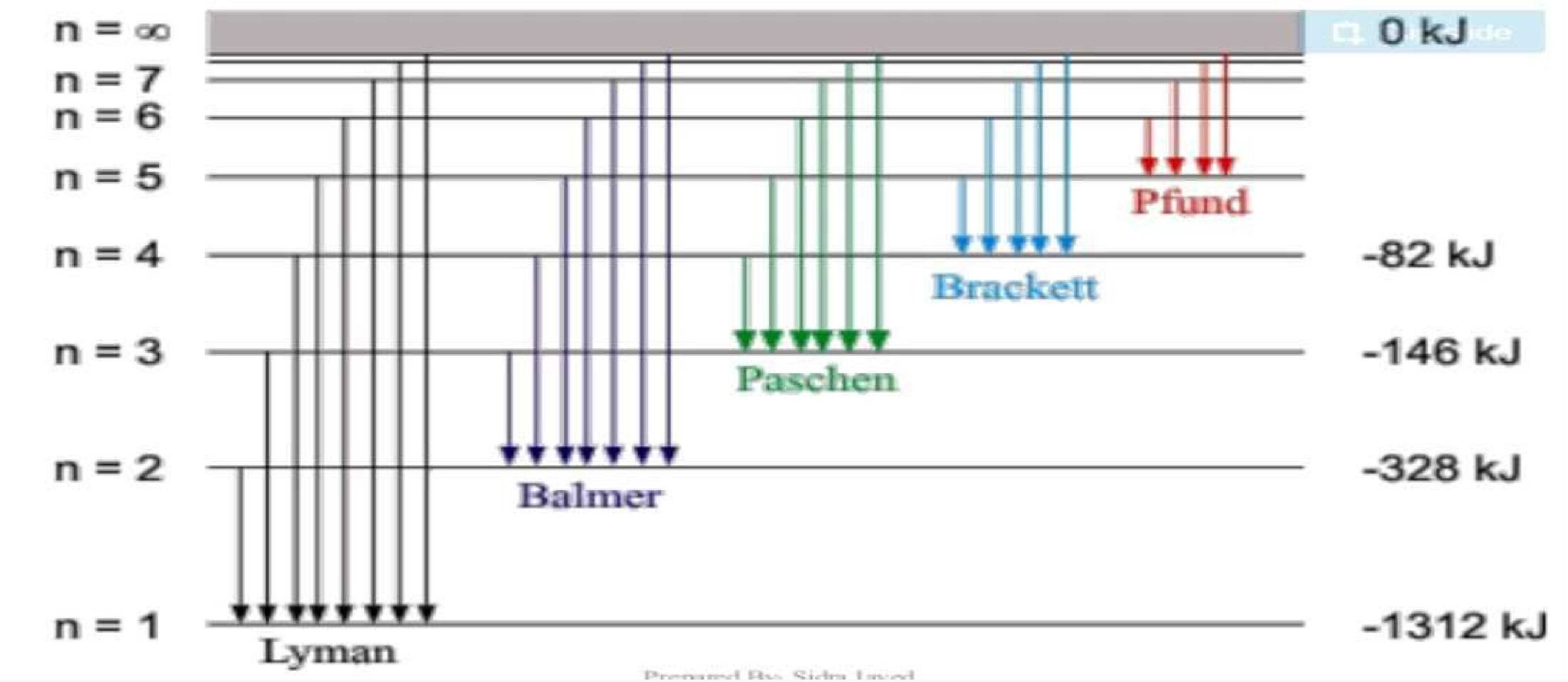
$$\bar{\upsilon} = 1.09678 \times 10^7 \left(\frac{1}{5^2} - \frac{1}{7^2} \right) = 2.148 \times 10^5 m^{-1}$$

Limiting Line: $n_1 = 5, n_2 = \infty$

$$\bar{\upsilon} = 1.09678 \times 10^7 \left(\frac{1}{5^2} - \frac{1}{\infty^2} \right) = 4.387 \times 10^5 m^{-1}$$



Series spectra of Hydrogen Atom



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