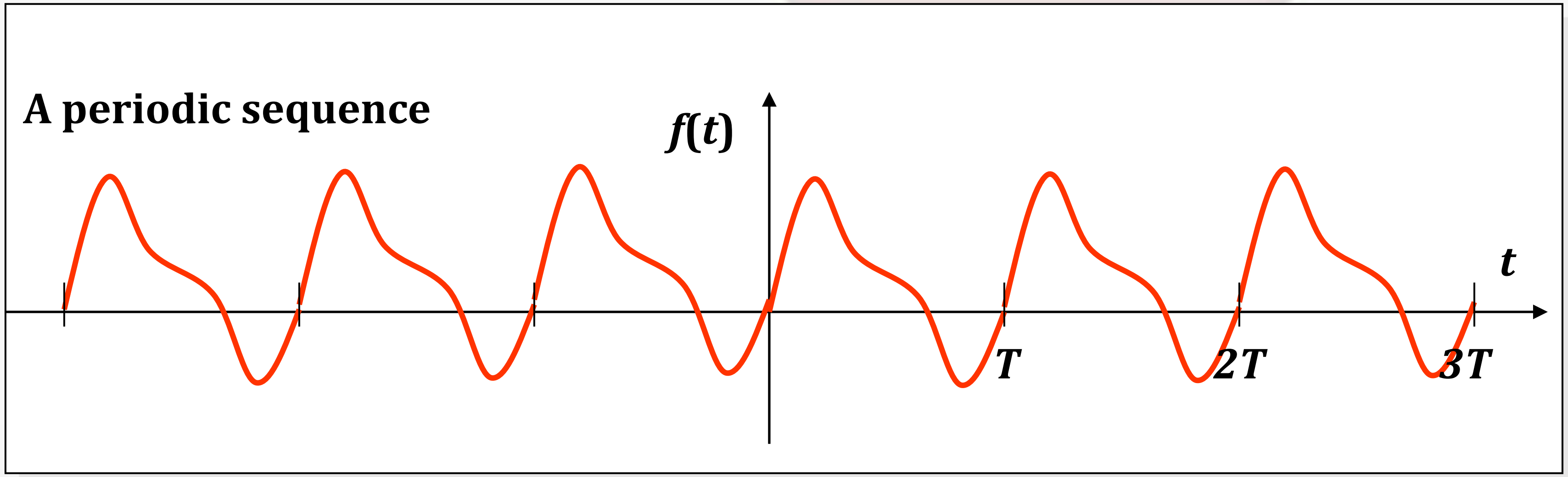
Fourier Series

Introduction

Decompose a periodic input signal into *primitive periodic components*.



Synthesis

$$f(t)=\underbrace{\frac{a_0}{2}}_{\text{DC Part}}+\underbrace{\sum_{n=1}^{\infty}a_n\cos\frac{2\pi nt}{T}}_{\text{Even Part}}+\underbrace{\sum_{n=1}^{\infty}b_n\sin\frac{2\pi nt}{T}}_{\text{Odd Part}}$$

T is a period of all the above signals

Let $\omega_0=2\pi/T$.

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos(n\omega_0 t)+\sum_{n=1}^{\infty}b_n\sin(n\omega_0 t)$$

Definition & Properties

Here we will express a non-sinusoidal periodic function into a fundamental and its harmonics. A series of sines and cosines of an angle and its multiples of the form.

$$\frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \ldots + a_n \cos nx + \ldots$$

$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \ldots + b_n \sin nx + \ldots$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

is called the Fourier series, where $a_0, a_1, a_2, \ldots a_n, \ldots b_1, b_2, b_3 \ldots b_n \ldots$ are constants.

Cont..

A periodic function $f(x)$ can be expanded in a Fourier Series. The series consists of the following:

(*i*) A constant term a_0 (called d.c. component in electrical work).

(*ii*) A component at the fundamental frequency determined by the values of a_1, b_1.

(*iii*) Components of the harmonics (multiples of the fundamental frequency) determined by a_2, ,a_3 ... b_2, b_3 .... And a_0, a_1, a_2 ... ,b_1, b_2 ... are known as *Fourier coefficients* or Fourier constants.

Cont..

DIRICHLET'S CONDITIONS FOR A FOURIER SERIES

If the function $f(x)$ for the interval $(-\pi, \pi)$

(1) is single-valued (2) is bounded

(3) has at most a finite number of maxima and minima.

(4) has only a finite number of discontinuous

(5) is $f(x + 2\pi) = f(x)$ for values of x outside $[-\pi, \pi]$, then

$$S_P(x) = \frac{a_0}{2} + \sum_{n=1}^{P} a_n \cos nx + \sum_{n=1}^{P} b_n \sin nx$$

converges to $f(x)$ as $P \to \infty$ at values of x for which $f(x)$ is continuous and the sum of the series is equal to $\frac{1}{2}[f(x+0) + f(x-0)]$ at points of discontinuity.

Advantages of Fourier Series

1. Discontinuous function can be represented by Fourier series. Although derivatives of the discontinuous functions do not exist. (This is not true for Taylor's series).

2. The Fourier series is useful in expanding the periodic functions since outside the closed interval, there exists a periodic extension of the function.

3. Expansion of an oscillating function by Fourier series gives all modes of oscillation (fundamental and all overtones) which is extremely useful in physics.

4. Fourier series of a discontinuous function is not uniformly convergent at all points.

5. Term by term integration of a convergent Fourier series is always valid, and it may be valid if the series is not convergent. However, term by term, differentiation of a Fourier series is not valid in most cases.

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