TOPICS COVERED

- Black-Body Radation Laws (Classical Approach)
  - The Wien Displacement Law
  - The Stefan-Boltzmann Law
  - The Rayleigh-Jeans Law

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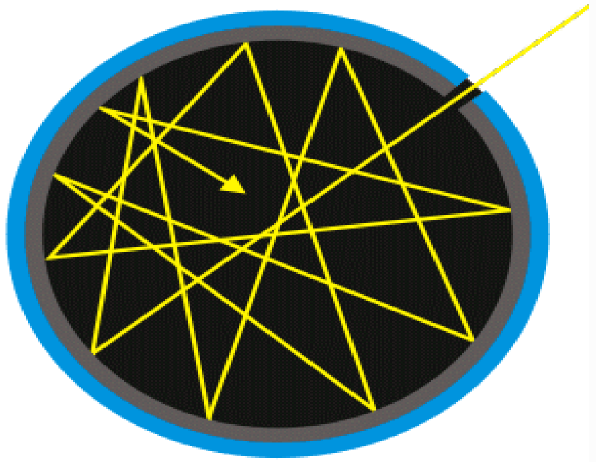
Definition of black body

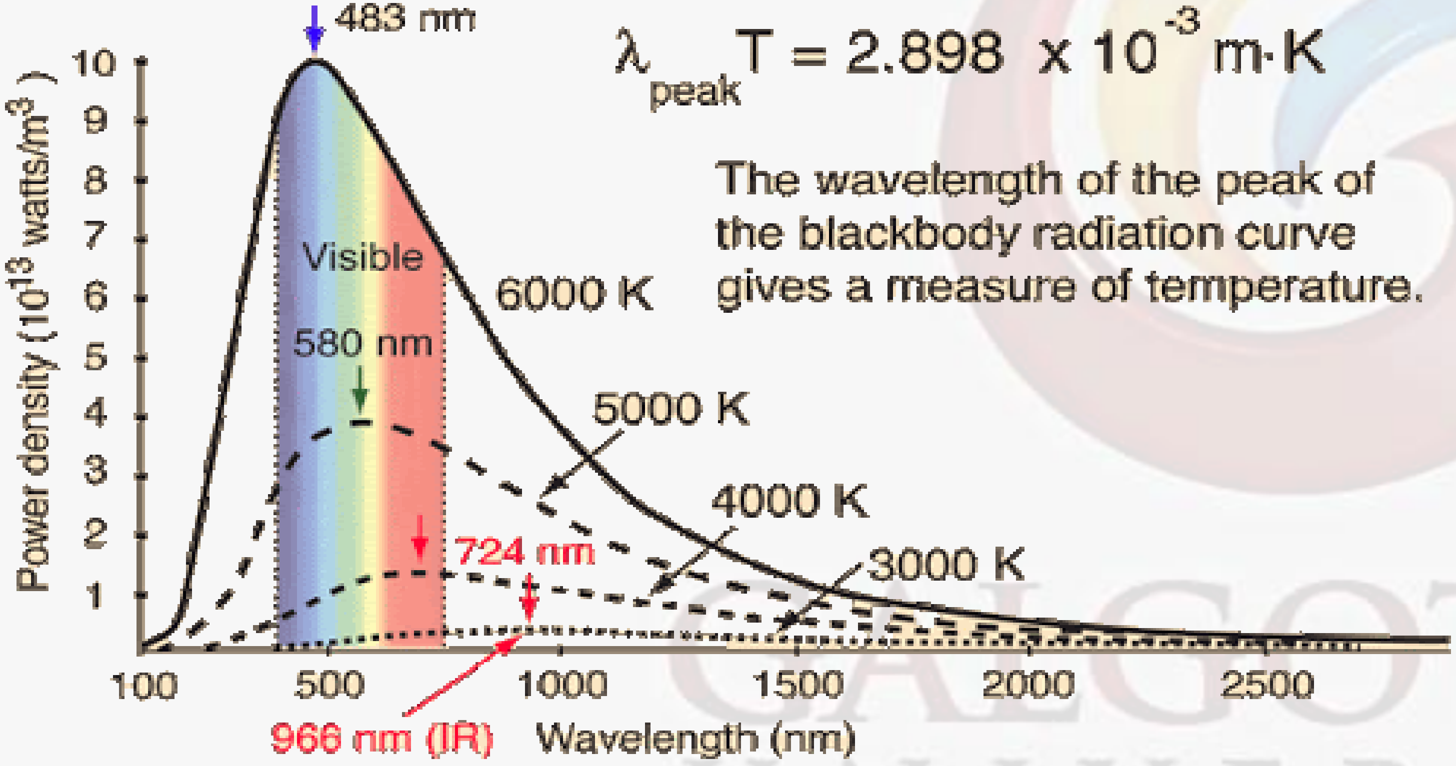
A black body is an ideal body which allows the whole of the incident radiation to pass into itself (without reflecting the energy) and absorbs within itself this whole incident radiation (without passing on the energy). This propety is valid for radiation corresponding to all wavelengths and to all angels of incidence. Therefore, the black body is an ideal absorber of incident radaition.

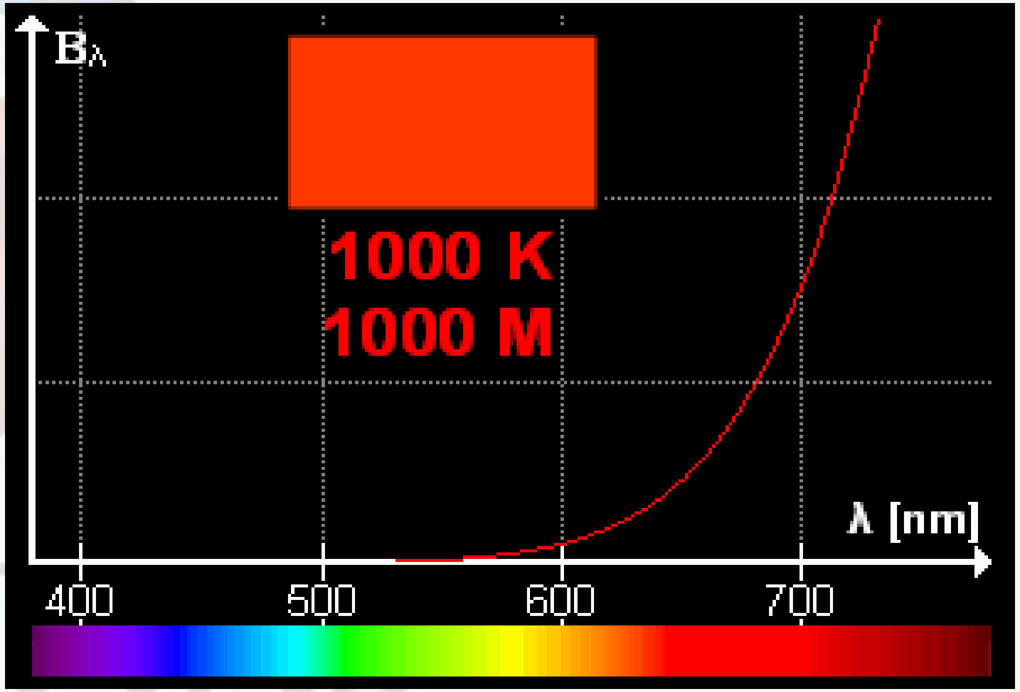


Conceptual Black Body

- Blackbody radiation is theoretically interesting because the radiation properties of the blackbody are independent of the particular material. Physicists can study the properties of intensity versus wavelength at fixed temperatures.

Wien's displacement law





Wien's displacement law: **The maximum of the distribution shifts to smaller wavelengths as the temperature is increased.**

Stefan-Boltzmann law

A number of attempts aimed at explaining the origin of the continuous character of this radiation were carried out. The most serious among such attempts, and which made use of classical physics, were due to Wilhelm Wien in 1889 and Rayleigh in 1900. In 1879 J. Stefan found *experimentally that the total intensity (or the total power per unit surface area) radiated* by a glowing object of temperature *T is given by*

$$\mathcal{P} = a\sigma T^4,$$

which is known as the Stefan–Boltzmann law, where $\sigma = 5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}K^{-4}}$ is the Stefan–Boltzmann constant, and a is a coefficient which is less than or equal to 1; in the case of a blackbody a= 1.

* Gives the total energy being emitted at all wavelengths by the blackbody (which is the area under the Planck Law curve).

* Explains the growth in the height of the curve as the temperature increases. Notice that this growth is very abrupt.

Wien's energy density distribution

Wien's energy density distribution

Using thermodynamic arguments, Wien took the Stefan–Boltzmann law and in 1894 he extended it to obtain the energy density per unit frequency of the emitted blackbody radiation:

$$u(\nu, T) = A\nu^3 e^{-\beta\nu/T},$$

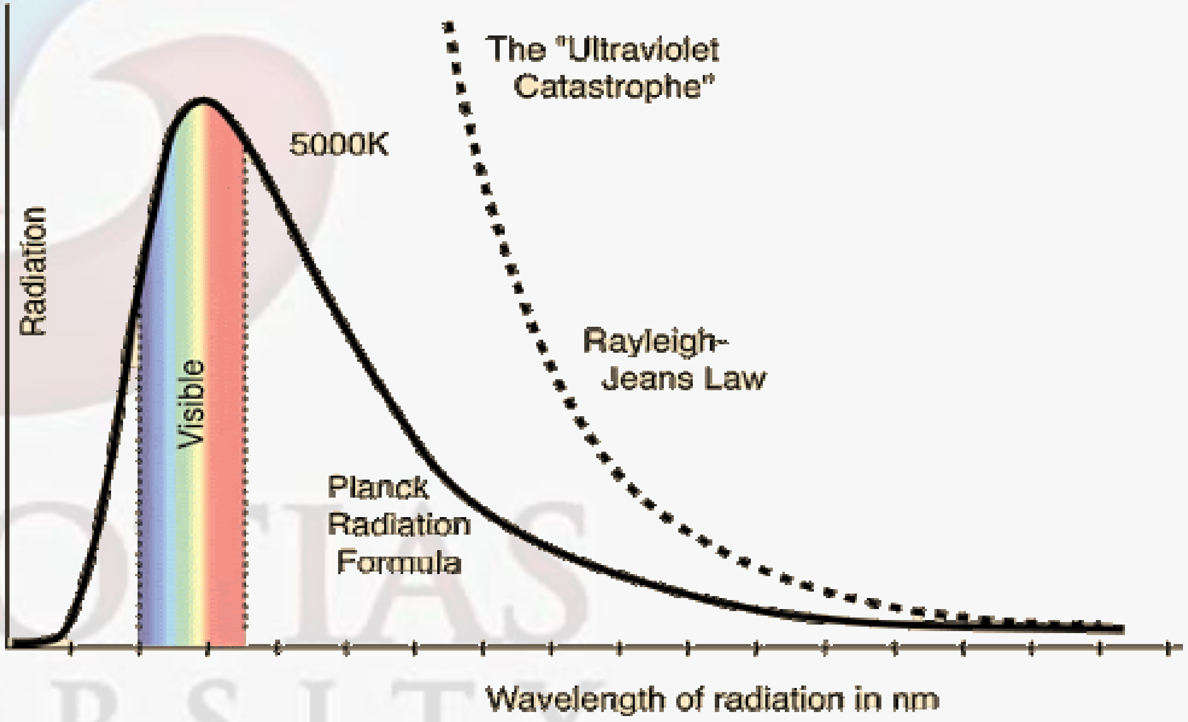
where A and β are empirically defined parameters (they can be adjusted to fit the experimental data). **Note:** $u(\nu, T)$ has the dimensions of an energy per unit volume per unit frequency; its SI units are $\mathrm{J\,m^{-3}\,Hz^{-1}}$. Although Wien's formula fits the high-frequency data remarkably well, it fails badly at low frequencies

Rayleigh-Jeans Law

* It agrees with experimental measurements for long wavelengths.

* It predicts an energy output that diverges towards infinity as wavelengths grow smaller.

* The failure has become known as the ultraviolet catastrophe.



Rayleigh-Jeans Energy density distribution

Rayleigh's energy density distribution

In his 1900 attempt, Rayleigh focused on understanding the nature of the electromagnetic radiation inside the cavity. He considered the radiation to consist of standing waves having a temperature T with nodes at the metallic surfaces. These standing waves, he argued, are equivalent to harmonic oscillators, for they result from the harmonic oscillations of a large number of electrical charges, electrons, that are present in the walls of the cavity. When the cavity is in thermal equilibrium, the electromagnetic energy density inside the cavity is equal to the energy density of the charged particles in the walls of the cavity; the average total energy of the radiation leaving the cavity can be obtained by multiplying the average energy of the oscillators by the number of modes (standing waves) of the radiation in the frequency interval ν to $\nu + d\nu$:

$$N(\nu) = \frac{8\pi\nu^2}{c^3},$$

where $c = 3 \times 10^8$ m s^{-1} is the speed of light; the quantity $(8\pi\nu^2/c^3)d\nu$ gives the number of modes of oscillation per unit volume in the frequency range ν to $\nu + d\nu$. So the electromagnetic energy density in the frequency range ν to $\nu + d\nu$ is given by

$$u(\nu, T) = N(\nu)\langle E\rangle = \frac{8\pi\nu^2}{c^3}\langle E\rangle,$$

where $\langle E\rangle$ is the average energy of the oscillators present on the walls of the cavity (or of the electromagnetic radiation in that frequency interval); the temperature dependence of $u(\nu, T)$ is buried in $\langle E\rangle$.

How does one calculate $\langle E\rangle$? According to the equipartition theorem of classical thermodynamics, all oscillators in the cavity have the same mean energy, irrespective of their frequencies[3]:

$$\langle E\rangle = \frac{\int_0^\infty E e^{-E/kT} dE}{\int_0^\infty e^{-E/kT} dE} = kT,$$

where $k = 1.3807 \times 10^{-23}$ J K^{-1} is the Boltzmann constant. An insertion of into leads to the Rayleigh–Jeans formula:

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3}kT.$$

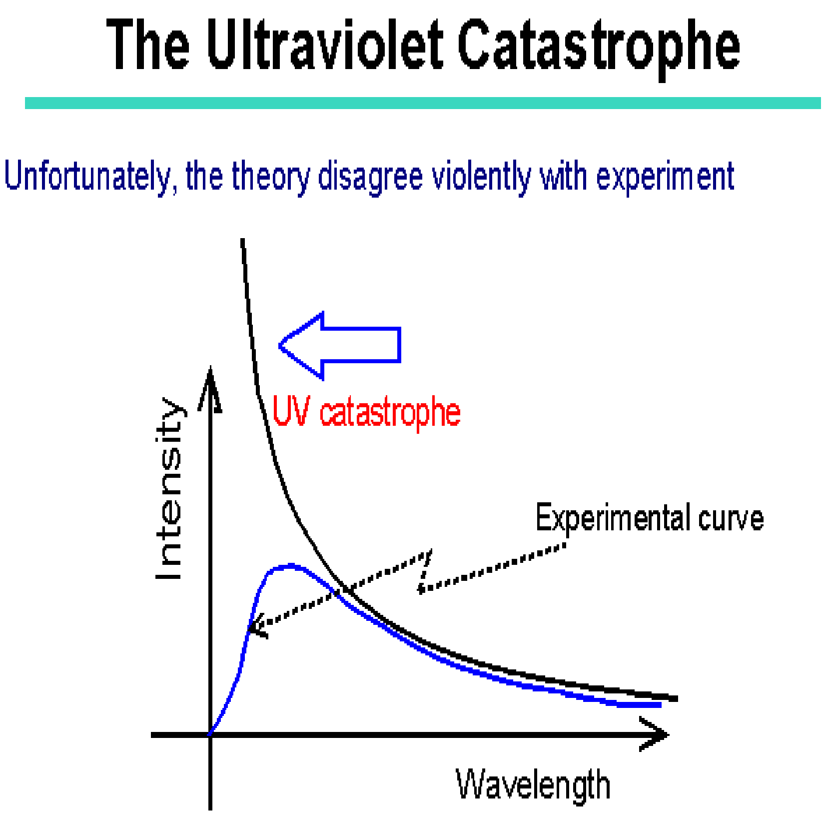
Except for low frequencies, this law is in complete disagreement with experimental data: $u(\nu, T)$ as given by *diverges* for high values of ν, whereas experimentally it must be finite . Moreover, if we integrate over all frequencies, the integral *diverges*. This implies that the cavity contains an *infinite* amount of energy. This result is absurd. Historically, this was called the *ultraviolet catastrophe*, diverges for *high* frequencies (i.e., in the *ultraviolet* range)—a real catastrophical failure of classical physics indeed! The origin of this failure can be traced to the derivation of the average energy . It was founded on an erroneous premise: the energy exchange between radiation and matter is *continuous*; any amount of energy can be exchanged.

Ultraviolet Catastrophe

$$I(\lambda,T)=\frac{2\pi ckT}{\lambda^4}$$

This formula also had a problem. The problem was the term in the denominator.

For large wavelengths it fitted the experimental data but it had major problems at shorter wavelengths.



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