The Planck's Law

Plank Radiation Formula

Planck's energy density distribution

By devising an ingenious scheme—interpolation between Wien's rule and the Rayleigh–Jeans rule—Planck succeeded in 1900 in avoiding the ultraviolet catastrophe and proposed an accurate description of blackbody radiation. In sharp contrast to Rayleigh's assumption that a standing wave can exchange *any* amount (continuum) of energy with matter, Planck considered that the energy exchange between radiation and matter must be *discrete*. He then *postulated* that the energy of the radiation (of frequency ν) emitted by the oscillating charges (from the walls of the cavity) must come *only* in *integer multiples* of $h\nu$:

$$E = nh\nu, \qquad n = 0,\ 1,\ 2,\ 3,\ \cdots, \tag{1.7}$$

where h is a universal constant and $h\nu$ is the energy of a *"quantum"* of radiation (ν represents the frequency of the oscillating charge in the cavity's walls as well as the frequency of the radiation emitted from the walls, because the frequency of the radiation emitted by an oscillating charged particle is equal to the frequency of oscillation of the particle itself). That is, the energy of an oscillator of natural frequency ν (which corresponds to the energy of a charge oscillating with a frequency ν) must be an *integral multiple* of $h\nu$; note that $h\nu$ is not the same for all oscillators, because it depends on the frequency of each oscillator. Classical mechanics, however, puts no restrictions whatsoever on the frequency, and hence on the energy, an oscillator can have. The energy of oscillators, such as pendulums, mass–spring systems, and electric oscillators, varies continuously in terms of the frequency. Equation (1.7) is known as *Planck's quantization rule* for energy or *Planck's postulate*.

So, assuming that the energy of an oscillator is quantized, Planck showed that the *correct* thermodynamic relation for the average energy can be obtained by merely replacing the integration —that corresponds to an energy continuum—by a *discrete* summation corresponding to the discreteness of the oscillators' energies[4]:

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} nh\nu e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-nh\nu/kT}} = \frac{h\nu}{e^{h\nu/kT} - 1}, \tag{1.8}$$

and hence, by inserting (1.8) into (1.4), the energy density per unit frequency of the radiation emitted from the hole of a cavity is given by

$$\boxed{u(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}.} \tag{1.9}$$

This is known as *Planck's distribution*. It gives an exact fit to the various experimental radiation distributions, The numerical value of h obtained by fitting (1.9) with the experimental data is $h = 6.626 \times 10^{-34}$ J s.

At high frequencies, $h\nu \gg kT$ and $e^{h\nu/kT} \to \infty$, which means that $u(\nu)\, d\nu \to 0$ as observed. No more ultraviolet catastrophe. At low frequencies, where the Rayleigh-Jeans formula is a good approximation to the data , $h\nu \ll kT$ and $h\nu/kT \ll 1$. In general,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

If x is small, $e^x \approx 1 + x$, and so for $h\nu/kT \ll 1$ we have

$$\frac{1}{e^{h\nu/kT} - 1} \approx \frac{1}{1 + \frac{h\nu}{kT} - 1} \approx \frac{kT}{h\nu} \qquad h\nu \ll kT$$

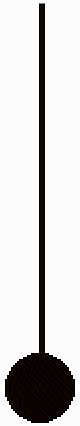
Thus at low frequencies Planck's formula becomes

$$u(\nu)\, d\nu \approx \frac{8\pi h}{c^3}\nu^3\left(\frac{kT}{h\nu}\right) d\nu \approx \frac{8\pi kT}{c^3}\nu^2\, d\nu$$

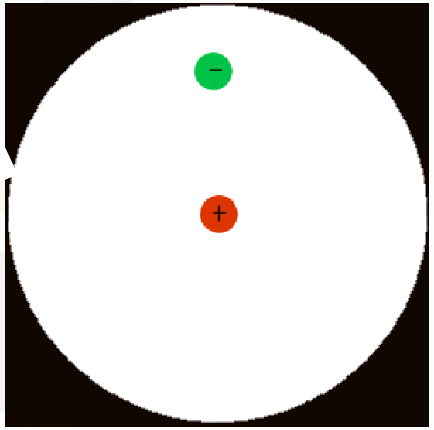
which is the Rayleigh-Jeans formula. Planck's formula is clearly at least on the right track; in fact, it has turned out to be completely correct.

Planck's quantum is small for "ordinary-sized" objects but large for atoms etc

"ordinary" pendulum
$f = 1$ Hz



Hydrogen atom
$f \approx 2\text{x}10^{14}$ Hz



about the same as the electron's KE

$E_{quant} = hf$ $=6.6\text{x}10^{-34}\text{Jsx}1\text{Hz}$

$=6.6\text{x}10^{-34}\text{J}$ ← very tiny

$E_{quant} = hf$

$=(6.6\text{x}10^{-34}\text{Js})\text{x}(2\text{x}10^{14}\text{Hz})$

$=(6.6 \text{ x } 2) \text{ x } 10^{-34+14}\text{J}$

$=1.3 \text{ x } 10^{-19}\text{J}$

Typical energies in “ordinary” life

Typical energy of a tot on a swing:

$E_{tot} = mgh_{max}$

$= 20kg \times 10 m/s^2 \times 1m$

$= 200\ kgm^2/s^2$

$= 200\ J$

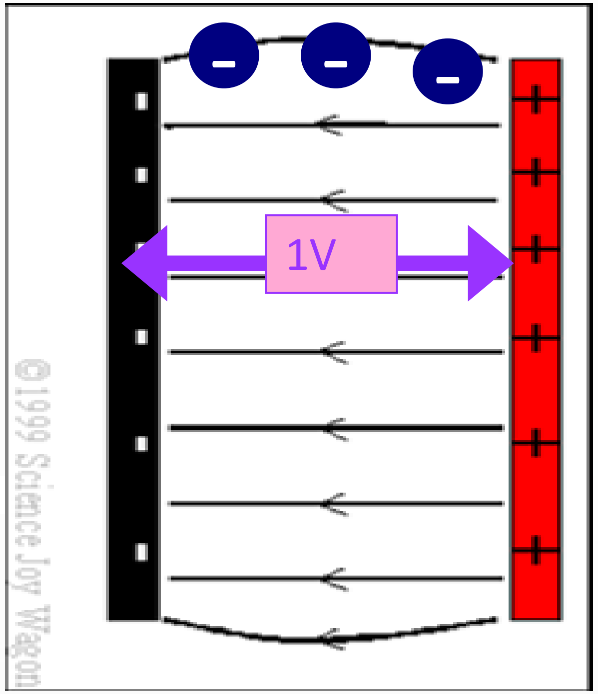
much, much larger than $E_{quant} = 6.6 \times 10^{-34} J$

h_{max}

Typical electron KE in an atom

1 “electron Volt”

Energy gained by an electron crossing a 1V voltage difference



Energy = q V

$$1eV = 1.6x10^{-19} C x 1V$$

$$= 1.6x10^{-19} \text{ Joules}$$

similar

$E_{quant} = 1.3 \times 10^{-19} J$ for $f \approx 2x10^{14}$ Hz

Key Points of Planck's Radiation Law

Planck assumed that the radiation in the cavity was emitted (and absorbed) by some sort of "oscillators" that were contained in the walls. He used Boltzman's statistical methods to arrive at the following formula that fit the blackbody radiation data.

$$\mathcal{I}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Planck's radiation law

Planck made two modifications to the classical theory:

1) The oscillators (of electromagnetic origin) can only have certain discrete energies determined by $E_n = nhf$, where n is an integer, f is the frequency, and h is called Planck's constant. $h = 6.6261 \times 10^{-34}$ J·s.
2) The oscillators can absorb or emit energy in discrete multiples of the fundamental quantum of energy given by

$$\Delta E = hf$$

Classical vs ***Quantum*** world

In everyday life, quantum effects can be safely ignored

This is because Planck's constant is so small

At atomic & subatomic scales, quantum effects are dominant & must be considered

Laws of nature developed without consideration of quantum effects do not work for atoms

Application for Black Body

- The area of Earth's disk as viewed from space is, Area = πr^2.
- The total energy incident on Earth is, Incident energy = $(\pi r^2)S_o$.
- The energy absorbed by the Earth/atmosphere system, as viewed from space is

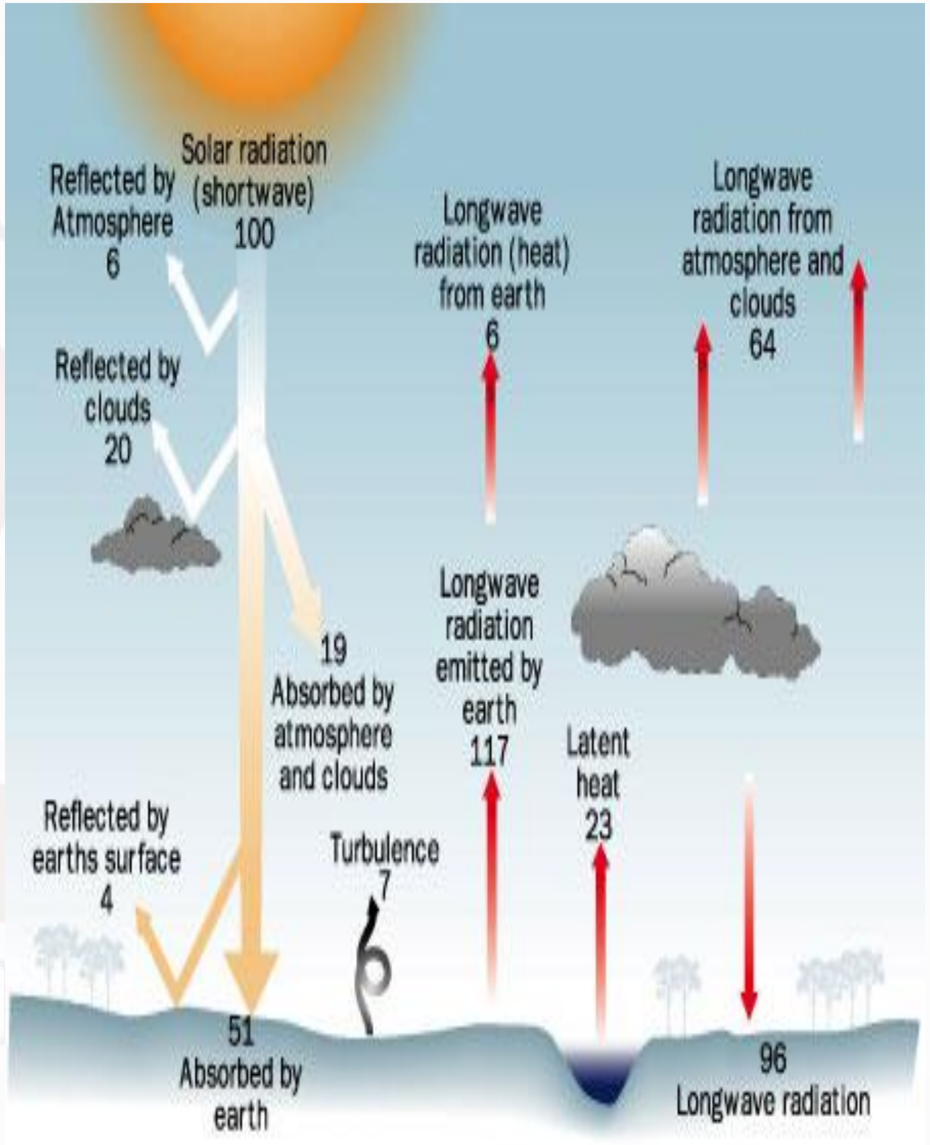
Absorbed energy = $(\pi r^2)S_o(1 - A)$. As we know that bodies must be in radiative equilibrium. The solar energy striking Earth's disk as viewed from space is re-emitted as thermal radiation by the surface of the entire globe, as described by the Stefan-Boltzmann Law, Emitted energy = $(4\pi r^2)\sigma T^4$.

- Set the absorbed energy equal to the emitted energy:

$(\pi r^2)S_o(1 - A) = (4\pi r^2)\sigma T_E^4$, Solving for T yields:

$T_E = [S_o(1 - A)/(4\sigma)]^{(1/4)}$

$= [1370 \cdot (1-0.3)/(4 \cdot 5.67 \times 10^{-8})]^{(1/4)} = 255$ K.

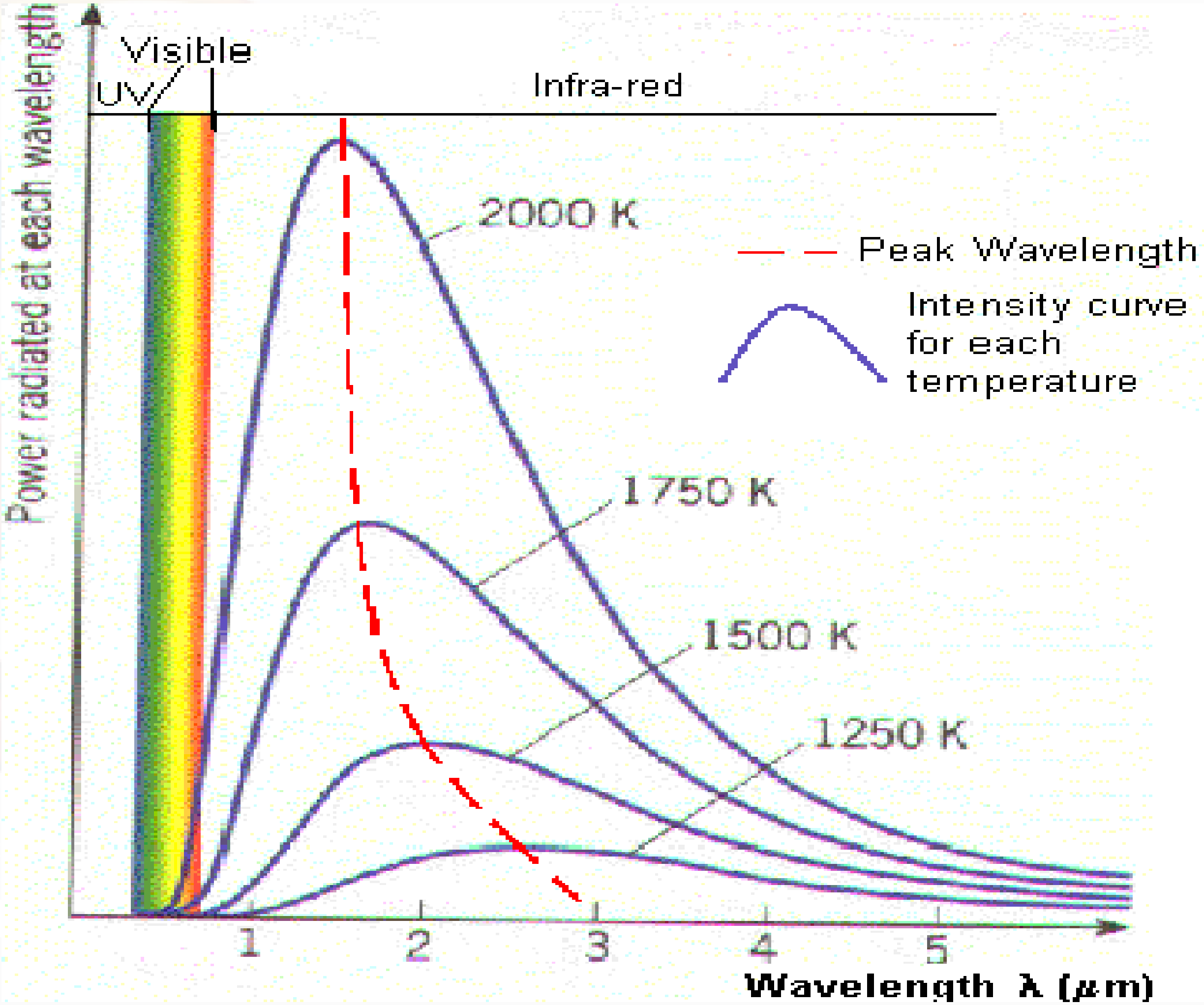


Conclusion

❖As the temperature increases, the peak wavelength emitted by the black body decreases.

❖As temperature increases, the total energy emitted increases, because the total area under the curve increases.

❖The curve gets infinitely close to the x-axis but never touches it.



Summary

- A black body is a theoretical object that absorbs 100% of the radiation that hits it. Therefore it reflects no radiation and appears perfectly black.

- Roughly we can say that the stars radiate like blackbody radiators. This is important because it means that we can use the theory for blackbody radiators to infer things about stars.

- At a particular temperature the black body would emit the maximum amount of energy possible for that temperature.

- Blackbody radiation does not depend on the type of object emitting it. Entire spectrum of blackbody radiation depends on only one parameter, the temperature, T.

REFERENCES

- Modern Physics, J.R. Taylor, C.D. Zafiratos, M.A. Dubson, 2004, PHI Learning.
- Theory and Problems of Modern Physics, Schaum`s outline, R. Gautreau and W. Savin, 2nd Edn, Tata McGraw-Hill Publishing Co. Ltd.
- Quantum Physics, Berkeley Physics, Vol.4. E.H.Wichman, Tata McGraw-Hill Co.
- Basic ideas and concepts in Nuclear Physics, K.Heyde, 3rd Edn., Institute of Physics Pub.