UNIT 4

Two Port Networks

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Two Port Networks

Going From Y to Z Parameters

For the Y parameters we have:

$$I = YV$$

For the Z parameters we have:

$$V = ZI$$

From above;

$$V = Y^{-1}I = ZI$$

Therefore

$$Z = Y^{-1} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{y_{22}}{\Delta_Y} & \frac{-y_{12}}{\Delta_Y} \\ \frac{-y_{21}}{\Delta_Y} & \frac{y_{11}}{\Delta_Y} \end{bmatrix}$$

where

$$\Delta_Y = \det|Y|$$

Two Port Parameter Conversions:

$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_Y} & \frac{-y_{12}}{\Delta_Y} \\ \frac{-y_{21}}{\Delta_Y} & \frac{y_{11}}{\Delta_Y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_H}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{22}}{\Delta_Z} & \frac{-z_{12}}{\Delta_Z} \\ \frac{-z_{21}}{\Delta_Z} & \frac{z_{11}}{\Delta_Z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_H}{h_{11}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_Z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_Y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_H}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
$\begin{bmatrix} \frac{\Delta_Z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_Y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

Two Port Parameter Conversions:

To go from one set of parameters to another, locate the set of parameters you are in, move along the vertical until you are in the row that contains the parameters you want to convert to – then compare element for element

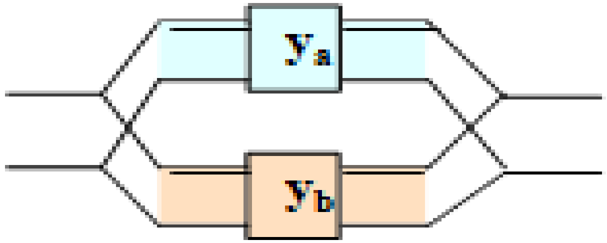
$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_Y} & \frac{-y_{12}}{\Delta_Y} \\ \frac{-y_{21}}{\Delta_Y} & \frac{y_{11}}{\Delta_Y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_H}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{22}}{\Delta_Z} & \frac{-z_{12}}{\Delta_Z} \\ \frac{-z_{21}}{\Delta_Z} & \frac{z_{11}}{\Delta_Z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_H}{h_{11}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_Z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_Y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_H}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
$\begin{bmatrix} \frac{\Delta_Z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_Y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

$$z_{11} = \frac{\Delta_H}{h_{22}}$$

Interconnection Of Two Port Networks

Three ways that two ports are interconnected:

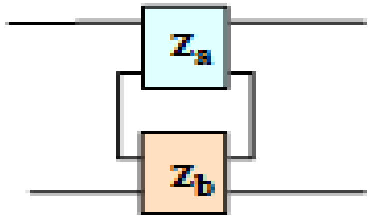
* Parallel



Y parameters

$$[y]=[y_a]+[y_b]$$

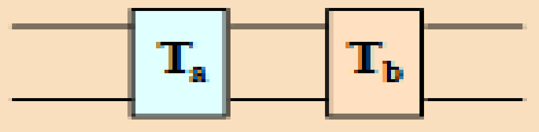
* Series



Z parameters

$$[z]=[z_a]+[z_b]$$

* Cascade



ABCD parameters

$$[T]=[T_a][T_b]$$

CONDITION FOR SYMMETRY AND RECIPROCITY OF Z PARAMETERS

Symmetry : For the network to be symmetrical, the voltage-to-current ratio at one port should be the same as the voltage-to-current ratio at the other port with one of the port short circuited.

Condition for symmetry

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

When the output port is open circuited, i.e., $I_2 = 0$

From the Z-parameter equation,

$$V_s = Z_{11} I_1$$
$$\frac{V_s}{I_1} = Z_{11}$$

When the input port is open circuited, i.e., $I_1 = 0$

From the Z-parameter equation,

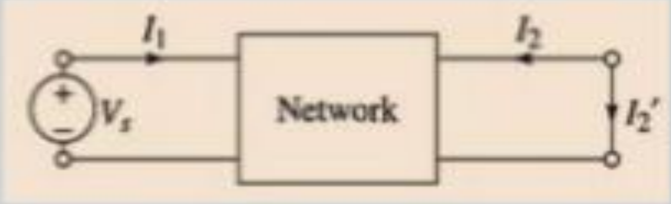
$$V_s = Z_{22} I_2$$
$$\frac{V_s}{I_2} = Z_{22}$$

Hence, for the network to be symmetrical,

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

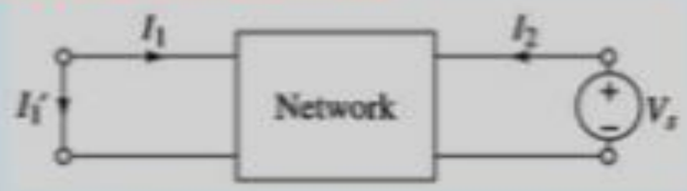
Reciprocity : A network is said to be reciprocal if the the ratio of excitation at one port to response at other port is same if excitation and responses are interchanged

Condition for Reciprocity



$V_1 = V_s$
$V_2 = 0$
$I_2 = -I_2'$

$$V_s = Z_{11} I_1 - Z_{12} I_2'$$
$$0 = Z_{21} I_1 - Z_{22} I_2'$$
$$I_1 = \frac{Z_{22}}{Z_{21}} I_2'$$
$$V_s = Z_{11} \frac{Z_{22}}{Z_{21}} I_2' - Z_{12} I_2'$$
$$\frac{V_s}{I_2'} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$



$V_2 = V_s$
$V_1 = 0$
$I_1 = -I_1'$

$$0 = -Z_{11} I_1' + Z_{12} I_2$$
$$V_s = -Z_{21} I_1' + Z_{22} I_2$$
$$I_2 = \frac{Z_{11}}{Z_{12}} I_1'$$
$$V_s = -Z_{21} I_1' + Z_{22} \frac{Z_{11}}{Z_{12}} I_1'$$
$$\frac{V_s}{I_1'} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{12}}$$

Hence, for the network to be reciprocal,

$$\frac{V_s}{I_1'} = \frac{V_s}{I_2'}$$
$$Z_{12} = Z_{21}$$

Parameter	Condition for Reciprocity	Condition for Symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
T	$AD - BC = 1$	$A = D$
T'	$A'D' - B'C' = 1$	$A' = D'$
h	$h_{12} = -h_{21}$	$h_{11}h_{22} - h_{12}h_{21} = 1$
g	$g_{12} = -g_{21}$	$g_{11}g_{22} - g_{12}g_{21} = 1$

Notes:

(1) To find *Z*-parameters of a 2-port network, apply KVL to the network.

(2) To find *Y*-parameters, apply KCL to the network.

(3) To find *h*-parameters and *ABCD* parameters, apply KVL or KCL to the given network. Convert the equations into the standard form of *Z*-parameters or *Y*-parameters respectively and then rearrange the equations to get the standard form of *h*-parameters and *ABCD*-parameter equations.

Z-Parameters in terms of H-Parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

Rewriting the second equation,

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$$

Comparing with

$$V_2 = Z_{21} I_1 + Z_{22} I_2,$$

$$Z_{21} = -\frac{h_{21}}{h_{22}}$$

$$Z_{22} = \frac{1}{h_{22}}$$

Also, $V_1 = h_{11} I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right] = h_{11} I_1 + \frac{h_{12}}{h_{22}} I_2 - \frac{h_{12}h_{21}}{h_{22}} I_1 = \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$

Comparing with $V_1 = Z_{11} I_1 + Z_{12} I_2,$

$$Z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

$$Z_{12} = \frac{h_{12}}{h_{22}}$$