UNIT 1

Graph Theory

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Properties of Tree:

i) It contains all the nodes of the graph.

ii) It contains (n_t-1) branches. Where 'n_t' is total number of nodes in the given graph.

iii) There are no closed paths.

Total number of tree branches, $n = (n_t-1)$

Where n_t = Total number of nodes

Total number of links, $l = (b-n)$

Where b = Total number of branches in the graph.

Degree of Node:

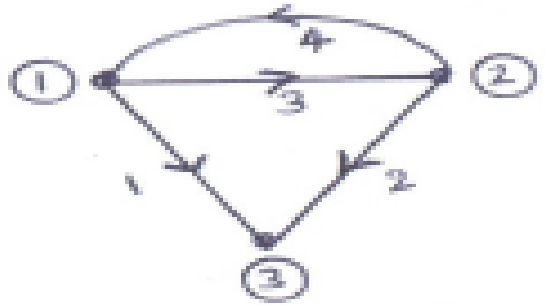
The number of branches attached to the node is degree of node.

II. Complete Incidence Matrix (A_a):

Incidence matrix gives us the information about the branches, which are joined to the nodes and the orientation of the branch, which may be towards a node or away from it.

Nodes of the graph form the rows and branches form the columns. If the branch is not connected to node, corresponding element in the matrix is given the value '0'. If a branch is joined, it has two possible orientations. If the orientation is away from the node, the corresponding matrix element is written as '+1'. If it is towards the node, the corresponding matrix element is written as '-1'.

Example: 1) Obtain complete incidence matrix for the graph shown



Solution:

$A_a =$

Nodes	Branches			
	1	2	3	4
1	1	0	1	-1
2	0	1	-1	1
3	-1	-1	0	0

$$A_a = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ -1 & -1 & 0 & 0 \end{bmatrix}$$

Properties of Incidence Matrix:

i) Each column has only two non-zero elements and all other elements are zero.

ii) If all the rows of 'A_a' are added, the sum will be a row whose elements equal zero.

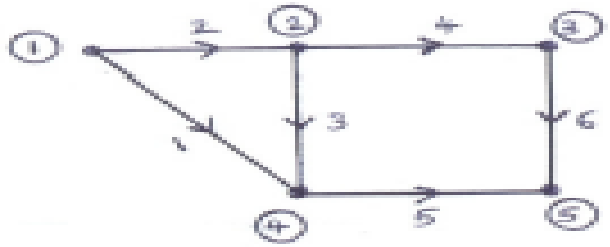
If the graph has '**b**' branches and 'n_t' nodes, the complete incidence matrix is of the order (n_t x b).

III. Reduced Incidence Matrix (A):

When one row is eliminated from the complete incidence matrix, the remaining matrix is called **reduced incidence matrix**

If the graph has '**b**' branches and 'n_t' nodes, the reduced incidence matrix is of the order (n_t-1) x b.

Example: 2) Write the complete and reduced incidence matrix for the given graph shown



Solution:

$A_a =$

Nodes	Branches					
	1	2	3	4	5	6
1	1	1	0	0	0	0
2	0	-1	1	1	0	0
3	0	0	0	-1	0	1
4	-1	0	-1	0	1	0
5	0	0	0	0	-1	-1

Complete Incidence Matrix, $A_a = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$

Reduced Incidence Matrix, $A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 & 1 & 0 \end{bmatrix}$

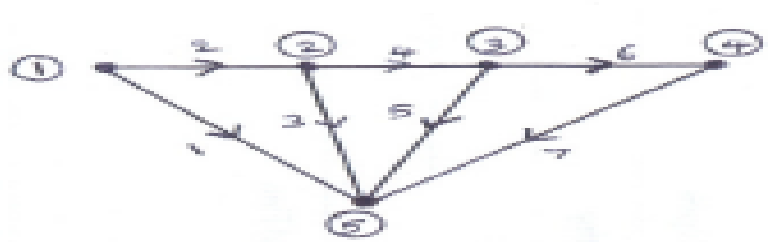
Example: 3) Draw the oriented graph of incidence matrix shown below

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Solution: The given matrix is a reduced incidence matrix. Obtain the complete incidence matrix in order to draw the oriented graph.

$$A_a = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & -1 & 0 & -1 & 0 & -1 \end{bmatrix}$$

Total number of nodes $= n_t = 5$
Total number of branches $= b = 7$



Oriented Graph

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5. K. V. V. Murthy and M. S. Kamath, “Basic Circuit Analysis”, Jaico Publishers, 1999.Syllabus

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