UNIT 5

Network Synthesis and Filters

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1. Positive Real Function

Positive-real functions, often abbreviated to **PR function** or **PRF**, are a kind of mathematical function that first arose in electrical network synthesis. They are complex functions, *Z*(*s*), of a complex variable, *s*. A rational function is defined to have the PR property if it has a positive real part and is analytic in the right halfplane of the complex plane and takes on real values on the real axis.

In symbols the definition is,

$$\Re[Z(s)] > 0 \quad \text{if} \quad \Re(s) > 0$$
$$\Im[Z(s)] = 0 \quad \text{if} \quad \Im(s) = 0$$

In electrical network analysis, *Z*(*s*) represents an impedance expression and *s* is the complex frequency variable, often expressed as its real and imaginary parts; $s = \sigma + i\omega$

in which terms the PR condition can be stated;

$$\Re[Z(s)] > 0 \quad \text{if} \quad \sigma > 0$$
$$\Im[Z(s)] = 0 \quad \text{if} \quad \omega = 0$$

The term *positive-real function* was originally defined by Otto Brune to describe any function *Z*(*s*) which

a) is rational (the quotient of two polynomials),

b) is real when *s* is real

c) has positive real part when *s* has a positive real part

Properties of Positive Real Function

1. The sum of two PR functions is PR.
2. The composition of two PR functions is PR. In particular, if $Z(s)$ is PR, then so are $1/Z(s)$ and $Z(1/s)$.
3. All the zeros and poles of a PR function are in the left half plane or on its boundary of the imaginary axis.
4. Any poles and zeroes on the imaginary axis are simple (have a multiplicity of one).
5. Any poles on the imaginary axis have real strictly positive residues, and similarly at any zeroes on the imaginary axis, the function has a real strictly positive derivative.
6. Over the right half plane, the minimum value of the real part of a PR function occurs on the imaginary axis (because the real part of an analytic function constitutes a harmonic function over the plane, and therefore satisfies the maximum principle).
7. For a rational PR function, the number of poles and number of zeroes differ by at most one.

Network Synthesis

Network synthesis is a design technique for linear electrical circuits. Synthesis starts from a prescribed impedance function of frequency or frequency response and then determines the possible networks that will produce the required response. The technique is to be compared to network analysis in which the response (or other behaviour) of a given circuit is calculated. Network synthesis was a great leap forward in circuit design. Prior to network synthesis, only network analysis was available, but this requires that one already knows what form of circuit is to be analysed. There is no guarantee that the chosen circuit will be the closest possible match to the desired response, nor that the circuit is the simplest possible. Network synthesis directly addresses both these issues. Network synthesis has historically been concerned with synthesising passive networks, but is not limited to such circuits.

Properties of RL Driving Point Impedance Functions:

1.The poles and zeros are simple. There are no multiple poles and zeros.
2.The poles and zeros are located on negative real axis.
3.The poles and zeros interlace each other on the negative real axis.
4.The poles and zeros are the critical frequencies. The critical frequency nearest to the origin is always a zero, which may be located at the origin.
5.The critical frequency at a greatest distance away from the origin is always a pole, which may be located at infinity also.
6.Partial fraction expansion of $Z_{RL}(s)$ gives the residues which are negative and real hence to obtain positive residues the expansion of $Z_{RL}(s)/s$ is obtained.
7.There can not be a pole at the origin.
8.The slope of the graph of z (σ) against σ is always positive.
9. The value of $Z_{RL}(s)$ at s = 0 is always less than the value of $Z_{RL}(s)$at s = ∞.

$$Z_{RL}(0) < Z_{RL}(\infty)$$

Properties of LC Immittance Functions:

The properties of $Z_{LC}(s)$ and $Y_{LC}(s)$ functions can be stated as,

The LC Immittance Function is always a ratio of odd to even or even to odd polynomials.

The poles and zeros are simple. There are no multiple poles or zeros either at origin or infinity or at any point.

The poles and zeros are located on the jω axis only.

The poles and zeros interlace (alternate) each other on the jω axis. There are no consecutive poles or zeros on the jω axis.

The imaginary poles and zeros occur in the form of complex conjugate pairs.

The highest powers of numerator and denominator must differ by unity.

The lowest powers of numerator and denominator must differ by unity.

There must be either a pole or zero at the origin and infinity. As the function is the ratio of even to odd polynomials, if the highest power of numerator is 2m then of denominator is 2m —1 which gives pole at ∞ or it can be 2m +1 which gives zero at ∞. And as lowest powers also differ by unity, there is pole or zero at the origin.

Residues at the imaginary axis poles are real and positive.

The number of elements required in any of the four forms of realization is equal to the highest power of s in the LC Immittance Function as a whole.

The slope of the graph of reactance against frequency is always positive.

Properties of RC Immittance Functions:

The various properties of RC Driving Point Impedance function can be stated as,

The poles and zeros are simple. There are no multiple poles and zeros.

The poles and zeros are located on negative real axis.

The poles and zeros interlace (alternate) each other on the negative real axis.

We know that the poles and zeros are called critical frequencies of the The critical frequency nearest to the origin is always a pole. This may be located at the origin.

The critical frequency at a greatest distance away from the origin is always a zero, which may be located at ∞ also.

The partial fraction expansion of $Z_{RC}(s)$ gives the residues which are always real and positive.

There is no pole located at infinity.

The slope of the graph of Z(σ) against σ is always negative.

There is no zero at the origin.

The value of $Z_{RC}(s)$ at s=0 is always greater than the value of $Z_{RC}(s)$ at s = ∞.

$$Z_{RC}(0) > Z_{RC}(\infty)$$

FOSTER REACTANCE THEOREM

Network synthesis involves the methods used to determine an electric circuit that satisfy certain specifications. Given an impulse response there are many techniques that can be used to synthesize a circuit with the specified response. Different methods may also be used to synthesize circuits, all of which may be optimal. Hence the solution to a network synthesis problem is never unique.

Many applications today use digital processing in lieu of analog processing and the GHz spectrum is finding increasing use in applications such as wireless communications. However, operation at high frequencies requires analog filtering and processing circuits simply because using digital techniques is neither realistic nor economical. Another advantage that analog devices have over their digital counterparts is their ability to operate with wide instantaneous bandwidths and moderately high dynamic ranges at microwave frequencies.

For a **Foster** 1 realisation the component values are given by the partial fraction expansion

$$Z(s) = K_{inf}s + \frac{K_0}{s} + \frac{K_1 s}{s^2 + w_1^2} + \frac{K_2 s}{s^2 + w_2^2} + \cdots + \frac{K_n s}{s^2 + w_n^2}$$

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