School of Computing Science and Engineering

Course Code : MATH2007 **Course Name: Discrete Mathematics**

Logic and logical Operators

By

Dr. Varsha Gautam

Galgotias University, Greater Noida(U.P.)

Name of the Faculty: Dr. Varsha Gautam **Program Name: B.Tech**

In this presentation we will discuss following things:

- Logic and Proposition
- Logical Operators
- Truth table

Mathematical Logic is a tool for working with complicated *compound* statements. It includes:

- A language for expressing them.
- A concise notation for writing them.
- A methodology for objectively reasoning about their truth or falsity.
- It is the foundation for expressing formal proofs in all branches of mathematics.

Proposition

A **proposition** is a declarative sentence (a sentence that declares a fact) that is either true or false, but not both.
Are the following sentences propositions?

- Toronto is the capital of Canada. (Yes)
- Read this carefully. (No)1+2=3(Yes)
- x+1=2(No)
- What time is it? (No)

Propositional Logic

Propositional Logic – the area of logic that deals with propositions.

Propositional Variables – variables that represent propositions: *p*, *q*, *r*, *s*

E.g. Proposition *p* – "Today is Friday."

Truth values – T, F

Operator/Connectives

An *operator* or *connective* combines one or more *operand* expressions into a larger expression. (*E.g.*, “+” in numeric exprs.)

Unary operators take 1 operand (*e.g.,* −3); *binary* operators take 2 operands (*eg* 3×4).

Propositional or *Boolean* operators operate on propositions or truth values instead of on numbers.

Negation

The unary *negation operator* "¬" (*NOT*) transforms a prop. into its logical *negation*.

E.g. If p = "I have brown hair."

then $\neg p$ = "I do **not** have brown hair."

Truth table for NOT:

p	$\neg p$
T	F
F	T

T :≡ True; F :≡ False

":≡" means "is defined as"

Conjunction

The binary *conjunction operator* "$\wedge$" (*AND*) combines two propositions to form their logical *conjunction*.

E.g. If *p*="I will have salad for lunch." and *q*="I will have steak for dinner.", then

$p \wedge q$="I will have salad for lunch **and** I will have steak for dinner."

Conjunction truth table

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Note that a conjunction $p_1 \wedge p_2 \wedge \ldots \wedge p_n$ of *n* propositions will have 2^n rows in its truth table.

Also: ¬ and ∧ operations together are sufficient to express *any* Boolean truth table!

Disjunction

The binary *disjunction operator* "$\vee$" (*OR*) combines two propositions to form their logical *disjunction*.

p="My car has a bad engine."

q="My car has a bad carburetor."

$p \vee q$="Either my car has a bad engine, **or** my car has a bad carburetor."

Disjunction truth table

Note that $p \lor q$ means that p is true, or q is true, **or both** are true!

So, this operation is also called *inclusive or,* because it **includes** the possibility that both p and q are true.

"¬" and "∨" together are also universal

p	q	$p \lor q$
F	F	F
F	T	**T**
T	F	**T**
T	T	T

Assignment

1.Which of these sentences are **propositions**? What are the **truth values** of those that are propositions?

a) 2 + 3 = 5. **b)** Answer this question!

2. Construct truth table for: $(p \to q) \leftrightarrow (\neg q \to \neg p)$

Video link: https://www.youtube.com/watch?v=q2eyZZKOIk&list=PLHXZ9OQGMqxersk8fUxiUMSIx0DBqsKZS&index=10

References: Discrete Mathematics and its application by Kenneth H Rosen

Thank you!!!!!