Course Code : MSCM303

Course Name: Integral equations and calculus of variation

Lecture-2

Classification of integral equations of Volterra and Fredholm types

The most general linear integral is of the form

 $g(x)y(x) = f(x) + \lambda \int_{a}^{b} K(x,t)y(t)dt$, ...(1)

where f(x), g(x), K(x,t) are known functions, y(x) is to be determined. K(x,t) is called kernel of integral equations.

Name of the Faculty: Dr. Leena Rani

Course Code : MSCM303

Course Name: Integral equations and calculus of variation

If a=constant, b=x The equation (1) is send to be Volterra Integral Equation

If a=constant, b=constant The equation (1) is send to be Fredholm Integral Equation

(*i*) Fredholm integral equation of the first kind. A linear integral equation of the form (by setting g(x) = 0 in (1))

is known as Fredholm integral equation of the first kind.

 $f(x) + \lambda \int_{x}^{x} K(x,t)y(t)dt = 0$,

Name of the Faculty: Dr. Leena Rani

Course Code : MSCM303

 $y(x) = f(x) + \lambda \int K(x,t)y(t)dt$

Course Name: Integral equations and calculus of variation

(*ii*) Fredholm integral equation of the second kind. A linear integral equation of the form (by setting g(x) = 1 in (1))

is known as *Fredholm integral equation of the second kind*. (*iii*) **Homogeneous Fredholm integral equation of the second kind.** A linear integral equation of the form (by setting f(x) = 0 in (2)).

(2)

$$y(x) = f(x) + \lambda \int_{a}^{x} K(x,t)y(t)dt \qquad \dots (3)$$

Name of the Faculty: Dr. Leena Rani

Course Code : MSCM303

Course Name: Integral equations and calculus of variation

is known as the homogeneous Fredholm integral equation of the second kind. (i) Volterra integral equation of the first kind.

A linear integral equation of the form (by setting g(x) = 0 in (1))

$$f(x) + \lambda \int_{a}^{x} K(x,t) y(t) dt = 0,$$
 ... (2)

is known as *Volterra integral equation of the first kind*. (*ii*) **Volterra integral equation of the second kind**. A linear integral equation of the form (by setting g(x) = 1)

$$y(x) = f(x) + \lambda \int_{a}^{x} K(x,t) y(t) dt,$$
 ... (3)

is known as Volterra integral equation of the second kind.

Name of the Faculty: Dr. Leena Rani

Course Code : MSCM303

Course Name: Integral equations and calculus of variation

(iii) Homogeneous Voterra integral equation of the second kind.

A linear integral equation of the form (by setting f(x) = 0 is (3))

 $y(x) = \lambda \int_{a}^{x} K(x,t) y(t) dt, \qquad \dots (4)$

is known as the homogeneous Volterra integral equation of the second kind.

Reference: https://nptel.ac.in/courses/111/107/111107103/

Name of the Faculty: Dr. Leena Rani