Lecture-2

Classification of integral equations of Volterra and Fredholm types

The most general linear integral is of the form

$$g(x)y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)dt, \qquad \ldots(1)$$

where f(x), g(x), K(x,t) are known functions, y(x) is to be determined. K(x,t) is called kernel of integral equations.

If a=constant , b=x The equation (1) is send to be Volterra Integral Equation

If a=constant , b=constant The equation (1) is send to be Fredholm Integral Equation

(*i*) **Fredholm integral equation of the first kind.**

A linear integral equation of the form (by setting g (x) = 0 in (1))

$$f(x)+\lambda\int_a^x K(x,t)y(t)dt=0 \ ,$$

is known as *Fredholm integral equation of the first kind.*

(*ii*) **Fredholm integral equation of the second kind.**

A linear integral equation of the form (by setting g (*x*) = 1 in (1))

$$y(x)=f(x)+\lambda\int_a^x K(x,t)y(t)dt \qquad (2)$$

is known as *Fredholm integral equation of the second kind.*

(*iii*) **Homogeneous Fredholm integral equation of the second kind.**

A linear integral equation of the form (by setting *f* (*x*) = 0 in (2)).

$$y(x)=f(x)+\lambda\int_a^x K(x,t)y(t)dt \qquad ...(3)$$

is known as the *homogeneous Fredholm integral equation of the second kind.*

(*i*) **Volterra integral equation of the first kind.**

A linear integral equation of the form (by setting g (*x*) = 0 in (1))

$$f(x)+\lambda\int_a^x K(x,t)\,y(t)\,dt=0, \qquad \ldots (2)$$

is known as *Volterra integral equation of the first kind.*

(*ii*) **Volterra integral equation of the second kind.**

A linear integral equation of the form (by setting g (x) = 1)

$$y(x)=f(x)+\lambda\int_a^x K(x,t)\,y(t)\,dt, \qquad \ldots (3)$$

is known as *Volterra integral equation of the second kind.*

(*iii*) **Homogeneous Voterra integral equation of the second kind.**

A linear integral equation of the form (by setting $f(x) = 0$ is (3))

$$y(x) = \lambda \int_a^x K(x,t)\, y(t)\, dt, \qquad \ldots (4)$$

is known as the *homogeneous Volterra integral equation of the second kind.*

Reference:

https://nptel.ac.in/courses/111/107/111107103/