Number System

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Number System

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	No

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POSITIONAL NUMBER SYSTEMS

In a positional number system, the position a symbol occupies in the number determines the value it represents. In this system, a number represented as:

$$\pm (S_{k-1} \ldots S_2\, S_1\, S_0 .\, S_{-1}\, S_{-2} \ldots S_{-l})_b$$

has the value of:

$$n = \pm \; S_{k-1} \times b^{k-1} + \ldots + S_1 \times b^1 + S_0 \times b^0 \;+\; S_{-1} \times b^{-1} + S_{-2} \times b^{-2} + \ldots + S_{-l} \times b^{-l}$$

in which S is the set of symbols, b is the base (or radix).

The decimal system (base 10)

The word decimal is derived from the Latin root decem (ten). In this system the base b = 10 and we use ten symbols

S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

The symbols in this system are often referred to as decimal digits or just digits.

Integers

$$N = \pm \; S_{k-1} \times 10^{k-1} + S_{k-2} \times 10^{k-2} + \ldots + S_2 \times 10^2 + S_1 \times 10^1 + S_0 \times 10^0$$

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	10^{k-1}	10^{k-2}	$\bullet\bullet\bullet$	10^2	10^1	10^0	Place values
$\pm$	S_{k-1}	S_{k-2}	$\bullet\bullet\bullet$	S_2	S_1	S_0	Number

$$N = \pm\, S_{k-1} \times 10^{k-1} + S_{k-2} \times 10^{k-2} + \bullet\bullet\bullet + S_2 \times 10^2 + S_1 \times 10^1 + S_0 \times 10^0 \quad \text{Values}$$

Fig: Place values for an integer in the decimal system

Example

The following shows the place values for the integer +224 in the decimal system.

			10^2		10^1		10^0	Place values
			2		2		4	Number
N	$=$	$+$	2×10^2	$+$	2×10^1	$+$	4×10^0	Values

Note that the digit 2 in position 1 has the value 20, but the same digit in position 2 has the value 200. Also note that we normally drop the plus sign, but it is implicit.

Example

The following shows the place values for the decimal number −7508. We have used 1, 10, 100, and 1000 instead of powers of 10.

			1000		100		10		1	Place values
			7		5		0		8	Number
N	$=$	$-($	7×1000	$+$	5×100	$+$	0×10	$+$	8×1	$)$ Values

Note that the digit 2 in position 1 has the value 20, but the same digit in position 2 has the value 200. Also note that we normally drop the plus sign, but it is implicit.

Reals

Integral part		Fractional part
$R = \pm$ $S_{k-1} \times 10^{k-1} + \ldots + S_1 \times 10^1 + S_0 \times 10^0$	+	$S_{-1} \times 10^{-1} + \ldots + S_{-l} \times 10^{-l}$

Example

The following shows the place values for the real number +24.13.

	10^1	10^0		10^{-1}	10^{-2}	Place values
	2	4	•	1	3	Number
$R = +$	2×10	+ 4×1	+	1×0.1	+ 3×0.01	Values

The binary system (base 2)

The word binary is derived from the Latin root bini (or two by two). In this system the base b = 2 and we use only two symbols,

S = {0, 1}

The symbols in this system are often referred to as binary digits or bits (binary digit).

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Integers

$$N = \pm\ S_{k-1} \times 2^{k-1} + S_{k-2} \times 2^{k-2} + \ldots + S_2 \times 2^2 + S_1 \times 2^1 + S_0 \times 2^0$$

	2^{k-1}	2^{k-2}	$\bullet\bullet\bullet$	2^2	2^1	2^0	Place values
$\pm$	S_{k-1}	S_{k-2}	$\bullet\bullet\bullet$	S_2	S_1	S_0	Number
	↓	↓		↓	↓	↓	

$$N = \pm\ S_{k-1} \times 2^{k-1} + S_{k-2} \times 2^{k-2} + \bullet\bullet\bullet + S_2 \times 2^2 + S_1 \times 2^1 + S_0 \times 2^0 \quad \text{Values}$$

Figure : Place values for an integer in the binary system

Example

The following shows that the number $(11001)_2$ in binary is the same as 25 in decimal. The subscript 2 shows that the base is 2.

		2^4		2^3		2^2		2^1		2^0	Place values
		1		1		0		0		1	Number
N	=	1×2^4	+	1×2^3	+	0×2^2	+	0×2^1	+	1×2^0	Decimal

The equivalent decimal number is N = 16 + 8 + 0 + 0 + 1 = 25.

References:

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