Quantum Mechanics

Covered Topics

- Properties of Matter Waves: Wave Function
- Davisson-Germer Experiment
- Matter Wave
- Expression of wave function
- References

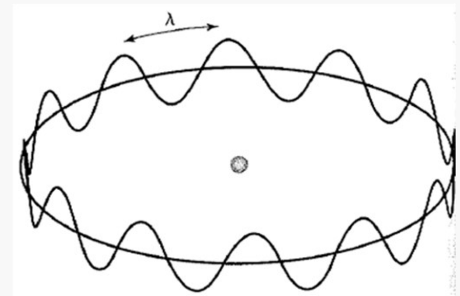
Properties of Matter Waves: Wave Function

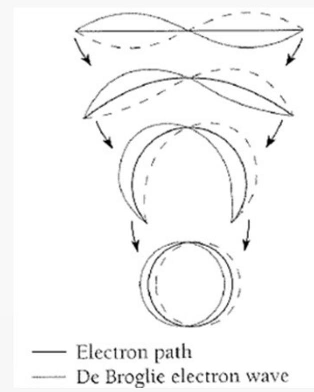
1. Lighter the particle, greater is the wavelength associated with it.
2. Smaller the velocity of the particle, greater is the wavelength associated with it. When v = 0 then $\lambda = \infty$, i. e., wavelength becomes indeterminate and if v = ∞ then $\lambda = 0$. This shows that matter waves are associated with particles in motion.
3. The quantity whose variation makes up matter waves is the wave function (ψ). The value of the wave function associated with a moving body at any point *(x,y,z)* in space at any time *(t)* is related to the probability of finding the body there at that time.
4. Since the magnitude of ψ oscillates between positive and negative values, the wave function ψ has no physical significance as probability of finding the particle at any place at any instant cannot be negative.
5. However is always positive and thus is physically significant.
6. $|\psi|^2$ dxdydx gives the probability of finding the particle at a time t in the volume element dxdydz in space.

$$\int_{all\ space} |\psi|^2 \, dxdydz = 1$$

$$L = n\hbar$$

$$2\pi r = n\lambda$$





De Broglie's Hypothesis:

predicts that one should see diffraction and interference of matter waves

For example we should observe

- Electron diffraction
- Atom or molecule diffraction

Wave nature of electron
big application :
Electron Microscope

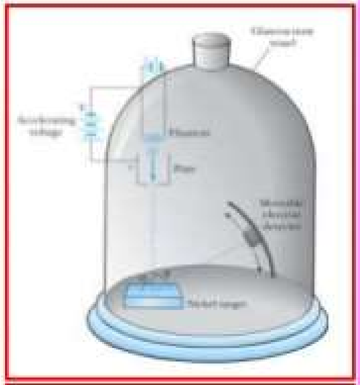
Davisson-Germer Experiment

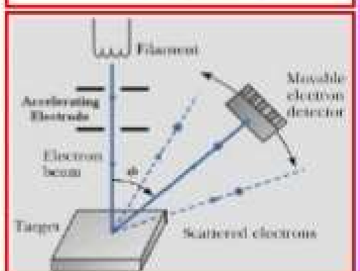
provides experimental confirmation of the matter waves proposed by de Broglie

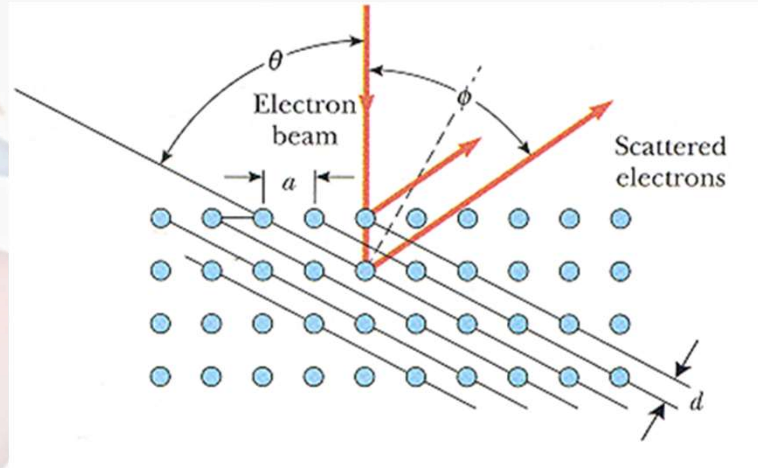
- If particles have a wave nature, then under appropriate conditions, they should exhibit diffraction
- Davisson and Germer measured the wavelength of electrons

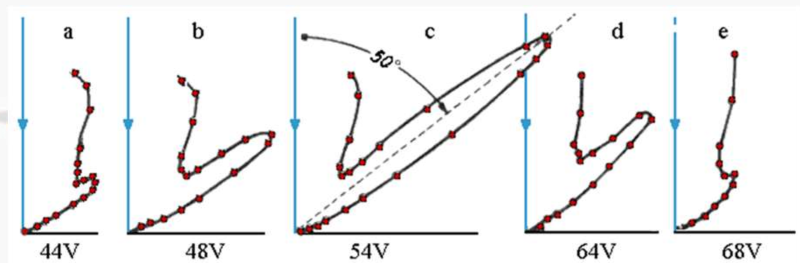
Electrons were directed onto nickel crystals

Accelerating voltage is used to control electron energy: $E = |e|V$









For φ ~ 50° the maximum is at ~54V

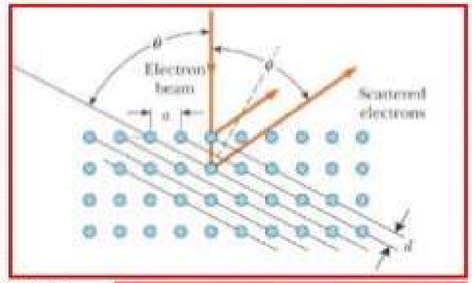
For X-ray Diffraction on Nickel

$$2d\sin\theta = \lambda$$

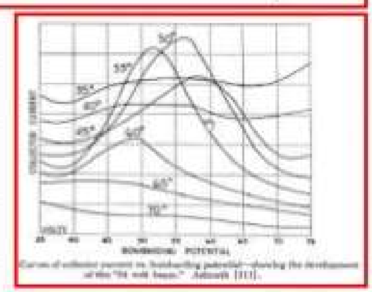
$$d_{<111>} = 0.91\,\mathring{A};\ \lambda_{X\text{-ray}} = 1.65\,\mathring{A}$$

$$\Downarrow$$

$$\theta = 65^{o} \Rightarrow \phi = 50^{o}$$



Ex: Assuming the wave nature of electrons **we can use de Broglie's approach** to calculate wavelengths of a matter wave corresponding to electrons in this experiment



$V = 54\text{ V} \Rightarrow E = 54\text{ eV} = 8.64\times10^{-18}\text{J}$

$$E = \frac{p^2}{2m},\ \ p = \sqrt{2mE},\ \ \lambda_B = \frac{h}{\sqrt{2mE}}$$

$$\lambda_B = \frac{6.63\times10^{-34}\,\text{J-sec}}{\sqrt{2\times9.1\times10^{-31}\text{kg}\times8.6\times10^{-18}\text{J}}} = 1.67\,\mathring{A}$$

excellent agreement with wavelengths of X-rays diffracted from Nickel!

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Matter Wave

What is a wave?

A wave is anything that moves!

Matter waves?

Quantities varies periodically:

Water waves: height of the water surface
Sound waves: pressure
Light waves: E and M fields

The quantity whose variations make up *matter waves* is called the wave function Ψ

Max Born (1926) extended this interpretation to the matter waves proposed by De Broglie, by assigning a mathematical function, $\Psi(\boldsymbol{r},t)$, called the wavefunction to every "material" particle

$\Psi(r,t)$ is what is "waving"

Normal Waves

- are a disturbance in space
- carry energy from one place to another
- often (but not always) will (approximately) obey the classical wave equation

Matter Waves

- disturbance is the wave function $\Psi(x, y, z, t)$
  - probability amplitude Ψ
  - probability density $p(x, y, z, t) = |\Psi|^2$

What could represent both wave and particle?

Find a description of a particle which is consistent with our notion of both particles and waves……

- Fits the "wave" description
- "Localized" in space

Definition of $\Psi(r,t)$

The probability $P(\boldsymbol{r},t)dV$ to find a particle associated with the wavefunction $\Psi(\boldsymbol{r},t)$ within a small volume dV around a point in space with coordinate $\boldsymbol{r}$ at some instant t is

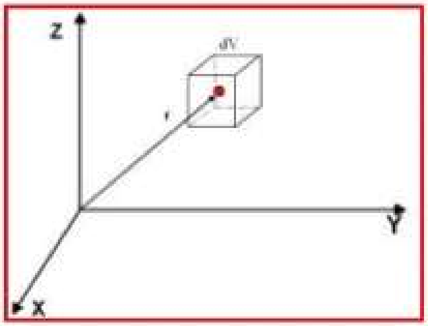
$$P(\mathbf{r},t)dV = |\Psi(\mathbf{r},t)|^2 dV$$

For one-dimensional case

$$P(x,t)dV = |\Psi(x,t)|^2 dx$$

Here $|\Psi(r,t)|^2 = \Psi^*(r,t)\Psi(r,t)$

$P(\mathbf{r},t)$ is the probability density



A large value of $|\Psi(\boldsymbol{r},t)|^2$ means the strong possibility of the particle's presence

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