Quantum Mechanics

Covered Topics

- Wave velocity (or phase velocity) of de Broglie waves
- Group Velocity
- Wave packet
- Wave packet, phase velocity and group velocity
- References

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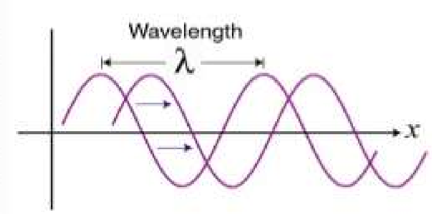
Wave velocity (or phase velocity) of de Broglie waves

- The velocities of the individual waves which superpose to produce the wave packet representing the particle are different - the wave packet as a whole has a different velocity from the waves that comprise it
- Phase velocity: The rate at which the phase of the wave propagates in space
- Group velocity: The rate at which the envelope of the wave packet propagates

The Phase Velocity

How fast is the wave traveling?

Velocity is a reference distance divided by a reference time.

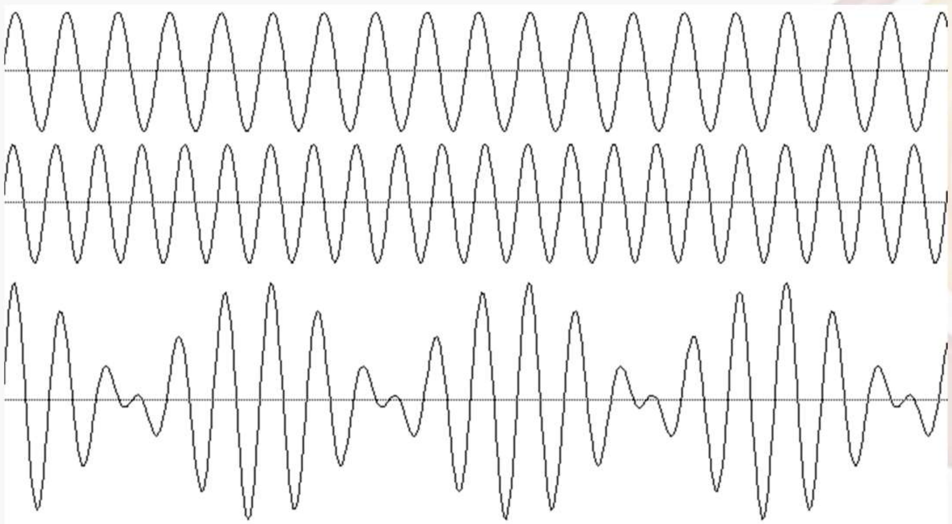


The phase velocity is the wavelength / period: $v = \lambda / \tau$

Since $f = 1/\tau$: $v = f\lambda$

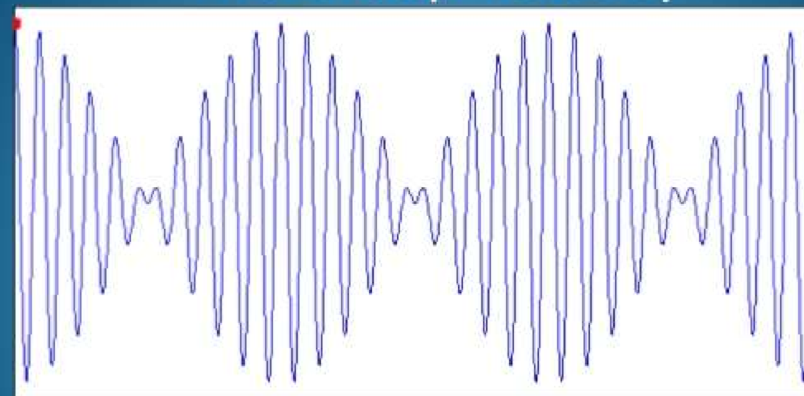
In terms of k, $k = 2\pi / \lambda$, and the angular frequency, $\omega = 2\pi / \tau$, this is: $v = \omega / k$

Group Velocity



Adding up waves of different frequencies.....

The Group Velocity

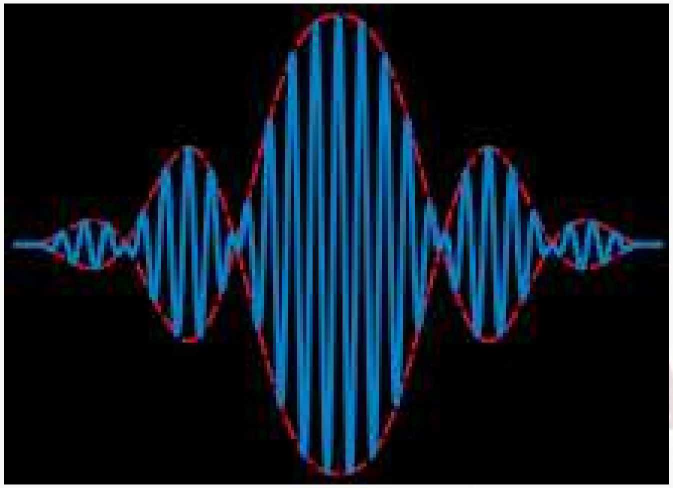


This is the velocity at which the overall shape of the wave's amplitudes, or the wave 'envelope', propagates. (= *signal velocity*)

Here, phase velocity = group velocity (the medium is *non-dispersive*)

Wave packet

If several waves of different wavelengths (frequencies) and phases are superposed together, one would get a resultant which is a localized wave packet



A wave packet is a group of waves with slightly different wavelengths interfering with one another in a way that the amplitude of the group (envelope) is non-zero only in the neighbourhood of the particle

A wave packet is localized – a good representation for a particle!

Wave packet, phase velocity and group velocity

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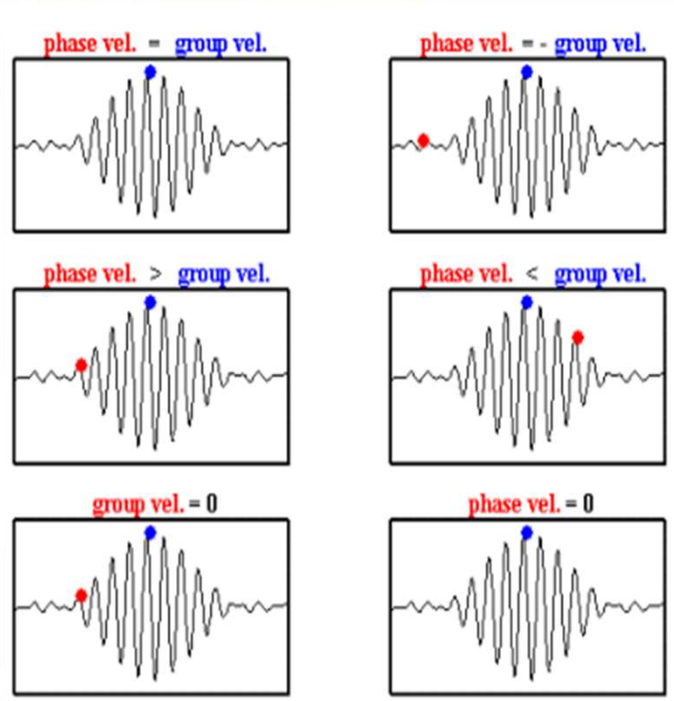

Phase velocity = Group Velocity
The entire waveform—the component waves *and* their envelope—moves as one. non-dispersive wave.

Phase velocity = -Group Velocity
The envelope moves in the *opposite direction* of the component waves.

Phase velocity > Group Velocity
The component waves move more quickly than the envelope.

Phase velocity < Group Velocity
The component waves move more slowly than the envelope.

Group Velocity = 0
The envelope is stationary while the component waves move through it.

Phase velocity = 0
Now only the envelope moves over stationary component waves.

Wave packet, phase velocity and group velocity

Phase velocity

$$v_p = \frac{\omega}{k} = \frac{E/\eta}{p/\eta} = \frac{E}{p}$$

$$v_p = \frac{E}{p} = \frac{\gamma m c^2}{\gamma m v} = \frac{c^2}{v} = \frac{c}{\beta}$$

$$v_p = \frac{c^2}{v}$$

Here C is the velocity of light and V is the velocity of the particle

$$v_g = v$$

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

Group velocity

$$v_g = \frac{\partial \omega}{\partial k} = \frac{\partial (E/\eta)}{\partial (p/\eta)} = \frac{\partial E}{\partial p}$$

$$v_g = \frac{\partial E}{\partial p} = \frac{\partial}{\partial p}\left(\frac{1}{2}\frac{p^2}{2m}\right)$$

$$= \frac{p}{m}$$

$$= v$$

$$v_g = \frac{\partial E}{\partial p} = \frac{\partial}{\partial p}\left(\sqrt{p^2c^2 + m^2c^4}\right)$$

$$= \frac{pc^2}{\sqrt{p^2c^2 + m^2c^4}}$$

$$= \frac{p}{m\sqrt{(p/(mc))^2 + 1}}$$

$$= \frac{p}{m\gamma}$$

$$= \frac{mv\gamma}{m\gamma}$$

$$= v.$$

References:

1. NouredineZettili,Quantum Mechanics: concepts and applications, 2nd Edition, Wiley, UK, 2009f
2. Introduction to Quantum Mechanics, D.J. Griffith, 2ndEd. 2005, Pearson Education
3. Quantum Mechanics, Robert Eisberg and Robert Resnick, 2ndEd., 2002, Wiley.
4. Quantum Mechanics, Leonard I. Schiff, 3rdEd. 2010, Tata McGraw Hill.
5. Modern Thermodynamics with Statistical Mechanics, Carl S. Helrich, Springer

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