Electricity and Magnetism

Topic: The Electric Field Due to a Continuous Charge Distribution

(worked examples)

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Instead of talking about electric fields of charge distributions, let's work some examples. We'll start with a "line" of charge.

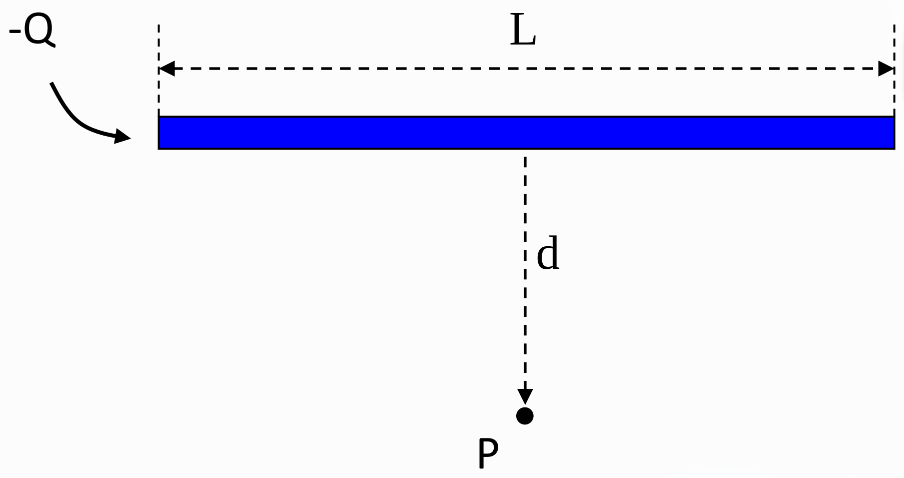
Example: A rod* of length L has a uniformly distributed total positive charge Q. Calculate the electric field at a point P located a distance d below the rod, along an axis through the left end of the rod and perpendicular to the rod.

Example: A rod* of length L has a uniformly distributed total negative charge -Q. Calculate the electric field at a point P located a distance d below the rod, along an axis through the center of and perpendicular to the rod.

I will work one of the above examples at the board in lecture. You should try the other for yourself.

*Assume the rod has negligible thickness.

Example: A rod of length L has a uniformly distributed total negative charge -Q. Calculate the electric field at a point P located a distance d below the rod, along an axis through the center of and perpendicular to the rod.



Starting equation: $$E = k\frac{|q|}{r^2}$$

"Legal" version of starting equation:

$$dE = k\frac{|dq|}{r^2}$$

This is "better" because it tells you how to work the problem! It also helps you avoid common vector mistakes.

You should begin electric field of charge distribution problems with this

$$dE = k\frac{|dq|}{r^2}$$

This is a “legal” version of a starting equation, so it is “official.”

because the equation “tells” you how to work the problem.

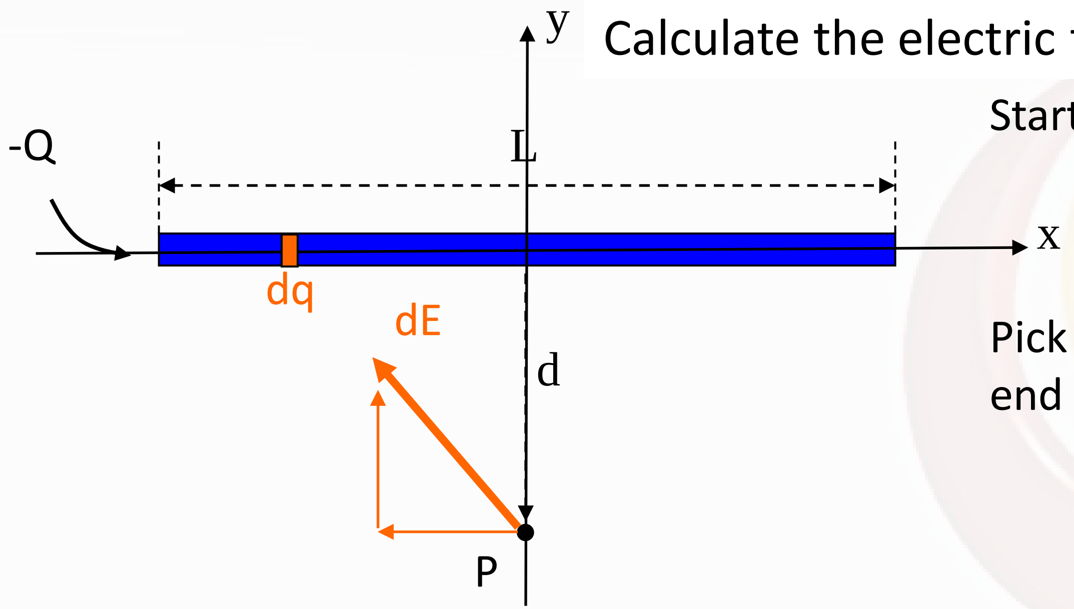
The equation says:

(1) pick a dq of charge somewhere in the distribution

(2) draw in your diagram the $d\vec{E}$ due to that dq

(3) draw the components of $d\vec{E}$

(4) for each component, check for simplifications due to symmetry, then integrate over the charge distribution.



Calculate the electric field at a point P.

Starting equation: $$dE = k\frac{|dq|}{r^2}$$

Pick a dq (best to not put it at either end or in the middle).

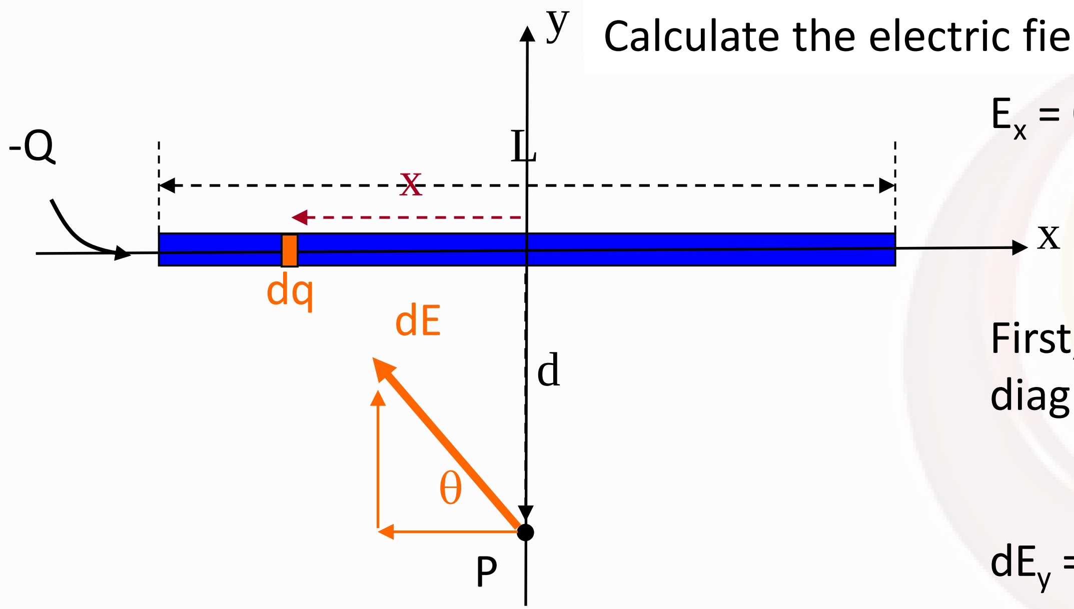
Draw the $\overrightarrow{dE}$ due to the dq.

Before I draw the components, I need to define axes!

Now draw the components.

Do you see why symmetry tells me that $E_x = 0$?

Calculate the electric field at a point P.



$E_x = 0$, so calculate E_y

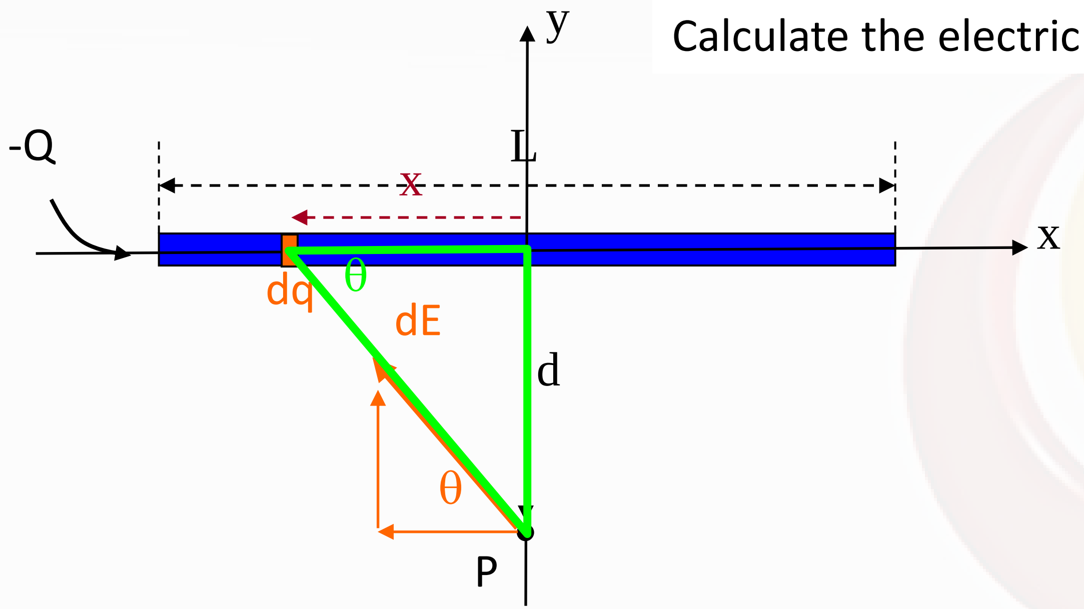
First, label an angle θ in the vector diagram.

$dE_y = +dE \sin\theta$

yes, the + sign conveys important information

To find $\sin\theta$, we need the x-coordinate of dq. If dq is at an arbitrary position along the x-axis, what is a good name for its coordinate? That's right, we'll call it x.

The diagram is getting rather "busy," but we are almost done with it.

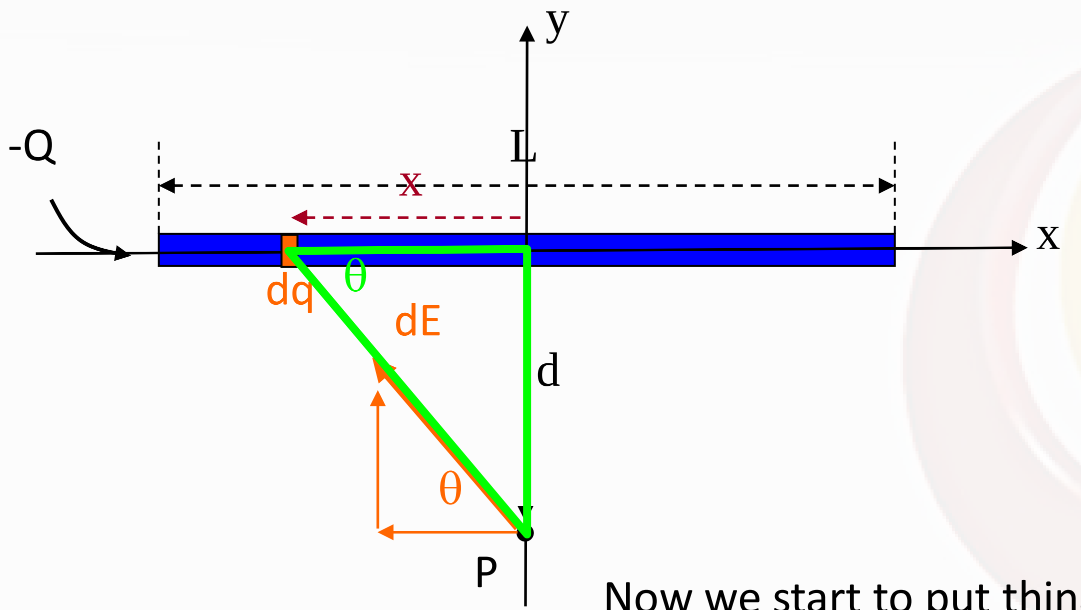


Calculate the electric field at a point P.

To find sin θ, look at the green triangle. The sides have length x and d, and hypotenuse r, where

$$r = \sqrt{x^2 + d^2}$$

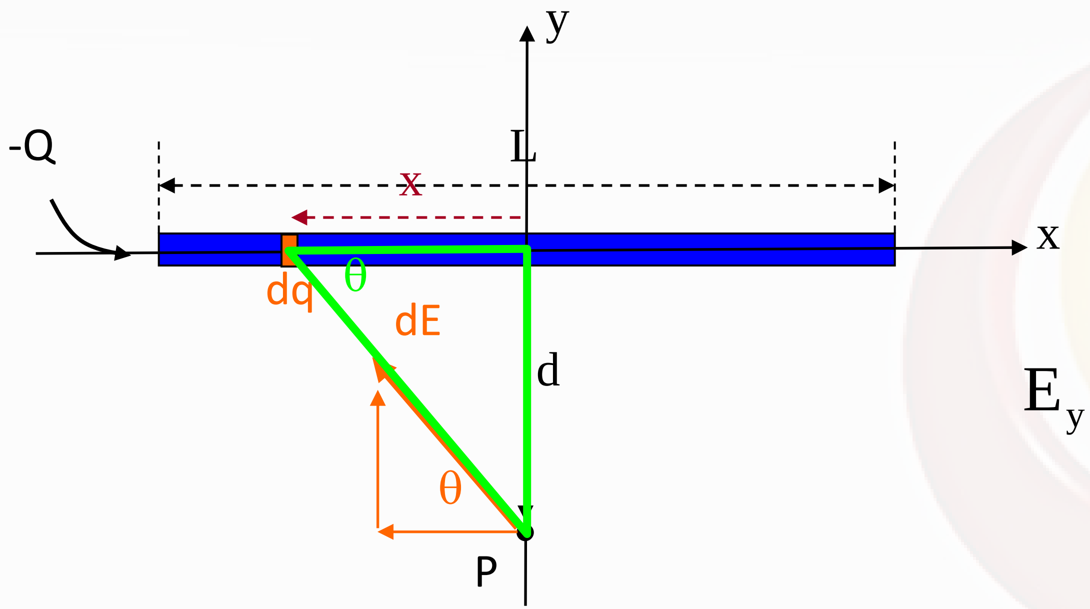
From the green triangle, we see that sin θ = d / r.



Now we start to put things together:

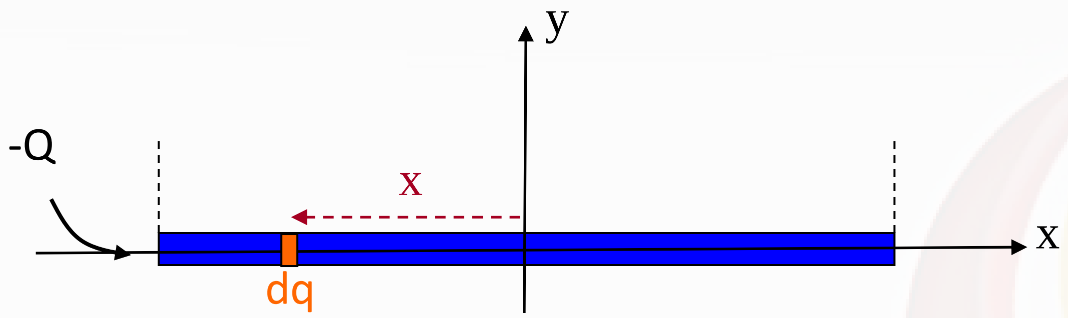
$$dE_y = +dE\sin\theta = +k\frac{|dq|}{r^2}\sin\theta = +k\frac{|dq|}{r^2}\frac{d}{r} = +k\frac{d\,|dq|}{r^3} = +kd\frac{|dq|}{\left(x^2+d^2\right)^{3/2}}$$

To find E_y we simply integrate from one end of the rod to the other (from –L/2 to L/2).



$$E_y = \int_{-L/2}^{L/2} k \frac{d|dq|}{\left(x^2 + d^2\right)^{3/2}}$$

But wait! We are integrating over the rod, which lies along the x-axis. Doesn't there need to be a dx somewhere?

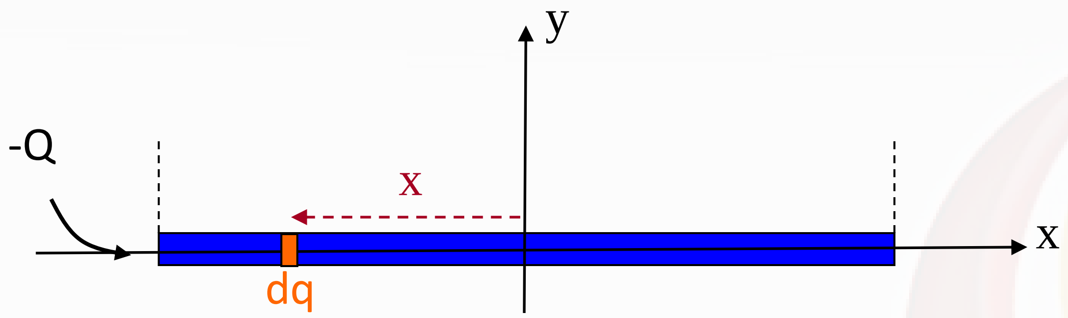


I removed un-needed "stuff" from the figure.

dq is a tiny bit of charge on the uniformly charged rod.

If the charge is uniformly distributed, then the amount of charge per length of rod is

$$\text{(linear charge density)} = \frac{\text{(charge)}}{\text{(length)}} \quad \text{or} \quad \lambda = \frac{Q}{L}$$



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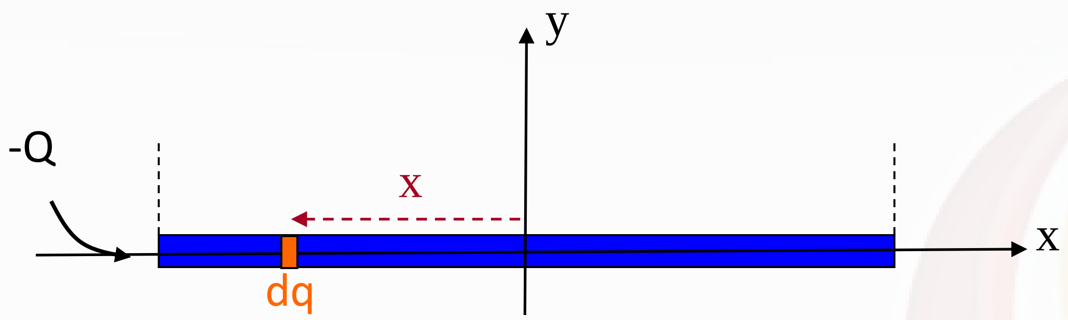
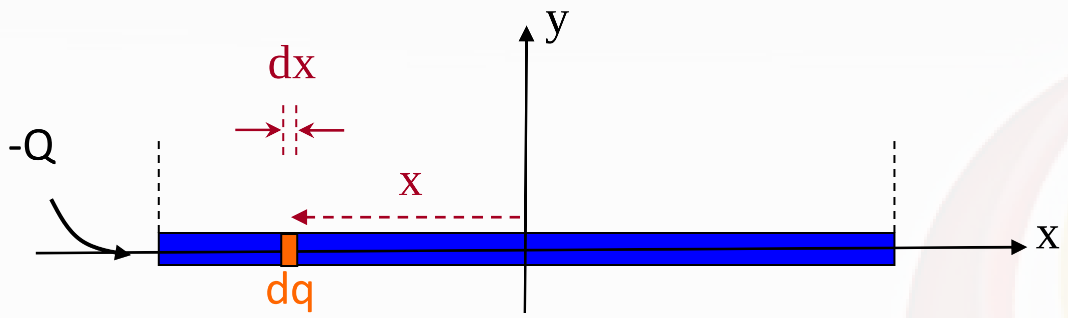


$$\lambda = \frac{Q}{L}$$

We use the symbol λ for linear charge density. You probably thought (based on Physics 1135) that λ is the symbol for wavelength. It is. But not today!

$$(\text{charge on segment of rod}) = \frac{(\text{charge})}{(\text{length})} \times (\text{length of segment of rod})$$

What would be a good name for an infinitesimal length of rod that lies along the x-axis? How about dx?



$$E_y = kd\left|\frac{Q}{L}\right|\int_{-L/2}^{L/2}\frac{dx}{\left(x^2+d^2\right)^{3/2}}$$

A note on the "just" math part. We expect you to remember derivatives and integrals of simple power and trig functions, as well as exponentials. The rest you can look up; on exams we will provide tables of integrals. We would provide you with the above integral. It is not one that I could do in 5 minutes, so I would not expect you to do it.