Electrodynamics

Topic Covered

- Energy-momentum 4-vector
- Newton's Law
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Energy-momentum 4-vector

Another quantity that follows quite straight forwardly from the 4-velocity is 4-momentum.

$$P^{\alpha} \equiv m_0 U^{\alpha} \; ; P^2 = -m_0{}^2 c^2$$

m_0 the rest mass of the particle.

Following previous example in which we see a spaceship to have $U^{\alpha} = (\gamma c, \gamma v, 0,0)$,

$$P^{\alpha} = (\gamma m_0 c, \gamma m_0 v, 0,0)$$

What is the first term in this vector?

By expanding to first order we see that $\gamma m_0 c \approx \frac{1}{c}\left(m_0 c^2 + \frac{1}{2} m_0 v^2\right)$ which simply is the rest mass energy plus the classical kinetic energy!

Therefore, we usually generalize this concept to define $P^{\alpha} = \left(\frac{E}{c}, P^x, P^y, P^z\right)$

Combining with $P^2 = -m_0{}^2 c^2$, we arrive at the equation $E^2 = P^2 c^2 + m_0{}^2 c^4$

Newton's Law

Similar with what we've done before, Newton's Law $F = \frac{\mathrm{dp}}{\mathrm{dt}}$ in 4-form then is :

$$F^{\mu} = \frac{\mathrm{dP}^{\mu}}{\mathrm{d\tau}}$$

Or in general,

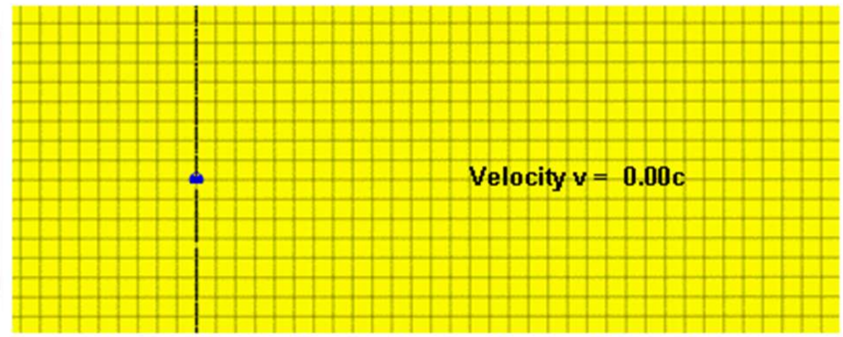
$$F^{\mu} = U^{\mu} P^{\alpha}{}_{,\alpha} \ ?$$

The Doppler factor

Just as in classical physics, waves exhibit Doppler shifts when viewed from frames with different velocities.

We define the doppler factor δ through $f = \delta f_0$

For relativistic velocities, $\delta = \frac{1}{\gamma(1-\beta\cos\theta)}$



Some cases:

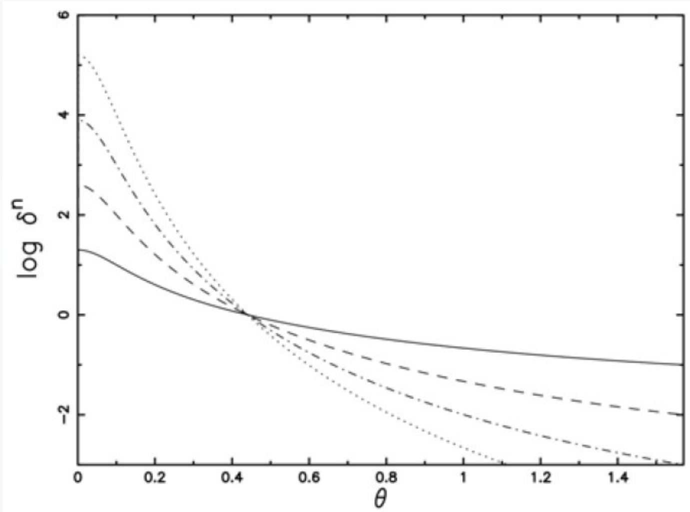
1. Source moving toward observer at v~c $\delta \sim 2\gamma$
2. Source receding from observer at v~c $\delta \sim \frac{1}{2\gamma}$
3. Sources not approaching nor receding $\delta = \frac{1}{\gamma}$ (time dilation)

Importance in astrophysics

The tremendous enhancement in the Doppler effect for relativistic motion is called "Doppler boosting" or "Doppler beaming". It affects many physical quantities (besides just frequency of the emitted light or radio waves) that are measured when astronomers observe relativistic jet flow. Some, such as brightness, depend on the Doppler factor to a high power like 3 or 4. Fig. 6.6 shows a plot of different powers of the Doppler factor as a function of the jet viewing angle θ for $\gamma = 10$. The Doppler factor does not drop formally to a value of 1 until it reaches an angle given by $\cos\theta = [(\gamma - 1)/(\gamma + 1)]^{1/2}$ (or $\theta = 0.440$ radians for $\gamma = 10$). However, we see from the figure that the effective angle of the Doppler enhancement for an exponent of $n = 3$ or 4 is only about $\theta \approx \theta_b = 0.1$ radians, where

$$\theta_b \equiv 1/\gamma \qquad (6.72)$$

is the effective "beaming angle". For travelers, or extragalactic radio jets, moving very near the speed of light with respect to us, it is very difficult to see them if they are moving at an angle $\theta > \theta_b$ to our line of sight.



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