

## Lecture-5

### Conversion of boundary value problem into integral equations

**Example:** Transform the BVP  $\frac{d^2y}{dx^2} + xy = 1$ ,  $y(0) = y(1) = 0$  into an integral equation.

**Solution:** We have

or 
$$\left(\frac{dy}{dx}\right)_0^x = \int_0^x 1 dx - \int_0^x xy(x)dx .$$

$$y'(x) = x - \int_0^x xy(x)dx + c , \quad (\text{let } y'(0)=c)$$

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Again integrating we get

$$y(x) = \frac{x^2}{2} - \int_0^x (x-t)ty(t)dt + cx \quad \dots(1)$$

Now, putting  $x = 1$

$$c = \int_0^1 (1-t)ty(t)dt - \frac{1}{2} .$$

Substituting the value of  $c$  in (1), we have

$$y(x) = \frac{x^2}{2} - \int_0^x (x-t)ty(t)dt + x \left( \int_0^1 (1-t)ty(t)dt - \frac{1}{2} \right),$$

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$$y(x) = \frac{x^2}{2} - \int_0^x (x-t)y(t)dt + x \left( \int_0^x (1-t)y(t)dt + \int_x^1 (1-t)y(t)dt \right) - \frac{1}{2}x$$
$$= \frac{x(x-1)}{2} + \int_0^1 K(x,t)y(t)dt.$$

where

$$K(x,t) = \begin{cases} t^2(1-x), & 0 \leq t < x \\ xt(1-t), & x \leq t \leq 1 \end{cases} .$$

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## Practice problem

1. Transform the BVP problem

$$\frac{d^2y}{dx^2} + y = x, y(0) = 0, y'(1) = 0$$

to an integral equation.

2. Transform the BVP problem

$$\frac{d^2y}{dx^2} + \lambda y = x, y(0) = 0, y(1) = 1.$$

to an integral equation.



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Reference:

<https://nptel.ac.in/courses/111/107/111107103/>



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