# Quantum Mechanics

## Covered Topics

- Dirac Notation
- References

## Dirac Notation

**Dirac** or **"bra-ket" notation** is a useful representation for QM equations:

$$\int \psi^*(x)\psi(x)dx = \langle\psi|\psi\rangle$$

$\psi^*$: complex conjugate; $\langle\psi|$: bra; $|\psi\rangle$: ket

**Useful relations**

$$\langle a\rangle = \langle\psi|\,\hat{A}\,|\psi\rangle$$

$$\langle\beta|\,\alpha\rangle = \langle\alpha|\,\beta\rangle^*$$

$$\langle\beta|\,\hat{T}\,|\alpha\rangle^* = \langle\alpha|\,\hat{T}^\dagger\,|\beta\rangle$$

**Schrödinger equation**

*TDSE:*

$$\hat{H}\,|\Psi\rangle = i\hbar\frac{\partial}{\partial t}\,|\Psi\rangle$$

*TISE:*

$$\hat{H}\,|\psi\rangle = E\,|\psi\rangle$$

Here are some rules for operator algebra:

1) If $\hat{\alpha} = \hat{\beta}$ then $\hat{\alpha} f = \hat{\beta} f$

2) $(\hat{\alpha} + \hat{\beta}) f = \hat{\alpha} f + \hat{\beta} f$

3) $\hat{\alpha}\hat{\beta} f = \hat{\alpha}(\hat{\beta} f)$

4) Operators corresponding to physical quantities are linear:

$$\hat{\alpha}[f(x) + g(x)] = \hat{\alpha} f(x) + \hat{\alpha} g(x)$$

5) and Hermitian:

$$\int \psi_i^* \hat{\alpha} \psi_j d\tau = \int \psi_j^* \hat{\alpha} \psi_i d\tau$$

## Properties of Operators

# Commutation Relation

Two operators commute if their order of application does not change the result (i.e. they share common eigenfunctions):

$$\hat{\alpha}\hat{\beta}f(x) - \hat{\beta}\hat{\alpha}f(x) = 0$$

Using another notation, we can say this as:

$$[\hat{\alpha}, \hat{\beta}] = 0$$

Examples of commuting operators:

$$[\hat{L}^2, \hat{L}_x^2] = [\hat{L}^2, \hat{L}_y^2] = [\hat{L}^2, \hat{L}_z^2] = 0$$

Total angular momentum and components commute.

$$[\hat{L}_z, \hat{L}_x] = i\hbar\hat{L}_y \qquad [\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z \qquad [\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x$$

But the components don't commute with each other.

**References:**

1. NouredineZettili,Quantum Mechanics: concepts and applications, 2nd Edition, Wiley, UK, 2009f
2. Introduction to Quantum Mechanics, D.J. Griffith, 2ndEd. 2005, Pearson Education
3. Quantum Mechanics, Robert Eisberg and Robert Resnick, 2ndEd., 2002, Wiley.
4. Quantum Mechanics, Leonard I. Schiff, 3rdEd. 2010, Tata McGraw Hill.
5. Modern Thermodynamics with Statistical Mechanics, Carl S. Helrich, Springer