UNIT 1: WAVE-PARTICLE DUALITY

Application of Uncertainty Principle

NON-EXISTENCE OF ELECTRON IN THE NUCLEUS

1. **Non-Existence of electrons in the nucleus**: The diameter of nucleus is of order 10^{-14} m. Therefore the uncertainty (Δx) in the position of any particle inside the nucleus will be of order 10^{-14} m. So the uncertainty in momentum will be $\Delta p_x = \hbar/(2\Delta x)$ = 0.527 X 10^{-20} kg m/s. If uncertainty in momentum is of this order, the momentum itself will be of same order. i.e., p ~ 0.527 X 10^{-20} kg m/s

For an electron, the rest energy $E_0 = m_0c^2$ = 511 keV

Value of pc for electron is 9.88MeV which is much greater than E_0. This means that we have to follow relativistic approach. Thus using $E^2 = p^2c^2 + m_0^2c^4$, the total energy comes out to be ~10MeV. If an electron exists inside the nucleus, it should come out with at least 10 MeV. But we know that during beta decay, electrons emitted have energies only of order 3 to 4 MeV. Thus it can be concluded that an electron cannot exist inside a nucleus.

NON-EXISTENCE OF ELECTRON IN THE NUCLEUS

For protons or neutrons, $m_0 = 1.67 \times 10^{-27}$ kg. So $E_0 = m_0c^2$ ~900 MeV, whereas pc ~ 9.88 MeV. Thus this is a non- relativistic case. So $E = p^2/2m$ =52 KeV. This energy is much less than the energies of particles emitted by nucleus. Hence protons, neutrons and other heavy particles can exist inside the nucleus.

RADIUS OF BOHR'S FIRST ORBIT CALCULATION

To estimate the radius of Bohr's first orbit, we will assume that the radius is of same order as the uncertainty in position; velocity and momentum of electron are of same order as the uncertainty in velocity and momentum. i.e., $\Delta x \sim x$, $\Delta p \sim p$ *and* $\Delta v \sim v$.

From Heisenberg's Uncertainty Principle, $\Delta x \Delta p \geq \frac{\hbar}{2} \Rightarrow \Delta x \Delta p \sim \hbar \Rightarrow \Delta p = \frac{\hbar}{\Delta x}$

The kinetic energy (K) and the potential energy (U) are given by

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow \Delta K = \frac{(\Delta p)^2}{2m} \sim \frac{\hbar^2}{2(\Delta x)^2}, \quad U = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{\Delta x}$$

Total energy is $E = K + V = \frac{\hbar^2}{2(\Delta x)^2} - \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{\Delta x}$

For the first orbit, E should be minimum i.e.,

$$\frac{dE}{dx} = \frac{dE}{d\Delta x} = 0 \Rightarrow \frac{d}{d\Delta x}\left[\frac{\hbar^2}{2(\Delta x)^2} - \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{\Delta x}\right] = 0 \Rightarrow \Delta x = \frac{4\pi\varepsilon_0\hbar^2}{mZe^2}$$

We confirm that $\frac{d^2E}{d^2\Delta x} = positive \quad at\ \Delta x = \frac{4\pi\varepsilon_0\hbar^2}{mZe^2}$

Hence a rough estimate of Bohr's first orbit is $\Delta x = \frac{4\pi\varepsilon_0\hbar^2}{mZe^2}$ *which gives 5.3 nm for Z=1. The energy of electron at this orbit comes out to be E ~ -13.6 eV. These numbers match very close to actual observed energies!!*

REFERENCES

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