UNIT 1: WAVE-PARTICLE DUALITY

COMPTON SCATTERING

COMPTON EFFECT

The scattering of a photon by an electron is called the Compton effect.

COMPLETE EXPLANATION & PROOF

According to the quantum theory of light, photons behave like particles except for their lack of rest mass. How far can this analogy be carried? For instance, can we consider a collision between a photon and an electron as if both were billiard balls?

Figure 2.22 shows such a collision: an x-ray photon strikes an electron (assumed to be initially at rest in the laboratory coordinate system) and is scattered away from its original direction of motion while the electron receives an impulse and begins to move. We can think of the photon as losing an amount of energy in the collision that is the same as the kinetic energy KE gained by the electron, although actually separate photons are involved. If the initial photon has the frequency ν associated with it, the scattered photon has the lower frequency ν', where

Loss in photon energy = gain in electron energy

$$h\nu - h\nu' = \text{KE} \tag{2.14}$$

Since the energy of a photon is $h\nu$, its momentum is

Photon momentum $$p = \frac{E}{c} = \frac{h\nu}{c} \tag{2.15}$$

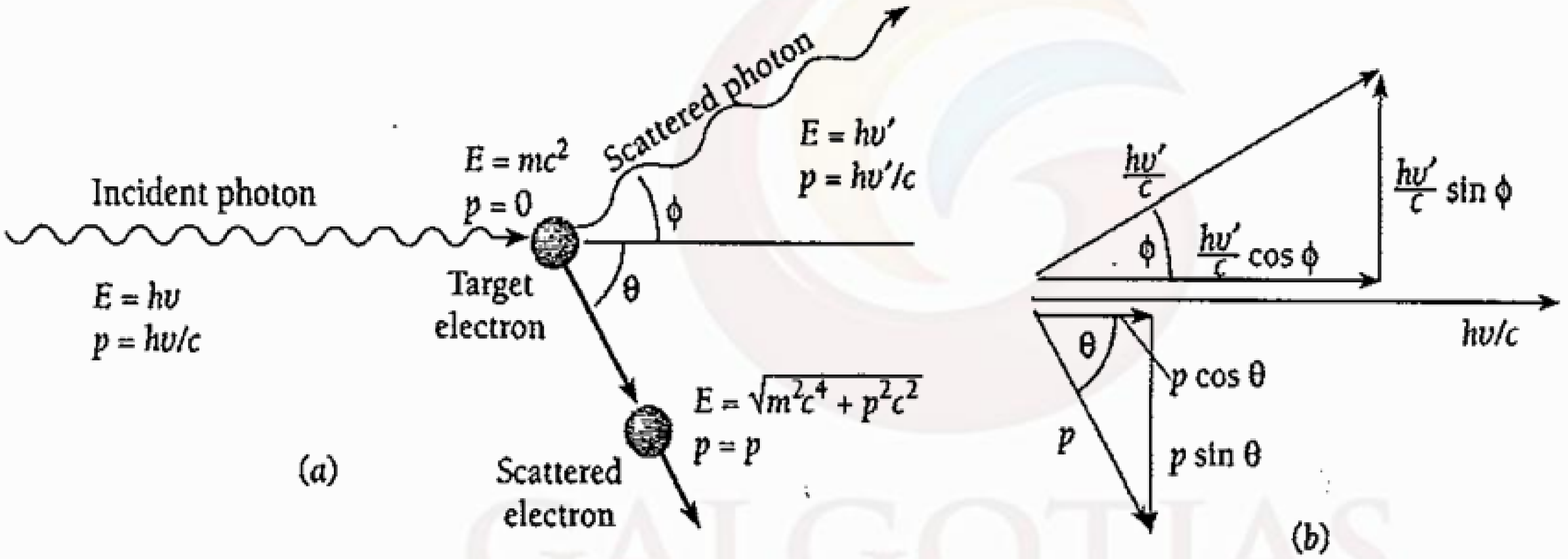

Figure 2.22 (*a*) The scattering of a photon by an electron is called the Compton effect. Energy and momentum are conserved in such an event, and as a result the scattered photon has less energy (longer wavelength) than the incident photon. (*b*) Vector diagram of the momenta and their components of the incident and scattered photons and the scattered electron.

Momentum, unlike energy, is a vector quantity that incorporates direction as well as magnitude, and in the collision momentum must be conserved in each of two mutually perpendicular directions. (When more than two bodies participate in a collision, momentum must be conserved in each of three mutually perpendicular directions.) The directions we choose here are that of the original photon and one perpendicular to it in the plane containing the electron and the scattered photon (Fig. 2.22).

The initial photon momentum is $h\nu/c$, the scattered photon momentum is $h\nu'/c$, and the initial and final electron momenta are respectively 0 and p. In the original photon direction

$$\text{Initial momentum} = \text{final momentum}$$

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta \tag{2.16}$$

and perpendicular to this direction

$$\text{Initial momentum} = \text{final momentum}$$

$$0 = \frac{h\nu'}{c} \sin \phi - p \sin \theta \tag{2.17}$$

The angle ϕ is that between the directions of the initial and scattered photons, and θ is that between the directions of the initial photon and the recoil electron. From Eqs. (2.14), (2.16), and (2.17) we can find a formula that relates the wavelength difference between initial and scattered photons with the angle ϕ between their directions, both of which are readily measurable quantities (unlike the energy and momentum of the recoil electron).

The first step is to multiply Eqs. (2.16) and (2.17) by c and rewrite them as

$$pc \cos \theta = h\nu - h\nu' \cos \phi$$

$$pc \sin \theta = h\nu' \sin \phi$$

By squaring each of these equations and adding the new ones together, the angle θ is eliminated, leaving

$$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\phi + (h\nu')^2 \tag{2.18}$$

the total energy of a particle

$$E = KE + mc^2$$

$$E = \sqrt{m^2c^4 + p^2c^2}$$

$$(KE + mc^2)^2 = m^2c^4 + p^2c^2$$

$$p^2c^2 = KE^2 + 2mc^2\,KE$$

Since

$$KE = h\nu - h\nu'$$

we have

$$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2 + 2mc^2(h\nu - h\nu') \qquad (2.19)$$

Substituting this value of p^2c^2 in Eq. (2.18), we finally obtain

$$2mc^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1 - \cos\phi) \qquad (2.20)$$

This relationship is simpler when expressed in terms of wavelength λ. Dividing Eq. (2.20) by $2h^2 c^2$,

$$\frac{mc}{h}\left(\frac{\nu}{c} - \frac{\nu'}{c}\right) = \frac{\nu}{c}\frac{\nu'}{c}(1 - \cos\phi)$$

and so, since $\nu/c = 1/\lambda$ and $\nu'/c = 1/\lambda'$,

$$\frac{mc}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{1 - \cos\phi}{\lambda\lambda'}$$

Compton effect

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi) \tag{2.21}$$

Equation (2.21) was derived by Arthur H. Compton in the early 1920s, and the phenomenon it describes, which he was the first to observe, is known as the **Compton effect.** It constitutes very strong evidence in support of the quantum theory of radiation.

Equation (2.21) gives the change in wavelength expected for a photon that is scattered through the angle ϕ by a particle of rest mass m. This change is independent of the wavelength λ of the incident photon. The quantity

Compton wavelength
$$\lambda_C = \frac{h}{mc} \tag{2.22}$$

is called the **Compton wavelength** of the scattering particle. For an electron $\lambda_C = 2.426 \times 10^{-12}$ m, which is 2.426 pm (1 pm = 1 picometer = 10^{-12} m). In terms of λ_C, Eq. (2.21) becomes

Compton effect
$$\lambda' - \lambda = \lambda_C(1 - \cos\phi) \tag{2.23}$$

The Compton wavelength gives the scale of the wavelength change of the incident photon. From Eq. (2.23) we note that the greatest wavelength change possible corresponds to $\phi = 180°$, when the wavelength change will be twice the Compton wavelength λ_C. Because $\lambda_C = 2.426$ pm for an electron, and even less for other particles owing to their larger rest masses, the maximum wavelength change in the Compton effect is 4.852 pm. Changes of this magnitude or less are readily observable only in x-rays: the shift in wavelength for visible light is less than 0.01 percent of the initial wavelength, whereas for x-rays of $\lambda = 0.1$ nm it is several percent. The Compton effect is the chief means by which x-rays lose energy when they pass through matter.

REFERENCES

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