UNIT 1: WAVE-PARTICLE DUALITY

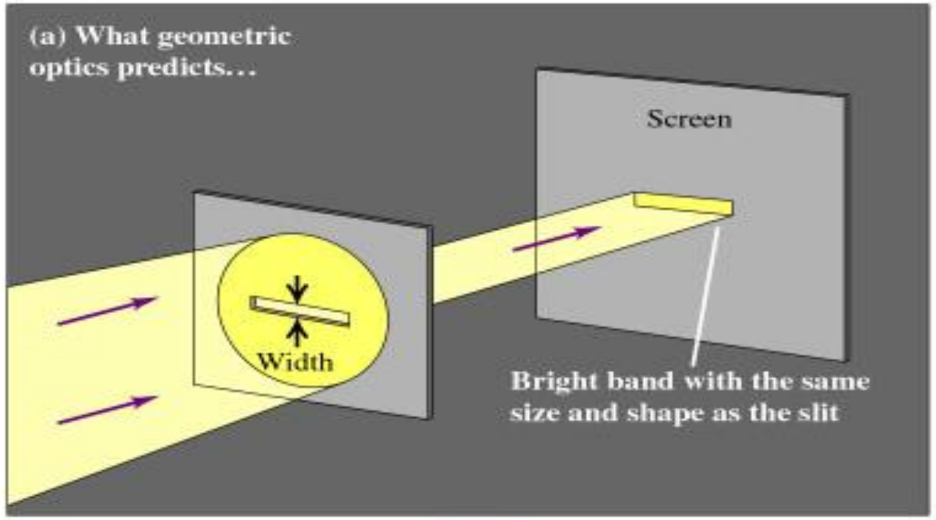
The Uncertainty Principle: Photon Picture

THE UNCERTAINTY PRINCIPLE

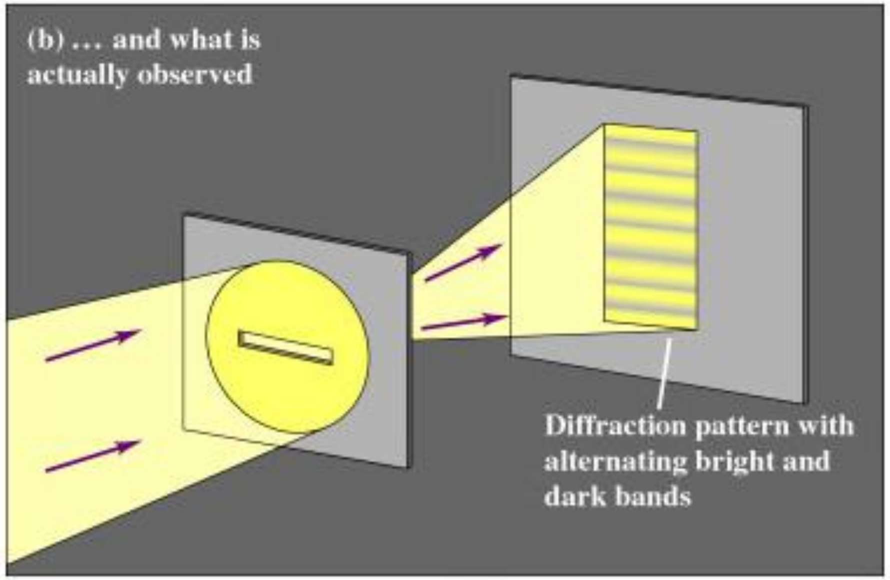
- One of the fundamental consequences of quantum mechanics is that it is IMPOSSIBLE to SIMULTANEOUSLY determine the POSITION and MOMENTUM of a particle with COMPLETE PRECISION

- Can be illustrated by a couple of “thought experiments”, for example the “photon picture” of single slit diffraction and the “Heisenberg Microscope”

SINGLE SLIT DIFFRACTION



INCORRECT



CORRECT

“geometrical” picture breaks down when slit wid
with wavelength

POSITION OF DARK FRINGES IN SINGLE-SLIT DIFFRACTION

$$\sin q = \frac{m/}{a}$$

If, like the 2-slit treatment we assume small angles, $\sin q \approx \tan q = y_{min}/R$, then

$$y_{\min} = \frac{Rm/}{a}$$

Positions of intensity <u>MINIMA</u> of diffraction pattern on screen, measured from central position .

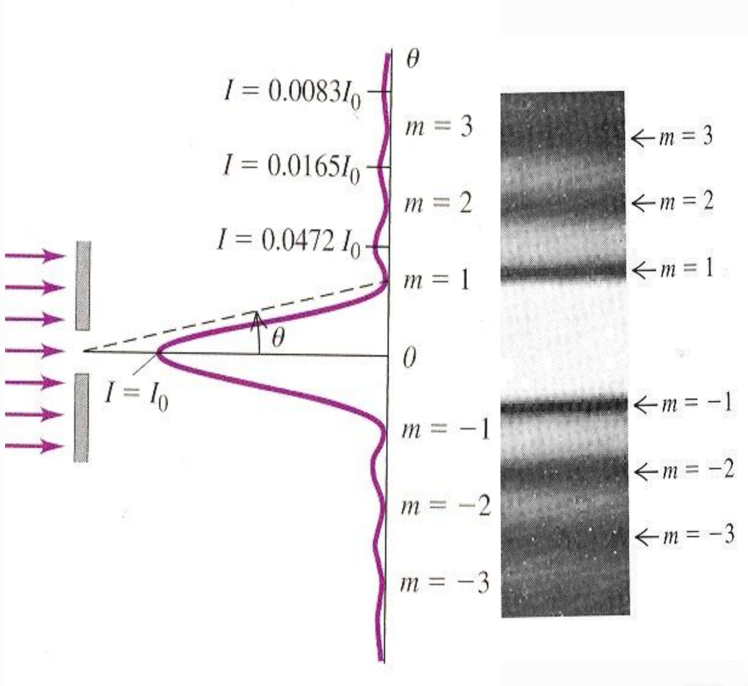
Very similar to expression derived for 2-slit experiment:

$$y_m = R\frac{n/}{d}$$

But remember, in this case y_m are positions of <u>MAXIMA</u>
In interference pattern

WIDTH OF CENTRAL MAXIMUM

- We can define the width of the central maximum to be the distance between the m = +1 minimum and the m=-1 minimum:



Intensity distribution

image of diffraction pattern

$$\Delta y = \frac{R\lambda}{a} - -\frac{R\lambda}{a} = \frac{2R\lambda}{a}$$

The narrower the slit, the more the diffraction pattern

SINGLE SLIT DIFFRACTION: PHOTON PICTURE

Since θ is small:

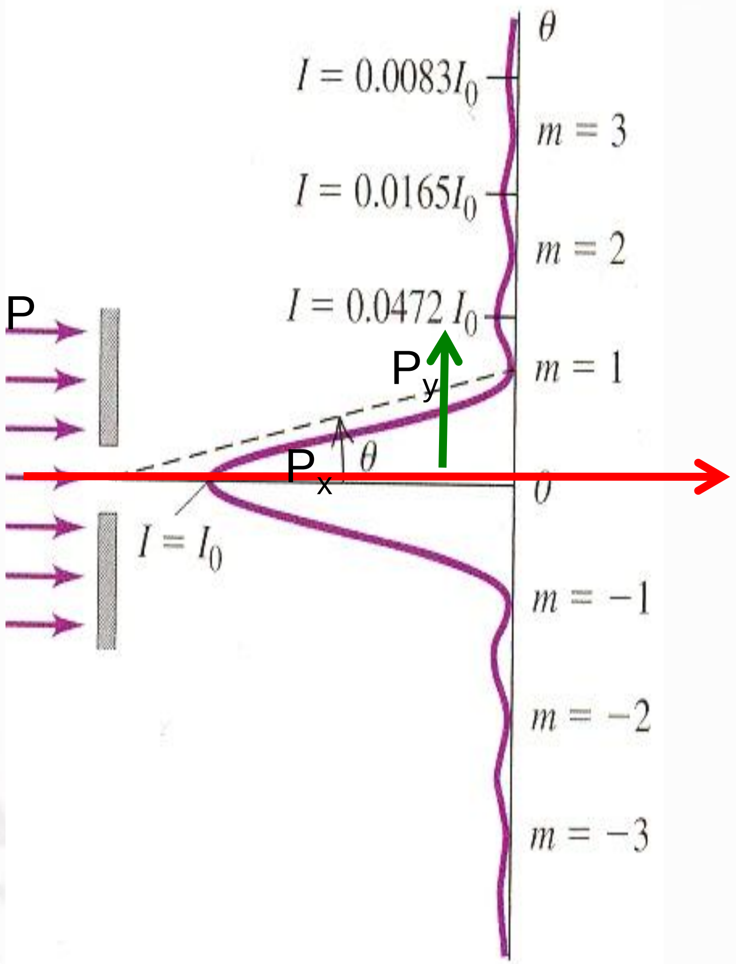
$$\sin\theta = \frac{\lambda}{a} \approx \theta = \frac{\lambda}{a}$$

Photons directed towards outer part of central maximum have momentum

$$\bar{p} = \bar{p}_x + \bar{p}_y$$

$$p_y = \theta p_x = p_x \frac{\lambda}{a} = p_x \frac{h}{p_x a} = \frac{h}{a}$$

ie, localizing photons in the y-direction to a slit of width *a* leads to a spread of y-momenta of at least h/a.



Intensity distribution

- **So, the more we seek to localize a photon (ie define its position) by shrinking the slit width, *a*, the more spread (uncertainty) we induce in its momentum:**
- **In this case, we have $Dp_y Dy \sim h$**

HEISENBERG MICROSCOPE

Suppose we have a particle, whose momentum is, initially, precisely known. For convenience assume initial p = 0.

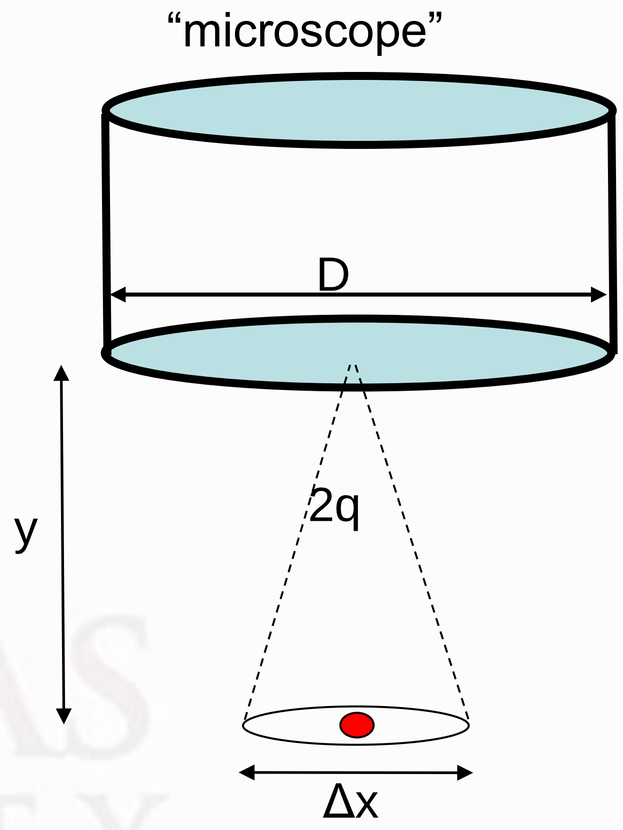
From wave optics (Rayleigh Criterion)

$$\sin\theta \approx \frac{\lambda}{D}$$

From our diagram:

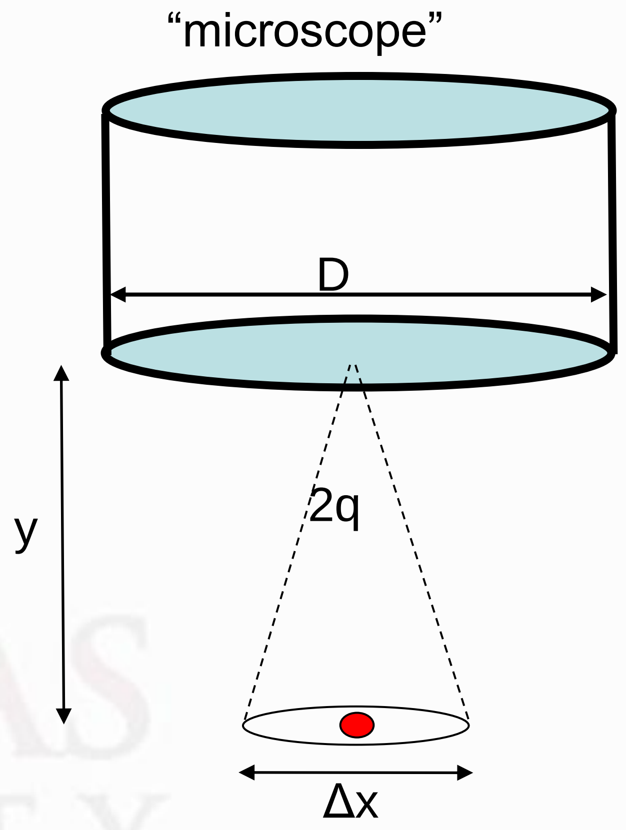
$$\sin\theta \approx \frac{\Delta x}{2} \cdot y = \frac{\Delta x}{2y}$$

$$\Delta x \approx \frac{2y\lambda}{D}$$



HEISENBERG MICROSCOPE

$$\Delta x \approx \frac{2y\lambda}{D}$$



Since this is a "thought experiment" we are free from any practical constraints, and we can locate the particle as precisely as we like by using radiation of shorter and shorter wavelengths.

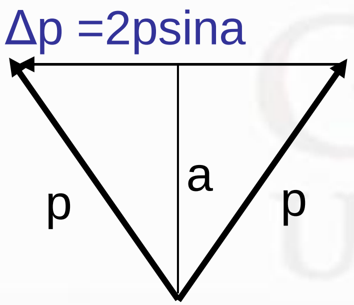
But what are the consequences of this?

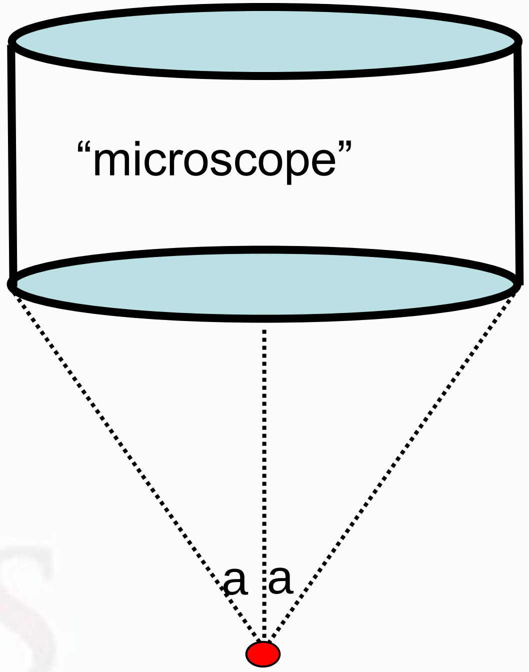
HEISENBERG MICROSCOPE

In order to see the particle, a photon must scatter off it and enter the microscope.

Thus process MUST involve some transfer of momentum to the particle…….

BUT there is an intrinsic uncertainty in the X-component of the momentum of the scattered photon, since we only know that the photon enters the microscope somewhere within a cone of half angle a:





By conservation of momentum, there must be the same uncertainty in the momentum of the observed particle……………

HEISENBERG MICROSCOPE: SUMMARY

Uncertainty in position of particle:

$$\Delta x \approx \frac{2y\lambda}{D}$$

Can reduce as much as we like by making **λ small** ... ...

Uncertainty in momentum of particle: $\Delta p \approx 2p_{photon}\sin\alpha \approx \frac{2h}{\lambda}\frac{D}{2y}$

So, if we attempt to reduce uncertainty in position by decreasing λ, we INCREASE the uncertainty in the momentum of the particle!!!!!!

Product of the uncertainties in position and momentum given by:

$$\Delta x \Delta p \approx \frac{2y\lambda}{D}\frac{Dh}{y\lambda} = 2h$$

THE UNCERTAINTY PRINCIPLE

Our microscope thought experiments give us a rough estimate for the uncertainties in position and momentum:

$$\Delta x \Delta p \sim h$$

"Formal" statement of the Heisenberg u

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

REFERENCES

- CONCEPTS OF MODERN PHYSICS, ARTHUR BEISER, MCGRAW-HILL.
- INTRODUCTION TO MODERN PHYSICS, RICH MEYER, KENNARD, COOP, TATA MCGRAW HILL
- INTRODUCTION TO QUANTUM MECHANICS, DAVID J. GRIFFITH, PEARSON EDUCATION.