UNIVERSITY



Course Code: BSCM301

Course Name: Real Analysis-I

Corollaries of Archimedean properties:

Corollary 1. Let y be any positive real number and x be any real number. Then, there exists a positive integer n (or natural number) such that ny > x.

Proof: Case 1. If y<x.

......v(given that y is positive).....x.....

Then, x is also positive.

Thus, x and y both are positive and y<x. Proof done by Archimedean property.

6

Course Code: BSCM301

Course Name: Real Analysis-I

Case 2. If 0 <x<y< th=""></x<y<>
0x y (given that y is positive)
Similar proof works.
Case 3. If x<0 <y< td=""></y<>
x
Proof of Archimedean property depends only on y not x.
Similar proof works.



Course Code: BSCM301

Course Name: Real Analysis-I

Corollary 2. For any real number x there exists an integer n such that n > x.

Proof: Put y=1 in Corollary 1. We will get the result.

Corollary 3. For any real number x there exists two integers m and n such that n < x < m.

Proof: For x < m we use corollary 2.(1)

If x is a real number (-1.2 is real number) **then** – x is also a real number (-(-1.2)=1.2 is also a real number).

By using Corollary 2, there exists an integer "p" such that p > -x.



6

Course Code : BSCM301

Course Name: Real Analysis-I

Operate "-" sign both the sides, we get

-p < x if p is an integer then -p = n (assume) is also an integer.

Thus, we get n < x.....(2)

From (1) and (2)

n < x < m. Proved.

Characterization of supremum and infimum:



Course Code: BSCM301

Course Name: Real Analysis-I

Characterization of supremum and infimum:

Theorem 1. Let α be a supremum of a subset S of R if and only if

- (i) $x \le \alpha \ \forall \ x \in S$ i.e., α is an upper bound for set S
- (ii) For any given \in > 0 there exists some $x \in S$ such that $x > \alpha \in$.

Proof: If part is sufficient and only if is necessary part.

Only if (necessary pat) i.e.,

Assume α is a supremum of a subset **S** of **R**.

Aim: Our aim is to prove that

 $a. x \le \alpha \ \forall \ x \in S$ i.e., α is an upper bound for set §

b. For any given \in > 0 there exists some $x \in S$ such that $x > \alpha - \in$.

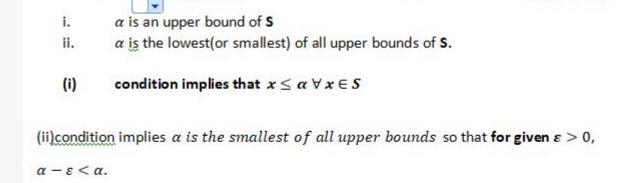
Galgotias University

(5

Course Code: BSCM301

Proved

Course Name: Real Analysis-I



There $\alpha - \varepsilon$ will not be an upper bound for **S**.

This implies that there exists some $x \in S$ such that $x > \alpha - \varepsilon$.

BY assumptid α a supremum of a subset **S** of **R**. Then, by the definition of supremum

UNIVERSITY

Galgotias University



Course Code: BSCM301

Course Name: Real Analysis-I

If part(sufficient)

Assume

a. $x \le \alpha \ \forall \ x \in S$ i.e., α is an upper bound for set §

b. For any given \in > 0 there exists some $x \in S$ such that $x > \alpha - \in$.

Aim: α is the supremum of S.

- (a) Condition implies that α is an upper bound
- **(b)** Condition implies that α is smallest BY def α is the supremum of **S**.

Proved.

Name of the Faculty: Dr. Pradeep Kumar

Program Name: B.Sc(H) in mathematics

Course Code: BSCM301 Course Name: Real Analysis-I



Theorem 1. Let β be a infimum of a subset S of R if and only if

- (i) $x \ge \beta \ \forall \ x \in S$ i.e., β is a lower bound for set S
- (ii) For any given \in > 0 there exists some $x \in S$ such that $x < \beta + \in$.

Proof: Similar proof as above

GALGOTIAS UNIVERSITY