

## Lecture-11

### Solution of Volterra integral equation of second kind by Resolvent kernels

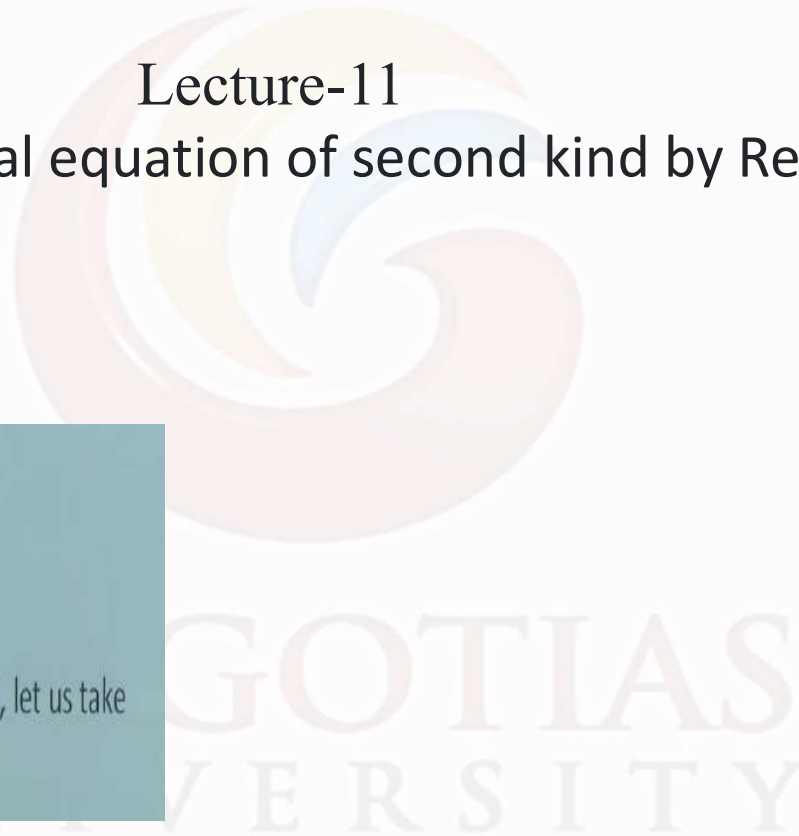
#### Neumann series

We have

$$y(x) = f(x) + \lambda \int_a^x K(x,t)y(t)dt.$$

As a zero order approximation to the required solution  $y(x)$ , let us take

$$y_0(x) = f(x).$$



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Then the first order approximation is given by

$$y_1(x) = f(x) + \lambda \int_a^x K(x,t)y_0(t)dt$$

and

$$y_2(x) = f(x) + \lambda \int_a^x K(x,t)y_1(t)dt$$

$$y_2(x) = f(x) + \lambda \int_a^x K(x,z)y_1(z)dz$$

$$= f(x) + \lambda \int_a^x K(x,z) \left\{ f(z) + \lambda \int_a^z K(z,t)y_0(t)dt \right\} dz$$

$$= f(x) + \lambda \int_a^x K(x,z)f(z)dz + \lambda^2 \int_a^x \left\{ \int_a^z K(x,z)K(z,t)y_0(t)dt \right\} dz$$

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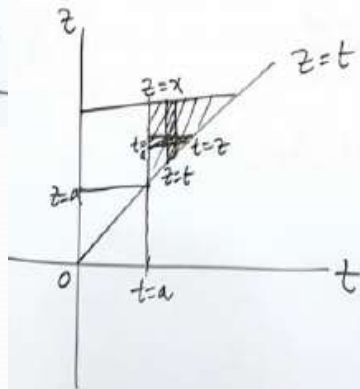
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On changing the order of the integration, we have

$$y_2(x) = f(x) + \lambda \int_a^x K(x,z) f(z) dz + \lambda^2 \int_a^x \left\{ \int_t^x K(x,z) K(z,t) dz \right\} f(t) dt$$

$$\int_a^x \int_a^z k(x,z) k(z,t) y_0(t) dt dz$$
$$= \int_a^x \left( \int_t^x k(x,z) k(z,t) dz \right) y_0(t) dt$$



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$$y_2(x) = f(x) + \lambda \int_a^x K(x,t)f(t)dt + \lambda^2 \int_a^x K_2(x,t)f(t)dt,$$

where

$$K_2(x,t) = \int_t^x K(x,z)K(z,t)dz$$

continuing this process

$$K_3(x,t) = \int_t^x K(x,z)K_2(z,t)dz$$

and in general

$$K_{n+1}(x,t) = \int_t^x K(x,z)K_n(z,t)dz, \quad \text{where } n = 1, 2, 3, \dots$$

and

$$K_1(x,t) = K(x,t)$$

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The functions  $K_n(x, t)$  are called iterated kernels.

By mathematical induction, we have

$$y_n(x) = f(x) + \sum_{m=1}^n \lambda^m \int_a^x K_m(x, t) f(t) dt .$$

As  $n \rightarrow \infty$ , we get the Neumann series

$$\begin{aligned} y(x) &= f(x) + \sum_{m=1}^{\infty} \lambda^m \int_a^x K_m(x, t) f(t) dt \\ &= f(x) + \int_a^x \left\{ \sum_{m=1}^{\infty} \lambda^m K_m(x, t) \right\} f(t) dt . \end{aligned}$$

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$$y(x) = f(x) + \lambda \int_a^x R(x,t;\lambda) f(t) dt .$$

where

$$R(x,t;\lambda) = \sum_{m=1}^{\infty} \lambda^{m-1} K_m(x,t) ,$$

and is called the **resolvent kernel**.

Hence the solution of the given integral equation may be written as

$$y(x) = f(x) + \lambda \int_a^x R(x,t;\lambda) f(t) dt.$$

## Reference:

<https://nptel.ac.in/courses/111/107/111107103/>

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