**Course Code : MSCM303** 

**Course Name: Integral equations and calculus of variation** 

Lecture-12

# Solution of Volterra integral equation of second kind by Resolvent kernels

**Example:** Find the resolvent kernel for the Volterra-type integral equation

$$y(x) = (1+x^{2}) + \int_{0}^{x} \left(\frac{1+x^{2}}{1+t^{2}}\right) y(t) dt.$$

and hence determine its solution.

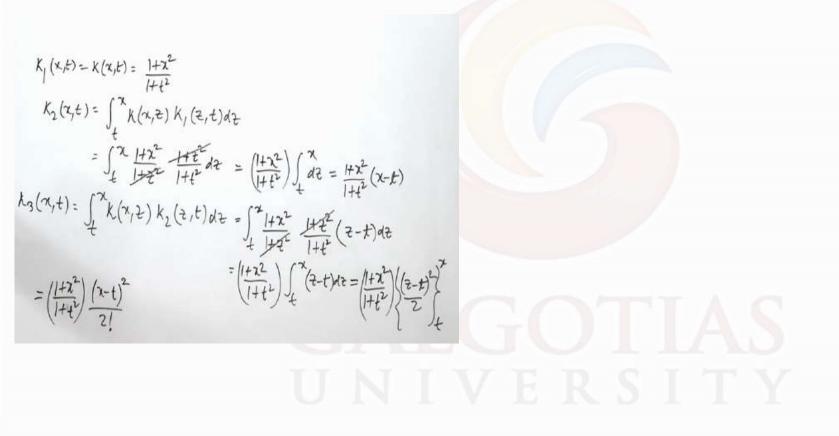
The resolvent kernel for given integral equation (1, 2)

$$R(x,t;\lambda) = \left(\frac{1+x^2}{1+t^2}\right)e^{x-t}$$

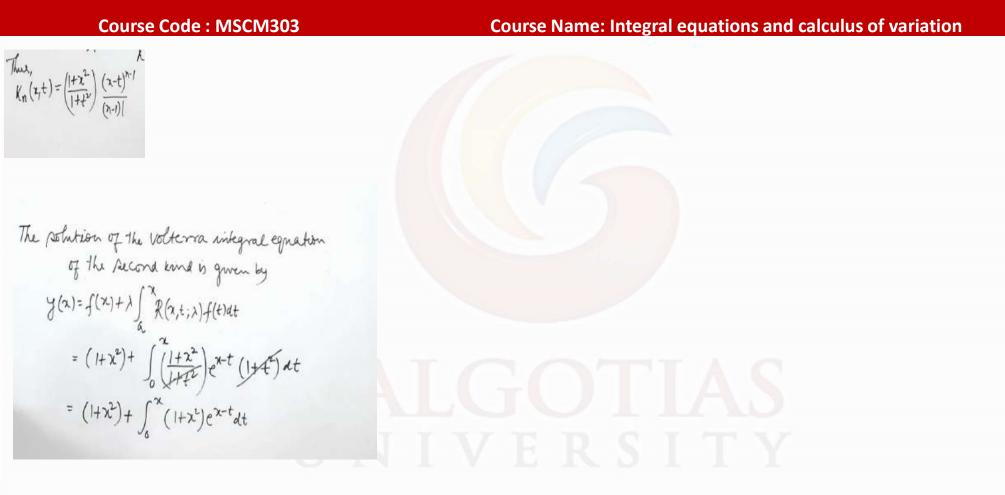
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**Particular form II** : Suppose that the kernel K(x, t) is a polynomial of degree (n - 1) in x, then we can represent it as

$$K(x,t) = b_0(t) + b_1(t)(t-x) + \dots + \frac{b_{n-1}(t)}{(n-1)!}(t-x)^{n-1}.$$
 ...(3)

Then the resolvent kernel  $R(x, t; \lambda)$  of (1) is given by

$$R(x,t;\lambda) = -\frac{1}{\lambda} \frac{d^n}{dt^n} h(x,t;\lambda)$$

where  $h(x, t; \lambda)$  is a solution of the differential equation

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$$\frac{d^{n}h}{dt^{n}} + \lambda \left[ b_{0}(t) \frac{d^{n-1}h}{dt^{n-1}} + b_{1}(t) \frac{d^{n-2}h}{dt^{n-2}} + \dots + b_{n-1}(t)h \right] = 0,$$

satisfying the conditions

$$h|_{t=x} = \frac{dh}{dt}\Big|_{t=x} = \dots = \frac{d^{n-2}h}{dt^{n-2}}\Big|_{t=x} = 0; \frac{d^{n-1}h}{dt^{n-1}}\Big|_{t=x} = 1.$$

The required solution is given by

$$y(x) = f(x) + \int^{x} R(x,t;\lambda)f(t)dt.$$

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Example: Consider  

$$y(x) = (\cos x - x - 2) + \int_{a}^{x} (t - x)y(t)dt.$$

## **Reference:**

https://nptel.ac.in/courses/111/107/111107103/



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