School of Computing Science and Engineering

Course Code : MATH2007 **Course Name: Discrete Mathematics**

Group

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Group

A non empty set G together with an operation ‘o’ is called a group if the following conditions are satisfied :

1. Closure axiom, $\forall\ a, b \in G \Rightarrow aob \in G$.
2. Associative axiom, $aob\ oc = ao(boc)\ \forall\ a, b, c \in G$
3. Existence of identity, $\exists$ an element $e \in G$, called identity $aoe = eoa = a\ \forall\ a \in G$.
4. Existence of inverse, $a \in G$, $\exists\ a^{-1} \in G\ s.t\ a^{-1}oa = aoa^{-1} = e$ This a^{-1} is called inverse of a.

Abelian Group

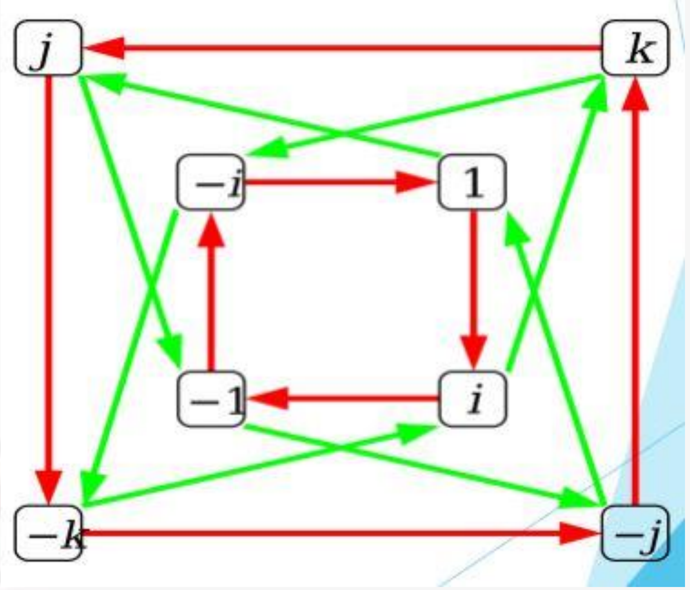
A group (G, o) is called abelian group or commutative group if aob = boa $\forall$ $a, b \in G$.

Examples :-

1. $(\mathbb{Z}, +)$, $(\mathbb{Q}, +)$, $(\mathbb{R}, +)$ all are abelian group.
2. $(\mathbb{Q}, \cdot)$, $(\mathbb{R}, \cdot)$ are commutative group. 1 is an identity ,$1/a$ is the inverse of a in each case.
3. The set of all $m \times n$ matrices (real and complex) with matrix addition as a binary operation is commutative group. The zero matrix is the identity element and the inverse of matric of A is –A.

Quaternion Group

G = { ±1, $\pm i$, $\pm j$, $\pm k$ } define a binary operation of multiplication as $i^2 = j^2 = k^2 = -1$, $ij = -jk = k$, $ki = -ik = j$, $jk = -kj = i$.



The red arrows represent multiplication on the right by i, and the green arrows represent multiplication on the right by j.

This is non abelian group for this operation. This is called Quaternion group.

Klein's four group

Let G = (e, a, b, c) with operation o defined by the following table :

O	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Theorem :- Uniqueness of identity

Statement - The identity e in a group always unique.
Proof - If possible, suppose that e and e' are two identity elements in a group G.
'e' is an identity element
$\Rightarrow ee' = e'e = e'$ [$ae = ea$ = a]
e' is an identity element
$\Rightarrow ee' = e'e = e$ [$ae' = e'a = a$]
these statements prove that $e = ee' = e'e = e'$
from which, we get $e = e'$.
Hence Proved

The cancellation laws

Statement - Suppose, a, b, c are arbitrary elements of a group G.

Then 1. $ab = ac \Rightarrow b = c$ (left cancellation)

2. $ba = ca \Rightarrow b = c$ (right cancellation)

Proof - Let e be the identity element in a group G.

Let $a, b, c \in G$ be arbitrary

$ab = ac$

$\Rightarrow a^{-1}\, ab = a^{-1}\,(ac)$

$\Rightarrow a^{-1}a\, b = a^{-1}a\, c$ [by associative law]

$\Rightarrow eb = ec$

$\Rightarrow b = c$

Again

$ba = ca$

$\Rightarrow ba\ a^{-1} = ca\ a^{-1}$

$\Rightarrow b\ aa^{-1} = c\ aa^{-1}$

$\Rightarrow be = ce$

$\Rightarrow b = c$

References:

1. Contemporary Abstract Algebra – Gallian
2. A Course in Abstract Algebra: Vijay K. Khanna and S. K. Bhambri