Strong-Field Stark Effect in Hydrogen Atom

Stark effect, the splitting of spectral lines observed when the radiating atoms, ions, or molecules are subjected to a strong electric field. The electric analogue of the Zeeman effect (*i.e.*, the magnetic splitting of spectral lines), it was discovered by a German physicist, Johannes Stark (1913).

Strong-Field Stark Effect

Earlier experimenters had failed to maintain a strong electric field in conventional spectroscopic light sources because of the high electrical conductivity of luminous gases or vapours. Stark observed the hydrogen spectrum emitted just behind the perforated cathode in a positive-ray tube.

Strong-Field Stark Effect

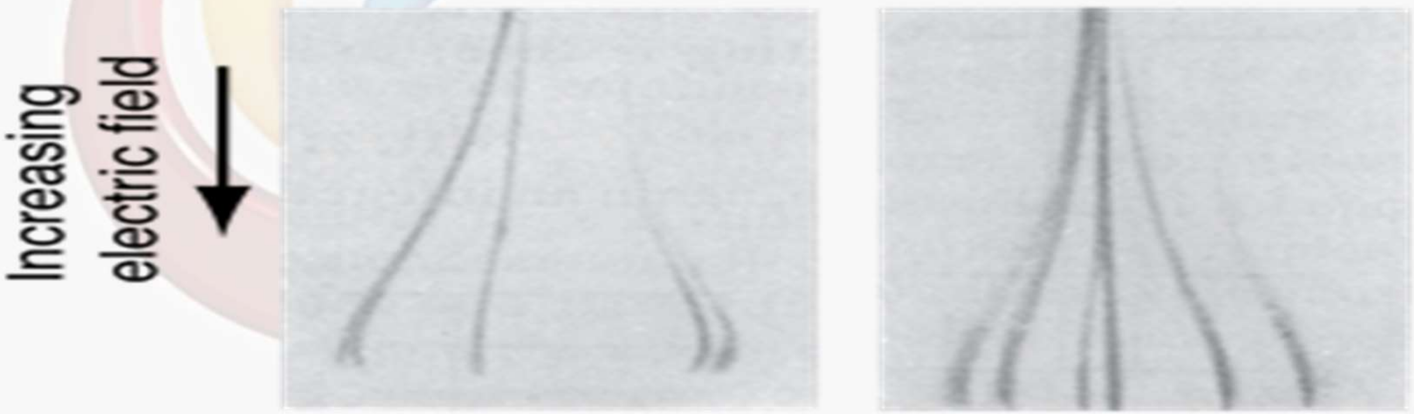
With a second charged electrode parallel and close to this cathode, he was able to produce a strong electric field in a space of a few millimeters. At electric field intensities of 100,000 volts per centimeter, Stark observed with a spectroscope that the characteristic spectral lines, called Balmer lines, of hydrogen were split into a number of symmetrically spaced components, some of which were linearly polarized (vibrating in one plane) with the electric vector parallel to the lines of force, the remainder being polarized perpendicular to the direction of the field except when viewed along the field.

Strong-Field Stark Effect

This transverse Stark effect resembles in some respects the transverse Zeeman effect, but, because of its complexity, the Stark effect has relatively less value in the analysis of complicated spectra or of atomic structure. Historically, the satisfactory explanation of the Stark effect (1916) was one of the great triumphs of early quantum mechanics.

Stark Effect in Atomic Spectra

Stark effect splitting of the helium transition at 438.8 nm.



Light polarized parallel to field

Light polarized perpendicular to electric field

Foster, J. S., J. Frank. Inst. 209, 585, (1930)

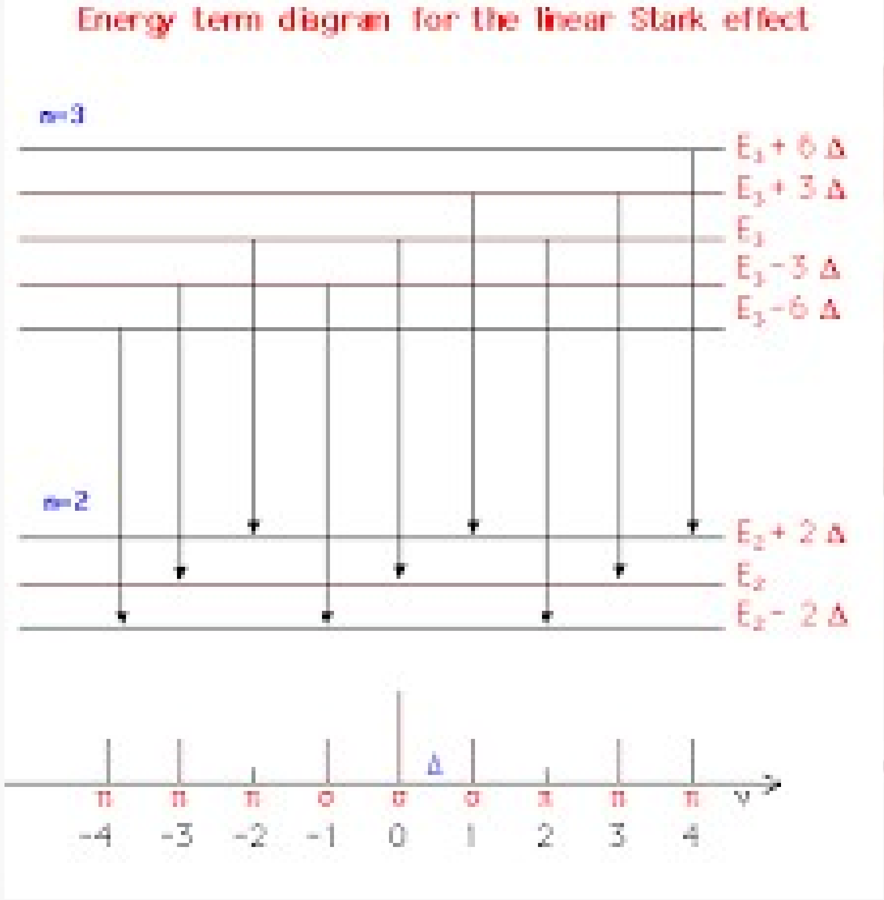
Stark Effect in Atomic Spectra

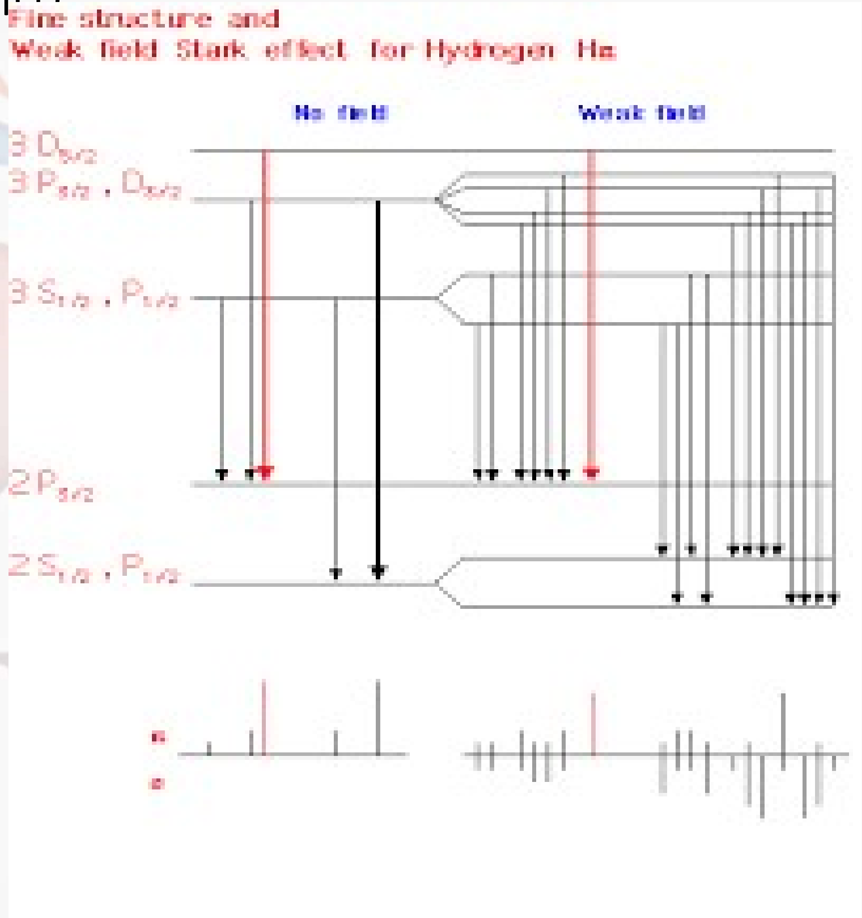
The splitting of atomic spectral lines as a result of an externally applied electric field was discovered by Stark, and is called the Stark effect. As the splitting of a line of the helium spectrum shows, the splitting is not symmetric like that of the Zeeman effect.

The splitting of the energy levels by an electric field first requires that the field polarizes the atom and then interacts with the resulting electric dipole moment. That dipole moment depends upon the magnitude of M_j, but not its sign, so that the energy levels show splitting proportional to quantum numbers J+1 or J+1/2, for integer and half-integer spins respectively.

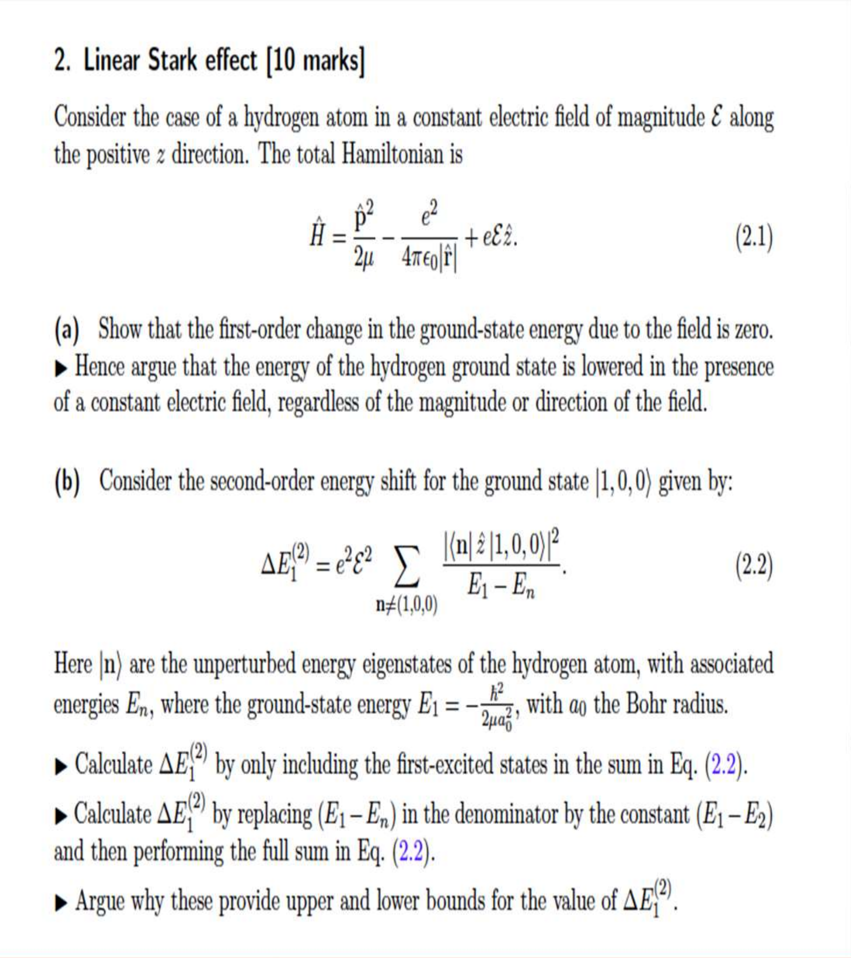
The Stark effect has been of marginal benefit in the analysis of atomic spectra, but has been a major tool for molecular rotational spectra.

Stark Effect(Strong Field)





Stark Effect(Strong Field)



2. Linear Stark effect [10 marks]

Consider the case of a hydrogen atom in a constant electric field of magnitude $\mathcal{E}$ along the positive z direction. The total Hamiltonian is

$$\hat{H} = \frac{\hat{\mathrm{p}}^2}{2\mu} - \frac{e^2}{4\pi\epsilon_0|\hat{\mathrm{r}}|} + e\mathcal{E}\hat{z}. \qquad (2.1)$$

(a) Show that the first-order change in the ground-state energy due to the field is zero.

▸ Hence argue that the energy of the hydrogen ground state is lowered in the presence of a constant electric field, regardless of the magnitude or direction of the field.

(b) Consider the second-order energy shift for the ground state $|1,0,0\rangle$ given by:

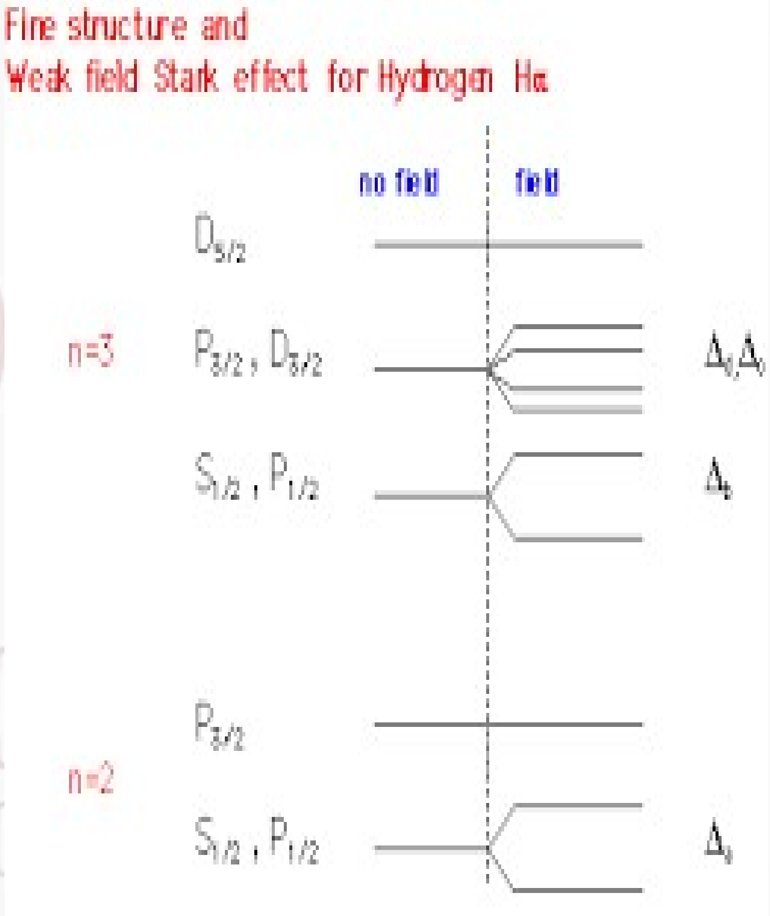
$$\Delta E_1^{(2)} = e^2\mathcal{E}^2 \sum_{\mathrm{n}\neq(1,0,0)} \frac{|\langle \mathrm{n}|\,\hat{z}\,|1,0,0\rangle|^2}{E_1 - E_n}. \qquad (2.2)$$

Here $|\mathrm{n}\rangle$ are the unperturbed energy eigenstates of the hydrogen atom, with associated energies E_n, where the ground-state energy $E_1 = -\frac{\hbar^2}{2\mu a_0^2}$, with a_0 the Bohr radius.

▸ Calculate $\Delta E_1^{(2)}$ by only including the first-excited states in the sum in Eq. (2.2).

▸ Calculate $\Delta E_1^{(2)}$ by replacing $(E_1 - E_n)$ in the denominator by the constant $(E_1 - E_2)$ and then performing the full sum in Eq. (2.2).

▸ Argue why these provide upper and lower bounds for the value of $\Delta E_1^{(2)}$.



Stark Effect(Strong Field)

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Light polarized parallel to field

Light polarized perpendicular to electric field

Foster, J. S., J. Frank. Inst. 209, 585, (1930)

References:

1. Raj Kumar, Atomic and Molecular Spectra: Laser, Kedear Nath Publication, Meerut, 2015
2. Arthur Beiser, Concepts of Modern Physics, 6th ed.,McGraw Hill, New Delhi, 2008.
3. Harvey Elliott White, Introduction to Atomic Spectra, McGraw Hill, 1963.
4. G M Barrow, Introduction to molecular spectroscopy, Tata McGraw Hill,Japan, 1962.
5. Internet

http://hyperphysics.phy-astr.gsu.edu/hbase/Atomic/stark.html