THE COMPTON EFFECT

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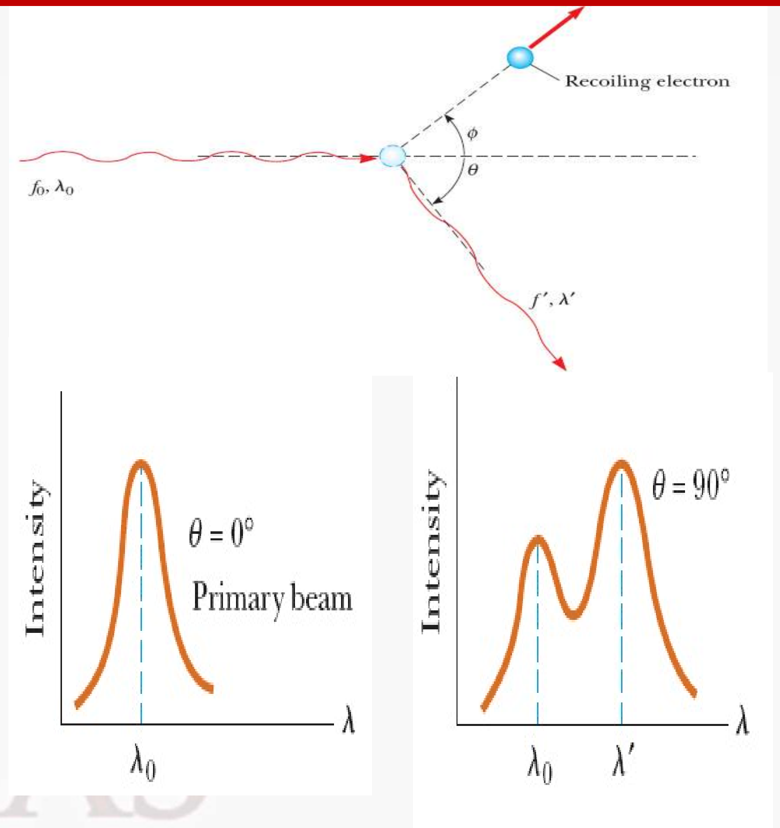
THE COMPTON EFFECT

What is Compton Effect ?

Arthur H. Compton (1923) measured intensity of scattered X-rays from solid target (scattering of X-rays from electrons), as function of wavelength for different angles.

In such a scattering, a shift in wavelength for the scattered X-rays takes place, which is known as Compton Effect.



THE COMPTON EFFECT

Classical Predictions

Oscillating electromagnetic waves of frequency f_0 incident on electrons should have two effects:

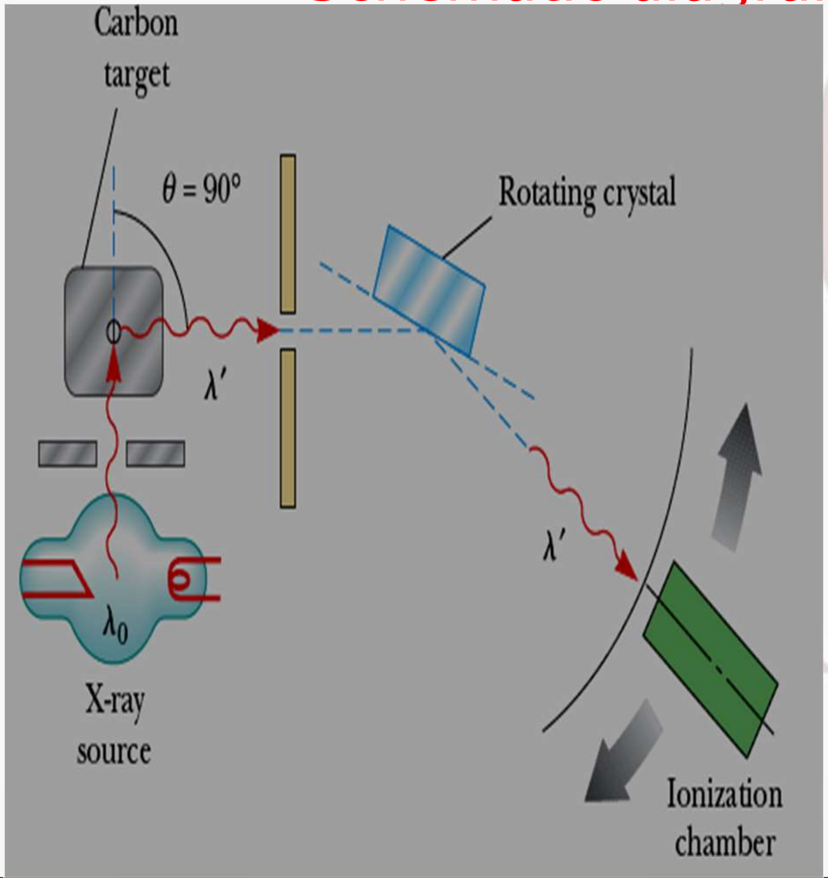
(a) oscillating electromagnetic field causes oscillations in electrons, which re-radiate in all directions

(b) radiation pressure should cause the electrons to accelerate in the direction of propagation of the waves.

Because different electrons will move at different speeds after the interaction, depending on the amount of energy absorbed from em waves, for a particular angle of incidence of the incoming radiation, the scattered wave frequency should show a distribution of Doppler-shifted values.

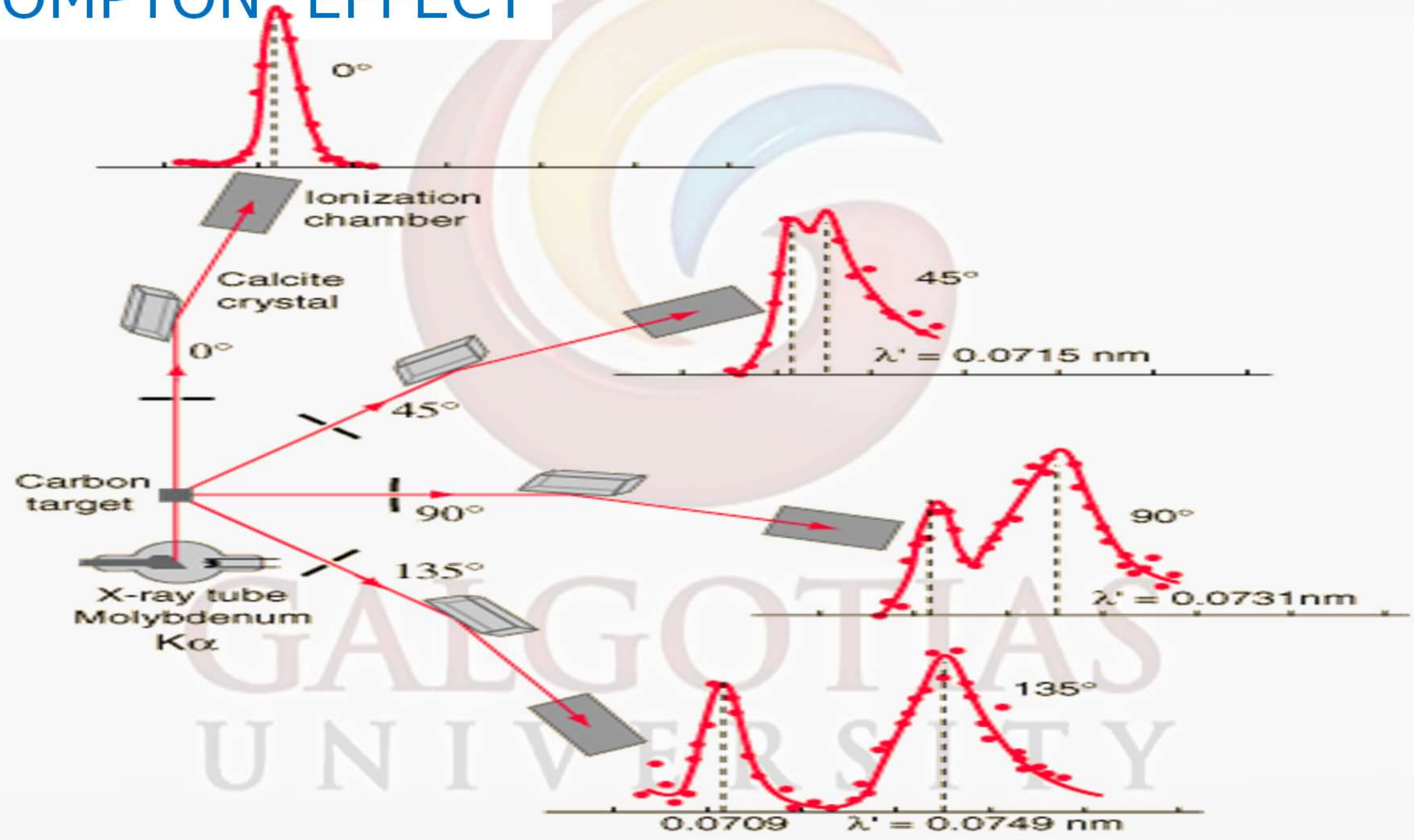
THE COMPTON EFFECT

Schematic diagram of Compton's apparatus



Here X- ray photons are scattered through 90° from a carbon (graphite) target. The wavelength is measured with a rotating crystal spectrometer using Bragg's law. Intensity of the scattered X-rays are measured using the ionization chamber.

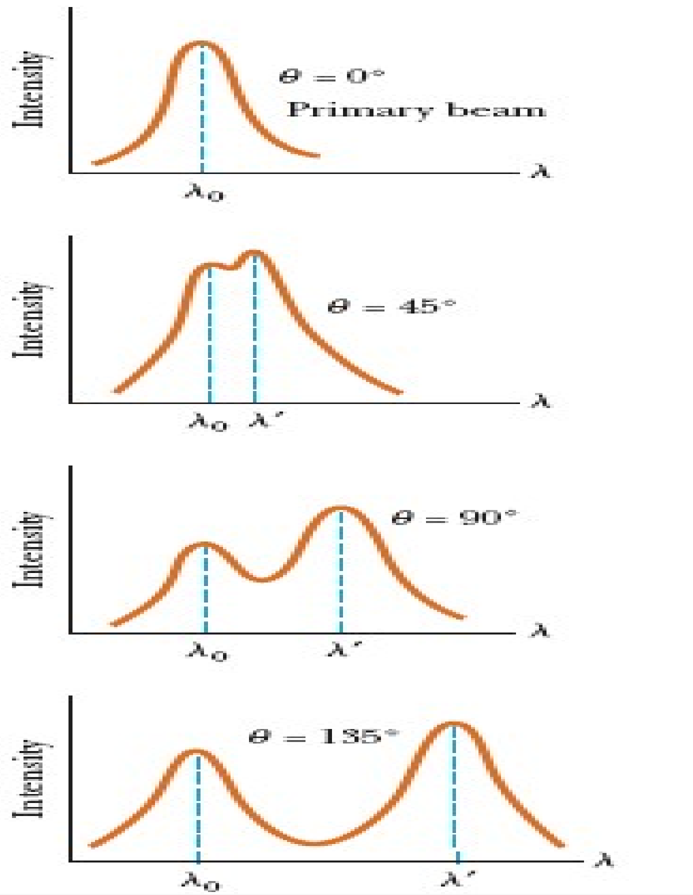
THE COMPTON EFFECT



THE COMPTON EFFECT

Experimental Observations

Contrary to the classical predictions where X-rays are treated as waves, in Compton experiment, at a given angle, only one frequency for scattered radiation is seen. This is shown in the figure, scattered x-ray intensity versus wavelength for Compton scattering at $\theta = 0°, 45°, 90°$, and $135°$



Compton could explain the experimental result by taking a "billiard ball" collisions between particles of light (X-ray photons) and electrons in the material.

THE COMPTON EFFECT

The graphs for three nonzero angles show two peaks, one at λ_0 and one at $\lambda' > \lambda_0$. The shifted peak at λ' is caused by the scattering of X-rays from free electrons. Shift in wavelength was predicted by Compton to depend on scattering angle as

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

This is known as Compton shift equation, and the factor

$$\frac{h}{m_e c} = 0.00243 \text{ nm}$$

is called the Compton wavelength.

Prediction were in excellent agreement with the experimental results.

THE COMPTON EFFECT

Relativistic formula relating energy and momentum of a particle having rest mass m

$$E^2 = p^2c^2 + m^2c^4$$

Relativistic expression for the momentum of a particle

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \textit{Lorentz factor}$$

$$P = \gamma m v$$

Relativistic expression for kinetic energy of a particle

$$K = (\gamma - 1) m c^2$$

For Photon $m = 0, \quad p = \frac{E}{c} = \frac{hf}{\lambda f} = \frac{h}{\lambda}$

Also we have, $E = K + m c^2$

Thus, $E = \gamma m c^2$

THE COMPTON EFFECT

FROM THE LAW OF CONSERVATION OF ENERGY

$$E_o + mc^2 = E' + E_e$$

where *m* is the rest mass of electron

$$E_o - E' + mc^2 = E_e \qquad \text{where } E_e^2 = p_e^2 c^2 + m^2c^4$$

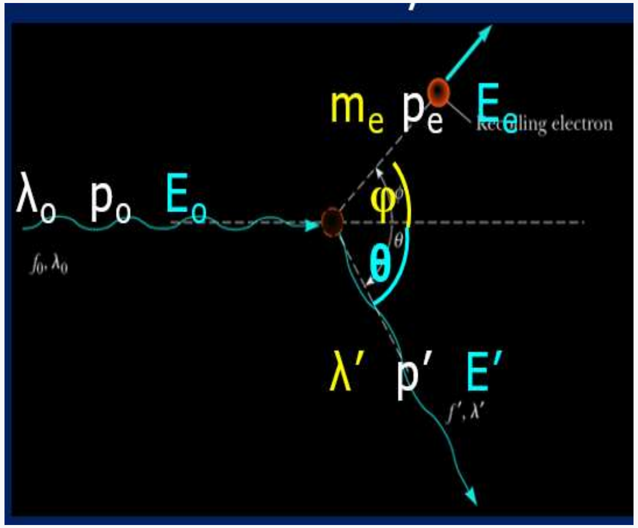
SQUARING

$$(E_o - E')^2 + m^2c^4 + 2(E_o - E')mc^2 = p_e^2 c^2 + m^2c^4$$

FROM THE LAW OF CONSERVATION OF MOMENTUM

x-COMPONENT, $p_o = p' \cos\theta + p_e \cos\varphi$

y-COMPONENT $0 = p' \sin\theta - p_e \sin\varphi$



Momentum vectors for the Compton scattering

THE COMPTON EFFECT

$$p_0 - p' \cos\theta = p_e \cos\varphi$$
$$p' \sin\theta = p_e \sin\varphi$$

SQUARE AND ADD

$$p_0^2 - 2p_0p' \cos\theta + p'^2 = p_e^2$$

SUBSTITUTE

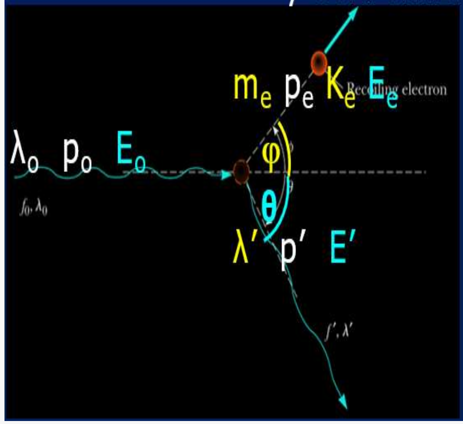
SQUARING

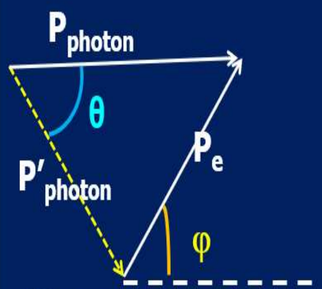
$$(E_0 - E')^2 + m^2c^4 + 2(E_0 - E')mc^2 = p_e^2c^2 + m^2c^4$$

$$(E_0 - E')^2 + 2(E_0 - E')mc^2 = [p_0^2 - 2p_0p' \cos\theta + p'^2]\, c^2$$

X-COMPONNET $p_0 = p' \cos\theta + p_e \cos\varphi$

Y-COMPONNET $0 = p' \sin\theta - p_e \sin\varphi$





Momentum vectors for the Compton scattering

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$$\left(\frac{\lambda' - \lambda_0}{\lambda_0\lambda'}\right) mc^2 = \frac{hc}{\lambda_0\lambda'}(1 - \cos\theta)$$

$$\lambda' - \lambda_0 = \frac{h}{mc}(1 - \cos\theta)$$

Equation is called the **Compton shift equation**.

$$(E_0 - E')^2 + 2(E_0 - E')mc^2 = [p_0^2 - 2p_0p' \cos\theta + p'^2]\, c^2$$

$$\left(\frac{hc}{\lambda_0} - \frac{hc}{\lambda'}\right)^2 + 2\left(\frac{hc}{\lambda_0} - \frac{hc}{\lambda'}\right) mc^2$$

$$= \left(\frac{hc}{\lambda_0}\right)^2 - 2\left(\frac{hc}{\lambda_0}\right)\left(\frac{hc}{\lambda'}\right)\cos\theta + \left(\frac{hc}{\lambda'}\right)^2$$

$$\left(\frac{hc}{\lambda_0}\right)^2 - 2\left(\frac{hc}{\lambda_0}\right)\left(\frac{hc}{\lambda'}\right) + \left(\frac{hc}{\lambda'}\right)^2 + 2hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right) mc^2$$

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The Compton Effect

Summary

X-rays are scattered at various angles by electrons in a target. In such a scattering, a shift in wavelength is observed for the scattered X-rays and the phenomenon is known as Compton effect.

Classical Physics does not predict the correct behaviour in this effect . If X-rays is treated as a photon, conservation of energy and liner momentum applied to the photon-electron collision yields for the Compton shift.

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

Where m_e is the mass of electron, c is the speed of light, and theta is the scattering angle.

Importance of Compton Effect

- In material physics, Compton scattering can be used to probe the wave function of the electrons in matter in the momentum representation.
- Compton scattering is an important effect in gamma spectroscopy which gives rise to the Compton edge, as it is possible for the gamma rays to scatter out of the detectors used.
- Compton scattering is of prime importance to radiobiology . As it happens to be the most probable interaction of high energy X-rays with atomic nuclei in living beings and is applied in radiation therapy.
- Compton scattering is an important effect in gamma spectroscopy which gives rise to the Compton edge, as it is possible for gamma rays to scatter out of the detectorsused.
- Compton suppression is used to detect stry scatter gamma rays to counteract this effect

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