

The logo of Galgotias University is a stylized circular emblem with three curved, overlapping bands in shades of yellow, blue, and red, resembling a globe or a dynamic swirl.

UNIT III

Fuzzy Set Theory

GALGOTIAS
UNIVERSITY

Introduction

- The word “fuzzy” means “vagueness (ambiguity)”.
- Fuzziness occurs when the boundary of a piece of information is not clear-cut.
- Fuzzy sets - 1965 Lotfi Zadeh as an extension of classical notation set.
- Classical set theory allows the membership of the elements in the set in **binary terms**.
- Fuzzy set theory permits membership function valued in the interval $[0,1]$.

GALGOTIAS
UNIVERSITY

Fuzzy Set

Example:

Words like young, tall, good or high are fuzzy.

- There is no single quantitative value which defines the term young.
- For some people, age 25 is young, and for others, age 35 is young.
- The concept young has no clean boundary.
- Age 35 has some possibility of being young usually depends on the context in which it is being considered.

□ Fuzzy set theory is an extension of classical set theory where elements have degree of membership.

□

□

Fuzzy Set

Introduction

Classical set theory

- Classes of objects with sharp boundaries.
- A classical set is defined by crisp(exact) boundaries, i.e., there is no uncertainty about the location of the set boundaries.
- Widely used in digital system design

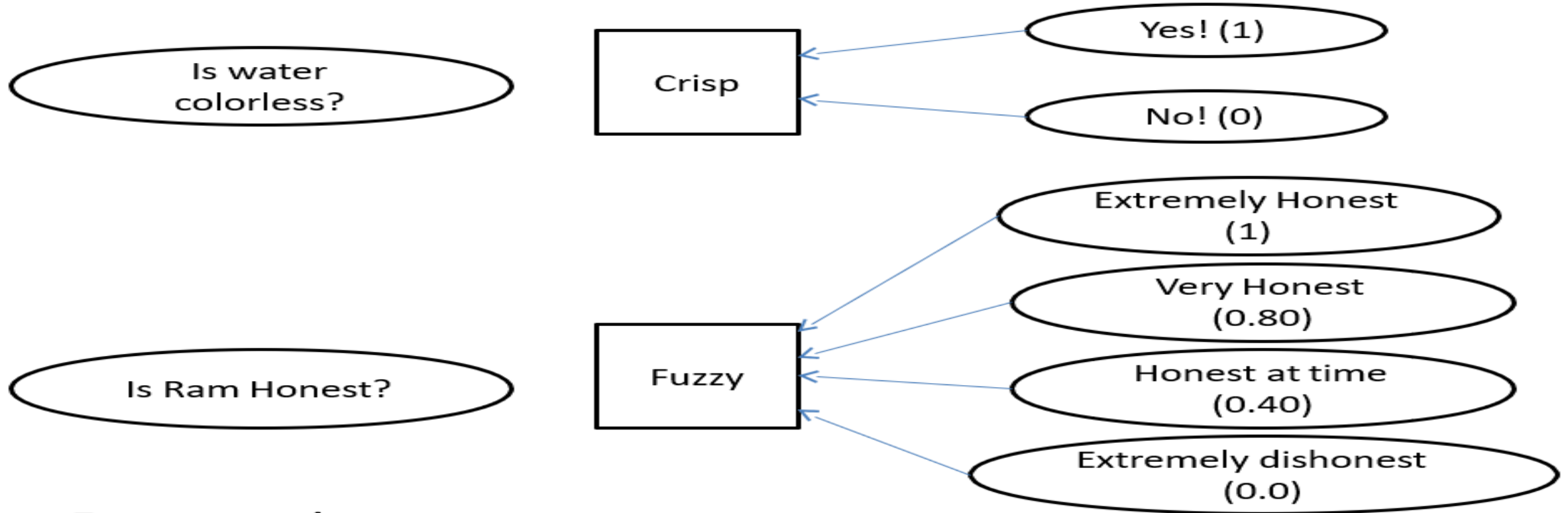
Fuzzy set theory

- Classes of objects with un-sharp boundaries.
- A fuzzy set is defined by its ambiguous boundaries, i.e., there exists uncertainty about the location of the set boundaries.
- Used in fuzzy controllers.

Fuzzy Set

Introduction (Continue)

Example



Fuzzy vs crips



Classical Set Theory

Classical set theory

- A Set is any well defined collection of objects.
- An object in a set is called an element or member of that set.
- Sets are defined by a simple statement
- Describing whether a particular element having a certain property belongs to that particular set.

$$A = \{a_1, a_2, a_3, \dots, a_n\}$$

- If the elements a_i ($i = 1, 2, 3, \dots, n$) of a set A are subset of universal set X , then set A can be represented for all elements $x \in X$ by its characteristics function

$$\mu_A(x) = 1 \text{ if } x \in X \text{ otherwise } 0$$



Operations on classical set theory

Union: the union of two sets A and B is given as

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$

Intersection: the intersection of two sets A and B is given as $A \cap B = \{ x \mid x \in A$
and $x \in B \}$

□

Complement: It is denoted by \tilde{A} and is defined as

$$\tilde{A} = \{ x \mid x \text{ does not belongs } A \text{ and } x \in X \}$$

□

□

Fuzzy Set

Fuzzy Sets

- **Fuzzy Logic** is derived from fuzzy set theory
- Many degree of membership (between 0 to 1) are allowed.
- Thus a membership function $\mu_A(x)$ is associated with a fuzzy
- Sets \tilde{A} such that the function maps every universe of discourse X to the interval $[0,1]$.
- The mapping is written as: $\mu_{\tilde{A}}(x): X \rightarrow [0,1]$.
- Fuzzy Logic is capable of handling inherently (vague or inexact or rough or inaccurate) concepts



Fuzzy Set

There are different shapes of membership functions;

- **Triangular**
- **Trapezoidal**
- **Gaussian etc**



GALGOTIAS
UNIVERSITY

Fuzzy Sets (Continue)

- **Trapezoid membership function**
- A *trapezoidal* membership function is specified by four parameters {a, b, c, d} as follows:

$$\mu_A(x) = \left\{ \begin{array}{ll} 0 & \text{if } x \leq a \\ \frac{x - a}{b - a} & \text{if } a \leq x \leq b \\ 1 & \text{if } b \leq x \leq c \\ \frac{d - x}{d - c} & \text{if } c \leq x \leq d \\ 0 & \text{if } d \leq x \end{array} \right.$$

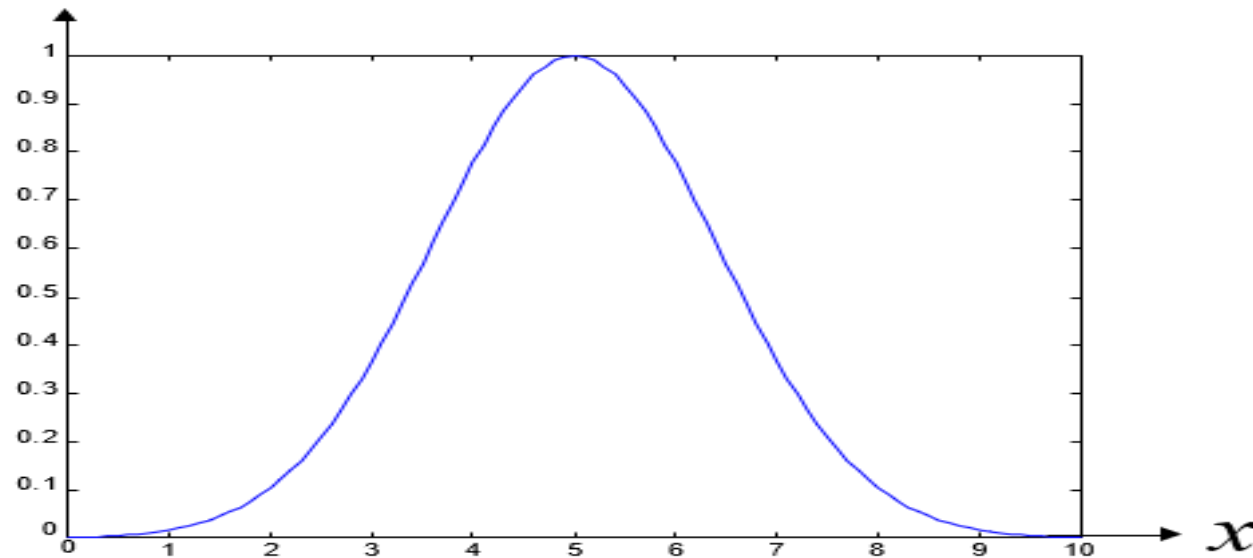
Fuzzy Set

- **Gaussian membership function**

$$\mu_A(x, c, s, m) = \exp\left\{-\frac{1}{2}\left|\frac{x-c}{s}\right|^m\right\}$$

- c : centre
- s : width
- m : fuzzification factor (e.g., $m=2$)

$\mu_A(x)$

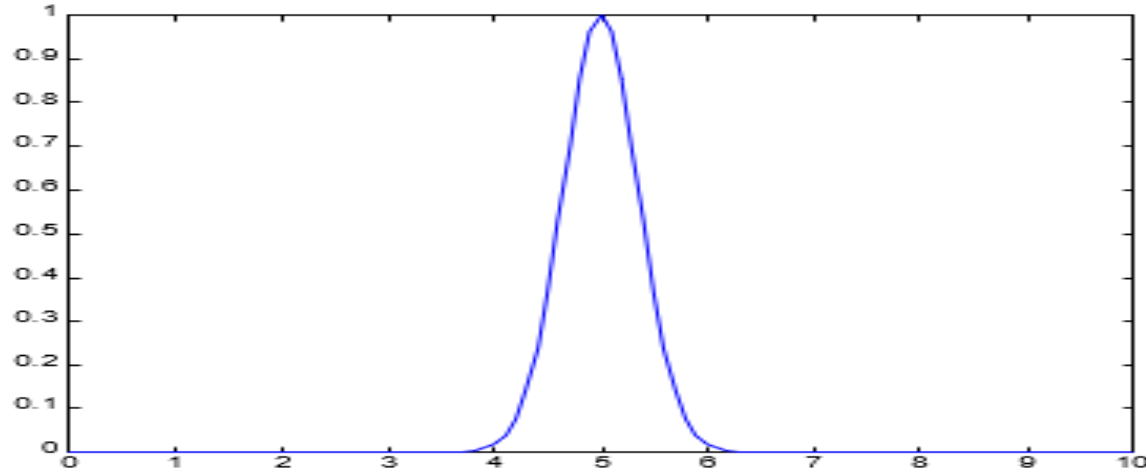


$c=5$

$s=2$

$m=2$

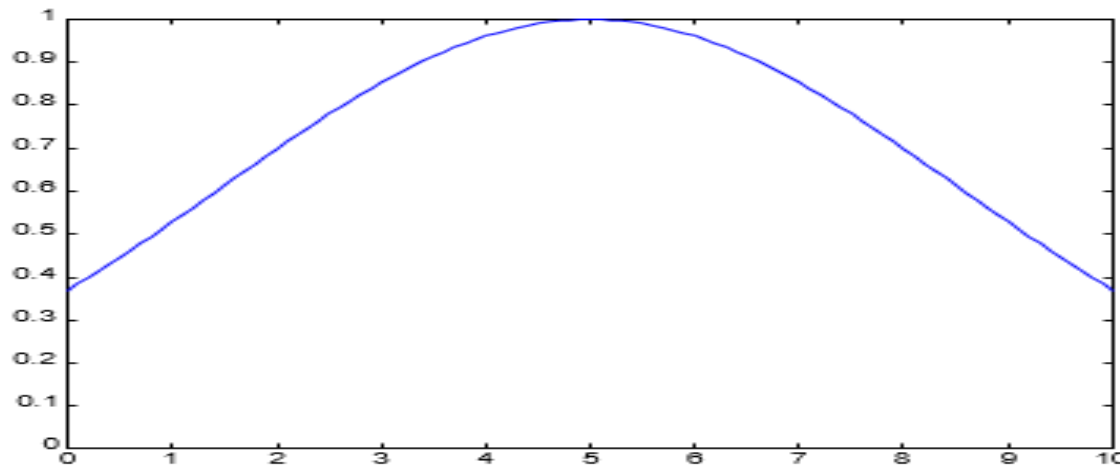
Fuzzy Set



$$c=5$$

$$s=0.5$$

$$m=2$$



$$c=5$$

$$s=5$$

$$m=2$$

Fuzzy Set

Fuzzy Set Operation

Given X to be the universe of discourse and \tilde{A} and \tilde{B} to be fuzzy sets with $\mu_A(x)$ and $\mu_B(x)$ are their respective membership function, the fuzzy set operations are as follows:

Union:

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

Intersection:

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

Complement:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

Fuzzy Set Operation

Example:

$$A = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0)\}$$

$$B = \{(x_1, 0.8), (x_2, 0.2), (x_3, 1)\}$$

Union:

$$A \cup B = \{(x_1, 0.8), (x_2, 0.7), (x_3, 1)\}$$

Because

$$\begin{aligned}\mu_{A \cup B}(x_1) &= \max(\mu_A(x_1), \mu_B(x_1)) \\ &= \max(0.5, 0.8) \\ &= 0.8\end{aligned}$$



$$\mu_{A \cup B}(x_2) = 0.7 \quad \text{and} \quad \mu_{A \cup B}(x_3) = 1$$



Fuzzy Interface

Mamdani fuzzy inference

- The Mamdani-style fuzzy inference process is performed in four steps:
- Fuzzification of the input variables
- Rule evaluation
- Aggregation of the rule outputs, and finally
- Defuzzification



GALGOTIAS
UNIVERSITY



Thank You