Unit 2:
L-2
Statistical Quality Control

Learning Objectives

A **control chart** is a graphical tool for monitoring the activity of an ongoing process. Control charts are sometimes referred to as **Shewhart control charts,** because Walter A. Shewhart first proposed their general theory. The values of the quality characteristic are plotted along the vertical axis, and the horizontal axis represents the samples, or subgroups (in order of time), from which the quality characteristic is found.

What are Control Charts?

- This versatile data collection and analysis tool can be used by a variety of industries and is considered one of the seven basic quality tools.
- The control chart is a graph used to study how a process changes over time. Data are plotted in time order. A control chart always has a central line for the average, an upper line for the upper control limit, and a lower line for the lower control limit.
- These lines are determined from historical data. By comparing current data to these lines, one can draw conclusions about whether the process variation is consistent (in control) or is unpredictable (out of control, affected by special causes of variation).
- It depicts whether there is any change on the characteristics of items since the start production run.

When To Use Control Charts:-

- When controlling ongoing processes by finding and correcting problems as they occur.
- When predicting the expected range of outcomes from a process.
- When determining whether a process is stable (in statistical control).
- When analyzing patterns of process variation from special causes (non-routine events) or common causes (built into the process).

ADVANTAGES :-

- It determines processes variability and detects unusual variations taking place in a process.
- It ensures product quality level.
- It builds up the reputation of the organization through customer's satisfaction.
- Provides information about the selection of process and setting of tolerance limits.

BASIC PROCEDURE:-

- Choose the appropriate control chart for given data.
- Determine the appropriate time period for collecting and plotting data.
- Collect data, construct chart and analyze the data.
- Look for "out-of-control signals" on the control chart. When one is identified, mark it on the chart and investigate the cause. Document investigation, learning, the cause and how it was corrected.
- Continue to plot data as they are generated. As each new data point is plotted, check for new out-of-control signals.
- When a new control chart is started, the process may be out of control. If so, the control limits calculated from the first 20 points are conditional limits. When we have at least 20 sequential points from a period when the process is operating in control, recalculate control limits.

TYPES OF CONTROL CHARTS:-

- **Variable** or **measurement** charts-
  - -)X-bar charts.
  - -)R-bar charts.
  - -)S-charts.
- **Attribute** charts-
  - -)'p' and 'np' charts.

AN EXAMPLE:-

Table 5-1 Flow Width Measurements (microns) for the Hard-Bake Process

Sample Number	Wafers 1	2	3	4	5	$\bar{x}_i$	R_i
1	1.3235	1.4128	1.6744	1.4573	1.6914	1.5119	0.3679
2	1.4314	1.3592	1.6075	1.4666	1.6109	1.4951	0.2517
3	1.4284	1.4871	1.4932	1.4324	1.5674	1.4817	0.1390
4	1.5028	1.6352	1.3841	1.2831	1.5507	1.4712	0.3521
5	1.5604	1.2735	1.5265	1.4363	1.6441	1.4882	0.3706
6	1.5955	1.5451	1.3574	1.3281	1.4198	1.4492	0.2674
7	1.6274	1.5064	1.8366	1.4177	1.5144	1.5805	0.4189
8	1.4190	1.4303	1.6637	1.6067	1.5519	1.5343	0.2447
9	1.3884	1.7277	1.5355	1.5176	1.3688	1.5076	0.3589
10	1.4039	1.6697	1.5089	1.4627	1.5220	1.5134	0.2658
11	1.4158	1.7667	1.4278	1.5928	1.4181	1.5242	0.3509
12	1.5821	1.3355	1.5777	1.3908	1.7559	1.5284	0.4204
13	1.2856	1.4106	1.4447	1.6398	1.1928	1.3947	0.4470
14	1.4951	1.4036	1.5893	1.6458	1.4969	1.5261	0.2422
15	1.3589	1.2863	1.5996	1.2497	1.5471	1.4083	0.3499
16	1.5747	1.5301	1.5171	1.1839	1.8662	1.5344	0.6823
17	1.3680	1.7269	1.3957	1.5014	1.4449	1.4874	0.3589
18	1.4163	1.3864	1.3057	1.6210	1.5573	1.4573	0.3153
19	1.5796	1.4185	1.6541	1.5116	1.7247	1.5777	0.3062
20	1.7106	1.4412	1.2361	1.3820	1.7601	1.5060	0.5240
21	1.4371	1.5051	1.3485	1.5670	1.4880	1.4691	0.2185
22	1.4738	1.5936	1.6583	1.4973	1.4720	1.5390	0.1863
23	1.5917	1.4333	1.5551	1.5295	1.6866	1.5592	0.2533
24	1.6399	1.5243	1.5705	1.5563	1.5530	1.5688	0.1156
25	1.5797	1.3663	1.6240	1.3732	1.6887	1.5264	0.3224
						$\Sigma\bar{x}_i = 37.6400$	$\Sigma R_i = 8.1302$
						$\bar{\bar{x}} = 1.5056$	$\bar{R} = 0.32521$

$$\overline{R} = \frac{\sum_{i=1}^{25} R_i}{25} = \frac{8.1302}{25} = 0.32521$$

$$\text{LCL} = \overline{R} D_3 = 0.32521(0) = 0$$

$$\text{UCL} = \overline{R} D_4 = 0.32521(2.114) = 0.68749$$

$$\overline{\overline{x}} = \frac{\sum_{i=1}^{25} \overline{x}_i}{25} = \frac{37.6400}{25} = 1.5056$$

$$\text{UCL} = \overline{\overline{x}} + A_2 \overline{R} = 1.5056 + (0.577)(0.32521) = 1.69325$$

$$\text{LCL} = \overline{\overline{x}} - A_2 \overline{R} = 1.5056 - (0.577)(0.32521) = 1.31795$$

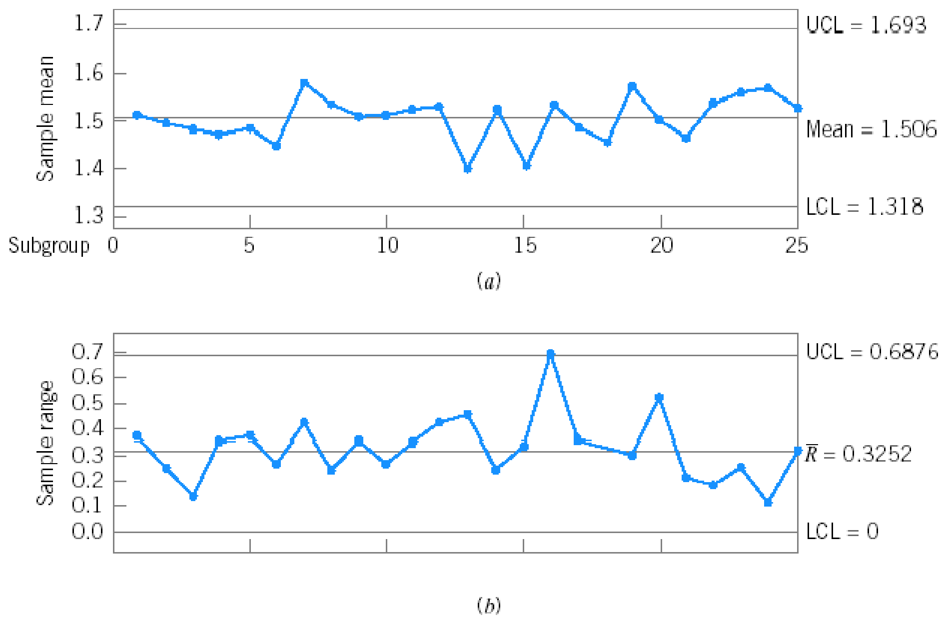

Figure 5-2 $\bar{x}$ and R charts (from Minitab) for flow width in the hard-bake process.

$$\hat{\sigma} = \frac{\overline{R}}{d_2} = \frac{0.32521}{2.326} = 0.1398 \text{ microns}$$

$$\begin{aligned} p &= P\{x < 1.00\} + P\{x > 2.00\} \\ &= \Phi\left(\frac{1.00 - 1.5056}{0.1398}\right) + 1 - \Phi\left(\frac{2.00 - 1.5056}{0.1398}\right) \\ &= \Phi(-3.61660) + 1 - \Phi(3.53648) \\ &\simeq 0.00015 + 1 - 0.99980 \\ &\simeq 0.00035 \end{aligned}$$

(Assume spec tolerance is 1.5 +/- 0.5 micron. Nonconformance probability)

C_p: Process Capability Ratio (PCR)

$$C_p = \frac{USL - LSL}{6\sigma} \qquad (5\text{-}11)$$

Note: 6σ spread is the basic definition of process capability. 3σ above mean and 3σ below.

If σ is unknown, we can use $\hat{\sigma}=^{R}$. $\hat{\sigma}$ in the example is 0.1398.

$$\hat{C}_p = \frac{2.00 - 1.00}{6(0.1398)} = \frac{1.00}{0.8388} = 1.192$$

$$P = \left(\frac{1}{C_p}\right)100\%$$

is simply the percentage of the specification band that the process uses up. For the hard-bake process an estimate of P is

$$\hat{P} = \left(\frac{1}{\hat{C}_p}\right)100\% = \left(\frac{1}{1.192}\right)100\% = 83.89$$

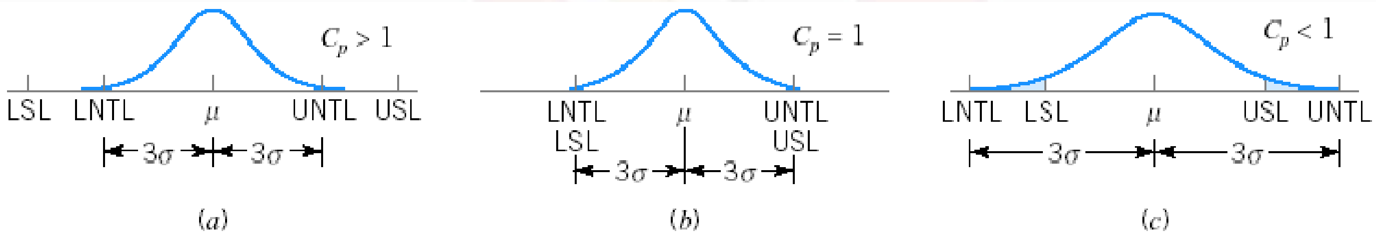

Figure 5-3 Process fallout and the process capability ratio C_p.

Revision of Control Limits and Center Lines

- Effective use of control charts requires periodic review and revision of control limits and center lines.
- Sometimes users replace the center line on the x chart with a target value.
- When *R* chart is out of control, out-of-control points are often eliminated to re-compute a revised value of *R* which is used to determine new limits and center line on *R* chart and new limits on x chart.

Summary:

This lecture has introduced the basic concepts of control charts for statistical process control. The benefits that can be derived from using control charts have been discussed. This lecture covers the statistical background for the use of control charts, the selection of the control limits, and the manner in which inferences can be drawn from the charts. The two types of errors that can be encountered in making inferences from control charts are discussed.

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- ASQ(1993). ANSI/ISO/ASQ. *Statistics—Vocabulary and Symbols-Statistical Quality Control,*A3534-2. Milwaukee, Wi: American Society for Quality.
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