

## Lecture-6

### Conversion of integral equations into differential equations

**Example:** Derive the integral equation from the differential equation:

$$y''(x) - \sin x y'(x) + e^x y(x) = x, \text{ where } y(0) = 1, y'(0) = -1. \quad \dots(1)$$

we may write the given differential equation as

$$y''(x) = y'(x) \sin x - e^x y(x) + x,$$

on integrating, we have

$$\int_0^x y''(x) dx = \int_0^x y'(x) \sin x dx - \int_0^x e^x y(x) dx + \left(\frac{x^2}{2}\right)_0^x$$
$$y'(x) = y(x) \sin x - \int_0^x (e^x + \cos x) y(x) dx + \frac{x^2}{2} - 1.$$

## School of Basic and Applied Sciences

Course Code : MSCM303

Course Name: Integral equations and calculus of variation

Again integrating, we get

$$\begin{aligned} [y(x)]_0^x &= \int_0^x y(x) \sin x dx - \int_0^x (x-t)(e^t + \cos t)y(t) dt + \left(\frac{x^3}{6} - x\right)_0^x \\ y(x) &= \frac{x^3}{6} - x + 1 + \int_0^x [-(x-t)(e^t + \cos t) + \sin t] y(t) dt \quad \dots(2) \end{aligned}$$

Which is a Volterra integral equation of second kind.  
Now let us recover IVP from integral equation

Differentiating (2) w. r. t.  $x$ , we get

$$y'(x) = \frac{x^2}{2} - 1 - \int_0^x (e^t + \cos t)y(t) dt + y(x) \sin x$$

# School of Basic and Applied Sciences

Course Code : MSCM303

Course Name: Integral equations and calculus of variation

At  $x = 0$ , we obtain  $y'(0) = -1$ .

Again differentiating with respect to  $x$

$$y''(x) = x - \left[ \int_0^x \frac{\partial}{\partial x} \{ (e^t + \cos t)y(t) \} dt + (e^x + \cos x)y(x) \frac{d}{dx} x - (e^0 + \cos 0)y(0) \frac{d}{dx} 0 \right] + y(x) \cos x + y'(x) \sin x$$

$$\Rightarrow y''(x) - y' \sin x + e^x y = x .$$

GOTIAS  
UNIVERSITY

Name of the Faculty: Dr. Leena Rani

Program Name: M.Sc(Mathematics)

# School of Basic and Applied Sciences

Course Code : MSCM303

Course Name: Integral equations and calculus of variation

## Conversion of integral equation into BVP

Similarly, we can recover the BVP

$$y'' + x y = 1, \quad y(0) = 0, \quad y(1) = 1$$

from the corresponding Fredholm integral equation

$$y(x) = \frac{x(x+1)}{2} + \int_0^1 K(x,t)y(t)dt \quad \dots(1)$$

where

$$K(x,t) = \begin{cases} t^2(1-x), & 0 \leq t < x \\ xt(1-t), & x \leq t \leq 1 \end{cases}$$

GOTIAS  
UNIVERSITY

Name of the Faculty: Dr. Leena Rani

Program Name: M.Sc(Mathematics)

## School of Basic and Applied Sciences

Course Code : MSCM303

Course Name: Integral equations and calculus of variation

Find  $y'(x)$  and  $y''(x)$  of equation (1) we will get the boundary value problem.

### Reference:

<https://nptel.ac.in/courses/111/107/111107103/>

GALGOTIAS  
UNIVERSITY