Intervals:

Open Intervals: Let a and b be any two **finite** real numbers such that $a < b$. Then, the **open** interval is defined as

$(a, b) = \{x \in \boldsymbol{R}: \boldsymbol{a} < x < b\}$ i.e., set of all real numbers lying between a and b but a and b are not included in the set.

....1...............x..................2..............................

(1,2)---open interval

Close interval: Let a and b be any two **finite** real numbers such that $a < b$. Then, the **close** interval is defined as

$[a, b] = \{x \in \boldsymbol{R}: \boldsymbol{a} \leq \boldsymbol{x} \leq \boldsymbol{b}\}$ i.e., set of all real numbers between lying between a and b but a and b are included in the set.

Semi-open or semi close intervals: These are all of the type $(a, b]$ and $[a, b)$ i.e., for 1st $a \notin (a, b]$ and $b \notin [a, b)$.

Infinite intervals: These are all defined as $(-\infty, b), (-\infty, b], (a, \infty), [a, \infty), (-\infty, \infty)$.

Or $(-\infty, b) = \{x \in \boldsymbol{R}: \boldsymbol{x} < b\}$

$$(a, \infty), = \{x \in \boldsymbol{R}:\ \boldsymbol{x} > a\}.$$

Supremum and infimum of intervals: The supremum and infimum of intervals are the boundary points.

Example. sup((1,2))=2 and inf((1,2))=1

-∞...........lower bounds.....................1].......................[2.............upper bounds..............................∞.

Example 2. S= $(a, \infty), = \{x \in \boldsymbol{R}: \ \boldsymbol{x} > a\}$

Sup S=does not exist

Inf S=a

$-\infty$...........**lower bounds**...................**a]**..........**given set**.......................[∞ ....upper bounds

Q. find sup and inf.

a. $A = \cap \left(1-\frac{1}{n}, 1+\frac{1}{n}\right), n \in N =$ (1-1,1+1) $\cap \left(1-\frac{1}{2}, 1+\frac{1}{2}\right) \cap \left(1-\frac{1}{3}, 1+\frac{1}{3}\right) \cap \ldots\ldots\ldots \cap \{1\}$

=(0,2) $\cap \left(\frac{1}{2}, \frac{3}{2}\right) \cap \ldots\ldots\ldots \cap \{1\}$

={1}

sup A=inf A=1

......(0.............(point 5............(point 6......1......1.3)......1.5)...............2)..

b. $B = \cup \left[\frac{1}{n}, 2 - \frac{1}{n}\right], n \in N$= [1,1]={1}$\cup \left[\frac{1}{2}, \frac{3}{2}\right] \cup \ldots\ldots\ldots\ldots\ldots\ldots\ldots.\cup (0,2) = (0,2)$

supB=2 infB=0

Q. C=$\left\{sin\frac{n\pi}{3}, n \in N\right\} = \{sin\frac{\pi}{3},\ sin\frac{2\pi}{3},\ sin\,\pi,\ sin\frac{4\pi}{3}, \ldots\ldots\ldots\ldots\}$

$=\{\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, 0, -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \ldots\ldots\ldots\ldots\ldots\ldots = \{-\frac{\sqrt{3}}{2}, 0, \frac{\sqrt{3}}{2}\}$

supC= $\frac{\sqrt{3}}{2}$ infC= -- $\frac{\sqrt{3}}{2}$

D. (i) H={$r \in Q$; $r < 3$}

supH=3, infH=does not exist

(ii) S={$r \in Q$; $r^2 < 3$} most important for competitive exams

=(- $\sqrt{3}$, $\sqrt{3}$) since $r^2 < 3$ implies $r^2 - (\sqrt{3})^2 < 0$ implies $-\sqrt{3} < r < \sqrt{3}$.

supS= $\sqrt{3}$ infS= --$\sqrt{3}$

Note: (i) If $a^2 < b^2$ implies $a^2 - b^2 < 0$ implies $(a-b)(a+b) < 0$ implies $-b < a < b$

(ii) If $a^2 > b^2$ implies $a^2 - b^2 > 0$ implies $(a-b)(a+b) > 0$ implies either $a < -b$

or $a > b$.

Reference book: Bansi Lal and Sanjay Arora; Introduction to Real Analysis, Satya Prakashan, 1st Vol (1991)