Lecture-19

Fredholm Integral Equation with Separable Kernels: Theory

$c_1 - \lambda c_1 a_{11} - \lambda c_2 a_{12} \cdots - \lambda c_n a_{1n} = b_1$

$c_2 - \lambda c_1 a_{21} - \lambda c_2 a_{22} \cdots - \lambda c_n a_{2n} = b_1$

$\vdots$

$c_n - \lambda c_1 a_{n1} - \lambda c_2 a_{n2} \cdots - \lambda c_n a_{nn} = b_n$

$$\begin{pmatrix} 1-\lambda a_{11} & -\lambda a_{12} & \cdots & -\lambda a_{1n} \\ -\lambda a_{21} & 1-\lambda a_{22} & \cdots & -\lambda a_{2n} \\ -\lambda a_{n1} & - & \cdot & 1-\lambda a_{nn} \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

$\boxed{D(\lambda)\, C = F}$ $\qquad D(\lambda) = [I - \lambda A]_{n\times n}$

$$x(x) = f(x) + \lambda \sum_{i=1}^{n} c_i a_i(x)$$

$$x(x) = \lambda \sum_{i=1}^{n} c_i a_i(x)$$

$f(x) \equiv 0$

$D(\lambda)\, C = 0$

✓ $|D(\lambda)| = 0$ nontrivial soln.

✓ $|D(\lambda)| \neq 0$ $\boxed{c_i = 0}$ $\forall\, i = 1, \ldots, n$

$x(x) = 0$

$$x(x) = \lambda \int_a^b K(x,t)\, x(t)\, dt$$

$$K(x,t) = \sum_{i=1}^{n} a_i(x)\, b_i(t)$$

However, if $D(\lambda) = 0$, at least one of the $c's$ can be assigned arbitrarily, and the remaining $c's$ can be accordingly determined. In this cases, infinitely many solutions of the integral equation (2) exist.
Those values of λ for which $D(\lambda) = 0$ are known as eigenvalues and any nontrivial solution of the homogeneous integral equation is called a corresponding eigenfunction. If k of the constants $c_1, c_2, \dots, c_n$ can be assigned arbitrarily for a given characteristic value of λ, then k linearly independent eigenfunctions are obtained.

$f(x) \not\equiv 0$

$$b_i = \int_a^b f(t)\, b_i(t)\, dt$$

$$b_i = 0 \quad \forall\, i = 1, \cdots n.$$

$f(t)$ is orthogonal to $b_1(t) \cdots b_n(t)$

$$D(\lambda)\, C = 0$$

$D(\lambda) = 0$ infinit no. of soln

$$X(x) = f(x) + \lambda \sum_{i=1}^{n} c_i\, a_i(x)$$

$D(\lambda) \neq 0$

$C = 0$

$$X(x) = f(x)$$

$f(x) \equiv 0$ ✓

$$D(\lambda)\, C = 0$$

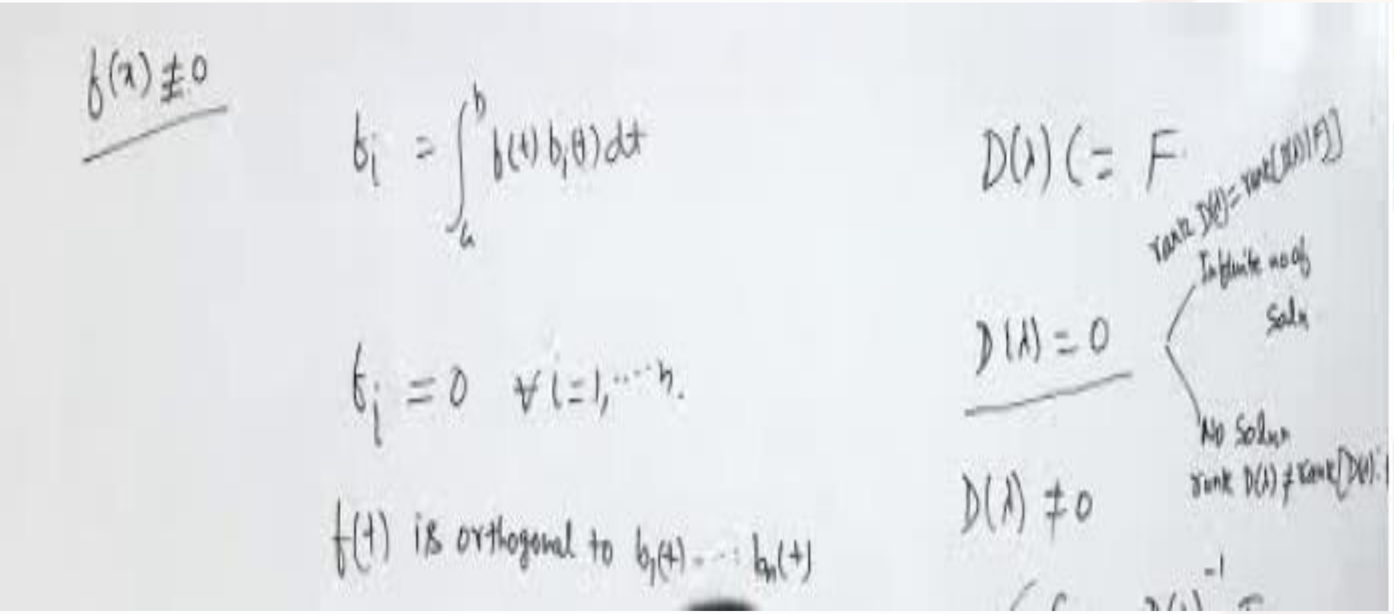
✓ $|D(\lambda)| = 0$ nontrivial soln.

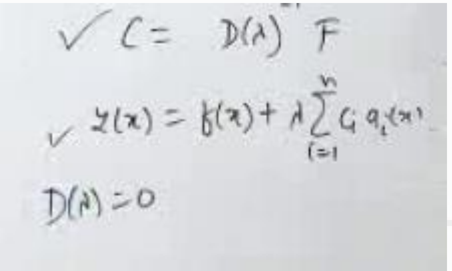
✓ $|D(\lambda)| \neq 0$ $\boxed{c_i = 0}$ $\forall\, i = 1, \cdots n$

$$X(x) = \lambda \sum_{i=1}^{n} c_i\, a_i(x) \qquad X(x) = 0$$

$$X(x) = \lambda \int_a^b K(x,t)\, X(t)\, dt$$

$$K(x,t) = \sum_{i=1}^{n} a_i(x)\, b_i(t)$$





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Reference:
https://nptel.ac.in/courses/111/107/111107103/