UNIT 5

Coordinate Systems and Transformation

Lecturer-1

Electrostatics

- **Electric charges at rest (static electricity)**
- **Involves electric charges, the forces between them, and their behavior in materials**

	DIFFERENTIAL FORM	INTEGRAL FORM
E-Gauss:	$\nabla \cdot \epsilon_o \vec{E} = \rho$	$\oiint_S \epsilon_o \vec{E} \cdot dS = \iiint_V \rho dV$
Faraday:	$\nabla \times \vec{E} = -\frac{\partial}{\partial t} \mu_o \vec{H}$	$\oint_C \vec{E} \cdot d\vec{C} = \iint_S -\frac{\partial}{\partial t} \mu_o \vec{H} \cdot d\vec{S}$
H-Gauss:	$\nabla \cdot \mu_o \vec{H} = 0$	$\oiint_S \mu_o \vec{H} \cdot d\vec{S} = 0$
Ampere:	$\nabla \times \vec{H} = \vec{J} + \frac{\partial}{\partial t} \epsilon_o \vec{E}$	$\oint_C \vec{H} \cdot d\vec{C} = \iint_S (\vec{J} + \frac{\partial}{\partial t} \mu_o \vec{H}) \cdot d\vec{S}$

Static arise when $\frac{\partial}{\partial t} \equiv 0$, and Maxwell's Equations split into **decoupled electrostatic** and **magnetostatic** eqns.

Electro-quasistatic and magneto-quasitatic systems arise when one (but not both) time derivative becomes important.

$$\oint_C A \cdot dC = \iint_S \nabla \times A \cdot dS$$

$$\oiint_S A \cdot dS = \iiint_V \nabla \cdot A \, dV$$

Charges and Currents

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial}{\partial t}\epsilon_o \vec{E}$$

$$\nabla \cdot \epsilon_o \vec{E} = \rho$$

$\Rightarrow$

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\oiint_S \vec{J} \cdot d\vec{S} + \iiint_V \frac{\partial \rho}{\partial t} dV = 0$$

Charge conservation and KCL for ideal nodes

There can be a nonzero charge density ρ in the absence of a current density J.

There can be a nonzero current density J in the absence of a charge density ρ.

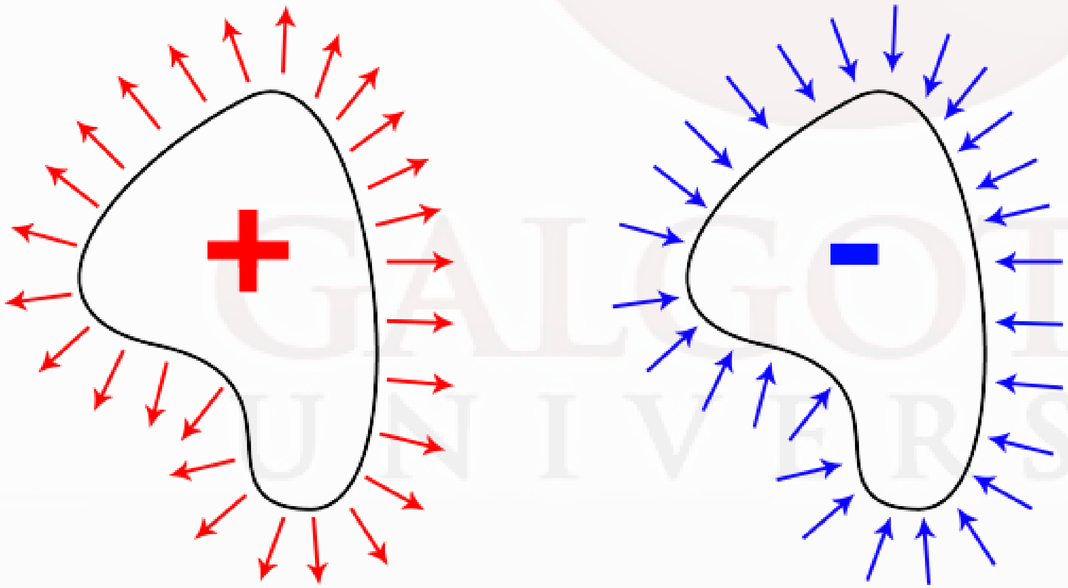
$$\vec{J} = \rho_+ \vec{v}_+ + \rho_- \vec{v}_-$$

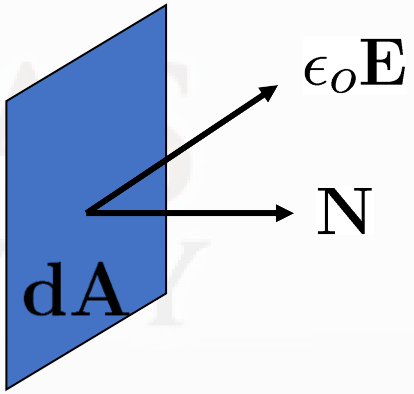
$$\rho = \rho_+ + \rho_-$$

Gauss' Law

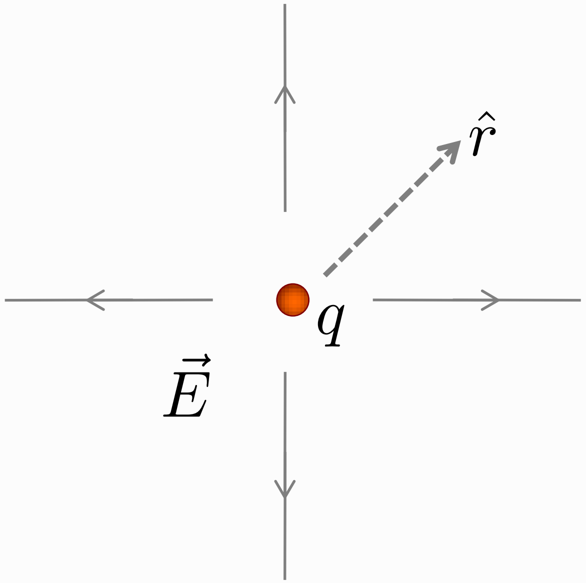
Flux of $\epsilon_o \mathbf{E}$ through closed surface ***S*** = net charge inside V

$$\int_S \epsilon_o \mathbf{E}\cdot \mathbf{dA} = \int_{\mathbf{V}} \rho \mathbf{dV} = Q_{enclosed}$$





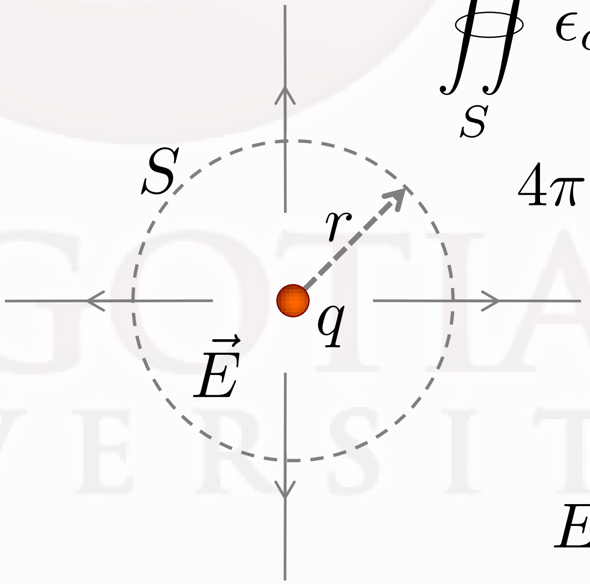
Point Charge Example



Apply Gauss' Law in integral form making use of symmetry to find $\vec{E}$

$$\oiint_S \epsilon_o \vec{E} \cdot dS = q$$

$$4\pi\epsilon_o E_r r^2 = q$$



- Assume that the image charge is uniformly distributed at $r = \infty$. Why is this important ?
- Symmetry

$$\Rightarrow \vec{E} = E_r(r)\,\hat{r}$$

$$E_r = \frac{q}{4\pi\epsilon_o r^2}$$

Gauss' Law Tells Us …

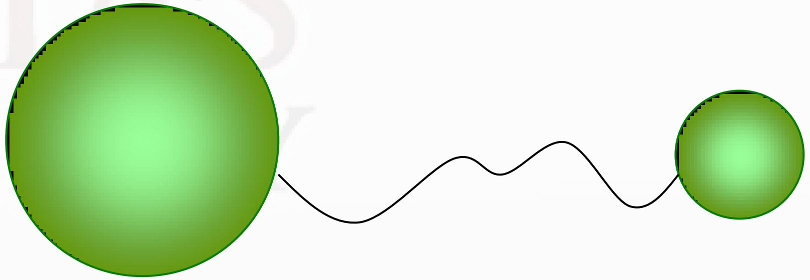
… the electric charge can reside only on the surface of the conductor.

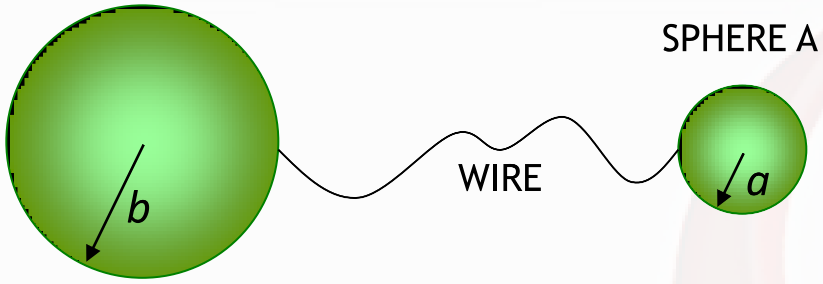
[If charge was present inside a conductor, we can draw a Gaussian surface around that charge and the electric field in vicinity of that charge would be non-zero ! A non-zero field implies current flow through the conductor, which will transport the charge to the surface.]

… there is no charge at all on the inner surface of a hollow conductor.

… that, if a charge carrying body has a sharp point, then the electric field at that point is much stronger than the electric field over the smoother part of the body.

Lets show this by considering two spheres of different size, connected by a long, thin wire …





Because the two spheres are far apart, we can assume that charges are uniformly distributed across the surfaces of the two spheres, with charge q_a *on the surface of sphere A and* q_b *on the surface of sphere B*

$$q_a + q_b = q$$

$$V_b = \frac{q_b}{4\pi\epsilon b} \qquad V_a = \frac{q_a}{4\pi\epsilon a}$$

$$since \quad V_b = V_a \quad then$$

$$q_b = q\frac{b}{a+b} \qquad q_a = q\frac{a}{a+b}$$

… and the E-field on the surface of the spheres is:

$$E_b = \frac{q_b}{4\pi\epsilon b^2} = \frac{q}{4\pi\epsilon(a+b)b}$$

$$E_a E_b = \frac{q_a}{4\pi\epsilon a^2} = \frac{q}{4\pi\epsilon(a+b)a}$$

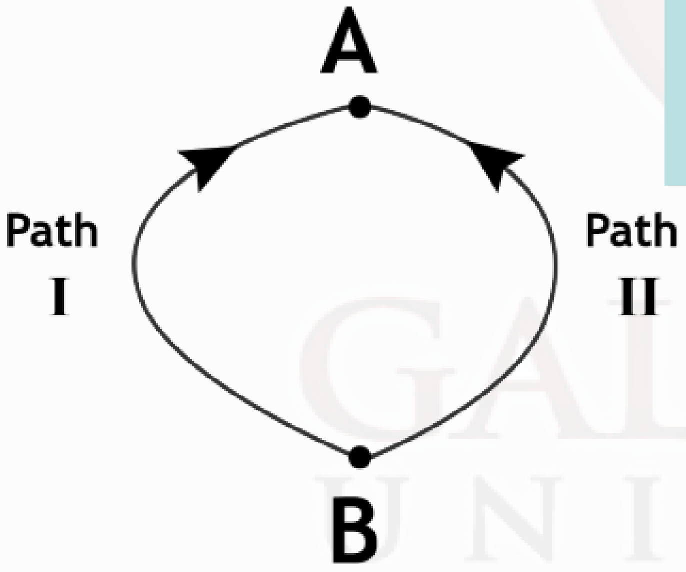
Note that $E_a >> E_b$ if $b >> a$

Faraday's Law

Dynamic form: $\nabla \times \vec{E} = \frac{\partial}{\partial t}\mu_o \vec{H}$

Static form: $\nabla \times \vec{E} = 0 \quad \Rightarrow \vec{E} = -\nabla\Phi$

and $\Phi_a - \Phi_b = \int_a^b \vec{E} \cdot d\vec{C}$



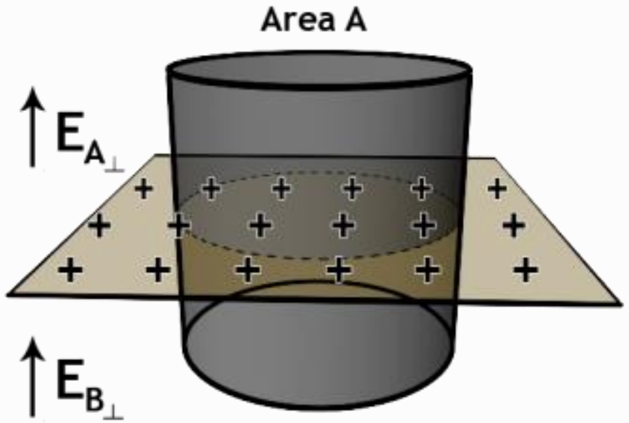
$$\oint \vec{E} \cdot d\vec{C} = 0 \quad (KVL)$$

$$\Rightarrow \underbrace{\int_b^a \vec{E} \cdot d\vec{C}}_{\text{Path I}} + \underbrace{\int_a^b \vec{E} \cdot d\vec{C}}_{\text{Path II}} = 0$$

$$\Rightarrow \underbrace{\int_b^a \vec{E} \cdot d\vec{C}}_{\text{Path I}} = \underbrace{\int_a^b \vec{E} \cdot d\vec{C}}_{\text{Path II}}$$

A unique path-independent potential may be defined if and only if $\frac{\partial B}{\partial t} = 0$

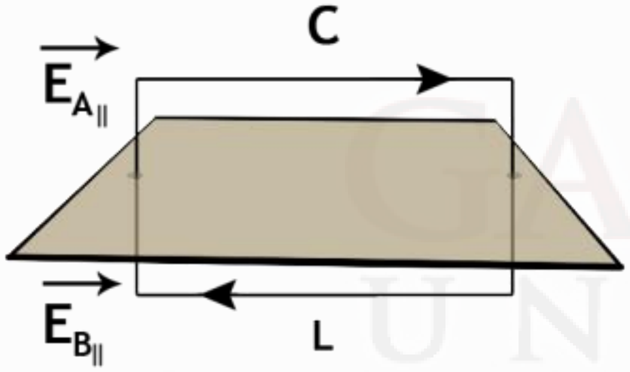
Boundary Conditions



$$\lim_{\delta \to 0} \; Gauss \Rightarrow (\epsilon_0 E_{A\perp} - \epsilon_0 E_{B\perp})A = \rho_s A$$

$$\hat{n} \cdot (\epsilon_0 E_A - \epsilon_0 E_B) = \rho_s$$

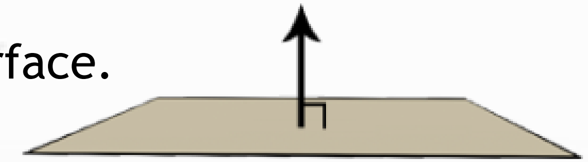
Normal $\vec{E}$ is discontinuous at a surface charge.



$$\lim_{\delta \to 0} \; Faraday \Rightarrow (E_{A\parallel} - E_{B\parallel})L = 0$$

$$\hat{n} \times (E_A - E_B) = 0$$

Tangential $\vec{E}$ is continuous at a surface.



A static field terminates perpendicularly on a conductor

References

1. H. Hayt and J. A. Buck, “Electromagnetic field theory”, 7th Edition, TATA Mc Graw Hill.

2. M. N. O. Sadiku, “Elements of Electromagnetics”, 5th Edition, Oxford University Press 2010