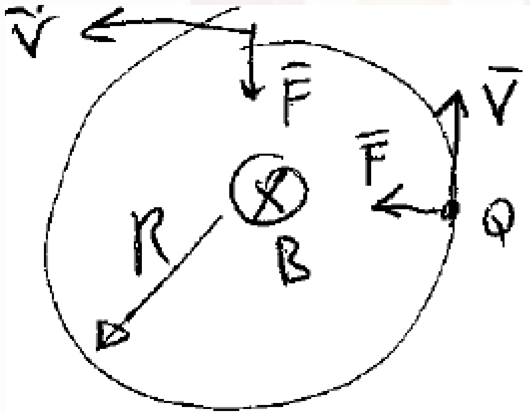
Electricity and Magnetism

Topic Covered: Magneto-static (Lorentz Force)

$$\bar{F} = Q\left[\underbrace{\bar{E}}_{\text{This term defines E.}} + \underbrace{(\bar{v} \times \bar{B})}_{\text{Lorentz force}}\right]$$

There are electromagnetic forces that depend on the velocity of a charge in given fields, and these forces are perpendicular to **v**. This term defines a field “B”, which is the macroscopic magnetic field, or “Magnetic Induction”.

Cyclotron motion: charge moving in a magnetic field.



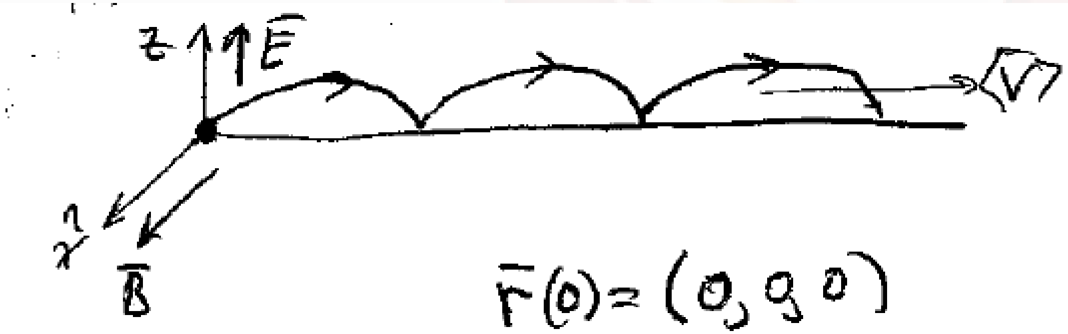
Lorentz force provides centripetal force

$$QvB = \frac{mv^2}{R}$$

Momentum

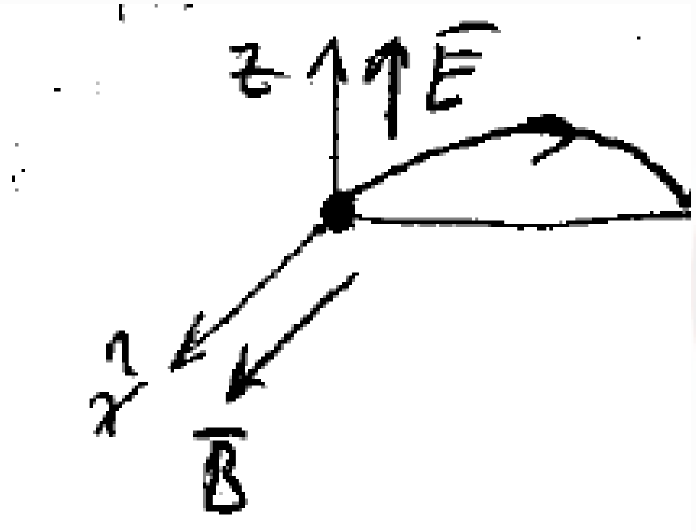
$$p = QBR$$

Cycloid motion. Charge accelerated by an **E**-field in a perpendicular magnetic field.



$\bar{F}(0)=(0,q\,0)$ since $\bar{F} \perp \bar{B}$

$\bar{V}(0)=(0,0,0)$ $F_x=0,\ V_x=0$



y

$$\bar{v} \times \bar{B} = (0, \dot{y}, \dot{z}) \times (B, 0, 0)$$

$$= [0, B\dot{z}, -B\dot{y}]$$

$$\bar{F} = Q(\bar{E} + \bar{v} \times \bar{B}) = [0, QB\dot{z}, Q(E - B\dot{y})]$$

Equation of motion, Newton's second law:

$$m\bar{a} = (0, m\ddot{y}, m\ddot{z}) = [0, B\dot{z}, E - B\dot{y}]$$

Define the "cyclotron frequency": $\omega \equiv \frac{QB}{m}$

$$\ddot{y} = \omega\dot{z}, \quad \ddot{z} = \omega\left(\frac{E}{B} - \dot{y}\right)$$

Take time derivative of the first equation and substitute into the second equation

$$\ddot{y} = \omega \dot{z} \qquad \ddot{z} = \omega\left(\frac{E}{B} - \dot{y}\right)$$

$$\dddot{y} = \omega \ddot{z}$$
$$= \omega^2\left(\frac{E}{B} - \dot{y}\right)$$

$$\dddot{y} + \omega^2 \dot{y} = \omega^2 \frac{E}{B}$$

$$\dot{y} = D_1 \sin\omega t + D_2 \cos\omega t + \frac{E}{B}$$

$$\boxed{y = C_1 \cos\omega t + C_2 \sin\omega t + \frac{E}{B} t + C_3}$$

$$\omega \dot{z} = \ddot{y} = -C_1 \omega^2 \cos\omega t - C_2 \omega^2 \sin\omega t$$

integrate

$$\boxed{z = -C_1 \sin\omega t + C_2 \cos\omega t + C_4}$$

Initial conditions

$$\left.\begin{aligned} \dot{y}(0) &= 0 \\ \dot{z}(0) &= 0 \\ y(0) &= 0 \\ z(0) &= 0 \end{aligned}\right\} \xrightarrow{\text{give}} C_1, C_2, C_3, C_4$$

$$y(0) = 0 = C_1 \cos 0 + C_2 \sin 0 + \frac{E}{B} 0 + C_3$$

$$z(0) = 0 = C_2 \cos 0 - C_1 \sin 0 + C_4$$

$$C_1 + C_3 = 0$$

$$C_2 + C_4 = 0$$

$$\dot{y}(0) = 0 = -C_1 \omega \sin 0 + C_2 \omega \cos 0 + \frac{E}{B} \rightarrow C_2 = \frac{-E}{\omega B} = -C_4$$

$$\dot{z}(0) = 0 = -C_2 \omega \sin 0 - C_1 \omega \cos 0 \rightarrow C_1 = 0$$

$$C_3 = 0$$

$$y = \frac{-E}{\omega B} \sin \omega t + \frac{E}{B} t$$

$$z = \frac{-E}{\omega B} \cos \omega t + \frac{E}{\omega B}$$

$$y(t) = R(\omega t - \sin\omega t)$$

$$z(t) = R(1 - \cos\omega t)$$

$$R \equiv \frac{E}{\omega B}$$

$$(y - R\omega t)^2 = R^2 \sin^2\omega t$$

$$(z - R)^2 = R^2 \cos^2\omega t \quad +$$

$$(y - R\omega t)^2 + (z - R)^2 = R^2$$

This is the equation of a circle with a moving center

No work is done by magnetic forces

d***l*** = **v** dt

$$dW_{mag} = \bar{F}_{mag} \cdot d\bar{l}$$

$$= Q(\bar{v} \times \bar{B}) \cdot \bar{v}\,dt$$

= 0

References:

- Griffiths, D. J. (1999). *Introduction to electrodynamics*. Upper Saddle River, N.J: Prentice Hall.
- *Concepts of Physics Part-2*, Bharati Bhawan Publishers & Distributors, 1992, ISBN
- Electricity and Magnetism, Edward M. Purcell, 1986 McGraw-Hill Education